MATH 425 Lecture 17

Chris Camano: ccamano@sfsu.edu

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The gram schmidt process

Given a basis $\{x_1, ..., x_p\}$ for a subspace W $\in \mathbb{F}^n$ let:

$$\begin{aligned}
&v_1 = x_1 \\
&u_1 = \frac{v_1}{||v_1||} = \frac{v_1}{r_{11}} \\
&v_2 = x_2 - (u_1 \cdot x_2)u1 = x_2 - r_{11}u_1 \\
&u_2 = \frac{v_2}{||v_2||} = \frac{v_2}{r_{22}} \\
&v_3 = v_3 - (x_3 \cdot u_1)u_1 - (x_3 \cdot u_2)u_2 \\
&u_3 = \frac{v_3}{||v_3||} = \frac{v_3}{r_{33}} \\
...& \\
&v_p = x_p - (x_p \cdot u_1)u_1 - ... - (x_p \cdot u_p)u_p \\
&u_p = \frac{u_p}{||u_P||} = \frac{v_3}{v_r r}
\end{aligned}$$

$$v_j = x_j - \sum_{k=1}^{j-1}(x_k \cdot x_j)u_k = x_j - \sum_{k=1}^{j-1} r_{kj}u_k$$

Without normalization:

$$x_j - \sum_{k=1}^{j-1} \frac{(x_k \cdot x_j)}{v_k \cdot v_k} v_k$$

This yeilds an orthonormal basis for W $\{u_1, ..., u_p\}$. Any non zero subspace W has an orthonormal basis.

$$\sum_{k=0}^{w} \binom{w}{2k}\binom{2k}{k}$$