Conduct and Testing For Conduct

Chris Conlon

Wednesday 15th October, 2025

Grad IO

Conduct Testing in Industrial Organization

Foundational Empirical IO Question: How do we observe data on price and quantity and infer which model of firm behavior generated those outcomes?

- Early work: Porter (1983), Bresnahan (1982,1987)
- Subsequent work defined the “menu” approach: Nevo (1998, 2001), Villas-Boas (2007)
- Recent revival of “internalization” parameters: Miller and Weinberg (2017), Crawford, Lee, Whinston, and Yurukoglu (2017), Pakes (2017)
- Parallel work by: Duarte, Magnolfi, Sølvsten, Sullivan (2022) which test is best (RV). Magnolfi, Quint, Sullivan, Waldfogel (2022) Should we test or estimate?
- Applications of our test: Starc and Wollman (2022), Scuderi (2022), others?

Is conduct testable? Berry and Haile (2014): yes.

- Absent additional restrictions, we cannot generally look at data on (P, Q) and decide whether or not collusion is taking place.
  - You say we started colluding at date t, I say we received a correlated shock to mc.
- We can make progress in two ways: (1) parametric restrictions on marginal costs; (2) exclusion restrictions on supply.
  - Most of the literature focuses on (1) by assuming something like: $\ln mc_{jt} = x_{jt}\gamma_1 + w_{jt}\gamma_2 + \omega_{jt}$.
  - In principle (2) is possible if we have instruments that shift demand for products but not supply. (These are much easier to come up with than “supply shifters”). (This is the point Berry Haile 2014 make).

A famous plot (Bresnahan 87)

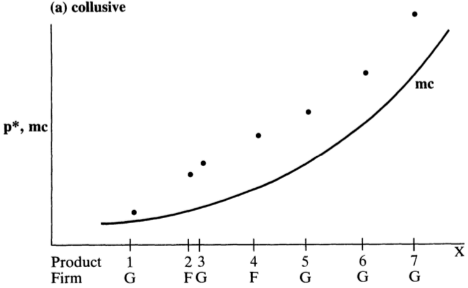


Figure 2(a)

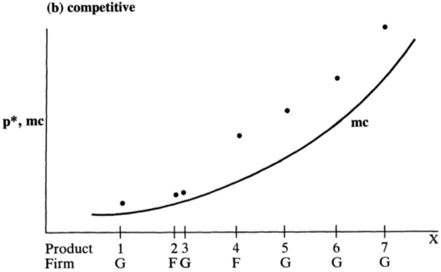


Figure 2(b)

Bresnahan (1980/1982) recognized this problem: we need “rotations of demand”.

Conduct Testing in Pictures (Berry Haile 2014)

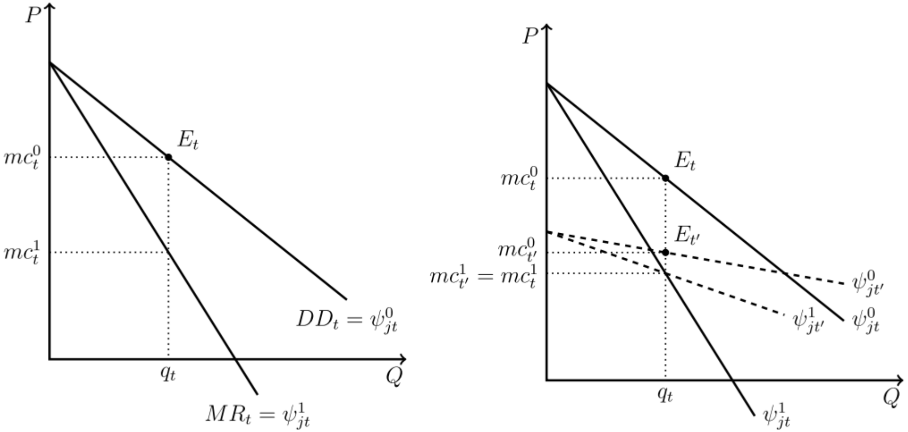


Figure 2(ab) from Berry and Haile (2014), Example 1.

We begin with a relatively standard BLP-style differentiated products setup.

- Markets t
- Products j
- Data $\chi_t = \{(\mathsf{x}_{jt}, \mathsf{v}_{jt}, \mathsf{w}_{jt})$ for all $j \in \mathcal{J}_t\}$.
- Market Shares $\mathcal{S}_{\mathbf{t}} = [s_{1t}, \ldots, s_{Jt}, s_{0t}]$.
- Prices $\mathbf{p}_t = [p_{1t}, \ldots, p_{Jt}]$.
- Consumers i with demographics y_{it} (income, presence of kids, etc.)

