MATH 425 Lecture 15

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Opening Notes

Professor Hosten will be visiting next class.

Orthogonality

Unit Vector
A unit vector is a vector of length one.
To find a unit vector that is a basis for a subspace you first take a given vector in the subspace and normalize. This returns a unit vector for the subspace.

Orthogonal subspace $W^{\perp}$

$$\forall \text{ Subspaces } W, \exists W^{\perp} : \forall w \in W, w^{\perp} \in W^{\perp} \quad w \cdot w^{\perp} = 0$$

This is to say for all subspaces there is an Orthogonal complement in which all vectors from the first subspace dotted with vectors in the orthogonal complement are 0.

Orthogonal complement
The orthogonal complement of a subspace is called the orthogonal complement. The orthogonal complement is the set of all vectors that are orthogonal to W.

$$\text{RowA}^{\perp} = \text{NulA} \quad \text{ColA}^{\perp} = \text{NulA}^{T}$$

Orthogonal sets
A set of vectors is an orthogonal set if each pair of distinct vectors from the set is orthogonal.

Orthonormal set
An orthonormal set is a set of orthogonal vectors that are normalized.

An mxn matrix U has orthonormal columns iff :

$$U^{T}U = \mathbb{I}_{n}$$

Proof:

$$\text{Let } U = \begin{bmatrix} | & | & | \\ \mathrm{u}_1 & \mathrm{u}_2 & ...\mathrm{u}_3 \\ | & | & | \end{bmatrix}$$

$u_i \in \mathbb{F}^n$

$$U^T U = \begin{bmatrix} u_1^T u_1 & ... & u_1^T u_m \\ . & & . \\ . & & . \\ u_m^T u_1 & ... & u_n^T u_n \end{bmatrix} = \mathbb{I}_n$$

as $\forall u_i, u_i \in U u_i^T u_i = 1$

Orthogonal matrix
A square matrixU for which $U^-1 = U^T$ is called an orthogonal matrix.

Orthongality and linear independence : If S $=\{u_1, ..., u_p\}$ is an on orthongal set of non zero vectors in $\mathbb{F}^n$ then S is linearly independent and is a basis for the subspace spanned by S.

Orthogonal basis
An orthogonal basis for a subspace W of $\mathbb{R}^n$ is a basis for W that is also an orthogonal set.
theorem:
Let U be an mxn matrix with Orthonormal columns and let x and y be in $\mathbb{R}^n$ then:

- $Ux \cdot Uy = x \cdot y$
- $||Ux||$=$||x||$
- $Ux \cdot Uy$=0