Frequently Asked Question in Quant Interviews

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Contents

1 Probability **1**
- 1.1 Probability and Expected Value Calculation . . . 1
- 1.2 Game Theory . . . 11
- 1.3 Stochastic Processes . . . 16
  - 1.3.1 Markov Chain . . . 16
  - 1.3.2 Random Walk . . . 17

2 Math **17**
- 2.1 Limits and Functions . . . 18
- 2.2 Calculus . . . 18
- 2.3 Linear Algebra . . . 18
- 2.4 Combinatorics . . . 18

3 Statistics and Econometrics Methods **20**
- 3.1 Statistical Properties . . . 21
- 3.2 Linear Regression . . . 22
- 3.3 Asset Pricing . . . 24

4 Machine Learning and Data Analytic **24**

5 Finance **25**
- 5.1 Stochastic Finance . . . 25
- 5.2 Derivatives . . . 26
- 5.3 Others . . . 26

6 Coding **27**
- 6.1 String Processing . . . 28
- 6.2 Dynamic Programming . . . 29
- 6.3 Divide and Conquer . . . 29
- 6.4 Data Structure . . . 30
- 6.5 Others . . . 30

7 Others **32**

1 Probability

1.1 Probability and Expected Value Calculation

Question 1.1: (HRT)

Two people are playing a game, and the current score is $4 : 7$. Player A has a probability of $\frac{1}{2}$ of winning each point. Assuming the total score goes to 11 points, what is the probability that A will win?"

Question 1.2: (HRT)

A and B are playing a game. A has a $\frac{2}{3}$ chance of winning each round. The winner is the first to have at least 3 points and be ahead by at least 2 points. What is the probability that A wins?

Question 1.3: (HRT)

What is the expected number of die rolls to get a 6, conditioned on the event that all rolls show even numbers? Compare this scenario with flipping 100 dice versus 600 coins, and find $P(X > Y)$.

Question 1.4: (Two Sigma)

A $3 \times 3 \times 3$ cube is made up of smaller blue cubes. The outer surface of the large cube is painted red. After disassembling the cube, I randomly pick a block and roll it, revealing 5 blue faces. What is the probability that the face touching the table is also blue?

Question 1.5: (Two Sigma)

Throw a needle of length 1 onto a 1×1 grid. What is the probability that the needle intersects the grid lines at two points?

Question 1.6: (Citadel GFI HRT)

Players take turns selecting numbers from $U(0, 1)$. The process stops when a number greater than 1 is selected. What is the expected number of rounds?

Question 1.7: (Citadel Sec Index Arb)

What is the expected value of the index of the first person who surpasses the one before them in a row of people?

Question 1.8: (Belvedere Trading QR Intern; 2024)

Please find the expected number of HHHHHHTTTTTT after flipping a coin infinite times.

Answer 1.1: The expected number is $+\infty$. If we treat every 12 flips as a group, then the expected number of HHHHHHTTTTTT within each group is non-negative. Clearly, the sum of expected value of these ho. is smaller than the one asked in the question, since there are also some "interaction terms" e.g. from 5 to 17 . But even for the smaller sum, the sum will be infinite.
The question is related to the **Infinite Monkey Theorem**. The probability that we flip a HHHHHHTTTTTT infinitely times will be 1 . Here is a proof. Suppose that we divide the sequence into multiple group with each group having 12 flip. Let E_k be the event that the k-th(group is HHHHHHTTTTTT. These events are independent. Then

$$\sum \mathbb{P}(E_k) = +\infty.$$

By Borel-Cantelli Lemma, we have $\mathbb{P}(\limsup E_k) = 1$.

Question 1.9: (HRT QR; 2024)

A and B roll two dices. If the sum of A's roll is 6, A wins. If the sum of B's roll is 7, B wins. A first rolls one round and B rolls. Starting at B, the two people roll two rounds each. What is the probability for A to win the game?

Answer 1.2: The answer is

$$\frac{5}{36} + \frac{31}{36}\sum_{n=1}^{\infty}\left(\frac{5}{6}\right)^{2n}\left(\frac{31}{36}\right)^{2n-2}\frac{5}{36}\left(1+\frac{31}{36}\right) \approx 0.457$$

Question 1.10: (Jane Street QT Intern; 2024)

Given a twelve-sided dice, you roll the dice repeatedly until the cumulative sum is odd. What is the number of the cumulative sum you can have with the highest probability?

Answer 1.3: Clearly, before your first odd number, the previous rolls must be event number. Note that the the probability of stopping at step n is $(1/2)^n$ and that for any number larger than 11 , you need at least 2 rolls. Thus we can only look at the first and the second roll. You can find the 11 will occurs with largest probability.

Question 1.11: (SIG QT Intern; 2024)

I have 1 red and 10 white balls. I have to put all the balls into two jars. Then I pick one jar at random and pick a ball at random. How should I put the balls into two jars so that I have the highest chance of picking the red ball?

Answer 1.4: It must be 1 red in a jar and the 10 white in another. The probability is 1/2.

Question 1.12: (SIG QT; 2024)

You are an owner of a large company. Now a small company want to sell their company to you. You know that the value of the small company will double after your company buy it (because you have a better management team). But you don't know the company's current value (the small company itself knows). Should you buy it? What is your strategy of bidding?

Answer 1.5: Yes, you should buy it. Each time when you offer a price x, if the company doesn't accept it, you know the value of the company y is larger than your x. Then you keep doubling your offer until the company accepts it, you can make sure that you won lose money unless your first offer is accepted. I think this question wants to test whether you are a Bayesian guy. If there are some other conditions e.g. the number of bids you can make, you can have a prior first and then update it with the information when your bid is rejected i.e. $y > x$.

Question 1.13: (SIG QR; 2025)

Randomly pick two points on a unit circle and what is the expected length of the arc formed by them.

Answer 1.6: Equivalent to randomly draw $x \sim$ Uniform $([0, 2\pi])$. Thus the expected arc is π.

Question 1.14: (SIG QR; 2025)

There are 8 seats in a round table. Now there are 3 people sitting there and what is the probability that at least two people sit together.

Answer 1.7: Label the seat from 1 to 8 . It is easy to see that the answer is

$$\frac{6 \times 8 + 2 \times 8 \times 4}{8 \times 7 \times 6} = \frac{1}{3}$$

Question 1.15: (InfiniteQuant QR Intern; 2024)

You have a thirty-sided and I have a twenty-sided dice. We roll both of them and whoever gets the higher roll wins. If we roll the same amount, I win. The loser pays the dollar amount of the winning roll. What is the expected value of this game for me? How does this value change if I have the option to re-roll once when losing the first roll?

Answer 1.8: We split the question into several sub-questions.

1. There are 20×30 situations in total. If we consider it as a 30×20 matrix and each position occurring with the same probability, we know in the upper 20×20 sub-matrix the upper triangle cancel with the lower triangle. Thus for the first 20 rows, only the diagonal elements matter. For the rows after 20, I lose money all the time. Thus the expected value is

$$\frac{1}{20 \times 30} \sum_{n=1}^{20} n - \frac{1}{30} \sum_{n=21}^{30} n = -8.15$$

2. The EV can be calculated using conditional expectation. You should note that if I lose, your roll cannot be 1 (and with small probability of rolling a small number).

$$\begin{aligned} &\mathbb{P}(\text{I win})\mathbb{E}\left(\text{value I get } \mid \text{I win}\right) + \mathbb{P}(\text{I lose})\mathbb{E}\left(\text{value I get} \mid \text{I lose}\right) \\ &= \frac{7}{20} \sum_{i=1}^{20} \frac{i^2}{210} - \frac{13}{20} \times \mathbb{E}\left(\text{value I get } \mid \text{I lose}\right). \end{aligned}$$

Here, I separate the $\mathbb{E}\left(\text{value I get} \mid \text{I lose}\right)$ term because it is not easy to calculate due to the change of the distribution of your first roll.

$$\begin{aligned} \mathbb{E}\left(\text{value I get} \mid \text{I lose}\right) &= \sum_{i=2}^{30} \mathbb{E}\left(\text{value I get} \mid \text{I lose and your roll is } i\right) \times \mathbb{P}(i \mid \text{I lose}) \\ &= \sum_{i=2}^{20} \frac{i-1}{390 \times 20} \left(\sum_{j=i}^{20} j - i(i-1) \right) - \sum_{i=21}^{30} \frac{20i}{390} \\ &\approx -15.37 \end{aligned}$$

Thus the new EV is around -5.2.

