

# The EPFL-EKV MOSFET Model Equations for Simulation

*Technical Report*

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## 1) Notes on the EPFL-EKV MOSFET model for circuit simulation

### 1.1 Introduction

The EPFL-EKV MOSFET model is a scalable and compact simulation model built on fundamental physical properties of the MOS structure. This model is dedicated to the design and simulation of low-voltage, low-current analog, and mixed analog-digital circuits using submicron CMOS technologies.

This report provides a description of the equations and parameters used for the computer simulation version of the EPFL-EKV MOSFET model. The description concentrates on the intrinsic part of the MOSFET, and is intended to give the model user information on parameter handling and the actual equations used in the computer simulation.

The extrinsic part of the MOSFET is handled as it is often made for other MOSFET models. The extrinsic model includes the series resistances of the source and drain diffusions, which are handled as external elements, as well as junction currents and capacitances. The model user should refer to the documentation provided with the simulator used.

### 1.2 Single equation model

The EPFL-EKV MOSFET model is in principle formulated as a 'single expression', which preserves continuity of first- and higher-order derivatives with respect to any terminal voltage, in the entire range of validity of the model. The analytical expressions of first-order derivatives as transconductances and transcapacitances are not presented in this document but are also available for computer simulation.

### 1.3 Effects modeled

The EPFL-EKV MOSFET model version 2.6 includes modeling of the following physical effects:

- basic geometrical and process related aspects as oxide thickness, junction depth, effective channel length and width
- effects of doping profile, substrate effect
- modeling of weak, moderate and strong inversion behavior
- modeling of mobility effects due to vertical and lateral fields, velocity saturation
- short-channel effects as channel-length modulation (CLM), source and drain charge-sharing (including for narrow channel widths), reverse short channel effect (RSCE)
- modeling of substrate current due to impact ionization
- quasistatic charge-based dynamic model
- thermal and flicker noise modeling
- a first-order non-quasistatic model for the transadmittances
- short-distance geometry- and bias-dependent device matching.

## 1.4 Coherence of static and dynamic models

All aspects regarding the static, the quasistatic and non-quasistatic dynamic and noise models are all derived in a coherent way from a single characteristic, the normalized transconductance-to-current ratio. Symmetric normalized forward and reverse currents are used throughout these expressions. For quasistatic dynamic operation, both a charge-based model for the node charges and trans-capacitances, and a simpler capacitances model are available. The dynamic model, including the time constant for the non-quasistatic model, is described in symmetrical terms of the forward and reverse normalized currents. The charge formulation is further used to express effective mobility dependence of local field.

## 1.5 Bulk reference and symmetry

Voltages are all referred to the local substrate:

$$V_G = V_{GB} \quad \text{Intrinsic gate-to-bulk voltage} \quad (1)$$

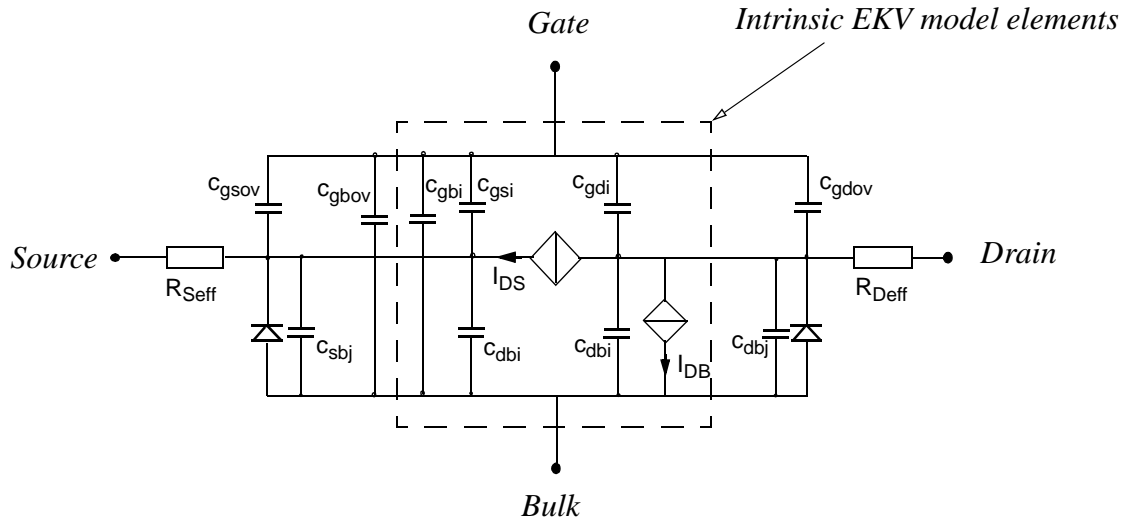
$$V_S = V_{SB} \quad \text{Intrinsic source-to-bulk voltage} \quad (2)$$

$$V_D = V_{DB} \quad \text{Intrinsic drain-to-bulk voltage} \quad (3)$$

$V_S$  and  $V_D$  are the intrinsic source and drain voltages, which means that the voltage drop over extrinsic resistive elements is supposed to have already been accounted for externally.  $V_D$  is the electrical drain voltage, and is chosen such that  $V_D \geq V_S$ . Bulk reference allows the model to be handled symmetrically with respect to source and drain, a symmetry that is inherent in common MOS technologies (excluding non-symmetric source-drain layouts).

NOTE: Intrinsic model equations are presented for an N-channel MOSFET. P-channel MOSFETs are dealt with as pseudo-N-channel, i.e. the polarity of the voltages ( $V_G$ ,  $V_S$ ,  $V_D$ , as well as VFB, VTO and TCV) is inversed prior to computing the current for P-channel, which is given a negative sign. No other distinctions are made between N-channel and P-channel, with the exception of the  $\eta$  factor for effective mobility calculation.

