CPE111 Discrete Mathematics for Computer Engineers

International Program
Homework #2, due on August 24, 2022

Chapter 2

1. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

a) the set of people who speak English, the set of people who speak English with an Australian accent

The second set is a subset of the first.

b) the set of fruits, the set of citrus fruits

The second set is a subset of the first.

c) the set of students studying discrete mathematics, the set of students studying data structures

Neither is a subset of the other.

2. Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$ True

b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$ True

c) $\{\emptyset\} \in \{\{\emptyset\}\}$ True

d) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$ True.

3. What is the cardinality of each of these sets?

a) $\emptyset$ 0 ; no element

b) $\{\emptyset\}$ 1 ; the empty set.

c) $\{\emptyset, \{\emptyset\}\}$ 2 elements

d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$ 3 elements.

4. Let $A = \{a, b, c, e, i, j\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

a) $A \cup B$ $= \{a, b, c, d, e, f, g, h, i, j\}$

b) $A \cap B$ $= \{a, b, c, e\}$

c) $A - B$ $= \{i, j\}$

d) $B - A$ $= \{d, f, g, h\}$

5. Let A, B, and C be sets. Use a Venn diagram or a truth table to show that

a) $(A - B) - C \subseteq A - C$

A	B	C	A - B	(A - B) - C	A - C
1	1	1	0	0	0
1	1	0	0	0	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

$\therefore (A-B)-C \subseteq A-C$ #

b) $(B \cup C) - A = (B - A) \cup (C - A)$

A	B	C	B ∪ C	(B ∪ C) - A	B - A	C - A	(B - A) ∪ (C - A)
1	1	1	1	0	0	0	0
1	1	0	1	0	0	0	0
1	0	1	1	0	0	0	0
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	1	0	1	1	1	0	1
0	0	1	1	1	0	1	1
0	0	0	0	0	0	0	0

$\therefore (B \cup C) - A = (B-A) \cup (C-A)$ #

6. Show that $A \oplus B = (B - A) \cup (A - B)$

A	B	A ⊕ B	B - A	A - B	(B - A) ∪ (A - B)
1	1	0	0	0	0
1	0	1	0	1	1
0	1	1	1	0	1
0	0	0	0	0	0

$\therefore A \oplus B = (B-A) \cup (A-B)$ #

7. Determine whether f is a function from **Z** to **R** if

a) $f(n) = \pm n$ This is not a function ; not well-defined.

b) $f(n) = \sqrt{n^2 + 1}$ This is a function ; well-defined real number.

c) $f(n) = 1 / (n^2 + 4)$ This is a function

d) $f(n) = 1 / (n^3 - 1)$ This is a function

8. Find these values

a) $\lfloor -1.1 \rfloor$ = -2

d) $\lceil -5.8 \rceil$ = -5

f) $\lceil -2.99 \rceil$ = -2

h) $\left\lceil \left\lfloor \frac{3}{2} \right\rfloor + \left\lceil \frac{5}{3} \right\rceil + \frac{1}{2} \right\rceil$ $= \left\lceil 1 + 2 + \frac{1}{2} \right\rceil = \left\lceil \frac{7}{2} \right\rceil = 4$ #

9. Determine whether each of these functions from **Z** to **Z** is one-to-one.

a) $f(n) = n - 1$ One-on-one ; since if $n_1 - 1 = n_2 - 1$, then $n_1 = n_2$

b) $f(n) = n^2 + 1$ not one-on-one ; since, for example, $f(2) = f(-2) = 5$

c) $f(n) = n^3$ one-on-one ; if $n_1^3 = n_2^3$, then $n_1 = n_2$ (take the cube root of each side)

d) $f(n) = \lceil n/2 \rceil$ not one-on-one ; for example, $f(3) = f(4) = 2$

10. Consider these functions from the set of students in a discrete mathematics class. Under what conditions is the function one-to-one if it assigns to a student.

a) mobile phone number

one-on-one ; 1 number per student

b) student identification number

one-on-one ; 1 number per student

c) final grade in the class

not one-on-one ; some students might have a same grade together (at least 2 students).

d) home town (There're two cases occuring in this problems)

1. one-on-one, in case of nobody shares the same hometown in the class
2. not one-on-one, in case of "at least two students" share the same hometown in the class.

11. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from **R** to **R**.