Testing Conduct: Multiproduct Bertrand Example

We generalize the $\mathcal{H}(\kappa)$ and derive multi-product Bertrand FOCs:

$$\arg\max_{p \in \mathcal{J}_f} \pi_f(\mathbf{p}) = \sum_{j \in \mathcal{J}_f} (p_j - c_j) \cdot q_j(\mathbf{p}) + \kappa_{fg} \sum_{j \in \mathcal{J}_g} (p_j - c_j) \cdot q_j(\mathbf{p})$$

$$\rightarrow 0 = q_j(\mathbf{p}) + \sum_{k \in (\mathcal{J}_f, \mathcal{J}_g)} \kappa_{fg} \cdot (p_k - c_k) \frac{\partial q_k}{\partial p_j}(\mathbf{p})$$

- Instead of 0's and 1's we now have $\kappa_{fg} \in [0, 1]$ representing how much firm f cares about the profits of g.
  - If f and g merge (or fully coordinated) then $\kappa_{fg} = 1$
  - Often in the real world firms cannot reach fully collusive profits and $\kappa_{fg} \in (0, 1)$.
  - Evidence that $\kappa_{fg} > 0$ is not necessarily evidence of malfeasance, just a deviation from static Bertrand pricing (e.g. Cournot, Dynamics, etc.)

- Recall the Δ matrix which we can write as $\Delta = \tilde{\Delta} \odot \mathcal{H}(\kappa)$, where $\odot$ is the element-wise or Hadamard product of two matrices.
  - $\tilde{\Delta}$ is the matrix of demand derivatives with $\Delta(j,k) = \frac{\partial q_j}{\partial p_k}$ for all elements.
  - $\mathcal{H}(\kappa) = \kappa_{fg}$ for products owned by (f,g) where $\kappa_{ff} = 1$ always.
- Mergers are about changing 0's to 1's in the $\mathcal{H}(\kappa)$ matrix.
- Matrix form of FOC: $q(\mathbf{p}) = \Delta(\mathbf{p},\kappa) \cdot (\mathbf{p} - \mathbf{mc})$
- $\mathbf{mc} = \mathbf{p} - \underbrace{\Delta(\mathbf{p},\theta_2,\kappa)^{-1} s(\mathbf{p})}_{\eta(\mathbf{p},\mathbf{s},\theta_2,\kappa)}$ where η_{jt} is the markup.

Reasons for Deviations from Multiproduct Bertrand

Biased estimates of own and cross price derivatives: For anything to work, you have correct estimates of $\tilde{\Delta}$. My prior is most papers underestimate diversion ratios for close substitutes.

Vertical Relationships: Who sets supermarket prices? Just the retailer? Just the manufacturer? Some combination of both? Retailers tend to soften downstream price competition.

Faulty Timing Assumptions: Bertrand is a simultaneous move pricing game. Lots of alternatives (Stackelberg leader-follower, Edgeworth cycles, etc.).

Dynamics and Dynamic Pricing: Forward looking firms or consumers might not set static Nash prices. [e.g. Temporary Sales, Switching Costs, Network Effects, etc.]

Unmodeled Supergame: Maybe firms are legally tacitly colluding, higher prices might be about what firms believe will happen in a price war.

Simultaneous Problem

Assume additivity, and write in terms of structural errors:

$$
\begin{aligned}
\delta_{jt}(\mathcal{S}_t, \mathsf{y}_t, \tilde{\theta}_2) + \alpha p_{jt} &= h_d(\mathsf{x}_{jt}, \mathsf{v}_{jt}, \theta_1) + \xi_{jt} \\
f\left(p_{jt} - \eta_{jt}(\mathbf{p}, \mathbf{s}, \theta_2, \kappa)\right) &= h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}, \theta_3) + \omega_{jt}
\end{aligned}
$$

- To simplify slides we let $f(x) = x$ (often $f(x) = \log(x)$) but we can put that in $h_s(\cdot)$.
- $h(\cdot)$ are often just linear relationships like: $\theta_1[\mathsf{x}_{jt}, \mathsf{v}_{jt}]$.
- Endogeneity Problem: p_{jt} and η_{jt} are functions of $(\boldsymbol{\xi}, \boldsymbol{\omega})$.
- (θ_2, κ) parameters that determine markups

Approach #1: Demand Side

1. Estimate θ_2 from demand alone.

$$\delta_{jt}(\mathcal{S}_t, \tilde{\theta}_2) + \alpha p_{jt} = h_d(\mathsf{x}_{jt}, \mathsf{v}_{jt}, \theta_1) + \xi_{jt}$$
$$E[\xi_{jt} | \mathsf{x}_t, \mathsf{v}_t, \mathsf{w}_t] = 0$$

2. Recover marginal costs $\widehat{\mathbf{mc}} = \mathbf{p} + \boldsymbol{\eta}$

$$\boldsymbol{\eta}(\mathbf{p}, \mathbf{s}, \theta_2, \kappa) \equiv \left(\mathcal{H}(\kappa) .* \tilde{\Delta}(\mathbf{p}, \theta_2)\right)^{-1} q(\mathbf{p})$$

Challenges:

- Given $[\mathbf{q}, \mathbf{p}, \tilde{\Delta}, \mathcal{H}(\kappa)]$ I can always produce a vector of marginal costs $\mathbf{mc}$ that rationalizes what we observe. [ie: J equations J unknowns].
- Nonparametrically we cannot identify κ without more restrictions (!).

What do people do?

Maybe some vectors of $\mathbf{mc}$ look less "reasonable" than others.

