MATH440 Homework Set 1

Chris Camano: ccamano@sfsu.edu

1/30/2022

Homework questions: 2.2.12, 2.2.16, 2.2.22,2.2.38 2.3.10, 2.3.12, 2.3.14

1 Chapter 2.2 Sample Spaces and the Algebra of Sets

Problem 2.2.12 Consider the experiment of choosing coefficients for the quadratic equation $ax^2 + bx + c = 0$. Characterize the values of a,b,and c associated with the event A: Equation has complex roots.

$$A = \{(a, b, c)|b^2 - 4ac < 0, a, b, c \in \mathbb{R}\}$$

Problem 2.2.16 Sketch the regions in the xy plane coresponding to $A \cup B$ and $A \cap B$ if:

$$A = \{(x, y)|0 < x < 3, 0 < y < 3\}$$
$$B = \{(x, y)|2 < x < 4, 2 < y < 4\}$$

Please see attached illustration for image of region(Still working on Tikz skills).

Problem 2.2.22 Suppose that each of the twelve letters in the word T E S S E L L A T I O N is written on a chip. Define the following events F,R and C as follows:

F: Letters in the first half of the alphabet
R: Letters that are repeated
V: Letters that are vowels
F={A,E,E,I,L,L}
R={T,T,L,L,E,E,S,S}
V={E,A,I,O,E}

a) What is F$\cap R \cap V$
$F \cap R = \{E, E\}$
$(F \cap R) \cap V = \{E, E\}$

b) What is F$^C \cap R \cap V^C$
$F^C = \{N, O, S, S, T, T\}$
$V^C = \{S, S, T, T, N, L, L\}$
$R^C = \{I, O, N, A\}$

$F^C \cap R = \{S, S, T, T\}$
$(F^C \cap R) \cap V^C = \{S, S, T, T\}$

c) What is F $\cap R^C \cap V$
$F \cap R^C = \{I, A\}$
$(F \cap R^C) \cap V = \{I, A\}$

Problem 2.2.38
Please see attached illustration figure 2.

Problem 2.3.10
An urn contains twenty-four chips, numbered 1 through 24. one is drawn at random. Let A be the event that the number is divisible by 2 and B be the event that the number is divisible by 3. Find $P(A \cup B)$

$$\begin{aligned} A &= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24\} \\ B &= \{3, 6, 9, 12, 15, 18, 21, 24\} \\ A \cup B &= \{2, 3, 4, 6, 8, 9, 12, 14, 15, 16, 18, 20, 21, 22, 24\} \\ P(A \cup B) &= \frac{|A \cup B|}{|S|} = \frac{16}{24} = \frac{2}{3} \end{aligned}$$

Problem 2.3.12
Events A_1 and A_2 are such that $A_1 \cup A_2 = S$ and $A_1 \cap A_2 = \emptyset$ find p_2 if $P(A_1) = p_1, P(A_2) = p_2,$ and $3p_1 - p_2 = \frac{1}{2}$

$$\begin{aligned} 3p_1 - p_2 &= \frac{1}{2} \\ p_1 + p_2 &= 1 \text{ as } A_1 \cup A_2 = S \end{aligned}$$

$$\begin{bmatrix} 3 & -1 & \frac{1}{2} \\ 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{3}{8} \\ 0 & 1 & \frac{5}{8} \end{bmatrix} \therefore p_1 = \frac{3}{8}, p_2 = \frac{5}{8}$$

Problem 2.3.14
Three events A B and C are defined on a sample space S given that:

$$P(A) = .2P(B) = .1P(C) = .3$$

What is the smallest possible value for:

$$P[(A \cup B \cup C^C)]$$

$$P[(A \cup B \cup C)^C)] = P(A^c \cap B^c \cap C^c)$$

by Demorgan's Rules. Strategy: find the union of all three sets then subtract that total from the sample space to represent the complement:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \therefore$$

$$P(A \cup B \cup C) = P(A \cup B) + P(C) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C)$$

The union of A and B are largest if they are disjoint therefore:

$$P(A \cup B) \leq P(A) + P(B)$$

$$P(A \cup B) \leq .2 + .1$$

$$P(A \cup B) \leq .3$$

Extending the same logic if $P(A \cup B)$ is disjoint from C in all ways then the union will be at its largest.

