9.5 Homework Questions:

- Find the equation of a plane containing the point $(-1, -3, -2)$ and the line of intersection of the planes: $-x+2y+2z=-17$,
$-x-2y+z=4$.

Solution:

Let t be some real number and suppose that $x=t$. Then,

$$\left.\begin{aligned} -t+2y+2z &= -17 \\ -t-2y+z &= 4 \end{aligned}\right\} \begin{aligned} 2y+2z &= -17+t \\ -2y+z &= 4+t. \end{aligned}$$

$$3z = -13+2t.$$

$$\Rightarrow z = -\frac{13}{3}+\frac{2}{3}t.$$

So, $-t+2y+2\left(-\frac{13}{3}t+\frac{2}{3}t\right)=-17$ and

$-t+2y-\frac{26}{3}t+\frac{4}{3}t=-17 \Rightarrow$

$2y=\frac{25}{3}t-17 \Rightarrow y=\frac{25}{6}t-\frac{17}{2}.$

$$\Rightarrow \vec{v}(t) = \langle t, \frac{25}{6}t - \frac{17}{2}, \frac{2}{3}t - \frac{13}{3} \rangle$$

is the equation of the line of the intersection.

Now, we need 3 points for an equation of a plane.

we will use: $A = (-1, -3, -2)$

$$B = v(0) = (0, -\frac{17}{2}, -\frac{13}{3})$$

$$C = v(1) = (1, \frac{25}{6} - \frac{17}{2}, \frac{2}{3} - \frac{13}{3})$$

$$= (1, -\frac{13}{3}, -\frac{11}{3})$$

So, $\vec{AB} = \langle 0-(-1), -\frac{17}{2} - (-3), -\frac{13}{3} - (-2) \rangle$

$$= \langle 1, -\frac{11}{2}, -\frac{7}{3} \rangle$$ and

$$\vec{AC} = \langle 1-(-1), -\frac{13}{3} - (-3), -\frac{11}{3} - (-2) \rangle$$

$$= \langle 2, -\frac{4}{3}, -\frac{5}{3} \rangle .$$

It follows that

$$\vec{AB} \times \vec{AC} = \cdots = \left\langle \frac{109}{18}, -3, \frac{29}{3} \right\rangle$$

So the equation of the plane should be:

$$\frac{109}{18}(x+1) - 3(y+3) + \frac{29}{3}(z+3) = 0 .$$