Homogenous Products

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Grad IO

One of the earliest exercises in econometrics is the estimation of supply and demand for a homogenous product

- According to Stock and Trebbi (2003) IV regression first appeared in a book by Phillip G. Wright in 1928 entitled The Tariff on Animal and Vegetable Oils [neatly tucked away in Appendix B: Supply and Demand for Butter and Flaxseed.]
- Lots of similar studies of simultaneity of supply + demand for similar agricultural products or commodities.

Supply and Demand For Coffee, everything is linear

$$\begin{aligned} Q_t^d &= \alpha_0 + \alpha_1 P_t + U_t \\ Q_t^s &= \beta_0 + \beta_1 P_t + V_t \\ Q_t^d &= Q_t^s \end{aligned}$$

Solving for P_t, Q_t:

$$\begin{aligned} P_t &= \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{V_t - U_t}{\alpha_1 - \beta_1} \\ Q_t &= \frac{\alpha_1\beta_0 - \alpha_0\beta_1}{\alpha_1 - \beta_1} + \frac{\alpha_1 V_t - \beta_1 U_t}{\alpha_1 - \beta_1} \end{aligned}$$

Price is a function of both error terms, and we can't use a clever substitution to cancel things out.

To make things really obvious:

$$Cov(P_t, U_t) = -\frac{Var(U_t)}{\alpha_1 - \beta_1}$$
$$Cov(P_t, V_t) = \frac{Var(V_t)}{\alpha_1 - \beta_1}$$

- When demand slopes down ($\alpha_1 < 0$) and supply slopes up ($\beta_1 > 0$) then price is positively correlated with demand shifter U_t and negatively correlated with supply shifter V_t.

Working (1927)

$$\begin{aligned} Cov(P_t, Q_t^d) &= \alpha_1 Var P_t + Cov(P_t, U_t) \\ Cov(P_t, Q_t^s) &= \beta_1 Var P_t + Cov(P_t, V_t) \end{aligned}$$

- Bias in OLS estimate (Demand) $Bias(\alpha_1) = \frac{Cov(P_t, U_t)}{Var P_t}$.
- Bias in OLS estimate (Supply) $Bias(\beta_1) = \frac{Cov(P_t, V_t)}{Var P_t}$.
- We can actually write both this way when $Cov(U_t, V_t) = 0$:

$$\text{OLS Estimate} = \frac{\alpha_1 Var(V_t) + \beta_1 Var(U_t)}{Var(V_t) + Var(U_t)}$$

- More variation in supply $V_t \rightarrow$ better estimate of demand.
- More variation in demand $U_t \rightarrow$ better estimate of supply.
- Led Working to say the statistical demand function (OLS) is not informative about the economic demand function (or supply function).

- For most of you, this was probably a review.
- We know what the solution is going to be to the simultaneity problem.
- We need an excluded instrument that shifts one curve without affecting the other.
- We can use this to form a 2SLS estimate.
- Instead let's look at something a little different...

Angrist, Imbens, and Graddy (ReStud 2000).

- Demand for Whiting (fish) at Fulton Fish Market
- Do not place functional form restrictions on demand (log-log, log-linear, linear, etc.).
- "What does linear IV regression of Q on P identify, even if the true (but unknown) demand function is nonlinear"
- Takes a program evaluation/treatment effects approach to understanding the "causal effect" of price on quantity demanded.
- Aside: Is there even such a thing as the causal effect of price on quantity demanded?

Ranked in increasing complexity

1. Linear system with constant coefficients

$$
\begin{aligned}
q_t^d(p,z,x) &= \alpha_0 + \alpha_1 p + \alpha_2 z + \alpha_3 x + \epsilon_t \\
q_t^s(p,z,x) &= \beta_0 + \beta_1 p + \beta_2 z + \beta_3 x + \eta_t
\end{aligned}
$$

2. Linear system with non-constant coefficients

$$
\begin{aligned}
q_t^d(p,z,x) &= \alpha_{0t} + \alpha_{1t} p + \alpha_{2t} z + \alpha_{3t} x + \epsilon_t \\
q_t^s(p,z,x) &= \beta_{0t} + \beta_{1t} p + \beta_{2t} z + \beta_{3t} x + \eta_t
\end{aligned}
$$

3. Nonlinear system with constant shape (separable)

$$
\begin{aligned}
q_t^d(p,z,x) &= q^d(p,z,x) + \epsilon_t \\
q_t^s(p,z,x) &= q^s(p,z,x) + \eta_t
\end{aligned}
$$

4. Nonlinear system with time-varying shape (non-separable)

$$
\begin{aligned}
q_t^d(p,z,x) &= q^d(p,z,x,\epsilon_t) \\
q_t^s(p,z,x) &= q^s(p,z,x,\eta_t)
\end{aligned}
$$

Two kinds:

1. Heterogeneity depending on value of p fixing t (only relevant in nonlinear models)
2. Heterogeneity across t, fixing p (cases 2 and 4).