Let: $P(A \cap C) = P(B \cap C) = P(A \cap B \cap C) = 0$

therefore:

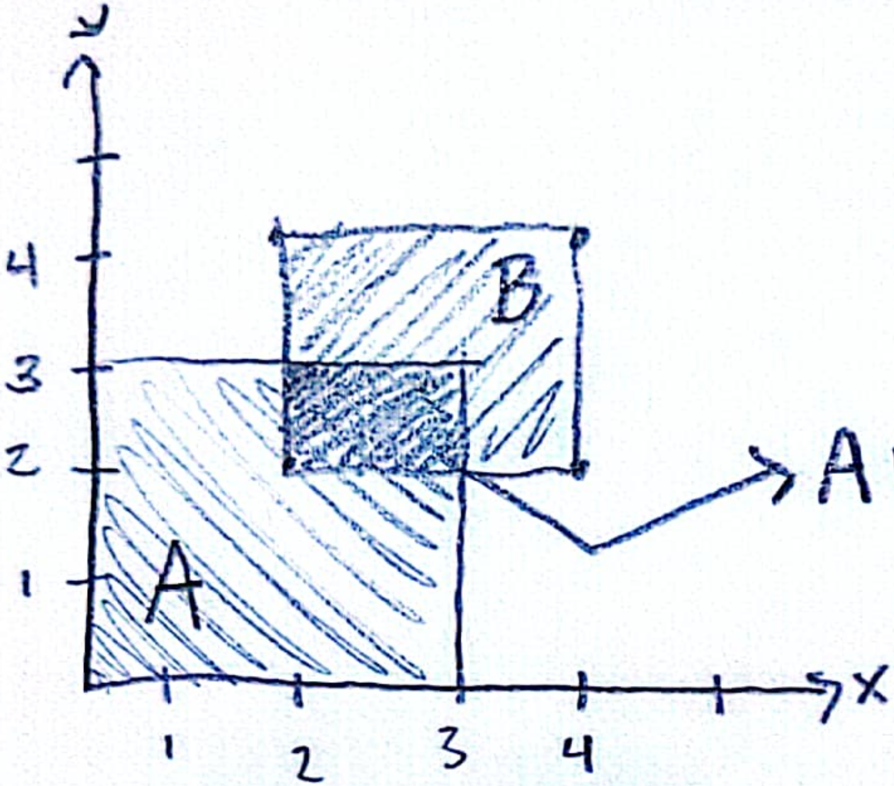
$$P(A \cup B \cup C) \leq P(A \cup B) + P(C) = .3 + .3$$

$$P(A \cup B \cup C) \leq .6$$

$$P[(A \cup B \cup C)^C] = 1 - .6 = .4$$

Therefore the smallest $P[(A \cup B \cup C)^C]$ could possibly be is .4 as this only occurs when the union of the three events is at its largest which occurs when the three events are disjoint.

