Part 2: Correlation

Building a GPS receiver from scratch

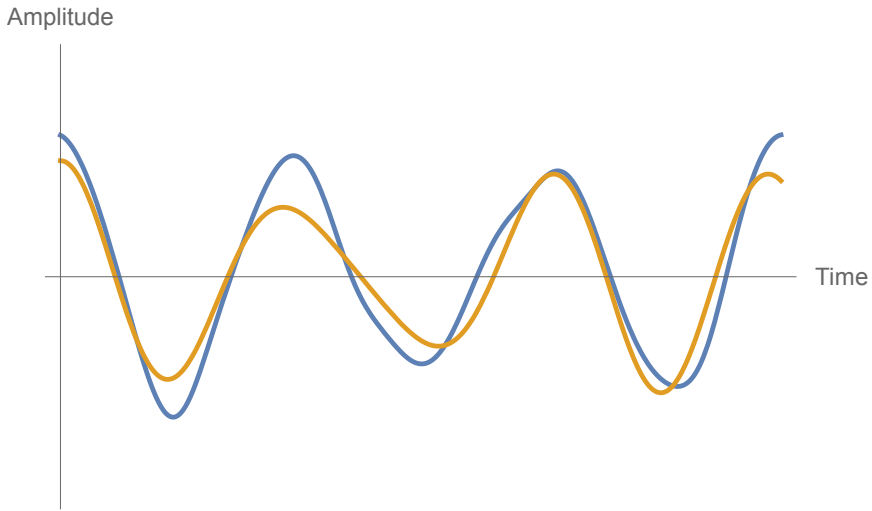
Chris Doble

Topics

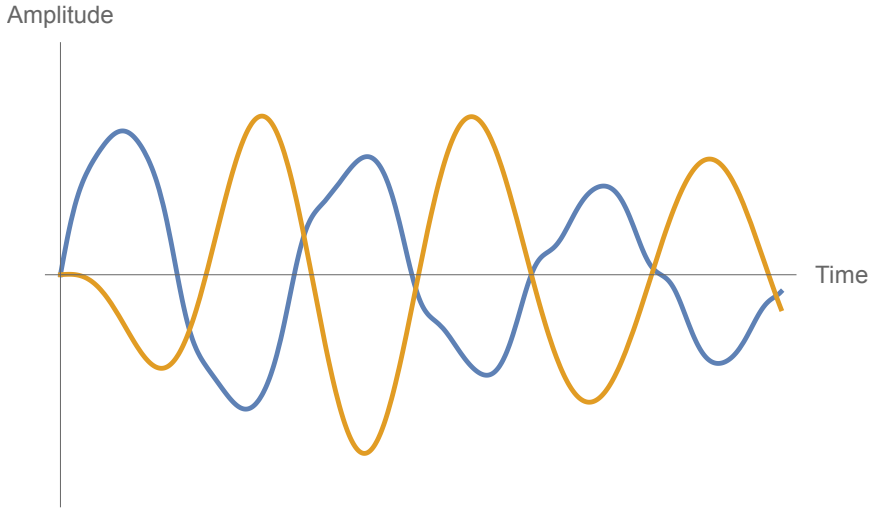
1. Correlation
2. Cross-correlation
3. Autocorrelation