Question 1.16: (Citadel Sec Trader Intern; 2024)

There are five boxes: one has \$15, the rest are empty. At any time, you may pay X dollars to open one of the boxes and keep the contents. Assuming you continue paying until you find the box with the money, what value of X (in dollars) makes the game fair?

Answer 1.9: You can find the \$15 in at most five steps since there is only five boxes. The expected step needed is 3 and the fair value should be \$5.

Question 1.17: (Citadel Sec Trader Intern; 2024)

12 people from 6 married couples are randomly assigned to seats around a round table. What is the expected number of couples that are seated next to each other?

Answer 1.10: Let I_i be the indicator r.v. for the event that the i-th couple sit together. Then the expected number of couples seated next to each other is

$$\mathbb{E}\left[\sum_{i=1}^{6} I_i\right] = 6 \times \mathbb{P}\Big(\text{the } i\text{-th couple seated together} = 6 \times \frac{2}{11} = \frac{12}{11}.$$

Here are some further questions that may be asked.

1. What is the probability that the i-th couple seated together conditional on the j-th couple seated together?

2. What is the probability that no couple is seated together?

For 1, we can think that the j-th couple break the circle and thus their are 10! ways of arrangement. If we further treat the i-th couple as a single person i.e. seated together, there are $2 \times 9!$ ways. So the probability is $1/5$. For 2, let C_i be the event that the i-th couple sit together. Based on 1, we can calculate $\mathbb{P}(C_1, C_2)$ and

$$\Pr(C_1 \mid C_2, C_3) = \frac{\Pr(C_1, C_2 \mid C_3)}{\Pr(C_2 \mid C_3)} = \frac{2}{9}.$$

Thus we have $\Pr(C_1, C_2, C_3)$. Following this calculation, we have $\Pr(C_1, C_2, C_3, C_4)$, $\Pr(C_1, C_2, C_3, C_4, C_5)$ and $\Pr(C_1, C_2, C_3, C_4, C_5, C_6)$. Then we can use $1 - \Pr(\cup C_i)$ and the expansion of $\Pr(\cup C_i)$ to calculate the probability.

Question 1.18: (Citadel Sec Trader Intern; 2024)

In a group of people, each person' favorite color is either red, blue, yellow, or green. If 15 people's favorite color isn't red, 23 people's favorite color isn't blue, 20 people's favorite color isn't yellow, an 17 people's favorite color isn't green, how many total people are there?

Answer 1.11: 75/3=25.

Question 1.19: (HRT Algo Dev Intern; 2024)

Given a thirty-sided dice, you roll the dice repeatedly until the cumulative sum is greater than or equal to 300 . What is the most likely result of the sum?

Answer 1.12: Let the probability of the cumulative sum being n to be p_n. Then we know

$$p_n = \sum_{i=1}^{30} \frac{p_{n-i}}{30}.$$

Then we have

$$\begin{aligned}
p_{300} &= \frac{1}{30}\left(p_{299} + p_{298} + \cdots + p_{270}\right) \\
p_{301} &= \frac{1}{30}\left(p_{299} + p_{298} + \cdots + p_{271}\right) \\
p_{302} &= \frac{1}{30}\left(p_{299} + p_{298} + \cdots + p_{272}\right) \\
&\cdots \\
p_{329} &= \frac{p_{299}}{30}.
\end{aligned}$$

By comparing the number of terms on RHS, we know 300 is the most likely result of the sum.

Question 1.20: (Jane Street QT Intern; 2024)

I give you a twelve-sided die and will pay you whatever the die lands on. If you are unhappy with the roll, you can choose to roll another two six-sided dice and I will pay the sum of the two dice. How much are you willing to pay to play the game?

Answer 1.13: Let's say we want to maximize the expected profit we can get from the game. If we choose to play the second roll, our expected profit will be 7 . So when our first roll is less than 7 , we will choose to attend the second round. Thus the expected value of this game will be

$$\frac{1}{2} \times 7 + \frac{1}{2} \times \sum_{i=7}^{12} \frac{i}{6} = 8.25.$$

Question 1.21: (Jane Street QT Intern; 2024)

You have a lottery ticket with 10 slots. The number behind each slot follows Uniform $[0, 1]$ and your payout is the maximum number between any of these slots. How much are you prepared to pay for the lottery ticket?

Answer 1.14: It tests your familiarity with rank statistics. The expectation of the max of n uniform distribution from 0 to 1 is $n/(n+1)$ and the min is $1/(n+1)$. Thus the answer is $10/11$.

Question 1.22: (Virtu QT Intern; 2024)

You have a deck of cards. What is the expected number of cards you have to draw to get cards of all four suites?

Answer 1.15: This problem is similar to the collecting-distinct-card problem on the Green book but it is more complicated since it is negative hypergeometric distribution instead of negative binomial distribution. If Y follows negative hypergeometric distribution with N trials and K success, then

$$\mathbb{E}[Y] = \frac{N+1}{K+1},$$

which can be calculated using the formula of expectation directly. Thus if we denote Y_i as the number of drawing to get the i-th suite after the (i-1)-th suite, we have the expectation of the total number to be

$$\mathbb{E}\left[Y_{\text{total}}\right] = \mathbb{E}\left[Y_1\right] + \mathbb{E}\left[Y_2\right] + \mathbb{E}\left[Y_3\right] + \mathbb{E}\left[Y_4\right].$$

Clearly, $\mathbb{E}\left[Y_1\right] = 1$

$$\mathbb{E}\left[Y_2 \mid Y_1\right] = \frac{52+1-Y_1}{39+1}.$$

Thus the expectation of Y_2 is $52/40$. Similarly, we can calculate the expectation of Y_3 and Y_4.

$$\mathbb{E}\left[Y_3\right] = \frac{52+1-1-52/40}{26+1} = \frac{169}{90}$$
$$\mathbb{E}\left[Y_4\right] = \frac{52+1-1-52/40-169/90}{13+1} = \frac{2197}{630}$$

Thus the total number is $4829/630$.

Question 1.23: (HRT Algo Dev Intern; 2024)

If we keep randomly sampling numbers from $[0, 1]$ and stop until the sequence is not increasing, what is $\mathbb{E}$ (the number of sampling)?

Answer 1.16: Let's analyze it step by step. What is the probability of doing only 2 sampling? It is

$$P_2 = \int_0^1 x dx = \frac{1}{2}$$

What about 3 steps?

$$P_3 = \int_0^1 \int_y^1 x \frac{1}{\text{vol}} dx dy = \frac{1}{6}$$

where the vol here indicates the volume of simplex $\{(x, y) \mid x, y \in [0, 1], y \geq x\}$. You may notice the pattern here: for $n \in \mathbb{Z}^+$,

$$P_n = \int_0^1 \int_{x_1}^1 \int_{x_2}^1 \cdots \int_{x_{n-1}}^1 \frac{x_{n-1}}{(n-1)!} dx_{n-1} \cdots dx_1 = \frac{1}{n!},$$

where the $(n-1)$! comes from the volume of $n-1$ dimension simplex. thus the expectation is e. This requires you know the volume of simplex from your calculus courses.

Question 1.24: (HRT Algo Dev Intern; 2024)

If we keep randomly sampling numbers from $[0, 1]$ and stop until the sum of the sampled number exceeds 1 , what is $\mathbb{E}$ (the number of sampling)?

