Chapter 1 Functions

Number system: real number $\mathbb{R}$

$\mathbb{Z}^+$ Positive integers (Natural number) $1, 2, 3, 4, \dots$
$\mathbb{Z}^0$ Zero 0
$\mathbb{Z}^-$ Negative integers $-1, -2, -3, -4, \dots$
} Integer $\mathbb{Z}$

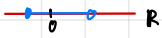
$\mathbb{Q}$ Rational numbers $\frac{p}{q}$ where $p, q \in \mathbb{Z}$

$\mathbb{R} \setminus \mathbb{Q}$ Irrational numbers $\pi, e, \sqrt{2}, \ln 2$

note: $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \approx 2.716$

Note sometimes $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ depending on the context.

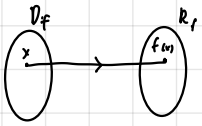
Interval



Interval is a connected subset of $\mathbb{R}$.

- $(a,b) = \{x \in \mathbb{R} \mid a < x < b\}$
- $[a,b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$
- $[a,b) = \{x \in \mathbb{R} \mid a \leq x < b\}$
- $(a,b] = \{x \in \mathbb{R} \mid a < x \leq b\}$
- $[a,\infty) = \{x \in \mathbb{R} \mid x \geq a\}$
- $(-\infty,b) = \{x \in \mathbb{R} \mid x < b\}$

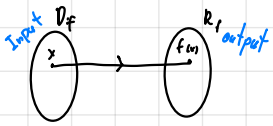
Definition
A function f is a rule where assign each x in the domain to only one corresponding value called $f(x)$



Domain $D_f = \{x \in \mathbb{R} \mid f(x) \text{ is defined}\}$

Range $R_f = \{f(x) \mid x \in D_f\}$

Suppose A and B are sets. A function f from A to B is a rule that assigns each $x \in A$ to only one corresponding value $f(x) \in B$.



we write $f: A \to B$ for a function from A to B

Ex. Real valued function: $f: \mathbb{R} \to \mathbb{R}$ $(a = \mathbb{R}, b = \mathbb{R})$

Ex. $f(x) = x^2$
$f(2) = 4$
$f(-1) = 1$

Ex. $f(x) = \sqrt{x}$ $f: A \to B$ $A = \mathbb{R}^+ = [0, \infty)$ $D_f = [0, \infty)$
$B = \mathbb{R}$ $R_f = [0, \infty)$

Ex. For each real number a, we define a line ℓ_a described by $y = ax + a^2$

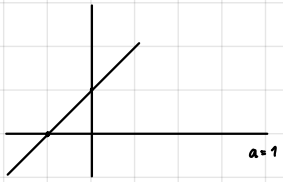
Input		Output
a	$\longrightarrow$	$\ell_a : y = ax + a^2$
real number		line

This is a function from the set of real numbers to set of lines

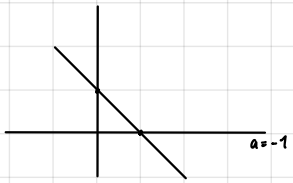
$f: \mathbb{R} \to \{\text{lines}\}$

(i) Plot ℓ_1 and ℓ_{-1}

Note $f(1) = \ell_1$ line
$f(-1) = \ell_{-1}$ line

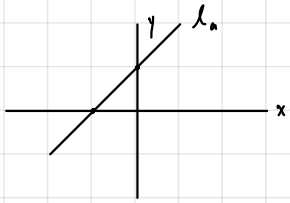


$\ell_1 : y = x + 1$



$\ell_{-1} : y = -x + 1$

(ii) Which points (x,y) on the place line on at least one line ℓ_a?



$\ell_a : y = ax + a^2$

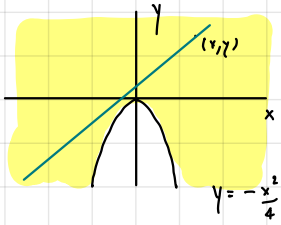
If (x,y) is a point of the line ℓ_a, then $y = ax + a^2$

$$a^2 + xa - y = 0$$

$$a = \frac{-x \pm \sqrt{x^2 + 4y}}{2}$$

Hence, we need $x^2 + 4y \geq 0$

$$y \geq -\frac{x^2}{4}$$



So $\{(x,y) \mid y \geq -\frac{x^2}{4}\}$ lines on at least one line ℓ_a