

Representative Consumer Models

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Grad IO

Let's start with the following as a benchmark:

- ▶ A **representative agent** demand system.
- ▶ The consumer chooses an **expenditure** level for each good and consumes at least a little of all goods.
- ▶ Which desirable properties?:
 - We want a fully flexible matrix of demand derivatives $\Delta(\mathbf{p})$.
 - Probably we want some flexibility so that $\Delta(\mathbf{p}) \neq \Delta(\mathbf{p}')$.
 - Would satisfy axioms of consumer theory (WARP, Slutsky Symmetry, etc.).

Brief Aside: Constant Elasticity Demand

One candidate from your first year course would be a **constant elasticity demand model**. Which we could micro-found with utility for consuming $q(\omega)$ for each of J goods:

$$U = \left(\int_0^J q(\omega)^\rho d\omega \right)^{\frac{1}{\rho}} \quad 0 \leq \rho \leq 1$$

We can solve Lagrangians and find (Frisch) demands:

$$q(\omega) = \left(\frac{\lambda p(\omega)}{\rho} \right)^{\frac{1}{\rho-1}}$$

With ratios:

$$\frac{q(\omega_1)}{q(\omega_2)} = \left(\frac{p(\omega_1)}{p(\omega_2)} \right)^{\frac{1}{\rho-1}}$$

Common substitution: $\sigma = \frac{1}{1-\rho}$. or $\rho = \frac{\sigma-1}{\sigma}$.

Brief Aside: Constant Elasticity Demand

Some CES algebra:

$$q(\omega_1) = q(\omega_2) \left(\frac{p(\omega_1)}{p(\omega_2)} \right)^{-\sigma}$$
$$\underbrace{\int_0^J p(\omega_1) q(\omega_1) d\omega_1}_{I \equiv \text{consumer income}} = \int_0^J q(\omega_2) p(\omega_1)^{1-\sigma} p(\omega_2)^\sigma d\omega_1$$

$$I = q(\omega_2) p(\omega_2)^\sigma \int_0^J p(\omega_1)^{1-\sigma} d\omega_1$$

Now we can solve for Marshallian Demand:

$$q(\omega_2) = \frac{I \cdot p(\omega_2)^{-\sigma}}{\underbrace{\int_0^J p(\omega_1)^{1-\sigma} d\omega_1}_{P^{1-\sigma}}} \quad \text{Where } P \text{ is the overall price index.}$$

Brief Aside: Constant Elasticity Demand

Using the overall price index $P = \left(\int_0^J p(\omega_1)^{1-\sigma} d\omega_1 \right)^{\frac{1}{\rho}}$, we can re-write Marshallian demand:

$$q(\omega) = p(\omega)^{-\sigma} P^{\sigma-1} I = \left(\frac{p(\omega)}{P} \right)^{-\sigma} \frac{I}{P}$$

We can establish the well-known **homotheticity** property of CES by plugging back into original equation for $U(\cdot)$ and noting that $e(P, u) = P \cdot u$.

$$\begin{aligned} U &= \left(\int_0^J q(\omega)^\rho d\omega \right)^{1/\rho} = \left(\int_0^J p(\omega)^{1-\sigma} I^\rho P^{(\sigma-1)\rho} d\omega \right)^{1/\rho} \\ &= IP^{\sigma-1} \left(\int_0^J p(\omega)^{1-\sigma} d\omega \right)^{\frac{\sigma}{\sigma-1}} = IP^{\sigma-1} P^{-\sigma} = \frac{I}{P}. \end{aligned}$$

- Utility is just income divided by the price index!
- Homothetic because consumption of all goods scales with I .

Brief Aside: Constant Elasticity Demand

Demand (and its derivative) for a single good:

$$\begin{aligned}q(p) &= p^{-\sigma} P^{\sigma-1} I \\ \frac{\partial q}{\partial p} &= -\sigma p^{-\sigma-1} P^{\sigma-1} I \\ \frac{-q}{\frac{\partial q}{\partial p}} &= \frac{p}{\sigma}\end{aligned}$$

So that monopoly markup becomes $p = \frac{mc}{\rho}$

- ▶ CES means one markup (and elasticity) for all goods.
- ▶ Hard to do IO here. Not so helpful in understanding strategic price setting behavior!
- ▶ Better left for Trade and Macro economists.

