List of References

For this project, [RMQ07] is most relevant to our algorithm. [Rou98] contains the Dupire Local Volatility (LV) derivation. [GHL11] discusses the Monte Carlo (MC) calibration of a particular Stochastic Local Volatility (SLV) model, including the Markovian projection. The detailed derivation in this paper can be found in [GHL13]. [Hom14] serves as a comprehensive reference document on the background.
For other aspects or background knowledge needed for this project, consider the following:

- Matching the marginal distribution of Stochastic Differential Equations (SDE): [Gyö86].
- Kolmogorov Forward Equation: [HC22] and [CLP08] address two-dimensional Kolmogorov forward equations.
- Local Volatility Model:
  - Original works by Dupire: [D$^+$94, Dup97].
  - Additional notes on derivations: [Rou98].
  - With stochastic interest rates: [Hu15] provides the corresponding derivation for the Dupire formula, also found in [ÖH23] under the foreign exchange market settings.
- Calibration: [HL].
- Markovian projection: [GL23] explores path-dependent volatility, and [Pit06] discusses its application with LIBOR, supplementing [GHL11].
- T-forward measure: [Cho03] includes lecture notes from Course Stat391/FinMath 346 that cover the change of Numéraire and T-forward measure. We apply this change of measure in the hybrid model described in [GHL11] and [GHL13].
- Volatility derivatives and volatility trading: [Hul21] contains the formula derivation for VIX, and [CM98] discusses volatility trading.
- McKean-Vlasov SDE and propagation of chaos: [GHL13] and [Szn91].

- Malliavin Calculus: Applied in [GHL11], particularly when the volatility representation $\mathbb{E}^{\mathbb{Q}^t}\left[\left(r_t - r_t^0\right)\mathbf{1}_{S_t>K}\right]$ and $K\partial_K^2\mathcal{C}(t,K)$ are very small for out-of-the-money strikes. Numerically, this $\frac{0}{0}$ ratio can be problematic. The Malliavin representation can mitigate this issue due to Malliavin's integration by parts.
  - Introduction notes: [Gu23, Gra03].
  - Clark-Ocone formula and its applications: [AS19], with an application in digital options involving the Malliavin derivative of the indicator function.
- Notes for PDEs: [Hun22], which we use for the definitions of distribution and distributional derivatives, applied in calculating local volatility in [Rou98, DKZ96].
- Finite difference method: Textbook covering explicit, fully implicit, and Crank-Nicolson methods, along with relevant stability and convergence analysis: [WDH93, WHD95].
- Alternating Direction Implicit (ADI) methods:
  - Implicit: Original works on the ADI method from the 1950s-1960s are covered by [BV59, BVY62].

    Peaceman-Rachford [PR55], **Douglas-Rachford** [Dou57], **Craig-Sneyd** [CS88], **Fairweather-Mitchell** [FM65].

    More recent works or relevant notes include [Ope05, Wu21, NN19].

    Two theses, [Lin08] and [dG12], apply the ADI method with the Heston model and more general applications in derivatives pricing (Heston and SABR), respectively.
  - Explicit: [PMR$^+$20].

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