Computational Details for IO Economists

Chris Conlon

Columbia University

October 4, 2010

Table of contents

Motivation

- Demand Estimation is one of the most important tools in the IO economists toolbox.
- BLP provide a parsimonious way to accurately represent substitution patterns with a simple model.
- BUT, Implementation can be tricky for number of reasons.
- Recent interest in IO about numerical properties/problems of these estimators.
- Estimating a structural models is a nonlinear optimization problem.

Motivation

Why Care about Computation at all?

- Still no standard implementation – “roll your own”.

Motivation

Why Care about Computation at all?

- Still no standard implementation – “roll your own”.
- We care about *replicability* of empirical studies.

Motivation

Why Care about Computation at all?

- Still no standard implementation – “roll your own”.
- We care about *replicability* of empirical studies.
- Results should not depend on implementation details.

Motivation

Why Care about Computation at all?

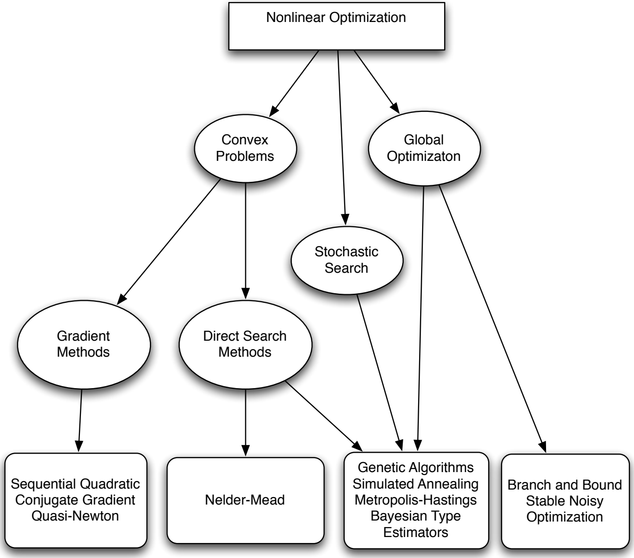
- Still no standard implementation – “roll your own”.
- We care about *replicability* of empirical studies.
- Results should not depend on implementation details.
- Performance Matters – (dynamics, robustness)

Motivation

Why Care about Computation at all?

- Still no standard implementation – "roll your own".
- We care about *replicability* of empirical studies.
- Results should not depend on implementation details.
- Performance Matters – (dynamics, robustness)
- Performance Matters even more when you get the wrong answer!

Optimization Methods



Recent Advances in Optimization Literature

Large Scale Algorithms

- Much focus has been on very large convex optimization problems – these have gotten really good.
- Most of these rely on first and second derivatives and quadratic approximations.
- Ways to do derivatives: analytic, numeric, symbolic and automatic (new!)
- Easy to solve 10,000+ parameter constrained problems often in less than 20 major iterations.
- Lots of industrial strength software packages.
- When in doubt express your problem as a convex one.
- Algorithm is polynomial $\approx O(k^3)$

Recent Advances in Optimization Literature

Non-convex Optimization

- Non-convex optimization is slower – most algorithms are exponential in number of parameters (Curse of Dimensionality). $\approx O(a^k)$
- Some stochastic search algorithms are faster (DE, etc.)
- In general, convergence properties of these algorithms are an open question (not good!).
- Non-smooth/Non-convex optimization is still hard – maybe 10 parameters for not too difficult objective functions.

Motivation

"Folk Theory" of Optimization in Economics

- Unconstrained Optimization is easier than Constrained Optimization

Consequences of Folk Theory

Motivation

"Folk Theory" of Optimization in Economics

- Unconstrained Optimization is easier than Constrained Optimization
- More parameters are harder

Consequences of Folk Theory

Motivation

“Folk Theory” of Optimization in Economics

- Unconstrained Optimization is easier than Constrained Optimization
- More parameters are harder
- Quasi-Newton Methods are unreliable

Consequences of Folk Theory

Motivation

"Folk Theory" of Optimization in Economics

- Unconstrained Optimization is easier than Constrained Optimization
- More parameters are harder
- Quasi-Newton Methods are unreliable

Consequences of Folk Theory

- Rewrite all problems as unconstrained optimization

Motivation

"Folk Theory" of Optimization in Economics

- Unconstrained Optimization is easier than Constrained Optimization
- More parameters are harder
- Quasi-Newton Methods are unreliable

