9.5 Exercises

1) $$\vec{r}(t) = \underbrace{(3+3t)}_{x(t)}\vec{i} + \underbrace{(-5-3t)}_{y(t)}\vec{j} + \underbrace{(-1+3t)}_{z(t).}\vec{k}$$

$$\left[\ \vec{r}(t) = \langle 3+3t,\ -5-3t,\ -1+3t \rangle.\ \right]$$

2) Let $P(4,2,4)$ be a point in $\mathbb{R}^3$.

Let $\vec{v} = 0\vec{i} - 3\vec{j} - 3\vec{k} = \langle 0,-3,-3\rangle$.

What are the expressions for the line through P that is parallel to $\vec{v}$?

i) [vectorized expression]

any real #.

$$\vec{r}(t) = \vec{r}_0 + t\vec{v} = \langle 4,2,4\rangle + t\langle 0,-3,-3\rangle.$$

ii) [parametric expression]

$$\langle 4,2,4\rangle + t\langle 0,-3,-3\rangle = \langle \underset{x(t)}{4},\ \underset{y(t)}{2-3t},\ \underset{z(t).}{4-3t}\rangle$$

3) Let $P(5,-2,-2)$ be a point in $\mathbb{R}^3$.

Let $\vec{w}$ be a line in $\mathbb{R}^3$ expressed in parametric equations:

$$\left.\begin{aligned} x(t) &= 1+2t \\ y(t) &= 2+2t \\ z(t) &= 3+6t. \end{aligned}\right\} \vec{w}(t) = \langle 1,2,3 \rangle + t\langle 2,2,6 \rangle.$$

$\Rightarrow \langle 2,2,6 \rangle$ is parallel to $\vec{w}$.

Hence, a line through P that is parallel to $\vec{w}$ is:

$$\vec{r}(t) = \langle 5,-2,-2 \rangle + t\langle 2,2,6 \rangle.$$

$$= \langle 5+2t, -2+2t, -2+6t \rangle.$$

Parametrically: $x(t) = 5+2t$

$$y(t) = -2+2t$$

$$z(t) = -2+6t$$

$\vec{r}$ intersects :

- xy plane when $z = 0 = -2 + 6t$

$$\Rightarrow t = \frac{1}{3}.$$

At $t = \frac{1}{3}$: $x(\frac{1}{3}) = 5 + 2(\frac{1}{3}) = \frac{17}{3}$

$$y(\tfrac{1}{3}) = -2 + 2(\tfrac{1}{3}) = -4/3.$$

$\Rightarrow$ intersects at $(17/3, -4/3, 0)$.

- xz plane when $y = 0 = -2 + 2t$

$$\Rightarrow t = 1.$$

At $t = 1$: $x(1) = 7$, $z(1) = 4$

$\Rightarrow$ intersects at $(7, 0, 4)$.

- yz plane when $x = 0 = 5 + 2t$

$$\Rightarrow t = -\frac{5}{2}.$$

At $t = -5/2$: $y(-5/2) = -2 + 2(-\frac{5}{2}) = -7.$

$$z(-5/2) = -2 + 6(-\tfrac{5}{2}) = -17.$$

$\Rightarrow$ intersects at $(0, -7, -17)$.

4) • Let $\vec{r}(t) = \langle -5, 5, 2\rangle + t\langle 5, -3, -3\rangle$.

Parametrically, we have:

$$x(t) = -5 + 5t$$

$$y(t) = 5 - 3t$$

$$z(t) = 2 - 3t.$$

• Let plane A be represented by

$$-2x + 4y + z = -18.$$

• Suppose that the line $\vec{r}$ intersects the plane A. This means there exists $t_0 \in \mathbb{R}$ such that

$$-2 \cdot x(t_0) + 4 \cdot y(t_0) + z(t_0) = -18.$$

So, $-2(-5 + 5t_0) + 4(5 - 3t_0) + (2 - 3t_0) = -18$

$\Rightarrow \quad 10 - 10t_0 + 20 - 12t_0 + 2 - 3t_0 = -18$

$\Rightarrow \quad -25t_0 = -50 \Rightarrow t_0 = 2.$

At $t = 2$: $\left.\begin{aligned} x(2) &= -5 + 10 = 5. \\ y(2) &= 5 - 6 = -1. \\ z(2) &= 2 - 6 = -4. \end{aligned}\right\}$ intersection @ $(5, -1, -4)$.

5) Algorithm for constructing an equation of a plane w/ three distinct points in $\mathbb{R}^3$:

i) Let A, B, C be the points and

Let $\vec{v} = \overrightarrow{AB}$ and $\vec{w} = \overrightarrow{AC}$.

ii) Compute $\vec{v} \times \vec{w}$. This is $\vec{n}$.

Let $\vec{n} = \langle a, b, c \rangle$.

iii) [Output] equation is given by:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0,$$

where $A = (x_0, y_0, z_0)$.

Let $A = (-3, 2, -4)$, $B = (-1, -2, -1)$, $C = (-1, -1, 1)$.

