MATH 440 Homework 7

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3.5) 6,14,20,32
3.6) 8,12,18,20

3.5.6

given a sample size **n=20** and a proability of defect **p=.04** the distibution of x can be modeled by a binomial distribution. Therefore

$$E(X) = np = 20(.04) = .8$$

3.5.14

Given a sample size of **n=15** observations we require a measurement of proability to discern the expected value within the given interval. the probability the continuous random variable x lies within the given interval can be found by integrating over the probability density function within the given bounds:

$$\int_{\frac{1}{2}}^{1} 3y^2 dy = 3\int_{\frac{1}{2}}^{1} y^2 dy = 3(\frac{7}{24}) = \frac{7}{8}$$

$$E(X) = np = 15(\frac{7}{8}) = 13.125$$

3.5.20

Given:

$$E(X) = \sum_{\forall k} 2^k p_X(2^k)$$

We can substitute the value for X represented as 2^k in the original example to yeild.

$$E(X) = \sum_{\forall k} c^k p_X(2^k)$$

as in this new problem the amount won each time can vary between 0 and 2 dollars raised to some power k.
Adjusting bounds yeilds:

$$E(X) = \sum_{k=1}^{\infty} c^k p_X(2^k) = \sum_{k=1}^{\infty} c^k \frac{1}{2^k} = \sum_{k=1}^{\infty} \left(\frac{c}{2}\right)^k$$

To use the sum of a geometric series rule we must start at k =0 therefore we will adjust the bounds of the sum.

$$\sum_{k=1}^{\infty} \left(\frac{c}{2}\right)^k = \left(\sum_{k=0}^{\infty} \left(\frac{c}{2}\right)^k\right) - 1$$

as the difference between the sum from 0 and 1 is only 1.. Applying the geometric rule for convergent series:

$$\frac{1}{1-\frac{c}{2}} - 1 = \frac{c}{2-c}$$

b)
Letting the corresponding value of x correlate with a logarithm yeilds:

$$E(X) = \sum_{\forall k} log2^k p_X(2^k)$$

Adjusting bounds again:

$$E(X) = \sum_{k=1}^{\infty} log2^k p_X(2^k)$$

Substituting probability:

$$E(X) = \sum_{k=1}^{\infty} log2^k \left(\frac{1}{2}\right)^k = \sum_{k=1}^{\infty} \frac{log2^k}{2^k} = \sum_{k=1}^{\infty} k\frac{log2}{2^k} = log2 \sum_{k=1}^{\infty} \frac{k}{2^k}$$

https://math.stackexchange.com/questions/337937/why-sum-k-1-infty-frack2k-2

3.5.32

3.6.8

a)

$$\int_1^{\infty} \frac{2}{y^3} = 1$$

$$\int_1^\infty \frac{2}{y^3} dy = 1$$
$$2 \lim_{b\to\infty} \int_1^b \frac{1}{y^3} dy = 1$$
$$2 \lim_{b\to\infty} \frac{-1}{2y^2}\Big|_1^b = 1$$
$$-\lim_{b\to\infty} \frac{1}{y^2}\Big|_1^b = 1$$
$$-\lim_{b\to\infty} \Big(\frac{1}{b^2} - 1) = 1$$
$$-\lim_{b\to\infty} \frac{1}{b^2} + \lim_{b\to\infty} 1 = 1$$
$$1 = 1$$

b)

$$E(X) = \int_1^\infty y(\frac{2}{y^3})dy = \int_1^\infty \frac{2}{y^2}dy = \lim_{b\to\infty}\int_1^b 2y^{-2}dy = \lim_{b\to\infty} \frac{-2}{y}\Big|_1^b = 2$$

c)

To calculate variance we must fisrst calculate the expected value for the random variable squared.

$$E(Y^2) = \int_1^b y^2(\frac{2}{y^3}dy) = \lim_{b\to\infty} 2ln(y)\Big|_1^b = \infty$$

since part of the required computation for the varianve is infinite the variance cannot be measured in a finite manner.

3.6.12

3.6.18

3.6.20