Representative Consumer Models

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Grad IO

A Benchmark

Let's start with the following as a benchmark:

- A representative agent demand system.
- The consumer chooses an expenditure level for each good and consumes at least a little of all goods.
- Which desirable properties?:
  - We want a fully flexible matrix of demand derivatives $\Delta(\mathrm{p})$.
  - Probably we want some flexibility so that $\Delta(\mathrm{p}) \neq \Delta(\mathrm{p}')$.
  - Would satisfy axioms of consumer theory (WARP, Slutsky Symmetry, etc.).

Brief Aside: Constant Elasticity Demand

One candidate from your first year course would be a constant elasticity demand model. Which we could micro-found with utility for consuming $q(\omega)$ for each of J goods:

$$U = \left(\int_0^J q(\omega)^\rho d\,\omega \right)^{\frac{1}{\rho}} \quad 0 \leq \rho \leq 1$$

We can solve Lagrangians and find (Frisch) demands:

$$q(\omega) = \left(\frac{\lambda p(\omega)}{\rho} \right)^{\frac{1}{\rho-1}}$$

With ratios:

$$\frac{q(\omega_1)}{q(\omega_2)} = \left(\frac{p(\omega_1)}{p(\omega_2)} \right)^{\frac{1}{\rho-1}}$$

Common substitution: $\sigma = \frac{1}{1-\rho}$. or $\rho = \frac{\sigma-1}{\sigma}$.

Some CES algebra:

$$q(\omega_1) = q(\omega_2)\left(\frac{p(\omega_1)}{p(\omega_2)}\right)^{-\sigma}$$

$$\underbrace{\int_0^J p(\omega_1)q(\omega_1)d\,\omega_1}_{I \equiv \text{ consumer income}} = \int_0^J q(\omega_2)p(\omega_1)^{1-\sigma}p(\omega_2)^{\sigma}d\,\omega_1$$

$$I = q(\omega_2)p(\omega_2)^{\sigma}\int_0^J p(\omega_1)^{1-\sigma}d\,\omega_1$$

Now we can solve for Marshallian Demand:

$$q(\omega_2) = \frac{I \cdot p(\omega_2)^{-\sigma}}{\underbrace{\int_0^J p(\omega_1)^{1-\sigma}d\,\omega_1}_{P^{1-\sigma}}}$$ Where P is the overall price index.

Brief Aside: Constant Elasticity Demand

Using the overall price index $P = \left(\int_0^J p(\omega_1)^{1-\sigma} d\,\omega_1\right)^{\frac{1}{\rho}}$, we can re-write Marshallian demand:

$$q(\omega) \;=\; p(\omega)^{-\sigma} P^{\sigma-1} I = \left(\frac{p(\omega)}{P}\right)^{-\sigma} \frac{I}{P}$$

We can establish the well-known homotheticity property of CES by plugging back into original equation for $U(\cdot)$ and noting that $e(P,u) = P \cdot u$.

$$\begin{aligned} U &= \left(\int_0^J q(\omega)^\rho d\,\omega\right)^{1/\rho} = \left(\int_0^J p(\omega)^{1-\sigma} I^\rho P^{(\sigma-1)\rho} d\,\omega\right)^{1/\rho} \\ &= I P^{\sigma-1} \left(\int_0^J p(\omega)^{1-\sigma} d\,\omega\right)^{\frac{\sigma}{\sigma-1}} = I P^{\sigma-1} P^{-\sigma} = \frac{I}{P}. \end{aligned}$$

- Utility is just income divided by the price index!
- Homothetic because consumption of all goods scales with I

Demand (and its derivative) for a single good:

$$
\begin{aligned}
q(p) &= p^{-\sigma} P^{\sigma-1} I \\
\frac{\partial q}{\partial p} &= -\sigma p^{-\sigma-1} P^{\sigma-1} I \\
\frac{-q}{\frac{\partial q}{\partial p}} &= \frac{p}{\sigma}
\end{aligned}
$$

So that monopoly markup becomes $p = \frac{mc}{\rho}$

- CES means one markup (and elasticity) for all goods.
- Hard to do IO here. Not so helpful in understanding strategic price setting behavior!
- Better left for Trade and Macro economists.