- Marginal costs ≤ 0 seem problematic. [Might just be that your estimates for demand are too inelastic...]
- or I have a parametric model of MC in mind.

$$f\left(p_{jt} - \eta_{jt}(\mathbf{p}, \mathbf{s}, \theta_2, \kappa)\right) = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}, \theta_3) + \omega_{jt}$$
$$E[\omega_{jt} | \mathsf{x}_{\mathbf{t}}, \mathsf{w}_{\mathbf{t}}, \mathsf{v}_{\mathbf{t}}] = 0$$

- Can test that model with GMM objective of mc_{jt} on regressors.
- Maybe marginal costs cannot deviate too much within product from period to period. (We can write these as moment restrictions too).

Approach #2: Simultaneous Supply and Demand

Estimate θ_2 using both supply and demand. The fit of my supply side will also inform my demand parameters, particularly α the price coefficient. [BLP 95 used this for additional power with lots of random coefficients and potentially weak instruments].

$$\delta_{jt}(\mathcal{S}_t, \tilde{\theta}_2) + \alpha p_{jt} = h_d(\mathsf{x}_{jt}, \mathsf{v}_{jt}, \theta_1) + \xi_{jt}$$
$$f\left(p_{jt} - \eta_{jt}(\mathbf{p}, \mathbf{s}, \theta_2, \kappa)\right) = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}, \theta_3) + \omega_{jt}$$

Challenges:

- Should I try to estimate κ? or just compare objective values at $\kappa_{fg} \in \{0, 1\}$?
- Am I testing conduct? Or am I testing the functional form for my supply model?
- Will a missing IV/restriction change whether or not I believe firms are colluding?

There are 2.5 ways to think about conduct:

1. Using moment conditions to estimate $\hat{\kappa}$ or $\mathcal{H}(\kappa)$ directly. (Wald)
    - Often with a small number of parameters (ie: $\kappa_{fg} = 0$ except for firms I know are in a cartel).
    - Can be challenging to tell similar values of κ_{fg} apart (under-powered).
2. "Menu Approach" / (LR)
    - Nevo (Economics Letters 1998)
    - Bresnahan (1987)
    - Compare some goodness-of-fit critieria across assumed values of κ (Bertrand vs. Collusion)
3. Rejecting (or failing to reject) a single model.
    - "We can reject monopoly pricing..."

Testing a single model of κ

Put the η_{jt} on the RHS and test whether $\lambda = 1$:

$$p_{jt} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}, \theta_3) + \lambda \cdot \eta_{jt}\left(\mathbf{p}, \mathbf{s}, \theta_2, \kappa\right) + \omega_{jt} \text{ with } \mathbb{E}[\omega_{jt} | \mathsf{x}_t, \mathsf{w}_t, \mathsf{z}_t] = 0s$$

- We are basically running 2SLS with IV for the endogenous η_{jt}
- "Informal" test of Villas Boas (2007): $\mathbb{E}[\omega_{jt} | \mathsf{x}_{jt}, \mathsf{w}_{jt}, \eta_{jt}] = 0.$
  - Considers different forms of $f(\cdot)$: linear, exponential, logarithmic.
  - WP discusses this more than Restud version.
- Pakes (2017) uses Wollman (2018) data and BLP IV $\mathbb{E}[\omega_{jt} | x_{jt}, w_{jt}, f(x_{-j})] = 0.$
- $\lambda \neq 0$ is hard to interpret.

Table 1: Wollman & Pricing Equilibrium.

Taken from Pakes, 2017, *Journal of Industrial Economics.*

	Price	(S.E.)	Price	(S.E.)
Gross Weight	.36	(0.01)	.36	(.003)
Cab-over	.13	(0.01)	.13	(0.01)
Compact front	-.19	(0.04)	0.21	(0.03)
long cab	-.01	(0.04)	0.03	(0.03)
Wage	.08	(.003)	0.08	(.003)
$\widehat{Markup}$	.92	(0.31)	1.12	(0.22)
Time dummies?	No	n.r.	Yes	n.r.
R^2	0.86	n.r.	0.94	n.r.

Note. There are 1,777 observations; 16 firms over the period 1992-2012. S.E.=Standard error.

These are somewhat reassuring:

- $\lambda \approx 1$ for multiproduct-oligopoly
- Fit is pretty good $R^2 > 0.8$ and $R^2 > 0.5$ for within vehicle regressions (not shown).
- As a behavioral model, multiproduct demand estimation seems successful.
- But, do we know that an alternative $\mathcal{H}(\kappa)$ would have a $\lambda \neq 1$ or a lower R^2, and if so how low before we can "reject" the model?

Goodness of Fit Tests

Another idea (Bonnet and Dubois, Rand 2010) runs the following regression:

$$\log\left(p_{jt} - \eta_{jt}(\mathbf{p}, \mathbf{s}, \hat{\theta}_2, \kappa)\right) = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}, \theta_3) + \omega_{jt}$$

- Run a regression for each κ and obtain $Q(\kappa) = \sum_{jt} \hat{\omega}_{jt}^2$
- Employ the non nested test of Rivers and Vuong (2002). Why?
- Working out the distribution of $Q(\kappa_1) - Q(\kappa_2) = T(\kappa_1, \kappa_2)$ is the hard part.
- Also this is OLS (or NLLS) and there are no instruments or exclusion restrictions for the supply side. Presumably we could add some and do GMM? (I think this is the "formal" test of Villas Boas (ReStud 2007)).