## 2) Equivalent circuit



This figure represents the intrinsic and extrinsic elements of the MOS transistor. For quasistatic dynamic operation, only the intrinsic capacitances from the simpler capacitances model are shown here. However a charge-based transcapacitances model is also available for computer simulation.

### 3) Device input variables

NAME	DESCRIPTION	UNITS <sup>1</sup>	DEFAULT
L	channel length	<i>m</i>	-
W	channel width	<i>m</i>	-
M or NP	parallel multiple device number	-	1.0
N or NS	series multiple device number	-	1.0

1) For length units selection, please refer to the documentation of your simulator.

### 4) EKV intrinsic model parameters

#### 4.1 Process related parameters

NAME	DESCRIPTION	UNITS <sup>1</sup>	DEFAULT	RANGE
COX <sup>2</sup>	gate oxide capacitance per unit area	<i>F/m<sup>2</sup></i>	0.7E-3	-
XJ	junction depth	<i>m</i>	0.1E-6	$\geq 1.0E-9$
DW <sup>3</sup>	channel width correction	<i>m</i>	0	-
DL	channel length correction	<i>m</i>	0	-

1) For the choice of length units, please refer to the documentation of your simulator.

2) The default value of COX can be calculated as function of TOX.

3) DL and DW parameters usually have a negative value; see effective length and width calculation.

#### 4.2 Basic intrinsic model parameters

NAME	DESCRIPTION	UNITS	DEFAULT <sup>1</sup>	RANGE
VTO <sup>2</sup>	long-channel threshold voltage	<i>V</i>	0.5	-
GAMMA	body effect parameter	$\sqrt{V}$	1.0	$\geq 0$
PHI	bulk Fermi potential (*2)	<i>V</i>	0.7	$\geq 0.1$
KP	transconductance parameter	<i>A/V<sup>2</sup></i>	50.0E-6	-
E0 (EO)	mobility reduction coefficient	<i>V/m</i>	1.0E12	$\geq 1E5$
UCRIT	longitudinal critical field	<i>V/m</i>	2.0E6	$\geq 1E5$

1) The default values of VTO, GAMMA, PHI, KP can be calculated as function of TOX, NSUB, UO, VFB for the purpose of statistical circuit simulation.

2) As  $V_G$ , VTO is also referred to the bulk.

#### 4.2.1 Optional parameters

The following parameters are introduced, to accommodate scaling behavior of the process and basic intrinsic model parameters, as well as statistical circuit simulation. Note that the parameters TOX, NSUB, VFB, UO, and VMAX are only used if COX, GAMMA and/or PHI, VTO, KP and UCRIT are *not* specified, respectively. Further, a simpler mobility reduction model due to vertical field is accessible. The mobility reduction coefficient THETA is only used if E0 is *not* specified.

NAME	DESCRIPTION	UNITS <sup>1</sup>	DEFAULT	RANGE
TOX <sup>2</sup>	oxide thickness	$m$	-	$\geq 0$
NSUB <sup>3</sup>	channel doping	$cm^{-3}$	-	$\geq 0$
VFB <sup>4</sup>	flat-band voltage	$V$	-	-
UO <sup>5</sup>	low-field mobility	$cm^2/(Vs)$	-	$\geq 0$
VMAX <sup>6</sup>	saturation velocity	$m/s$	-	$\geq 0$
THETA <sup>7</sup>	mobility reduction coefficient	$1/V$	0	$\geq 0$

1) Note the choice of  $cm$  as basic unit for NSUB and UO, while TOX and VMAX are in  $m$ .

2) Optional parameter used to calculate COX.

3) Optional parameter accounting for the dependence of GAMMA on COX, as well as for calculation of PHI.

4) Optional parameter used to calculate VTO as a function of COX, GAMMA, PHI.

5) Optional parameter accounting for the dependence of KP on COX.

6) Optional parameter used to calculate UCRIT.

7) Optional parameter for mobility reduction due to vertical field.

#### 4.3 Channel length modulation and charge sharing parameters

NAME	DESCRIPTION	UNITS	DEFAULT	RANGE
LAMBDA	depletion length coefficient (channel length modulation)	-	0.5	$\geq 0$
WETA	narrow-channel effect coefficient	-	0.25	-
LETA	short-channel effect coefficient	-	0.1	-

#### 4.4 Reverse short-channel effect parameters

NAME	DESCRIPTION	UNITS	DEFAULT	RANGE
Q0 (QO)	reverse short channel effect peak charge density	$A \cdot s/m^2$	0	-
LK	reverse short channel effect characteristic length	$m$	0.29E-6	$\geq 1.0E-8$

#### 4.5 Impact ionization related parameters

NAME	DESCRIPTION	UNITS	DEFAULT	RANGE
IBA	first impact ionization coefficient	$1/m$	0	-
IBB	second impact ionization coefficient	$V/m$	3.0E8	$\geq 1.0E8$
IBN	saturation voltage factor for impact ionization	-	1.0	$\geq 0.1$

#### 4.6 Intrinsic model temperature parameters

NAME	DESCRIPTION	UNITS	DEFAULT
TCV	threshold voltage temperature coefficient	$V/K$	$1.0E-3$
BEX	mobility temperature exponent	-	$-1.5$
UCEX	longitudinal critical field temperature exponent	-	$0.8$
IBBT	temperature coefficient for IBB	$1/K$	$9.0E-4$

#### 4.7 Matching parameters

NAME	DESCRIPTION	UNITS	DEFAULT
AVTO	area related threshold voltage mismatch parameter	$V_m$	$0^1$
AKP	area related gain mismatch parameter	$m$	$0$
AGAMMA	area related body effect mismatch parameter	$\sqrt{V_m}$	$0$

1) NOTE: only DEV values are applicable to the statistical matching parameters AVTO, AGAMMA, AKP for Monte-Carlo type simulations. Default is 1E-6 for all three parameters in some implementations, to allow sensitivity analysis to be performed on the matching parameters. LOT specifications should not be used for AVTO, AGAMMA, AKP.