Answer 1.17: This is similar to the Question 1.23. Here, we provide a method using a fact that if X is non-negative,

$$\mathbb{E}[X] = \int_{\mathbb{R}^+} \Pr(X > t)dt$$

You may see this conclusion from a more advanced probability course. Thus

$$\begin{aligned}
\mathbb{E}\left(\text{the number of sampling}\right) &= 1 + \sum_{i=2}^{\infty} \Pr\left(\text{the number of sampling} > i\right) \\
&= 1 + \sum_{i=2}^{\infty} \Pr\left(x_1 + x_2 + \cdots x_i < 1\right) \\
&= 1 + \sum_{i=2}^{\infty} \frac{1}{i!} \\
&= e.
\end{aligned}$$

The answer is also the same as the Question 1.23.

Question 1.25: (Akuna QR Intern; 2024)

You throw a dice until the sum of your throws can be divided by 3 . What is the number of throws on average?

Answer 1.18: Note that adding 1 to 6 , can always generate 2 numbers that can be divided by 3). So the expected step is

$$\sum_{n=1}^{\infty} n \left(\frac{2}{3}\right)^{n-1} \frac{1}{3} = 3.$$

Question 1.26: (Virtu QT Intern; 2024)

You flip a coin four times. If you at first flip a head, you win one dollar. If you flip a consecutive head, you win double your previous winnings up to that point. What is the expected value of your winnings?

Answer 1.19: You have the following cases.

1. Four heads = 27;
2. Three heads = $4 \times \frac{1}{4}6 \times \frac{1}{4} + 9 \times \frac{2}{4} = 7$;
3. Two heads = $2 \times \frac{3}{6} + 3 \times \frac{3}{6} = 2.5$;
4. One heads = 1.

Thus the EV is $27 \times \frac{1}{16} + 7 \times \frac{1}{4} + 2.5 \times \frac{6}{16} + 1 \times \frac{1}{4} = 4.625$.

Question 1.27: (HRT Algo Dev Intern; 2024)

If X and Y i.i.d. normal, what is $\Pr(Y > 3X)$? What is $\Pr(Y > 3X \mid X > 0)$?

Answer 1.20: $1/2$ and $1/2 - \pi^{-1} \arctan 3$. Standard Gaussian is polar symmetric.

Question 1.28: (Belvedere Trading QR Intern; 2024)

X and Y i.j.d follow standard normal. What is $\text{var}[X \mid X + Y = u]$ given u ?

Answer 1.21: $1/2$. Better remember the distribution for of conditional Gaussian.

Question 1.29: (Belvedere Trading QR Intern;2024)

Could you change the numbers on two six-sided dice to other positive numbers so that the probability distribution of their sum remains unchanged?

Answer 1.22: Here is a method borrow from Stack Exchange by combining polynomials and the sum of dices problem (or combinatorics problem). A standard dice corresponds to a polynomial

$$D(z) = z + z^2 + z^3 + z^4 + z^5 + z^6.$$

Then the sum 6 t two dices becomes

$$D^2(z) = z + 2z^2 + 3z^3 + 4z^4 + 5z^5 + 6z^7 + 5z^8 + 4z^9 + 3z^{10} + 2z^{11} + z^{12}.$$

So we want to find two polynomials D_1 and D_2 s.t. $D_1(z)D_2(z) = D^2(z)$ and also satisfy that $D_1(1) = D_2(z) = 6, D_1(0) = D_2(0) = 0$. Note that $D(z)$ can be factorized as

$$D(z) = z(z+1)\left(z^2 - z + 1\right)\left(z^2 + z + 1\right)$$

To ensure $D_1(1) = D_2(1) = 6$ and $D_1(0) = D_2(0) = 0$, we can keep $z(z+1)\left(z^2 + z + 1\right)$ and change $z^2 - z + 1$. This provides the two polynomials,

$$D_1(z) = z(z+1)\left(z^2 + z + 1\right) = z + z^3 + z^4 + z^5 + z^6 + z^8$$
$$D_2(z) = z(z+1)\left(z^2 - z + 1\right)^2\left(z^2 + z + 1\right) = z + 2z^2 + 2z^3 + z^4$$

So the two dices will be $\{1, 3, 4, 5, 6, 8\}$ and $\{1, 2, 2, 3, 3, 4\}$.

Question 1.30: (HRT Algo Dev Intern; 2024)

You randomly select a person on street. Let's say it is X. Then you keep randomly selecting other people on street. What is the expected number of people you select before you select a person taller than X.

Answer 1.23: Let's say the p.d.f. and c.d.f. of the height is f and F. We know the number of people we need to meet conditional on X is a geometric distribution with probability $1 - F(X)$ (let's say we use X to represent the height of X). So the expected number will be

$$\mathrm{E}(X) = \mathrm{E}\left(\mathbb{E}(N \mid X)\right) = \int_0^\infty \frac{dF(x)}{1 - F(x)} = \ln(1 - F(x))|_0^\infty = \infty.$$

Question 1.31: (Cubist QR; 2024)

For a fair dice, what is, the average throw it will take to have 1 to 6 occurring at least 2 times?

Answer 1.24: Let's start with a two-face dices(0 or 1). For the first two throws, it can be $00, 01, 11, 10$. Conditional on 00 or 11 the problem is reduced to throw the other one twice (Pascal's distribution) and the value is 4 . Conditional on 10 or 01 , the problem is reduced to throw 0 and at least once and the value is 3 . So for this simple case, the answer is 5.5 . You may get a feeling that the solution requires consideration of the path. Indeed, starting with 66 is different from 56 . The former one requires 5 more numbers with two times occurrence and the latter requires 4 more numbers with two times occurrence and 2 more numbers with one times occurrence. So we can solve it by DP. Define $f(x, y)$ as the expected number of throws with need to have x numbers occurring once (group A) and y numbers occurring twice (group B). Clearly, $x + y \leq 6$. Then

$$\begin{aligned} f(x, y) =& \mathbb{E}\left(x\text{numbers occurring once and } y \text{ numbers occurring twice}\right) \\ &= \mathbb{E}\left(* \mid \text{group } A \text{ occurs first}\right) \Pr(\text{ group } A \text{ occurs first }) + \\ &\mathbb{E}\left(* \mid \text{group } B \text{ occurs first }\right) \Pr(\text{group } B \text{ occurs first}). \end{aligned}$$

It is easy to calculate that

$$\mathbb{E}\left(* \mid \text{group } A \text{ occurs first}\right) = \mathbb{E}\left(* \mid \text{group } B \text{ occurs first }\right) = 6/(x + y)$$

and

$$\Pr(\text{group } A \text{ occurs first }) = x/(x + y), \Pr(\text{ group } y \text{ occurs first }) = y/(x + y).$$

1.2 Game Theory

Question 1.32: (HRT)

Blackjack (21) game: Write a strategy to win.

Question 1.33: (Citadel GFI)

30 sided die, A pick a number ,B pick a different number, who closer to the dice number wins, what is A's strategy?

Question 1.34: (Citadel GFI)

An infinite set of numbers $\sim U(0, k)$, where each day you choose one number, but you can only keep m numbers. Each day, you can discard one number. How can you maximize the sum of the m numbers?

Question 1.35: (Optiver QT Intern; 2024)

Now I have a deck of cards and said that each card is worth it's face values. I deal out three cards face down and told you that I am willing to trade them with me for \$15. Will you trade? How many times will you trade with me?

Answer 1.25: This is a relatively open question and it tests whether you can properly evaluate a trade.
Firstly, you need to figure out what is the expected value of this trade or the sum of the three values. It is 21 . So you have an edge and you should trade. Secondly, what is the maximal profit and loss? The maximal profit is 24 while the maximal loss is 12 . So will you bet if you only have 15? You bankruptcy if the dealer draws three aces...
My suggestion is to consider VaR. If you an accept 100 dollar 95%VaR of your capital, you should bet around 10 times (-10) should be around 5 percent quantiles). I think this is not related to Kelly's criterion since it is not a reinvesting scenarios.

Question 1.36: (SIG QT Intern; 2024)

We play a game of guessing an integer x uniformly distributed on 1,20]. The one that guesses the closer numbers to x wins x dollars. Will you go first gr second and what is your strategy? What is you probability of wining?