Villas Boas (2007)

TABLE 7

Percentage price-cost margins (PCM) by scenario

Wholesale, retail, and total PCM	Median	S.D.	Min	Max
Given random coefficient demand				
Model 1: Simple linear pricing: T-PCM	43·8	25·4	25·5	136·5
Model 2: Hybrid model: T-PCM	41·6	28·6	15·8	136·7
Model 3.1: Zero wholesale margin: R-PCM	21·1	9·0	12·5	50·0
Model 3.2: Zero retail margin: W-PCM	20·6	4·7	12·8	45·8
Model 4: Wholesale collusion: T-PCM	72·8	22·4	38·8	253·0
Model 5: Retail collusion: T-PCM	69·9	25·8	23·4	263·2
Model 6: Monopolist: T-PCM	40·6	11·5	21·8	103·7

PCM = $(p-c)/p$ where p is price and c is marginal cost. S.D., standard deviation; W, wholesale; R, retail; T, total.

Source: My calculations.

TABLE 8

Sample statistics of recovered costs and informal testing

	Recovered retail and wholesale costs				
	Mean	S.D.	Min	Max	Percentage <0
Model 1: Simple linear pricing	0·2730	0·1552	−0·1532	0·5364	1·35
Model 2: Hybrid model	0·2757	0·1681	−0·1541	0·5364	1·45
Model 3.1: Zero wholesale margin	0·3796	0·0907	0·1201	0·6298	0
Model 3.2: Zero retail margin	0·3815	0·0895	0·1302	0·6279	0
Model 4: Wholesale collusion	0·1352	0·1114	−0·4590	0·4409	13·12
Model 5: Retail collusion	0·1530	0·1329	−0·4895	0·5053	12·6
Model 6: Monopolist	0·2953	0·0951	−0·0089	0·5628	0·08

Notes: Recovered costs = p − epcm where p is retail price and epcm are the estimated margins. Last column has the percentage of cases with recovered estimated costs being negative.

Source: My calculations.

- Try out different models of price setting behavior, compute η markups.
- Unsurprisingly models with higher markups also have more costs $MC < 0$.
- Is this evidence of incorrect conduct assumption or inelastic demand?

TABLE 10

p-Values for pairwise non-nested comparisons

	Alternative models						
H_0 model	1	2	3·1	3·2	4	5	6
1: Simple linear pricing	–	0·50	0·00	0·50	0·24	0·00	0·50
2: Hybrid	0·00	–	0·50	0·50	0·12	0·00	0·50
3.1: Zero wholesale margin	0·41	0·29	–	0·05	0·50	0·39	0·07
3.2: Zero retail margin	0·39	0·40	0·05	–	0·50	0·39	0·17
4: Wholesale collusion	0·49	0·48	0·50	0·50	–	0·48	0·50
5: Retail collusion	0·00	0·00	0·50	0·50	0·22	–	0·50
6: Monopolist	0·34	0·35	0·17	0·31	0·48	0·34	–
Chain size weighted							
1: Simple linear pricing	–	0·08	0·01	0·06	0·08	0·00	0·00
2: Hybrid	0·17	–	0·15	0·22	0·00	0·06	0·14
3.1: Zero wholesale margin	0·08	0·15	–	0·11	0·15	0·12	0·00
3.2: Zero retail margin	0·01	0·07	0·00	–	0·09	0·01	0·00
4: Wholesale collusion	0·00	0·05	0·04	0·09	–	0·00	0·02
5: Retail collusion	0·00	0·02	0·03	0·11	0·02	–	0·00
6: Monopolist	0·10	0·20	0·00	0·15	0·20	0·14	–

Notes: *p*-Values reported from non-nested, Cox-type (Smith, 1992) test statistics of the null model in a row being true against the specified alternative model in a column. Bottom part is a robustness check. It has the same format as above, but the non-nested comparisons are based on estimates for the case when the portion of the manufacturer's profit due to each retailer is weighted by the retailer's chain size.

Source: My calculations.

- Cox test has pairwise rejection problem (4) rejects (5) and (5) rejects (4).
- Likewise for 3.1 and 3.2 in top panel.

Recap

So far three approaches to exploit $E[\omega_{jt}|\mathsf{x}_t, \mathsf{w}_t, \mathsf{z}_t] = 0$

1. Put the markup on RHS and instrument for it to test $\lambda = 1$ (Wald)

$$p_{jt} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}, \theta_3) + \lambda \cdot \eta_{jt}\left(\mathbf{p}, \mathbf{s}, \widehat{\theta_2}, \kappa\right) + \omega_{jt}$$

2. Put the markup on LHS assuming $\lambda = 1$ and test goodness of fit of supply equation (Anderson Rubin)

$$p_{jt} - \eta_{jt}\left(\mathbf{p}, \mathbf{s}, \widehat{\theta_2}, \kappa\right) = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}, \theta_3) + \omega_{jt}$$

3. Estimate supply and demand simultaneously $[\theta_1, \theta_2, \theta_3]$ and compare goodness of fit for different κ. (Likelihood Ratio)

Backus Conlon Sinkinson (2022-?)

