Part 7: Decoding

Building a GPS receiver from scratch

Chris Doble

Navigation message structure

Name	Number of bits	Emitted every
Pseudosymbol	1/20	1 ms

Navigation message structure

Name	Number of bits	Emitted every
Pseudosymbol	1/20	1 ms
Pseudobit / bit	1	20 ms

Navigation message structure

Name	Number of bits	Emitted every
Pseudosymbol	1/20	1 ms
Pseudobit / bit	1	20 ms
Word	30	0.6 s

Navigation message structure

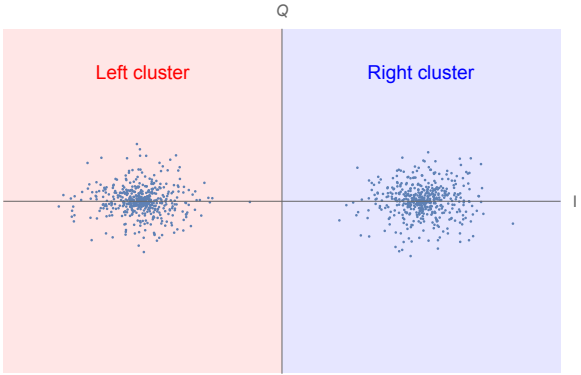
Name	Number of bits	Emitted every
Pseudosymbol	1/20	1 ms
Pseudobit / bit	1	20 ms
Word	30	0.6 s
Subframe	300	6 s

Navigation message structure

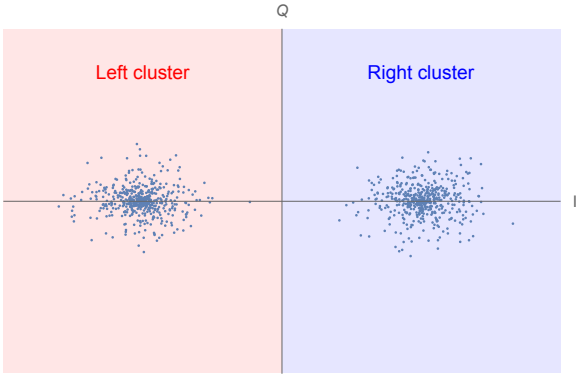
Name	Number of bits	Emitted every
Pseudosymbol	1/20	1 ms
Pseudobit / bit	1	20 ms
Word	30	0.6 s
Subframe	300	6 s
Frame	1,500	30 s

Topics

Pseudosymbol integration



Pseudosymbol integration



Pseudosymbol integration

$$-1\ -1\ +1\ +1\ +1\ +1\ -1\ -1\ -1\ -1\ -1\ -1\ -1\ -1\ +1\ +1\ +1\ +1$$

Pseudosymbol integration

−1 −1 +1 +1 | +1 +1 −1 −1 | −1 −1 −1 −1 | −1 −1 +1 +1 +1 +1

Pseudosymbol integration

−1 −1 +1 +1 | +1 +1 −1 −1 | −1 −1 −1 −1 | −1 −1 +1 +1 | +1 +1

Offset	Score
0	
1	
2	
3	

Pseudosymbol integration

0	0	−4	0		
−1 −1 +1 +1	+1 +1 −1 −1	−1 −1 −1 −1	−1 −1 +1 +1	+1	+1

Offset	Score
0	
1	
2	
3	

Pseudosymbol integration

```
 0 => 0        0 => 0        -4 => 4       0 => 0
-1 -1 +1 +1   +1 +1 -1 -1   -1 -1 -1 -1   -1 -1 +1 +1  +1 +1
```

Offset	Score
0	
1	
2	
3	

Pseudosymbol integration

```
 0 => 0      0 => 0      -4 => 4      0 => 0
-1 -1 +1 +1 +1 +1 -1 -1 -1 -1 -1 -1 -1 -1 +1 +1 +1 +1
```