Answer 1.26: Clearly, you should go second since you will have more information (the first one's guess). Your criterion here is to find y to minimize $\mathbb{E}(|y-x|)$. So you should always pick 10 or 11 if you go first. If your opponent is rational, he or she will pick the other one and you have 50% probability wining the game.
If you go second and your opponent pick 10 or 11 , you can pick the another one. If your opponent pick a number n other than 10 or 11 , you can pick $n+1$ if $n<10$ (probability of wining being $1-n/20$) and $n-1$ if $n>11$ (probability of wining being $(n-1)/20$).

Question 1.37: (Jane Street QT Intern; 2024)

You and I play a game with two dices, one ten-sided and one six-sided. You guess the sum of the numbers after we roll them. If your guess is correct, you get the sum of the numbers in dollars. Else you get nothing. How would you make the best guess?

Answer 1.27: Be careful: you are not maximising the probability. You are maximising the expected profit you can get! So your best guess x should maximise xp_x, where p_x is the probability of having the sum to be x.
Note that the probability will be symmetric and centered at 9 (expectation of the sum). You also know that you will always choose the number larger or equal to 9 . So after checking $9, 10, 11, 12$ you will find that xp_x is maximised at 11.

Question 1.38: (Jane Street QT Intern; 2024)

There are two boxes A and B. At each round, you can choose either place 1 dollar in a random box or cash out a random box. Now you can play this game for 100 rounds. How do you maximize your profit?

Answer 1.28: It is always good to start with something simple. 100 rounds are too many and let's start with 2 rounds. Clearly, the optimal strategy is first place and then withdrawal, which earns $1/2$ dollar expectedly. Let's make it a bit more complicated with 4 rounds. In total, there should be 2^4 possible strategies but some are trivial i.e. (i) you won't withdraw without placing money and (ii) you won't place money all the time. It turns out that there are the following possible strategies and the expected profits are also calculated.

$$\begin{aligned}
&\text{place withdrawal withdrawal withdrawal} &&= \$\frac{7}{8}\\
&\text{place place withdrawal withdrawal} &&= \$\frac{3}{2}\\
&\text{place place place withdrawal} &&= \$\frac{3}{2}
\end{aligned}$$

In the above calculation process, we can see that there are actually two variables that control the difficulty of the problems: (i) number of placing, which determines the maximal amount of dollar you can earn and (ii) the order of placing and withdrawaling. The (i) should be easier to consider (binomial distribution is easier than order statistics). So we can fixed the order and consider the number only.

For $2N$ rounds game (if you can solve 100 rounds, you should solve the general case and vice versa), if we always first do placing and then do withdrawaling for the rest rounds, what is the maximal profits we can earn? i.e. what is the best x where x is the number of placing in the strategy? This should not be hard to calculate since things are very structured. After x placing, if there are x_1 in Box A and x_2 in Box B, then after the $2N - x$ withdrawaling, the expected profits P are

$$\mathbb{E}\left(P \mid x_1, x_2\right) = x_1\left(\frac{1}{2}\right)^{2N-x} + x_2\left(\frac{1}{2}\right)^{2N-x} + (x_1 + x_2)\left(1 - 2\left(\frac{1}{2}\right)^{2N-x}\right) = x\left(1 - \left(\frac{1}{2}\right)^{2N-x}\right).$$

Thus the unconditional expected profit is also the same value. It is not hard to find the maxima of the expected profit by taking derivatives for any given N.

The next step is to consider the ordering of placing and withdrawaling. This may be a bit tricky. If we have x placing and $2N - x$ withdrawaling, we label the $2N - x$ withdrawaling as $w_1, w_2, \cdots, w_{2N-x}$. Then the total expected profit is actually the sum of the expected profit from each withdrawaling i.e.

$$\mathbb{E}(P) = \sum_{i=1}^{2N-x} \mathbb{E}\left(P_{w_i}\right).$$

For each withdrawaling, the expected profit $\mathbb{E}\left(P_{w_i}\right)$ depends on current status of the two boxes (and also the previous withdrawaling). Intuitively, the more placing before a withdrawaling, the higher $\mathrm{E}\left[P_{w_i}\right]$ will be. Thus for each fixed an the optimal ordering is to first place and then to withdrawal for the rest rounds which means that the maximal profits is from the above strategy.

The above results can be easily verified on the four-round case. For $N = 50$, the maximal profit is \$92.53125 achieved at $x = 94$.

Question 1.39: (Jane Street QT Intern 2024)

A company is valued at $V \sim$ Uniform $[0, 1]$. You are to place a bid B for the company. If $B < V$, your bid loses and you get nothing. If $B > V$ you get to purchase the company at price B, and the company will end up being worth 1.5 V . What price B should you bid to maximize your profit?

Answer 1.29: Let's say our criterion is to maximize the expected profit. Then we are maximizing

$$\max_B \mathbb{E}\left((1.5V - B)I\{B > V\}\right)$$

The expectation is $-0.25B^2$. Don't bid if you don't have any information better than a bound condition!

Question 1.40: (Jane Street QT Intern; 2024

I give you 100 blank cards, and you can write a positive integer on each card. I look at the cards when you are done, then I shuffle the deck. I guess the top card of the deck, and if I am right, I make the dollar which is written on the card. What numbers should you write on the cards to minimize the expected return of mine?

Answer 1.30: Obviously, you cannot write 1 on each card because doing so you lose 1 dollar every time.
Also, you won't want to write a number larger than 100 since you can always find a better number while keeping the distribution.
From my perspective, after I inspect the cards, I know I will guess the number satisfying

$$\max_x xp_x.$$

So from your perspective, you want to minimize my maximum i.e.

$$\min_{\{x,p_x\}} \max_x xp_x$$

Now the question boils down to how to solve the minimax problem. You may notice that for any set of number, the max is minimized when all xp_x equals to each other, or else we can always improve the min by moving some "weight" from one number to another. So

$$p_1 = 2p_2 = \cdots = Np_N$$
$$\sum_{n=1}^{N} p_n = 100$$
$$p_i \in \mathbb{Z}_{\bigcap}^{+}$$

So we want to find N and p_1 to minimize p_1 given the above constraints. To solve this combinatorial problem, you can first find p_1 s.t.

$$p_1\left(1 + \frac{1}{2} + \cdots + \frac{1}{100}\right) = 100$$

and then do some rounding to reduce your search space. The final answer should be $p_1 = 28$.

Question 1.41: (JPMC Sys Trading Intern; 2024)

You throw a hundred face die without any cost either you take the number that shows on the face or leave. Else you pay a dollar for next throw and see what you get. For every successive throw you will need a dollar. But once you choose to take the payout the game stops. What should show up as a number on a die for you to exit?

Answer 1.31: If you do a lot of Green book exercises, you may feel that what you need to do here is to compare the expected value of next throw i.e. \$51.5 and the current value. But you actually miss something i.e. your loss in previous round. Let's say the threshold is s. The actually objective function here should be

$$\max_s \left(\mathbb{E}\left(X \mid X \geq s\right) - \mathbb{E}\left(T_s\right)\right) = \max_s \left(\frac{100+s}{2} - \frac{100}{101-s}\right)$$

where T_s is the stopping time you first get $X \geq s$ (you can use geometric distribution to get the expectation). The optimal value should be around 86.857 . So you need to further compare 86 with 87.

Question 1.42: (Jump Trading QT Intern; 2024)

You are in a one-on-one coin-flip gambling game. Your opponent has 1 million dollar on hand (the max you can bet is 1 million dollar). If the result is tail, you win 2 x your bet; else, you lose your bet. How much would you bet?

Answer 1.32: This question is similar to Question 3.25. Basically, you need to define your utility function here and maximize it. Say you have 1 dollar and then you want to maximize

$$\max_x \frac{1}{2}u(1+2x) + \frac{1}{2}u(1-x)$$

You can choose $u(x) = \log x$ to achieve maximal long term growth i.e. Kelly's criterion, which will give you $x = 0.25$.

Question 1.43: (JPMC QA Intern; 2024)

Which one do ^{v04} prefer, return from a continuous uniform distribution on $[0, 100]$ or a discrete one on $[0, 100]$?