Consequences of Folk Theory

- Rewrite all problems as unconstrained optimization
- Use fixed points and multi-step procedures to reduce parameter space

Motivation

"Folk Theory" of Optimization in Economics

- Unconstrained Optimization is easier than Constrained Optimization
- More parameters are harder
- Quasi-Newton Methods are unreliable

Consequences of Folk Theory

- Rewrite all problems as unconstrained optimization
- Use fixed points and multi-step procedures to reduce parameter space
- Use Nelder-Mead/Simplex methods for optimization

Motivation

Thanks to recent advances in optimization:

More Accurate Description of Optimizaiton

1. *Shape* of the problem is what matters – convexity is really important

Consequences of State of the Art Optimization

Motivation

Thanks to recent advances in optimization:

More Accurate Description of Optimizaiton

1. *Shape* of the problem is what matters – convexity is really important
2. Constrained Problems are not much more difficult

Consequences of State of the Art Optimization

Motivation

Thanks to recent advances in optimization:

More Accurate Description of Optimizaiton

1. *Shape* of the problem is what matters – convexity is really important
2. Constrained Problems are not much more difficult
3. More parameters can make the problem easier (or harder)

Consequences of State of the Art Optimization

Motivation

Thanks to recent advances in optimization:

More Accurate Description of Optimizaiton

1. *Shape* of the problem is what matters – convexity is really important
2. Constrained Problems are not much more difficult
3. More parameters can make the problem easier (or harder)

Consequences of State of the Art Optimization

- Tested stable Newton-routines are very reliable.

Motivation

Thanks to recent advances in optimization:

More Accurate Description of Optimizaiton

1. *Shape* of the problem is what matters – convexity is really important
2. Constrained Problems are not much more difficult
3. More parameters can make the problem easier (or harder)

Consequences of State of the Art Optimization

- Tested stable Newton-routines are very reliable.
- Good Solvers handle 10,000+ parameters

Motivation

Thanks to recent advances in optimization:

More Accurate Description of Optimizaiton

1. *Shape* of the problem is what matters – convexity is really important
2. Constrained Problems are not much more difficult
3. More parameters can make the problem easier (or harder)

Consequences of State of the Art Optimization

- Tested stable Newton-routines are very reliable.
- Good Solvers handle 10,000+ parameters
- Computational burden are Jacobian and Hessian (and storage)

Convexity

An optimization problem is convex if

$$\min_x f(\mathbf{x}) \quad s.t. \quad h(\mathbf{x}) \leq 0 \quad A\mathbf{x} = 0$$

- $f(\mathbf{x}), h(\mathbf{x})$ are convex (PSD second derivative matrix)
- Equality Constraint is affine

Some helpful identities about convexity

- Compositions and sums of convex functions are convex.
- Norms $||$ are convex, max is convex, log is convex
- $\log(\sum_{i=1}^n \exp(x_i))$ is convex.
- Fixed Points can introduce non-convexities.
- Globally convex problems have a unique optimum

Properties of Convex Optimization

- If a program is globally convex then it has a unique minimizer that will be found by convex optimizers.
- If a program is not globally convex, but is convex over a region of the parameter space, then most convex optimization routines find any local minima in the convex hull
- Convex optimization routines are unlikely to find local minima (including the global minimum) if they do not begin in the same convex hull as the optimum (starting values matter!).
- Most good commercial routines are clever about dealing with multiple starting values and handling problems that are well approximated by convex functions.
- Good Routines use information about sparseness of Hessian – this generally determines speed.

Nested Logit Model

FIML Nested Logit Model is Non-Convex

$$\min_{\theta} \sum_{j} q_j \ln P_j(\theta) \quad \text{s.t.} \quad P_j(\theta) = \frac{e^{x_j \beta/\lambda} (\sum_{k \in g_l} e^{x_j \beta/\lambda})^{\lambda - 1}}{\sum_{\forall l'} (\sum_{k \in g_l'} e^{x_j \beta/\lambda})^{\lambda}}$$

This is a pain to show but the problem is with the cross term $\frac{\partial^2 P_j}{\partial \beta \partial \lambda}$ because $\exp[x_j \beta/\lambda]$ is not convex.

