MATH 425 Lecture 6

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continuous random variables

A discrete random variable is a variable of the form:

$$X : \mathcal{S} \mapsto \mathbb{R}$$

Where $\mathcal{S}$ is a discrete sample space

$$P(x_i) = P(X = x_i) = P(\{s : X(s) = x_i\})$$

For continuous random variable this means that

$$\mathcal{S} = \mathbb{R} \text{ or } \mathcal{S} \subset \mathbb{R}$$

Example
Let $y_1, y_2, .., y_n$ be a set of measurements . Let n =40. Assigning arbitrary data for these points we can construct a histogram. This is used to discretize data and visualize frequency.
frequency The number of times an event occurs.
Relative frequency (w)

$$\frac{f}{\mathcal{S}}$$

where f is frequency
Density

$$\frac{w}{l}$$

where l is the length of each bin on ths histogram.
When you decrease the size of the bins in a progressively shrinking density function the behavior of the histogram can be described by a function;;
When this is accomplished we can describe a function f(x) such that :

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$f(x) \geq 0$$

$$\int_{a}^{b} f(x)dx = P(a \leq y \leq b)$$

Continuous random variable

Let Y be a function from a sample space S to $\mathbb{R}$. The function Y is called a continuous random variable if there exists a function f(y) such that for all real numbers a and b such that $a < b$

$$P(a \leq y \leq b) = \int_a^b f(y)dy$$

In this event f(y) is called a probability density function (pdf) for a continuous random variable

The function f(y) is a pdf for some continuous random variable y $\iff$ the following conditions are met.

$$f(y) \geq 0$$
$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Common distributions

Uniform distribution

$$f(y) = c \quad \forall y : a \leq y \leq b \qquad f(y) = 0 \text{ otherwise}$$

Exponential distribution

$$f(y) = \lambda e^{-\lambda y} \text{ if } y \geq 0$$

and zero otherwise. $\lambda > 0$

Normal distribution

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

cumulative distribution function

$$F_y(t) = P(y \leq t) = \int_{-\infty}^{t} f(y)dy$$