Offset	Score
0	1.0
1	
2	
3	

Pseudosymbol integration

```
                  4 => 4          -4 => 4          -4 => 4          4 => 4
-1 -1 +1 +1 +1 +1 -1 -1 -1 -1 -1 -1 -1 -1 +1 +1 +1 +1
```

Offset	Score
0	1.0
1	
2	4.0
3	

Pseudosymbol integration

−1 −1 +1 +1 +1 +1 −1 −1 −1 −1 −1 −1 −1 −1 +1 +1 +1 +1

Offset	Score
0	1.0
1	2.5
2	4.0
3	2.6

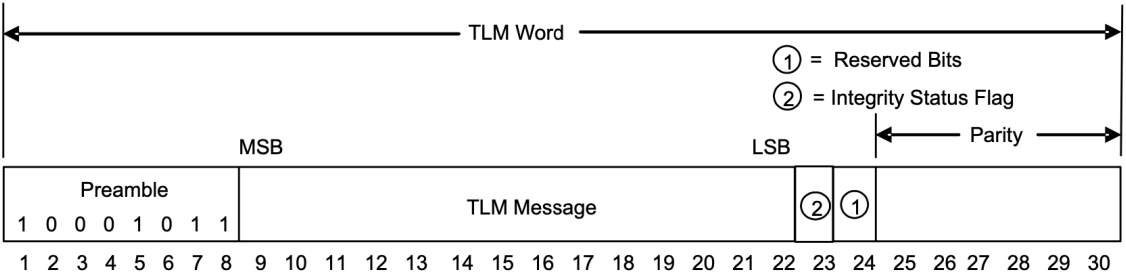
Pseudosymbol integration

−1 −1 +1 +1 +1 +1 −1 −1 −1 −1 −1 −1 −1 −1 +1 +1 +1 +1

Offset	Score
0	1.0
1	2.5
2	**4.0**
3	2.6

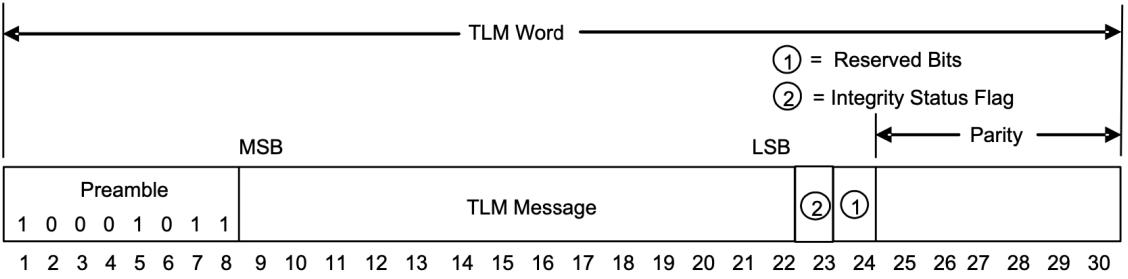
Topics

Pseudobit integration



Source: Figure 20-2, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

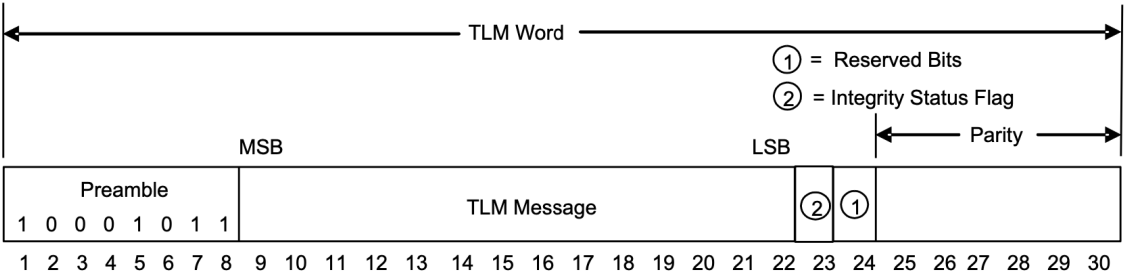
Pseudobit integration



Source: Figure 20-2, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

$$-1 \;\; +1 \;\; +1 \;\; +1 \;\; -1 \;\; +1 \;\; -1 \;\; -1 \Rightarrow -1 \text{ maps to } 1, +1 \text{ maps to } 0$$

