a Draw the project diagram.

b Determine the length of the critical path and the critical activities for this project.

9 Explain why an activity's free float can never exceed the activity's total float.

10 A project is complete when activities A–E are completed. The predecessors of each activity are shown in Table 25. Draw the appropriate project diagram. (*Hint:* Don't violate rule 4.)

11 Determine the probabilities that 1–2–4 and 1–3–4 are critical paths for Figure 37.

12 Given the information in Table 26, **(a)** draw the appropriate project network, and **(b)** find the critical path.

13 The government is going to build a high-speed computer in Austin, Texas. Once the computer is designed (D), we can select the exact site (S), the building contractor (C), and the operating personnel (P). Once the site is selected, we can begin erecting the building (B). We can start manufacturing the computer (COM) and preparing the operations manual (M) only after contractor is selected. We can begin training the computer operators (T) when the operating manual and personnel selection are completed. When the computer and the building are both finished, the computer may be installed (I). Then the computer is considered operational. Draw a project network that could be used to determine when the project is operational.

14 Write a LINGO program that can be used to crash the project network of Example 6 with the crashing costs given in Table 14.

15 Consider the project diagram in Figure 42. This project must be completed in 90 days. The time required to complete each activity can be reduced by up to five days at the costs given in Table 27.

Formulate an LP whose solution will enable us to minimize the cost of completing the project in 90 days.

16–17 Find the critical path, total float, and free float for each activity in the project networks of Figures 43 and 44.

TABLE 25

Activity	Predecessors
A	—
B	A
C	A
D	B
E	B, C

TABLE 26

Activity	Immediate Predecessors	Duration (Days)
A	—	3
B	—	3
C	—	1
D	A, B	3
E	A, B	3
F	B, C	2
G	D, E	4
H	E	3

FIGURE 42

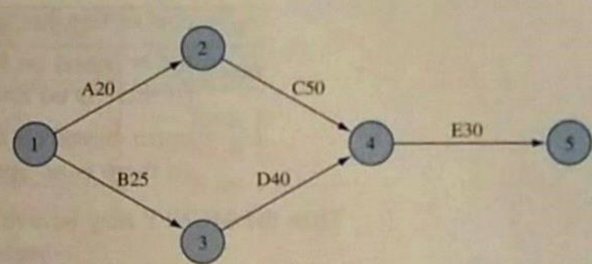


TABLE 27

Activity	Cost of Reducing Activities Duration by 1 Day ($)
A	300
B	200
C	350
D	260
E	320

FIGURE 43

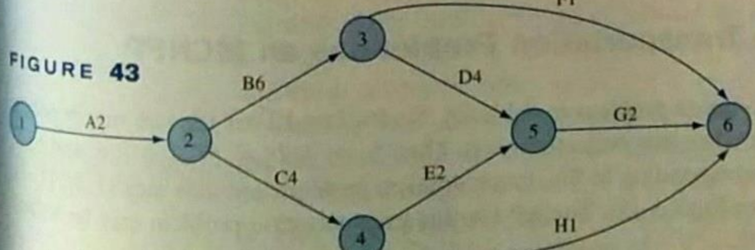
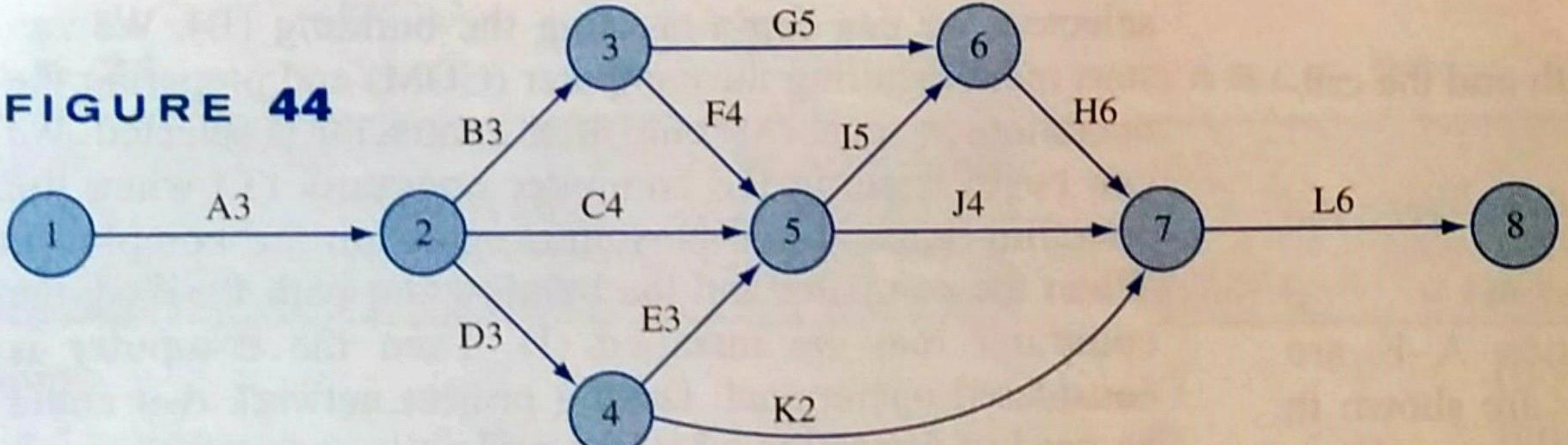


