Journal of Applied Psychology
1994, Vol. 79, No. 3, 402–411

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0021-9010/94/$3.00

Utility Analysis for the One-Cohort Selection–Retention Decision With a Probationary Period

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This article provides a method for estimating the utility of a choice between alternative predictors for selecting a single cohort of new employees when some of these employees are later judged to perform at an unacceptable level and are therefore dismissed at the end of the probationary period. The method is shown to apply to both the traditional selection utility situation, in which the objective is to obtain a fixed number of hires, and the situation in which the objective is to obtain a fixed number of employees who successfully survive the initial probationary period. The utility estimation method proposed applies J. W. Boudreau and C. J. Berger's (1985) conceptual model to a single-cohort probationary case and extends that model by providing an estimation method to predict the utility of alternative predictors before selection takes place. The limitations of the proposal as well as its relationship to recruitment and separation costs are also discussed.

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In personnel selection, the concept of utility refers to the net gain in productivity (i.e., the gain minus the costs) that results from using a particular selection device. Brogden (1949) developed the basic equations for estimating this utility, and most of the current research on the topic (e.g., Boudreau, 1983a; Martin & Raju, 1992; Murphy, 1986; Raju, Burke, & Normand, 1990) has been derived from his pioneering work. At present, the most elaborated version of the classic utility model is the external-employee-movement framework presented by Boudreau and Berger (1985) and further discussed by Boudreau (1991). Although the framework can handle a variety of decision situations, Boudreau asserted that the required complexity level of any utility model ultimately depends on the specific nature of the decision problem, and Boudreau and Berger summed up the conditions that allow their proposal to be simplified.

In the present article, I focus on one such special case of Boudreau and Berger's (hereinafter referred to as B & B) model. More specifically, the proposed formulation addresses the choice among alternative predictors for selecting a single cohort of new employees when some of these employees are later judged to perform at an unacceptable level and are therefore dismissed at the end of the training and probationary period. For further reference, this situation is hereinafter referred to as *selection with a probationary period* (or *probationary selection* for short), and the new model can be regarded as an alternative statement of the B & B one-cohort selection–retention model, in which it is assumed that (a) dismissal is the only form of separation and (b) the retention-decision variable (i.e., job performance rating) correlates with the selection predictor.

Although the new selection–retention model is less general than the external-employee-movement approach, it still offers a significant contribution to the existing utility literature because it overcomes a limitation of the B & B proposal. The limitation relates to the estimation procedure that B & B proposed to predict the value of some of the crucial utility parameters. Although the B & B model offers a conceptually adequate framework to study the probationary selection problem, I show in the next section that their estimation procedure cannot be applied in the present context because of the correlation between the selection variable and the retention-decision variable.

Besides providing a solution to the parameter estimation problem, the present proposal also meets the criteria of theoretical and practical relevance discussed by Boudreau (1991). The theoretical importance of the new model derives from the fact that the proposal integrates the former Taylor–Russell approach (Taylor & Russell, 1939) with the standard Brogden-Cronbach-Gleser (BCG) formulation (Brogden, 1949; Cronbach & Gleser, 1965). The practical value is linked to two observations. First, the model relates to a typical decision situation in that misselection and the subsequent dismissal of unsatisfactory employees is a fact of life in selection: Many new employees are first hired on a probationary basis, and only at the end of this period is it decided (usually on the basis of observed job performance) whether they will remain on the job (Brewster, Hegewisch, Holden, & Lockhart, 1992). Second, the proposed model can be applied to another important selection situation that also cannot be dealt with in the B & B and BCG frameworks. More specifically, the model can be extended to estimate the utility of a predictor when, at the end of the probationary period, a fixed quota of successfully performing selectees is needed. As observed by one of the reviewers for this article, the latter selection situation may be even more realistic than the traditional fixed-quota-of-selectees decision.

I thank William Whitely and two particularly helpful anonymous reviewers for their useful comments on a draft of the article.

A computer program (written in FORTRAN 77) that performs the utility calculations described in this article is available on request. Because the program calls routines from the Numerical Algorithms Group library, interested readers should first check whether they have this library (or a similar one) at their disposal.

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Fixed-Quota Selection With a Probationary Period

In the fixed-quota probationary selection condition, the new selection–retention model estimates the utility of a predictor when a given number (N) of employees must be hired from an applicant pool of a fixed number of candidates (n) with the understanding that the unsuccessful selectees will be dismissed after the probationary period, whereas the successful employees will remain on the job. Whether a selectee is considered successful or not depends only on his or her performance rating at the end of the probation period. Therefore, performance rating is the retention-decision variable (cf. the variable q in the B & B model, 1985, p. 590) and, assuming top-down selection, the selectees have a predictor score (X) that is at least equal to x_c—the predictor cutoff score that corresponds to the selection ratio N/n—whereas the successful selectees are characterized by a performance rating R at least equal to r_c, the critical criterion rating. Equating the duration of one time period with the time spent on the job by the unsuccessful selectees (i.e., the length of the trial period, excluding the training time), the net utility of the predictor-selected workforce (U_p) can then be written as follows:

$$U_p = N[\mu_{Y(x_c)} - \mu_s] + (T-1)NS_p[\mu_{Y(x_c r_c)} - \mu_s] - NC_t - nC_p, \quad (1)$$

where T is the average number of time periods on the job for the successful selectees; S_p is the success ratio of the predictor (i.e., S_p indicates the proportion of selectees whose performance rating at the end of the trial period is at least equal to r_c); Y denotes the dollar-valued payoff of job performance; $\mu_{Y(x_c)}$ represents the average payoff of all the selectees; $\mu_{Y(x_c r_c)}$ expresses the average payoff of the successful selectees; μ_s refers to the average service cost (i.e., salary and costs other than compensation) associated with the job performance; C_p indicates the cost per candidate of using the predictor; and C_t is the training cost per hire.

Apart from the omission of the separation cost, which will be included at a later stage, the definition of U_p in Equation 1 is a faithful reflection of the conceptualization proposed by Boudreau and Berger (1985, p. 584) in their Figure 1. More specifically, the first term of the equation represents the utility of the new cohort of selectees (cf. the additions in Boudreau & Berger's, 1985, terminology) for Time Period 1, whereas the transaction costs of the acquisitions are summarized by the last two terms. Finally, the utility of the retained part of the cohort, over the periods 2 to T, is given by the second term of the formula, which (as indicated by Boudreau & Berger, 1985) consists of the product of (a) the average length of stay on the job (after the trial period) of the retained selectees, (b) the quantity of retentions (i.e., the product NS_p), and (c) the quality minus the performance cost (the term $\mu_{Y(x_c r_c)} - \mu_s$) of these retained employees.

