MATH 440 Homework 6

Chris Camano: ccamano@sfsu.edu

3/10/2022

3.4.6

Let n be postive integer, show that $f(y) = (n+2)(n+1)y^n(1-y), 0 \leq yleq1$ is a pdf

$$\begin{aligned}
&\int_0^1 (n+2)(n+1)y^n(1-y)dy = 1 \\
&(n+2)(n+1)\int_0^1 y^n(1-y)dy = 1 \\
&(n+2)(n+1)\int_0^1 y^n - y^{n+1}dy = 1 \\
&(n+2)(n+1)\Big[\int_0^1 y^n dy - \int_0^1 y^{n+1}dy\Big] = 1 \\
&(n+2)(n+1)\Big[\frac{y^{n+1}}{n+1}|_0^1 - \frac{y^{n+2}}{n+2}|_0^1\Big] = 1 \\
&(n+2)(n+1)\Big[\frac{1}{n+1} - \frac{1}{n+2}\Big] = 1 \\
&(n+2)(n+1)\Big[\frac{n+2-n+1}{(n+1)(n+2)}\Big] = 1 \\
&n+2-n+1 = 1 \\
&1 = 1
\end{aligned}$$

3.4.9

$$F_Y(y) = \int_{-\infty}^{y} f_Y(t))dy$$

$$\forall y > 0$$
$$F_Y(y) = \int_{-1}^{y} 1 - |t| dt$$
$$\int_{-1}^{0} 1 + t dt + \int_{0}^{y} 1 - t dt$$
$$F_Y(y) = (t + \frac{t^2}{2}\Big|_{-1}^{0}) + (t - \frac{t^2}{2}\Big|_{0}^{y})$$
$$F_Y(y) = 1 - \frac{1}{2} + y - \frac{y^2}{2}$$

$$\forall y \leq 0$$
$$F_Y(y) = \int_{-1}^{y} 1 - |t| dt$$
$$\int_{-1}^{y} 1 + t dt$$
$$F_Y(y) = t + \frac{t^2}{2}\Big|_{-1}^{y}$$
$$F_Y(y) = \frac{1}{2} + y + \frac{y^2}{2}$$

Thus, $F_Y(y)$=0 $\quad \forall y < -1$
$F_Y(y) = \frac{1}{2} + y + \frac{y^2}{2} \quad \forall y, -1 \leq y \leq 0$
$F_Y(y) = 1 - \frac{1}{2} + y - \frac{y^2}{2} \quad \forall y, 0 < y \leq 1$
and 1 for all y greater than one

3.4.14

$$
\begin{aligned}
&F_Y(y) = \int_0^y te^{-t}dt \\
&let u = t, du = 1, dv = e^{-t}, v = -e^{-t} \\
&F_Y(y) = \int_0^y te^{-t}dt = -te^{-t} + \int e^{-t}dt \\
&F_Y(y) = \int_0^y te^{-t}dt = -te^{-t} - e^{-t} \\
&F_Y(y) = -te^{-t} - e^{-t}\Big|_0^y \\
&F_Y(y) = -ye^{-y} - e^{-y} + 1)
\end{aligned}
$$