9.5 Homework Questions:

- Find the equation of a plane containing the point $(-1,-3,-2)$ and the line of intersection of the two planes:

$$-x+2y+2z=-17 \quad (1)$$
$$-x-2y+z=4 \quad (2).$$

Solution:

Let $\vec{n}_1 = \langle -1, 2, 2 \rangle$ and

$\vec{n}_2 = \langle -1, -2, 1 \rangle$.

[Normal vectors to the planes (1) & (2)].

Notice the following:

- Vectors orthogonal to $\vec{n}_1$ are on plane (1)
- Vectors orthogonal to $\vec{n}_2$ are on plane (2)

$\Rightarrow$ vectors orthogonal to both $\vec{n}_1$ and $\vec{n}_2$ are on both (1) and (2)!

$\rightarrow$ This is precisely the intersection of the planes.

$\Longrightarrow$

$$\vec{n}_1 \times \vec{n}_2 = \cdots = \langle 6, -1, 4 \rangle.$$

- Let's find a point on the intersection line. Let $x = 0$.

$$-x + 2y + 2z = -17 \rightarrow 2y + 2z = -17$$

$$-x - 2y + z = 4 \rightarrow -2y + z = 4$$

$$3z = -13$$

$$\Longrightarrow z = -13/3.$$

$$\rightarrow 2y + 2(-13/3) = -17$$

$$\rightarrow 2y - \frac{26}{3} = -17 \Rightarrow 2y = \frac{-25}{3}$$

$$\Longrightarrow y = \frac{-25}{6}.$$

$\Longrightarrow \left(0, -25/6, -13/3\right)$ is a point on the intersection.

- Vector connecting
  $(0, -25/6, -13/3)$ and $(-1, -3, -2)$:

$$\langle -1-0, -3+\frac{25}{6}, -2+\frac{13}{3} \rangle$$

$$= \langle -1, \frac{7}{6}, \frac{7}{3} \rangle.$$

- vector orthogonal to
  $\langle -1, \frac{7}{6}, \frac{7}{3} \rangle$ and $\langle 6, -1, 4 \rangle$

$\Rightarrow \langle -1, \frac{7}{6}, \frac{7}{3} \rangle \times \langle 6, -1, 4 \rangle$

$= \cdots$ (cross prod.) $\cdots = \langle 7, 18, -6 \rangle$.

$\Longrightarrow 7(x+1) + 18(y+3) - 6(z+2) = 0$

is the equation of the plane containing the intersection line and the point $(-1, -3, -2)$.