Almost Ideal Demand System: Deaton & Muellbauer (1980)

Recall our desirable properties:

- ▶ We want a fully flexible matrix of demand derivatives $\Delta(\mathbf{p})$.
- ▶ Probably we want some flexibility so that $\Delta(\mathbf{p}) \neq \Delta(\mathbf{p}')$.
- ▶ Would satisfy axioms of consumer theory (WARP, Slutsky Symmetry, etc.).
- ▶ Key ideas: **separable preferences** and **multi-stage budgeting**.
 - Allocating expenditures within a group: Index can be calculated without knowing what you choose within the group.
 - Other products respond only to the *index price* not to individual prices!

Begin by defining an expenditure function:

$$\log e(u, \mathbf{p}) = (1 - u) \log \underbrace{a(\mathbf{p})}_{\text{subsistence}} + u \cdot \log \underbrace{b(\mathbf{p})}_{\text{bliss}}$$

We assume a particular functional form for $a(\mathbf{p})$, $b(\mathbf{p})$ that is second-order flexible.

Almost Ideal Demand System: Deaton & Muellbauer (1980)

Here is the form of the expenditure function:

$$\log e(u, \mathbf{p}) = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{kj}^* \log p_k \log p_j + u \beta_0 \prod_k p_k^{\beta_k}$$

- ▶ Estimate $(\alpha_i, \beta_i, \gamma_{ij}^*)$ from data.
- ▶ We usually require $\sum_i \alpha_i = 1, \sum_k \gamma_{jk}^* = \sum_j \beta_j = 0$ so that demand is linearly homogenous in \mathbf{p} .
- ▶ Also often impose that $\gamma_{jk}^* = \gamma_{kj}^*$.
 - Sometimes we impose this *ex-ante*, other times we test for it *ex post*.
- ▶ We can also see that we have at least one parameter for each of the first two own and cross price derivatives of $e(\cdot)$.

Almost Ideal Demand System: Deaton & Muellbauer (1980)

After applying Shepard's Lemma and logarithmic differentiation, we can obtain the expenditure share for good i :

$$\begin{aligned}w_i &= \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i u \beta_0 \prod_k p_k^{\beta_k} \quad \text{with} \quad \gamma_{ij} = \frac{1}{2}(\gamma_{ij}^* + \gamma_{ji}^*) \\ &= \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(x/P)\end{aligned}$$

- ▶ x represents total expenditure within group, P is the price index for the group.
- ▶ Two price indices are commonly used ("Exact" and Stone 1954's linear approximate index):

$$\begin{aligned}\log P &= \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_j \sum_k \gamma_{kj} \log p_k \log p_j \\ \log P &= \sum_k w_k \log p_k\end{aligned}$$

- ▶ AIDS seemed like a better name in 1980 than it does today!
- ▶ Gets used often in international trade or macro-consumption literature.
 - Product categories are often: durables, non-durables, housing, utilities, etc. from CEX data.
- ▶ Can use it for IO purposes (each “group” contains a single product).
- ▶ If p_k changes demand for good j (it does!) then we need an instrument for every price!
- ▶ We still have J^2 possible elasticities or $J \times (J + 1)/2$.
 - Can simplify with multi-stage budgeting. (but we have to know what segments are)
 - Massive data requirements: $J = 45$ in a vending machine means we need over 2000 observations.

Beer Example: Hausman, Leonard, Zona (1994)

Goals:

- Estimate demand for beer in the US.
- Analyze a merger, test assumptions about firm conduct

Three stages:

1. Brand-Level (AIDS): 5 brands per segment.

$$\underbrace{w_i}_{\text{brand expenditure share}} = \alpha_i + \sum_j \alpha_{ij} \log p_j + \beta_i \log \left(\frac{x}{P} \right) + \varepsilon_1$$

2. Segment-Level (log-log): Premium, Light, Popular.

$$\underbrace{\log q_m}_{\text{seg. quantity}} = \beta_m \underbrace{\log y_B}_{\text{beer expenditure}} + \sum_k \sigma_k \log \underbrace{\pi_k}_{\text{segment price index}} + \alpha_m + \varepsilon_2$$

3. Top: Beer Expenditure vs. other goods (wine, spirits, hamburgers).

$$\log e_t = \beta_0 + \beta_1 \underbrace{\log y_t} + \beta_2 \underbrace{\log P_b} + Z_t \delta + \varepsilon_3$$

Identification: Hausman, et.al (1994)