Then, $\vec{v} = \overrightarrow{AB} = \langle -1-(-3), -2-2, -1-(-4) \rangle$

$= \langle 2, -4, 3 \rangle$ and

$\vec{w} = \overrightarrow{AC} = \langle -1+3, -1-2, 1+4 \rangle = \langle 2, -3, 5 \rangle$.

Now, $\vec{n} = \vec{v} \times \vec{w}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -4 & 3 \\ 2 & -3 & 5 \end{vmatrix} = \begin{vmatrix} -4 & 3 \\ -3 & 5 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & -4 \\ 2 & -3 \end{vmatrix}\vec{k}$$

$$= (-20+9)\vec{i} - (10-6)\vec{j} + (-6+8)\vec{k}$$

$$= \langle \underset{a}{-11}, \underset{b}{-4}, \underset{c}{2} \rangle$$. With $A = (\underset{x_0}{-3}, \underset{y_0}{2}, \underset{z_0}{-4})$,

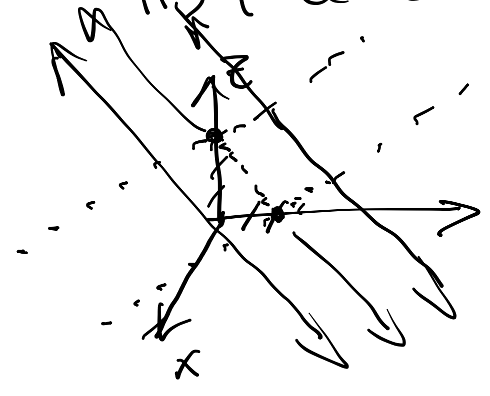
the equation of the plane is :

$$-11(x+3) - 4(y-2) + 2(z+4) = 0 .$$

6) Find an equation of a plane containing :

i) line on xy-plane with $y=2$

ii) line on xz-plane with $z=3$.



$$\Longrightarrow \quad z = -\frac{3}{2}y + 3$$

[think of a line connecting (2,0) and (0,3) on yz-plane]

Homework Questions:

#5) parametric equation of a line

through $A(7,-2,-7)$ and

perpendicular to the plane

$$\underset{\substack{\uparrow \\ a}}{-8}x + \underset{\substack{\uparrow \\ b}}{9}y - \underset{\substack{\uparrow \\ c}}{\frac{8}{3}}z = 8.$$

$$\Rightarrow \underbrace{\vec{n} = \langle -8, 9, \tfrac{-8}{3} \rangle}$$

$\vec{n}$ is orthogonal to the plane.

$$\Rightarrow \vec{r}(t) = \langle 7, -2, -7 \rangle + t\langle -8, 9, \tfrac{-8}{3} \rangle$$

$$\Rightarrow \quad x(t) = 7 - 8t$$

$$y(t) = -2 + 9t$$

$$z(t) = -7 - \frac{8}{3}t$$

For the intersection with xy-plane,

let $z = 0$: $\quad 0 = -7 - \frac{8}{3}t \Rightarrow t = \frac{-21}{8}.$

$$\Rightarrow x\left(\tfrac{-21}{8}\right) = 7 - 8\left(\tfrac{-21}{8}\right) = 28.$$

$$y\left(\frac{-21}{8}\right) = -2 + 9\left(\frac{-21}{8}\right) = \frac{-16}{8} + \frac{-189}{8}$$

$$= \frac{205}{8}.$$

$\Rightarrow \left(28, \frac{205}{8}, 0\right)$ is the intersection.

9) Perpendicular to $\vec{n} = \langle 6, 3, 1 \rangle$, containing $(5, -2, 7)$:

$$6(x-5) + 3(y+2) + (z-7) = 0.$$

Is $(7, 0, -7)$ a point on the plane?

$$\Rightarrow 6(7-5) + 3(0+2) + (-7-7)$$

$$= 12 + 6 - 14 \neq 0.$$ NO.

$$P_2: 6(x-8) + 3y + (z+5) = 0.$$

12) Let A be $(2,8,5)$ [point]

p be $9x+2y+6z=-90$ [plane].

a) is A on p?

$\Rightarrow 9(2)+2(8)+6(5)$

$= 18+16+30 \neq -90$, NO.

b) $\vec{n} = \langle 9,2,6 \rangle$. [a, b, c — the coefficients].

c) "point on z-axis" means

$x=0$ and $y=0$.

$\Rightarrow 9(0)+2(0)+6z=-90$

$\Rightarrow z = \frac{-90}{6} = -15$.

$\Rightarrow (0,0,-15) = B$.

d) $\vec{AB} = \langle 0-2, 0-8, -15-5 \rangle$

$= \langle -2, -8, -20 \rangle$.

$\Rightarrow |\vec{AB} \cdot \vec{n}| = |(-2)(9) + (-8)(2) + (-20)(6)|$

$= |-18 - 16 - 120| = 154$.

e) $\|\vec{n}\| = \sqrt{9^2 + 2^2 + 6^2}$

$= \sqrt{81 + 4 + 36} = \sqrt{121} = 11$.

$\Rightarrow \dfrac{154}{11} = 14$.