Like all utility expressions, the formulation of Equation 1 makes several simplifying assumptions (cf. Boudreau, 1991). It is assumed that both the mean service cost (μ_s) and the training investment (C_t) are the same for successful and unsuccessful employees. In addition, it is also assumed that the mean service cost and the mean payoff, $\mu_{Y(x_c r_c)}$, do not change over time; but the eventual time dependency of both parameters can be accounted for if their value at time $t+1$ is a simple function of the corresponding quantities at time t. Finally, the model does not provide for the impact of financial and economic correction parameters, such as variable cost, taxes, and discounting (cf. Boudreau, 1983a). These parameters could easily be included in the present model, however, and their inclusion would have the effect of reducing the utility values reported here. They have been omitted simply to focus on the main tenet of the proposal.

Utility Estimation Problem

For Utility Equation 1 to be of any real practical value as a decision aid in choosing between alternative predictors, one should be able to estimate its value before the selection is actually performed. Thus, besides values for the usual parameters—T, C_p, and $\mu_{Y(x_c)}$—advance estimates of S_p, $\mu_{Y(x_c r_c)}$, C_t, and μ_s are also needed. The latter two parameters pose no major estimation problems, however, because Boudreau and Berger (1985) offered several adequate suggestions to determine μ_s and referred to the human resource accounting literature to evaluate C_t.

To determine the remaining two quantities—S_p and $\mu_{Y(x_c r_c)}$—one might consider another proposal by Boudreau and Berger (1985). More specifically, these authors suggested the use of concurrent or past empirical data to estimate the number $N(1 - S_p)$ of separations (N_s, in their notation) and the difference in average quality between the retained and the preseparation workforce (the difference $[\mu_{Y(x_c r_c)} - \mu_{Y(x_c)}]$ in the present article), where the latter difference would be obtained as the product of (a) the average standardized score of the retained group on the retention-decision variable ($\bar{Z}_{qr}$, in Boudreau and Berger's notation), (b) the correlation between this variable and dollar-valued payoff in the group of selectees ($r_{q,sv}$), and (c) the value of a one-standard-deviation difference in payoff in the selected cohort ($SD_{sv_{i_0}}$; cf. Boudreau & Berger, 1985, pp. 591, 596).

Unfortunately, the B & B model cannot be applied in the present context of selection with a probation period because the situation is characterized by a substantial correlation between the selection-decision and the retention-decision variables. In fact, this correlation (i.e., between performance rating and predictor scores) is often used as a proxy for the correlation between the selection predictor and the dollar-valued performance (cf. Raju et al., 1990), and, because of this association, past empirical records will typically produce wrong estimates of all four quantities: N_s, $\bar{Z}_{qr}$, $r_{q,sv}$, and $SD_{sv_{i_0}}$. Only data that relate to a predictor with the same validity and that are used with an identical selection ratio will not lead to such biased estimates. But it is most unlikely for both conditions to hold simultaneously, especially when the selection decision, as is typically the case, involves the introduction of a new predictor. Finally, using current empirical data is no solution either because the present objective is to predict the utility of selection devices that have not yet been applied in a particular situation to decide between the alternative possibilities.

Alternative Utility Model

To overcome the estimation difficulties described above, I propose a new utility model. The model, which integrates the earlier Taylor–Russell framework with the classic BCG approach, is based on two assumptions. First, it is assumed that, after standardization, the observed predictor scores (X) and the observed performance ratings (R) are jointly distributed in the applicant population according to the bivariate standard normal density, with ρ_{XR} the correlation between X and R. This assumption is identical to the one that Taylor and Russell (1939) introduced to estimate the success ratio of a selection, and I discuss its justification at length in the closing section of this article.

The second assumption asserts that the regression of dollar-valued payoff of performance (Y) on performance rating (R) is linear: $E(Y|R = r) = \mu_Y + \rho_{YR}\sigma_Y r$; with μ_Y and σ_Y the mean and the standard deviation (over one time period) of Y in the applicant population, respectively, and with ρ_{YR} the correlation between Y and R. Although the assumption is essentially similar to the one proposed by Boudreau and Berger in their discussion of nonrandom retentions (cf. Boudreau & Berger, 1985, pp. 590–591), it should be noted that the linearity of the regression is presumed for the applicant population, because this feature makes it possible to estimate the newly introduced correlation parameter ρ_{YR} by using the procedure suggested by Hsu (1982), in which R is considered as an alternative predictor.[1]

Given the previous two assumptions, the new utility model no longer requires the type of empirical data proposed by Boudreau and Berger to estimate the success ratio (S_p) and the average quality of the retained (successful) selectees, $\mu_{Y(x_c r_c)}$. More specifically, it follows from the first assumption that the success ratio of the predictor is equal to $\Phi_2(x_c, r_c)/\Phi_1(x_c)$, where $\Phi_2(\cdot, \cdot)$ and $\Phi_1(\cdot)$ refer to the bivariate and univariate standard normal distributions,[2] respectively. In turn, the second assumption implies that the average quality of the retained selectees is obtained as

$$\mu_{Y(x_c r_c)} = \mu_Y + \rho_{YR}\sigma_Y \mu_{R(x_c r_c)}, \tag{2}$$

with $\mu_{R(x_c r_c)}$ the mean performance rating of the successful selectees. A similar equation relates $\mu_{Y(x_c)}$ to $\mu_{R(x_c)}$, the mean performance rating of all selectees.

The bivariate assumption also provides an analytic expression for the average rating scores because Muthén (1990) and Tallis (1961) have shown that

$$\mu_{R(x_c r_c)} = \frac{\left[\phi_1(r_c)\Phi_1\!\left(\frac{x_c - \rho r_c}{\sqrt{1-\rho^2}}\right) + \rho\phi_1(x_c)\Phi_1\!\left(\frac{r_c - \rho x_c}{\sqrt{1-\rho^2}}\right)\right]}{\Phi_2(x_c, r_c),} \tag{3}$$

where $\phi_1(\mathrm{a})$ denotes the univariate normal density function, and the shorter notation, ρ, is used to indicate the correlation ρ_{XR}. Finally, $\mu_{R(x_c)}$ equals $\rho\mu_{X(x_c)}$, whereas the mean predictor score of the selectees, $\mu_{X(x_c)}$, equals $\phi_1(x_c)/\Phi_1(x_c)$ because the bivariate assumption implies that the regression of R on X is linear and that the marginal distributions of R and X are both normal.

