List of References

For this project, [RMQ07] is most relevant to our algorithm. [Rou98] contains the Dupire Local Volatility (LV) derivation. [GHL11] discusses the Monte Carlo (MC) calibration of a particular Stochastic Local Volatility (SLV) model, including the Markovian projection. The detailed derivation in this paper can be found in [GHL13]. [Hom14] serves as a comprehensive reference document on the background.
For other aspects or background knowledge needed for this project, consider the following:

- Matching the marginal distribution of Stochastic Differential Equations (SDE): [Gyö86].
- Kolmogorov Forward Equation: [HC22] and [CLP08] address two-dimensional Kolmogorov forward equations.
- Local Volatility Model:
  - Original works by Dupire: [D$^+$94, Dup97].
  - Additional notes on derivations: [Rou98].
  - With stochastic interest rates: [Hu15] provides the corresponding derivation for the Dupire formula, also found in [ÖH23] under the foreign exchange market settings.
- Calibration: [HL].
- Markovian projection: [GL23] explores path-dependent volatility, and [Pit06] discusses its application with LIBOR, supplementing [GHL11].
- T-forward measure: [Cho03] includes lecture notes from Course Stat391/FinMath 346 that cover the change of Numéraire and T-forward measure. We apply this change of measure in the hybrid model described in [GHL11] and [GHL13].
- Volatility derivatives and volatility trading: [Hul21] contains the formula derivation for VIX, and [CM98] discusses volatility trading.
- McKean-Vlasov SDE and propagation of chaos: [GHL13] and [Szn91].

- Malliavin Calculus: Applied in [GHL11], particularly when the volatility representation $\mathbb{E}^{\mathbb{Q}^t}\left[\left(r_t - r_t^0\right)\mathbf{1}_{S_t>K}\right]$ and $K\partial_K^2\mathcal{C}(t,K)$ are very small for out-of-the-money strikes. Numerically, this $\frac{0}{0}$ ratio can be problematic. The Malliavin representation can mitigate this issue due to Malliavin's integration by parts.
  - Introduction notes: [Gu23, Gra03].
  - Clark-Ocone formula and its applications: [AS19], with an application in digital options involving the Malliavin derivative of the indicator function.
- Notes for PDEs: [Hun22], which we use for the definitions of distribution and distributional derivatives, applied in calculating local volatility in [Rou98, DKZ96].
- Finite difference method: Textbook covering explicit, fully implicit, and Crank-Nicolson methods, along with relevant stability and convergence analysis: [WDH93, WHD95].
- Alternating Direction Implicit (ADI) methods:
  - Implicit: Original works on the ADI method from the 1950s-1960s are covered by [Dou57, BV59, BVY62, PR55]. More recent works or relevant notes include [Unk05, Wu21]. Two theses, [Lin08] and [dG12], apply the ADI method with the Heston model and more general applications in derivatives pricing (Heston and SABR), respectively.
  - Explicit: [PMR$^+$20].

References

[AS19] Takuji Arai and Ryoichi Suzuki. A clark-ocone type formula via ito calculus and its application to finance. *arXiv preprint arXiv:1906.06648*, 2019.

[BV59] Garrett Birkhoff and Richard S Varga. Implicit alternating direction methods. *Transactions of the American Mathematical Society*, 92(1):13–24, 1959.

[BVY62] Garrett Birkhoff, Richard S Varga, and David Young. Alternating direction implicit methods. In *Advances in computers*, volume 3, pages 189–273. Elsevier, 1962.

[Cho03] Yuan K. Chou. Course materials for stat 391: Introduction to stochastic processes, 2003. Accessed: 2024-06-16.

[CLP08] Antoine Conze, Nicolas Lantos, and Olivier Pironneau. The forward kolmogorov equation for two dimensional options. *Chinese Annals of Mathematics*, 2008.

[CM98] Peter Carr and Dilip Madan. Towards a theory of volatility trading. *Volatility: New estimation techniques for pricing derivatives*, 29:417–427, 1998.

[D+94] Bruno Dupire et al. Pricing with a smile. *Risk*, 7(1):18–20, 1994.

