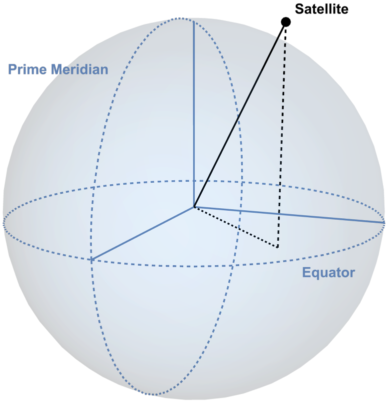
Part 8: Solving

Building a GPS receiver from scratch

Chris Doble

Topics

Coordinate system



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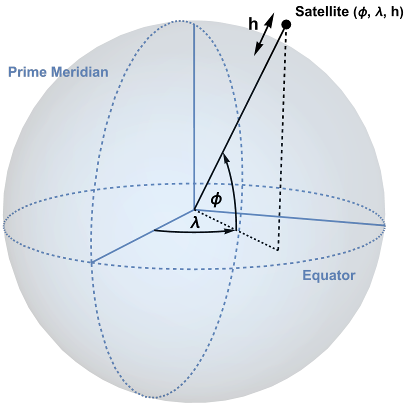

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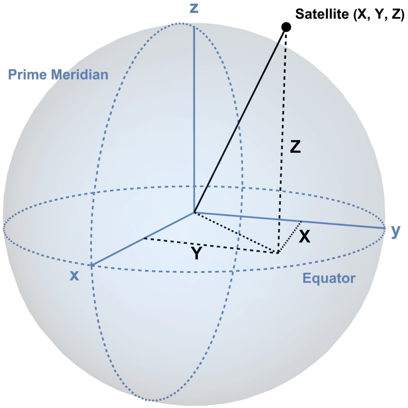

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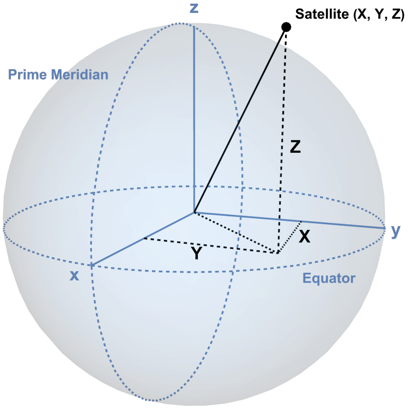
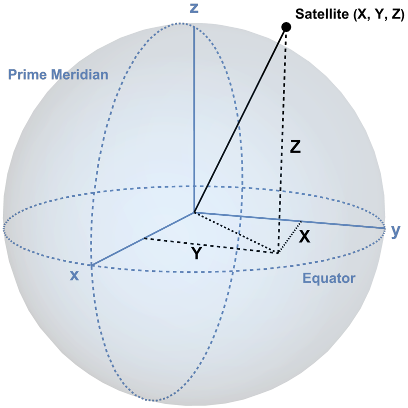


Transit time T

Coordinate system



Transit time $T \Rightarrow$ distance cT

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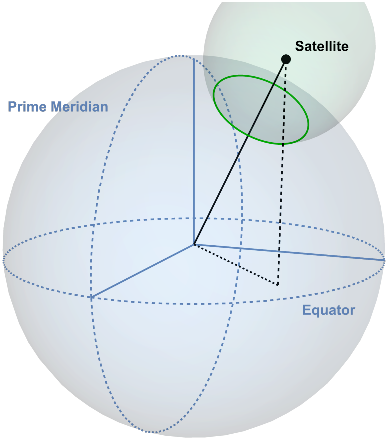

Transit time $T \Rightarrow$ distance cT

The pseudorange equation

$$\sqrt{(X-x)^2+(Y-y)^2+(Z-z)^2}=cT$$

where (x, y, z) is our unknown location.

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$$\sqrt{(X-x)^2+(Y-y)^2+(Z-z)^2}-cT=0$$

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Topics

1. The pseudorange equation
2. Satellite location and transit time
3. System of equations
   - Definition
   - Solving

Transmission time

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The user shall correct the time received from the SV with the equation (in seconds)

$$t = t_{sv} - \Delta t_{sv} \qquad (1)$$

where

t = GPS system time (seconds),

t_{sv} = effective SV PRN code phase time at message transmission time (seconds),

Δt_{sv} = SV PRN code phase time offset (seconds).

