

Part 2: Correlation

Building a GPS receiver from scratch

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Topics

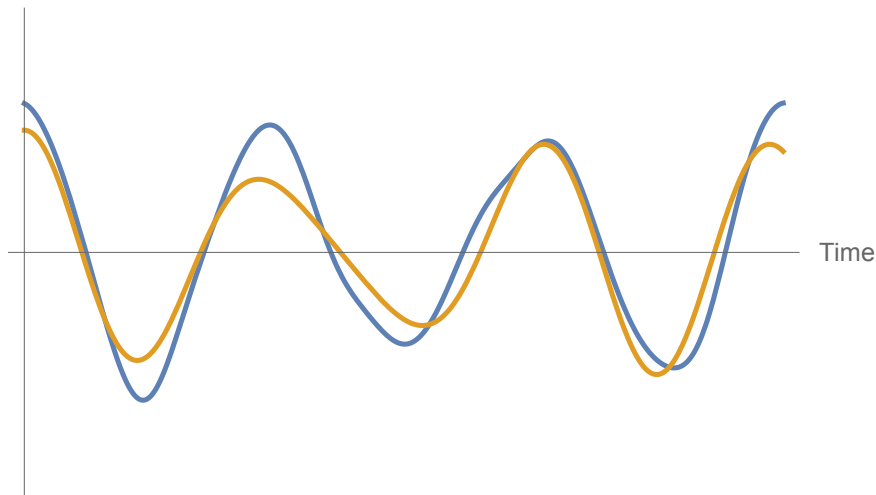
- 1 Correlation
- 2 Cross-correlation
- 3 Autocorrelation

Topics

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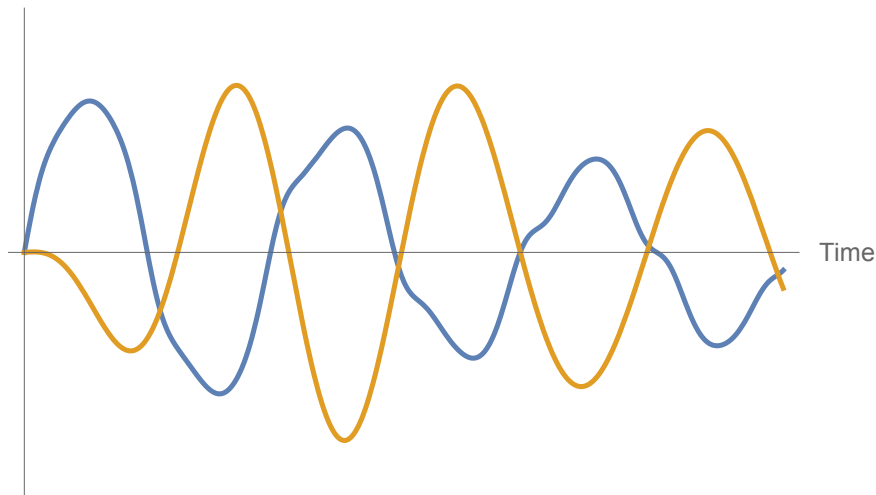
Positive correlation

