Part 4: Sampling

Building a GPS receiver from scratch

Chris Doble

Topics

1. Hardware
2. Parameters
   - Centre frequency
   - Bandwidth
   - Sampling rate
3. I/Q samples
   - Definition
   - Different frequencies
   - Complex values

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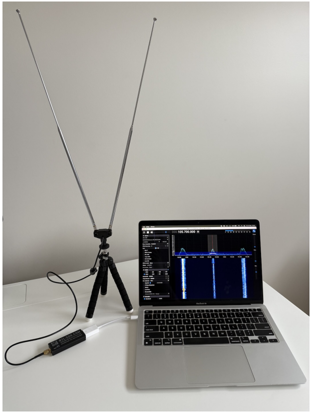
Hardware

- Software defined radio (SDR) dongle



Hardware

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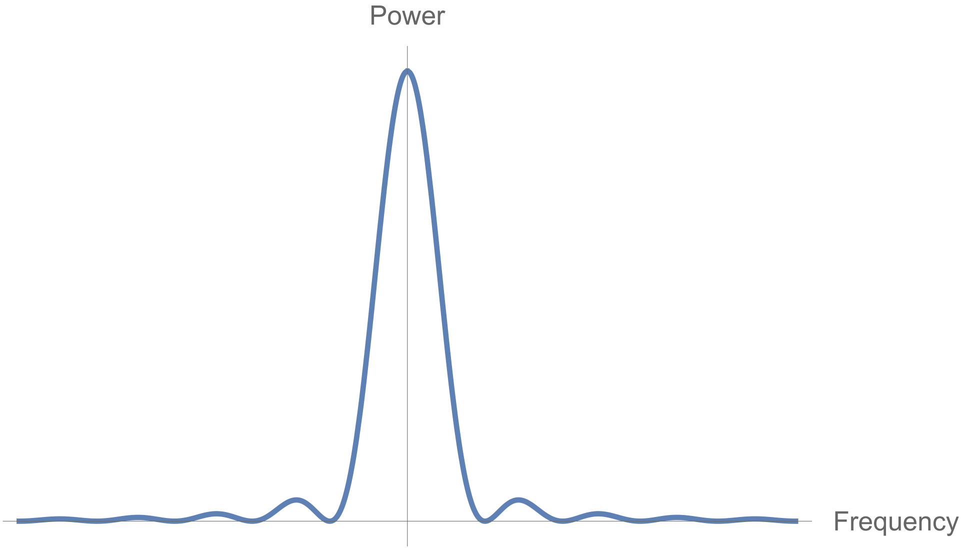
Topics

Centre frequency

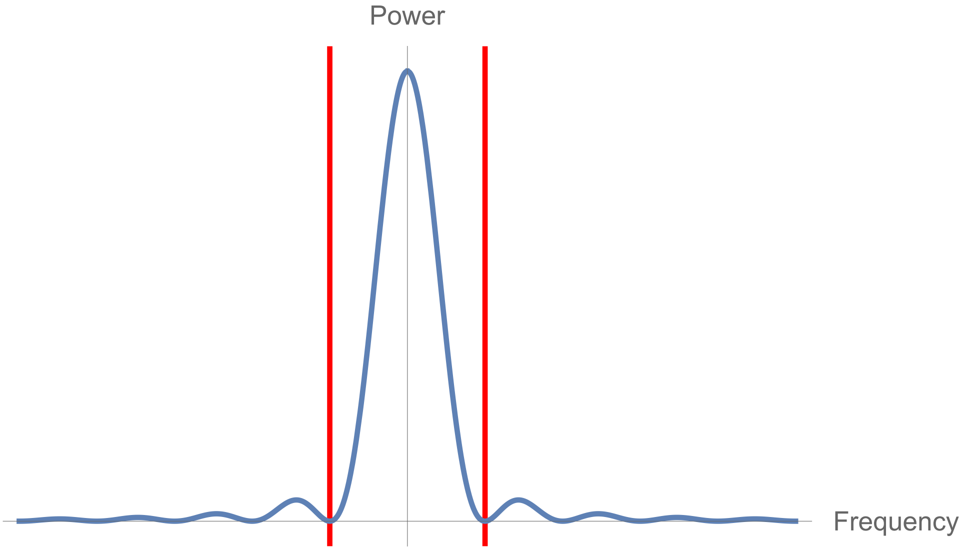
$$f = 1575.42\,\mathrm{MHz}$$

Topics

The power spectrum of BPSK



The power spectrum of BPSK



Bandwidth

$$B = 2.046\,\mathrm{MHz}$$

Topics

Sampling rate

$$f_{L1} = 1575.42\,\text{Mhz} \approx 1.6\,\text{GHz}$$

The Nyquist-Shannon sampling theorem

If the maximum frequency contained within a signal is f_{max},
then the signal can be determined from its samples
if the sampling rate is greater than $2f_{\mathrm{max}}$.