Answer 1.33: The mean is the same $50, \text{Bet}$ the variance of continuous is $10000/12 \approx 833$ while the discrete one is 850.

Question 1.44: (Jump Trading QR Intern; 2024)

There are 10 balls and you want to find the heaviest ball by randomly select two balls and determine which of the rocks is heavier. What is the expected number of weighing you need?

Answer 1.34: Note that it is not asking you to find the heaviest ball using least number of steps(should be finished in at most 9 steps). We know there are in total $\binom{9}{2} = 45$ pairs we can have but we want the 9 pairs to be selected (the pairs including the heaviest one). So using geometric distribution and the idea of collecting distinct objects, we have

$$EV = \sum_{n=1}^{9} \frac{45}{n}.$$

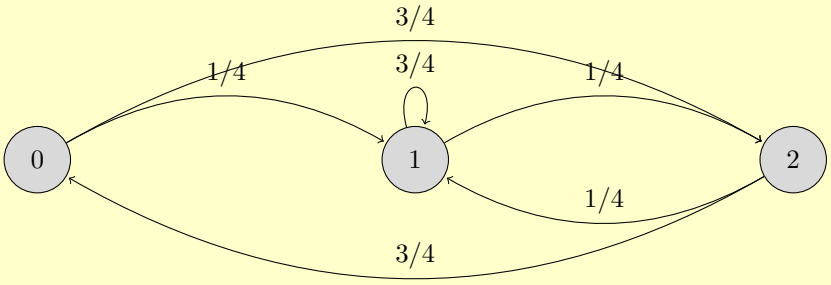
1.3 Stochastic Processes

1.3.1 Markov Chain

Question 1.45: (SIG QR; 2024)

I have a total of two umbrellas that I keep in the office or at home. When it rains, I bring an umbrella with me on my commute. Otherwise I never bring an umbrella. If the probability of rain on any leg of the commute is 1/4, what is the probability that it rains and I won't have an umbrella with me?

Answer 1.35: Let's say I change my location each step and let X_n be the number of umbrellas with me at step n. Clearly, X_n is a Markov Chain with the following transition graph.



Thus based on The transition matrix is

$$P = \begin{bmatrix} 0 & 1/4 & 3/4 \\ 0 & 3/4 & 1/4 \\ 3/4 & 1/4 & 0 \end{bmatrix}.$$

Let $\pi P = \pi$. We have $\pi = [\frac{3}{14}, \frac{1}{2}, \frac{2}{7}]$. Thus the probability is $\frac{3}{14}$.

1.3.2 Random Walk

Question 1.46: (JPMC Sys Trading Intern; 2024)

Consider a random walk S_n in $\mathbb{R}^3$. What is the magnitude of $|S_n|$ in terms of n?

Answer 1.36: By CLT, it should be $O_p(\sqrt{n})$.

Question 1.47: (HRT Algo Trader)

Flip a coin, H win $1, Tlose1$, the game ends when $+10 or -20$, what's the probability you will win?

2 Math

Question 2.1: (HRT Algo Trader)

There are 9 points with integer coordinates in $x - y - z$ space. Prove that there exists at least one pair of points such that their midpoint also has integer coordinates

2.1 Limits and Functions

2.2 Calculus

2.3 Linear Algebra

Question 2.2: (Citadel Sec Index Arb)

How many solutions will the linear equation $\mathbf{AX} = \mathbf{B}$ have? Suppose $\mathbf{A}$ is an $m \times n$ matrix. What happens if $m > n$, and how can we ensure a unique solution when $m < n$?

Question 2.3: (Jump Trading QR Intern; 2024)

Let A be a $n \times n$ covariance matrix such that all diagonal elements are equal to 1 , and the non-diagonal ones to ρ with $-1 \leq \rho \leq 1$. Which range of values is admissible for?

Answer 2.1: We know that if all X_i are the same, then $\rho = 1$. So the tightest upper bound is 1. In addition, we have $\text{var}\left(\sum_{i=1}^{n} X_i\right) \geq 0$, which indicates that

$$n + n(n-1)\rho \geq 0 \Rightarrow \rho \geq -\frac{1}{n - \mathbb{Y}}$$

This bound is achievable by Y_i if X_i 's are i.i.d. and $Y_i = X_i - \bar{X}$. Thus the bound for ρ is $[-1/(n-1), 1]$.

Question 2.4: (Jump Trading QR Intern 2024)

When A and B are both positive definite, what is the condition that makes AB positive definite?

Answer 2.2: The condition is $AB = BA$. Under this condition,

1. AB is symmetric
2. AB is positive definitive using the fact that A and B can be diagonalized at the same time and a matrix D is positive definite $\Leftrightarrow$ it's eigenvalues are all positives.

How to prove that $AB = BA \Rightarrow A$ and B can be diagonalized at the same time? Basically, we can observe that B preserve the eigenspaces of A. So we can restrict and diagonalize B on each eigenspace of A (on the eigenspace of A, A is actually equivalent to identity matrix).

2.4 Combinatorics

Question 2.5: (Virtu QT Intern; 2024)

If you have a deck of cards and put them in another stack, how many cards do you have to put down to guarantee a Three of a Kind?

Answer 2.3: 27 by the Pigeonhole Principle.

Question 2.6: (Akuna QR; 2025)

How many trailing 0 is there for 100!?

Answer 2.4: There are 245 's if we factorize 100!. So there will be 24 trailing zeroes.

Question 2.7: (Akuna QR; 2025)

How many ways you can divide 30 distinct people into two groups?

Answer 2.5: $2^30 - 2$. Don't forget the -2 .

Question 2.8: (Cubist QR; 2024)

You start with \$1. We play a game by flipping a coin. If the head occurs you lose 20% of your wealth and if the tail occurs you win 20% of wealth. Is it possible that you end up with \$1 ?

Answer 2.6: We want for some integers n, m,

$$\left(\frac{4}{5}\right)^n \left(\frac{6}{5}\right)^m$$

This is impossible because

$$\frac{n}{m_1} = \frac{\ln 5 - \ln 6}{\ln 4 - \ln 5}$$

is not a rational number.

Question 2.9: (SIG QT 2024)

You have a tricycle and you want to travel on it for 1,000 miles. You also have two spare tires with you. If you want them to be worn the same by the end of the trip, how much will each of them be used? What is the minimum number of stops you have to make in order to achieve this?

Answer 2.7: Total tire miles is $3 \times 1000 = 3000$ miles and you need to allocate them to 5 tires. It is 600 miles. You need 3 stops.

$$(1,2,3) \times 400 \text{ miles } \to (3,4,5) \times 200 \text{ miles } \to (1,4,5) \times 200 \text{ miles } \to (2,4,5) \times 200 \text{ miles}$$

Question 2.10: (SIG QR; 2024)

A frog is travelling from point A $(0,0)$ to point B (x,y) but each step can only be 1 unit up or 1 unit to the right. Also, the frog refuses to move three steps in the same direction consecutively. Compute the number of ways the frog can move from A to B.

Answer 2.8: The idea is from Math Stack Exchange. One way of doing it is by Jackson-Goulden cluster method, which I believe only less than 0.1% quant interviewees know what it is. Another approach is by integer partion. Let's say we can partition x and y into

$$x = a_1 + a_2 + \cdots + a_n$$
$$y = b_1 + b_2 + \cdots + b_m$$

with $a_i, b_j < 3$. If $|m - n| > 1$, clearly we cannot use the partitioning to form a path. Why? Because there will be consecutive a_i or b_j block in the sequence (which brings repeated counting). If $m = n + 1 (n = m + 1)$, then the frog must go up (right) first to avoid consecutive blocks. If $m = n$, either direction can be the first. Then the remaining question is about orderings. Solving this problem requires some manual calculation.

Question 2.11: (JPMC Sys Trading Intern; 2024)

If I write down all of the numbers from one to one million on a page, how many times do I write down a two?

Answer 2.9: If containing leading 0 , from 0 to 999999 , all numbers ($0-9$) should occur uniformly. Thus the times you write down a two should be 600,000.

