MATH425 Lecture 6

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1 Discussion of linear independence and Span

Given a set of vectors of the form

$$\{v_1, v_2, v_3\}$$

Span$\{v_1, v_2, v_3\}$ is the region spanned by all linear combination of these vectors.
Discussion of linear systems: Given two equations of the from

$$2x - y = 1, x + y = 5$$

We can augment these equations to a matrix of the form

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

1.1 Row space view

understanding this matrix throught the perspective of row space we can understand the matrix as a series of rows$\{v_{11}, v_{21}, .., v_{n1}\}$ dotted with $\begin{bmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{bmatrix}$ such that

$$\{v_{1,1}, v_{2,1}, .., v_{n-1,1}\} \cdot \begin{bmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{bmatrix} = y_{1,n}$$

In the context of this exmple we have

$$\begin{bmatrix} 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1$$

Through the row space view we can think of each row as a linear equation with the intersection of the rows representing the solution to the system of linear equations. //

1.2 Column Space view

Express the columns of the matrix as a linear combination such that the final vector is the product of the sum of these weighted columns:

$$x \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

Essentially this is the process of thinking of stretching out vectors such that the sum of their now stretched version equates to the desired vector.//

1.3 Span

Span is the vector space formed by all linear combinations of a set of vectors.

$$\text{Let } \{v_1, v_2, ..., v_n\} \in \mathbb{F}^n.$$

$$\text{Span } \{v_1, v_2, ..., v_n\}$$

is the set of all vectors of the form:

$$c_1 v_1 + c_2 v_2 + ... + c_n v_n$$

$$c_1, c_2, ...c_n \in \mathbb{F}^n$$

$$\text{Span } \{v_1, v_2, ..., v_n\} \subset \mathbb{F}^n$$