- ▶ Price is correlated with both **unobserved product quality** and **unobserved demand shocks**.
- ▶ Finding brand level instruments is the challenge.
- ▶ The famous **Hausman instrument**: use prices in one city to instrument for prices in another

$$\log p_{jnt} = \delta_j \log c_{jt} + \alpha_{jn} + \omega_{jnt}$$

- ▶ Instruments tend to be **strong** but **exclusion** can be questionable.
- ▶ Key is that ω_{jnt} are independent of each other (is this believable?).
 - People mostly complain about national ad campaigns (this is beer after all!)
- ▶ What about other instruments? (Input prices, taxes, etc.).
- ▶ Specification Test: brand price in other segments should not have an effect controlling for the price index of other segments.

TABLE 1

Beer Segment Conditional Demand Equations.

	Premium	Popular	Light
Constant	0.501 (0.283)	-4.021 (0.560)	-1.183 (0.377)
log (Beer Exp)	0.978 (0.011)	0.943 (0.022)	1.067 (0.015)
log (P _{PREMIUM})	-2.671 (0.123)	2.704 (0.244)	0.424 (0.166)
log (P _{POPULAR})	0.510 (0.097)	-2.707 (0.193)	0.747 (0.127)
log (P _{LIGHT})	0.701 (0.070)	0.518 (0.140)	-2.424 (0.092)
Time	-0.001 (0.000)	-0.000 (0.001)	0.002 (0.000)
log (# of Stores)	-0.035 (0.016)	0.253 (0.034)	-0.176 (0.023)

Number of Observations = 101.

TABLE 2

Brand Share Equations: Premium.

	1 Budweiser	2 Molson	3 Labatts	4 Miller	5 Coors
Constant	0.393 (0.062)	0.377 (0.078)	0.230 (0.056)	-0.104 (0.031)	-
Time	0.001 (0.000)	-0.000 (0.000)	0.001 (0.000)	0.000 (0.000)	-
log (Y/P)	-0.004 (0.006)	-0.011 (0.007)	-0.006 (0.005)	0.017 (0.003)	-
log (P _{Budweiser})	-0.936 (0.041)	0.372 (0.231)	0.243 (0.034)	0.150 (0.018)	-
log (P _{Molson})	0.372 (0.231)	-0.804 (0.031)	0.183 (0.022)	0.130 (0.012)	-
log (P _{Labatts})	0.243 (0.034)	0.183 (0.022)	-0.588 (0.044)	0.028 (0.019)	-
log (P _{Miller})	0.150 (0.018)	0.130 (0.012)	0.028 (0.019)	-0.377 (0.017)	-
log (# of Stores)	-0.010 (0.009)	0.005 (0.012)	-0.036 (0.008)	0.022 (0.005)	-
Conditional Own	-3.527	-5.049	-4.277	-4.201	-4.641
Price Elasticity	(0.113)	(0.152)	(0.245)	(0.147)	(0.203)

$$\Sigma = \begin{Bmatrix} 0.000359 & -1.436E-05 & -0.000158 & -2.402E-05 \\ - & 0.000109 & -6.246E-05 & -1.847E-05 \\ - & - & 0.005487 & -0.000392 \\ - & - & - & 0.000492 \end{Bmatrix}$$

Note: Symmetry imposed during estimation.

TABLE 3

Brand Share Equations: Popular Price.

	1 Old Milwaukee	2 Genesee	3 Milwaukee's Best	4 Busch	5 Piels Lager
Constant	0.287 (0.062)	0.225 (0.067)	-0.019 (0.063)	0.531 (0.079)	-
Time	-0.000 (0.000)	-0.001 (0.000)	0.000 (0.000)	0.001 (0.000)	-
log (Y/P)	0.014 (0.006)	-0.018 (0.007)	0.001 (0.007)	0.004 (0.008)	-
log (P _{Old Milwaukee})	-0.979 (0.028)	0.235 (0.021)	0.369 (0.022)	0.257 (0.030)	-
log (P _{Genesee})	0.235 (0.021)	-0.698 (0.029)	0.222 (0.022)	0.205 (0.030)	-
log (P _{Milwaukee's Best}) . . .	0.369 (0.022)	0.222 (0.022)	-1.048 (0.036)	0.388 (0.035)	-
log (P _{Busch})	0.257 (0.030)	0.205 (0.030)	0.388 (0.035)	-0.892 (0.062)	-
log (# of Stores)	-0.044 (0.010)	0.122 (0.011)	-0.023 (0.010)	-0.091 (0.012)	-
Conditional Own	-4.789	-3.832	-5.813	-5.704	-3.956
Price Elasticity	(0.109)	(0.120)	(0.164)	(0.329)	(0.465)

$$\Sigma = \begin{Bmatrix} 0.000603 & -0.000123 & -0.000137 & -8.289E-05 \\ - & 0.000515 & -0.000143 & -7.136E-05 \\ - & - & 0.000758 & -0.000109 \\ - & - & - & 0.00262 \end{Bmatrix}$$