1. $(f \circ g)(x) = f(g(x))$
$= f(x+2)$
$= (x+2)^2 + 1$
$= x^2 + 4x + 4 + 1$
$= x^2 + 4x + 5$ #

2. $(g \circ f)(x) = g(f(x))$
$= g(x^2+1)$
$= x^2 + 1 + 2$
$= x^2 + 3$ #

12. Find the first four terms of the sequence defined by each of these recurrence relations and initial conditions.

a) $a_n = -3a_{n-1}$, $a_0 = -1$

$a_0 = -1$
$a_1 = -3(a_{1-1}) = -3a_0 = 3$
$a_2 = -3(a_{2-1}) = -3a_1 = -3(3) = -9$
$a_3 = -3(a_{3-1}) = -3a_2 = -3(-9) = 27$
$a_4 = -3(a_{4-1}) = -3a_3 = -3(27) = -81$

b) $a_n = na_{n-1} + a^2_{n-2}$, $a_0 = -1$, $a_1 = 0$

$a_0 = -1$
$a_1 = 0$
$a_2 = 2a_1 + a_0^2 = 2(0) + (-1)^2 = 1.$
$a_3 = 3a_2 + a_1^2 = 3(1) + (0)^2 = 3$
$a_4 = 4a_3 + a_2^2 = 4(3) + 1^2 = 13$

c) $a_n = a_{n-1} + a_{n-2} + a_{n-3}$, $a_0 = 1$, $a_1 = 1$, $a_2 = 2$

$a_0 = 1$

$a_1 = 1$

$a_2 = 2$

$a_3 = a_2 + a_1 + a_0 = 2+1+1 = 4$

$a_4 = a_3 + a_2 + a_1 = 4+2+1 = 7$.

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54/354 12 **13.** Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if

a) $a_n = 0$ $\quad -3a_{n-1} + 4a_{n-2} = -3\cdot 0 + 4\cdot 0$

$= 0$

$= a_n$

b) $a_n = 1$ $\quad -3a_{n-1} + 4a_{n-2} = -3\cdot 1 + 4\cdot 1$

$= 1$

$= a_n$

c) $a_n = (-4)^n$ $\quad = -3(-4)^{n-1} + 4(-4)^{n-2}$

$= (-4)^{n-2}((-3)(-4)+4)$

$= (-4)^{n-2}(16)$

$= (-4)^{n-2}(-4)^2 = (-4)^n = a_n$ #

d) $a_n = 2(-4)^n + 3$ $\quad -3a_{n-1} + 4a_{n-2} = -3(2(-4)^{n-1}+3) + 4(2(-4)^{n-2}+3)$

$= (-4)^{n-2}((-6)(-4)+4\cdot 2) - 9 + 12$

$= (-4)^{n-2}(32) + 3$

$= (-4)^{n-2}(-4)^2(2) + 3$

$= 2(-4)^n + 3 = a_n$ #

14. Find the value of each of these sums

a) $\sum_{j=0}^{5}(1 + (-1)^j)$

$2+0+2+0+2+0 = 6$ #

b) $\sum_{j=0}^{6}(3^j - 2^j)$

$\frac{3^7-1}{3-1} - \frac{2^7-1}{2-1} = 1093 - 127 = 966$ #

15. Find $\prod_{j=0}^{4}(j! + 2)$

$= (0!+2)(1!+2)(2!+2)(3!+2)(4!+2)$

$= 3 \times 3 \times 4 \times 8 \times 26$

$= 7488$ #

16. Let $\mathbf{A} = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$

a) Find $\mathbf{A}^2$

$= \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1+2 & -2+6 \\ -1+3 & 2+9 \end{bmatrix}$

$= \begin{bmatrix} 3 & 4 \\ 2 & 11 \end{bmatrix}$ #

b) Find $\mathbf{A}^3$ $= A^2 \cdot A$

$= \begin{bmatrix} 3 & 4 \\ 2 & 11 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -3+4 & 6+12 \\ -2+11 & 4+33 \end{bmatrix}$

$= \begin{bmatrix} 1 & 18 \\ 9 & 37 \end{bmatrix}$ #

17. Let A = $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find

a) A∨ B $= \begin{bmatrix} 1 \vee 0 & 1 \vee 1 \\ 0 \vee 1 & 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ #

b) A∧ B $= \begin{bmatrix} 1 \wedge 0 & 1 \wedge 1 \\ 0 \wedge 1 & 0 \wedge 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ #

c) A⊙B $= \begin{bmatrix} (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 0) \vee (0 \wedge 1) & (0 \wedge 1) \vee (0 \wedge 0) \end{bmatrix}$

$= \begin{bmatrix} 0 \vee 1 & 1 \vee 0 \\ 0 \vee 0 & 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ #

18. Find the Boolean product of A and B, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3\times 3}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}_{3\times 2}$$

$A \odot B = \begin{bmatrix} 1 \vee 0 \vee 0 & 0 \vee 0 \vee 0 \\ 0 \vee 0 \vee 1 & 0 \vee 1 \vee 1 \\ 1 \vee 0 \vee 0 & 0 \vee 1 \vee 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ #