MATH425 Lecture 2

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1 The field

A field $\mathbb{F}$ is a set containing at least two distinct elements, an additive identity or $\{0\}$. And a multiplicative Identity $\{1\}$, and the elements of the set that satisfy the following properties

2 Properties of a Field

$$\text{Commutativity: } \alpha + \beta = \beta + \alpha \qquad \forall \alpha\beta \in \mathbb{F}$$

$$\text{Associativity: } \alpha + (\beta + \lambda) = (\alpha + \beta) + \lambda \qquad (\alpha\beta)\lambda = \alpha(\beta\lambda) \qquad \forall \alpha\beta \in \mathbb{F}$$

$$\text{Identities:} \lambda + 0 = \lambda \text{ and } \lambda(1) = \lambda \quad \forall \lambda \in \mathbb{F}$$

$$\text{Additive inverse: } \forall \alpha \in \mathbb{F} \exists \beta \in \mathbb{F} \text{ such that } \alpha + \beta = 0$$

$$\text{Multiplicative inverse:} \forall \alpha \in \mathbb{F} \text{ with} \alpha \neq 0, \exists \beta \in \mathbb{F} \text{ such that } \alpha\beta = 1$$

$$\text{Distributive property} \lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta \quad \forall \lambda, \alpha, \beta \in \mathbb{F}$$

Examples of Fields
$\mathbb{R}$ set of real numbers
GF(2): Galois field of order 2. finite field: integers modulo 2 = {0,2}

Characteristics of GF2
Since GF2 has two elemets it forms a 2 dimensions matri for addition with zeros on the principal diagonal and 1s on the other indicies.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Multiplication for GF(2) also forms a 2x2 matrix as again, there are only two elements.

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

In GF(2) you take the modulous of anything larger than 2 becuase the set in constrained to two elements.

3 Re-Introduction to Complex Numbers

The complex plane is a field that consists of the set of complex numbers.

$$\mathbb{C} = \{a + bi, a, b \in \mathbb{R}\}$$

Where i= $\sqrt{-1}$.
In python j is used in place of i as a fragement of applications in electrical engineering.

$$z = a + bi = Re(z) + iIm(z)$$

Re(z)=a, Im(z)=b