Dynamic Demand II: Durable Goods

C.Conlon

Monday 10^{th} November, 2025

Grad IO

Today's Readings

- Melnikov (Yale PhD Thesis 2001)
- Gowrisankaran Rysman (JPE)
- Hendel and Nevo (Econometrica)

We can formally write down a dynamic programming problem that consumers solve:

$$
\begin{aligned}
V_i(f_{i0t}, \varepsilon_{it}, \Omega_t) \quad = \quad & \text{max}\{f_{i0t} + \beta \mathbb{E}_{\Omega}[\mathbb{E}_{\varepsilon} V_i(f_{i0t}, \varepsilon_{it}, \Omega_{t+1})|\Omega_t], \\
& \max_j f_{ijt} - \alpha_i p_{jt} + \beta \mathbb{E}_{\Omega}[\mathbb{E}_{\varepsilon} V_i(f_{ijt}, \varepsilon_{it}, \Omega_{t+1})|\Omega_t]\}
\end{aligned}
$$

For a dynamic model to make sense we may want to place some restrictions:

- Rational Expectations
- Dynamic Consistency
- Law of motion for consumer types: $w_{i,t+1} = h(w_{i,t}, s_{ijt})$

This Bellman has defined a *Replacement Problem*.

- You own a single durable good with the option to *upgrade* each period.
- When you upgrade you throw away the old durable and get nothing in exchange.
- After a purchase j you receive flow utility $f_{i0t+1} = f_{ijt}$ each period if you don't make a new purchase.
- We could add in depreciation or probabilistic failure if we wanted to.
- No resale market (reasonable for high-tech).

Inclusive Value

Helpful to write: $EV_i(\Omega_t) = \int V_i(\varepsilon_{it}, \Omega_t) f(\varepsilon)$ Rust's Trick

$$\begin{aligned} V_i(f_{i0t}, \varepsilon_{it}, \Omega_t) &= \max\{f_{i0t} + \beta \mathbb{E}_\Omega [EV_i(f_{i0t}, \Omega_{t+1})|\Omega_t] + \varepsilon_{i0t}, \\ & \max_j f_{ijt} - \alpha_i p_{jt} + \beta \mathbb{E}_\Omega [EV_i(f_{ijt}, \Omega_{t+1})|\Omega_t] + \varepsilon_{ijt}\} \end{aligned}$$

We can write the ex-ante expected utility of purchasing in period t without having to condition on which good you purchase:

$$\begin{aligned} \delta_i(\Omega_t) &= \mathbb{E}_\varepsilon [\max_j f_{ijt} - \alpha_i p_{jt} + \beta \mathbb{E}_\Omega [EV_i(f_{ijt}, \Omega_{t+1})|\Omega_t] + \varepsilon_{ijt}] \\ &= \log \left(\sum_j \exp[f_{ijt} - \alpha_i p_{jt} + \beta \mathbb{E}_\Omega [EV_i(f_{ijt}, \Omega_{t+1})|\Omega_t]] \right) \end{aligned}$$

$$EV_i(f_{i0}, \Omega) = \log\left(\exp[f_{i0} + \beta \mathbb{E}_{\Omega'}[EV_i(f_{i0}, \Omega')|\Omega]] + \exp(\delta_i(\Omega))\right) + \eta$$

where $\eta = 0.577215665$ (Euler's Constant).

The fact that the expected value function depends recursively on itself and $\delta_i(\Omega_t)$ (Inclusive Value) leads to the following assumption.

Inclusive Value Sufficiency

If $\delta_i(\Omega) = \delta_i(\tilde{\Omega})$ then $g(\delta_i(\Omega')|\Omega) = g(\delta_i(\tilde{\Omega}')|\tilde{\Omega})$ for all $\Omega, \tilde{\Omega}$.

- The idea is that δ tells me everything about the future evolution of the states
- More restrictive than it looks. δ is low because quality is low? or because prices are high? Is this the result of a dynamic pricing equilibrium? (No!)

Inclusive Value Sufficiency

Under IVS the problem reduces to

$$\begin{aligned} EV_i(f_{i0}, \delta_i) &= \log\left[\exp(f_{i0} + \beta \mathbb{E}_{\Omega'}[EV_i(f_{i0}, \delta_i')|\delta_i]) + \exp(\delta_i)\right] \\ \delta_i &= \log\left(\sum_j \exp[f_{ijt} - \alpha_i p_{jt} + \beta \mathbb{E}_{\delta'}[EV_i(f_{ijt}, \delta_i')|\delta_i]\right) \end{aligned}$$

The idea is that the inclusive value δ_{it} IS the state space, along with his current holding of the durable f_{i0t}.

