Residual Reduction Algorithm (RRA)

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1 Introduction

2 Reducing Residuals

Throughout this paper, when we use the term “reduce a residual”, or “reduce” a DC offset, it really means “try to eliminate” a residual or DC offset. That is, our strategy is to compute parameter alterations that would, in theory, completely eliminate the DC offset of a particular residual. But since the theoretical solutions we compute are not entirely true in practice since there are many factors affecting a particular residual other than just the one parameter we are altering. However, this failure to completely eliminate the DC offset is actually a good thing: we know that there are unmodeled forces in our system (for example, our model has no arms), so we actually do want some small DC offsets to remain for our residuals just to make us feel like we haven’t eliminated these unmodeled forces.

To reduce a particular residual R with DC offset $d_R \in \mathbb{R}$ by altering a particular parameter p by an amount Δp, we must compute Δp using the following equation:

$$R_{old} - R_{new} = d_R$$

We require that R_{new} and R_{old} be expressible in terms of inertial parameters and possibly joint variables, and that R_{new} also be expressed in terms of Δp. Then the above equation can be solved for Δp in terms of the inertial parameters and the DC offset d_R. It is easier to see why this equation is true if we look at it this way:

$$R_{new} = R_{old} - d_R$$

Here, R_{old} is the original residual with the DC offset d_R. If we *remove the DC offset* from R_{old}, i.e. if we subtract the DC offset from R_{old}, we get R_{new}.

2.1 Reducing Forward-Backward Rocking

We will reduce the residual MZ by independently altering two parameters: the torso center of mass x-coordinate by an amount Δt_x and the lumbar extension angle by an amount Δl_e.

2.1.1 Altering the Torso Center of Mass

Here we will compute an amount Δt_x by which to alter the x-coordinate of the torso center of mass in order to balance the DC offset of the MZ residual. Let m be the mass of the torso and let $\mathbf{g}$ denote acceleration due to gravity. Let d_{MZ} be the DC offset of the MZ residual. Let $\mathbf{r_0}$ be the moment arm (lever arm) of the torso, which we define to be the vector pointing from the pelvis center of mass to the torso center of mass. Note that $\mathbf{r_0}$ varies as the torso position varies, but its magnitude stays fixed. Then we have that the original value of MZ at any torso position is:

$$MZ_{old} = \mathbf{r_0} \times m\mathbf{g}$$

Let $\mathbf{r_1}$ be the torso moment arm after the center of mass has been displaced in the x direction by Δt_x. Note that $\mathbf{r_1}$ may not have the same magnitude as $\mathbf{r_0}$. Then the new value of MZ is:

$$\begin{aligned}MZ_{new} &= \mathbf{r_1} \times m\mathbf{g} \\ &= (\mathbf{r_0} + (\Delta t_x, 0, 0)) \times m\mathbf{g} \\ &= \mathbf{r_0} \times m\mathbf{g} + (\Delta t_x, 0, 0) \times m\mathbf{g}\end{aligned}$$

The last step is correct since the cross product distributes over addition. Let $\mathbf{d_{MZ}} = (0, 0, d_{MZ})$, i.e. $\mathbf{d_{MZ}}$ is a vector representation of the DC offset. Now we plus the above expressions into the equation $MZ_{old} - MZ_{new} = \mathbf{d_{MZ}}$:

$$\mathbf{r_0} \times m\mathbf{g} - (\mathbf{r_0} \times m\mathbf{g} + (\Delta t_x, 0, 0) \times m\mathbf{g}) = \mathbf{d_{MZ}}$$

The $\mathbf{r_0} \times m\mathbf{g}$ expressions cancel out on both sides, and the value of the remaining cross product is

$$(\Delta t_x, 0, 0) \times m\mathbf{g} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \Delta t_x & 0 & 0 \\ 0 & -mg & 0 \end{vmatrix} = (0, 0, -mg\Delta t_x).$$

So we are left with

$$-(0, 0, -mg\Delta t_x) = (0, 0, d_{MZ})$$

or looking at just the z-coordinates

$$\begin{aligned} mg\Delta t_x &= d_{MZ} \\ \Delta t_x &= \frac{d_{MZ}}{mg}. \end{aligned} \tag{1}$$

So, in order to reduce the DC offset of the MZ residual, i.e. to reduce the average forward-backward rocking motions of a walking model, our computation suggests that we should alter the torso center of mass x-coordinate by an amount d_{MZ}/mg.