A Simple Substitution Saves the Day: let $\gamma = \beta/\lambda$

$$\min_{\theta} \sum_{j} q_j \ln P_j(\theta) \quad \text{s.t.} \quad P_j(\theta) = \frac{e^{x_j \gamma} (\sum_{k \in g_l} e^{x_j \gamma})^{\lambda - 1}}{\sum_{\forall l'} (\sum_{k \in g_l'} e^{x_j \gamma})^{\lambda}}$$

This is much better behaved and easier to optimize.

Nested Logit Model

	Original[1]	Substitution[2]	No Derivatives[3]
Parameters	49	49	49
Nonlinear λ	5	5	5
Likelihood	2.279448	2.279448	2.27972
Iterations	197	146	352
Time	59.0 s	10.7 s	192s

Discuss Nelder-Meade

[1] KNITRO-AMPL
[2] KNITRO-AMPL
[3] fminunc-MATLAB

Computing Derivatives

A key aspect of any optimization problem is going to be computing the derivatives (first and second) of the model. There are some different approaches

- Numerical: Often inaccurate and error prone (why?)
- Pencil and Paper: this tends to be mistake prone – but often actually the fastest
- Automatic (AMPL): Software brute forces through a chain rule calculation at every step (limited language).
- Symbolic (Maple/Mathematica): software "knows" derivatives of certain objects and can do its own simplification. (limited language).

Extremum Estimators

Often faced with extremum estimator problems in econometrics (ML, GMM, MD, etc.) that look like:

$$\hat{\theta} = \arg\max_{\theta} Q_n(\theta), \quad \theta \in \Theta \tag{1}$$

Many economic problems contain constraints, such as: market clearing (supply equals demand), consumer's consume their entire budget set, or firm's first order conditions are satisfied. A natural way to represent these problems is as constrained optimization.

Constrained Problems

MPEC

$$\hat{\theta} = \arg\max_{\theta} Q_n(\theta, P), \quad \text{s .t.} \quad \Psi(P, \theta) = 0, \quad \theta \in \Theta$$

Constrained Problems

MPEC

$$\hat{\theta} = \arg\max_{\theta} Q_n(\theta, P), \qquad \text{s .t.} \qquad \Psi(P, \theta) = 0, \quad \theta \in \Theta$$

Fixed Point / Implicit Solution

In much of the literature the tradition has been to express the solutions $\Psi(P, \theta) = 0$ implicitly as $P(\theta)$:

$$\hat{\theta} = \arg\max_{\theta} Q_n(\theta, P(\theta)), \quad \theta \in \Theta$$

MPEC

Su and Judd (2008) show a combination of the MPEC approach and modern optimization software change the rules of the game:

New Rules:

1. Number of parameters isn't the major concern
2. Constraints are OK
3. Good shape (nice Hessian) is the key determinant of difficulty

MPEC

Su and Judd (2008) show a combination of the MPEC approach and modern optimization software change the rules of the game:

New Rules:

1. Number of parameters isn't the major concern
2. Constraints are OK
3. Good shape (nice Hessian) is the key determinant of difficulty

The calculus is different

- Not just about better computation for existing methods
- Given the MPEC framework what else is possible?
- What are the *best methods* now?

BLP Demand Example

BLP 1995

The estimator solves the following mathematical program:

$$
\begin{aligned}
\min_{\theta_2} & \quad g(\xi(\theta_2))' W g(\xi(\theta_2)) \quad \text{s.t.} \\
g(\xi(\theta_2)) &= \frac{1}{N} \sum_{\forall j,t} \xi_{jt}(\theta_2)' z_{jt} \\
\xi_{jt}(\theta) &= \delta_j(\theta_2) - x_{jt}\beta - \alpha p_{jt} \\
s_{jt}(\delta, \theta_2) &= \int \frac{\exp[\delta_j(\theta_2) + \mu_{ij}]}{1 + \sum_k \exp[\delta_j(\theta_2) + \mu_{ik}]} f(\mu|\theta_2) \\
\log(S_{jt}) &= \log(s_{jt}(\delta, \theta_2)) \quad \forall j, t
\end{aligned}
$$

BLP Algorithm

The estimation algorithm is generally as follows:

1. Guess a value of nonlinear parameters θ_2
2. Compute $s_{jt}(\delta, \theta_2)$ via integration
3. Iterate on $\delta_{jt}^{h+1} = \delta_{jt}^{h} + \log(S_{jt}) - \log(s_{jt}(\delta^h, \theta_2))$ to find the δ that satisfies the share equation
4. IV Regression δ on observable X and instruments Z to get residual ξ.
5. Use ξ to construct $g(\xi(\theta_2))$.
6. Construct GMM Objective

Analysis

Pros

- Plugging in constraints and using the contraction mapping means you can dramatically reduce the parameter space for nonlinear search (particularly for product dummies case).
- Without large-scale optimization software – this may be your only hope if you have a lot of linear parameters.

