

CPE 111 Discrete Mathematics for Computer Engineers
International Program, 2022
Homework 1, due on LEB2 at noon on 17 Aug 2022

Chapter 1

Sec 1.1

1. Which of these are propositions? What are the truth values of those that are propositions?

- a) Do not eat in the classroom.
- b) What time is it?
- c) There is pollution in Bangkok.
- d) $4 + x = 5$.
- e) The moon is made of green cheese
- f) $2n \geq 50$.

2. Suppose that Smartphone A has 256MB RAM and 32GB ROM, and the resolution of its camera is 8 MP; Smartphone

B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

- b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
- d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.

3. Determine whether these biconditionals are true or false.

a) $2 + 2 = 4$ if and only if $1 + 2 = 3$.

b) $1 + 1 = 2$ if and only if $2 + 3 = 5$.

c) $1 + 1 = 3$ if and only if monkeys can fly.

d) $0 > 1$ if and only if $2 > 1$.

4. Construct a truth table for each of these compound propositions.

c) $q \oplus (p \wedge q)$

| p | q | $p \wedge q$ | $q \oplus (p \wedge q)$ |
|----------|----------|--------------------------------|---|
| T | T | | |
| T | F | | |
| F | T | | |
| F | F | | |

e) $(q \rightarrow \neg p) \rightarrow (p \leftrightarrow q)$

| p | q | $\neg p$ | $q \rightarrow \neg p$ | $p \leftrightarrow q$ | $(q \rightarrow \neg p) \rightarrow (p \leftrightarrow q)$ |
|----------|----------|----------------------------|--|---|--|
| T | T | F | | | |
| T | F | F | | | |
| F | T | T | | | |
| F | F | T | | | |

5. Evaluate each of these expressions.

a) $(1\ 1011 \oplus 1\ 1001) \oplus 1\ 1010$

b) $(1\ 0011 \vee 0\ 1000) \wedge (1\ 0001 \vee 1\ 1010)$

Sec 1.2

6. Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output

$(p \wedge \neg r) \wedge (\neg q \vee r)$ from input bits p, q, and r.

Sec 1.3

7. Use De Morgan's laws to find the negation of each of the following statements.

a) Kwame will take a job in industry or go to graduate school.

Given $p \vee q$

$\neg (p \vee q) = ?$

b) James is young and strong.

Given $p \wedge q$

$\neg (p \wedge q) = ?$

8. Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

| p | q | $p \rightarrow q$ | $(\neg p \wedge (p \rightarrow q))$ | $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ |
|---|---|-------------------|-------------------------------------|--|
| T | T | | | |
| T | F | | | |
| F | T | | | |
| F | F | | | |

9. Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

| p | q | r | $\neg p$ | $q \rightarrow r$ | $p \vee r$ | $\neg p \rightarrow (q \rightarrow r)$ | $q \rightarrow (p \vee r)$ |
|---|---|---|----------|-------------------|------------|--|----------------------------|
| T | T | T | | | | | |
| T | T | F | | | | | |
| T | F | T | | | | | |
| T | F | F | | | | | |
| F | T | T | | | | | |
| F | T | F | | | | | |
| F | F | T | | | | | |
| F | F | F | | | | | |

Sec 1.4

9. Let $Q(x)$ be the statement " $x + 10 > 2x$." If the domain consists of all integers, what are these truth values?

- a) $Q(5)$
- b) $\exists x Q(x)$
- c) $\forall x Q(x)$
- g) $\exists x \neg Q(x)$

10. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

- a) $\exists x(x^2 = 2)$
- b) $\exists x(x^3 = -1)$
- c) $\forall x(x^2 + 1 \geq 2)$
- d) $\exists x(x^2 \neq x)$
- e) $\forall x(x^2 > x)$

Sec 1.5

11. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

- a) $\forall x \exists y (x = y^2)$
- b) $\exists x \forall y (xy = 0)$
- c) $\exists x \exists y (x - y = y - x)$
- d) $\exists x \forall y (y = 0 \rightarrow xy = 1)$
- e) $\forall x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$
- f) $\forall x \forall y \exists z (z = (x + y) / 2)$