

My OpenMath 5.4.13

Suppose that $f(2) = 3$ and $f(6) = 7$,

$$\int_2^6 e^{-3x} f(x) dx = 18.$$

What is $\int_2^6 e^{-3x} f'(x) dx$?

Solution : Notice that

$$\int_2^6 e^{-3x} f(x) dx = \int_2^6 f(x) e^{-3x} dx.$$

$\underbrace{}_{u} \underbrace{}_{dv}$

Integration by Parts : $\int u dv = uv - \int v du$

so, in order to use I.B.P,

$$\text{Let } u = f(x) \text{ and } dv = e^{-3x} dx.$$

Differentiate
↓

$$du = f'(x) dx$$

Integrate
↓

$$v = \int e^{-3x} dx = -\frac{1}{3} e^{-3x}$$

$$\text{Hence, } uv - \int v du =$$

$$f(x) \left(-\frac{1}{3} e^{-3x} \right) - \int \left(-\frac{1}{3} e^{-3x} \right) (f'(x) dx)$$

$\underbrace{f(x)}_{u} \quad \underbrace{-\frac{1}{3} e^{-3x}}_{v} \quad \underbrace{\int}_{du} \quad \underbrace{(f'(x) dx)}_{du}$

$$= -\frac{1}{3} f(x) e^{-3x} + \frac{1}{3} \int e^{-3x} f'(x) dx.$$

Now, let us consider the bounds of the integration. By Integration by parts above, we see that

$$\int_2^6 e^{-3x} f(x) dx = -\frac{1}{3} \cdot f(x) e^{-3x} \Big|_2^6 + \frac{1}{3} \int_2^6 e^{-3x} f'(x) dx.$$

With $\int_2^6 e^{-3x} f(x) dx = 18$ from the beginning, it follows that

$$18 = -\frac{1}{3} \cdot f(x) e^{-3x} \Big|_2^6 + \frac{1}{3} \int_2^6 e^{-3x} f'(x) dx.$$

Let us "solve" the integral by isolating it:

$$\frac{1}{3} \int_2^6 e^{-3x} f'(x) dx = (8 + \frac{1}{3} \cdot f(x) e^{-3x}) \Big|_2^6$$

↓
multiply by 3 to
both sides

$$\int_2^6 e^{-3x} f'(x) dx = 48 + f(x) \cdot e^{-3x} \Big|_2^6$$

$$f(6)e^{-18} - f(2)e^{-6}$$

7 3
↑ ↑
from the
beginning

$$\Rightarrow \int_2^6 e^{-3x} f'(x) dx = 48 + 7e^{-18} - 3e^{-6}$$

I hope this is
right.



My OpenMath 5.4.16 (?)

Suppose that $\int e^{3x} \cos(3x) dx =$

$$F(x) - \int G(x) dx.$$

What are F and G ?

Solution:

Notice that $F(x) - \int G(x) dx$

looks a lot like the integration

by parts formula:

$$\int u dv = \underbrace{uv}_{F(x)} - \underbrace{\int v du}_{G(x) dx}.$$

$$F(x) \quad G(x) dx.$$

There are multiple ways to solve
this problem, depending on

the choices for u and dv .

Let $u = e^{3x}$, $dv = \cos(3x) dx$.

Then, $du = 3e^{3x} dx$ and

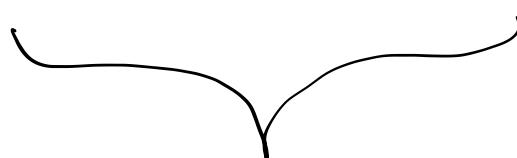
$$v = \int \cos(3x) dx = \frac{1}{3} \cdot \sin(3x)$$

[Disregard the const. + C for now]

Then, $uv - \int v du$

$$= e^{3x} \left(\frac{1}{3} \cdot \sin(3x) \right) - \int \frac{1}{3} \cdot \sin(3x) \cdot 3e^{3x} dx$$

$$= \frac{1}{3} e^{3x} \cdot \sin(3x) - \int e^{3x} \cdot \sin(3x) dx$$

 $F(x)$

 $G(x)$

Now, let us evaluate

$\int e^{3x} \cdot \sin(3x) dx$ in a similar manner.

To use integration by parts, let

$$u = e^{3x} \quad \text{and} \quad dv = \sin(3x) dx.$$

Then, $du = 3e^{3x} dx$ and

$$v = \int \sin(3x) dx = -\frac{1}{3} \cos(3x).$$

Hence, $uv - \int v du$

$$\begin{aligned} &= -\frac{1}{3} e^{3x} \cos(3x) - \int -\frac{1}{3} \cos(3x) \cdot 3e^{3x} dx \\ &= -\frac{1}{3} e^{3x} \cos(3x) + \int e^{3x} \cos(3x) dx. \end{aligned}$$

So, $\int e^{3x} \cos(3x) dx = \frac{1}{3} e^{3x} \sin(3x)$
 $\quad \quad \quad - (-\frac{1}{3} e^{3x} \cos(3x))$
 $\quad \quad \quad + \int e^{3x} \cos(3x) dx$

$$\begin{aligned} &= \frac{1}{3} e^{3x} \sin(3x) + \frac{1}{3} e^{3x} \cos(3x) \\ &\quad - \int e^{3x} \cos(3x) dx \end{aligned}$$

Add $\int e^{3x} \cos(3x) dx$ to both sides
to obtain

$$2 \int e^{3x} \cos(3x) dx = \frac{1}{3} e^{3x} (\sin(3x) + \cos(3x))$$
$$\Rightarrow \int e^{3x} \cos(3x) dx = \frac{1}{6} e^{3x} (\sin(3x) + \cos(3x)) + C,$$

C is constant.