Note: Symmetry imposed during estimation.

TABLE 5

Overall Elasticities.

	Elasticity	Standard Error
Budweiser	-4.196	0.127
Molson	-5.390	0.154
Labatts	-4.592	0.247
Miller	-4.446	0.149
Coors	-4.897	0.205
Old Milwaukee	-5.277	0.118
Genesee	-4.236	0.129
Milwaukee's Best	-6.205	0.170
Busch	-6.051	0.332
Piels	-4.117	0.469
Genesee Light	-3.763	0.072
Coors Light	-4.598	0.115
Old Milwaukee Light	-6.097	0.140
Lite	-5.039	0.141
Molson Light	-5.841	0.148

Light Segment Own and Cross Elasticities.

	Genesee Light	Coors Light	Old Milwaukee Light	Lite	Molson Light
Genesee Light	-3.763 (0.072)	0.464 (0.060)	0.397 (0.039)	0.254 (0.043)	0.201 (0.037)
Coors Light	0.569 (0.085)	-4.598 (0.115)	0.407 (0.058)	0.452 (0.075)	0.482 (0.061)
Old Milwaukee Light . .	1.233 (0.121)	0.956 (0.132)	-6.097 (0.140)	0.841 (0.112)	0.565 (0.087)
Lite	0.509 (0.095)	0.737 (0.122)	0.587 (0.079)	-5.039 (0.141)	0.577 (0.083)
Molson Light	0.683 (0.124)	1.213 (0.149)	0.611 (0.093)	0.893 (0.125)	-5.841 (0.148)

- ▶ Relatively large own and cross price elasticities.
- ▶ Authors simulated **partial merger analysis**.
 - Hold prices of all non-merging parties fixed.
 - Solving for best-response of single-product.
 - How would full equilibrium analysis differ?
- ▶ Merger of Coors and Labatt's: Coors Markup 19.9% \rightarrow 23.2% (small).
- ▶ Claim is that presence of other competitors constraints potential to raise prices. How? Why?

Hausman (1997) aka *The Apple Cinnamon Cheerios War*.

- ▶ What is the value of a new good? How should we adjust CPI?
- ▶ Potentially HUGE issue. Why?
- ▶ Weekly cereal data.. 7 cities, 137 weeks. Three segments (adults, kids, family) with max 9 brands.
- ▶ Calculate $e(p_{-n}, p_n^*, u) / e(\mathbf{p}, u)$. Find a **virtual price** p^* (or choke price) that leaves consumers as well off as a world without Apple-Cinnamon Cheerios.
- ▶ Virtual price is about $2\times$ actual price. CPI may be overstated by as much as 25% for all cereal brands (tons of new products).

Chaudhuri, Goldberg, Jia (AER 2006)

- ▶ Indian market for antibiotics: (foreign vs. domestic) (licensed vs. unlicensed producers).
- ▶ Different brands, packages, etc. also different active ingredients ($J = 300$ they aggregate to four active ingredients \times country of origin).
- ▶ Monthly sales data (SKU level) for 4 regions in India (Market Research firm).
- ▶ What would prices and quantities look like if intellectual property rights were enforced and unlicensed producers were shut down?

Issues

- ▶ Products enter and exit the market. How do we model this?
- ▶ Dosages differ across products. How do we construct Q ?
- ▶ Don't treat licensed v. unlicensed as different products. Why?

Results

- ▶ Estimate AIDS demand aggregated across demands
- ▶ Get upper and lower bounds on marginal costs
 - Assume that $p = mc$
 - Assume monopoly pricing.
- ▶ Calculate the **virtual price** or “choke price” that makes expenditures zero on unlicensed products.
- ▶ Get changes in consumer surplus (integrated demand curve) and producer profits without unlicensed firms.