2.1.2 Altering the Lumbar Extension Angle

Now we wish to compute an amount Δl_e by which to alter the lumbar extension angle (throughout the entire time interval, not just at the initial time) so that the DC offset for MZ is reduced. We can represent the alteration of the lumbar extension angle with the following geometry: consider the triangle consisting of two vectors $\mathbf{r_0}$ and $\mathbf{r_1}$ with equal length r_0 and with a common starting point with an angle Δl_e between them. Suppose the vectors are oriented so that Δl_e is drawn in a positive sense (counterclockwise) when it is drawn from $\mathbf{r_0}$ to $\mathbf{r_1}$. Let $\mathbf{\Delta l} = \mathbf{r_1} - \mathbf{r_0}$. Assuming Δl_e is small, we can apply an approximation from biomechanics which states that the moment arm (lever arm) of a muscle is equal to $\delta l/\delta\theta$ where δl is the change in length of the muscle when the joint spanned by the muscle rotates by a small angle $\delta\theta$. Applying this approximation to our triangle, we have that

$$r_0 = \Delta l/\Delta l_e$$

$$\Delta l = r_0 \Delta l_e,$$

where $\Delta l = \|\mathbf{\Delta l}\|$. We will show how to compute the (direction of) the vector $\mathbf{\Delta l}$ later. As before, we define $MZ_{new} = \mathbf{r_1} \times m\mathbf{g}$ and $MZ_{old} = \mathbf{r_0} \times m\mathbf{g}$. From the definition of $\mathbf{\Delta l}$, we know that $\mathbf{r_1} = \mathbf{r_0} + \mathbf{\Delta l}$. Plugging into the equation $MZ_{old} - MZ_{new} = \mathbf{d_{MZ}}$, we have

$$\mathbf{r_0} \times m\mathbf{g} - (\mathbf{r_0} \times m\mathbf{g} + \mathbf{\Delta l} \times m\mathbf{g}) = (0, 0, d_{MZ})$$

$$-\mathbf{\Delta l} \times m\mathbf{g} = (0, 0, d_{MZ}).$$

If we write $\mathbf{\Delta l} = (\Delta l_x, \Delta l_y, 0)$, then we have

$$\mathbf{\Delta l} \times m\mathbf{g} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \Delta l_x & \Delta l_y & 0 \\ 0 & -mg & 0 \end{vmatrix} = (0, 0, -mg\Delta l_x).$$

Subsituting into the previous equation, we have

$$mg\Delta l_x = d_{MZ}.$$

Now if we write $\Delta l_x = \Delta l \cos\theta$ where θ is the angle representing the orientation of the vector $\mathbf{\Delta l}$ relative to the positive x-axis (counterclockwise is positive), and since we know the length of the vector is $\Delta l = r_0 \Delta l_e$, we have that $\Delta l_x = r_0 \Delta l_e \cos\theta$, so substituting into the above equation yields

$$mgr_0 \Delta l_e \cos\theta = d_{MZ}$$

$$\Delta l_e = \frac{d_{MZ}}{mgr_0 \cos\theta}.$$

Let α be the angle representing the orientation of $\mathbf{r_0}$, measured in a positive (counterclockwise) sense starting from the positive x-axis. The angle l_e is the orientation of $\mathbf{r_0}$ as measured in a positive (counterclockwise) sense starting from the positive y-axis. So $\alpha = l_e + 90°$. Since we assumed that Δl_e is small, the vector $\mathbf{\Delta l}$ is approximately tangent to the circle with radius r_0 centered at the pelvis center of mass, i.e. we can assume that $\mathbf{\Delta l}$ is just $\mathbf{r_0}$ rotated counterclockwise by $90°$ and scaled. Since we defined θ to be the angle swept counterclockwise from the positive x-axis to $\mathbf{\Delta l}$, then we can assume that $\theta = \alpha + 90° = l_e + 180°$. Hence we have

$$\cos\theta = \cos(l_e + 180°) = \cos l_e \cos 180° - \sin l_e \sin 180° = -\cos l_e$$

so

$$\Delta l_e = -\frac{d_{MZ}}{mgr_0 \cos l_e}. \qquad (2)$$

2.2 Reducing Left-Right Rocking

We will reduce the residual MX by independently altering two parameters: the torso center of mass z-coordinate by an amount Δt_z and the lumbar bending angle by an amount Δl_b.

