# MATH 425 Lecture 11

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3/3/2022

## Review of homework

You can only talk about eigenvalues in the context of square matrices or linear operators. **Try researching linear transformations that operate as dimensionality reduction**This is in some ways a projection to a lower dimensional subspace.

## Homegenous Coordinates

One methodology for evaluating whether or not a transformation is a linear transforamtion is examining the effect the transformation has on the zero vector. A transforamtion is not linear if the zero vector is mapped to some other location.
**Homogenus Coordinates**: A method for preforming translation via matrix multiplication. This can be used to translate a two dimensional place through a three dimensional space. :

$$\text{Given } x, y, \in \mathbb{R}^2 \quad \forall a, z \in \mathbb{R}, z \neq 0 \quad (xz, yz, z)$$

are the Homogenus Coordinates of xy.
Suppose $(x, y) = (2, 3)$ the Homogenus Coordinates of (2,3) is (2,3,1)
This concept generalizes to higher dimensions as well:

$$\text{Given } x, y, z \in \mathbb{R}^3 \quad \forall h \in \mathbb{R}, h \neq 0 \quad (xh, yh, zh, h)$$

are the Homogenus Coordinates of xyz.

Translation can be understood as a form of matrix multiplication:
Recall for a translation by some vector $v = \begin{bmatrix} h \\ k \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix} = \begin{bmatrix} x+h \\ y+k \end{bmatrix}$$

which is the transformation that desicribes the translation of x by vector v
The matrix multiplication is:

$$\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+h \\ y+k \\ 1 \end{bmatrix}$$

Let H=

$$\begin{bmatrix} \mathbb{I} & v \\ \vec{0}^T & 1 \end{bmatrix}$$

Any linear transformation on $\mathbb{R}^n$ is represented with respect to Homogenus Coordinates by a partitioned matrix of the form:

$$H = \begin{bmatrix} A & \vec{0} \\ \\ \vec{0}^T & 1 \end{bmatrix}$$

Where A is the nxn matrix of the transformationin $\mathbb{R}^n$

**2d scalar matrix in Homogenus Coordinates**

let A= $\begin{bmatrix} k & 0 \\ 0 & h \end{bmatrix}$

$$\begin{bmatrix} A & \vec{0} \\ \\ \vec{0}^T & 1 \end{bmatrix}$$

**2d rotation matrix by theta in Homogenus Coordinates**

let A= $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

$$\begin{bmatrix} A & \vec{0} \\ \\ \vec{0}^T & 1 \end{bmatrix}$$

**2d rotation matrix by theta in Homogenus Coordinates**