Recall our desirable properties:

- We want a fully flexible matrix of demand derivatives $\Delta(\mathrm{p})$.
- Probably we want some flexibility so that $\Delta(\mathrm{p}) \neq \Delta(\mathrm{p}')$.
- Would satisfy axioms of consumer theory (WARP, Slutsky Symmetry, etc.).
- Key ideas: separable preferences and multi-stage budgeting.
    - Allocating expenditures within a group: Index can be calculated without knowing what you choose within the group.
    - Other products respond only to the index price not to individual prices!

Begin by defining an expenditure function:

$$\log e(u, \mathrm{p}) = (1-u) \log \underbrace{a(\mathrm{p})}_{\text{subsistence}} + u \cdot \log \underbrace{b(\mathrm{p})}_{\text{bliss}}$$

We assume a particular functional form for $a(\mathrm{p}), b(\mathrm{p})$ that is second-order flexible.

Here is the form of the expenditure function:

$$\log e(u, \mathrm{p}) = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j + u\beta_0 \prod_k p_k^{\beta_k}$$

- Estimate $(\alpha_i, \beta_i, \gamma^*_{ij})$ from data.
- We usually require $\sum_i \alpha_i = 1$, $\sum_k \gamma^*_{jk} = \sum_j \beta_j = 0$ so that demand is linearly homogenous in p.
- Also often impose that $\gamma^*_{jk} = \gamma^*_{kj}$.
  - Sometimes we impose this ex-ante, other times we test for it ex post.
- We can also see that we have at least one parameter for each of the first two own and cross price derivatives of $e(\cdot)$.

After applying Shepard's Lemma and logarithmic differentiation, we can obtain the expenditure share for good i:

$$\begin{aligned} w_i &= \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i u \beta_0 \prod_k p_k^{\beta_k} \quad \text{with} \quad \gamma_{ij} = \frac{1}{2}(\gamma^*_{ij} + \gamma^*_{ji}) \\ &= \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(x/P) \end{aligned}$$

- x represents total expenditure within group, P is the price index for the group.
- Two price indices are commonly used ("Exact" and Stone 1954's linear approximate index):

$$\begin{aligned} \log P &= \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_j \sum_k \gamma_{kj} \log p_k \log p_j \\ \log P &= \sum_k w_k \log p_k \end{aligned}$$

- AIDS seemed like a better name in 1980 than it does today!
- Gets used often in international trade or macro-consumption literature.
  - Product categories are often: durables, non-durables, housing, utilities, etc. from CEX data.
- Can use it for IO purposes (each "group" contains a single product).
- If p_k changes demand for good j (it does!) then we need an instrument for every price!
- We still have J^2 possible elasticities or $J \times (J+1)/2$.
  - Can simplify with multi-stage budgeting. (but we have to know what segments are)
  - Massive data requirements: $J = 45$ in a vending machine means we need over 2000 observations.

Beer Example: Hausman, Leonard, Zona (1994)

Goals:

- Estimate demand for beer in the US.
- Analyze a merger, test assumptions about firm conduct

Three stages:

1. Brand-Level (AIDS): 5 brands per segment.

$$\underbrace{w_i}_{\text{brand expenditure share}} = \alpha_i + \sum_j \alpha_{ij} \log p_j + \beta_i \log\left(\frac{x}{P}\right) + \varepsilon_1$$

2. Segment-Level (log-log): Premium, Light, Popular.

$$\underbrace{\log q_m}_{\text{seg. quantity}} = \beta_m \underbrace{\log y_B}_{\text{beer expenditure}} + \sum_k \sigma_k \log \underbrace{\pi_k}_{\text{segment price index}} + \alpha_m + \varepsilon_2$$

3. Top: Beer Expenditure vs. other goods (wine, spirits, hamburgers).

$$\log e_t = \beta_0 + \beta_1 \log y_t + \beta_2 \log P_b + Z_t \delta + \varepsilon_3$$

Identification: Hausman, et.al (1994)

- Price is correlated with both unobserved product quality and unobserved demand shocks.
- Finding brand level instruments is the challenge.
- The famous Hausman instrument: use prices in one city to instrument for prices in another

$$\log p_{jnt} = \delta_j \log c_{jt} + \alpha_{jn} + \omega_{jnt}$$

- Instruments tend to be strong but exclusion can be questionable.
- Key is that ω_{jnt} are independent of each other (is this believable?).
  - People mostly complain about national ad campaigns (this is beer after all!)
- What about other instruments? (Input prices, taxes, etc.).
- Specification Test: brand price in other segments should not have an effect controlling for the price index of other segments.