#### 4.8 Flicker noise parameters

NAME	DESCRIPTION	UNITS	DEFAULT
KF	flicker noise coefficient	$-1$	$0$
AF	flicker noise exponent	-	$1$

1) Unit of KF may depend on flicker noise model chosen if options are available.

#### 4.9 Setup parameters

NAME	DESCRIPTION	UNITS	DEFAULT
NQS <sup>1</sup>	Non-Quasi-Static (NQS) operation switch	-	$0$
SATLIM <sup>2</sup>	ratio defining the saturation limit $i_f/i_r$	-	$\exp(4)$
XQC <sup>3</sup>	charge/capacitance model selector	-	$0.4$

- 1) NQS=1 switches Non-Quasi-Static operation on, default is off (NQS model option may not be available in all implementations).
- 2) Only used for operating point information. (SATLIM option may not be available in all implementations).
- 3) Selector for charges/transcapacitances (default) or capacitances only model. XQC=0.4: charges/transcapacitances model; XQC=1: capacitances only model. (XQC model option may not be available in all implementations).

## 5) Static intrinsic model equations

### 5.1 Basic definitions

$$\epsilon_{si} = SCALE \cdot 104.5 \times 10^{-12} [F/m] \quad \text{Permittivity of silicon} \quad (4)$$

$$\epsilon_{ox} = SCALE \cdot 34.5 \times 10^{-12} [F/m] \quad \text{Permittivity of silicon dioxide} \quad (5)$$

$$q = 1.602 \times 10^{-19} [C] \quad \text{Magnitude of electron charge}$$

$$k = 1.3807 \times 10^{-23} [JK^{-1}] \quad \text{Boltzmann constant}$$

$$T_{ref} = 300.15 [K] \quad \text{Reference temperature} \quad (6)$$

$$T_{nom} [K] \quad \text{Nominal temperature of model parameters}$$

$$T [K] \quad \text{Model simulation temperature}$$

$$V_t(T) = \frac{k \cdot T}{q} \quad \text{Thermal voltage} \quad (7)$$

$$E_g(T) = \left( 1.16 - 0.000702 \cdot \frac{T^2}{T + 1108} \right) [eV] \quad \text{Energy gap} \quad (8)$$

$$n_i(T) = 1.45 \times 10^{16} \cdot \left( \frac{T}{T_{ref}} \right) \cdot \exp \left( \frac{E_g(T_{ref})}{2 \cdot V_t(T_{ref})} - \frac{E_g(T)}{2 \cdot V_t(T)} \right) [m^{-3}] \quad \text{Intrinsic carrier concentration} \quad (9)$$

### 5.2 Parameter preprocessing

#### 5.2.1 Handling of model parameters for P-channel MOSFETs

For P-channel devices, the sign of VFB, VTO and TCV is inverted before processing. Therefore, VTO and TCV are usually positive and VFB negative for N-channel, and vice versa for P-channel MOSFETs.

#### 5.2.2 Intrinsic parameters initialization

The basic intrinsic model parameters COX, GAMMA, PHI, VTO, KP and UCRIT are related to the fundamental process parameters TOX, NSUB, VFB, UO, VMAX, respectively, similarly as in early SPICE models. For the purpose of statistical circuit simulation, it is desirable to introduce parameter variations on the level of the latter parameters. These dependencies are also of interest if device scaling is to be analyzed, and are useful when parameter sets should be obtained from other MOSFET models. Therefore the possibility is introduced to use the following relations:

If COX is not specified, then it is initialized as:

$$COX = \begin{cases} \epsilon_{ox}/TOX & \text{for: } TOX > 0 \\ \text{default} & \text{otherwise} \end{cases} \quad (10)$$

If GAMMA is not specified, then it is initialized as:

$$GAMMA = \begin{cases} \frac{\sqrt{2q\epsilon_{si} \cdot (NSUB \cdot 10^6)}}{COX} & \text{for: } NSUB > 0 \\ \text{default} & \text{otherwise} \end{cases} \quad (11)$$

If  $\text{PHI}$  is not specified, then it is initialized as:

$$\text{PHI} = \begin{cases} 2V_t(T_{nom}) \cdot \ln\left(\frac{\text{NSUB} \cdot 10^6}{n_i(T_{nom})}\right) & \text{for: NSUB} > 0 \\ \text{default} & \text{otherwise} \end{cases} \quad (12)$$

If  $\text{VTO}$  is not specified, then it is initialized as:

$$\text{VTO} = \begin{cases} \text{VFB} + \text{PHI} + \text{GAMMA} \cdot \sqrt{\text{PHI}} & \text{if VFB specified} \\ \text{default} & \text{otherwise} \end{cases} \quad (13)$$

If  $\text{KP}$  is not specified, then it is initialized as:

$$\text{KP} = \begin{cases} (\text{UO} \cdot 10^{-4}) \cdot \text{COX} & \text{for: UO} > 0 \\ \text{default} & \text{otherwise} \end{cases} \quad (14)$$

If  $\text{UCRIT}$  is not specified, then it is initialized as:

$$\text{UCRIT} = \begin{cases} \text{VMAX}/(\text{UO} \cdot 10^{-4}) & \text{for: VMAX} > 0, \text{UO} > 0 \\ \text{default} & \text{otherwise} \end{cases} \quad (15)$$

If  $\text{E0}$  is not specified, then a simplified mobility model is used with the parameter  $\text{THETA}$ :

$$\text{E0} = \begin{cases} 0 & \text{if THETA specified} \\ \text{default} & \text{otherwise} \end{cases} \quad (16)$$

Note: the value zero is given to  $\text{E0}$  here, indicating that the simplified mobility model is used in conjunction with  $\text{THETA}$  instead of the standard mobility model.

Note that optional parameters may not be available in all implementations.

### 5.2.3 Default values and parameter ranges

Model parameters which are not defined in the model parameter sets are either initialized according to the above relations, or set to their default values. For certain parameters, their numerical range has to be restricted to avoid numerical problems such as divisions by zero. If a parameter given in a parameter set falls outside the specified range (see range column in the parameter tables) then its value is set to the closest acceptable value.