Pseudobit integration



Source: Figure 20-2, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

$$-1\ +1\ +1\ +1\ -1\ +1\ -1\ -1 \Rightarrow -1 \text{ maps to } 1, +1 \text{ maps to } 0$$
$$+1\ -1\ -1\ -1\ +1\ -1\ +1\ +1 \Rightarrow +1 \text{ maps to } 1, -1 \text{ maps to } 0$$

Topics

Parity bits

$$
\begin{aligned}
D_1 &= d_1 \oplus D_{30}^{\star} \\
D_2 &= d_2 \oplus D_{30}^{\star} \\
D_3 &= d_3 \oplus D_{30}^{\star} \\
&\bullet \\
&\bullet \\
&\bullet \\
&\bullet \\
D_{24} &= d_{24} \oplus D_{30}^{\star} \\
D_{25} &= D_{29}^{\star} \oplus d_1 \oplus d_2 \oplus d_3 \oplus d_5 \oplus d_6 \oplus d_{10} \oplus d_{11} \oplus d_{12} \oplus d_{13} \oplus d_{14} \oplus d_{17} \oplus d_{18} \oplus d_{20} \oplus d_{23} \\
D_{26} &= D_{30}^{\star} \oplus d_2 \oplus d_3 \oplus d_4 \oplus d_6 \oplus d_7 \oplus d_{11} \oplus d_{12} \oplus d_{13} \oplus d_{14} \oplus d_{15} \oplus d_{18} \oplus d_{19} \oplus d_{21} \oplus d_{24} \\
D_{27} &= D_{29}^{\star} \oplus d_1 \oplus d_3 \oplus d_4 \oplus d_5 \oplus d_7 \oplus d_8 \oplus d_{12} \oplus d_{13} \oplus d_{14} \oplus d_{15} \oplus d_{16} \oplus d_{19} \oplus d_{20} \oplus d_{22} \\
D_{28} &= D_{30}^{\star} \oplus d_2 \oplus d_4 \oplus d_5 \oplus d_6 \oplus d_8 \oplus d_9 \oplus d_{13} \oplus d_{14} \oplus d_{15} \oplus d_{16} \oplus d_{17} \oplus d_{20} \oplus d_{21} \oplus d_{23} \\
D_{29} &= D_{30}^{\star} \oplus d_1 \oplus d_3 \oplus d_5 \oplus d_6 \oplus d_7 \oplus d_9 \oplus d_{10} \oplus d_{14} \oplus d_{15} \oplus d_{16} \oplus d_{17} \oplus d_{18} \oplus d_{21} \oplus d_{22} \oplus d_{24} \\
D_{30} &= D_{29}^{\star} \oplus d_3 \oplus d_5 \oplus d_6 \oplus d_8 \oplus d_9 \oplus d_{10} \oplus d_{11} \oplus d_{13} \oplus d_{15} \oplus d_{19} \oplus d_{22} \oplus d_{23} \oplus d_{24}
\end{aligned}
$$

Where

$d_1, d_2, \ldots, d_{24}$ are the source data bits;

the symbol $\star$ is used to identify the last 2 bits of the previous word of the subframe;