Thus, in comparison with the BCG approach and except for the already discussed quantities ρ_{YR}, μ_s, and C_t, the application of the new utility model requires the specification of only two additional parameters. The evaluation of the first of these new parameters, μ_Y, is, however, part of the judgmental procedure to estimate σ_Y described by Schmidt, Hunter, McKenzie, and Muldrow (1979). Moreover, in a recent study, Judiesch, Schmidt, and Mount (1992) concluded that "under most conditions, a reasonable estimate of $\bar{Y}$. . . can be computed" (p. 248).

Finally, the specification of the performance cutoff is basically similar to the determination of x_c because the values of both parameters are the result of managerial decisions that define the characteristics of the utility problem. The meaning of being a successful performer is something for the organization to decide, and the present model allows the translation of this decision in terms of meaningful economic utility parameters. From the assumption that the regression of dollar-valued payoff on observed performance is linear, it follows that r_c can be expressed as $r_c = [E(Y|R = r_c) - \mu_Y]/(\rho_{YR}\sigma_Y)$. So, if the organization decides that a successful employee must contribute, for example, at least as much as his or her service cost—that is, it is decided that $E(Y|R = r_c)$ should be equal to μ_s—then r_c is chosen equal to $(\mu_s - \mu_Y)/(\rho_{YR}\sigma_Y)$. Alternatively, management might decide that to be successful a selectee must perform as well as the average of the present, tenured employees (μ_{Y_t}); in this case, r_c would equal $(\mu_{Y_t} - \mu_Y)/(\rho_{YR}\sigma_Y)$ and so on. Also, observe that the decided criterion cutoff value can easily be implemented at the end of the probationary period because the present assumptions imply that the successful employees can be identified by simply ranking the selectees according to their performance ratings. The top percentage of the thus-ranked performers ($S_p \times 100$) then corresponds to the percentage of successful selectees.

Predictor Utility

As noted above, Equation 1 indicates the utility of the predictor-selected workforce and not the utility of using the predictor. To obtain the latter type of utility estimate, it is common practice to calculate ΔU defined as the difference between U_p and U_0, the net utility of a randomly selected workforce. Following the same logic as before, this random utility can be written as

$$U_0 = N[\mu_Y - \mu_s] + (T-1)NS_0[\mu_{Y(r_c)} - \mu_s] - NC_t, \tag{4}$$

where $S_0 = \Phi_1(r_c)$ is the success ratio of a random selection and $\mu_{Y(r_c)}$ corresponds to the mean payoff of the randomly selected successful selectees. This average can be computed as $\mu_{Y(r_c)} = \mu_Y + \rho_{YR}\sigma_Y\mu_{R(r_c)}$, with the mean performance rating of the randomly successful selectees, $\mu_{R(r_c)}$, equal to $\phi_1(r_c)/\Phi_1(r_c)$.

[1] To apply the procedure of Hsu (1982), the two basic assumptions of the present model must be combined with the assumption that the regression of Y on X is linear (this is the basic assumption in the Brogden-Cronbach-Gleser model (Brogden, 1949; Cronbach & Gleser, 1965) and that the pairs Y, R and Y, X have a bivariate exponential type of distribution.

[2] More specifically, $\Phi_2(x_c, r_c)$ is defined as $\int_{x_c}^{+\infty}\int_{r_c}^{+\infty}\phi_2(X, R)dRdX$, whereas $\Phi_1(x_c)$ is equal to $\int_{x_c}^{+\infty}\phi_1(X)dX$, with $\phi_2(\mathrm{a, b})$ and $\phi_1(\mathrm{a})$ the bivariate and univariate standard normal densities, respectively.

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Subtracting U_0 from U_p and rearranging terms then leads to the following expression for the predictor utility:

$$\Delta U \equiv U_p - U_0 = N[\mu_{Y(x_c)} - \mu_Y]$$
$$+ (T-1)N[S_p\mu_{Y(x_c r_c)} - S_0\mu_{Y(r_c)} + (S_0 - S_p)\mu_s] - nC_p. \quad (5)$$

From the equation, it can be seen that the parameters required to evaluate U_p also suffice to estimate ΔU.

Relationship to the BCG Utility Model

The classic BCG framework does not distinguish between successful and unsuccessful selectees. As a consequence, the framework cannot be applied to the present problem of selection with a probationary period. The relationship between the BCG and the new selection–retention model is straightforward, however, in that the latter formulation of the utility of a predictor-selected workforce essentially differs from the classic proposal in only two respects. First, the total of all selectees, N, is divided into the numbers S_pN and $(1 - S_p)N$ of successful (retained) and unsuccessful (dismissed) hires, respectively, and the expected length of stay is adjusted accordingly. Successful selectees are expected to remain T time periods on the job, whereas the unsuccessful selectees are fired after one time period. Second, the expected payoff is also differentiated: During the first period, the same payoff, $\mu_{Y(x_c)}$, is used as in the BCG approach, whereas in the following periods the payoff $\mu_{Y(x_c r_c)}$ is introduced.

The analysis outlined above shows that the present selection–retention model becomes equivalent to the BCG formulation only if one assumes that all selected individuals remain with the organization for the full duration of the T time periods. Because this assumption does not hold in many practical selection decisions (and certainly not in the present, probationary selection situation), it follows that utility models that incorporate employee separation—such as the B & B model and the present elaboration of this framework—are generally to be preferred.

Fixed Quota of Successful Selectees

The situation addressed here differs from the traditional fixed-quota selection problem in that the quota is not in terms of people to be hired but in terms of successful selectees (cf. Kao & Rowan, 1959). Thus, as is often the case in practical situations, the organization needs to add a fixed number (N) of successful employees to the workforce; and the problem is to predict the utility of using a predictor to perform the selection, again with the understanding that the unsatisfactorily performing employees will be dismissed at the end of the probationary period. As before, the utility of the predictor is obtained by comparing U_p, the utility of the predictor-selected workforce (cf. Equation 1), with U_0, the utility of a randomly hired group of employees (cf. Equation 4), but this time the computation of both U_p and U_0 is complicated by the requirement that N successful employees must ultimately be added to the workforce after selection and separations occur.