Topics

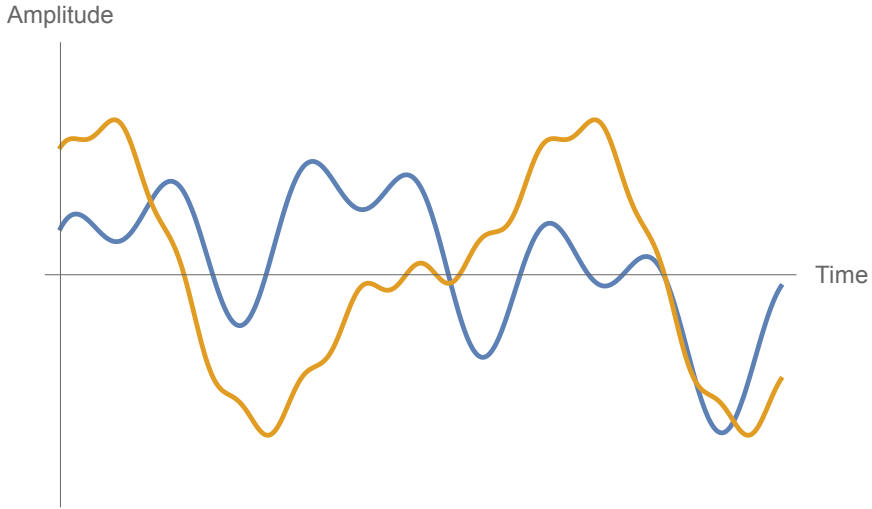
Positive correlation



Negative correlation



Near-zero correlation

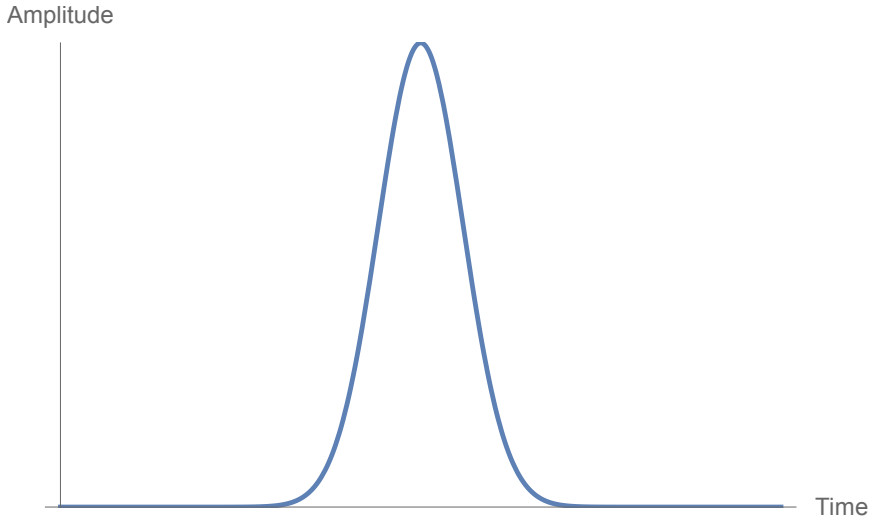


Definition

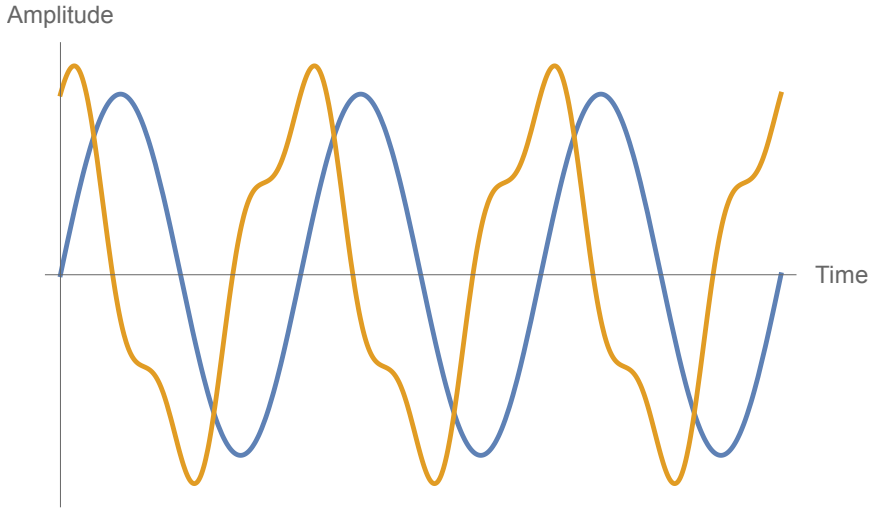
The correlation of two signals $f(t)$ and $g(t)$ is defined as