2.2.1b

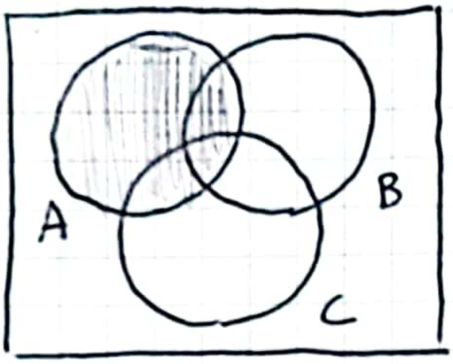


$A = \{(x,y) \mid 0 < x < 3, 0 < y < 3\}$

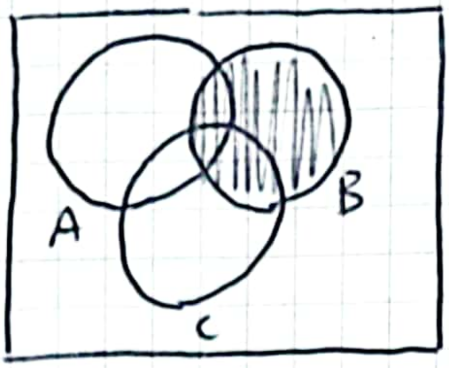
$B = \{(x,y) \mid 2 < x < 4, 2 < y < 4\}$

$A \cup B = \{(x,y) \mid 2 < x < 3, 2 < y < 3\}$

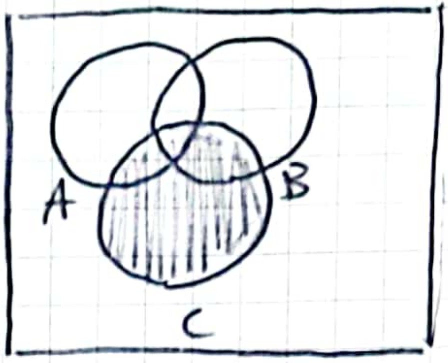
2.2.38) $S = \{A, B, C\}$



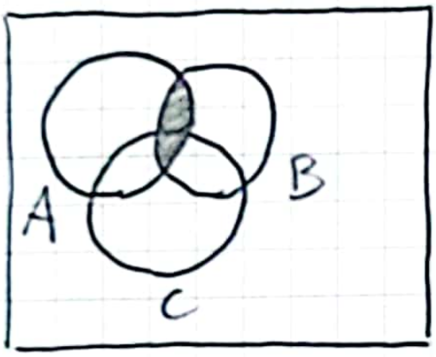
$N(A)$



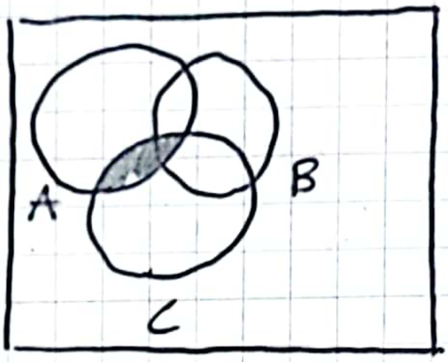
$N(B)$



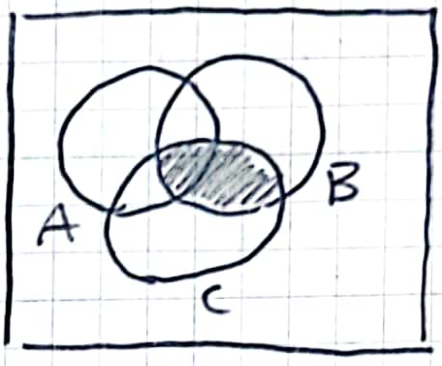
$N(C)$



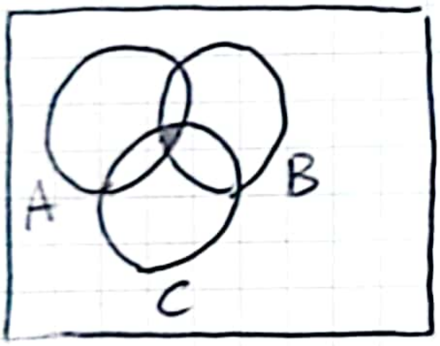
$N(A \cap B)$



$N(A \cap C)$



$N(B \cap C)$



$N(A \cap B \cap C)$

$$\begin{aligned} N(A \cup B \cup C) = {} & N(A) + N(B) + N(C) \\ & - N(A \cap B) - N(A \cap C) \\ & - N(B \cap C) + N(A \cap B \cap C) \end{aligned}$$