A different number of people must be hired to satisfy the requirement, depending on whether the predictor is used or not. More specifically, the pair of assumptions introduced earlier implies that the number of predictor-selected employees, N_p, should equal N/S_p, whereas N_0—the number of randomly selected hires—must equal N/S_0. Replacing the number of selectees with N_p and N_0 in Equations 1 and 4, respectively; rewriting both N_pS_p and N_0S_0 as N; and subtracting the thus-modified formula for U_0 from the equally updated expression for U_p, it follows that the utility of the predictor, ΔU, is

$$\Delta U \equiv U_p - U_0 = N_p\mu_{Y(x_c)} - N_0\mu_Y - (N_p - N_0)\mu_s$$
$$+ (T-1)N[\mu_{Y(x_c r_c)} - \mu_{Y(r_c)}] - (N_p - N_0)C_t - nC_p, \quad (6)$$

where the predictor cutoff (x_c) is the solution of $\Phi_2(x_c, r_c) = N/n$, provided that $n \geq N/\Phi_1(r_c)$. For, if the latter condition is not fulfilled, then it is impossible to obtain at least N successful selectees.

For the fixed-quota-of-selectees problem, it has been shown that the BCG framework and the present proposal can be made equivalent by recognizing that BCG assumes that all employees are treated as successful and that all have average tenure of T periods. The BCG model treats the fixed-quota-of-selectees problem and the fixed-quota-of-successful-hires problem precisely the same, whereas the present procedure recognizes that they are different. The B & B model can encompass the fixed-quota-of-hires problem, but, as noted above, the suggested estimation approach of the B & B model is likely to be inaccurate for the probationary selection situation. Thus, the proposed estimation approach improves on the BCG model by recognizing the difference between the two problems, and it provides a method for estimating parameters necessary to apply the B & B conceptualization to the probationary situation. Also, it seems likely that many organizations define their selection goal as filling a number of vacancies with acceptable employees, rather than simply hiring that number of new employees (Milkovich & Boudreau, 1991). Thus, the present proposal is both theoretically and practically interesting.

To further strengthen the relevance of the new selection–retention utility analysis, I show in the next section how two additional decision parameters can be integrated into the framework. These new parameters are the cost associated with firing an unsuccessful selectee (C_s; cf. the separation cost discussed by Boudreau & Berger, 1985) and the recruitment expenses.

Separation and Recruitment Costs

The separation costs are easily accounted for. In case the organization needs N successful selectees, $(N_p - N)$ predictor-selected employees will have to be fired after the first time period, whereas the number of dismissals will equal $(N_0 - N)$ when the selection is performed randomly. Thus, to adjust the predictor utility for the separation costs, the term $(N_p - N_0)C_s$ must be subtracted from the right-hand side of Equation 6. Because N_p will be smaller than N_0 when a valid predictor is used, this will always result in a higher value of ΔU. On the other hand, when the decision situation corresponds to the common fixed-quota selection scenario, the quantities $N(1 - S_p)C_s$ and $N(1 - S_0)C_s$ must be subtracted from the right-hand side of Equations 1 and 4, respectively, and the predictor utility, corrected for separation

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costs, is obtained by adding the term $N(S_p - S_0)C_s$ to the expression for ΔU in Equation 5.

To assess the impact of the recruitment cost, one can adopt a procedure similar to the one described by Martin and Raju (1992). That is, if n is the number of applicants and $C_{r(n)}$ represents the average recruitment cost per applicant, then the utility of the predictor-selected workforce—adjusted for the recruitment expenses—is found by subtracting the quantity $nC_{r(n)}$ from the utility U_p. Also, observe that the notation $C_{r(n)}$ clearly indicates that the average recruitment cost is conceived as a function of the number of recruited applicants (cf. Martin & Raju, 1992).

The same recruitment cost does not apply to the randomly selected workforce, however. Assuming—as the majority of the utility models do (Boudreau, 1991; Martin & Raju, 1992)—that the characteristics of the applicant pool (i.e., the value of μ_Y and σ_Y) are independent of the size of the pool, it follows that the utility of the randomly selected employees is not related to the selection ratio. Hence, the recruitment cost in random selection must be based on the number of selectees, which means that this cost is equal to $NC_{r(N)}$ for the fixed-quota selection decision and equal to $N_0C_{r(N_0)}$ when N successful selectees are needed. Furthermore, the assumption implies that the predictor utility is almost always inflated when the recruitment cost is not accounted for, regardless of whether unsuccessful selectees are dismissed or not. That is, the utility of a valid predictor is typically bought at the expense of a higher recruitment cost because such a predictor is useful only when more people than needed are recruited. The only exception to this occurs when the total recruitment cost does not depend on the number of recruitments. Although rare, this may happen when, for example, an organization has a sufficient number of spontaneous walk-ins.

To summarize, the utility of the predictor, corrected for both the recruitment and separation costs, is equal to

$$\Delta U = N[\mu_{Y(x_c)} - \mu_Y] + (T-1)N[S_p\mu_{Y(x_c r_c)} - S_0\mu_{Y(r_c)} + (S_0 - S_p)\mu_s] - n[C_p + C_{r(n)}] + NC_{r(N)} + N(S_p - S_0)C_s \quad (7)$$

in the common fixed-quota selection situation. Alternatively, when the quota relates to N successful selectees, the corrected predictor utility is

$$\Delta U = N_p\mu_{Y(x_c)} - N_0\mu_Y - (N_p - N_0)\mu_s + (T-1)N[\mu_{Y(x_c r_c)} - \mu_{Y(r_c)}] - (N_p - N_0)(C_s + C_t) - n[C_p + C_{r(n)}] + N_0C_{r(N_0)}. \quad (8)$$

Observe that Equations 7 and 8 both reduce to the classic utility formula (adjusted for training, separation, and recruitment costs) when it is assumed that all selectees are successful.

Illustrative Application

The purpose of the following example is to illustrate the utility equations derived above and to clarify some of the computational aspects of the formulas. The example is based on data gathered during a workshop on utility held by the selection division of the Belgian Association of Work and Organizational Psychology. As a preparatory step, the participants—all of whom had several years of experience in recruitment and selection—were asked to summarize a number of firm-specific utility figures with regard to the hiring of recently graduated university students for an entry-level management position. In what follows, only the data related to the selection of junior managers in the financial sector are used to construct a typical selection scenario for this position.