The SV PRN code phase offset is given by

$$\Delta t_{sv} = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 + \Delta t_r \qquad (2)$$

$$\vdots$$

Thus, the user who utilizes the L1 C/A signal only shall modify the code phase offset in accordance with paragraph 20.3.3.3.3.1 with the equation

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Transmission time

$$t_{sv} = \mathrm{TOW} \times 6\,\mathrm{s}$$

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3. Calculate the number of seconds since GPS started operating
4. Calculate the remainder when divided by the number of seconds in a GPS week

Location

$\mu = 3.986005 \text{ x } 10^{14}$ meters^3/sec^2

$\dot{\Omega}_e = 7.2921151467 \text{ x } 10^{-5}$ rad/sec

$A = \left(\sqrt{A}\right)^2$

$n_0 = \sqrt{\frac{\mu}{A^3}}$

$t_k = t - t_{oe}$*

$n = n_0 + \Delta n$

$M_k = M_0 + nt_k$

$E_0 = M_k$

$$E_j = E_{j-1} + \frac{M_k - E_{j-1} + e \sin E_{j-1}}{1 - e \cos E_{j-1}}$$

$E_k = E_j$

$$\nu_k = 2 \tan^{-1}\left(\sqrt{\frac{1+e}{1-e}} \tan \frac{E_k}{2}\right)$$

$\Phi_k = \nu_k + \omega$

$\delta u_k = c_{us}\sin 2\Phi_k + c_{uc}\cos 2\Phi_k$
$\delta r_k = c_{rs}\sin 2\Phi_k + c_{rc}\cos 2\Phi_k$
$\delta i_k = c_{is}\sin 2\Phi_k + c_{ic}\cos 2\Phi_k$

$u_k = \Phi_k + \delta u_k$

$r_k = A(1 - e \cos E_k) + \delta r_k$

$i_k = i_0 + \delta i_k + (IDOT)\, t_k$

$x_k' = r_k \cos u_k$
$y_k' = r_k \sin u_k$

$\Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e)\, t_k - \dot{\Omega}_e\, t_{oe}$

$x_k = x_k' \cos\Omega_k - y_k' \cos i_k \sin\Omega_k$
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Definition

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$$\sqrt{(X_4 - x)^2 + (Y_4 - y)^2 + (Z_4 - z)^2} - c(T_4 - t) = 0$$

Topics

The Newton-Raphson method

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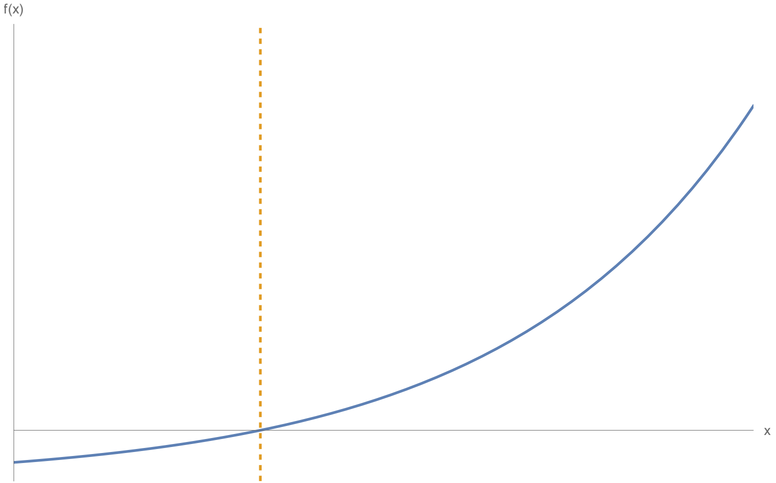

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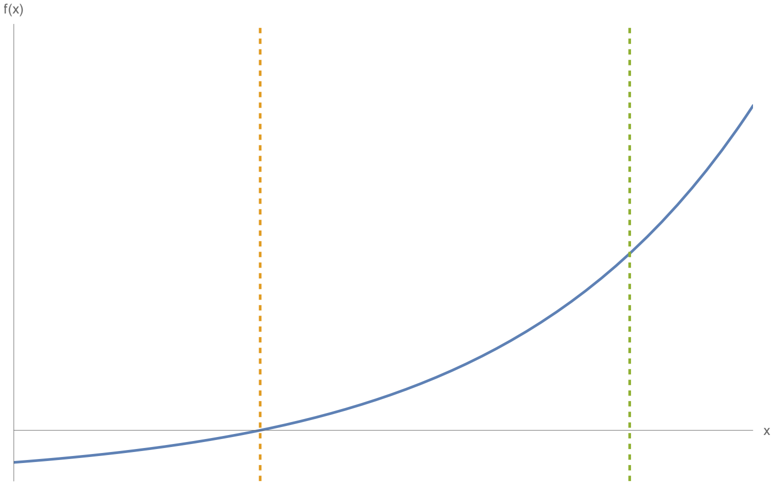