- The problem is that we don't generality know which kind of heterogeneity we face.
- Is case (4) hopeless? Or what can we expect to learn?
- Even econometricians struggle with non-linear non-separable models (!)

AIG: Assumptions

Assume binary instrument $z_t \in \{0, 1\}$ to make things easier.

1. Regularity conditions on $q_t^d, q_t^s, p_t, z_t, w_t$ first and second moment and is stationary, etc.
    - $q_t^d(p, z, x)$, $q_t^s(p, z, x)$ are continuously differentiable in p.
2. z_t is a valid instrument in q_t^d
    - Exclusion: for all p, t

      $$q_t^d(p, z = 1, x_t) = q_t^d(p, z = 0, x_t) \equiv q_t^d(p, x_t)$$

      ie: conditioning on p_t means no dependence on z_t
    - Relevance: for some period t: $q_t^s(p_t, 1, x_t) \neq q_t^s(p_t, 0, x_t)$.
      ie: z_t actually shifts supply somewhere!
    - Independence: ϵ_t, η_t, z_t are mutually independent conditional on x_t.

Focus on the simple case:

- $z \in \{0, 1\}$ where 1 denotes "stormy at sea" and 0 denotes "calm at sea"
- Idea is that offshore weather makes fishing more difficult but doesn't change onshore demand.
- Ignore x (for now at least) or assume we condition on each value of x.

$$\hat{\alpha}_{1,0} \rightarrow^{p} \frac{\mathbb{E}[q_t|z_t = 1] - \mathbb{E}[q_t|z_t = 0]}{\mathbb{E}[p_t|z_t = 1] - \mathbb{E}[p_t|z_t = 0]} \equiv \alpha_{1,0}$$

- If we are in case (1) then we are good. In fact, any IV gives us a consistent estimate of α_1
- If we are in case (4) then $\alpha_{1,0}$ the object we recover, is not an estimator of a structural parameter.
- Moreover, this is at best a LATE, and thus it differs depending on which

- Should we divorce structural estimation from estimating “deep” population parameters (as suggested by Lucas critique)?
- Authors make the point that IV estimator identifies something about relationship between p and q, without identifying deep structural parameters?
- In IO this is a somewhat heretical idea (especially to start the course with).

In order to interpret the Wald estimator $\alpha_{1,0}$ we make some additional economic assumptions on the structure of the problem:

1. Observed price is market clearing price $q_t^d(p_t) = q_t^s(p_t, z_t)$ for all t. (This means no frictions!).
2. "Potential prices": for each value of z there is a unique market clearing price

$$\forall z, t : \tilde{p}(z,t) \text{ s.t. } q_t^d(\tilde{p}(z,t)) = q_t^s(\tilde{p}(z,t), z).$$

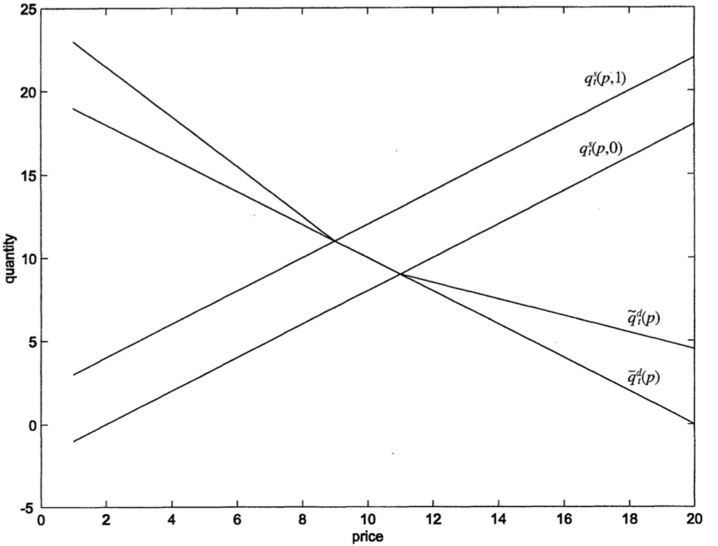
$\tilde{p}(z,t)$ is the potential price under any counterfactual (z,t)

- Just like in IV we need denominator to be nonzero so that $E[p_t|z_t = 1] \neq E[p_t|z_t = 0]$.
- Other key assumption is the familiar monotonicity assumption
  - $\tilde{p}(z, t)$ is weakly increasing in z.
  - Just like in program evaluation this is the key assumption. There it rules out "defiers" here it allows us to interpret the average slope as $\alpha_{1,0}$.
  - Assumption is untestable because you do not observe both potential outcomes $\tilde{p}(0, t)$ and $\tilde{p}(1, t)$ (same as in program evaluation).
  - Any story about IV is just a story! (Always the case!) unless we have repeated observations on the same individual.