Table 1
Selection Decision Parameters for the Illustrative Application

Parameter description	Notation	Value
No. of applicants	n	136
No. of selectees	N	17
Average tenure (time periods) for successful selectees[a]	T	11
Predictor validity (predictor–observed performance r)	ρ_{XR} or ρ	0.35
Correlation between money-valued payoff (Y) and observed (rated) performance (R)	ρ_{YR}	0.85
Predictor cost per applicant	C_p	$200
Training cost per selectee	C_t	$10,000
Average payoff of job performance (one time period) in the applicant population	μ_Y	$25,000
SD	σ_Y	$7,000
Average service cost per employee	μ_s	$22,500
Separation cost per unsuccessful employee	C_s	$1,000
Recruitment cost per applicant		
Random selection[b]	$C_{r(N_0)}$	$100
Predictor selection	$C_{r(n)}$	$400

[a] Average tenure for unsuccessful selectees was one time period.
[b] Fixed-quota-of-successful-selectees condition.

Table 1 shows a complete overview of the parameters and the parameter values of the scenario. From the table, it can be seen that the situation refers to the selection of 17 selectees from a total of 136 applicants, using a predictor (a general ability test) with validity ρ_{XR} equal to 0.35 and assuming a correlation ρ_{YR} of 0.85 between observed performance and the dollar value of that performance. First, the utility of the predictor-selected workforce (U_p) and the utility of using the predictor (ΔU) are calculated when a fixed quota of 17 graduates is hired on a probationary basis. Next, these quantities are computed for the case that 17 successful selectees are needed at the end of the trial period.

Traditional Fixed-Quota Selection Situation

The data in Table 1 indicate that the selection ratio, $\Phi_1(x_c)$, is $17/136 = 0.125$ in the fixed-quota situation. Hence, the critical predictor score, x_c, equals 1.150.[3] As to the value of the crite-

[3] All computations were performed in double precision, and the reported values are accurate to the nearest third decimal or the nearest integer. Because of the truncation, some results may be slightly different from those obtained by straight calculation. For example, $\mu_{R(1.150,-0.420)}$ is in fact equal to 0.8315061 and $\mu_{Y(1.150,-0.420)}$ = $29,947.46. Hence, the value of $\mu_{Y(1.150,-0.420)}$ is reported as 29,947, whereas the straight

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Table 2
Calculation of the Average Performance Payoff Predictor and Randomly Selected Workforce

Quantity	Computational formula	Value
$\mu_{R(1.150,-0.420)}$	$(\phi_1(-0.42)\times\Phi_1\{[(1.15-0.35\times-0.42)]/\sqrt{1-0.35^2}\}+0.35\times\phi_1(1.15)\times\Phi_1[(-0.42-0.35\times1.15)/\sqrt{1-0.35^2}])/\Phi_2(1.15,-0.42)$	0.832
$\mu_{Y(1.150,-0.420)}$	$25{,}000+0.85\times7{,}000\times0.832$	29,947
$\mu_{X(1.150)}$	$\phi_1(1.150)/\Phi_1(1.150)$	1.647
$\mu_{R(1.150)}$	0.35×1.647	0.576
$\mu_{Y(1.150)}$	$25{,}000+0.85\times7{,}000\times0.576$	28,430
	Calculation of S_0 and $\mu_{Y(-0.420)}$	
S_0	$\Phi_1(-0.420)$	0.663
$\mu_{R(-0.420)}$	$\phi_1(-0.420)/\Phi_1(-0.420)$	0.551
$\mu_{Y(-0.420)}$	$25{,}000+0.85\times7{,}000\times0.551$	28,279

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rion cutoff (r_c) it is decided that a selectee is successful when his or her expected payoff is at least equal to the expected service cost. So, r_c can be computed as $(22{,}500-25{,}000)/(7{,}000\times0.85)=-0.420$. Using this value and the value of x_c, one can calculate the success ratio of the predictor (S_p) as $\Phi_2(1.150,-0.420)/\Phi_1(1.150)=0.853$.

To determine the utility of the predictor-selected workforce (U_p; see Equation 1), the values of $\mu_{Y(1.150,-0.420)}$ and $\mu_{Y(1.150)}$ are also required. The calculations that lead to the determination of both these quantities are summarized in the first part of Table 2. The first column of the table indicates the quantity that is to be computed, the second column gives the computational formula, and the final column contains the resulting value. To evaluate the expressions $\Phi_1(a)$ and $\Phi_2(a, b)$, which occur in some of the calculations, routines from the Numerical Algorithms Group FORTRAN library (Numerical Algorithms Group [NAG], 1990) are used. As a final result, it is found that $\mu_{Y(1.150,-0.420)}=\$29{,}947$ and that $\mu_{Y(1.150)}=\$28{,}430$. So, the utility of the predictor-selected workforce, uncorrected for separation and recruitment costs, is equal to (see Equation 1)

$$U_p=17\times(28{,}430-22{,}500)+10\times17\times0.853\times(29{,}947-22{,}500)-17\times10{,}000-136\times200=\$983{,}955. \quad (9)$$

In a similar way, and by using the quantities that are tabulated in the second half of Table 2, one can calculate the (equally uncorrected) utility of the random-selected workforce (U_0; see Equation 4):

$$U_0=17\times(25{,}000-22{,}500)+10\times17\times0.663\times(28{,}279-22{,}500)-17\times10{,}000=\$523{,}635. \quad (10)$$

Hence, the utility of using the predictor (ΔU; see Equation 5) for the fixed-quota selection amounts to $983{,}955-523{,}635=\$460{,}320$ when neither the separation nor the recruitment costs are accounted for.

Inspection of Equations 9 and 10 reveals that the predictor utility is essentially derived from three sources. First, the average quality of the predictor-based hires exceeds that of the randomly selected employees; that is, compare the average, $\mu_{Y(x_c)}$, of \$28,430 to $\mu_Y\approx\$25{,}000$. Second, using the predictor also results in a higher average quality of the retained selectees; compare $\mu_{Y(x_c r_c)}=\$29{,}947$ to $\mu_{Y(r_c)}=\$28{,}279$. Finally, a larger quantity of employees is retained when the selection is based on the predictor (i.e., compare $17\times0.853=14.501$ to $17\times0.663=11.271$).

