

MATH440 Homework Set 1

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Homework questions: 2.2.12, 2.2.16, 2.2.22, 2.2.38 2.3.10, 2.3.12, 2.3.14

1 Chapter 2.2 Sample Spaces and the Algebra of Sets

Problem 2.2.12 Consider the experiment of choosing coefficients for the quadratic equation $ax^2 + bx + c = 0$. Characterize the values of a,b, and c associated with the event A: Equation has complex roots.

$$A = \{(a, b, c) | b^2 - 4ac < 0, a, b, c \in \mathbb{R}\}$$

Problem 2.2.16 Sketch the regions in the xy plane corresponding to $A \cup B$ and $A \cap B$ if:

$$\begin{aligned} A &= \{(x, y) | 0 < x < 3, 0 < y < 3\} \\ B &= \{(x, y) | 2 < x < 4, 2 < y < 4\} \end{aligned}$$

Please see attached illustration for image of region(Still working on Tikz skills).

Problem 2.2.22 Suppose that each of the twelve letters in the word T E S S E L L A T I O N is written on a chip. Define the following events F, R and C as follows:

F: Letters in the first half of the alphabet

R: Letters that are repeated

V: Letters that are vowels

$$F = \{A, E, E, I, L, L\}$$

$$R = \{T, T, L, L, E, E, S, S\}$$

$$V = \{E, A, I, O, E\}$$

a) What is $F \cap R \cap V$

$$F \cap R = \{E, E\}$$

$$(F \cap R) \cap V = \{E, E\}$$

b) What is $F^C \cap R \cap V^C$

$$F^C = \{N, O, S, S, T, T\}$$

$$V^C = \{S, S, T, T, N, L, L\}$$

$$R^C = \{I, O, N, A\}$$

$$F^C \cap R = \{S, S, T, T\}$$

$$(F^C \cap R) \cap V^C = \{S, S, T, T\}$$

c) What is $F \cap R^C \cap V$

$$F \cap R^C = \{I, A\}$$

$$(F \cap R^C) \cap V = \{I, A\}$$

Problem 2.2.38

Please see attached illustration figure 2.

Problem 2.3.10

An urn contains twenty-four chips, numbered 1 through 24. one is drawn at random. Let A be the event that the number is divisible by 2 and B be the event that the number is divisible by 3. Find $P(A \cup B)$

$$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24\}$$

$$B = \{3, 6, 9, 12, 15, 18, 21, 24\}$$

$$A \cup B = \{2, 3, 4, 6, 8, 9, 12, 14, 15, 16, 18, 20, 21, 22, 24\}$$

$$P(A \cup B) = \frac{|A \cup B|}{|S|} = \frac{16}{24} = \frac{2}{3}$$

Problem 2.3.12

Events A_1 and A_2 are such that $A_1 \cup A_2 = S$ and $A_1 \cap A_2 = \emptyset$ find p_2 if $P(A_1) = p_1$, $P(A_2) = p_2$, and $3p_1 - p_2 = \frac{1}{2}$

$$3p_1 - p_2 = \frac{1}{2}$$

$$p_1 + p_2 = 1 \text{ as } A_1 \cup A_2 = S$$

$$\begin{bmatrix} 3 & -1 & \frac{1}{2} \\ 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{3}{8} \\ 0 & 1 & \frac{1}{8} \end{bmatrix} \therefore p_1 = \frac{3}{8}, p_2 = \frac{5}{8}$$

Problem 2.3.14

Three events A B and C are defined on a sample space S given that:

$$P(A) = .2, P(B) = .1, P(C) = .3$$

What is the smallest possible value for:

$$P[(A \cup B \cup C^c)]$$

$$P[(A \cup B \cup C)^c] = P(A^c \cap B^c \cap C^c)$$

by Demorgan's Rules. Strategy: find the union of all three sets then subtract that total from the sample space to represent the complement:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \therefore$$

$$P(A \cup B \cup C) = P(A \cup B) + P(C) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C)$$

The union of A and B are largest if they are disjoint therefore:

$$P(A \cup B) \leq P(A) + P(B)$$

$$P(A \cup B) \leq .2 + .1$$

$$P(A \cup B) \leq .3$$

Extending the same logic if $P(A \cup B)$ is disjoint from C in all ways then the union will be at its largest.

$$\text{Let: } P(A \cap C) = P(B \cap C) = P(A \cap B \cap C) = 0$$

therefore:

$$P(A \cup B \cup C) \leq P(A \cup B) + P(C) = .3 + .3$$

$$P(A \cup B \cup C) \leq .6$$

$$P[(A \cup B \cup C)^c] = 1 - .6 = .4$$

Therefore the smallest $P[(A \cup B \cup C)^c]$ could possibly be is .4 as this only occurs when the union of the three events is at its largest which occurs when the three events are disjoint.