Mastermind

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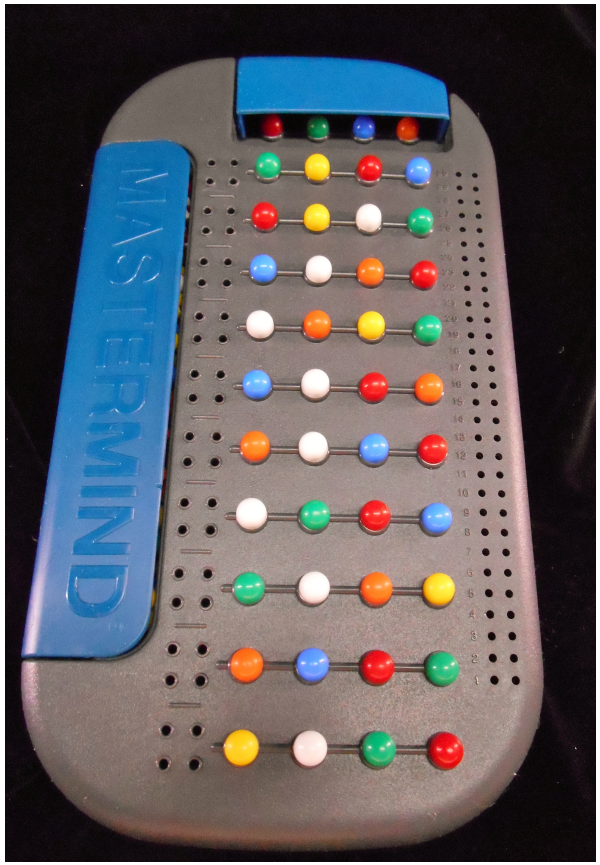
Mastermind

i. Codemaker vs. Codebreaker

ii. Queries: Guess a vector from $[k]^n$. Get two-color feedback.



Knuth Paper – 1976

i. Original Mastermind: Four spots and six colors
ii. Deterministic strategy, wins in at most five turns
iii. Minimax algorithm

Generalizations

i. Basic Extension: n spots, k colors

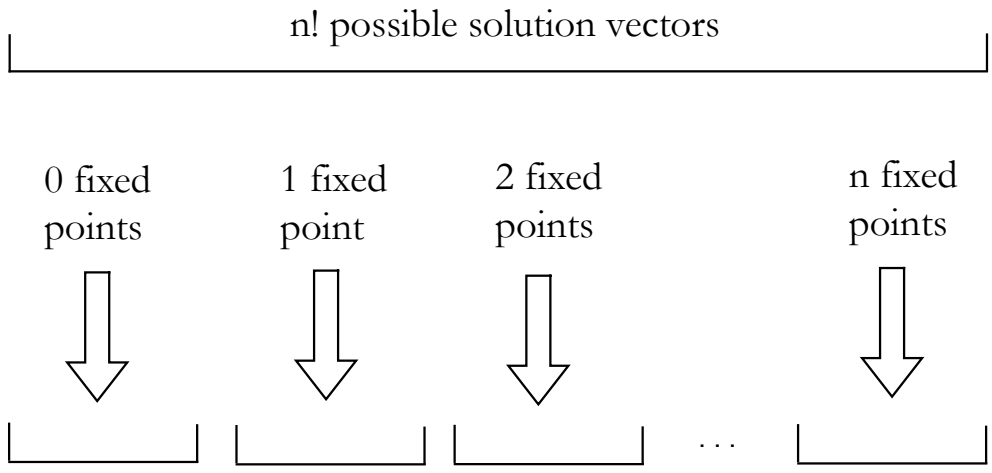
ii. Repeats vs. no repeats

iii. Non-adaptive vs. adaptive strategies

Known Bounds

1. Lower Bounds: Try to beat $O(n)$.
2. Upper Bounds: Try to achieve/beat $O(n \log k)$

Deriving $n - \log\log n$ Lower Bound



i. Uses more information than previous proofs, using "buckets."

ii. Bucket $i \Longleftrightarrow i$ fixed points

iii. Size of buckets:

$$|B_i| = \binom{n}{i} \cdot D(n-i)$$

$$= \binom{n}{i} \cdot (n-i)! \cdot \sum_{j=0}^{n-i} \frac{(-1)^j}{j!}$$

Deriving $n \log \log n$ Upper Bound when $k = n$

[Doerr et. al., 2013]

i. Split hidden vector into "coins" (subvectors).

ii. Use coin weighing problem to eliminate colors.

Basics of Entropy

Definition: Let X be a random variable with domain D.

$$H(X) = -\sum_{x \in D} \mathbb{P}[X = x] \cdot \log_2 \left(\mathbb{P}[X = x]\right)$$

Key Properties:

i. If random variables X, Y uniquely determine each other's outcomes, then $H(X) = H(Y)$.

ii. *Subadditivity:* A vector $X = (X_1, X_2, \ldots, X_n)$ of random variables has

$$H(X) \leq \sum_{i=1}^{n} H(X_i)$$

Use these properties to give lower bound on non-adaptive strategies.

Non-Adaptive Strategies

i. Random Guessing

ii. Evaluate probability of success