Although the first two of these utility components are self-evident, one might object to the last component because it does not reflect the actual practice of probationary selection in organizations. According to the objection, organizations would replace the unsuccessful selectees at the end of the probationary period with new hires until they have enough qualified people to produce the required output. As a consequence, the difference between the utility of a predictor-selected versus a randomly selected workforce would be substantially smaller than the one obtained in the present proposal because of the reduced difference in the quantity of contributing employees.

This is an important objection, not because it implies that the proposed calculation is incorrect but because it points out that the initial formulation of the utility question as a single-cohort decision problem may sometimes be inadequate. Obviously, when the selection proceeds according to the scenario depicted in the objection, then the present single-cohort proposal must be extended to a multiple-cohort formulation to account for the fact that the predictor will be used repeatedly. Such a generalization is easily achieved, however, by adopting a procedure that is similar to the one proposed by Boudreau (1983b).

Alternatively, when it is feasible to hire enough workers to ensure that the required number of successful selectees is obtained at once, the objection suggests that the utility question should, right from the start, be framed as a decision related to the hiring of a fixed quota of successful selectees. The next example shows that such a formulation is indeed possible within the new utility model. The example deals also with the separation and recruitment costs that were left unaccounted for in the previous illustration.

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calculation $25{,}000+0.85\times7{,}000\times0.832$ (see top of Table 2) results in the slightly different value of \$29,950.40.

Fixed Quota of Successful Selectees

This example focuses on the utility of selecting a fixed number of 17 successful selectees from the applicant pool. To determine the predictor utility in the fixed-quota-of-successful-selectees condition, it is necessary to first know the required number of selectees: N_p (predictor selection) and N_0 (random selection). The latter number is easily obtained, however, because N_0 can be calculated as N/S_0, where S_0 depends only on the value of r_c. Applied to the present example, in which $r_c = -0.420$ and $N = 17$, this results in $N_0 = 17/\Phi_1(-0.420) = 25.648$.

The derivation of the number N_p is somewhat more difficult in that it requires solving the following nonlinear equation in x_c: $\Phi_2(x_c, r_c) = N/n$. Thus, in the example, $N_p = 136 \times \Phi_1(x_c)$, where x_c is the solution of $\Phi_2(x_c, -0.420) = 17/136$. To solve the nonlinear equation, a routine from the NAG library (NAG, 1990) is used, which returns a value of 1.046 for x_c. Hence, N_p equals $136 \times \Phi_1(1.046) = 20.099$.

Because the decision situation relates to the selection of exactly 17 successful selectees, the numbers N_0 and N_p are typically noninteger valued. To overcome this problem, the decision objective might be rephrased to the requirement that at least 17, but less than 18, successful employees are selected. The calculations could then proceed with N_0 and N_p equal to 26 and 21, respectively. Such rounding to the next integer will only marginally affect the utilities U_0, U_p, and ΔU, however; they were therefore omitted from further computations.

Given the values of x_c, N_0, and N_p, the calculation of the expected utility of the predictor usage (see Equation 6) poses no other problems than those encountered in the computation of the predictor utility for the common fixed-quota selection. Thus, it is found that $\mu_{Y(1.046, -0.420)} = \$29{,}837$ and that $\mu_{Y(1.046)} = \$28{,}253$, which implies that

$$\Delta U = 20.099 \times 28{,}253 - 25.648 \times 25{,}000 - (20.099 - 25.648) \times 22{,}500 + 10 \times 17 \times (29{,}837 - 28{,}279) - (20.099 - 25.648) \times 10{,}000 - 136 \times 200 = \$344{,}774. \quad (11)$$

To adjust for the separation and recruitment costs, the terms $(20.099 - 25.648) \times 1{,}000$ ($C_s = \$1{,}000$, see Table 1) and $136 \times 400 - 25.648 \times 100$ ($C_{r(n)} = \$400$ and $C_{r(N_0)} = \$100$; cf. Table 1) must be subtracted from this utility value. The corrected utility of the predictor is then estimated to be \$298,487.

Comparing the present utility figure of \$344,774 (i.e., the value of ΔU calculated in Equation 11) to the previously obtained estimate of \$460,320, one can see that the utility of the predictor decreases substantially when the decision problem is framed in terms of hiring a fixed quota of successful candidates. This is not surprising, however, because now the number of randomly selected employees that contribute to the utility in Time Period 1 is higher than the corresponding number of predictor-selected hires. Also, although it is still true that the average quality of both the initially selected and subsequently retained selectees is higher when the predictor is used, there is no difference anymore with regard to the quantity of the retained hires. Whether random or predictor selection is performed, enough applicants are hired to guarantee that 17 of the selectees are successful and, hence, are retained. Finally, both effects are not compensated for by (a) the higher acquisition cost associated with the random selection (i.e., 25.648 instead of 20.099 selectees must be trained in the random-selection condition) and (b) the higher total performance cost of the random selectees during the first period.

This lack of compensation may very well generalize to situations other than the present selection situations when the average length of stay on the job for successful selectees is substantially higher than the duration of the probationary period. Hence, the new selection–retention model may offer an additional explanation for the observation that the usual utility models overestimate the real gain of selection (cf. Becker, 1989; Boudreau, 1991; Murphy, 1986) because these models typically address the utility issue in the context of the common fixed-quota selection condition and not in terms of the fixed-quota-of-successful-selectees situation (cf. Boudreau, 1991, p. 698).

Discussion

I have described a new utility model for dealing with the one-cohort selection–retention decision problem. Unlike the B & B model, the new approach allows for a correlation to exist between the selection variable and the retention variable. Because of this feature, the new estimation procedure can be used to predict the selection utility for the one-cohort probationary period situation, under both the goal of obtaining a fixed quota of new hires and the goal of obtaining a fixed quota of successful employees. Hence, the new utility model conforms to the view that utility analysis is a decision-making tool that enables industrial and organizational psychologists to anticipate the future consequences of "programs designed to enhance the value of the work force to the employing organization" (Boudreau, 1991, p. 623).