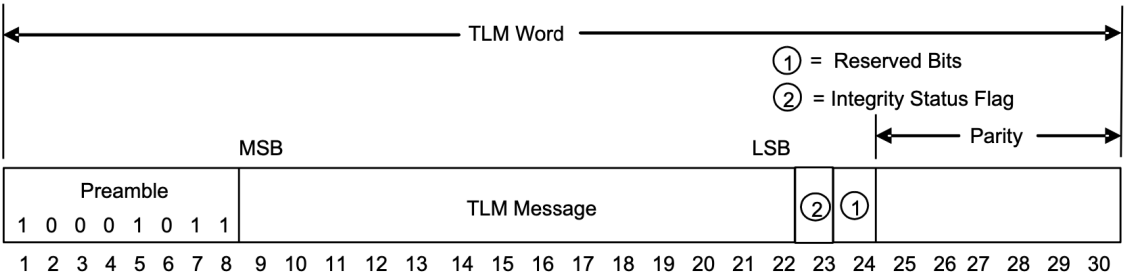
$D_{25}, D_{26}, \ldots, D_{30}$ are the computed parity bits;

$D_1, D_2, \ldots, D_{29}, D_{30}$ are the bits transmitted by the SV;

$\oplus$ is the "modulo-2" or "exclusive-or" operation.

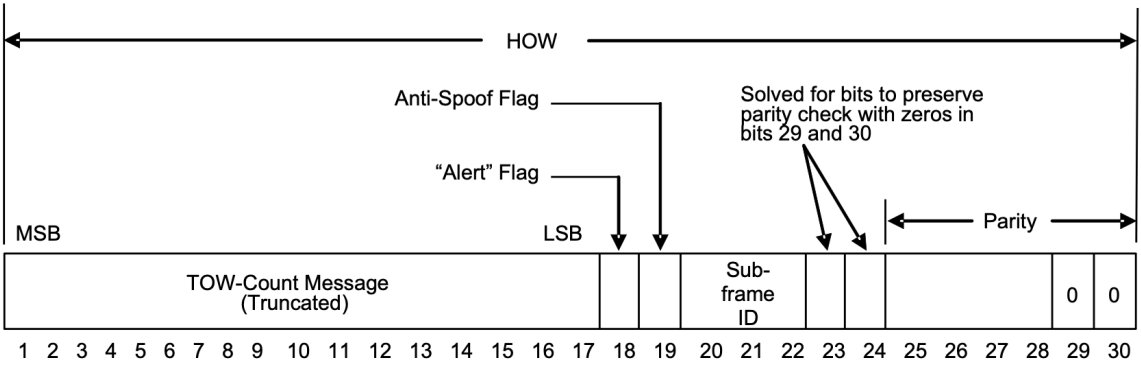
Source: Table 20-XIV, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

The telemetry word



Source: Figure 20-2, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

The handover word



Source: Figure 20-2, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

The TOW count

- GPS started operating at midnight UTC on the night of Saturday January 5, 1980

The TOW count

- GPS started operating at midnight UTC on the night of Saturday January 5, 1980
- The number of weeks that have passed since that night is called the GPS week number

The TOW count

- GPS started operating at midnight UTC on the night of Saturday January 5, 1980
- The number of weeks that have passed since that night is called the GPS week number
- The time-of-week count (TOW count) is the number of 1.5 s periods that have elapsed since the start of the current GPS week (since midnight UTC Saturday night)

The TOW count

- GPS started operating at midnight UTC on the night of Saturday January 5, 1980
- The number of weeks that have passed since that night is called the GPS week number
- The time-of-week count (TOW count) is the number of 1.5 s periods that have elapsed since the start of the current GPS week (since midnight UTC Saturday night)
- The TOW count is a 19 bit number

The TOW count

- GPS started operating at midnight UTC on the night of Saturday January 5, 1980
- The number of weeks that have passed since that night is called the GPS week number
- The time-of-week count (TOW count) is the number of 1.5 s periods that have elapsed since the start of the current GPS week (since midnight UTC Saturday night)
- The TOW count is a 19 bit number
- The truncated TOW count is the 17 most significant bits of the TOW count as it will appear at the time the next subframe begins transmission

