

# MATH 440 Lecture 7

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## Joint densities

Assume we have two random variables. X with some pdf  $f_x(x)$  and y with pdf  $f_y(y)$ . It is not enough to have both of these pdfs when describing a relationship but when they are dependent on each other we will need a multivariate distribution function.

### Discrete Case:

Suppose S is a discrete sample space. This is to say finite or infinite countable, on which two random variables x and y are defined.

The joint Probability Density Function of X and Y is defined as follows:

$$P_{XY}(X, Y) = P(\{s : X(s) = x, Y(s) = y\})$$

$$P(X = x, Y = y)$$

Properties of joint probability:

$$\begin{aligned} p(x, y) &\geq 0 \\ \sum_{\forall x \forall y} p(x, y) &= 1 \end{aligned}$$

### Marginal pdf

The marginal Probability Density Function is the distribution just for x or just for y. Suppose that  $P_{XY}(x, y)$  is a joint pdf of discrete random variables (x, y). Then

$$P_X(x) = \sum_{\forall y} P(x, y)$$

$$P_Y(y) = \sum_{\forall x} p(x, y)$$

Where  $P_X(x)$  and  $P_Y(y)$  are marginal pdfs.

Two random variables defined on the same sample space S of real numbers are jointly continuous if there exists a function  $f(x, y)$  such that for any region  $D \in \mathbb{R}^2$ :

$$P((X, Y) \in D) = \int \int_D f_{XY}(x, y) dx dy$$

Where  $f_{XY}(x, y)$  is the joint pdf for (x,y)

The function  $f(x,y)$  is a joint for some pair (x,y) of random variables iff :

$$f(x, y) \geq 0$$
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$$