This document contains my notes on Stanford's online course GPS: An Introduction to Satellite Navigation. Each section corresponds to the video of the same title.

## Contents



# 1 GPS How and Why

- In order to calculate the receiver's position we need to know:

  1. the time at which a satellite transmitted a radio signal,
  2. the location of the satellite when it transmitted the signal,
  3. the speed of the radio transmission (close to the speed of light), and
  4. the time at which the radio signal is received.

- If we can obtain these four pieces of information from at least four satellites, we can solve an equation for four unknowns: the offset of the user's clock from the satellites' clocks, and the user's x, y, and z coordinates.

- The offset of the user's clock from the satellites' clocks is a single unknown rather than one for each satellite because all the satellites' clocks are synchronised.

# 2 Satellites

- GPS satellites are in medium Earth orbit (MEO).

- A single GPS satellite can typically see one third of the Earth's surface.

- There are additional satellites in geostationary orbit (GEO) above various countries to augment GPS data.

## 3 Navigation Messages

- The navigation message tells us the location of the satellite and the time at which it broadcast the navigation message.
- An **ephemeris** is the orbital data for a satellite.
- This information is broadcast by each satellite at around 50 bps.
- A full GPS message consists of 25 pages. Each page consists of 5 subframes. Each subframe is 300 bits. Thus, it takes 6 s to transmit a subframe, 30 s to transmit a page, and 12.5 min to transmit a message.
- Each page consists of information about the broadcasting satellite, ephemeris parameters, and a page of the almanac.
- The ephemeris parameters are expressed as Keplerian elements. These can be split into three categories:
  - The first describes the shape of the elliptical orbit itself. It doesn't position the ellipse relative to the Earth. This category includes:
    * the semi-major axis $a$ which determines the size of the ellipse, and
    * and the eccentricity $e$ which determines how circular or elliptical the orbit is.

    GPS orbits are close to circular ($e = 0$), but not quite. To increase accuracy we must account for the eccentricity of the orbit.
  - The next describes how the orbit is oriented relative to the Earth. The Earth is positioned at one of the foci of the ellipse. This category includes:
    * the inclination $i$ which is the angle the orbital plane makes with the equatorial plane,
    * the right ascension of the ascending node (RAAN) $\Omega$ which is the angle between the vernal equinox and the ascending node of the orbit in the equatorial plane in the direction of the Earth's rotation, and
    * the angle of perigee $\omega$ which is the angle between the ascending node and perigee in the orbital plane.
  - The last describes the satellite's position in the orbit. This category contains only the true anomaly $\nu$ which is the angle between perigee and the satellite in the orbital plane.

## 4 Navigation Signals

- There is a unique code for each satellite.

- Each 0 or 1 in a satellite's code is known as a **chip**.
- Satellites transmit at 1.023 Mcps (million chips per second).
- The L1 frequency is the most used civilian frequency.
- The code for each satellite has good autocorrelation properties (i.e. it's easy to see when the receiver has aligned its code with the transmitted code) and low cross-correlation with other satellites.

# 5 Pseudoranging

- If you take the difference in time between when the satellite transmitted the message and when the receiver received it and multiply that by the speed of light, you get the distance between the satellite and the receiver

$$t_{\text{received}} - t_{\text{sent}} = \frac{d}{c}.$$

- The **replica** is the receiver's copy of the chipping code that "slides along" the received signal to find correlation.
- The **late replica** is a fraction of a chip later than the received signal, the **prompt replica** is equal to the received signal, and the **early replica** is a fraction of a chip earlier than the received signal. The receiver tries to keep the three replicas positioned such that the difference in correlation values between the early and late replicas is 0.
- The clocks of all the satellites are synchonised, but the receiver's clock may differ. For this reason we introduce the **clock bias** $b_u$ such that the adjusted time of receival is

$$t_{\text{u}} = t_{\text{received}} + b_u$$

and the actual distance can be calculated from

$$t_{\text{u}} - t_{\text{send}} = \frac{d}{c} + b_u.$$

- Ideally we would only need to solve for $x$, $y$, and $z$ to determine out position, but we also need to solve for $b_u$ to determine the receiver's clock bias. For this reason at least four satellites need to be in view.

# 6 GPS Performance a First Look

- You can determine the velocity of the receiver based on the doppler shift in the received signals.

## 7 Pseudoranges Including Errors

- The pseudorange equation is

$$\tau^{(n)} = \sqrt{(x_u - x^{(n)})^2 + (y_u - y^{(n)})^2 + (z_u - z^{(n)})^2} + b_u + \nu_u^{(n)}$$

  where $\tau^{(n)}$ is the pseudorange between the user and the $n$th satellite, $(x, y, z)_u$ are the user's coordinates, $(x, y, z)^{(n)}$ are the $n$th satellite's coordinates, $b_u$ is the user's clock offset, and $\nu_u^{(n)}$ is the error suffered by the user in the measurement from the $n$th satellite.

- When a signal travels through the ionosphere and the troposphere its speed reduces and this causes error. If a satellite is directly overhead this is minimised, but if it's on the horizon the signal spends more time in the troposphere and error is increased.

- The updated pseudorange equation including these errors is

$$\tau_C = \left(d_u^{(k)} + b_u - \delta B^{(k)}\right) + \delta I_u^{(k)} + \delta T_u^{(k)} + \nu_u^{(k)}$$
$$d_u^{(k)} = \sqrt{(x_u - x^{(k)})^2 + (y_u - y^{(k)})^2 + (z_i - z^{(k)})}$$

  where $d_u^{(k)}$ is the distance between the user and the $k$th satellite, $b_u$ is the user's clock bias, $\delta B^{(k)}$ is the clock bias of the $k$th satellite, $\delta I_u^{(k)}$ is the ionospheric error experienced by the user in measurements from the $k$th satellite, $\delta T_u^{(k)}$ is the tropospheric error experienced by the user in measurements from the $k$th satellite, and $\nu_u^{(k)}$ is all other error experienced by the user in measurements from the $k$th satellite.