### 5.2.4 Intrinsic parameters temperature dependence

$$\text{VTO}(T) = \text{VTO} - \text{TCV} \cdot (T - T_{nom}) \quad (17)$$

$$\text{KP}(T) = \text{KP} \cdot \left(\frac{T}{T_{nom}}\right)^{\text{BEX}} \quad (18)$$

$$\text{UCRIT}(T) = \text{UCRIT} \cdot \left(\frac{T}{T_{nom}}\right)^{\text{UCEX}} \quad (19)$$

$$\text{PHI}(T) = \text{PHI} \cdot \frac{T}{T_{nom}} - 3 \cdot V_t \cdot \ln\left(\frac{T}{T_{nom}}\right) - E_g(T_{nom}) \cdot \frac{T}{T_{nom}} + E_g(T) \quad (20)$$

$$\text{IBB}(T) = \text{IBB} \cdot [1.0 + \text{IBBT} \cdot (T - T_{nom})] \quad (21)$$

### 5.3 Bulk referenced intrinsic voltages

Voltages are all referred to the local substrate (See “Bulk reference and symmetry” on page 2.):

$$V_G = V_{GB} = V_{GS} - V_{BS} \quad \text{Intrinsic gate-to-bulk voltage} \quad (22)$$

$$V_S = V_{SB} = -V_{BS} \quad \text{Intrinsic source-to-bulk voltage} \quad (23)$$

$$V_D = V_{DB} = V_{DS} - V_{BS} \quad \text{Intrinsic drain-to-bulk voltage} \quad (24)$$

For P-channel devices, all signs of the above voltages are inverted prior to processing.

### 5.4 Effective channel length and width

$$W_{eff} = W + DW \quad (25)$$

$$L_{eff} = L + DL \quad (26)$$

NOTE: Contrary to the convention adopted in other MOSFET models, DL and DW usually do have a negative value due to the above definition.

### 5.5 Short distance matching

Random mismatch between two transistors with identical layout and close to each other is in most cases suitably described by a law following the inverse of the square root of the transistors' area. The following relationships have been adopted:

$$VTO_a = VTO + \frac{AVTO}{\sqrt{NP \cdot W_{eff} \cdot NS \cdot L_{eff}}} \quad (27)$$

$$KP_a = KP \cdot \left( 1 + \frac{AKP}{\sqrt{NP \cdot W_{eff} \cdot NS \cdot L_{eff}}} \right) \quad (28)$$

$$GAMMA_a = GAMMA + \frac{AGAMMA}{\sqrt{NP \cdot W_{eff} \cdot NS \cdot L_{eff}}} \quad (29)$$

These model equations are only applicable in Monte-Carlo and sensitivity simulations.

NOTE: since negative values for both  $KP_a$  and  $GAMMA_a$  are not physically meaningful, these are clipped at zero.

### 5.6 Reverse short-channel effect (RSCE)

$$C_\epsilon = 4 \cdot (22 \times 10^{-3})^2 \quad C_A = 0.028 \quad (30)$$

$$\xi = C_A \cdot \left( 10 \cdot \frac{L_{eff}}{LK} - 1 \right) \quad (31)$$

$$\Delta V_{RSCE} = \frac{2 \cdot Q0}{COX} \cdot \frac{1}{\left[ 1 + \frac{1}{2} \cdot (\xi + \sqrt{\xi^2 + C_\epsilon}) \right]^2} \quad (32)$$

### 5.7 Effective gate voltage including RSCE

$$V_G' = V_G - VTO_a - \Delta V_{RSCE} + PHI + GAMMA_a \sqrt{PHI} \quad (33)$$



## 5.8 Effective substrate factor including charge-sharing for short and narrow channels

### 5.8.1 Pinch-off voltage for narrow-channel effect:

$$V_{P0} = \begin{cases} V_{G'} - \text{PHI} - \text{GAMMA}_a \left( \sqrt{V_{G'} + \left( \frac{\text{GAMMA}_a}{2} \right)^2} - \frac{\text{GAMMA}_a}{2} \right) & \text{for: } V_{G'} > 0 \\ -\text{PHI} & \text{for: } V_{G'} \leq 0 \end{cases} \quad (34)$$

### 5.8.2 Effective substrate factor accounting for charge-sharing:

$$V'_{S(D)} = \frac{1}{2} \cdot [V_{S(D)} + \text{PHI} + \sqrt{(V_{S(D)} + \text{PHI})^2 + (4V_t)^2}] \quad (35)$$

NOTE: Eqn. 35 prevents the argument of the square roots in the subsequent code from becoming negative.

$$\gamma^\circ = \text{GAMMA}_a - \frac{\epsilon_{si}}{\text{COX}} \cdot \left[ \frac{\text{LETA}}{L_{eff}} \cdot (\sqrt{V'_S} + \sqrt{V'_D}) - \frac{3 \cdot \text{WETA}}{W_{eff}} \cdot \sqrt{V_{P0} + \text{PHI}} \right] \quad (36)$$

$$\gamma' = \frac{1}{2} \cdot (\gamma^\circ + \sqrt{\gamma^{\circ 2} + 0.1 \cdot V_t}) \quad (37)$$

NOTE: The purpose of Eqn. 37 is to prevent the effective substrate factor from becoming negative.

## 5.9 Pinch-off voltage including short- and narrow-channel effects

$$V_P = \begin{cases} V_{G'} - \text{PHI} - \gamma' \cdot \left( \sqrt{V_{G'} + \left( \frac{\gamma'}{2} \right)^2} - \frac{\gamma'}{2} \right) & \text{for: } V_{G'} > 0 \\ -\text{PHI} & \text{for: } V_{G'} \leq 0 \end{cases} \quad (38)$$

Note: the pinch-off voltage accounts for channel doping effects such as threshold voltage and substrate effect. For long-channel devices,  $V_P$  is a function of gate voltage; for short-channel devices, it becomes also a function of source and drain voltage due to the charge-sharing effect.