Question 2.12: (Jump Trading QR Intern; 2024).

There is a survey sent to all passengers on a number of different planes (same model). The survey asks each person how full their plane was how full was the average plane? There is 50% people saying their planes are 80% full and 20% people saying their planes are 20% full, how full was the plane on average?

Answer 2.10: Assume that there are 100 seats on a plane. So those people answering 80% full must be on a plane with 80 seats occupied, while those people answering 20% must be on a plane with 20 seated occupied. Therefore, there must be four-times as many planes that were 20% full compared to 80% full. Thus the average is

$$0.2 * 0.8 + 0.8 * 0.2 = 0.32.$$

3 Statistics and Econometrics Methods

Question 3.1: (Citadel GFI)

High frequency estimation of correlation, what's the issue?

Question 3.2: (Citadel Sec Index Arb)

Consider n data points uniformly distributed in a d-dimensional unit ball centered at th origin. What is the median distance from the origin to the closest data point?

Answer 3.1: Check The Essentials of Statistical Learning and discussion on StackOverFlow.

3.1 Statistical Properties

Question 3.3: (JPMC QA Intern; 2024)

If the sample average of X is strictly larger than Y in period one and two respectively, is it possible that the sample average of Y is strictly larger than X if combing period one and two?

Answer 3.2: This is the Simpson's paradox. Here is an example. In period one, the number of sample of X and Y is 100 and the average of X is 1 , while the average of Y is 99/100. In period two, the number of sample of X is 100 and the average is 51/100, while the number of sample of Y is 2 and the average is 1/2. Then combining period one and two, the average of X is 151/200 while the average of Y is 100/102.

Question 3.4: (Arrowstreet QR; 2024)

Given a random variable $X \sim \mathcal{N}(0,1)$, after truncating it by -1 and +1 i.e. $X^* = \max(\min(X,1),-1)$, will the kurtosis increase or decrease? Prove your argument.

Answer 3.3: This is very intuitive: after truncation, you have no tail. Thus the kurtosis (characterizing tail behavior) will decrease. If the interviewers buy this argument, you can just stop here. If you write down the kurtosis formula after truncation, it is equivalent to

$$\kappa = \frac{\left(\int_0^1 x^4\phi(x)dx\right)\left(\int_0^1 \phi(x)dx\right)}{\left(\int_0^1 x^2\phi(x)dx\right)^2}$$

Then you only need to calculate the three integrals separately. It shouldn't be too hard.
What if your interviewers keep grilling you: if it is not a standard normal but a symmetric uni-modal distribution e.g. t-distribution, is it correct? Here is a method you can use. Let

$$g_{a,b}(t) = \left(\int_0^t x^a p(x)dx\right)\left(\int_0^t x^b p(x)dx\right)$$
$$f(t) = \frac{g_{4,0}(t)}{g_{2,2}(t)}$$

We would like to show $f(t)/f(\infty) < \mathrm{N}$ (Note that

$$\frac{g_{a,b}(t)}{g_{a,b}(\infty)} = \frac{\left(\int^t x^a p(x)dx\right)\left(\int_0^t x^b p(x)dx\right)}{\left(\int_0^\infty x^a p(x)dx\right)\left(\int_0^\infty x^b p(x)dx\right)}$$

Thus we know it is maximized when $a = b$ using $2ab \le a^2 + b^2$. Thus $f(t)/f(\infty) < 1$.

3.2 Linear Regression

Question 3.5: (Arrowstreet QR; 2024)

We have $\text{corr}(Y, X_1) > 0, \text{corr}(Y, X_2) = 0$ and $\text{corr}(X_1, X_2) > 0$. If considering two models, $Y = aX_1 + \epsilon$ and $Y = \beta_1 X_1 + \beta_2 X_2 + \epsilon$. Which is larger, $|a|$ or $|\beta_1|$?

Answer 3.4: Note that

$$\begin{aligned} \text{cov}(Y, X_1) &= a\,\text{cov}(X_1, X_1) \\ \text{cov}(Y, X_1) &= \beta_1\,\text{cov}(X_1, X_1) + \beta_2\,\text{cov}(X_2, X_1) \\ 0 = \text{cov}(Y, X_2) &= \beta_1\,\text{cov}(X_1, X_2) + \beta_2\,\text{cov}(X_2, X_2) \end{aligned}$$

Thus we have

$$a = \beta_1 \left(1 - \rho^2\right),$$

where $\rho = \text{corr}(X_1, X_2)$. Thus $|a| \geq |\beta_1|$.

Question 3.6: (InfiniteQuant QR Intern; 2024)

You have random variables X and Y. You run regression $Y \sim X$ and $X \sim Y$. What is the relationship between $\beta_{Y\sim X}$ and $\beta_{X\sim Y}$?

Answer 3.5: $\beta_{Y\sim X} = \text{cov}(Y, X)/\,\text{var}(X)$ and $\beta_{X\sim Y} = \text{cov}(Y, X)/\,\text{var}(Y)$. Thus $\beta_{Y\sim X}\beta_{X\sim Y} = \text{corr}(X, Y)^2$.

Question 3.7: (Arrowstreet QR Intern; 2024)

What are the assumptions of linear regression? What is the impact to our OLS estimate if the assumptions are not satisfied?

Answer 3.6: Here are the assumptions.

1. Linear relationship. The expectation of the dependent variable Y conditional on the independent variables satisfy $\mathbb{E}[Y \mid X] = X\beta$.
2. Normality on error. $\epsilon \mid X$ follows Gaussian distribution.
3. Homoscedasticity. $\epsilon_i \mid X$ has the same variance.
4. Independence. $\epsilon_i \mid X$ are independent.

Let's discuss the impact of not satisfying these assumptions except the first one.

1. No normality. Our OLS estimate $\hat{\beta}$ is still the best linear unbiased estimator (BLUE) and $\hat{s}$ is still unbiased. The test distribution is no longer t and F distributed.
2. Heteroscedasticity. Our OLS estimate $\hat{\beta}$ is still the unbiased linear estimator but $\hat{s}$ is biased now (usually underestimate the variance). $\hat{\beta}$ is no longer BLUE. The test distribution is no longer t and F distributed. But if the heteroscedasticity is known, it can be corrected using GLS.
3. No independence. Same as heteroscedasticity since the error is Gaussian distributed.

Question 3.8: (Arrowstreet QR Intern; 2024)

Please discuss how measurement error affects the OLS estimators $\left(\beta, \sigma^2\right)$.

Answer 3.7: Consider the univariate model $y = \alpha + \beta x + \epsilon$. If we observe $\tilde{y} = y + u$ and $\tilde{x} = x + v$ where u and v are Gaussian noises, we have

$$\tilde{\beta} = \frac{\operatorname{cov}(\beta x + \alpha + u, x + v)}{\operatorname{var}(x + v)} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_v^2}\beta$$

We can see that the measurement error on y won't affect the estimation on β (In fact, it only affects the estimation of σ^2). When there is measurement error on x, our β estimate is no longer unbiased. So does the estimate of α.

Question 3.9: (Arrowstreet QR Intern; 2024)

How will you resolve collinearity in linear regression? Which methods are better?

Answer 3.8: Ridge and Lasso. If this question is asked in a problem/related to portfolio management or factor models, here is a comparison.

- Lasso: the result model is sparse and more interpretable;
- Ridge: the coefficients of each covariate is not zero. If these covariates are factor premiums, it results in a more diverse model (portfolio), which should be more robust in reality.

Question 3.10: (Citadel Sec Index Arb)

Pros and Cons of ridge regression and Lasso, also compare with PCA.

Question 3.11: (Citadel Sec Index Arb)

Assume very large data size,design an online linear regression.

3.3 Asset Pricing

Question 3.12: (Arrowstreet QR Intern; 2024)

What problem does the Fama-Macbeth regression model want to resolve?

Answer 3.9: It resolves measurement error and the cross-sectional heteroskedasticity issue when testing factor models (time-series heteroskedasticity is less severe than the cross-sectional one), Here is the procedure of FM regression.

