# MATH440 Lecture 3

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## 1 2.5 Independence of events

review: conditional probability.

$$\begin{aligned}
\text{Let s be a sample space with events } A, B \subset S \\
P(B) > 0 \\
P(A|B) = \frac{P \cap B}{P(B)} \therefore \\
P(A \cap B) = P(A|B)P(B)
\end{aligned}$$

**Independence**
Two events A and B are indepepdent if

$$P(A|B) = P(A)$$

and dependent otherwise.

This is to say that the probability of A does not depend on whether or not event B occurs.
Two events A and B are also indepepdent if

$$P(A \cap B) = P(A)P(B)$$

This is because we know the following logic:

$$\begin{aligned}
P(A|B) = P(A) \\
P(A \cap B) = P(A|B)P(B) = P(A)P(B)
\end{aligned}$$

**Theorem**
Let A and B be two indepepdent events. Then $A^C, B^C$ are also indepepdent.

$$P(A \cap B) = P(A)P(B)$$

We have to then show the following to proof the indepepdence of the complements:

$$P(A^C \cap B^C) = P(A^C)P(B^C)$$

$$\begin{aligned}
P(A^C \cap B^C) &= P((A \cup B)^C) \\
P(A^C \cap B^C) &= 1 - P(A \cup B) \\
P(A^C \cap B^C) &= 1 - [P(A) + P(B) - P(A \cap B)] \\
P(A^C \cap B^C) &= 1 - [P(A) + P(B) - P(A)P(B)] \\
P(A^C \cap B^C) &= 1 - P(A) - P(B) + P(A)P(B)] \\
P(A^C \cap B^C) &= [1 - P(A)] - P(B) + P(A)P(B)] \\
P(A^C \cap B^C) &= P(A^C) - [P(B)(1 - P(A)]] \\
P(A^C \cap B^C) &= P(A^C) - [P(B)(P(A^C)]] \\
P(A^C \cap B^C) &= P(A^C)[1 - P(B)] \\
P(A^C \cap B^C) &= P(A^C)P(B^C)
\end{aligned}$$

Indepdent Events:

$$P(A \cap B) = P(A) + P(B)$$

Mutually exclusive events:

$$P(A \cap B) = \emptyset$$

The major take away is that when two events are independent the intersection is the product of the two events **Independence of more than two events**

Given $A_1, ..., A_n$ independent events are said to be independent if for every set of indicies $i_1, ..., i_k$

$$P(A_{i_1} \cap, ..., \cap A_{i_k}) = P(A_{i_1})P(A_{i_2})...P(A_{i_k})$$