MATH425 Homework 1

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1 Problem 1:

Solve the following system:

$$\begin{aligned} u+v+w &= -2 \\ 3u+3v-w &= 6 \\ u-v+w &= -1 \end{aligned}$$

expressed as an augmented matrix this system of linear equations is:

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 3 & 3 & -1 & 6 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

the following matrix when reduced to row echelon form reveals the solution to the system of linear equations.

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 3 & 3 & -1 & 6 \\ 1 & -1 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & \frac{3}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

implying that the solution to the system is that:

$$\begin{aligned} u &= \frac{3}{2} \\ v &= -\frac{1}{2} \\ w &= -3 \end{aligned}$$

2 Problem 2:

Choose h and k such that the system below has (a) no solution, (b) a unique solution, and (c) many solutions. Give separate answers for each part.

$$\begin{aligned} x_1 + 3x_2 &= 2 \\ 3x_1 + hx_2 &= k \end{aligned}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & h & k \end{bmatrix}$$

(a) No solutions:

$$\mathbf{R2{=}{-}3R1{+}R2}$$
$$\begin{bmatrix}1 & 3 & 2\\ 3 & h & k\end{bmatrix} = \begin{bmatrix}1 & 3 & 2\\ 0 & h-9 & k-6\end{bmatrix}$$

Thus the system is inconsistent when **h=9** and $k \neq 6$

(b) A unique solution

by the logic of the reduction in part a the system is consistent for all values of h not equal to 9 as there will be a pivot in row 2 if this is the case.

(c) Infinite solutions

By the logic of the reduction in part a the system is consistent with Infinite solutions when h=9 and k =6.

3 Problem 3:

Consider the system below:

$$\begin{aligned} 4x_1 + x_2 + 3x_3 &= 9\\ x_1 - 7x_2 - 2x_3 &= 12\\ 8x_1 + 6x_2 - 5x_3 &= 15 \end{aligned}$$
$$\begin{bmatrix}4 & 1 & 3 & 9\\ 1 & -7 & -2 & 12\\ 8 & 6 & -5 & 15\end{bmatrix}$$

(a) Column-space view: Find the vectors $v_1, v_2, v_3$and write the system as a vector equation:

$$x_1v_1 + x_2v_2 + x_3v_3 = \begin{bmatrix}9\\12\\15\end{bmatrix}$$
$$x_1\begin{bmatrix}1\\-7\\6\end{bmatrix} + x_2\begin{bmatrix}3\\-2\\-5\end{bmatrix} + x_3\begin{bmatrix}3\\-2\\-5\end{bmatrix} = \begin{bmatrix}9\\12\\15\end{bmatrix}$$

(b) Row-space view: Find the vectors w_1, w_2, w_3 and x such that the system is equivalent to:

$$\begin{aligned} w_1 \cdot x &= 9\\ w_2 \cdot x &= 12\\ w_3 \cdot x &= 15 \end{aligned}$$

$$w_1 = [4,1,3]$$
$$w_2 = [1,-7,-2]$$
$$w_3 = [8,6,-5]$$

$$[4,1,3] \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = 9$$
$$[1,-7,-2] \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = 12$$
$$[8,6,-5] \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = 15$$

Let x be the following solution to the system to validate this claim. Solution obtained using row operations. $\begin{bmatrix} \frac{328}{121} \\ \\ \frac{-14}{11} \\ \\ \frac{-23}{121} \end{bmatrix}$

$$[4,1,3] \cdot \begin{bmatrix} \frac{328}{121} \\ \\ \frac{-14}{11} \\ \\ \frac{-23}{121} \end{bmatrix} = 9$$
$$[1,-7,-2] \cdot \begin{bmatrix} \frac{328}{121} \\ \\ \frac{-14}{11} \\ \\ \frac{-23}{121} \end{bmatrix} = 12$$
$$[8,6,-5] \cdot \begin{bmatrix} \frac{328}{121} \\ \\ \frac{-14}{11} \\ \\ \frac{-23}{121} \end{bmatrix} = 15$$

4 Problem 4:

Determine if **b** is a linear combination of the vectors formed from the columns of the matrix **A** .

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 6 \\ 1 & -2 & 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 6 & -5 \\ 1 & -2 & 5 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & \frac{241}{33} \\ 0 & 1 & 0 & \frac{-43}{33} \\ 0 & 0 & 1 & \frac{-2}{11} \end{bmatrix}$$

A unique solution exists for the given existence problem therefore yes **b** is a linear combinations of the vectors in **A**

5 Problem 5

Let f(z) = az + b where $z \in \mathbb{C}$. Find a and b if f(z) translates z one unit up and one unit to the right, rotates the result by $\frac{\pi}{2}$ clockwise and scales the resulting complex number by 2.

Since the input value z is rotate around clockwise b y$\frac{\pi}{2}$ this means that part of the coefficient must be $e^{-\frac{\pi}{2}i}$ so to start we have:

$$f(z) = e^{-\frac{\pi}{2}i}z + b$$

Next we must scale the complex number by 2 modifying the function to be

$$f(z) = 2e^{-\frac{\pi}{2}i}z + b$$

Finally to move each output up one and one to the right we must add $1 + i$ to the result for a final description of

$$f(z) = 2e^{-\frac{\pi}{2}i}z + (1 + i))$$