Basic Setup

We start with marginal revenue and marginal cost (unobserved ω, observed $h(\cdot)$)

$$\psi_{jt}^{m} = mc_{jt}$$
$$p_{jt} - \eta_{jt}^{m} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}) + \omega_{jt}^{m}$$

- Let's be vague/flexible with $h_s(\cdot)$ for now, but I don't know the production function.
- Assume: Demand and hence η_{jt}^{m} are known (given conduct).
- Idea (η^A, η^B) are monopoly/perfect competition or Cournot/Bertrand.

Setup: Challenges

The true model for markups (conduct) will satisfy the CMR: $\mathbb{E}[\omega_{jt}|z^s_{jt}] = 0$

$$p_{jt} - \eta^{(m)}_{jt} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}; \theta_3) + \omega_{jt}$$

Goal is test two competing markups $\eta^{(A)}_{jt}, \eta^{(B)}_{jt}$, but there are challenges:

1. Test will depend on how we choose unconditional moment restrictions $\mathbb{E}[\omega_{jt} \cdot A(z^s_{jt})] = 0$
2. Test may depend on how we specify $h_s(\cdot)$
   - All tests are basically joint tests of the specification for observed marginal costs and the exclusion restriction.
   - Villas Boas (2007) tries log, linear, exponential in $x\beta$
3. Choice of $\eta^{(m)}_{jt}$ will affect our choice of weighting matrix and thus the test. (Hall Pelletier (2011))

The Question

Two competing markups $(\eta_{jt}^A, \eta_{jt}^B)$: which fits the data better?
(both may be misspecified)

$$p_{jt} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}) + \tau\, \eta_{jt}^A + (1-\tau)\, \eta_{jt}^B + \omega_{jt}$$

Model is defined by a conditional moment restriction $\mathbb{E}[\omega_{jt} | z_{jt}^s] = 0$

- $H_0 : \tau = 1$ vs $H_a : \tau = 0$
- This is a model selection problem or a non nested testing problem.
  - We might want to compare more than two alternatives (too bad).
- Obvious endogeneity problem with η_{jt}!

Our Idea #1: Optimal IV (again)

Given the CMR $\mathbb{E}[\omega_{jt}|z^s_{jt}] = 0$ we can ask what is the optimal IV for moment $\mathbb{E}[\omega'_{jt} A(z^s_{jt})] = 0$:

Solve for ω_{jt}

$$\omega_{jt} = p_{jt} - h_s(\mathrm{x}_{jt}, \mathrm{w}_{jt}) + \tau\, \eta^A_{jt} + (1-\tau)\, \eta^B_{jt}$$

- What is $\frac{\partial \omega_{jt}}{\partial \tau}$? : $\quad A(z_t) = \mathbb{E}[\eta^A_{jt} - \eta^B_{jt} \mid \mathbf{x_t}, \mathbf{w_t}, \mathbf{v_t}, \mathrm{y}_t]$
- This looks like a first stage (nonparametric) regression of markup difference on instruments/all exogenous variables. (Newey 1990).
- Instruments are supposed to explain differences in endogenous markups beyond the regressors $h_s(\mathrm{x}_{jt}, \mathrm{w}_{jt})$ (this is the usual first-stage F-stat).

Our Idea: Motivation #2 (Misspecification)

Index the true model by 0. Then,

$$p_{jt} - \eta^0_{jt} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}) + \omega^0_{jt}.$$

To motivate a useful test, we ask what happens when we estimate supply with the wrong conduct model (1):

$$p_{jt} - \eta^1_{jt} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}) + \underbrace{\underbrace{\eta^0_{jt} - \eta^1_{jt}}_{\equiv \Delta\eta^{0,1}_{jt}} + \omega^0_{jt}}_{\omega^1_{jt}}.$$

- Misspecifying conduct introduces an omitted variable: the difference in markups.
- Our test is premised on detection of this omitted variable.

Our Innovation: How does this help?

The model is given by

$$p_{jt} - \eta^m_{jt} = h_s(\cdot) + \omega^m_{jt}, \text{ and } \mathbb{E}[\omega^{(m)}_{jt} \cdot A(z_t)] = 0.$$

We suggest $A(z_t) = \mathbb{E}[\eta^1_{jt} - \eta^2_{jt} | \chi_t, z_{jt}]$; several advantages:

- Reduces potentially many moments ($\mathbb{E}[\omega'_{jt} z_t] = 0$) to a single, scalar moment. No need for a weighting matrix, or associated problems.
- Testing is reduced to two prediction exercises: $\mathbb{E}[\eta^1_{jt} - \eta^2_{jt} \mid \mathsf{x}_t, \mathsf{w}_t, \mathsf{v}_t, \mathsf{y}_t, z_{jt}] - \mathbb{E}[\eta^1_{jt} - \eta^2_{jt} \mid \mathsf{x}_{jt}, \mathsf{w}_{jt}]$ and $\widehat{\omega}^{(m)}_{jt}$.
- Show in the paper that this leads to the most powerful test (maximizes distance between two GMM objective functions conditional on weight matrix).
- Downside: Our choice of instrument is model specific! UMP is not going to happen.

