Introduction to chapter three: 3.2 and 3.3

3.3 Random Variables

Discrete Random Variables
Random variables are used to inject concepts of calculus into the study of probability. The Discrete case is the simple case for random variables. Our sample space in these contexts can be finite, countable infinite, or continuous.

Random variable
A random variable is a function from a sample space into $\mathbb{R}$

$$X : \mathcal{S} \mapsto \mathbb{R}$$

discrete random variable
A discrete random variable is a function whose domain is a sample space $\mathcal{S}$ and whose values are from a finite or countable set of real numbers is called a discrete random variable .

We need to define a probability function for a new sample space.
Where X is a random variable and x is an observed value.

Given a sample space $\mathcal{S} = \{s_1, s_2, ...s_,\}$ we map each s to some value x $x \in \mathbb{R}$.

$$P(x) = P(X = x) = P(\{s : X(s) = x\})$$

The function listed above is caleld a discrete probability function. recall the rules of probability:

$$\forall A, P(A) \geq 0$$
$$P(\S) = 1$$
$$A \cap B = \emptyset : P(A \cup B) = P(A) + P(B)$$

if:

$$A_1, A_2, ...A_n, A_i \cap A_j = \emptyset \forall i, j$$
$$P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$$

The probability denisty function of a discrete random variable :
The probability denisty function of a discrete random variable is given by :

$$P(x) = P(X = x) = P(\{s : X(s) = x\})$$

A function P(x) is a probability denisty function of a random variable x $\iff$

$$p(x) \geq 0$$
$$\sum_{\forall x} P(x) = 1$$