$\left\{ \left(\frac{1}{3n}\right)^{\left(\frac{5}{\ln(4n)}\right)} \right\}$ converge? diverge?

$$\lim_{n\to\infty} \left(\frac{1}{3n}\right)^{\left(\frac{5}{\ln(4n)}\right)} = ?$$

Notice that For all $x>0$, $x = e^{\ln(x)}$.

For all $n = 1, 2, 3, \ldots$, $\frac{1}{3n} > 0$. So,

$$\frac{1}{3n} = e^{\ln\left(\frac{1}{3n}\right)} = e^{\ln(1)-\ln(3n)} = e^{-\ln(3n)}.$$

Then, $\left(\frac{1}{3n}\right)^{\left(\frac{5}{\ln(4n)}\right)} = \left(e^{-\ln(3n)}\right)^{\left(\frac{5}{\ln(4n)}\right)} = e^{-5\cdot\frac{\ln(3n)}{\ln(4n)}}$.

Hence, $\lim_{n\to\infty} \left(\frac{1}{3n}\right)^{\left(\frac{5}{\ln(4n)}\right)} = \lim_{n\to\infty} e^{-5\cdot\frac{\ln(3n)}{\ln(4n)}} = e^{-5\cdot\lim_{n\to\infty}\frac{\ln(3n)}{\ln(4n)}}$.

Here, $\lim_{n\to\infty} \underbrace{\frac{\ln(3n)}{\ln(4n)}}_{\frac{\infty}{\infty}\text{: indeterminate}} = \lim_{n\to\infty} \frac{\frac{1}{3n}\cdot 3}{\frac{1}{4n}\cdot 4}$ by L'Hôpital's Rule

$$= \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} 1 = 1.$$

Thus, $e^{-5\cdot\lim_{n\to\infty}\frac{\ln(3n)}{\ln(4n)}} = e^{-5\cdot 1} = e^{-5}$.