Testing Environment

Compare violations of unconditional moments under $(\eta^A_{jt}, \eta^B_{jt})$ and $A(z^s_{jt})$:

$$p_{jt} - \eta^A_{jt} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}) + \omega^A_{jt}$$
$$p_{jt} - \eta^B_{jt} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}) + \omega^B_{jt}$$

These are just nonparametric regressions.

Which gives us

$$g_A = \frac{1}{N}\sum_{jt} \omega^A_{jt}\, A(z^s_{jt}), \quad g_B = \frac{1}{N}\sum_{jt} \omega^B_{jt}\, A(z^s_{jt})$$
$$Q_m = g_m'\, W_m\, g_m$$

Now consider a Rivers Vuong (2002) type test $T_{RV} = \sqrt{n}\left(\frac{Q_A - Q_B}{\sigma_{Q_A - Q_B}}\right) \sim N(0,1)$ $H_0 : Q_A - Q_B = 0$ vs. $H_A : Q_A > Q_B$ or $Q_A < Q_B$.

Getting the SD of the difference is hard $\rightarrow$ bootstrap

Algorithm

(1) Split the sample by markets t into 70% *test* and 30% *train*.

(2) On the *training sample*:

  (a) Approximate the optimal instruments $a(z^s_{jt}) = \mathbb{E}[\Delta\eta^{(1,2)}_{jt} \mid z^s_{jt}]$ as the fitted values from:

$$\Delta\eta^{1,2}_{jt} = g(z^s_{jt}) + \zeta_{jt}.$$

  (b) Estimate the marginal cost function, under models 1 and 2 to obtain residuals $\widehat{\omega^1_{jt}}$ and $\widehat{\omega^2_{jt}}$:

$$p_{jt} - \eta^m_{jt} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}; \theta_3) + \omega^m_{jt}.$$

(3) On the *test sample*:

  (a) For each candidate model, compute the value of the scalar moment:[1]

$$Q(\eta^m) = \left(\sum_{j,t} \hat{\omega}^m_{jt} \cdot \hat{g}(\mathbf{z_t}) \right)^2 .$$

  (b) Repeat the previous step on bootstrapped samples and estimate $\hat{\sigma}/\sqrt{n}$ the standard error of the difference $\tilde{Q}(\eta^1) - \tilde{Q}(\eta^2)$.

  (c) Compute the test statistic

$$T = \frac{\sqrt{n}\,(Q(\eta^1) - Q(\eta^2))}{\hat{\sigma}} \sim \mathcal{N}(0,1).$$

Note: Steps 2(a) and 2(b) can be done in any order via non-parametric regression.

Comparison to Literature

- Bresnahan (1987): Did LR test to determine collusion vs. competition in 1955 automobile price war
  - No IV, errors were measurement in P, Q.
- Bonnet and Dubois (2010): RV test
  - But no IV – maximum likelihood with normally distirbuted ω_{jt}'s.
- Villas Boas (2007): Cox test to determine double marginalization or not in yogurt
  - GMM objective, unclear what if any IV are used.
  - Need to "know" the true model.
- Duarte, Magnolfi, Solvsten, Sullivan (2022): RV beats Cox pretty badly in Monte Carlo.

Limitations

Not everything is testable:

- If $\Delta\eta_{jt}$ cannot be explained by z^s_{jt} beyond contents of $(\mathrm{x}_j, \mathrm{w}_j)$ we have nothing
- Flexible demand models are required to generate cross sectional variation in markups
- Discuss plain logit
- Beware of “accidental” exclusion restrictions.
- Moments are about correlation not levels.

Possible Exclusion Restrictions

We are looking for variables which affect demand but not supply:

$$\sigma_j^{-1}(\mathcal{S}_t, \mathbf{p_t}, \mathsf{y_t}, \mathsf{x}_t, \mathsf{v}_t, \tilde{\theta}_2) = h_d(\mathsf{x}_{jt}, \mathsf{v}_{jt}; \theta_1) - \alpha\, p_{jt} + \lambda \, \mathsf{log}(\mathsf{ad}_{jt}) + \xi_{jt}$$
$$p_{jt} - \eta_{jt}(\mathcal{S}_t, \mathbf{p_t}; \theta_2, \mathcal{H}_t(\kappa)) = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}; \theta_3) + \omega_{jt}$$

Things we use:

- Obvious choice: v_{jt} (things like product recalls are relatively weak)
- Demographics (enter nonlinearly): y_t (chain-level income works well)
- Characteristics of other goods: $f(\mathsf{x}_{-j,t})$ (BLP instruments).
- Characteristics of other goods: $\mathsf{w}_{-j,t}$ (commodity price of oats for Rice Krispies)

Things we don't use:

- Unobserved demand shocks ξ_{jt} (see MacKay Miller 2020 for $Cov(\xi_j, \omega_j) = 0$).
- Observable κ conduct shifters (financial mergers/events, see Miller Weinberg (2018))