The present proposal is not without limitations, however. First, the model pertains to the one-cohort selection–retention decision problem only, and even this topic is not dealt with exhaustively because certain decision parameters or outcome attributes (cf. Boudreau, 1991) are not included in the model. Thus, the equations do not anticipate the effects of rejected job offers on the utility of the selection. To integrate these effects, an extension of the proposals by Hogarth and Einhorn (1976) and by Murphy (1986) may be considered, however. In particular, one could assume that the probability of accepting a job offer—$P(A = 1)$, with A the dichotomous job-acceptance variable—can be estimated from the logistic regression of job acceptance on X (cf. Raju, Steinhaus, Edwards, & DeLessio, 1991, who use logistic regression to express the probability of job success): $P(A = 1) = [e^{(b_0+b_1X)}]/[1 + e^{(b_0+b_1X)}]$. From the assumption it then follows that, for example, the average payoff of the successful selectees who accept the job offer, $\mu_{Y(x_c r_c a)}$, is equal to $\mu_Y + \rho_{YR}\sigma_Y\mu_{R(x_c r_c a)}$, with

$$\mu_{R(x_c r_c a)} = \frac{\int_{x_c}^{+\infty}\int_{r_c}^{+\infty} R\left[\frac{e^{b_0+b_1X}}{1+e^{b_0+b_1X}}\right]\phi_2(X,R)\,dR\,dX}{\int_{x_c}^{+\infty}\int_{r_c}^{+\infty}\left[\frac{e^{b_0+b_1X}}{1+e^{b_0+b_1X}}\right]\phi_2(X,R)\,dR\,dX}. \quad (12)$$

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The other quantities in Equations 1 and 4 could be estimated in a similar way.

Some of the practical problems related to this extension are comparable to those that characterized Hogarth and Einhorn's (1976) and Murphy's (1986) proposals; that is, the assumption concerning the logistic relationship between job acceptance and X may be erroneous, or it may be less straightforward to obtain advance estimates of the parameters b_0 and b_1. Finally, the analytic formulas to compute, for example, S_p and $\mu_{R(x_c r_c)}$, must be replaced by the numerical evaluation of double integrals of the type shown above for the determination of $\mu_{R(x_c r_c a)}$.

Another limitation of the present proposal is that no translation of the utility outcomes is given in terms of the capital budgeting framework (cf. Cascio & Morris, 1990). Such a translation is easily achieved, however, because it only requires substituting the BCG type of utility values with the new estimates in the formulas presented by, among others, Boudreau (1983a) and Cascio and Morris (1990).

A more serious problem with the new utility model relates to the assumption that the observed predictor and criterion scores follow a bivariate normal distribution. Even though Schmidt et al. (1979) mentioned the studies by Tiffin and Vincent (1960) and by Surgent (1947)—both of which substantiated the validity of the bivariate normal model to describe test–criterion sample data—the assumption may still be judged as questionable. Remember, however, that the main purpose of the distributional assumption is to enable the estimation of the utility before the selection actually takes place. In this respect, Schmidt et al. (1979) observed that "for most purposes, there is no need for the utility estimates to be accurate down to the last dollar" (p. 617). Their second remark—that one should also consider the alternative situation—is also relevant here because, as shown earlier here, neither the BCG nor the B & B models offer an adequate approach to the presently discussed selection-decision problems.

The argumentation above is not really conclusive, however, and I therefore decided to conceive of a generalized version of the new selection–retention utility model. The generalization is based on the work by Raju et al. (1990) in that a distinction is made between true and observed predictor–criterion scores. A full account of the extension is given in the Appendix. For present purposes, it suffices to observe that the generalization permits one to estimate the robustness of the utility outcomes for violations of the assumption that the observed predictor and criterion scores are bivariate normally distributed. To mimic the level of violation, the variance of the measurement errors e_X and e_R is manipulated relative to the variance of the true score components X_t and R_t. The robustness of the utility outcomes can then be assessed by comparing the values of U_p, U_0, and ΔU obtained in the zero-error condition, to the corresponding estimates in the nonzero conditions.

Table 3 shows the results of such a comparison with respect to the U_p, U_0, and ΔU values that were calculated in the example given above for the common fixed-quota selection problem. The rows of the table correspond to the ratio of the variance of the error components ($\sigma^2_{e_X}$ and $\sigma^2_{e_R}$, respectively) to the variance of the true score components. (In all cases, the two error variances were assumed equal.) The columns of the table refer to the utilities U_p, U_0, and ΔU, and the upper entry of each cell indicates the value of the appropriate quantity. Although these values indicate a steady decrease in the utilities for increasing levels of error variance, it should be noted that the value changes are related not only to the incremental violation of the bivariate assumption but also to the decline of the correlation between the observed scores. In case that, for example, $\sigma^2_{e_X}/\sigma^2_{X_t} = \sigma^2_{e_R}/\sigma^2_{e_{R_t}} = 0.3$, it follows that the correlation between X_O and R_O decreases from .35 (i.e., the correlation in the zero-error condition) to $.35/\sqrt{1.3 \times 1.3} = .269$. To neutralize this unwanted effect, one can modify the extended bivariate utility model in a very simple way. In particular, it suffices to fix the correlation of the observed scores at the initial value of .35 for each error condition by adjusting the correlation between the true score components. With this modification, the error conditions differ only in the form of the observed score distribution and differ no longer in the magnitude of the observed score correlation. This allows for a better baseline with which to evaluate the robustness issue.

Table 3
Utility Values for the Extended Bivariate Model

Ratio $\sigma^2_{e_X}/\sigma^2_{X_t}$	U_p	U_0	ΔU
.00	983,955	523,635	460,320
	983,955	523,635	460,320
.10	932,590	505,829	426,761
	982,183	505,829	476,354
.30	845,419	473,886	371,533
	976,790	473,886	502,904
.50	773,537	446,198	327,339
	968,625	446,198	522,427
1.00	638,543	391,528	247,015
	937,896	391,528	546,368

The results that correspond to the modification are also summarized in Table 3. For each error condition, the lower cell entry indicates the appropriate utility value, and by comparing these values to the utilities of the zero-error condition one can see that the new utility model is quite robust for mild violations of the distributional assumption. Also, because the U_0 estimates are unaffected by the modification, whereas the U_p values decrease only slowly, it follows that the predictor utilities, ΔU, increase for higher error levels. But the overestimation is quite small when both the predictor and the criterion are measured in a reasonably reliable way.