## 5.10 Slope factor

$$n = 1 + \frac{\text{GAMMA}_a}{2 \cdot \sqrt{V_P + \text{PHI} + 4V_t}} \quad (39)$$

Note: the slope factor (or body effect factor), which is primarily a function of the gate voltage, is linked to the weak inversion slope.

## 5.11 Large signal interpolation function

$F(v)$  is the large-signal interpolation function relating the normalized currents to the normalized voltages. A simple and accurate expression for the transconductance-to-current ratio allows a consistent formulation of the static large-signal interpolation function, the dynamic model for the intrinsic charges (and capacitances) as well as the intrinsic time constant and the thermal noise model for the whole range of current from weak to strong inversion:

$$\frac{g_{ms} \cdot V_t}{I_{DS}} = \frac{1}{\sqrt{0.25 + i} + 0.5} \quad (40)$$

Large-signal interpolation function:

$$y = \sqrt{0.25 + i} - 0.5 \quad (41)$$

$$v = 2y + \ln(y) \quad (42)$$

Unfortunately Eqn. 42 cannot be inverted analytically. However it can be inverted using a Newton-Raphson iterative scheme. Currently a simplification of this algorithm that avoids iteration is used, leading to a continuous expression for the large signal interpolation function. The (inverted) large signal interpolation function has the following asymptotes in strong and weak inversion respectively:

$$F(v) = \begin{cases} (v/2)^2 & \text{for: } v \gg 0 \\ \exp(v) & \text{for: } v \ll 0 \end{cases} \quad (43)$$

### 5.12 Forward normalized current

$$i_f = F\left[\frac{V_P - V_S}{V_t}\right] \quad (44)$$

### 5.13 Velocity saturation voltage

$$V_C = \text{UCRIT} \cdot \text{NS} \cdot L_{eff} \quad (45)$$

NOTE: Eqn. 45 accounts for multiple series device number NS .

$$V_{DSS} = V_C \cdot \left[ \sqrt{\frac{1}{4} + \frac{V_t}{V_C} \cdot \sqrt{i_f}} - \frac{1}{2} \right] \quad (46)$$

NOTE: The variable  $V_{DSS}$  in this formulation for computer simulation is half the value of the actual saturation voltage.

### 5.14 Drain-to-source saturation voltage for reverse normalized current

$$V_{DSS}' = V_C \cdot \left[ \sqrt{\frac{1}{4} + \frac{V_t}{V_C} \cdot \left( \sqrt{i_f} - \frac{3}{4} \cdot \ln(i_f) \right)} - \frac{1}{2} \right] + V_t \cdot \left[ \ln\left(\frac{V_C}{2V_t}\right) - 0.6 \right] \quad (47)$$

### 5.15 Channel-length modulation

$$\Delta V = 4 \cdot V_t \cdot \sqrt{\text{LAMBDA} \cdot \left( \sqrt{i_f} - \frac{V_{DSS}}{V_t} \right) + \frac{1}{64}} \quad (48)$$

$$V_{ds} = \frac{V_D - V_S}{2} \quad (49)$$

$$V_{ip} = \sqrt{V_{DSS}^2 + \Delta V^2} - \sqrt{(V_{ds} - V_{DSS})^2 + \Delta V^2} \quad (50)$$

$$L_C = \sqrt{\frac{\epsilon_{si}}{\text{COX}}} \cdot \text{XJ} \quad (51)$$

$$\Delta L = \text{LAMBDA} \cdot L_C \cdot \ln\left(1 + \frac{V_{ds} - V_{ip}}{L_C \cdot \text{UCRIT}}\right) \quad (52)$$

### 5.16 Equivalent channel length including channel-length modulation and velocity saturation

$$L' = NS \cdot L_{eff} - \Delta L + \frac{V_{ds} + V_{ip}}{UCRIT} \quad (53)$$

$$L_{min} = NS \cdot L_{eff} / 10 \quad (54)$$

NOTE: Eqn. 53 and Eqn. 54 account also for multiple series device number NS .

$$L_{eq} = \frac{1}{2} \cdot (L' + \sqrt{L'^2 + L_{min}^2}) \quad (55)$$

NOTE: Eqn. 55 prevents the equivalent channel length to become zero or negative.

### 5.17 Reverse normalized current

#### 5.17.1 Reverse normalized current:

$$i_r' = F \left[ \frac{V_P - V_{ds} - V_S - \sqrt{V_{DSS}'^2 + \Delta V^2} + \sqrt{(V_{ds} - V_{DSS}')^2 + \Delta V^2}}{V_t} \right] \quad (56)$$

#### 5.17.2 Reverse normalized current for mobility model, intrinsic charges/capacitances, thermal noise model and NQS time-constant:

$$i_r = F \left[ \frac{V_P - V_D}{V_t} \right] \quad (57)$$

### 5.18 Transconductance factor and mobility reduction due to vertical field

$$\beta_0 = KP_a \cdot \frac{NP \cdot W_{eff}}{L_{eq}} \quad (58)$$

NOTE: the use of the device parameter NP (or M) gives accurate results for simulation of parallel devices, whereas the use of NS (or N) for series devices is only approximate. Note that  $L_{eq}$  accounts for multiple series device number NS .

$$\eta = \begin{cases} 1/2 & \text{for } NMOS \\ 1/3 & \text{for } PMOS \end{cases} \quad (59)$$

$$q_{B0} = GAMMA_a \cdot \sqrt{PHI} \quad (60)$$

$$\beta_0' = \beta_0 \cdot \left( 1 + \frac{COX}{EO \cdot \epsilon_{si}} \cdot q_{B0} \right) \quad (61)$$

$$\beta = \frac{\beta_0'}{1 + \frac{COX}{EO \cdot \epsilon_{si}} \cdot V_t \cdot |q_B + \eta \cdot q_I|} \quad (62)$$

For the definition of the normalized depletion and inversion charges  $q_B$  and  $q_I$  refer to the section on the node charges. The use of  $\beta_0'$  ensures that  $\beta \approx \beta_0$  when  $q_I \ll q_B$ . The formulation of  $\beta$  arises from the integration of the local effective field as a function of depletion and inversion charge densities along the channel. No substrate bias dependency is needed due to the inclusion of depletion charge. Note that the resulting mobility expression also depends on  $V_{DS}$ .

