Part 3: GPS signals

Building a GPS receiver from scratch

Chris Doble

Topics

Topics

GPS frequencies

GPS Frequencies

Band	Frequency (MHz)	Phase	Original usage	Modernized usage
L1	1575.42 (10.23 × 154)	I	Encrypted precision P(Y) code	
		Q	Coarse/acquisition (C/A) code	C/A, L1 Civilian (L1C), and Military (M) code
L2	1227.60 (10.23 × 120)	I	Encrypted precision P(Y) code	
		Q	unmodulated carrier	L2 Civilian (L2C) code and Military (M) code
L3	1381.05 (10.23 × 135)		used by Nuclear Detonation (NUDET) Detection System Payload (NDS): signals nuclear detonations/ high-energy infrared events. Used to enforce nuclear test ban treaties.	
L4	1379.9133... (10.23 × 1214/9)		—	being studied for additional ionospheric correction[46]:607
L5	1176.45 (10.23 × 115)	I	—	Safety-of-Life (SoL) Data signal
		Q		Safety-of-Life (SoL) Pilot signal

Source: "GPS signals" from Wikipedia, CC BY-SA 4.0, https://en.wikipedia.org/wiki/GPS_signals

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- $D_i(t)$ represents the navigation message bit of satellite number i at time t

Topics

Modulation

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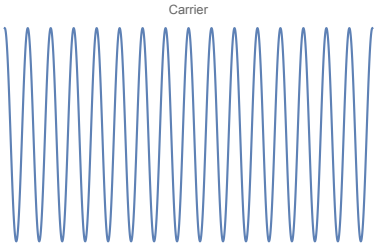

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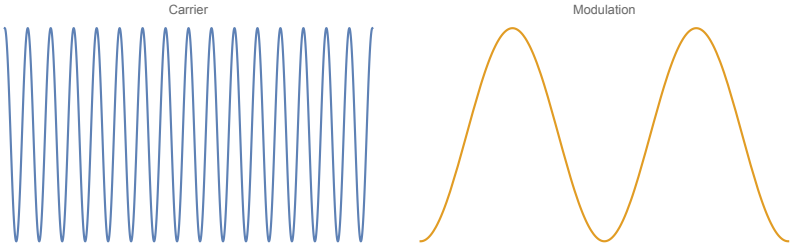

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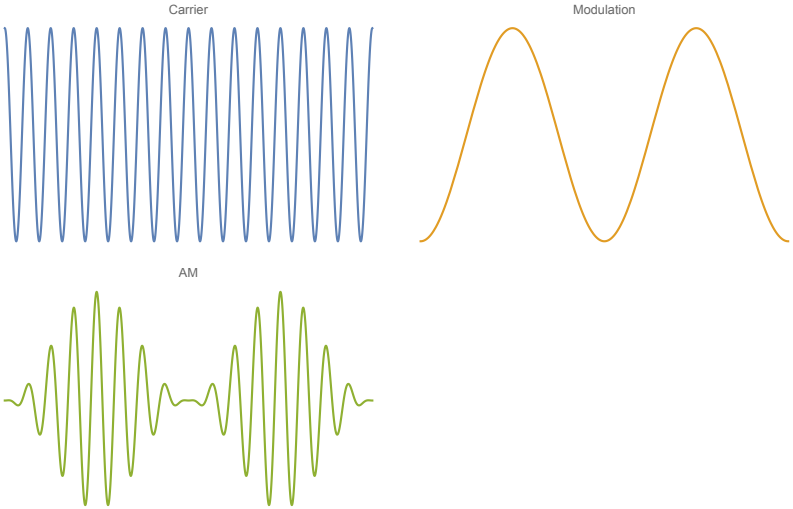

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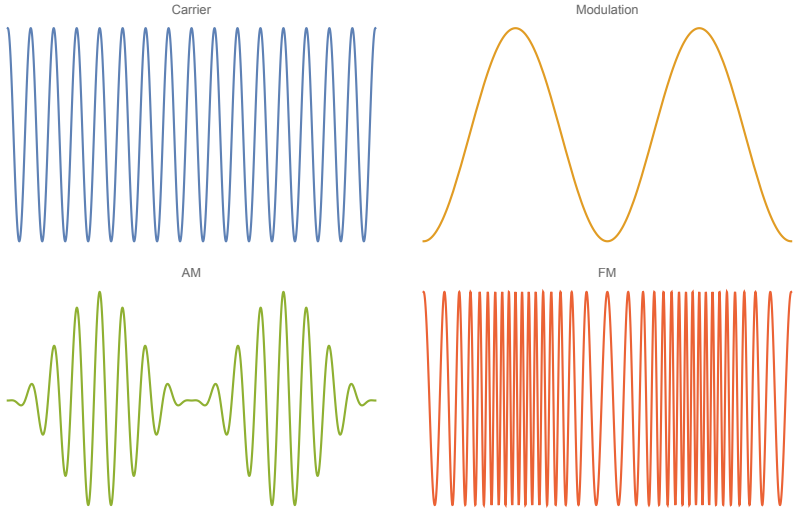

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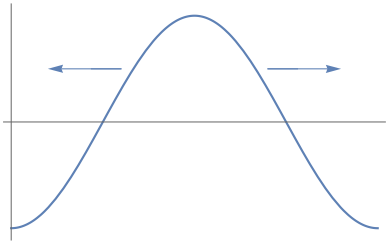

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$$D_i = \begin{array}{cccccc} 0 & 1 & 0 & 0 & 1 & 1 \end{array}$$

