MyOpenMath 5.4.13

Suppose that $f(2)=3$ and $f(6)=7$,

$$\int_2^6 e^{-3x} f(x)\,dx = 18.$$

What is $\int_2^6 e^{-3x} f'(x)\,dx$?

Solution: Notice that

$$\int_2^6 e^{-3x} f(x)\,dx = \int_2^6 \underbrace{f(x)}_{u}\, \underbrace{e^{-3x}\,dx}_{dv}.$$

Integration by Parts: $\int u\,dv = uv - \int v\,du$

So, in order to use I.B.P,

let $u = f(x)$ and $dv = e^{-3x}\,dx$.

Differentiate $\qquad\qquad$ Integrate

$du = f'(x)\,dx \qquad\qquad v = \int e^{-3x}\,dx = -\frac{1}{3}e^{-3x}$

Hence, $uv - \int v\,du =$

$$\underbrace{f(x)}_{u}\underbrace{\left(-\tfrac{1}{3}e^{-3x}\right)}_{v} - \int \underbrace{\left(-\tfrac{1}{3}e^{-3x}\right)}_{v}\underbrace{\left(f'(x)\,dx\right)}_{du}$$

$$= -\frac{1}{3}f(x)e^{-3x} + \frac{1}{3}\int e^{-3x}f'(x)\,dx.$$

Now, let us consider the bounds of the integration. By integration by parts above, we see that

$$\int_2^6 e^{-3x}f(x)\,dx = -\frac{1}{3}\cdot f(x)e^{-3x}\Big|_2^6 + \frac{1}{3}\int_2^6 e^{-3x}f'(x)\,dx.$$

With $\int_2^6 e^{-3x}f(x)\,dx = 18$ from the beginning, it follows that

$$18 = -\frac{1}{3}\cdot f(x)e^{-3x}\Big|_2^6 + \frac{1}{3}\int_2^6 e^{-3x}f'(x)\,dx.$$

Let us "solve" the integral by isolating it:

$$\frac{1}{3}\int_2^6 e^{-3x} f'(x)\,dx = 18 + \frac{1}{3} f(x)e^{-3x}\Big|_2^6$$

multiply by 3 to both sides

$$\int_2^6 e^{-3x} f'(x)\,dx = 48 + \underbrace{f(x)\cdot e^{-3x}\Big|_2^6}$$

$$\underbrace{f(6)}_{7}e^{-18} - \underbrace{f(2)}_{3}e^{-6}$$

From the beginning

$$\Rightarrow \int_2^6 e^{-3x} f'(x)\,dx = 48 + 7e^{-18} - 3e^{-6}.$$

I hope this is right. 😌

My OpenMath 5.4.16(?)

Suppose that $\int e^{3x} \cos(3x)\, dx =$

$$F(x) - \int G(x)\, dx.$$

What are F and G?

<u>Solution</u>:

Notice that $F(x) - \int G(x)\, dx$

looks a lot like the Integration

by parts formula:

$$\int u\, dv = \underbrace{uv}_{F(x)} - \int \underbrace{v\, du}_{G(x)\, dx}.$$

There are multiple ways to solve

this problem, depending on

the choices for u and dv.

Let $u = e^{3x}$, $dv = \cos(3x)\,dx$.

Then, $du = 3e^{3x}\,dx$ and

$$v = \int \cos(3x)\,dx = \frac{1}{3}\cdot\sin(3x)$$

[Disregard the const. $+C$ for now]

Then, $uv - \int v\,du$

$$= e^{3x}\left(\frac{1}{3}\cdot\sin(3x)\right) - \int \frac{1}{3}\cdot\sin(3x)\cdot 3e^{3x}\,dx$$

$$= \underbrace{\frac{1}{3}e^{3x}\cdot\sin(3x)}_{F(x)} - \int \underbrace{e^{3x}\cdot\sin(3x)}_{G(x)}\,dx$$

Now, let us evaluate

$\int e^{3x} \cdot \sin(3x)\, dx$ in a similar manner.

To use integration by parts, let

$u = e^{3x}$ and $dv = \sin(3x)\, dx$.

Then, $du = 3e^{3x}\, dx$ and

$v = \int \sin(3x)\, dx = -\frac{1}{3}\cos(3x)$.

Hence, $uv - \int v\, du$

$= -\frac{1}{3}e^{3x}\cos(3x) - \int -\frac{1}{3}\cos(3x) \cdot 3e^{3x}\, dx$

$= -\frac{1}{3}e^{3x}\cos(3x) + \int e^{3x}\cos(3x)\, dx$.

So, $\int e^{3x}\cos(3x)\, dx = \frac{1}{3}e^{3x}\sin(3x)$

$- \left(-\frac{1}{3}e^{3x}\cos(3x) + \int e^{3x}\cos(3x)\, dx\right)$

$= \frac{1}{3}e^{3x}\sin(3x) + \frac{1}{3}e^{3x}\cos(3x)$

$- \int e^{3x}\cos(3x)\, dx$

Add $\int e^{3x}\cos(3x)\,dx$ to both sides to obtain

$$2\int e^{3x}\cos(3x)\,dx = \frac{1}{3}e^{3x}\left(\sin(3x)+\cos(3x)\right)$$

$$\Rightarrow \int e^{3x}\cos(3x)\,dx = \frac{1}{6}e^{3x}\left(\sin(3x)+\cos(3x)\right) + C,$$

C is constant.