BCS in one slide

Two competing markups (markdowns) $(\eta_{jt}^A, \eta_{jt}^B)$. Which fit data better? (both may be misspecified)

$$p_{jt} = h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}) + \tau\, \eta_{jt}^A + (1-\tau)\, \eta_{jt}^B + \omega_{jt} \text{ where } \mathbb{E}[\omega_{jt} | \mathsf{x}_{jt}, \mathsf{w}_{jt}, z_{jt}] = 0$$

The only IV that matters is the predicted markup difference, which we can get in a first stage

$$\eta_{jt}^A - \eta_{jt}^B = g_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}, z_{jt}) + \zeta_{jt}^{A-B}$$

As long as we have some exclusion restrictions z_{jt} that:

- **Relevance**: Predict markups beyond the cost shifters $(\mathsf{x}_{jt}, \mathsf{w}_{jt})$.
- **Exlcusion**: Do not affect marginal cost. (exclusion)

Rest of paper: make $h(\cdot)$ and $g(\cdot)$ as flexible as possible, and get a Vuong $\mathcal{N}(0,1)$ test.

BCS in one slide

Two competing markups (markdowns) $(\eta_{jt}^A, \eta_{jt}^B)$. Which fit data better? (both may be misspecified)

$$
\begin{aligned}
p_{jt} - \eta_{jt}^A &= h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}) + \omega_{jt} \text{ under } H_0 : \tau = 1 \\
p_{jt} - \eta_{jt}^A &= h_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}) + \omega_{jt} \text{ under } H_1 : \tau = 0
\end{aligned}
$$

The only IV that matters is the predicted markup difference, which we can get in a first stage

$$\eta_{jt}^A - \eta_{jt}^B = g_s(\mathsf{x}_{jt}, \mathsf{w}_{jt}, z_{jt}) + \zeta_{jt}^{A-B}$$

As long as we have some exclusion restrictions z_{jt} that:

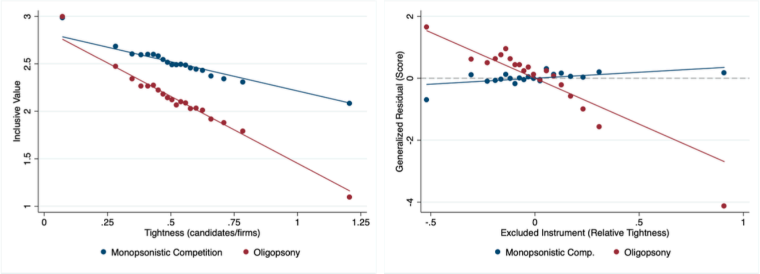
- **Relevance**: Predict markups beyond the cost shifters $(\mathsf{x}_{jt}, \mathsf{w}_{jt})$.
- **Exlcusion**: Do not affect marginal cost. (exclusion)

Rest of paper: make $h(\cdot)$ and $g(\cdot)$ as flexible as possible, and get a Vuong $\mathcal{N}(0,1)$ test.

Examples

Scuderi Roussille (2025): Models of Labor Supply

Figure 5: Vuong Test



(a) First Stage **(b)** Visualizing the Vuong Test

Note: Panel (a) plots the "first stage" relationship between the model-implied inclusive values Λ_i and Λ_i^{-j} and the instrumental variable t_{ij}, conditional on firm covariates z_j and candidate covariates x_i and two-week period dummies. Panel (b) plots the relationship between generalized residuals and the excluded instrument for the non-predictive monopsonistic competition and oligopsony models. Under proper specification, the correlation of the generalized residuals and the excluded instrument should be zero (the dashed line). The larger the deviation from zero, the greater the degree of mis-specification of the model.

- Offered wages for online job platform
- Compares Monopsony vs. oligopolistic competition vs. perfect comp.
- Compares tailored offers vs. not (price discrimination).

Scuderi JMP 2022: Models of Labor Supply

- Firms ignore competitors (Monopsony)
- Firms offer wages independent of candidate characteristics (experience, demographics).
- Firms are definitely NOT paying MPL.

Table 4: Non-Nested Model Comparison Tests (Rivers and Vuong, 2002)

Model	(1) Monopsonistic Comp.	(2)	(3) Oligopsony	(4)
	Not Predictive	Type Predictive	Not Predictive	Type Predictive
Perfect Competition	-54.84	-54.40	-39.92	-39.91
Monopsonistic, Not Predictive	–	7.83	3.98	2.69
Monopsonistic, Type Predictive		–	2.77	1.54
Oligopsony, Not Predictive			–	-3.67
Oligopsony, Type Predictive				–

Note: This table reports test statistics from the Rivers and Vuong (2002) non-nested model comparison procedure. Positive values imply the row model is preferred to the column model. Under the null of model equivalence, the test statistics are asymptotically normal with mean zero and unit variance.

specification. In the figure, the generalized residuals for the monopsonsitic competition alternative are closely aligned with the x-axis, while the generalized residuals for the oligopsony alternative are strongly negatively related to tightness.

Our tests therefore suggest that models of firm behavior in which firms both ignore strategic interactions in wage setting and do not tailor wage offers to candidates on the basis of predictable preference variation are closer approximations to firms' true bidding behavior on the platform than are models in which firms act strategically and tailor offers. In Appendix F, we report the testing results using the original Vuong (1989) likelihood comparison test, which yield qualitatively identical model comparisons. In the following analysis, we adopt the not-predictive monopsonostic competition model as our preferred model of conduct.