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 & \downarrow & & \downarrow & \downarrow & & \\
 & 1 & & 1 & 1 & &
\end{array}
$$

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$$\hat{D}_i(t) f_i(t)$$

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$$0 \rightarrow 1,\ 1 \rightarrow -1$$

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-1	-1	1

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$\times$	1	-1
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- The signal transmitted by a satellite is $\hat{D}_i(t)\hat{PRN}_i(t)f_i(t)$

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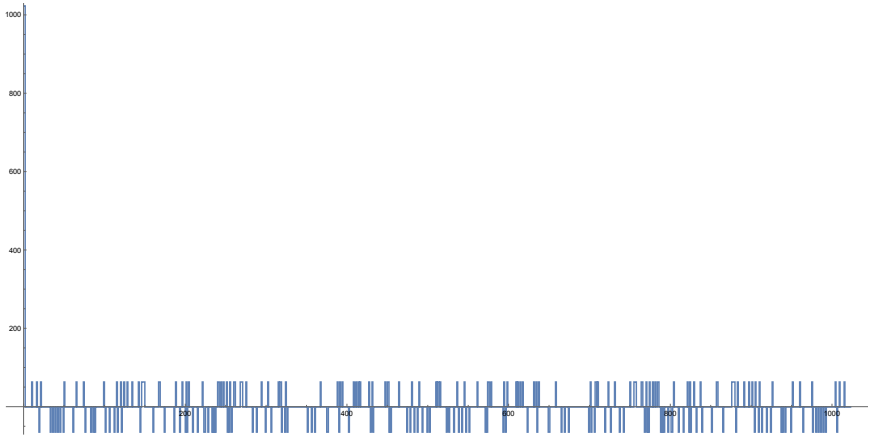

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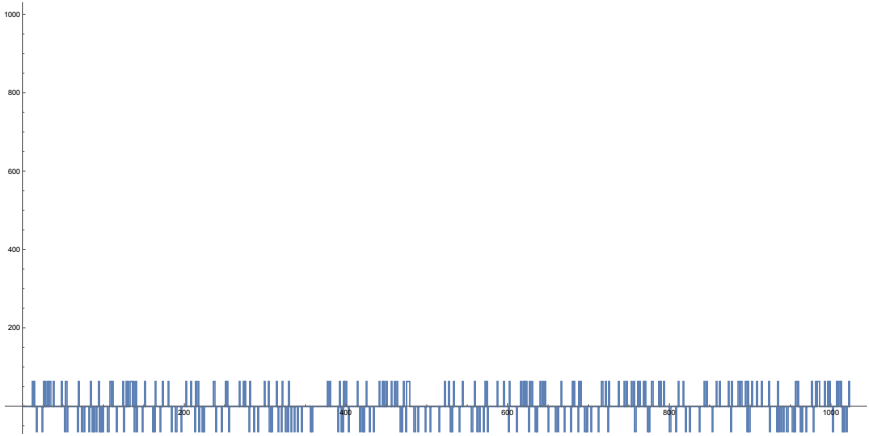

Decoding a bit from a satellite

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$$\hat{D}_i(t)P\hat{R}N_i(t)f_i(t)$$

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$$\hat{D}_1(t)P\hat{R}N_1(t)f_1(t) + \hat{D}_2(t)P\hat{R}N_2(t)f_2(t)$$

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$$\hat{D}_1(t)P\hat{R}N_1(t)f_1(t) + \hat{D}_2(t)P\hat{R}N_2(t)f_2(t) + N_1(t)$$

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Decoding a bit from a satellite

$$c_1 \hat{D}_1(t) P \hat{R} N_1(t) + c_2 \hat{D}_2(t) P \hat{R} N_2(t) + N_2(t)$$

Decoding a bit from a satellite

$$\int_0^T [c_1 \hat{D}_1(t) P \hat{R} N_1(t) + c_2 \hat{D}_2(t) P \hat{R} N_2(t) + N_2(t)] P \hat{R} N_1(t)\, dt$$

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$$\int_0^T c_1 \hat{D}_1(t) P\hat{R}N_1(t) P\hat{R}N_1(t-\tau)\,dt + \int_0^T c_2 \hat{D}_2(t) P\hat{R}N_2(t) P\hat{R}N_1(t-\tau)\,dt + \int_0^T N_2(t) P\hat{R}N_1(t-\tau)\,dt$$

Decoding a bit from a satellite

$$
\begin{aligned}
c_1 \int_0^T & \hat{D}_1(t) P \hat{R} N_1(t) P \hat{R} N_1(t-\tau)\, dt \\
& + c_2 \int_0^T \hat{D}_2(t) P \hat{R} N_2(t) P \hat{R} N_1(t-\tau)\, dt \\
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\end{aligned}
$$

Decoding a bit from a satellite

$$
\begin{aligned}
& c_1 \hat{D}_1(0) \int_0^T P\hat{R}N_1(t) P\hat{R}N_1(t-\tau)\, dt \\
& \quad + c_2 \hat{D}_2(0) \int_0^T P\hat{R}N_2(t) P\hat{R}N_1(t-\tau)\, dt \\
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Decoding a bit from a satellite

$$\alpha \hat{D}_1(0) + \int_0^T N_2(t) P \hat{R} N_1(t)\, dt$$

Decoding a bit from a satellite

$$\alpha \hat{D}_1(0) + \beta$$

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- The PRN code is XOR-ed with the navigation message and repeats once per ms
- To decode the navigation message bit of satellite number i:
  - Record the received signal for 1 ms
  - Calculate its correlation with an aligned copy of satellite number i's PRN code