FIGURE 44



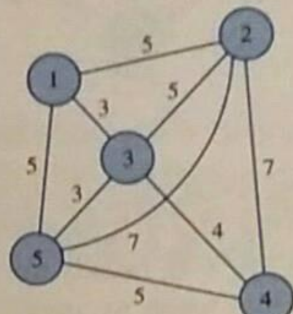
PROBLEMS

Group A

1 The distances (in miles) between the Indiana cities of Gary, Fort Wayne, Evansville, Terre Haute, and South Bend are shown in Table 38. It is necessary to build a state road system that connects all these cities. Assume that for political reasons no road can be built connecting Gary and Fort Wayne, and no road can be built connecting South Bend and Evansville. What is the minimum length of road required?

2 The city of Smalltown consists of five subdivisions. Mayor John Lion wants to build telephone lines to ensure that all the subdivisions can communicate with each other. The distances between the subdivisions are given in Figure 50. What is the minimum length of telephone line required? Assume that no telephone line can be built between subdivisions 1 and 4.

FIGURE 50
Network for Problem 2



Group B

3 In this problem, we explain why the MST algorithm works. Define

S = minimum spanning tree

C_t = nodes connected after iteration t of MST algorithm has been completed

C'_t = nodes not connected after iteration t of MST algorithm has been completed

A_t = set of arcs in minimum spanning tree after t iterations of MST algorithm have been completed

Suppose the MST algorithm does not yield a minimum spanning tree. Then, for some t, it must be the case that all arcs in A_{t-1} are in S, but the arc chosen at iteration t (call it a_t) of the MST algorithm is not in S. Then S must contain some arc a'_t that leads from a node in C_{t-1} to a node in C'_{t-1}). Show that by replacing arc a'_t with arc a_t, we can obtain a shorter spanning tree than S. This contradiction proves that all arcs chosen by the MST algorithm must be in S. Thus, the MST algorithm does indeed find a minimum spanning tree.

4 **a** Three cities are at the vertices of an equilateral triangle of unit length. Flying Lion Airlines needs to supply connecting service between these three cities. What is the minimum length of the two routes needed to supply the connecting service?

b Now suppose Flying Lion Airlines adds a hub at the "center" of the equilateral triangle. Show that the length of the routes needed to connect the three cities has decreased by 13%. (*Note:* It has been shown that no matter how many "hubs" you add and no matter how many points must be connected, you can never save more than 13% of the total distance needed to "span" all the original points by adding hubs.)†

TABLE 38

	Gary	Fort Wayne	Evansville	Terre Haute	South Bend
Gary	—	132	217	164	58
Fort Wayne	132	—	290	201	79
Evansville	217	290	—	113	303
Terre Haute	164	201	113	—	196
South Bend	58	79	303	196	—