CPE111 Discrete Mathematics for Computer Engineers

International Program
Homework #2, due on August 24, 2022

Chapter 2

1. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

a) the set of people who speak English, the set of people who speak English with an Australian accent

b) the set of fruits, the set of citrus fruits

c) the set of students studying discrete mathematics, the set of students studying data structures

2. Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$

b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

c) $\{\emptyset\} \in \{\{\emptyset\}\}$

d) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

3. What is the cardinality of each of these sets?

a) $\emptyset$

b) $\{\emptyset\}$

c) $\{\emptyset, \{\emptyset\}\}$

d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

4. Let $A = \{a, b, c, e, i, j\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

a) $A \cup B$

b) $A \cap B$

c) $A - B$

d) $B – A$

5. Let A, B, and C be sets. Use a Venn diagram or a truth table to show that

a) $(A - B) - C \subseteq A - C$

b) $(B \cup C) - A = (B - A) \cup (C - A)$

6. Show that $A \oplus B = (B - A) \cup (A - B)$

7. Determine whether f is a function from **Z** to **R** if

a) $f(n) = \pm n$

b) $f(n) = \sqrt{n^2 + 1}$

c) $f(n) = 1 / (n^2 + 4)$

d) $f(n) = 1 / (n^3 - 1)$

8. Find these values

a) $\lfloor -1.1 \rfloor$

d) $\lceil -5.8 \rceil$

f) $\lceil -2.99 \rceil$

h) $\left\lceil \left\lfloor \frac{3}{2} \right\rfloor + \left\lceil \frac{5}{3} \right\rceil + \frac{1}{2} \right\rceil$

9. Determine whether each of these functions from **Z** to **Z** is one-to-one.

a) $f(n) = n - 1$

b) $f(n) = n^2 + 1$

c) $f(n) = n^3$

d) $f(n) = \lceil n/2 \rceil$

10. Consider these functions from the set of students in a discrete mathematics class. Under what conditions is the function one-to-one if it assigns to a student.

a) mobile phone number

b) student identification number

c) final grade in the class

d) home town

11. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from **R** to **R**.

12. Find the first four terms of the sequence defined by each of these recurrence relations and initial conditions.

a) $a_n = -3a_{n-1}$, $a_0 = -1$

b) $a_n = na_{n-1} + a^2_{n-2}$, $a_0 = -1$, $a_1 = 0$

c) $a_n = a_{n-1} + a_{n-2} + a_{n-3}$, $a_0 = 1$, $a_1 = 1$, $a_2 = 2$

13. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if
a) $a_n = 0$

b) $a_n = 1$

c) $a_n = (-4)^n$

d) $a_n = 2(-4)^n + 3$

14. Find the value of each of these sums

a) $\sum_{j=0}^{5}(1+(-1)^j)$

b) $\sum_{j=0}^{6}(3^j - 2^j)$

15. Find $\prod_{j=0}^{4}(j! + 2)$

16. Let **A** = $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$
a) Find $\mathbf{A}^2$

b) Find $\mathbf{A}^3$

17. Let A = $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find

a) A∨ B

b) A∧ B

c) A⊙B

18. Find the Boolean product of A and B, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$