Cons

- The rate on the contraction mapping is SLOW compared to modern optimization procedures. (DFS 2007)
- Fixed points tend create highly non-convex problems.
- Unless contraction mapping begins for fixed δ^0 for every guess of θ then objective function will not always produce the same value for different guesses of θ (no numeric gradients).
- Enforces constraints regarding marketshares and linear parameters hold not just at the optimum, but at every guess which severely restricts the path optimization routine may

Dube Fox Su 2009

An MPEC Algorithm for computing the same estimator as BLP has been suggested:

BLP-MPEC

The estimator solves the following mathematical program:

$$\begin{aligned}
\min_{\theta,\xi,s,g} \quad & g(\xi)'Wg(\xi) \quad \text{s.t.} \\
g(\xi) &= \frac{1}{N}\sum_{\forall j,t} \xi'_{jt} z_{jt} \\
s_{jt}(\theta) &= \int \frac{\exp[x_{jt}\beta_i + \xi_{jt} - \alpha_i p_{jt}]}{1+\sum_k \exp[x_{kt}\beta_i + \xi_{kt} - \alpha_i p_{kt}]} f(\beta_i|\theta) \\
\log(S_{jt}) &= \log(s_{jt}(\beta,\alpha,\xi,\theta)) \quad \forall j,t
\end{aligned}$$

(Knittel Metaxoglou 2008)

Recent paper (just updated) takes 10 algorithms, 50 starting values and uncovers 100+ parameter estimates and Nevo code/data:

- *a local minimum may yield parameter values that are close to the true values but have an objective function value that is very different. Therefore we focus on the economic meaning of the variation in parameter estimates...*
- *... researchers will need to use multiple starting values, at least 50 and multiple algorithms*
- Mistakes abound
- What weight matrix is used?

Nevo Results

	Nevo
Price	-28.189
σ_p	0.330
σ_{const}	2.453
σ_{sugar}	0.016
σ_{mushy}	0.244
$\pi_{p,inc}$	15.894
$\pi_{p,inc2}$	-1.200
$\pi_{p,kid}$	2.634
$\pi_{c,inc}$	5.482
$\pi_{c,age}$	0.2037
GMM	29.3611
EL	
Time	28 s

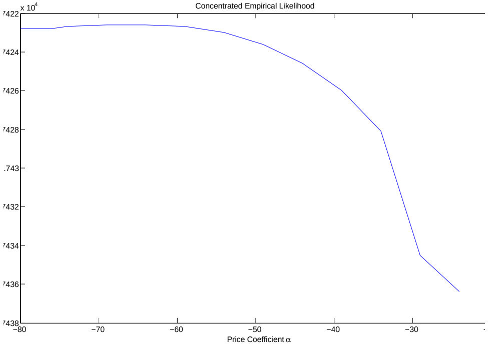
Nevo Results

	Nevo	BLP-MPEC
Price	-28.189	-62.726
σ_p	0.330	0.558
σ_{const}	2.453	3.313
σ_{sugar}	0.016	-0.006
σ_{mushy}	0.244	0.093
$\pi_{p,inc}$	15.894	588.206
$\pi_{p,inc2}$	-1.200	-30.185
$\pi_{p,kid}$	2.634	11.058
$\pi_{c,inc}$	5.482	2.29084
$\pi_{c,age}$	0.2037	1.284
GMM	29.3611	4.564
EL		
Time	28 s	12s

Nevo Results

	Nevo	BLP-MPEC	EL
Price	-28.189	-62.726	-61.433
σ_p	0.330	0.558	0.524
σ_{const}	2.453	3.313	3.143
σ_{sugar}	0.016	-0.006	0
σ_{mushy}	0.244	0.093	0.085
$\pi_{p,inc}$	15.894	588.206	564.262
$\pi_{p,inc2}$	-1.200	-30.185	-28.930
$\pi_{p,kid}$	2.634	11.058	11.700
$\pi_{c,inc}$	5.482	2.29084	2.246
$\pi_{c,age}$	0.2037	1.284	1.37873
GMM	29.3611	4.564	
EL			-17422
Time	28 s	12s	19s