The key result establishes that the numerator of $\alpha_{1,0}$:

$$\mathbb{E}[q_t|z_t = 1] - \mathbb{E}[q_t|z_t = 0] = \mathbb{E}_t\left[\int_{\tilde{p}(0,t)}^{\tilde{p}(1,t)} \frac{\partial q_t^d(s)}{\partial s} d\, s\right]$$

- For each t we average over the slope of demand curve among the two potential prices: $\int_{\tilde{p}(0,t)}^{\tilde{p}(1,t)} \frac{\partial q_t^d(s)}{\partial s} d\, s$
- This range could differ for each t.
- Then we average this average over all t.



25
20
15
10
5
0
-5
quantity
0
2
4
6
8
10
12
14
16
18
20
price
$q_i^s(p,1)$
$q_i^s(p,0)$
$\tilde{q}_i^d(p)$
$\bar{q}_i^d(p)$

FIGURE 1

AGI: Takeaways

What did we learn?

- $\alpha_{1,0}$ only provides information about demand curve in range of potential price variation induced by the instrument.
- Don't know anything about demand curve outside this range!
- For different instruments z, $\alpha_{1,0}$ has a different interpretation like the LATE does. (Different from the linear model where anything works!).
- This is a bit weird: different cost shocks could trace out different paths along the demand curve– why do we care if price change came from a tax change or an input price change? Are they tracing out different subpopulations?
- We need monotonicity so that we know the range of integration $\tilde{p}(0,t) \to \tilde{p}(1,t)$ instead of $\tilde{p}(1,t) \to \tilde{p}(0,t)$
- Observations where $\tilde{p}(0,t) = \tilde{p}(1,t)$ don't factor into the average but we don't know what these observations are because potential prices are unobserved! What is the relevant sub-sample?

$$
\begin{aligned}
\alpha_{1,0} &= \frac{\mathbb{E}[\int_{\tilde{p}(0,t)}^{\tilde{p}(1,t)} \frac{\partial q_t^d(s)}{\partial s} d\, s]}{\mathbb{E}\tilde{p}(1,t) - \mathbb{E}\tilde{p}(0,t)} \\
&\to \int_0^\infty E\left[\frac{\partial q_t^d(s)}{\partial s} | s \in [\tilde{p}(0,t), \tilde{p}(1,t)]\right] \omega(s) ds
\end{aligned}
$$

- given t average the slope of q_t^d from $\tilde{p}(0,t)$ to $\tilde{p}(1,t)$
- given price $s \in [\tilde{p}(0,t), \tilde{p}(1,t)]$ average $q_t^d(s)$ across t. (randomness is due to ϵ_t).
- Weight $\omega(s)$ is not a function of t but it is largest for prices most likely to fall between $\tilde{p}(0,t)$ and $\tilde{p}(1,t)$.
- Case (2): $q_t^d(p) = \alpha_{0t} + \alpha_{1t} p + \epsilon_t$.

$$
\alpha_{1,0} = \frac{\mathbb{E}[\alpha_{1t}(\tilde{p}(1,t) - \tilde{p}(0,t))]}{\mathbb{E}\tilde{p}(1,t) - \mathbb{E}\tilde{p}(0,t)} \neq \mathbb{E}\alpha_{1,t}
$$

We need mean independence

- Suppose we had a continuous z instead, now we can do a full nonparametric IV estimator.

$$a(z) = \lim_{\nu \to 0} \frac{\mathbb{E}(q_t|z) - \mathbb{E}(q_t|z-\nu)}{\mathbb{E}(p_t|z) - \mathbb{E}(p_t|z-\nu)}$$

- Use a kernel to estimate $\widehat{q}|z$ and $\widehat{p}|z$

$$\alpha'(z) = \frac{\widehat{q}'(z)}{\widehat{p}'(z)} \approx \frac{\widehat{q}'(z+h) - \widehat{q}(z)}{\widehat{p}'(z+h) - \widehat{p}(z)}$$

- When you have a parametric model, you don't need these results because we can define whatever (nonlinear) parametric functional form we want.
- There we will focus on parsimonious and realistic parametric functional forms. (this is the rest of the course)
- If we don't have a parametric model, then these show us that linear IV estimators give us some average (a particular one!) of slopes.
- Caveat: this only works for a single product. In the multi-product case things are a lot more complicated
    - For multiproduct oligopoly it is much harder to satisfy the monotonicity condition. Why?