### 5.18.1 Simple mobility reduction model

For reasons of compatibility with the former EKV model versions (EKV model versions prior to v2.6), a possibility is introduced to choose the simpler mobility reduction model which uses the parameter THETA. In case the model parameter E0 is *not* specified (see parameter preprocessing), the simpler mobility model is taken into account according to:

$$V_P' = \frac{1}{2} \cdot (V_P + \sqrt{V_P^2 + 2V_t^2}) \quad (63)$$

$$\beta = \frac{\beta_0}{1 + \text{THETA} \cdot V_P'} \quad (64)$$

For availability of this option please check with the documentation in your simulator.

### 5.19 Specific current

$$I_S = 2 \cdot n \cdot \beta \cdot V_t^2 \quad (65)$$

### 5.20 Drain-to-source current

$$I_{DS} = I_S \cdot (i_f - i_r') \quad (66)$$

For P-channel devices,  $I_{DS}$  is given a negative sign.

NOTE: this drain current expression is a single equation, valid in all operating regions: weak, moderate and strong inversion, non-saturation and saturation. It is therefore not only continuous among all these regions, but also continuously derivable.

### 5.21 Transconductances

The transconductances are obtained through derivation of the drain current:

$$g_{mg} \equiv \frac{\partial I_{DS}}{\partial V_G} \quad g_{ms} \equiv -\frac{\partial I_{DS}}{\partial V_S} \quad g_{md} \equiv \frac{\partial I_{DS}}{\partial V_{DS}}$$

Note the following relationships with the derivatives where the source is taken as reference:

$$g_m \equiv \frac{\partial I_{DS}}{\partial V_{GS}} = g_{mg} \quad g_{mbs} \equiv \frac{\partial I_{DS}}{\partial V_{BS}} = g_{ms} - g_{mg} - g_{md} \quad g_{ds} \equiv \frac{\partial I_{DS}}{\partial V_{DS}} = g_{md}$$

The analytic derivatives are available with the model code.

### 5.22 Impact ionization current

$$V_{ib} = V_D - V_S - \text{IBN} \cdot 2 \cdot V_{DSS} \quad (67)$$

$$I_{DB} = \begin{cases} I_{DS} \cdot \frac{\text{IBA}}{\text{IBB}} \cdot V_{ib} \cdot \exp\left(\frac{-\text{IBB} \cdot L_C}{V_{ib}}\right) & \text{for: } V_{ib} > 0 \\ 0 & \text{for: } V_{ib} \leq 0 \end{cases} \quad (68)$$

NOTE: the factor 2 in the expression for  $V_{ib}$  accounts for the fact that the numerical value of  $V_{DSS}$  is half the actual saturation voltage. Further note that the substrate current is intended to be treated as a component of the total extrinsic drain current, flowing from the drain to the bulk. The total drain current is therefore expressed as  $I_D = I_{DS} + I_{DB}$ .

The substrate current therefore also affects the total extrinsic conductances, in particular the drain conductance.

## 6) Quasi-static model equations

Both a charge-based model for transcapacitances, allowing charge-conservation during transient analysis, and a simpler capacitances based model are available. Note that the charges model is formulated in symmetric terms of the forward and reverse normalized currents, i.e. symmetrical for both drain and source sides.

Further note that short-channel effects, as charge-sharing and reverse short-channel effects, are included in the dynamic model through the pinch-off voltage.

### 6.1 Dynamic model for the intrinsic node charges

$$n_q = 1 + \frac{\text{GAMMA}_a}{2 \cdot \sqrt{V_P + \text{PHI} + 10^{-6}}} \quad (69)$$

#### 6.1.1 Normalized intrinsic node charges:

$$x_f = \sqrt{\frac{1}{4} + i_f} \quad (70)$$

$$x_r = \sqrt{\frac{1}{4} + i_r} \quad (71)$$

$$q_D = -n_q \cdot \left( \frac{4}{15} \cdot \frac{3x_r^3 + 6x_r^2x_f + 4x_rx_f^2 + 2x_f^3}{(x_f + x_r)^2} - \frac{1}{2} \right) \quad (72)$$

$$q_S = -n_q \cdot \left( \frac{4}{15} \cdot \frac{3x_f^3 + 6x_f^2x_r + 4x_fx_r^2 + 2x_r^3}{(x_f + x_r)^2} - \frac{1}{2} \right) \quad (73)$$

$$q_I = q_S + q_D = -n_q \cdot \left( \frac{4}{3} \cdot \frac{x_f^2 + x_fx_r + x_r^2}{x_f + x_r} - 1 \right) \quad (74)$$

$$q_B = \begin{cases} (-\text{GAMMA}_a \cdot \sqrt{V_P + \text{PHI} + 10^{-6}}) \cdot \frac{1}{V_t} - \left( \frac{n_q - 1}{n_q} \right) \cdot q_I & \text{for: } V'_G > 0 \\ -V'_G \cdot \frac{1}{V_t} & \text{for: } V'_G \leq 0 \end{cases} \quad (75)$$

$$q_G = -q_I - q_{OX} - q_B \quad (76)$$

$q_{OX}$  is a fixed oxide charge assumed to be zero. The above equation expresses the charge conservation among the four nodes of the transistor.