Amplitude



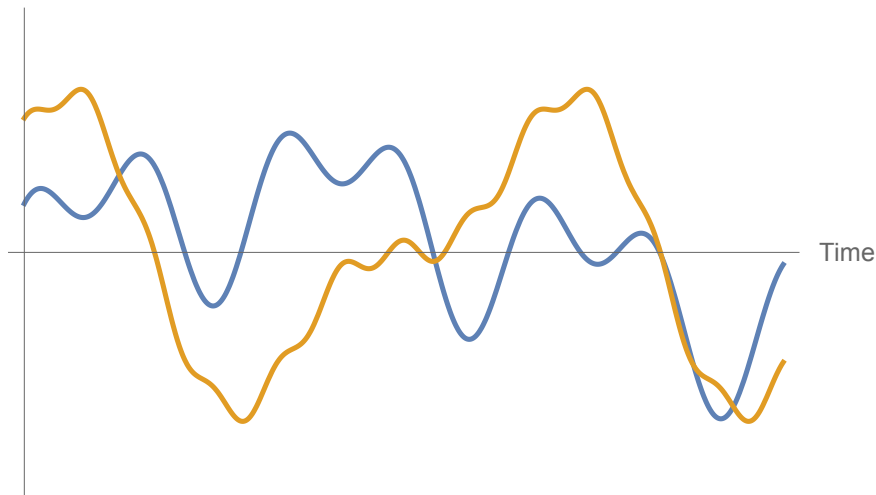
Negative correlation

Amplitude



Near-zero correlation

Amplitude

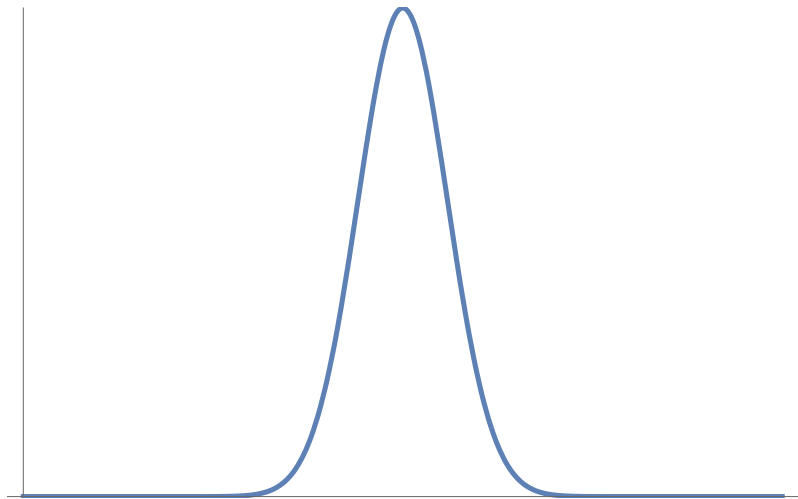


The correlation of two signals $f(t)$ and $g(t)$ is defined as

$$\int_{-\infty}^{+\infty} f(t)g(t) dt.$$

$$f(t) \rightarrow 0 \text{ as } |t| \rightarrow \infty$$

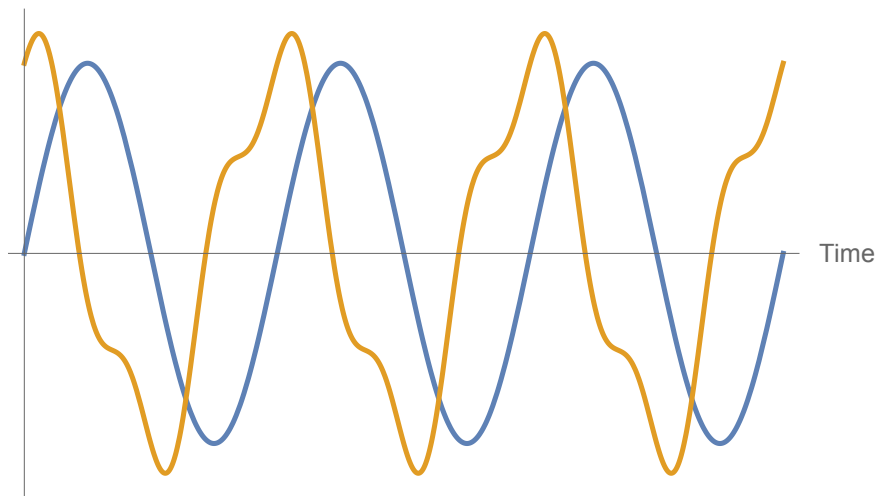
Amplitude



Time

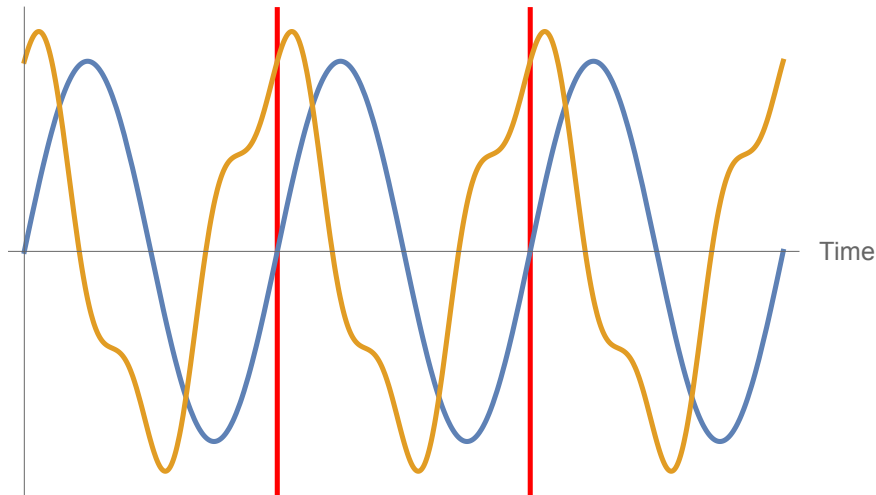
Periodic signals

Amplitude



Periodic signals

Amplitude



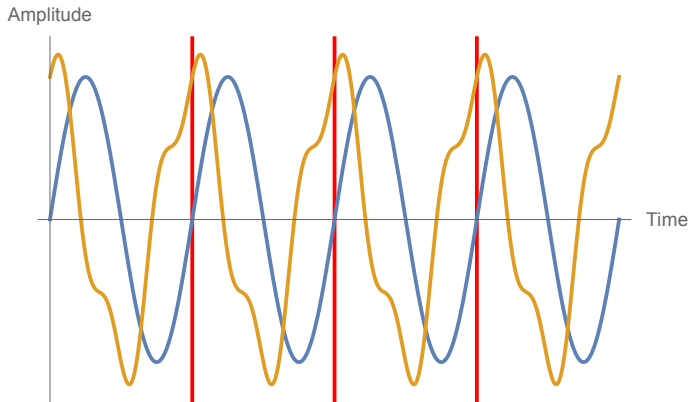
Definition

