Math 425 Applied and Computational Linear Algebra Spring 2022

hw2 , <u>PART A</u>: **due on iLearn by 12:30pm on Thursday, February 17**

1. List five vectors in Span $\{\mathbf{v_1}, \mathbf{v_2}\}$. For each vector, show the weights on $\mathbf{v_1}$, $\mathbf{v_2}$ used to generate the vector and list the three entries of the vector. Do not make a sketch.

$$\mathbf{v_1} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$$

2. Decide whether the following sets of vectors are linearly independent or linearly dependent. Give reasons for your choices.

a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} -3 \\ 2 \\ -7 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -11 \\ -8 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} -3 \\ 2 \\ -7 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -11 \\ -8 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} -3 \\ 2 \\ -7 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -11 \\ -8 \end{bmatrix}, \quad \begin{bmatrix} 9 \\ 12 \\ 13 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 4 \\ 9 \\ 10 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 5 \\ 7 \\ 1 \\ 0 \end{bmatrix} \right\}$

3. Determine if the columns of the matrix form a linearly independent set. Justify each answer.

a) $\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$, $\qquad$ b) $\begin{bmatrix} 1 & -3 & 3 & -2 \\ -3 & 7 & -1 & 2 \\ 0 & 1 & -4 & 3 \end{bmatrix}$

4. Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix}$, and $\mathbf{v_3} = \begin{bmatrix} 2 \\ -9 \\ h \end{bmatrix}$.

a) For what values of h is $\mathbf{v_3}$ in Span $\{\mathbf{v_1}, \mathbf{v_2}\}$?

b) For what values of h is $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ linearly *dependent* ?

5. Given $A = \begin{bmatrix} 4 & 1 & 6 \\ -7 & 5 & 3 \\ 9 & -3 & 3 \end{bmatrix}$, observe that the first column plus twice the second column equals the third column. Find a nontrivial solution of $A\mathbf{x} = \mathbf{0}$.

6. Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ when n is less than m?

7. Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$. Is $\mathbf{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A? Why or why not?

8. Let $A = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Show that the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for all possible $\mathbf{b}$, and describe the set of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ does have a solution.

9. Let $B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$. Do the columns of B span $\mathbb{R}^4$? Does the equation $B\mathbf{x} = \mathbf{y}$ have a solution for each $\mathbf{y} \in \mathbb{R}^4$?

<u>PART B</u>: **due on Gradescope by 12:30pm on Thursday, February 17**

Please remember you can submit your code several times before the deadline.

A few must-dos:

- Complete your code in the template **hw2.py**.
- **Do not** change the name of the template. You must submit it as **hw2.py**
- **Do not** change the name of the procedures or their inputs.

Complete the procedure `bitter_rivals`, to find which two senators disagree the most. The description of the inputs and outputs are provided in the template **hw2.py**.