1. Tun time series regression to get the beta $\hat{\beta}_i$;
2. For each t, run cross-sectional regression $r_{i,t} = \gamma_t + \lambda_t \hat{\beta}_i + \epsilon_{i,t}$ and get the estimators $\hat{\gamma}_t$ and $\hat{\lambda}_t$;
3. Calculate the mean and std of $\hat{\gamma}_t$ and $\hat{\lambda}_t$, and use them to do hypothesis testing.

The idea is actually like bootstrapping.

Question 3.13: (Arrowstreet QR Intern; 2024)

How do you estimate the covariance matrix for a large number of assets?

Answer 3.10: This is the answer that the interviewer provides me. If you directly use the time series data (say T periods) for each asset (say N assets in total), you probably end up with $O\left(N^2T\right)$ operations. But you can actually leverage your factor nodels. If you have F factors, you only need $O\left(F^2T + NF^2\right)$.

Question 3.14: (Citadel Sec Index Arb)

What is BIC? What is the significance of Bayesian method?

4 Machine Learning and Data Analytic

Question 4.1: (HRT)

Where should an ATM be placed in the United States? (open-ended question)

Question 4.2: (Two Sigma)

How can we determine the age of a tree without cutting it down? The approach should involve open-ended modeling, identifying features, and building a model.

Question 4.3: (Two Sigma)

Given GDP and CPI data, how can you build signals?

Question 4.4: (Two Sigma)

Given Twitter data, how can you build alpha signals? Be very specific in identifying features and modeling. There are many relevant follow-ups, such as year-on-year data changes, seasonality, etc.

Question 4.5: (Two Sigma)

Given analyst data in the form of -1, 0, and 1, how do you build stock positions? You need to consider the analyst's forecasting ability, prediction range, and other factors.

5 Finance

5.1 Stochastic Finance

Question 5.1: (Barclays QA Intern; 2024)

What is the risk-neutral measure? How to use it?

Answer 5.1: Under the risk-neutral measure, the expected return of all risky assets (stocks, options, etc.) is the risk-free rate. In terms of how to use it, we mainly use it in derivative pricing. If the stock price follows the geometric Brownian motion

$$S_t = \mu dt + \sigma dW_t,$$

we can do change of measure and consider the problem under risk-neutral measure Q

$$S_t = r dt + \sigma dW_t.$$

Under $\mathbb{Q}$ any derivative with expected value $h(S_T, T)$ at time T can be priced as $e^{-r(T-t)}h(S_T, T)$ at time t.

Question 5.2: (Barclays QA Intern; 2024)

What is the Ito formula? How to use it?

Answer 5.2: I will say it's just Taylor's expansion.

$$df(t, S_t) = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}dS_t + \frac{1}{2}\frac{\partial^2 f}{\partial^2 x}(dS_t)^2$$

5.2 Derivatives

Question 5.3: (BofA SnT Intern; 2024)

Why an option is not free?

Answer 5.3: The most simple answer is that the expected pay-off of an option is always positive.

Question 5.4: (MS SnT Intern; 2024)

There are three call options on the market, with the same expiry and with strikes 10,20 , and 30 . Suppose the call option with strike 10 costs \$12, the call option with strike 20 costs \$7, and the call option with strike 30 costs \$1. Is there an arbitrage opportunity?

Answer 5.4: The call price is a convex function. So we can calculate the call with strike price 20 based on the ones with 10 and 30.

$$7 = C(20) = C\left(\frac{10+30}{2}\right) \leq \frac{1}{2}(C(10) + C(30)) = 6.5$$

So we know the $C(20)$ is more expensive than it should be. So you can short the 20 one and long the 10 and 30 ones.

Question 5.5: (MLP QR Intern; 2024)

What is the difference between forward and futures? What is the delta of them?

Answer 5.5: Here are the differences.

1. Forward is traded OTC while futures is traded at exchange;
2. Future is mark-to-market. This mechanism is to reduce the counterparty risk for the exchange;
3. Theoretically, the delta of future and forward are 1. But since future is traded at exchange, the delta of future is not exactly 1.

5.3 Others

Question 5.6: (InfiniteQuant QR Intern; 2024)

What is your estimation of the fair value of a stock if you know the first level price on the limit order book? What if you know two levels?

Question 5.7: (InfiniteQuant QR Intern; 2024)

Which one is harder to earn money, market making or market taking?

Question 5.8: (Jiaqi QR; 2024)????

Please design an equity portfolio based on analysts' recommendation score.

Question 5.9: (WorldQuant QR; 2024)

You have an alpha A performs better on large cap and an alpha B performs better on small cap. How to form a better alpha to perform well on both large and small cap companies?

Question 5.10: (SquarePoint QR; 2024)

How to reduce your turnover in your alpha research?

Answer 5.6: Moving average.

Question 5.11: (Citadel Sec Index Arb)

How to develop an Alpha signal from provided options data?

Question 5.12: (Citadel Sec Index Arb)

How to build risk models using provided volatility surface data? Potential strategies for improvements.

Question 5.13: (Two Sigma)

What is the principle of market making? How do you provide liquidity?

6 Coding

Question 6.1: (HRT)

An $O(V+E)$ algorithm to find all Articulation Points (APs) involves using Depth First Search (DFS). In DFS, we explore the graph in the form of a DFS tree. In this tree, a vertex **u** is considered the parent of another vertex **v** if **v** is discovered by **u** (which implies **v** is adjacent to **u** in the original graph). A vertex **u** is identified as an articulation point if it meets one of the following conditions:

1. **u** is the root of the DFS tree and has at least two children.
2. **u** is not the root, and it has a child **v** such that no vertex in the subtree rooted at **v** has a back edge to any of u's ancestors in the DFS tree.

Question 6.2: (HRT)

Given a weighted directed graph, find the second shortest path from A to B. What is the algorithm and its complexity?

6.1 String Processing

Question 6.3: (BofA QR Intern; 2025)

Given two strings s_1 and s_2, please design an algorithm to check whether s_2 is a substring of s_1.

Answer 6.1: Let's say $\text{len}(s_1) = n$ and $\text{len}(s_2) = m$ and assume $m < n$ (or it is trivial). The naive solution is to do double loop, which turns out to be $O(mn)$ A more efficient way is to apply **Knuth-Morris-Pratt (KMP)** algorithm. Note that this is just one of those efficient algorithms for string matching and it can also be used for finding all substring matching.

There are two components of KMP algorithm.

1. Calculate the length of the longest proper prefix that is also a suffix i.e. LPS for each substring of s_2 i.e. $s_2[:i], \forall i$. This can be realized in $O(m)$ by the following algorithm.

```
def construct_lps(s2):
    m = len(s2)
    lps = [0 for _ in range(m)]
    l, i = 0, 1
    while i < m:
        if s2[i] == s2[l]:
            l += 1
            lps[i] = l
            i += 1
        elif l == 0:
            lps[i] = 0
            i += 1
        else:
            l = lps[l - 1]
    return lps
```

2. Utilize the LPS information. When encountering a mismatch for $s1[i]$, we don't need to start from scratch for $s1[i+1]$. Instead, we find the suffix that is the longest prefix. The time complexity of this part is $O(n+m)$.

```
    def check_substring(s1, s2, lps):
        n, m = len(s1), len(s2)
        p1, p2 = 0, 0
        while p1 < n:
            if p2 == m:
                return True
            if s1[p1] =m s2[p2]:
                p1 += 1
                p2 += 1
            elif p2 == 0: p1 += 1
            else:
                p2 = 1ps[p2 - 1]
        return False
```

What we learn from this algorithm? Make use of information from previous calculation (However, this may sound easy but actually hard in practice...). When traversing a sequence, we actually gain short-term memory on the sequence but mid-term memory on the summarization. The latter part is what we need to utilize.

6.2 Dynamic Programming

Question 6.4: (Citadel GFI)

Given an $m \times n$ grid, find the shortest path from one point to another (some cells are impassable)

Question 6.5: (SIG QR; 2024)

A frog is travelling from point A$(0, 0)$ to point B (x, y) but each step can only be 1 unit up or 1 unit to the right. Also, the frog refuses to move three steps in the same direction consecutively. Compute the number of ways the frog can move from A to B.