TABLE 1

Beer Segment Conditional Demand Equations.

	Premium	Popular	Light
Constant	0.501	−4.021	−1.183
	(0.283)	(0.560)	(0.377)
log (Beer Exp)	0.978	0.943	1.067
	(0.011)	(0.022)	(0.015)
log ($P_{PREMIUM}$)	−2.671	2.704	0.424
	(0.123)	(0.244)	(0.166)
log ($P_{POPULAR}$)	0.510	−2.707	0.747
	(0.097)	(0.193)	(0.127)
log (P_{LIGHT})	0.701	0.518	−2.424
	(0.070)	(0.140)	(0.092)
Time	−0.001	−0.000	0.002
	(0.000)	(0.001)	(0.000)
log (# of Stores)	−0.035	0.253	−0.176
	(0.016)	(0.034)	(0.023)

Number of Observations = 101.

TABLE 2

Brand Share Equations: Premium.

	1 Budweiser	2 Molson	3 Labatts	4 Miller	5 Coors
Constant	0.393	0.377	0.230	−0.104	–
	(0.062)	(0.078)	(0.056)	(0.031)	–
Time	0.001	−0.000	0.001	0.000	–
	(0.000)	(0.000)	(0.000)	(0.000)	–
log (Y/P)	−0.004	−0.011	−0.006	0.017	–
	(0.006)	(0.007)	(0.005)	(0.003)	–
log ($P_{Budweiser}$)	−0.936	0.372	0.243	0.150	–
	(0.041)	(0.231)	(0.034)	(0.018)	–
log (P_{Molson})	0.372	−0.804	0.183	0.130	–
	(0.231)	(0.031)	(0.022)	(0.012)	–
log ($P_{Labatts}$)	0.243	0.183	−0.588	0.028	–
	(0.034)	(0.022)	(0.044)	(0.019)	–
log (P_{Miller})	0.150	0.130	0.028	−0.377	–
	(0.018)	(0.012)	(0.019)	(0.017)	–
log (# of Stores)	−0.010	0.005	−0.036	0.022	–
	(0.009)	(0.012)	(0.008)	(0.005)	–
Conditional Own	−3.527	−5.049	−4.277	−4.201	−4.641
Price Elasticity	(0.113)	(0.152)	(0.245)	(0.147)	(0.203)

$$\Sigma = \begin{Bmatrix} 0.000359 & -1.436E-05 & -0.000158 & -2.402E-05 \\ - & 0.000109 & -6.246E-05 & -1.847E-05 \\ - & - & 0.005487 & -0.000392 \\ - & - & - & 0.000492 \end{Bmatrix}$$

Note: Symmetry imposed during estimation.

TABLE 3

Brand Share Equations: Popular Price.

	1 Old Milwaukee	2 Genesee	3 Milwaukee's Best	4 Busch	5 Piels Lager
Constant	0.287	0.225	−0.019	0.531	–
	(0.062)	(0.067)	(0.063)	(0.079)	–
Time	−0.000	−0.001	0.000	0.001	–
	(0.000)	(0.000)	(0.000)	(0.000)	–
log (Y/P)	0.014	−0.018	0.001	0.004	–
	(0.006)	(0.007)	(0.007)	(0.008)	–
log ($P_{\text{Old Milwaukee}}$)	−0.979	0.235	0.369	0.257	–
	(0.028)	(0.021)	(0.022)	(0.030)	–
log (P_{Genesee})	0.235	−0.698	0.222	0.205	–
	(0.021)	(0.029)	(0.022)	(0.030)	–
log ($P_{\text{Milwaukee's Best}}$)	0.369	0.222	−1.048	0.388	–
	(0.022)	(0.022)	(0.036)	(0.035)	–
log (P_{Busch})	0.257	0.205	0.388	−0.892	–
	(0.030)	(0.030)	(0.035)	(0.062)	–
log (# of Stores)	−0.044	0.122	−0.023	−0.091	–
	(0.010)	(0.011)	(0.010)	(0.012)	–
Conditional Own	−4.789	−3.832	−5.813	−5.704	−3.956
Price Elasticity	(0.109)	(0.120)	(0.164)	(0.329)	(0.465)

