

Part 5: Acquisition

Building a GPS receiver from scratch

Chris Doble

1 Parameters

- PRN code phase
- Carrier wave frequency shift

2 Finding parameters

3 Parameter space

4 Determining presence

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- To decode satellite i 's navigation message bit, calculate

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where t_0 is a point in time and $r(t)$ is the received signal

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- In order to align them, we need to know the phase

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- Undo the rotation by multiplying each sample by $e^{-j2\pi\Delta ft}$

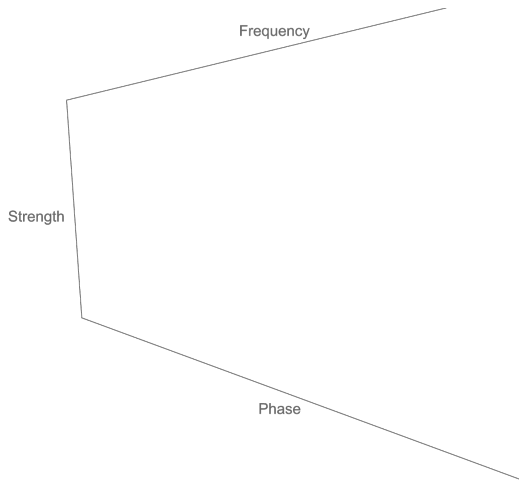
Recovering the signal

- Undo the rotation by multiplying each sample by $e^{-j2\pi\Delta f t}$
- This is called carrier wipeoff and it's why we need to know Δf

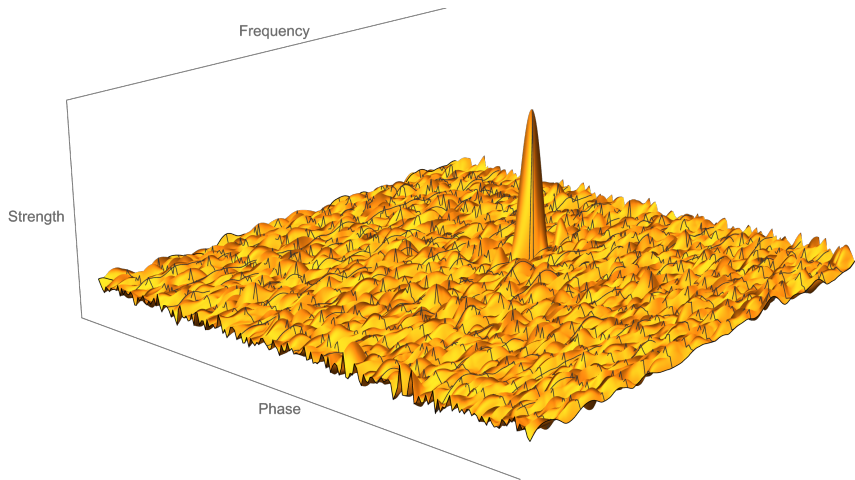
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- Calculate correlation over 10×1 ms periods, add their magnitudes
- This is called non-coherent integration

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- There are 2046 phases to check

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- $\Delta f_{\text{total}} \approx \pm 7.5$ kHz

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- ➋ Compare the signal strength with a threshold
- ➌ If it's greater \Rightarrow present
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- ➎ Periodically try to acquire satellites we're not tracking

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 - 3 If above the threshold \Rightarrow present
 - 4 If below the threshold \Rightarrow absent, check again later