Profile Empirical Likelihood

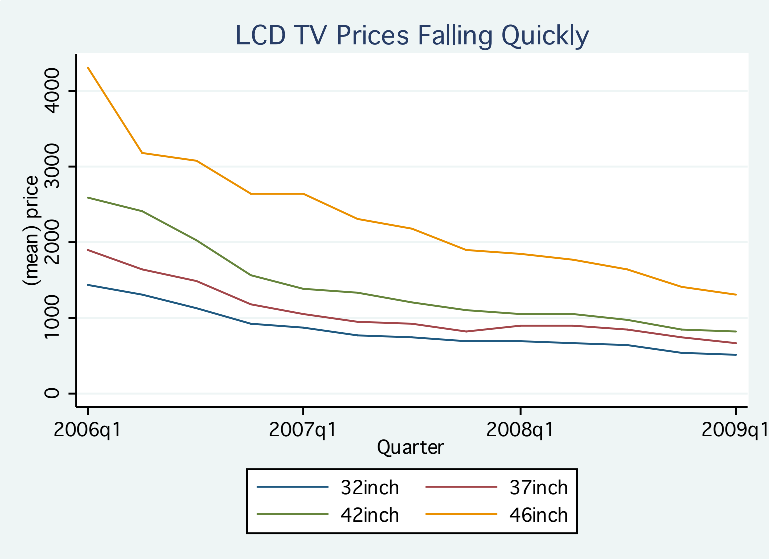


Introduction

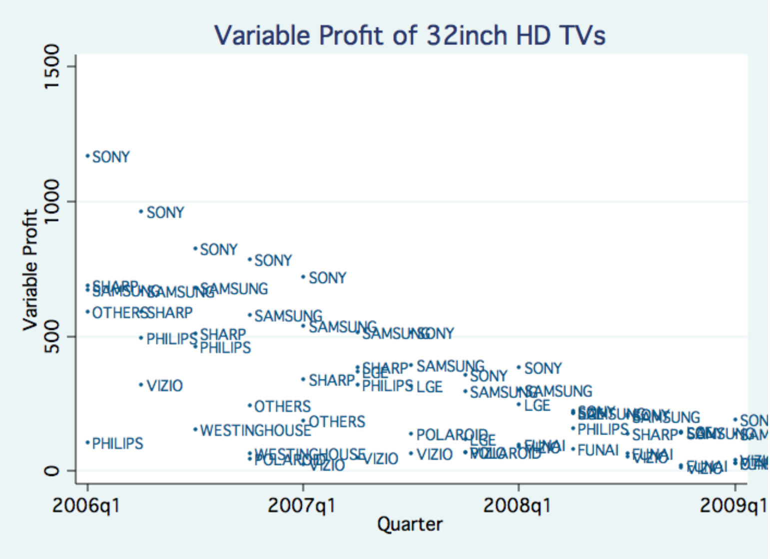
I study the relationship between *dynamic consumer behavior* and *prices* firms charge in equilibrium.

More specifically, I examine recent price declines in the market for LCD Televisions and the inter-temporal tradeoffs faced by consumers.

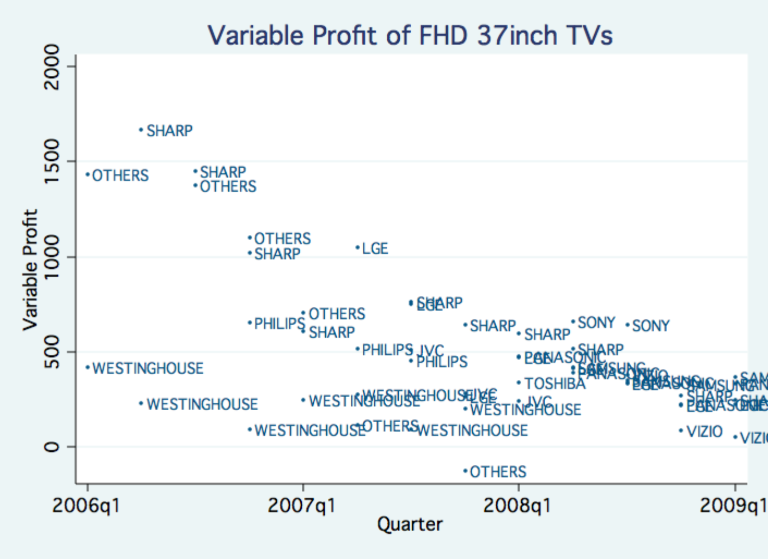
Price Declines in LCD TV's



Price Declines in LCD TV's



Price Declines in LCD TV's



Reasons for Low Prices

There are four explanations for why prices might be low:

1. Costs are low
2. Competition
3. High value consumers have left the market
4. Consumers have strategic option to delay purchase

Reasons for Low Prices

There are four explanations for why prices might be low:

1. Costs are low
2. Competition
3. High value consumers have left the market
4. Consumers have strategic option to delay purchase

Reasons for Low Prices

There are four explanations for why prices might be low:

1. Costs are low
2. Competition
3. High value consumers have left the market
4. Consumers have strategic option to delay purchase

These factors have different implications for:

1. Research & Development
2. Antitrust Policy

Literature

There is a large theoretical literature on these inter-temporal tradeoffs.

- Coase Conjecture: Coase (1972)
- Related literature Stokey (1982), Bulow (1982), etc.
- Theory literature mostly focuses on analytic results for single-product monopoly case

There is also a small but growing empirical literature:

- Melnikov (2001), Carranza (2007), Zhao (2008), Lee(2008)
- Gowrisankaran & Rysman (2009)
- Nair (2007)
- Erdem, Imai, & Keane (2003), Hendel & Nevo (2007)

Outline

1. Write down a dynamic model of consumer behavior.
2. Estimate the model.
3. Recompute prices in the absence of an option to wait.
4. Recompute prices in when the distribution of consumers is fixed.

Outline

1. Write down a dynamic model of consumer behavior.
2. Estimate the model.
3. Recompute prices in the absence of an option to wait.
4. Recompute prices in when the distribution of consumers is fixed.

This differs from past approaches in the literature in the following ways:

1. Focus on pricing problem not just demand side
2. Better data on manufacturer costs
3. Better computational technique (MPEC)
4. Improved statistical procedure (EL)

Model

Each consumer type is subscripted by i, and chooses a product j in period t to maximize utility:

$$\begin{aligned} u_{ijt} &= \alpha_i^x x_{jt} - \alpha_i^p p_{jt} + \xi_{jt} + \varepsilon_{ijt} \\ u_{i0t} &= \overline{u}_{i0t} + \varepsilon_{i0t} \end{aligned}$$

Model

Each consumer type is subscripted by i, and chooses a product j in period t to maximize utility:

$$\begin{aligned}
u_{ijt} &= \alpha_i^x x_{jt} - \alpha_i^p p_{jt} + \xi_{jt} + \varepsilon_{ijt} \\
u_{i0t} &= \overline{u}_{i0t} + \varepsilon_{i0t}
\end{aligned}$$

Consumers have a continuation value which depends on their existing television stock u_{i0t}.

$$\begin{aligned}
V_i(u_{i0t}, \varepsilon_{it}, \Omega_t) &= \max\{u_{i0t} + \beta E[E_\varepsilon V_i(u_{i0t}, \varepsilon_{it}, \Omega_{t+1})|\Omega_t], \\
&\quad \max_j u_{ijt} + \beta E[E_\varepsilon V_i(u_{ijt}, \varepsilon_{it}, \Omega_{t+1})|\Omega_t]\}
\end{aligned}$$

Assumptions

Assumption: No Upgrades

We rule out upgrades. After making a purchase consumers exit the market. $V_i(u_{ijt}, \cdot) = 0$ when $j \neq 0$.

Assumptions

Assumption: No Upgrades

We rule out upgrades. After making a purchase consumers exit the market. $V_i(u_{ijt}, \cdot) = 0$ when $j \neq 0$.