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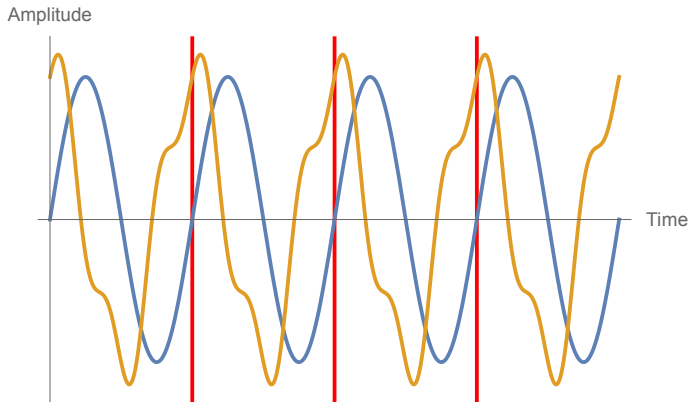
$$\int_{t_0}^{t_0+T} f(t)g(t) dt$$

where t_0 is an arbitrary point in time and T is their shared period.

Multiple periods



Multiple periods



$$\int_{t_0}^{t_0+nT} f(t)g(t) dt = n \int_{t_0}^{t_0+T} f(t)g(t) dt$$

Definition vs. intuition

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- Not similar at all \Rightarrow positive and negative products \Rightarrow sums cancel

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- Autocorrelation
 - The cross-correlation of a signal with itself