

CPE 111 Discrete Mathematics for Computer Engineers
International Program, 2022
Homework 1, due on LEB2 at noon on 17 Aug 2022

Chapter 1

Sec 1.1

1. Which of these are propositions? What are the truth values of those that are propositions?

a) Do not eat in the classroom.

Not a proposition ; it's a command.

b) What time is it?

Not a proposition ; it's a question.

c) There is pollution in Bangkok.

This is a proposition that is true ; there have sound and air pollution.

d) $4 + x = 5$.

Not a proposition ; its truth value depends on x

e) The moon is made of green cheese

This is a proposition that is false.

f) $2n \geq 50$.

Not a proposition ; its truth value depends on n

2. Suppose that Smartphone A has 256MB RAM and 32GB ROM, and the resolution of its camera is 8 MP; Smartphone

B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.

It's true because C has 5 MP resolution compared to B is only 4 MP.

d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.

It's false due to the hypothesis is true but the conclusion is false.

3. Determine whether these biconditionals are true or false.

a) $2 + 2 = 4$ if and only if $1 + 2 = 3$.

$$\text{True} \leftrightarrow \text{True} = \text{True} \#$$

b) $1 + 1 = 2$ if and only if $2 + 3 = 5$.

$$\text{True} \leftrightarrow \text{True} = \text{True} \#$$

c) $1 + 1 = 3$ if and only if monkeys can fly.

$$\text{False} \leftrightarrow \text{False} = \text{True} \#$$

d) $0 > 1$ if and only if $2 > 1$.

$$\text{False} \leftrightarrow \text{True} = \text{False} \#$$

4. Construct a truth table for each of these compound propositions.

c) $q \oplus (p \wedge q)$

p	q	$p \wedge q$	$q \oplus (p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	F

#

e) $(q \rightarrow \neg p) \rightarrow (p \leftrightarrow q)$

p	q	$\neg p$	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \rightarrow (p \leftrightarrow q)$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

5. Evaluate each of these expressions.

a) $(1\ 1011 \oplus 1\ 1001) \oplus 1\ 1010$

Sol'n

$$\begin{array}{r} 1\ 1011 \\ 1\ 1001 \\ \hline 0\ 0010 \end{array} \oplus \begin{array}{r} 0\ 0010 \\ 1\ 1010 \\ \hline 1\ 1000 \end{array} \#$$

b) $(1\ 0011 \vee 0\ 1000) \wedge (1\ 0001 \vee 1\ 1010)$

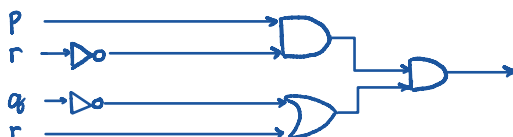
Sol'n

$$\begin{array}{r} 1\ 0011 \\ 0\ 1000 \\ \hline 1\ 1011 \end{array} \vee \begin{array}{r} 1\ 0001 \\ 1\ 1010 \\ \hline 1\ 1011 \end{array} \wedge \begin{array}{r} 1\ 1011 \\ 1\ 1011 \\ \hline 1\ 1011 \end{array} \#$$

Sec 1.2

6. Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output

$(p \wedge \neg r) \wedge (\neg q \vee r)$ from input bits p, q, and r.



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Sec 1.3

7. Use De Morgan's laws to find the negation of each of the following statements.

a) Kwame will take a job in industry or go to graduate school.

Given $p \vee q$ Kwame will not take a job in industry and will not go to graduate school
 $\neg(p \vee q) = ?$ $\neg(p \vee q) = (\neg p) \wedge (\neg q)$ #

b) James is young and strong.

Given $p \wedge q$ James is not young, or he is not strong.
 $\neg(p \wedge q) = ?$ $\neg(p \wedge q) = (\neg p) \vee (\neg q)$ #

8. Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology. \therefore Proof that it is not a tautology #

p $\neg p$	q $\neg q$	$p \rightarrow q$	$(\neg p \wedge (p \rightarrow q))$	$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$
T F	T F	T	F	T
T F	F T	F	F	F
F T	T F	T	T	T
F T	F T	T	T	T

9. Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

p	q	r	$\neg p$	$q \rightarrow r$	$p \vee r$	$\neg p \rightarrow (q \rightarrow r)$	$q \rightarrow (p \vee r)$
T	T	T	F	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	T

\therefore They are logically equivalent. #

Sec 1.4

9. Let $Q(x)$ be the statement " $x + 10 > 2x$." If the domain consists of all integers, what are these truth values?

- a) $Q(5)$ *True*
- b) $\exists x Q(x)$ *True*
- c) $\forall x Q(x)$ *False*
- g) $\exists x \neg Q(x)$ *True*

10. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

- a) $\exists x(x^2 = 2)$ *True*
- b) $\exists x(x^3 = -1)$ *True*
- c) $\forall x(x^2 + 1 \geq 2)$ *True*
- d) $\exists x(x^2 \neq x)$ *True*
- e) $\forall x(x^2 > x)$ *False*

Sec 1.5

11. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

- a) $\forall x \exists y (x = y^2)$ *False*
- b) $\exists x \forall y (xy = 0)$ *True*
- c) $\exists x \exists y (x - y = y - x)$ *True*
- d) $\exists x \forall y (y = 0 \rightarrow xy = 1)$ *False .*
- e) $\forall x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$ *False*
- f) $\forall x \forall y \exists z (z = (x + y) / 2)$ *True .*