$$\int_{-\infty}^{+\infty} f(t)g(t)\,dt.$$

$f(t) \to 0$ as $|t| \to \infty$



Periodic signals



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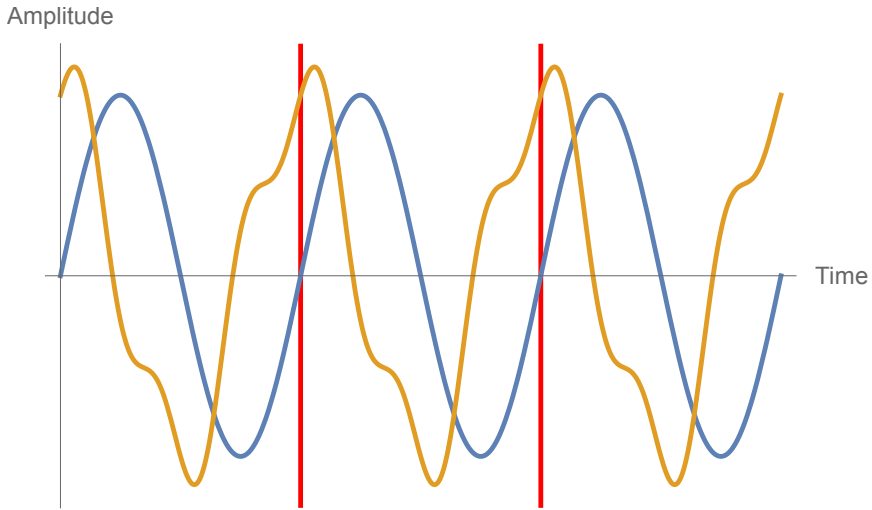


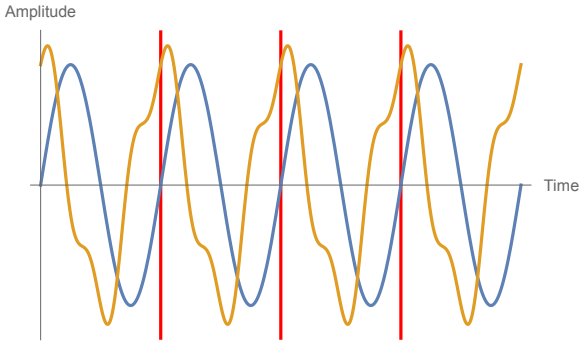
Definition

The correlation of two periodic signals $f(t)$ and $g(t)$ is defined as

$$\int_{t_0}^{t_0+T} f(t)g(t)\,dt$$

where t_0 is an arbitrary point in time and T is their shared period.

Multiple periods



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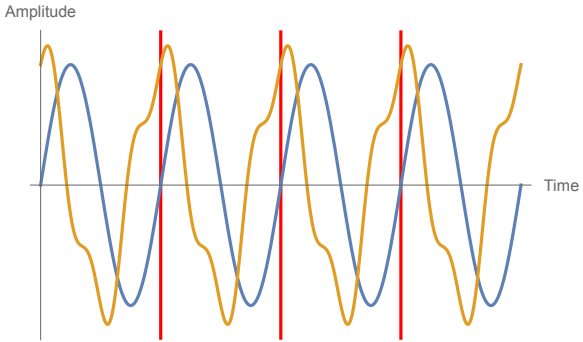

$$\int_{t_0}^{t_0+nT} f(t)g(t)\,dt = n\int_{t_0}^{t_0+T} f(t)g(t)\,dt$$

Definition vs. intuition

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- Not similar at all $\Rightarrow$ positive and negative products $\Rightarrow$ sums cancel

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- Cross-correlation
  - The correlation of two signals at different shifts
- Autocorrelation
  - The cross-correlation of a signal with itself