Math 425 Applied and Computational Linear Algebra Spring 2022

hw3 , PART A: **due on iLearn by 12:30pm on Thursday, March 3**

1. Let $T(\mathbf{x}) = A\mathbf{x}$. If $A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 3 & -5 & -9 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 6 \\ -7 \\ -9 \end{bmatrix}$, find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$, and determine whether $\mathbf{x}$ is unique.

2. Let $A = \begin{bmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 4 \end{bmatrix}$. Is $\mathbf{b}$ in the range of the linear transformation $T(\mathbf{x}) = A\mathbf{x}$?

3. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set in $\mathbb{R}^n$. Explain why the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent.

4. Consider a linear transformation from $T : \mathbb{R}^3 \to \mathbb{R}^2$, where

$$T\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}, \quad T\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}, \quad \text{and} \quad T\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -13 \\ 17 \end{bmatrix}.$$

Find the standard matrix A of the transformation T.

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that
$T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2)$. Find $\mathbf{x}$ such that $T(\mathbf{x}) = (-1, 4, 9)$.

6. Find the standard matrix for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which is a horizontal shear transformation that leaves $\mathbf{e}_1$ unchanged and maps $\mathbf{e}_2$ into $\mathbf{e}_2 + 3\mathbf{e}_1$.

7. The color of light can be represented in a vector $\begin{bmatrix} R \\ G \\ B \end{bmatrix}$ where R = amount of red, G = amount of green, and B = amount of blue. The human eye and the brain transform the incoming signal into the signal $\begin{bmatrix} I \\ L \\ S \end{bmatrix}$, where

$$\begin{aligned} \text{intensity} \quad I &= \frac{R+G+B}{3} \\ \text{long-wave signal} \quad L &= R - G \\ \text{short-wave signal} \quad S &= B - \frac{R+G}{2}. \end{aligned}$$

(a) Find the matrix P representing the transformation from $\begin{bmatrix} R \\ G \\ B \end{bmatrix}$ to $\begin{bmatrix} I \\ L \\ S \end{bmatrix}$

(b) Consider a pair of yellow sunglasses for water sports which cuts out all blue light and passes all red and green light. Find the matrix A which represents the transformation incoming light undergoes as it passes through the sunglasses.

(c) Find the matrix for the composite transformation which light undergoes as it first passes through the sunglasses and then the eye.

8. Let $\mathbf{v}$ be a fixed vector in $\mathbb{R}^n$ and let $T : \mathbb{R}^n \to \mathbb{R}$ be the mapping defined by $T(\mathbf{x}) = \mathbf{v}^T\mathbf{x}$ (i.e. the standard inner product).

(a) Is T a linear operator?

(b) Is T a linear transformation?

9. Find the 3×3 matrices that produce the described composite 2D transformations, using homogeneous coordinates. Apply the transformations to the **letter N** data, "letterN.pny" and submit the corresponding plots as well.

(a) Translate by $(-2, 3)$, and then scale the x-coordinate by 0.8 and the y-coordinate by 1.2

(b) Rotate points $\frac{\pi}{6}$, and then reflect through the x-axis.