$$\Sigma = \begin{Bmatrix} 0.000603 & -0.000123 & -0.000137 & -8.289\text{E}-05 \\ - & 0.000515 & -0.000143 & -7.136\text{E}-05 \\ - & - & 0.000758 & -0.000109 \\ - & - & - & 0.00262 \end{Bmatrix}$$

Note: Symmetry imposed during estimation.

Hausman, et.al (1994)

Table 5

Overall Elasticities.

	Elasticity	Standard Error
Budweiser	−4.196	0.127
Molson	−5.390	0.154
Labatts	−4.592	0.247
Miller	−4.446	0.149
Coors	−4.897	0.205
Old Milwaukee	−5.277	0.118
Genesee	−4.236	0.129
Milwaukee's Best	−6.205	0.170
Busch	−6.051	0.332
Piels	−4.117	0.469
Genesee Light	−3.763	0.072
Coors Light	−4.598	0.115
Old Milwaukee Light	−6.097	0.140
Lite	−5.039	0.141
Molson Light	−5.841	0.148

Light Segment Own and Cross Elasticities.

	Genesee Light	Coors Light	Old Milwaukee Light	Lite	Molson Light
Genesee Light	−3.763	0.464	0.397	0.254	0.201
	(0.072)	(0.060)	(0.039)	(0.043)	(0.037)
Coors Light	0.569	−4.598	0.407	0.452	0.482
	(0.085)	(0.115)	(0.058)	(0.075)	(0.061)
Old Milwaukee Light	1.233	0.956	−6.097	0.841	0.565
	(0.121)	(0.132)	(0.140)	(0.112)	(0.087)
Lite	0.509	0.737	0.587	−5.039	0.577
	(0.095)	(0.122)	(0.079)	(0.141)	(0.083)
Molson Light	0.683	1.213	0.611	0.893	−5.841
	(0.124)	(0.149)	(0.093)	(0.125)	(0.148)

- Relatively large own and cross price elasticities.
- Authors simulated partial merger analysis.
  - Hold prices of all non-merging parties fixed.
  - Solving for best-response of single-product.
  - How would full equilibrium analysis differ?
- Merger of Coors and Labatt's: Coors Markup 19.9% $\rightarrow$ 23.2% (small).
- Claim is that presence of other competitors constraints potential to raise prices. How? Why?

Hausman (1997) aka The Apple Cinnamon Cheerios War.

- What is the value of a new good? How should we adjust CPI?
- Potentially HUGE issue. Why?
- Weekly cereal data.. 7 cities, 137 weeks. Three segments (adults, kids, family) with max 9 brands.
- Calculate $e(p_{-n}, p_n^*, u)/e(\mathrm{p}, u)$. Find a virtual price p^* (or choke price) that leaves consumers as well off as a world without Apple-Cinnamon Cheerios.
- Virtual price is about $2\times$ actual price. CPI may be overstated by as much as 25% for all cereal brands (tons of new products).

Chaudhuri, Goldberg, Jia (AER 2006)

- Indian market for antibiotics: (foreign vs. domestic) (licensed vs. unlicensed producers).
- Different brands, packages, etc. also different active ingredients ($J = 300$ they aggregate to four active ingredients $\times$ country of origin).
- Monthly sales data (SKU level) for 4 regions in India (Market Research firm).
- What would prices and quantities look like if intellectual property rights were enforced and unlicensed producers were shut down?

Issues

- Products enter and exit the market. How do we model this?
- Dosages differ across products. How do we construct Q?
- Don't treat licensed v. unlicensed as different products. Why?

Results

- Estimate AIDS demand aggregated across demands
- Get upper and lower bounds on marginal costs
  - Assume that $p = mc$
  - Assume monopoly pricing.
- Calculate the virtual price or "choke price" that makes expenditures zero on unlicensed products.
- Get changes in consumer surplus (integrated demand curve) and producer profits without unlicensed firms.