MATH440 Lecture 2

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1 2.4 Conditional Probability

Example 1: Assume you roll a dice once:

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let event A be the number 6 occurs thus:

$$P(A) = \frac{1}{6}$$

Let event B be the event even number occurs.

$$P(B) = \frac{1}{2}$$

$$P(A|B) = \frac{1}{3}$$

Using conditional probability one constructs a subset of the sample space. The condition probability notation is read probability of A given B occurs.

Let A and B be any two events defined on a sample space such that $P(B) > 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

1.1 Multiplication rule

As an extension of this property we define the multiplication rule as:

$$P(A \cap B) = P(B|A)P(A)$$

Or

$$P(A \cap B) = P(A|B)P(B)$$

Due to the commutatitivy of sets.
Additionally:

$$P(A \cap B \cap C) = P(A)P(B|A)P(C|B \cap A)$$

For a collection of three sets.
Given

$$A_1, A_2, ..., A_n$$

$$P\Big(\bigcap_{i=1}^{n} A_n\Big) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)...P\Big(A_n|\bigcap_{i=1}^{n-1} A_i\Big)$$

1.2 Baye's Theorem