We still have the expectation to deal with:

$$\mathbb{E}_{\delta'}[EV_i(f_{ijt}, \delta_i')|\delta_i]$$

We need to take a stand on $g_i(\delta_i'|\delta_i)$ the anticipated law of motion for δ_i. G&R assume it follows an $AR(1)$ process.

$$\delta_{it+1} = \gamma_0 + \gamma_1 \delta_{it} + \nu_{it} \text{ with } \nu_{it} \sim N(0, \sigma_\nu^2)$$

If we see δ_{it} we could just run the $AR(1)$ regression to get consumer belief's $\hat{\gamma}$

I still haven't told you how to compute

$$\mathbb{E}_{\delta'}[EV_i(f_{ijt},\delta_i')|\delta_i,\gamma] = \int EV_i(f_{ijt},\delta_i')g(\delta'|\delta,\gamma)$$

1. We need to integrate $EV(f_{ijt},\delta_i)$ (a function) over a normal density.
2. But we don't observe $EV(f_{ijt},\delta_i)$ everywhere, only on the grid points of our state space.
3. We can fit a linear function, cubic spline, etc. over δ_i to EV_i at each value of f_{ijt} on our grid.
4. We need to interpolate $\widehat{EV}_i(\delta_i^s)$ (Linear, Cubic Spline, etc.)
5. We might as well interpolate the function at the *Gauss-Hermite* quadrature nodes and weights, recentered at $\gamma_0 + \gamma_1\delta$ in order to reduce the number of places we interpolate $\widehat{EV}_i$.

Rational Expectations-Alternative

There is an alternative method that is likely to be less accurate

$$\mathbb{E}_{\delta'}[EV_i(f_{ijt},\delta_i')|\delta_i,\gamma] = \int EV_i(f_{ijt},\delta_i')g(\delta'|\delta,\gamma)$$

1. We need to integrate $EV(f_{ijt},\delta_i)$ (a function) over a normal density but we only see it at the grid points of our state space.
2. We could discretize $g(\delta'|\delta,\gamma)$ so that it is a valid markov transition probability matrix (TPM) evaluated only at the grid points.
3. Now computing the expectation is just matrix multiplication.

I am a bit nervous about whether two discrete approximations will get the continuous integral correct.

The Estimation Problem

We need to solve $\forall i, t$:

$$
\begin{aligned}
S_{jt} &= \sum_i w_i s_{ijt}(f_{i0t}, \delta_{it}) \\
f_{ijt} &= \overline{\alpha} x_{jt} + \xi_{jt} + \sum_l \sigma_l x_{jl} \nu_{il} \\
s_{ijt}(f_{i0t}, \delta_{it}) &= \frac{\exp[f_{ijt} - \alpha_i p_{jt} + \beta \mathbb{E}_{\Omega'}[EV_i(f_{ijt}, \delta_i')|\delta_i]}{\exp[EV_i(f_{i0t}, \delta_{it})]} \\
EV_i(f_{i0}, \delta_i) &= \log\left[\exp(f_{i0} + \beta \mathbb{E}_{\Omega'}[EV_i(f_{i0}, \delta_i')|\delta_i]) + \exp(\delta_i)\right] \\
\delta_i(EV_i) &= \log\left(\sum_j \exp[f_{ijt} - \alpha_i p_{jt} + \beta \mathbb{E}_{\delta'}[EV_i(f_{ijt}, \delta_i')|\delta_i]\right) \\
\mathbb{E}[\delta_{it+1}|\delta_{it}] &= \gamma_0 + \gamma_1 \delta_{it} \\
w_{i,t+1} &= h(w_{i,t}, s_{ijt})
\end{aligned}
$$

1. Like BLP we guess the nonlinear parameters of the model θ
2. For a guess of the ξ_{jt}'s we can solve for EV_i by iteratively computing δ, and running the γ regression for each i and spline/interpolating to compute $E[EV_i]$. (Inner Loop)
3. G&R show how the contraction mapping of BLP can be modified to find a fixed point of the δ, ξ, γ relationship to find f_{ijt} (Middle Loop).
4. We need to make sure to update the $w_{i,t}$ via $h(\cdot)$. (This is a TPM that tells maps the transition probabilities of type i holding f_{i0t} to $f_{i0,t+1}$).
5. Once we've solved this whole system of equations, we use ξ to form moments just like BLP and do GMM. (Outer Loop)

Log LCD size	.003 (.002) *	.000 (.141)	-.073 (.093)	.004 (.005)
Media: DVD	.033 (.006) *	.004 (1.16)	.074 (.332)	.060 (.019) *
Media: tape	.012 (.005) *	-.005 (.683)	-.667 (.318) *	.015 (.018)
Media: HD	.036 (.009) *	-.002 (1.55)	-.647 (.420)	.057 (.022) *
Lamp	.005 (.002) *	-.001 (.229)	-.219 (.061) *	.002 (.003)
Night shot	.003 (.001) *	.004 (.074)	.430 (.060) *	.015 (.004) *
Photo capable	-.007 (.002) *	-.002 (.143)	-.171 (.173)	-.010 (.006)
Standard deviation coefficients ($\Sigma^{1/2}$)				
Constant	.079 (.021) *	.038 (1.06)	.001 (1147)	.087 (.038) *
Log price	.345 (.115) *	.001 (1.94)	-.001 (427)	.820 (.084) *

Standard errors in parentheses; statistical significance at 5% level indicated with *. All models include brand dummies, with Sony excluded. There are 4436 observations.