- Right assumption for the industry (multiple purchases, utility differences)
- Simplifies state space (don't keep track of TV stocks)
- Not necessary for estimation

Solving the Model

If ε_{ijt} is IID and Extreme Value, the model can be simplified. Note that the expected utility that i receives from making a purchase in period t does not depend on which product j is purchased.

$$\begin{aligned}\delta_{it} = E[\max_j u_{ijt}] &= \log \sum_j \exp(x_{jt}\alpha_i^x - \alpha_i^p p_{jt} + \xi_{jt}) \\ V_i(\Omega_t) &= \int V_i(\varepsilon_{ijt}, \Omega_t) f(\varepsilon)\end{aligned}$$

(Standard Abuse of Notation)

Solving the Model (2)

Now we can use Rust (1987) to simplify the dynamic stopping problem.

$$
\begin{aligned}
V_i(\varepsilon_{it}, \Omega_t) &= \max\{u_{i0t} + \beta E[E_\varepsilon V_i(\varepsilon_{it}, \Omega_{t+1})|\Omega_t], \max_j u_{ijt}\} \\
V_i(\Omega_t) &= \log\left(\exp(\beta E[V_i(\Omega_{t+1})|\Omega_t]) + \exp(\delta_{it})\right) + \eta
\end{aligned}
$$

Challenges

The key remaining challenge is that Ω_t is infinite dimensional.

Literature: IVS Assumption

The literature exploits the fact that $V_i(\Omega_t)$ recursively depends on itself and the inclusive value to assume that $\Omega_t = \delta_{it}$ and $P(\Omega_{t+1}|\Omega_t) = P(\delta_{i,t+1}|\delta_{it})$.

Challenges

The key remaining challenge is that Ω_t is infinite dimensional.

Literature: IVS Assumption

The literature exploits the fact that $V_i(\Omega_t)$ recursively depends on itself and the inclusive value to assume that $\Omega_t = \delta_{it}$ and $P(\Omega_{t+1}|\Omega_t) = P(\delta_{i,t+1}|\delta_{it})$.

- A common choice of functional form is that δ_{it} follows an AR(1).
- The problem with this is that it is not the result of economic behavior.

Assumption 2

I make a different assumption on the beliefs of consumers:

Perfect Foresight

$$
\begin{aligned}
v_{i,t+1} &= E[V_i(\Omega_{t+1})|\Omega_t]) \\
v_{it} &= \log\left(\exp(\beta v_{i,t+1}) + \exp(\delta_{it})\right) + \eta
\end{aligned}
$$

Assumption 2

I make a different assumption on the beliefs of consumers:

Perfect Foresight

$$
\begin{aligned}
v_{i,t+1} &= E[V_i(\Omega_{t+1})|\Omega_t]) \\
v_{it} &= \log\left(\exp(\beta v_{i,t+1}) + \exp(\delta_{it})\right) + \eta
\end{aligned}
$$

This may be more reasonable than it seems:

- Consumers know future value of market exactly (not all characteristics of all products)
- Just deviation in utility of outside option
- Already have ε_{ijt} and ξ_{jt}.

Solving the Model (3)

Now we can write the purchase probabilities for type i

$$s_{ijt} = \frac{e^{\delta_{it}}}{e^{v_{it}}} \cdot \frac{e^{\beta_i x_{jt} - \alpha_i p_{jt} + \xi_{jt}}}{e^{\delta_{it}}}$$

Solving the Model (3)

Now we can write the purchase probabilities for type i

$$s_{ijt} = \frac{e^{\delta_{it}}}{e^{v_{it}}} \cdot \frac{e^{\beta_i x_{jt} - \alpha_i p_{jt} + \xi_{jt}}}{e^{\delta_{it}}}$$

Each type has an associated weight w_{it} in each period. Consumers leave the market after making a purchase so that:

$$w_{i,t+1} = w_{i,t} s_{i0t}$$

The Estimation Problem

$$
\begin{aligned}
\min_{(\rho_{jt}, v_{it}, w_{it}, s_{ijt}, \delta_{it}, \xi_{jt}, \theta)} \sum_{j,t} \quad & g(\xi)' W g(\xi) \quad \text{s.t.} \quad S_{jt} = s_{jt} \\
g(\xi) &= \frac{1}{N} \sum_{\forall j,t} \xi'_{jt} z_{jt} \\
s_{jt} &= \sum_i w_{i,t} s_{ijt} \\
w_{i,t+1} &= w_{i,t}(1 - \sum_j s_{ijt}) \\
\exp[\delta_{it}] &= \sum_j \exp[x_{jt}\alpha_i^x - \alpha_i^p p_{jt} + \xi_{jt}]) \\
s_{ijt} &= \exp[x_{jt}\alpha_i^x - \alpha_i^p p_{jt} + \xi_{jt} - v_{it}] \\
v_{it} &= \log(\exp(\delta_{it}) + \exp(\beta v_{i,t+1}))
\end{aligned}
$$