Sampling rate

$$f_{\text{max}} = f + \frac{B}{2}$$

Sampling rate

$$
\begin{aligned}
f_{\mathrm{max}} &= f + \frac{B}{2} \\
&= 1575.42\,\mathrm{MHz} + \frac{2.046\,\mathrm{MHz}}{2}
\end{aligned}
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Sampling rate

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\begin{aligned}
f_{\text{max}} &= f + \frac{B}{2} \\
&= 1575.42\,\text{MHz} + \frac{2.046\,\text{MHz}}{2} \\
&= 1576.443\,\text{MHz} \\
&\approx 1.6\,\text{GHz}
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&= 1576.443\,\text{MHz} \\
&\approx 1.6\,\text{GHz} \\
f_s &= 2 f_{\text{max}} \\
&\approx 3.2\,\text{GHz}
\end{aligned}
$$

What is the RTL-SDRs sample rate?

The maximum sample rate is 3.2 MS/s (mega samples per second). However, the RTL-SDR is unstable at this rate and may drop samples. The maximum sample rate that does not drop samples is 2.56 MS/s, however some people have had luck with 2.8MS/s and 3.2 MS/s working well on some USB 3.0 ports.

https://www.rtl-sdr.com/about-rtl-sdr/

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- Large enough that aliases don't overlap
- $1/770$ of the L1 frequency $\Rightarrow$ alias at $0\,\text{Hz}$
- $f_{\text{max}} = 1.023\,\text{MHz} \Rightarrow f_{\text{Nyquist}} = 2.046\,\text{MHz} \Rightarrow$ within SDR dongle's capabilities

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The definition of I/Q samples

$$f(t) = A(t)\cos(2\pi ft + \phi(t))$$

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The definition of I/Q samples

$$f(t) = A(t)[\cos(2\pi ft)\cos(\phi(t)) + \cos(2\pi ft + \pi/2)\sin(\phi(t))]$$

The definition of I/Q samples

$$
\begin{aligned}
f(t) &= A(t)[\cos(2\pi ft)\cos(\phi(t)) + \cos(2\pi ft + \pi/2)\sin(\phi(t))] \\
&= A(t)\cos(\phi(t))\cos(2\pi ft) + A(t)\sin(\phi(t))\cos(2\pi ft + \pi/2)
\end{aligned}
$$

The definition of I/Q samples

$$\begin{aligned} f(t) &= A(t)[\cos(2\pi ft)\cos(\phi(t)) + \cos(2\pi ft + \pi/2)\sin(\phi(t))] \\ &= A(t)\cos(\phi(t))\cos(2\pi ft) + A(t)\sin(\phi(t))\cos(2\pi ft + \pi/2) \\ &= I(t)\cos(2\pi ft) + Q(t)\cos(2\pi ft + \pi/2) \end{aligned}$$

where $I(t) = A(t)\cos(\phi(t))$ and $Q(t) = A(t)\sin(\phi(t))$.

Determining a signal's phase

$$Q/I =$$

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$$Q/I = \frac{A(t)\sin(\phi(t))}{A(t)\cos(\phi(t))}$$

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&= \tan(\phi(t))
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$$

Determining a signal's phase

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\begin{aligned}
Q/I &= \frac{A(t)\sin(\phi(t))}{A(t)\cos(\phi(t))} \\
&= \tan(\phi(t)) \\
\phi(t) &= \arctan(Q/I)
\end{aligned}
$$

Topics

Sampling a signal of a different frequency

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f_1

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$$f_1$$

$$f_2 = f_1 + \Delta f$$

Sampling a signal of a different frequency

$$
\begin{aligned}
f_1 & \\
f_2 &= f_1 + \Delta f \\
f(t) &= A\cos(2\pi f_2 t)
\end{aligned}
$$

Sampling a signal of a different frequency

$$
\begin{aligned}
f_1 & \\
f_2 &= f_1 + \Delta f \\
f(t) &= A\cos(2\pi f_2 t) \\
&= A\cos(2\pi(f_1 + \Delta f)t) \\
&= A\cos(2\pi f_1 t + 2\pi \Delta f t)
\end{aligned}
$$

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\begin{aligned}
f_1 & \\
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f(t) &= A\cos(2\pi f_2 t) \\
&= A\cos(2\pi(f_1 + \Delta f)t) \\
&= A\cos(2\pi f_1 t + 2\pi\Delta f t) \\
&= A\cos(2\pi f_1 t + \phi(t))
\end{aligned}
$$

where $\phi(t) = 2\pi\Delta f t$.

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$$\begin{aligned}
\hat{D}_i(t)P\hat{R}N_i(t) &= \pm 1 \\
z\hat{D}_i(t)P\hat{R}N_i(t) &= z(\pm 1) \\
&= Ae^{j\phi}(\pm 1) \\
&= \pm Ae^{j\phi}
\end{aligned}$$

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- I/Q samples
    - Pairs of numbers that describe the signal
    - Often expressed as a single complex number
    - If the signal has a different frequency, the I/Q samples will continually rotate