MATH 425 Lecture 21

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SVD part 2

The best way to find the rank of a matrix is by computing the number of singular values in the sigma matrix of the singular value decomposition.
we can orthogonally diagonalize AA^T or A^TA and work with which ever is smaller to find the singular values.

Let A be an mxn matix and $\{v_1, ..., v_n\}$ be orthonormal vectors of A^TA corresponding to $\lambda_1, .., \lambda_n$ ordered such that $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda + n$ If A has r non zero singular values.Then :

$$\{Av_1, Av_2, ..., Av_r\}$$

is an orthogonal basis for the row space for the columns space of A and rank A is equal to r.

$$\{\frac{Av_1}{\sigma_1}, ..., \frac{Av_r}{\sigma_r}\}$$

Forms an orthogonormal basis of ColA.
ColA will have dim r nul A^T will have dim m-r
Orthonormal basis for $NulA^T$ Gives : $\{u_{r+1}, ..., u_m$

The four fundemental subspaces

The U matrix in SVD :

$$U = [U_rU_{m-r}] \quad U_r = [u_1, ..., u_r] \quad U_{m-r} = [u_{r+1}, ..., u_m]$$

U_r is an orthogonormal basis for ColA
U_{m-r} is an orthonormal basis for NulA^T

$$V = [V_rV_n - r] \quad V_R = [v_1, .., v_r] \quad V[v_{r+1}, ..., v_n]$$

V_r is an orthonormal basis for Rowspace of A
V_{n-r} is an orthonormal basis for NulA