The MPEC Approach

Estimate via the MPEC approach of Judd and Su (2008)

- Solve the problem directly using constrained optimization
- Key is that constraints only need to hold at optimum
- Problem has a LOT of parameters
- Problem is nearly convex
- Problem is highly sparse
- Dynamics are NOT approximated

Removing Perfect Foresight

Two options to relax perfect foresight

- Macroeconomics: Add an error to the outside good utility v_{it} and do a Hansen Singleton (IV) procedure.
- Put beliefs on $E[v_{i,t+1}|\delta_{it}]$. (Usually an AR(1)). (Carranza, G+R, etc.)

Multiple Purchases

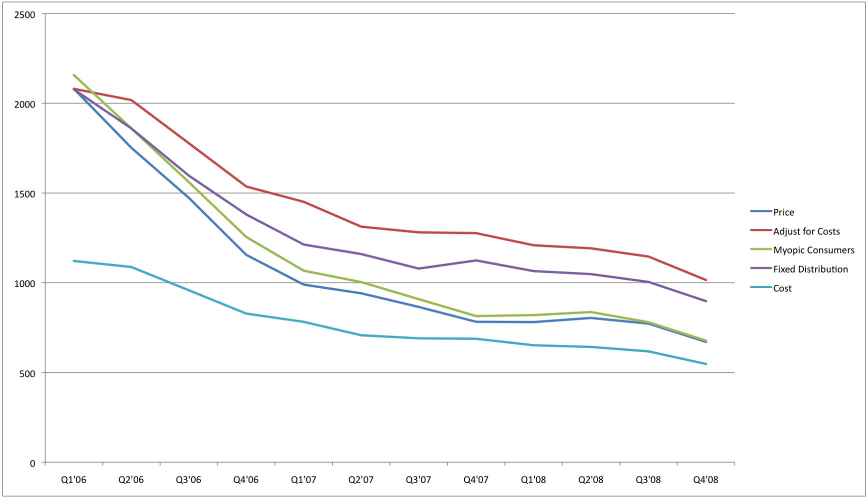
G&R (2009) allow for multiple (replacement) purchases

We can accommodate this by expanding the type space.
Your type is now your $(\beta_i \times \delta_{ijt})$ or your flow utility from holding good j in period t.
This complicates the transition rule.

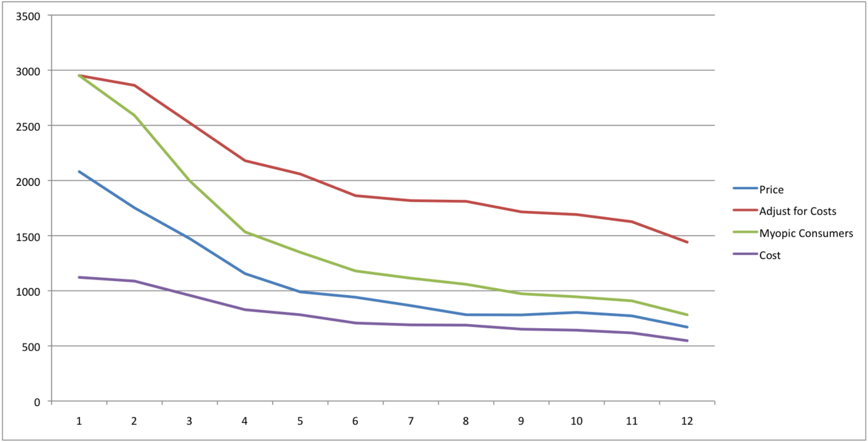
Hendel and Nevo (2007): Consumer Inventories

Now we also keep track of how much of each product consumer holds. (Even larger state space).

Price Declines in LCD TV's



Price Declines in LCD TV's



How do I do empirical work?

- SQL: Large empirical datasets often live in big databases
- AMPL: A modeling language with automatic differentiation. Easy to express problems- fast to code. Not so good about memory (Why?)
- Matlab: When AMPL doesn't work
- Fortran/C: for the parts of Matlab that are too slow

Implementation Tips

- Write Design Documents First!
- Test on Fake Data!
- Separate Data from Algorithms!
- Fewer Lines!
- Examples...