

# Etude 4: Desert Crossing

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## 1 Tasks

### 1.1 Using the vehicle without refueling, how far into the desert can you safely go?

No refueling means only using the 60L of petrol in the truck, so to travel safely we can only use 30L which is 360Km.

### 1.2 Describe a procedure whereby you could cross the desert in the vehicle.

To cross the desert we first found the point where you could make it to the end with a full tank + 4 cans in the truck (733Km from start spot. We then decided that we want to drop 2 cans of petrol on every trip, the max distance we could go to do that trip is 600Km. We make 3.5 trips to this spot so we have 50 Litres left in the tank and 8 cans of petrol. 733Km is 133 Km away from where we are at this point. Each trip to 733 will use 22.2 litres return. We can carry all cans in 2 trips, with a mix of filling up the truck and carrying cans. At 733Km we can make the final trip to the end of the desert. We had to leave 20L at 733Km and we have 5.7L left in the truck. All together this method I took 560L of petrol.

### 1.3 Describe a procedure whereby you could cross the desert and return in the vehicle.

The procedure for crossing the desert and returning requires one to leave enough fuel at each drop point to refuel the truck completely exactly twice, this is the goal state. Each drop point was chosen to be 480km, equivalent to 40L of fuel, apart (at 480, 960 and 1440km) with the exception of the final drop point at 1680km sitting 240km, equivalent to 20L of fuel, from the third drop point. Therefore at each point, to cross the desert and return, there must be 80L except for the final point which will have 40L.

The procedure for doing so requires a repeated stage of delivering fuel to the first two drop points. The truck and 3 cans are filled giving 120L of fuel available. These are then driven to the first point (480km) where 40L is deposited before returning to the home base. This is repeated twice leaving 80L at the first point. The truck and cans are then filled with 120L and driven to

the second drop point (960km), stopping at the first point to refuel (taking 40L to leave 40L remaining). At this point 40L is deposited and the truck returns to home base, again refueling at the first point (leaving 0L remaining). The steps are then repeated to get 80L at the second point, with the first step being repeated an extra time to get 80L at the first point. This stage makes a total of 8 trips using 120L each time, using a total of 960L of fuel. At this point it should be noted that the suggested solution is not the most efficient.

After this stage is complete the truck and 4 cans are filled giving 140L of fuel available. The truck is then driven to the third drop point stopping at both preceding points to refuel (taking 40L to leave 40L). 60L is then deposited and the truck returns again refueling with 40L from the first two points (leaving 0L at both). The first stage and this step are repeated to leave 120L at the third point, with the first stage being repeated an additional time to get 80L at the first and second point.

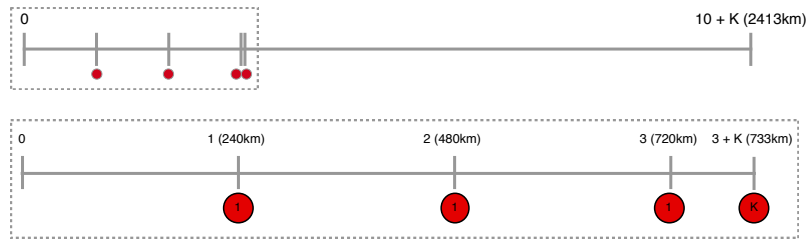
At this point the truck is filled with 120L and driven to the fourth and final drop point stopping at the first two drops to refuel with 40L (leaving 40L at both) and the third to refuel with 20L (leaving 100L). 40L is deposited and the truck returns to base refueling with 20, 40 and 40L from the third (80L), second (0L) and first (0L) points respectively. The first stage is repeated a final time to reach the goal state.

For the desert crossing journey the the truck and 2 cans are filled with an additional can being filled with 2.17L. Similar to the processes described above the truck is driven through each point refueling at each. In the end there is a total of 0.003L left in the truck and a total of 4362.17L has been used.  
 $(4 * \text{stage 1 (960L)} + 2 * 140L + 2 * 120L + 2.17L)$