2.2.1 Altering the Torso Center of Mass

Here we will compute an amount Δt_z by which to alter the z-coordinate of the torso center of mass in order to balance the DC offset of the MX residual. Let d_{MX} be the DC offset of the MX residual. The original value of MX at any torso position is:

$$MX_{old} = \mathbf{r_0} \times m\mathbf{g}$$

Let $\mathbf{r_1}$ be the torso moment arm after the center of mass has been displaced in the z direction by Δt_z. Note that $\mathbf{r_1}$ may not have the same magnitude as $\mathbf{r_0}$. Then the new value of MX is:

$$\begin{aligned} MX_{new} &= \mathbf{r_1} \times m\mathbf{g} \\ &= (\mathbf{r_0} + (0,0,\Delta t_z)) \times m\mathbf{g} \\ &= \mathbf{r_0} \times m\mathbf{g} + (0,0,\Delta t_z) \times m\mathbf{g} \end{aligned}$$

Let $\mathbf{d_{MX}} = (d_{MX}, 0, 0)$, i.e. $\mathbf{d_{MX}}$ is a vector representation of the DC offset. Now we plus the above expressions into the equation $MX_{old} - MX_{new} = \mathbf{d_{MX}}$:

$$\mathbf{r_0} \times m\mathbf{g} - (\mathbf{r_0} \times m\mathbf{g} + (0,0,\Delta t_z) \times m\mathbf{g}) = \mathbf{d_{MX}}$$

The $\mathbf{r_0} \times m\mathbf{g}$ expressions cancel out on both sides, and the value of the remaining cross product is

$$(0,0,\Delta t_z) \times m\mathbf{g} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \Delta t_z \\ 0 & -mg & 0 \end{vmatrix} = (mg\Delta t_z, 0, 0).$$

So we are left with

$$-(mg\Delta t_z, 0, 0) = (d_{MX}, 0, 0)$$

or looking at just the x-coordinates

$$-mg\Delta t_z = d_{MX}$$
$$\Delta t_z = -\frac{d_{MX}}{mg}. \tag{3}$$

So, in order to reduce the DC offset of the MX residual, i.e. to reduce the average forward-backward rocking motions of a walking model, our computation suggests that we should alter the torso center of mass z-coordinate by an amount $-d_{MX}/mg$.

2.2.2 Altering the Lumbar Bending Angle

Now we wish to compute an amount Δl_b by which to alter the lumbar bending angle (throughout the entire time interval, not just at the initial time) so that the DC offset for MX is reduced. We can represent the alteration of the lumbar bending angle with the following geometry: consider the triangle consisting of two vectors $\mathbf{r_0}$ and $\mathbf{r_1}$ with equal length r_0 and with a common starting point with an angle Δl_b between them. Suppose the vectors are oriented so that Δl_b is drawn in a positive sense (counterclockwise) when it is drawn from $\mathbf{r_0}$ to $\mathbf{r_1}$. Let $\mathbf{\Delta l} = \mathbf{r_1} - \mathbf{r_0}$. Assuming Δl_b is small, we can apply an approximation from biomechanics which states that the moment arm (lever arm) of a muscle is equal to $\delta l/\delta\theta$ where δl is the change in length of the muscle when the joint spanned by the muscle rotates by a small angle $\delta\theta$. Applying this approximation to our triangle, we have that

$$r_0 = \Delta l/\Delta l_b$$
$$\Delta l = r_0 \Delta l_b,$$

where $\Delta l = \|\mathbf{\Delta l}\|$. We will show how to compute the (direction of) the vector $\mathbf{\Delta l}$ later. As before, we define $MX_{new} = \mathbf{r_1} \times m\mathbf{g}$ and $MX_{old} = \mathbf{r_0} \times m\mathbf{g}$. From the definition of $\mathbf{\Delta l}$, we know that $\mathbf{r_1} = \mathbf{r_0} + \mathbf{\Delta l}$. Plugging into the equation $MX_{old} - MX_{new} = \mathbf{d_{MX}}$, we have

$$\mathbf{r_0} \times m\mathbf{g} - (\mathbf{r_0} \times m\mathbf{g} + \mathbf{\Delta l} \times m\mathbf{g}) = (d_{MX}, 0, 0)$$
$$-\mathbf{\Delta l} \times m\mathbf{g} = (d_{MX}, 0, 0).$$

If we write $\mathbf{\Delta l} = (0, \Delta l_y, \Delta l_z)$, then we have

$$\mathbf{\Delta l} \times m\mathbf{g} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \Delta l_y & \Delta l_z \\ 0 & -mg & 0 \end{vmatrix} = (mg\Delta l_z, 0, 0).$$