Answer 6.2: The idea is from Math Stack Exchange. We need a 3D DP where 2D for location 1D for states (this kind of remind me of the Buy and Sell Stock Leetcode problem). We define states as follows. 1. Moves up 1 unit consecutively; 2. Moves right 1 unit consecutively; 3. Moves up 2 unit consecutively; 4. Moves right 2 unit consecutively. The state transition equation is then

$$\begin{aligned} f(i, j, 1) &= f(i, j-1, 2) + f(i, j-1, 4) \\ f(i, j, 2) &= f(i-1, j, 1) + f(i-1, j, 3) \\ f(i, j, 3) &= f(i, j-1, 1) \\ f(i, j, 4) &= f(i-1, j, 2) \end{aligned}$$

The initial condition is

$$f(0, 1, 1) = f(1, 0, 2) = f(0, 2, 3) = f(2, 0, 4) = 1$$

Useful link: math.bot.

Question 6.6: (SquarePoint QR Intern; 2024)

You have a stock price sequence p_t. If you can only make k transactions, find the maximum profit you can achieve.

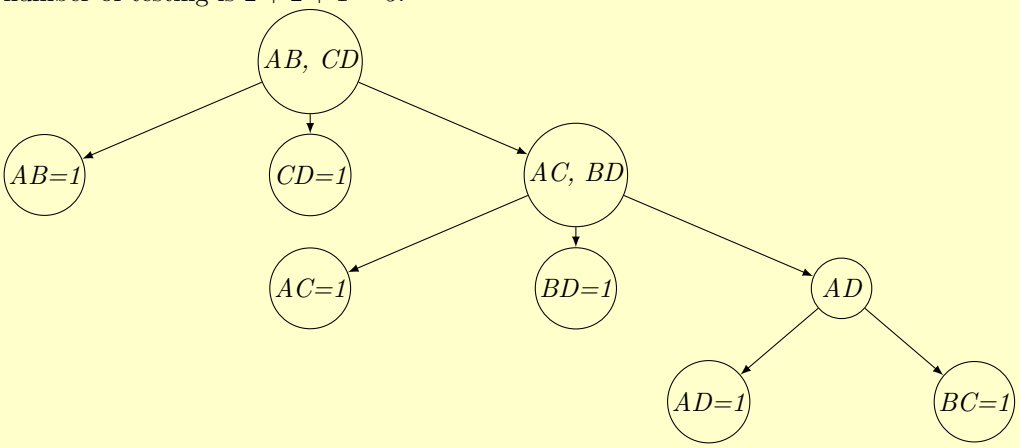
Answer 6.3: This is actually Leetcode 188 and there is a list of other buy and sell stock problems on Leetcode worth practicing.

6.3 Divide and Conquer

Question 6.7: (Jump Trading QR Intern; 2024)

You have a toy requiring two well functioned batteries to function. Now you have 50 good and 50 bad batteries in front of you but you don't know which one is good or bad. Please design an algorithm to find two good batteries for your toy to function.

Answer 6.4: Let's start with a simple case with 4 batteries (2 good +2 bad). What is the smallest number of test you need to find out the two good batteries? The naive upper bound is 6 . If we label them as A, B, C and D, here is another path of testing (forgive me for the sloppy notations). The number of testing is $2 + 2 + 1 = 5$.



This is one step less because in the last testing $AD = 0 \Leftrightarrow BC = 1$. Is it the best? Can we find the two good batteries in just 4 steps? I don't know...

For the 100 batteries case, it's quite similar. If we divide the batteries into 50 pairs and test all of them (50 times) and if none of them works, we know all of them must be 1 good +1 bad. Now pick two pairs, it is reduce to the 4 batteries case, starting at layer 2 above. So the total number of test is 53 . (Is it the minimal? I don't know...) This blog traces the problem back to 2905 Brazilian Mathematical Olympiad.

6.4 Data Structure

Question 6.8: (HRT)

Data structure problem: Design a structure to implement push and random pop.
Follow-up 2: For each push($x_i, weight_i$), $how can we implement random pop with weighted probability$?
Follow-up 3: How can we improve the efficiency?

6.5 Others

Question 6.9: (JPMC Sys Trading Intern; 2024)

How will you simulate a six-sided dice using a coin? How many flips do you need (on average)?

Answer 6.5: This question is similar to how to simulate an unbiased coin using a biased coin. Since fixing number of flips k, the possible outcomes of coin flips is 2^k, we need to discard some cases to keep the other 6 outcomes with equal probability. The least number of flips is 3 and here is a possible correspondence. $TTT = 1, TTH = 2, THT = 3, THH = 4, HTT = 5, HTH = 6$. If the first two outcomes we flip is a HH (the rest two cases HHT and HHH), we can stop and re-flip another 3 times. The expected number of flips can be calculated below.

$$EV = \frac{3}{4} * 3 + \frac{1}{4} * (2 + EV) \Rightarrow EV = \frac{11}{3}$$

Question 6.10: (Millennium QR Intern; 2024)

Implement n choose m function. Use this function to generate permutation.

Answer 6.6: The most naive method is to build a factorial look-up table and directly use the formula of $\binom{n}{m}$ to implement. But the problem here is that factorial may be too large and a 64 -bit integer cannot store the number. Another related problem is to generate all combinations (say n choose m) of a list. This can be implemented using recursion which turns out to be $O\left(\binom{n}{m}\right)$.

Question 6.11: (InfiniteQuant QR Intern; 2024)

How to implement rolling median? How to implement arbitrary rolling quantiles? What is the time complexity?

Answer 6.7: Rolling statistics can directly be implemented using balanced binary search tree (bBST) by adding an attribute indicating the size of left and right children, which turns out to be $O(\log N)$ time complexity for each retrieval In C + +, you can use policy-based data structure to implement it.

```
#include <ext/pb_ds/assoc_container,hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<
    std::pair<int, int>,
    null_type,
    std::less<std: :pair<int, int>>,
    rb_tree_tag,
    tree_order_statistics_node_update> ordered_set;
... // Define your window
auto median_it = window.find_by_order(median_idx);
double p = 0.25;// Fqn bye 25th percentile
int quantile_idx = \tavic_cast<int>(p * (k - 1));
auto quantile_it \window.find_by_order(quantile_idx);
```

Question 6.12: (GTS QT Intern; 2024)

You have a file and you can read each line sequentially.) If you want to choose a line from the file with equal probability, what should you do?

Answer 6.8: This requires your understanding of conditional probability. What you can do is:

1. Choose the first line (current selection);
2. Replace the current selection with the second line with probability 1/2;
3. Replace the current selection with the third line with probability 1/3;
4. ...
5. Terminate when reaching EOF.

This ensures equal probability since the probability of choosing line i is $1/i*(i/(i+1))*\cdots*(n-1)/n = 1/n$.

Question 6.13: (Akuna QR; 2025)

Given is a binary string s consisting of only 0 s and 1 s . In a single operation, any one " 1 " from the string can be chosen and moved to the right until it reaches the end of the string or another " 1 ". The cost of the operation is 1+ the number of places the one is moved. For example, in the string " 100010^n, the first one can be moved three places to the right in cost 4. It is mandatory to move a "1 " to the maximum possible position to the right. Given a binary string s, find the maximum possible number of operations possible to segregate the given string.

Answer 6.9: Answer 4.9. Since each move can provide one extra point, you need to maximise the number of moves. The optimal moving strategy is given by proving the "one" block from left to right to form a new block until the block cannot be moved any more. Here is the code (adding 1 to the right of the string to collect the last move).

```
num0 = num1 = ans = 0
s = s.lstrip("0") + "1"
for c in s:
    if c == "0": num0 +=1
    else:
        if num0: ans += num1 * (num0+1)
        num1 +=1
        num0 = 0
```

7 Others

Question 7.1: BofA QR Intern; 2024)

How will you design an elevator system?