#### **1.4 Describe a procedure whereby you could cross the desert in the vehicle using the minimum amount of fuel.**

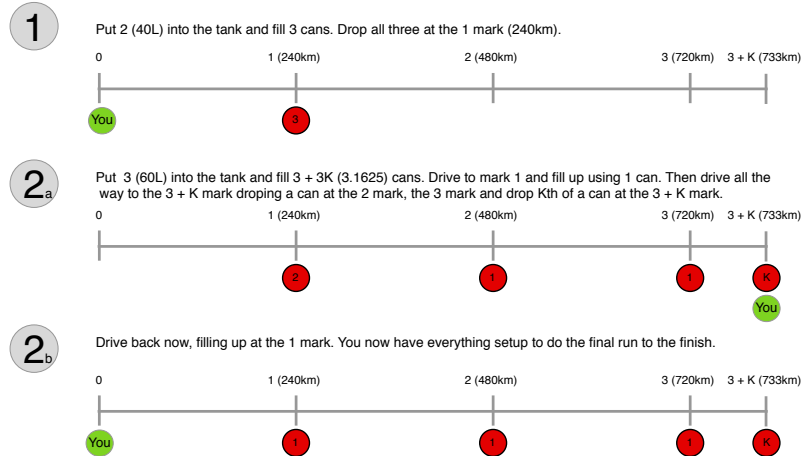
First we simplify the units to make thinking about this problem easier. We need to cross 2413km, and each liter is equivalent to 12km, so the total distance can be seen as 201.083L instead of 2413km. Each can hold 20L so we can see the 201.083L as 10.054166667 Cans. now 10.054166667 has an annoying 0.054166667 at the end. So we are going to call that  $K = 0.054166667$ . therefore making the total distance  $10 + K$  cans Where the cars tank can hold 3 cans (60L/20L) in the tank and 4 extra cans.

So we have to place cans in such a way where we can go to the finish line. This is how we want the desert to look, The red circles are fuel cans that are dropped off where the number is the amount dropped off.



With this setup we can get from the start all the way to the finish if we fill up with a fuel tank and fill up at each point. This is because the fuel we use to get to 1 mark is then fill back up by the 1 tank. At the 3 mark we have a full fuel tank as we have just filled up and we travel  $K$  so we have to fill up  $K$  again. We are now at the  $3 + K$  mark with a full fuel tank and 4 Cans. This means we can go the 7 to get to  $10 + K$  (2413km).

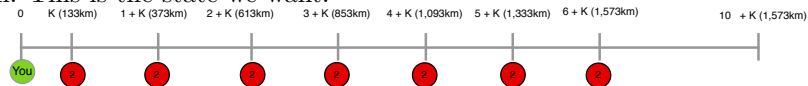
So how do we put theses fuel tanks there in the first place?



### 1.5 Describe a procedure whereby you could cross the desert and return in the vehicle using the minimum amount of fuel.

We are still going to use the same conversion as before where each can is one unit and each unit is equal to 240km. In this instance  $k$  is equal to 133km, or 11.08L of petrol.

To do this we need to get the desert to a state in which we can do it in one run. This is the state we want:



To this we are going to have to make multiple trips. Here is each step. Fill up at every 1 unit whenever you pass over. And drop at each point to make it the amount needed. Any cans sitting at distance  $k$  have 11.06L in them as that is the required amount to make it to that spot.

k	1 + k	2 + k	3 + k	4 + k	5 + k	6 + k	required fuel
4							66.50L
2	4						120.00L
0	2	4					120.00L
4	2	4					66.50L
2	0	2	4				120.00L
0	4	2	4				120.00L
4	4	2	4				66.50L
2	2	0	2	4			120.00L
0	0	4	2	4			120.00L
4	0	4	2	4			66.50L
2	4	4	2	4			120.00L
0	2	2	0	2	4		120.00L
4	2	2	0	2	4		66.50L
2	0	0	4	2	4		120.00L
0	4	0	4	2	4		120.00L
4	4	0	4	2	4		66.50L
2	2	4	4	2	4		120.00L
0	0	2	2	2	2	2	120.00L
2	2	2	2	2	2	2	124.32L

This method only requires you to take 120L of petrol with you, 40L in the tank and 4 cans, for each full trip. You can do this because at each stop you fill your car with one can, until you reach the point where you want to drop cans, where you drop all 4 cans. Then you refill what you use on the way back again, meaning you only need 40L as you only don't refill twice, the final leg and the movement to where you drop your cans off. And all cans at k hold the exact amount measured at the start of 11.06L (the required amount to travel K). The penultimate trip requires the tank to be filled with 51.08L, 3 full cans and a fourth can of 13.24L. The final trip (not show) requires a full tank and 4 full cans of petrol. Setting up for the final trip uses 1963.32L, and in total 2103.32L of petrol is used.