Subsituting into the previous equation and extracting just the x-coordinates, we have

$$-mg\Delta l_z = d_{MX}.$$

Now if we write $\Delta l_z = \Delta l \cos\theta$ where θ is the angle representing the orientation of the vector $\mathbf{\Delta l}$ relative to the positive y-axis, and since we know the length of the vector is $\Delta l = r_0 \Delta l_b$, we have that $\Delta l_z = r_0 \Delta l_b \sin\theta$, so substituting into the above equation yields

$$-mgr_0 \Delta l_b \sin\theta = d_{MX}$$
$$\Delta l_b = -\frac{d_{MX}}{mgr_0 \sin\theta}.$$

The angle l_b is the orientation of $\mathbf{r_0}$ as measured in a positive (counterclockwise) sense starting from the positive y-axis. Since we assumed that Δl_b is small, the vector $\mathbf{\Delta l}$ is approximately tangent to the circle with radius r_0 centered at the pelvis center of mass, i.e. we can assume that $\mathbf{\Delta l}$ is just $\mathbf{r_0}$ rotated counterclockwise by 90° and scaled. Since we defined θ to be the angle swept counterclockwise from the positive y-axis to $\mathbf{\Delta l}$, then we can assume that $\theta = l_b + 90°$. Hence we have

$$\sin\theta = \sin(l_b + 90°) = \sin l_b \cos 90° + \cos l_b \sin 90° = \cos l_b$$

so

$$\Delta l_b = -\frac{d_{MX}}{mgr_0 \cos l_b}. \tag{4}$$

2.3 Residual Reduction Algorithm (RRA)

The following simple algorithm will reduce the MX and MZ residuals if it is executed after the inverse kinematics stage of the simulation pipeline and before the CMC stage.

1. Run CMC once to compute the residuals.
2. Compute Δt_x and Δt_z using equations 1 and 3.
3. Compute Δl_e and Δl_b using equations 2 and 4 for each value of l_e and l_b given in the input motion file.
4. Check whether thresholds are exceeded for each correction. The thresholds are $|\Delta t_x|, |\Delta t_z| \leq 0.1$ meter and $|\Delta l_e|, |\Delta l_b| \leq 10°$.
5. Omit any correction that exceeds its threshold.
6. If all corrections exceed their thresholds, then give up and tell the user that this data is too bad to be corrected.

I actually have omitted the l_e and l_b corrections completely, since performing those changes *in addition to* the torso center of mass changes actually caused an increase in the MX and MZ residuals. Perhaps it would be effective to change *either* the back angles *or* the torso center of mass, but not both simultaneously. It appears though that changing the torso center of mass is effective enough, so the current implementation does exactly that, and completely ignores the back angles l_e and l_b.

3 Results

For su900061, the original DC offsets after the first step of RRA were

$$\begin{aligned} d_{FX} &= 1.54907 \\ d_{FY} &= -2.31422 \\ d_{FZ} &= -0.0353541 \\ d_{MX} &= 4.29873 \\ d_{MY} &= -0.369015 \\ d_{MZ} &= -9.47111 \end{aligned}$$

and the DC offsets after RRA and the second pass of CMC were

$$\begin{aligned} d_{FX} &= 1.29935 \\ d_{FY} &= -3.14646 \\ d_{FZ} &= 0.453855 \\ d_{MX} &= 1.07497 \\ d_{MY} &= -0.452011 \\ d_{MZ} &= 0.346921. \end{aligned}$$

For s26, the original DC offsets after the first step of RRA were

$$\begin{aligned} d_{FX} &= 11.9906 \\ d_{FY} &= 23.4048 \\ d_{FZ} &= 2.75953 \\ d_{MX} &= -4.45362 \\ d_{MY} &= 0.895433 \\ d_{MZ} &= 3.13189 \end{aligned}$$

and the DC offsets after RRA and the second pass of CMC were

$$\begin{aligned} d_{FX} &= 11.9219 \\ d_{FY} &= 23.7031 \\ d_{FZ} &= 2.87774 \\ d_{MX} &= -0.802673 \\ d_{MY} &= 1.04612 \\ d_{MZ} &= 0.838793. \end{aligned}$$

Interestingly, under the old algorithm where one or both of the back angles were also changed in the second pass of RRA, the resulting DC offsets for the same subject were

$$\begin{aligned} d_{FX} &= 12.0841 \\ d_{FY} &= 23.9657 \\ d_{FZ} &= 2.81026 \\ d_{MX} &= 23.0526 \\ d_{MY} &= 1.19643 \\ d_{MZ} &= -22.5084. \end{aligned}$$

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