One final issue remains to be settled. The issue concerns the complexity of the present utility model. More specifically, one might ask—as Boudreau (1991) did—whether the increased complexity is really worth it. I believe that the main arguments in favor of such complex models, put forward by Boudreau, also apply to the present utility framework. In addition, it should be noted that (except for the generalized version) all of the relevant formulas of the model have been around for at least 30 years. Alternatively, when a highly valid predictor is available or the predictor cutoff is highly negative, then the benefits of the more complex framework will be rather marginal. However, in most practical selection situations, predictors are moderately valid and the cutoff levels are near or above the average. In such situ-

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ations, the estimation approach suggested here allows selection decision makers to more accurately apply Boudreau and Berger's conceptual framework and to anticipate selection utility in the one-cohort probationary period situation, when the selection-decision variables and retention-decision variables correlate substantially.

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Appendix

Extension of the New Utility Model

The extension of the new utility model was inspired by the proposal of Raju, Burke, and Normand (1990) in that a distinction is made between true and observed scores for both the retention-decision measure (i.e., rated job performance) and the predictor measure. In addition, the extension restricts the assumption of bivariate normality to the joint distribution of the true predictor and performance scores. More specifically, the observed predictor and performance scores, X_O and R_O, respectively, are decomposed into

$$X_O = X_t + e_X, \quad \text{and}$$
$$R_O = R_t + e_R, \tag{A1}$$

where X_t and R_t denote the true scores and e_X and e_R denote the corresponding error components that are independent of X_t and R_t, and where it is assumed that $(X_t, R_t) \sim N_2(\mu, \Sigma)$, with $\mu' = (0, 0)$ and 1 and ρ (the correlation between true predictor and true performance scores) are the diagonal and off-diagonal elements, respectively, of Σ. Next, it is assumed that the regression of job performance payoff, Y, on true performance rating is linear: $E(Y|R_t) = \mu_Y + \rho_{YR_t}\sigma_Y R_t$, with ρ_{YR_t} the correlation between Y and R_t. (The simulation results, reported in Table 3, we obtained under the condition that ρ_{YR_t} is also equal to 0.85.) Finally, e_X and e_R are considered to be independent, rectangularly distributed random variables with average values equal to 0 and variance $\sigma^2_{e_X} = 1/12c^2_X$ and $\sigma^2_{e_R} = 1/12c^2_R$. The joint density of X_t, R_t, e_X, and e_R can then be written as

$$f(X_t, R_t, e_X, e_R) = c_X c_R \phi_2(X_t, R_t), \tag{A2}$$

with

$$-\infty \le X_t, R_t \le +\infty,$$
$$\frac{-1}{2c_X} \le e_X \le \frac{1}{2c_X}, \quad and$$
$$\frac{-1}{2c_R} \le e_R \le \frac{1}{2c_R}.$$

Applying the transformation $X_t = X_t$, $R_t = R_t$, $X_O = X_t + e_X$ and $R_O = R_t + e_R$ and integrating out X_t, the joint density of R_t, X_O, and R_O is

$$g(R_t, X_O, R_O) = c_X c_R \phi_1(R_t)\left[\Phi_1\left(\frac{X_O - 1/2c_X - \rho R_t}{\sqrt{1-\rho^2}}\right) - \Phi_1\left(\frac{X_O + 1/2c_X - \rho_{R_t}}{\sqrt{1-\rho^2}}\right)\right], \tag{A3}$$

with

$$-\infty \le X_O, R_O \le +\infty, \quad \text{and}$$
$$R_O - 1/2c_R \le R_t \le R_O + 1/2c_R.$$

The density function outlined above makes it possible to estimate, for example, the mean performance payoff for the successful predictor-selected candidates, where selection and success are both related to the corresponding observed score:

$$\mu_{Y(x_c r_c)} = \mu_Y + \rho_{YR_t}\sigma_Y \mu_{R_t(x_c r_c)}, \tag{A4}$$

with

$$\mu_{R_t(x_c r_c)} = \frac{\int_{x_c}^{+\infty}\int_{r_c}^{+\infty}\int_{R_O-1/2c_R}^{R_O+1/2c_R} R_t g(R_t, X_O, R_O)\,dR_t\,dR_O\,dX_O}{\int_{x_c}^{+\infty}\int_{r_c}^{+\infty}\int_{R_O-1/2c_R}^{R_O+1/2c_R} g(R_t, X_O, R_O)\,dR_t\,dR_O\,dX_O}. \tag{A5}$$

To evaluate the triple integrals in the latter quotient, a combination of quadrature routines from the NAG FORTRAN library (Numerical Algorithms Group, 1990) is implemented.

Similar developments lead to the estimation of the other components of Utility Equations 1 and 4. To compute, for example, the mean payoff of the successful random selectees, the joint distribution of R_t and R_O,

$$h(R_t, R_O) = c_R\phi_1(R_t), \quad R_O - 1/2c_R \le R_t \le R_O + 1/2c_R,$$
$$-\infty \le R_O \le +\infty, \tag{A6}$$

is considered to first obtain the mean true performance score of the random selectees whose observed performance is at least equal to r_c:

$$\mu_{R_t(r_c)} = \frac{\int_{r_c}^{+\infty}\int_{R_O-1/2c_R}^{R_O+1/2c_R} R_t h(R_t, R_O)\,dR_t\,dR_O}{\int_{r_c}^{+\infty}\int_{R_O-1/2c_R}^{R_O+1/2c_R} h(R_t, R_O)\,dR_t\,dR_O}$$
$$= \frac{\Phi_1(r_c - 1/2c_R) - \Phi_1(r_c + 1/2c_R)}{\int_{r_c}^{+\infty}[\Phi_1(R_O - 1/2c_R) - \Phi_1(R_O + 1/2c_R)]\,dR_O}. \tag{A7}$$

The mean payoff of the successful randomly selected candidates, $\mu_{Y(r_c)}$, is then equal to $\mu_Y + \rho_{YR_t}\sigma_Y\mu_{R_t(r_c)}$.

Although the present generalization assumes that the measurement errors are rectangularly distributed, it is obvious that any other distribution—for example, a triangular or a normal distribution—can be considered for e_X and e_R. Alternatively, all assumptions may be dropped, and the utility calculations could be based on the joint density of the observed predictor and criterion scores as estimated from previously gathered sample data. Such an approach seems hardly practical, however, especially when the selection decision relates to the introduction of a new predictor.

Received December 17, 1992
Revision received September 17, 1993
Accepted September 17, 1993 ■

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