#### 6.1.2 Total charges:

$$C_{ox} = \text{COX} \cdot \text{NP} \cdot W_{eff} \cdot \text{NS} \cdot L_{eff} \quad (77)$$

$$Q_{(I, B, D, S, G)} = C_{ox} \cdot V_t \cdot q_{(I, B, D, S, G)} \quad (78)$$

## 6.2 Intrinsic capacitances

### 6.2.1 Transcapacitances:

The intrinsic capacitances are obtained through derivation of the node charges with respect to the terminal voltages:

$$C_{xy} = \pm \frac{\partial}{\partial V_y}(Q_x) \quad x, y = G, D, S, B$$

where the positive sign is chosen when  $x = y$  and the negative sign otherwise. This results in simple and continuous analytical expressions for all the transcapacitances in terms of  $x_f$ ,  $x_r$ , the pinch-off voltage and the slope factor, and derivatives thereof, from weak to strong inversion and non-saturation to saturation.

### 6.2.2 Normalized intrinsic capacitances:

A simplified capacitive dynamic model, using the five intrinsic capacitances corresponding to the equivalent circuit shown in section 2) on page 2, can be obtained when neglecting the slight bias dependence of the slope factor  $n$ , resulting in the following simple functions:

$$c_{gs} = \frac{2}{3} \cdot \left( 1 - \frac{x_r^2 + x_r + \frac{1}{2}x_f}{(x_f + x_r)^2} \right) \quad (79)$$

$$c_{gd} = \frac{2}{3} \cdot \left( 1 - \frac{x_f^2 + x_f + \frac{1}{2}x_r}{(x_f + x_r)^2} \right) \quad (80)$$

$$c_{gb} = \left( \frac{n_q - 1}{n_q} \right) \cdot (1 - c_{gs} - c_{gd}) \quad (81)$$

$$c_{sb} = (n_q - 1) \cdot c_{gs} \quad (82)$$

$$c_{db} = (n_q - 1) \cdot c_{gd} \quad (83)$$

Note: this simplified capacitances model can be chosen by setting XQC=1 (availability may depend on the simulator used; please refer to the documentation of your simulator).

### 6.2.3 Total intrinsic capacitances:

$$C_{(gs, gd, gb, sb, db)} = C_{ox} \cdot c_{(gs, gd, gb, sb, db)} \quad (84)$$

## 7) Non-Quasi-Static (NQS) model equations

The EKV model includes a first order NQS model for small-signal (.AC) simulations. The expression of the NQS drain current is obtained from the quasi-static value of the drain current which is then 1st-order low-pass filtered. NQS is a flag (model parameter) allowing to disable the NQS model and  $\tau$  is the bias dependent characteristic time constant.

$\tau_0$  is the intrinsic time constant defined as:

$$\tau_0 = \frac{(\text{NS} \cdot L_{\text{eff}})^2}{2 \cdot \mu_{\text{eff}} \cdot V_t} = \frac{C_{ox}}{2 \cdot V_t \cdot \beta} \quad (85)$$

$$\tau = \tau_0 \cdot \frac{4}{15} \cdot \frac{(x_f^2 + 3x_f x_r + x_r^2)}{(x_f + x_r)^3} \quad (86)$$

$$I_{DS}(s) = \frac{I_{DS}}{1 + \text{NQS} \cdot s \cdot \tau} \quad (87)$$

The corresponding small-signal (.AC) transadmittances are then given by:

$$Y_{mg}(s) = \frac{g_{mg}}{1 + \text{NQS} \cdot s \cdot \tau} \quad (88)$$

$$Y_{ms}(s) = \frac{g_{ms}}{1 + \text{NQS} \cdot s \cdot \tau} \quad (89)$$

$$Y_{md}(s) = \frac{g_{md}}{1 + \text{NQS} \cdot s \cdot \tau} \quad (90)$$

$$Y_{mbs}(s) = Y_{ms}(s) - Y_{mg}(s) - Y_{md}(s) \quad (91)$$

NOTE: availability of NQS model is simulator-dependent. Please refer to the documentation of your simulator.

## 8) Intrinsic noise model equations

The noise is modeled by a current source  $I_{NDS}$  between intrinsic source and drain. It is composed of a thermal noise component and a flicker noise component and has the following Power Spectral Density (PSD):

$$S_{INDS} = S_{thermal} + S_{flicker} \quad (92)$$

### 8.1 Thermal noise

The thermal noise component PSD is given by:

$$S_{thermal} = 4kT \cdot \frac{\mu_{\text{eff}}}{(\text{NS} \cdot L_{\text{eff}})^2} \cdot |Q_I| = 4kT \cdot \beta \cdot |q_I| \quad (93)$$

Note that the above thermal noise expression is *valid in all regions of operation*, including for small  $V_{DS}$ .

### 8.2 Flicker noise

The flicker noise component PSD is given by:

$$S_{flicker} = \frac{\text{KF} \cdot g_{mg}^2}{\text{NP} \cdot W_{\text{eff}} \cdot \text{NS} \cdot L_{\text{eff}} \cdot \text{COX} \cdot f^{\text{AF}}} \quad (94)$$

Note: in some implementations, different expressions are accessible. Please refer to the documentation in your simulator.

## 9) Operating point information

### 9.1 Information displayed

At operating points, the following information should be displayed as a help for circuit design:

#### 9.1.1 Numerical values of model internal variables:

$V_G, V_S, V_D, I_{DS}, I_{DB}, g_{mg}, g_{ms}, g_{mbs}, g_{md}, V_P, n, \beta, I_S, i_f, i_r, i_r', \tau, \tau_0$   
intrinsic charges/capacitances

#### 9.1.2 Transconductance efficiency factor:

$$tef = g_{ms} \cdot V_t / I_{DS}$$

#### 9.1.3 'Early voltage':

$$VM = I_{DS} / g_{md}$$

#### 9.1.4 'Overdrive voltage':

$$n \cdot (V_P - V_S) \approx V_G - V_{TOa} - n \cdot V_S$$

For P-channel devices,  $n \cdot (V_P - V_S)$  is given a negative sign.

