Math 425 Applied and Computational Linear Algebra Spring 2022

hw4 , **due on iLearn by 12:30pm on Thursday, April 7**

1. Prove that the set $\{(x_1, x_2, 0) : x_1,\ x_2 \in \mathbb{F}\}$ is a subspace of $\mathbb{F}^3$.

2. Let $A = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 2 & -3 & -1 & -4 \\ 3 & -5 & -1 & -1 \end{bmatrix}$.

   (i) Find a basis for Row$A$ and Nul$A$.

   (ii) Find the inner product of each vector in the basis of Row$A$ with each vector in the basis of Nul$A$.

3. Let $\mathbf{u}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$. Show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal basis for $\mathbb{R}^3$ then express $\mathbf{x}$ as a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$.

4. Suppose W is a subspace of $\mathbb{R}^n$ spanned by n nonzero orthogonal vectors. Explain why $W = \mathbb{R}^n$.

5. Let $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix} \right\}$ and $\mathbf{y} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$. Write $\mathbf{y}$ as vector in W and a vector in $W^{\perp}$.

6. Let $\mathbf{z} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \\ -3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 5 \\ -2 \\ 4 \\ 2 \end{bmatrix}$. Find the best approximation to $\mathbf{z}$ by vectors of the form $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$.

7. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

a) Show that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is a basis for $W = \text{Span}\ \{\mathbf{u}_1, \mathbf{u}_2\}$. Do not use row reduction.

b) Show that $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ is not in W.

c) Use the fact that $\mathbf{u}_3$ is not in W to construct a nonzero vector v in $\mathbb{R}^3$ that is orthogonal to $\mathbf{u}_1$ and $\mathbf{u}_2$.

8. Find an orthogonal basis for the column space of the matrix $A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$.

9. Find an orthonormal basis for the column space of the matrix $A = \begin{bmatrix} 3 & -3 & 0 \\ -4 & 14 & 10 \\ 5 & -7 & -2 \end{bmatrix}$.

10. Let $\mathbf{u}_1, \ldots, \mathbf{u}_p$ be an orthogonal basis for the subspace W of $\mathbb{R}^n$, and let $T : \mathbb{R}^n \to \mathbb{R}^n$ be defined by $T(\mathbf{x}) = \text{proj}_W \mathbf{x}$. Show that T is a linear transformation.