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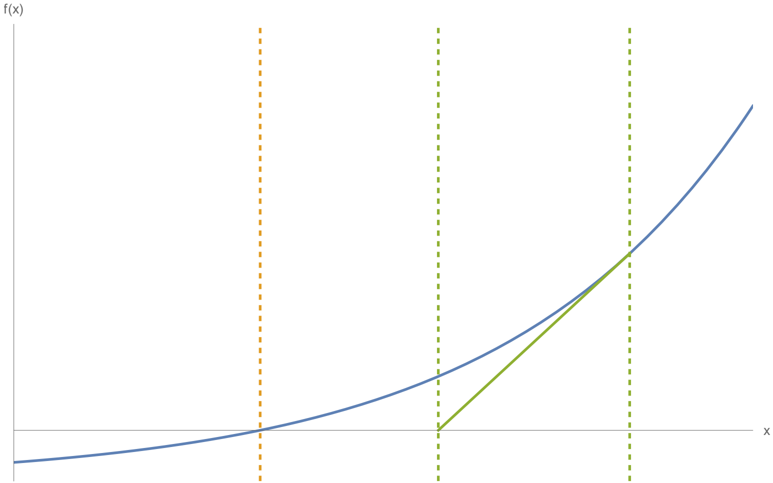

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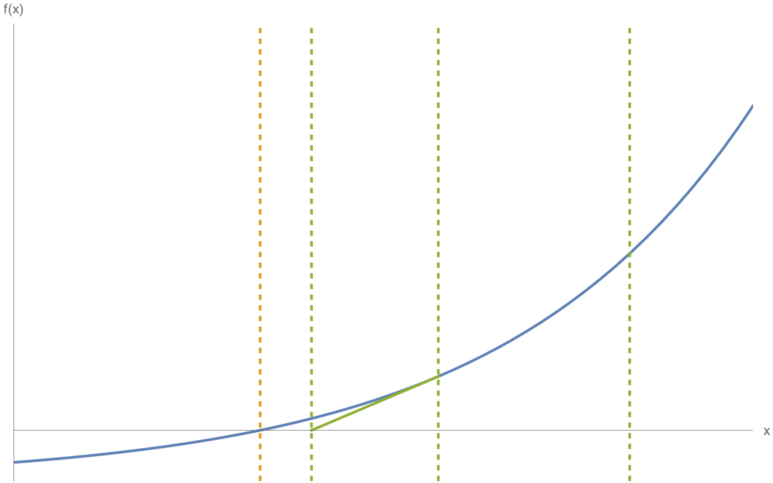

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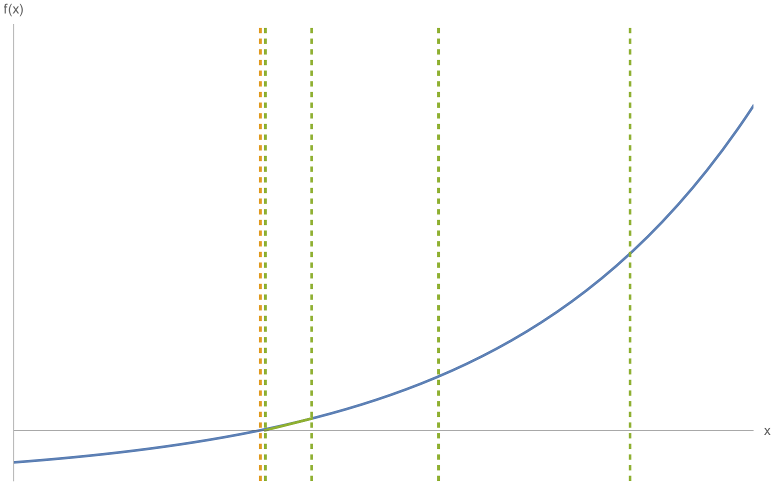

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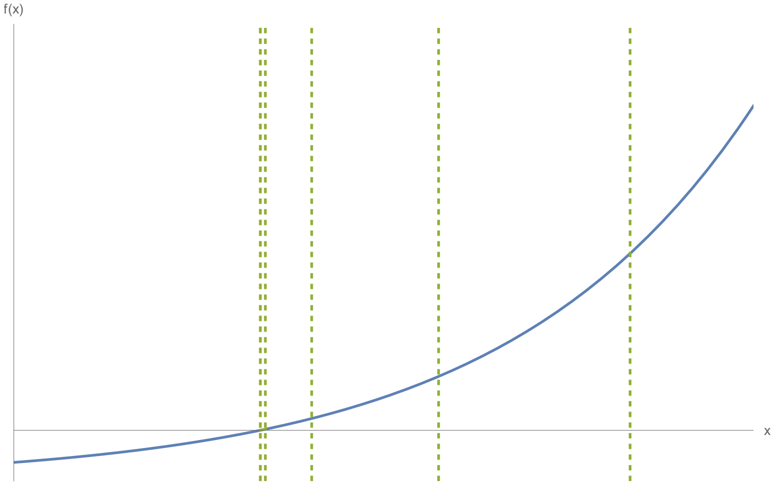

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- Convert from ECEF to geodetic coordinates using Bowring's method

Hooray!

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- We need a system of at least four pseudorange equations to solve for x, y, z, and t
- The Gauss-Newton algorithm is used to estimate x, y, z, and t

Thank you!

https://github.com/chrisdoble/gps-receiver