[dG12] CSL de Graaf. Finite difference methods in derivatives pricing under stochastic volatility models. *Master's thesis, Leiden University*, 2012.

[DKZ96] Emanuel Derman, Iraj Kani, and Joseph Z Zou. The local volatility surface: Unlocking the information in index option prices. *Financial analysts journal*, 52(4):25–36, 1996.

[Dou57] Jim Douglas. A note on the alternating direction implicit method for the numerical solution of heat flow problems. *Proceedings of the American Mathematical Society*, 8(2):409–412, 1957.

[Dup97] Bruno Dupire. Pricing and hedging with smiles. *Mathematics of derivative securities*, 1(1):103–111, 1997.

[GHL11] Julien Guyon and Pierre Henry-Labordere. The smile calibration problem solved. *Available at SSRN 1885032*, 2011.

[GHL13] Julien Guyon and Pierre Henry-Labordère. *Nonlinear option pricing*. CRC Press, 2013.

[GL23] Julien Guyon and Jordan Lekeufack. Volatility is (mostly) path-dependent. *Quantitative Finance*, 23(9):1221–1258, 2023.

[Gra03] Martino Grasselli. Introduction to malliavin calculus, 2003. Accessed: 2024-06-16.

[Gu23] Yueyu Gu. Course lectures, 2023. Accessed: 2024-06-16.

[Gyö86] István Gyöngy. Mimicking the one-dimensional marginal distributions of processes having an itô differential. *Probability theory and related fields*, 71(4):501–516, 1986.

[HC22] Maxime Holmes-Cerfon. Lecture 10 handout for asa 22, 2022. Accessed: 2024-06-16.

[HL] P Henry-Labordere. Calibration of local stochastic volatility models: A monte-carlo approach, risk magazine, sept.(2009). *Extended version: http://ssrn. com/abstract*, 1493306.

[Hom14] Cristian Homescu. Local stochastic volatility models: calibration and pricing. *Available at SSRN 2448098*, 2014.

[Hu15] Bing Hu. Local volatility model with stochastic interest rate. 2015.

[Hul21] John C. Hull. Technical note 22: An introduction to martingales, 2021. Accessed: 2024-06-16.

[Hun22] John K. Hunter. Notes on partial differential equations, 2022. Accessed: date-of-access.

[Lin08] Sensen Lin. Finite difference schemes for heston model. 2008.

[ÖH23] Orcan Ögetbil and Bernhard Hientzsch. Extensions of dupire formula: Stochastic interest rates and stochastic local volatility. *SIAM Journal on Financial Mathematics*, 14(2):452–474, 2023.

[Pit06] Vladimir Piterbarg. Markovian projection method for volatility calibration, 2006.

[PMR+20] Somayeh Pourghanbar, Jalil Manafian, Mojtaba Ranjbar, Aynura Aliyeva, and Yusif S Gasimov. An efficient alternating direction explicit method for solving a nonlinear partial differential equation. *Mathematical Problems in Engineering*, 2020(1):9647416, 2020.

[PR55] Donald W Peaceman and Henry H Rachford, Jr. The numerical solution of parabolic and elliptic differential equations. *Journal of the Society for industrial and Applied Mathematics*, 3(1):28–41, 1955.

[RMQ07] Yong Ren, Dilip Madan, and M Qian Qian. Calibrating and pricing with embedded local volatility models. *RISK-LONDON-RISK MAGAZINE LIMITED-*, 20(9):138, 2007.

[Rou98] Fabrice Douglas Rouah. Derivaton of local volatility, 1998.

[Szn91] Alain-Sol Sznitman. Topics in propagation of chaos. *Ecole d'été de probabilités de Saint-Flour XIX—1989*, 1464:165–251, 1991.

[Unk05] Unknown. Alternating direction implicit (adi) method, 2005. Accessed: 2024-06-16.

[WDH93] Paul Wilmott, Jeff Dewynne, and Sam Howison. Option pricing: mathematical models and computation. *(No Title)*, 1993.

[WHD95] Paul Wilmott, Sam Howison, and Jeff Dewynne. *The mathematics of financial derivatives: a student introduction*. Cambridge university press, 1995.

[Wu21] Ray Wu. Alternating direction implicit methods for black-scholes equations. 2021.