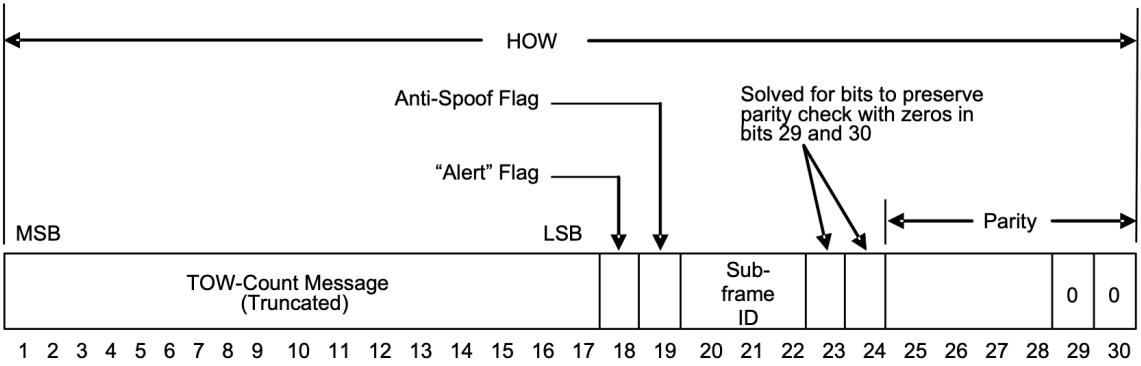
The TOW count

- GPS started operating at midnight UTC on the night of Saturday January 5, 1980
- The number of weeks that have passed since that night is called the GPS week number
- The time-of-week count (TOW count) is the number of 1.5 s periods that have elapsed since the start of the current GPS week (since midnight UTC Saturday night)
- The TOW count is a 19 bit number
- The truncated TOW count is the 17 most significant bits of the TOW count as it will appear at the time the next subframe begins transmission
- This corresponds to a 6 s period — how long it takes to transmit a subframe — so the truncated TOW count will increment by 1 with each subframe we receive

The TOW count

- GPS started operating at midnight UTC on the night of Saturday January 5, 1980
- The number of weeks that have passed since that night is called the GPS week number
- The time-of-week count (TOW count) is the number of 1.5 s periods that have elapsed since the start of the current GPS week (since midnight UTC Saturday night)
- The TOW count is a 19 bit number
- The truncated TOW count is the 17 most significant bits of the TOW count as it will appear at the time the next subframe begins transmission
- This corresponds to a 6 s period — how long it takes to transmit a subframe — so the truncated TOW count will increment by 1 with each subframe we receive
- We can use this to calculate the signal's transmission time

The handover word



Source: Figure 20-2, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

Subframes

Subframes

- Subframe 1

Subframes

- Subframe 1
  - Clock parameters

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted
    - Correct for atomic clock drift

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted
    - Correct for atomic clock drift
  - Health information

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted
    - Correct for atomic clock drift
  - Health information
- Subframes 2 and 3

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted
    - Correct for atomic clock drift
  - Health information
- Subframes 2 and 3
  - Orbital parameters

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted
    - Correct for atomic clock drift
  - Health information
- Subframes 2 and 3
  - Orbital parameters
    - Calculate a satellite's position

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted
    - Correct for atomic clock drift
  - Health information
- Subframes 2 and 3
  - Orbital parameters
    - Calculate a satellite's position
- Subframes 4 and 5

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted
    - Correct for atomic clock drift
  - Health information
- Subframes 2 and 3
  - Orbital parameters
    - Calculate a satellite's position
- Subframes 4 and 5
  - Parameters change every frame over 25 frames (12.5 minutes)

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted
    - Correct for atomic clock drift
  - Health information
- Subframes 2 and 3
  - Orbital parameters
    - Calculate a satellite's position
- Subframes 4 and 5
  - Parameters change every frame over 25 frames (12.5 minutes)
    - Other satellites

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted
    - Correct for atomic clock drift
  - Health information
- Subframes 2 and 3
  - Orbital parameters
    - Calculate a satellite's position
- Subframes 4 and 5
  - Parameters change every frame over 25 frames (12.5 minutes)
    - Other satellites
    - Earth's atmospheric conditions

Subframes

- Subframe 1
  - Clock parameters
    - Calculate when signals were transmitted
    - Correct for atomic clock drift
  - Health information
- Subframes 2 and 3
  - Orbital parameters
    - Calculate a satellite's position
- Subframes 4 and 5
  - Parameters change every frame over 25 frames (12.5 minutes)
    - Other satellites
    - Earth's atmospheric conditions
    - Etc.