Do Cartels encourage entry with high prices?

> *in 2013, Teva Pharmaceuticals, the largest generic firm, hired NP, a marketing executive with especially strong industry relationships, and tasked her with "price increase implementation."1 Over an 18-month period, industry participants exchanged thousands of calls and texts—alongside countless LinkedIn, Facebook, and WhatsApp messages and face-to-face conversations—with contacts at rival firms to coordinate the increases (Complaint, page 322).2 Following this period, prescription drug expenditures by governments, private insurers, and individuals rose sharply by billions of dollars.*

Starc Wollmann: Generic Pharma Cartel

- NP organizes the cartel and prices go up
- Slightly less in large markets (which are more likely to see entry)

Figure I: *Prices rise sharply following cartel formation*

This figure plots the average log price of cartelized and uncartelized drugs on the y-axis against calendar quarter on the x-axis. The vertical red line corresponds to the first quarter of 2013—the period in which NP joined Teva. Prices are normalized to zero in that quarter.

Starc Wollmann: Baseline Scenario

- all firms set competitive prices in uncartelized markets
- all firms set competitive prices in cartelized markets before cartel formation;
- and after cartel formation, members set prices that maximize their joint profits while nonmembers best respond

Starc Wollmann: Testing Results

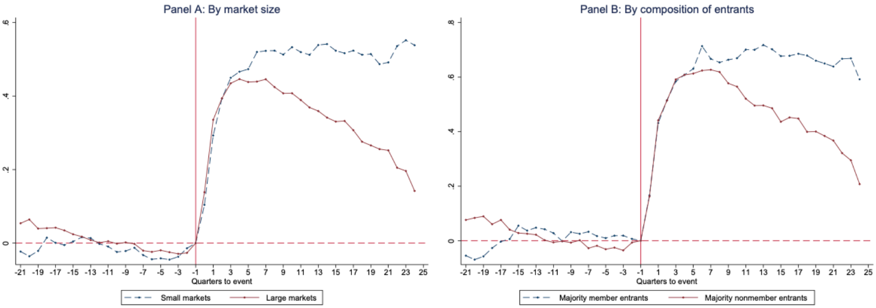


Figure III: *Prices after cartel formation*

In this figure, we plot estimates of β^τ, which are derived from equation 1, against x^τ, which represents event time in quarters. For Panel A, we proxy for market size by computing the number of prescriptions filled in the period just prior to NP joining Teva, and we distinguish between large and small cartelized markets based on how this figure relates to the median value. For Panel B, we calculate the fraction of entrants that arrive as a result of ANDAs filed after NP joins Teva and identify markets in which a majority of those firms are members and nonmembers, respectively.

Table IV: *Results of conduct tests*

	$\tilde{Q} \times 100$ Baseline	Alternative	Test statistic	p-value
Test A. Model vs. competition (pre-investigation)	.16247	.20688	-3.03	.001
Test B. Model vs. competition (post-investigation)	.20286	.34235	-4.63	$< .0001$
Test C. Model vs. nonmembers comply	1.3979	4.6533	-5.57	$< .0001$
Test D. Model vs. member entrants do not comply	.00008	.00008	-3.06	.001

This table reports the results of the testing procedure proposed by Backus et al. (2021) for pairwise comparison described in the text. The test statistic is distributed standard normal. The standard error of the differenc between $\tilde{Q}_1$ and $\tilde{Q}_2$ is obtained via bootstrapping.

An Internalization Parameter

Let κ represent the weight a firm places on competitors and τ the internalization of those weights.

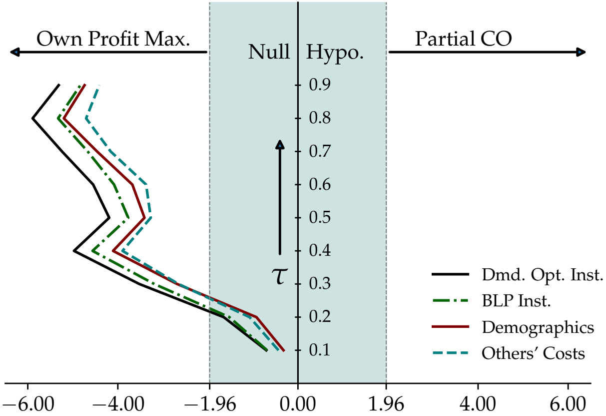
$$arg \max_{p_j : j \in \mathcal{J}_f} \sum_{j \in \mathcal{J}_f} (p_j - mc_j) \cdot s_j(\mathbf{p}) + \sum_{g \neq f} \tau \kappa_{fg} \sum_{j \in \mathcal{J}_g} (p_k - mc_k) \cdot s_k(\mathbf{p})$$

Now,

- $\tau = 0$ implies own-profit maximization
- $\tau = 1$ implies common ownership pricing
- τ in between is..? Agency?

We test $\tau \in (0.1, \ldots, 0.9)$ against own-profit maximization.

Internalization Parameter Results



Thanks!