G&R Robustness

	products		coefficients			
	(1)	(2)	(3)	(4)	(5)	(6)
Mean coefficients (α)						
Constant	-.098 (.026) *	-.129 (.108)	-.103 (.037) *	-.170 (.149)	-6.61 (.815) *	-.114 (.024) *
Log price	-3.31 (1.04) *	-2.53 (.940) *	-3.01 (.717) *	-6.94 (.822) *	-.189 (.079) *	-3.06 (.678) *
Log size	-.007 (.001) *	-.006 (.001) *	-.015 (.007) *	.057 (.008) *	-.175 (.049) *	-.007 (.001) *
Log pixel	.010 (.003) *	.008 (.001) *	.009 (.002) *	.037 (.012) *	-.288 (.053) *	.010 (.002) *
Log zoom	.005 (.002) *	.004 (.002) *	.004 (.002)	-.117 (.012) *	.609 (.074) *	.005 (.002)*
Log LCD size	.004 (.002) *	.004 (.001) *	.004 (.002) *	.098 (.010) *	-.064 (.088)	.003 (.001) *
Media: DVD	.033 (.006) *	.025 (.004) *	.044 (.018) *	.211 (.053) *	.147 (.332)	.031 (.005) *
Media: tape	.013 (.005) *	.010 (.004) *	.024 (.016)	.200 (.051) *	-.632 (.318) *	.012 (.004) *
Media: HD	.036 (.009) *	.026 (.005) *	.047 (.019) *	.349 (.063) *	-.545 (.419)	.034 (.007) *
Lamp	.005 (.002) *	.003 (.001) *	.005 (.002) *	.077 (.011) *	-.200 (.058) *	.004 (.001) *
Night shot	.003 (.001) *	.004 (.001) *	.003 (.001) *	-.062 (.008) *	.427 (.058) *	.003 (.001) *
Photo capable	-.007 (.002) *	-.005 (.002) *	-.007 (.002) *	-.061 (.019) *	-.189 (.142)	-.007 (.008)
Standard deviation coefficients ($\Sigma^{1/2}$)						
Constant	.085 (.019) *	.130 (.098)	.081 (.025) *	.022 (.004) *		.087 (.013) *
Log price	.349 (.108) *	2.41e-9 (.919)	1.06e-7 (.522)	1.68 (.319) *		.287 (.078) *
Log size			-.011 (.007)			
Log pixel			1.58e-10 (.002)			

Standard errors in parentheses; statistical significance at 5% level indicated with *. All models include brand dummies, with Sony excluded. There are 4436 observations, except in the yearly model, in which there are 505.

Results

- Contrary to the static model, price coefficient is negative (as one would expect).
- Coefficients on many product characteristics are intuitively appealing.
- Allowing for repeated purchases generates more “sensible” results.
- “Better results” from a dynamic model may be due to the fact that people wait to purchase because of the expectations of price declines and not directly because of high prices.
- Unlike the static model, in dynamic setup the explanation of waiting does not conflict with consumers buying relatively high-priced products.
- A variety of robustness measures show that the major simplifying assumptions about the dynamics in the model are broadly consistent with the data.

Perfect Foresight

G&R Report similar elasticities in the perfect foresight case. We make the following simplification

$$\mathbb{E}_{\Omega'}[EV_i(f_{i0}, \delta_{i,t+1})|\delta_{i,t}])] = EV_i(f_{i0}, \delta_{i,t+1})$$

This saves us a lot of headaches:

- No more integration/interpolation
- We can solve the problem on the grid!
- No more belief regressions

Recall our objective:

- Plug in an unbiased estimate for the “no-purchase” utility.
- Under perfect foresight this is just the inclusive value of tomorrow’s market $\delta_{i,t+1}$ appropriately discounted: $\sum_{k=1}^{T-k} \beta^{t+k} \delta_{i,t+k}$.
- Different ways to think about rational expectations
  - Expectational error of some or all of $\delta_{i,t+k}$’s.
  - Expectational error in today’s reservation utility.

Endogeneity and Instruments

- Dynamics mean we lean harder on the assumption of exogenous product characteristics
- In one period we can take characteristics as given, but in many periods this becomes less palatable (Do cameras exogenously improve over time?).
- Endogeneity: price is endogenous while other product characteristics are not, i.e. x_{jt}. (Size,Resolution, etc.)
- Price is chosen by the firms possibly after observing ξ_{jt} and, hence, is endogenous.
- Instruments: use variables that affect the price-cost margin, e.g. measures of how crowded a product is in characteristics space, which effects price-cost margin and the substitutability across products.
    1. all of the product characteristics in x;
    2. mean product characteristics for a given firm;
    3. mean product characteristics for all firms;
    4. the count of products offered by the firm and by all firms.
    5. changes in costs over time?