Topics

Decoding numbers

Parameter	No. of Bits**	Scale Factor (LSB)	Valid Range***	Units
IODE	8			(see text)
C_{rs}	16*	2^{-5}		meters
Δn	16*	2^{-43}		semi-circles/sec
M_0	32*	2^{-31}		semi-circles
C_{uc}	16*	2^{-29}		radians
e	32	2^{-33}	0.0 to 0.03	dimensionless
C_{us}	16*	2^{-29}		radians
$\sqrt{A}$	32	2^{-19}	2530 to 8192	$\sqrt{\text{meters}}$
t_{oe}	16	2^{4}	0 to 604,784	seconds
C_{ic}	16*	2^{-29}		radians
Ω_0	32*	2^{-31}		semi-circles
C_{is}	16*	2^{-29}		radians
i_0	32*	2^{-31}		semi-circles
C_{rc}	16*	2^{-5}		meters
ω	32*	2^{-31}		semi-circles
$\dot{\Omega}$	24*	2^{-43}	-6.33E-07 to 0	semi-circles/sec
IDOT	14*	2^{-43}		semi-circles/sec

* Parameters so indicated shall be two's complement, with the sign bit (+ or -) occupying the MSB;
** See Figure 20-1 for complete bit allocation in subframe;
*** Unless otherwise indicated in this column, valid range is the maximum range attainable with indicated bit allocation and scale factor.

Source: Table 20-III, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

Decoding numbers

Parameter	No. of Bits**	Scale Factor (LSB)	Valid Range***	Units
IODE	8			(see text)
C_{rs}	16*	2^{-5}		meters
Δn	16*	2^{-43}		semi-circles/sec
M_0	32*	2^{-31}		semi-circles
C_{uc}	16*	2^{-29}		radians
e	32	2^{-33}	0.0 to 0.03	dimensionless
C_{us}	16*	2^{-29}		radians
$\sqrt{A}$	32	2^{-19}	2530 to 8192	$\sqrt{\text{meters}}$
t_{oe}	16	2^{4}	0 to 604,784	seconds
C_{ic}	16*	2^{-29}		radians
Ω_0	32*	2^{-31}		semi-circles
C_{is}	16*	2^{-29}		radians
i_0	32*	2^{-31}		semi-circles
C_{rc}	16*	2^{-5}		meters
ω	32*	2^{-31}		semi-circles
$\dot{\Omega}$	24*	2^{-43}	-6.33E-07 to 0	semi-circles/sec
IDOT	14*	2^{-43}		semi-circles/sec

* Parameters so indicated shall be two's complement, with the sign bit (+ or -) occupying the MSB;
** See Figure 20-1 for complete bit allocation in subframe;
*** Unless otherwise indicated in this column, valid range is the maximum range attainable with indicated bit allocation and scale factor.

Source: Table 20-III, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

Decoding numbers

Parameter	No. of Bits**	Scale Factor (LSB)	Valid Range***	Units
IODE	8			(see text)
C_{rs}	16*	2^{-5}		meters
Δn	16*	2^{-43}		semi-circles/sec
M_0	32*	2^{-31}		semi-circles
C_{uc}	16*	2^{-29}		radians
e	32	2^{-33}	0.0 to 0.03	dimensionless
C_{us}	16*	2^{-29}		radians
$\sqrt{A}$	32	2^{-19}	2530 to 8192	$\sqrt{\text{meters}}$
t_{oe}	16	2^{4}	0 to 604,784	seconds
C_{ic}	16*	2^{-29}		radians
Ω_0	32*	2^{-31}		semi-circles
C_{is}	16*	2^{-29}		radians
i_0	32*	2^{-31}		semi-circles
C_{rc}	16*	2^{-5}		meters
ω	32*	2^{-31}		semi-circles
$\dot{\Omega}$	24*	2^{-43}	-6.33E-07 to 0	semi-circles/sec
IDOT	14*	2^{-43}		semi-circles/sec

* Parameters so indicated shall be two's complement, with the sign bit (+ or -) occupying the MSB;
** See Figure 20-1 for complete bit allocation in subframe;
*** Unless otherwise indicated in this column, valid range is the maximum range attainable with indicated bit allocation and scale factor.

Source: Table 20-III, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

Decoding numbers

Parameter	No. of Bits**	Scale Factor (LSB)	Valid Range***	Units
IODE	8			(see text)
C_{rs}	16*	2^{-5}		meters
Δn	16*	2^{-43}		semi-circles/sec
M_0	32*	2^{-31}		semi-circles
C_{uc}	16*	2^{-29}		radians
e	32	2^{-33}	0.0 to 0.03	dimensionless
C_{us}	16*	2^{-29}		radians
$\sqrt{A}$	32	2^{-19}	2530 to 8192	$\sqrt{\text{meters}}$
t_{oe}	16	2^{4}	0 to 604,784	seconds
C_{ic}	16*	2^{-29}		radians
Ω_0	32*	2^{-31}		semi-circles
C_{is}	16*	2^{-29}		radians
i_0	32*	2^{-31}		semi-circles
C_{rc}	16*	2^{-5}		meters
ω	32*	2^{-31}		semi-circles
$\dot{\Omega}$	24*	2^{-43}	-6.33E-07 to 0	semi-circles/sec
IDOT	14*	2^{-43}		semi-circles/sec

* Parameters so indicated shall be two's complement, with the sign bit (+ or -) occupying the MSB;
** See Figure 20-1 for complete bit allocation in subframe;
*** Unless otherwise indicated in this column, valid range is the maximum range attainable with indicated bit allocation and scale factor.

Source: Table 20-III, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

Decoding numbers

Decoding numbers

1. Parse the bits as if they were a normal integer

Decoding numbers

1. Parse the bits as if they were a normal integer
2. Convert the integer from two's complement representation (if necessary)

Decoding numbers

1. Parse the bits as if they were a normal integer
2. Convert the integer from two's complement representation (if necessary)
3. Multiply by the scale factor

Decoding numbers

Parameter	No. of Bits**	Scale Factor (LSB)	Valid Range***	Units
IODE	8			(see text)
C_{rs}	16*	2^{-5}		meters
Δn	16*	2^{-43}		semi-circles/sec
M_0	32*	2^{-31}		semi-circles
C_{uc}	16*	2^{-29}		radians
e	32	2^{-33}	0.0 to 0.03	dimensionless
C_{us}	16*	2^{-29}		radians
$\sqrt{A}$	32	2^{-19}	2530 to 8192	$\sqrt{\text{meters}}$
t_{oe}	16	2^{4}	0 to 604,784	seconds
C_{ic}	16*	2^{-29}		radians
Ω_0	32*	2^{-31}		semi-circles
C_{is}	16*	2^{-29}		radians
i_0	32*	2^{-31}		semi-circles
C_{rc}	16*	2^{-5}		meters
ω	32*	2^{-31}		semi-circles
$\dot{\Omega}$	24*	2^{-43}	-6.33E-07 to 0	semi-circles/sec
IDOT	14*	2^{-43}		semi-circles/sec

* Parameters so indicated shall be two's complement, with the sign bit (+ or -) occupying the MSB;
** See Figure 20-1 for complete bit allocation in subframe;
*** Unless otherwise indicated in this column, valid range is the maximum range attainable with indicated bit allocation and scale factor.

Source: Table 20-III, IS-GPS-200M, https://www.gps.gov/technical/icwg/IS-GPS-200M.pdf

Recap

Recap

- Pseudosymbols $\rightarrow$ bits $\rightarrow$ words $\rightarrow$ subframes $\rightarrow$ frames

Recap

- Pseudosymbols → bits → words → subframes → frames
- Group pseudosymbols into pseudobits

Recap

- Pseudosymbols → bits → words → subframes → frames
- Group pseudosymbols into pseudobits
  1. Collect several bits worth of pseudosymbols

Recap

- Pseudosymbols $\rightarrow$ bits $\rightarrow$ words $\rightarrow$ subframes $\rightarrow$ frames
- Group pseudosymbols into pseudobits
  1. Collect several bits worth of pseudosymbols
  2. Calculate a score for each possible grouping

Recap

- Pseudosymbols $\rightarrow$ bits $\rightarrow$ words $\rightarrow$ subframes $\rightarrow$ frames
- Group pseudosymbols into pseudobits
  1. Collect several bits worth of pseudosymbols
  2. Calculate a score for each possible grouping
  3. Choose the one with the greatest score

Recap

- Pseudosymbols $\rightarrow$ bits $\rightarrow$ words $\rightarrow$ subframes $\rightarrow$ frames
- Group pseudosymbols into pseudobits
  1. Collect several bits worth of pseudosymbols
  2. Calculate a score for each possible grouping
  3. Choose the one with the greatest score
- Map pseudobits to bits

Recap

- Pseudosymbols $\rightarrow$ bits $\rightarrow$ words $\rightarrow$ subframes $\rightarrow$ frames
- Group pseudosymbols into pseudobits
  1. Collect several bits worth of pseudosymbols
  2. Calculate a score for each possible grouping
  3. Choose the one with the greatest score
- Map pseudobits to bits
  1. Collect several subframes worth of pseudobits

Recap

- Pseudosymbols $\rightarrow$ bits $\rightarrow$ words $\rightarrow$ subframes $\rightarrow$ frames
- Group pseudosymbols into pseudobits
  1. Collect several bits worth of pseudosymbols
  2. Calculate a score for each possible grouping
  3. Choose the one with the greatest score
- Map pseudobits to bits
  1. Collect several subframes worth of pseudobits
  2. Search for the telemetry word preamble (or its inverse)

Recap

- Pseudosymbols $\rightarrow$ bits $\rightarrow$ words $\rightarrow$ subframes $\rightarrow$ frames
- Group pseudosymbols into pseudobits
  1. Collect several bits worth of pseudosymbols
  2. Calculate a score for each possible grouping
  3. Choose the one with the greatest score
- Map pseudobits to bits
  1. Collect several subframes worth of pseudobits
  2. Search for the telemetry word preamble (or its inverse)
- Obtain subframes' data bits by applying the parity algorithm

Recap

- Pseudosymbols $\rightarrow$ bits $\rightarrow$ words $\rightarrow$ subframes $\rightarrow$ frames
- Group pseudosymbols into pseudobits
  1. Collect several bits worth of pseudosymbols
  2. Calculate a score for each possible grouping
  3. Choose the one with the greatest score
- Map pseudobits to bits
  1. Collect several subframes worth of pseudobits
  2. Search for the telemetry word preamble (or its inverse)
- Obtain subframes' data bits by applying the parity algorithm
- TOW count in handover word tells when the next subframe begins transmission

Recap

- Pseudosymbols → bits → words → subframes → frames
- Group pseudosymbols into pseudobits
  1. Collect several bits worth of pseudosymbols
  2. Calculate a score for each possible grouping
  3. Choose the one with the greatest score
- Map pseudobits to bits
  1. Collect several subframes worth of pseudobits
  2. Search for the telemetry word preamble (or its inverse)
- Obtain subframes' data bits by applying the parity algorithm
- TOW count in handover word tells when the next subframe begins transmission
- Different subframes contain different parameters — we need 1, 2, and 3