#### 9.1.5 'SPICE-like' threshold voltage:

$$V_{TH} = V_{TOa} + \Delta V_{RSCE} + \gamma' \cdot \sqrt{V'_S} - \text{GAMMA}_a \cdot \sqrt{\text{PHI}}$$

NOTE: this expression is the 'SPICE-like' threshold voltage, referred to the source. It accounts also for charge-sharing and reverse short-channel effects on the threshold voltage.

For P-channel devices,  $V_{TH}$  is given a negative sign.

#### 9.1.6 Saturation voltage:

$$V_{DSAT} = 2V_{DSS} + 4V_t$$

For P-channel devices,  $V_{DSAT}$  is given a negative sign.

#### 9.1.7 Saturation / non-saturation flag:

$$\begin{array}{ll} \text{'SAT' or '1'} & \text{for } \frac{i_f}{i_r} > \text{SATLIM} \\ \text{'LIN' or '0'} & \text{for } \frac{i_f}{i_r} \leq \text{SATLIM} \end{array}$$

Note: implementation of operating point information may differ in some simulators (i.e. not all of the information may be available). Please refer to the documentation provided with your simulator.



## 10) Estimation and limits of static intrinsic model parameters

The EKV intrinsic model parameters can roughly be estimated from Spice level 2/3 parameters as indicated in the table below, if no parameter extraction facility is available. Attention has to be paid to units of the parameters. This estimation method can be helpful and generally gives reasonable results. Nevertheless be aware that the underlying modeling in Spice level 2/3 and in the EKV model is not the same, even if the names and the function of several parameters are similar. Therefore it is preferred if parameter extraction is made directly from measurements.

Lower and upper limits indicated in the table should give an idea on the order of magnitude of the parameters but do not necessarily correspond to physically meaningful limits, nor to the range specified in the parameter tables. These limits may be helpful for obtaining physically meaningful parameter sets when using nonlinear optimization techniques to extract EKV model parameters.

NAME	UNITS	DEFAULT	EXAMPLE	'LOWER'	'UPPER'	ESTIMATION <sup>1</sup>
COX	F/m <sup>2</sup>	0.7E-3	3.45E-3	-	-	$\epsilon_{ox}/\text{TOX}$
XJ	m	0.1E-6	0.15E-6	0.01E-6	1E-6	XJ
VTO	V	0.5	0.7	0	2	VTO
GAMMA	$\sqrt{V}$	1.0	0.7	0	2	$\sqrt{2q\epsilon_{si} \cdot \text{NSUB}/\text{COX}}$
PHI <sup>2</sup>	V	0.7	0.5	0.3	2	$2V_t \cdot \ln(\text{NSUB}/n_i)$
KP	A/V <sup>2</sup>	50E-6	150E-6	10E-6	-	$\text{UO} \cdot \text{COX}$
E0	V/m	1.0E12	200E6	$0.1/(0.4 \cdot \text{TOX})$	-	$0.2/(\text{THETA} \cdot \text{TOX})$
UCRIT	V/m	2.0E6	2.3E6	1.0E6	25E6	$\text{VMAX}/\text{UO}$
DL	m	0	$-0.15 \cdot L_{\min}$	$-0.5 \cdot L_{\min}$	$0.5 \cdot L_{\min}$	$\text{XL} - 2 \cdot \text{LD}$
DW	m	0	$-0.1 \cdot W_{\min}$	$-0.5 \cdot W_{\min}$	$0.5 \cdot W_{\min}$	$\text{XW} - 2 \cdot \text{WD}$
LAMBDA	-	0.5	0.8	0	3	-
LETA	-	0.1	0.3	0	2	-
WETA	-	0.25	0.2	0	2	-
Q0	As/m <sup>2</sup>	0.0	230E-6	0	-	-
LK	m	0.29E-6	0.4E-6	0.05E-6	2E-6	-
IBA	1/m	0.0	2.0E8	0.0	5.0E8	$\text{ALPHA} \cdot \text{VCR}/L_c$
IBB	V/m	3.0E8	2.0E8	1.8E8	4.0E8	$\text{VCR}/L_c$
IBN	-	1.0	0.6	0.4	1.0	-

1) Also compare with optional process parameters.

2) The minimum value of PHI also determines the minimum value of the pinch-off voltage. Due to the intrinsic temperature dependence of PHI, a lower value results for higher temperature, limiting the range of simulation for small currents.

$$\begin{aligned}
 \epsilon_{ox} &= 0.0345\text{E-}9 \text{ F/m} & q &= 1.609\text{E-}19 \text{ C} & k &= 1.381\text{E-}23 \text{ J/K} & L_c &= \sqrt{\epsilon_{si} \cdot \text{XJ}/\text{COX}} \\
 \epsilon_{si} &= 0.104\text{E-}9 \text{ F/m} & n_i &= 1.45\text{E}16 \text{ m}^{-3} & V_t &= kT/q = 0.0259 \text{ V (at room temperature)}
 \end{aligned}$$

NOTE: Parameters in this table suppose m (meter) has been chosen as length unit.  $L_{\min}$  and  $W_{\min}$  are the minimum drawn length and width of the transistors. Example values are indicated for enhancement N-channel devices.

## 11) Model updates description

Throughout the use of the EKV v2.6 MOSFET model by many designers, several enhancements have appeared to be necessary from the model formulation point of view, or desirable from the point of view of the application of the model. This paragraph provides a summary of the updates to the EKV v2.6 model formulation and documentation since its first release. Wherever possible, backward compatibility with former formulations is maintained. The user should find most implementations in commercial simulators on the latest level of description.

### 11.1 Revision I, September 1997

*Description:* Narrow channel effect on the substrate factor is revised to improve the transcapacitances behavior. The narrow channel effect is not anymore a function of the source voltage  $V_S$ , but of the pinch-off voltage  $V_P$ .

*Consequence:* the narrow channel effect parameters WETA, DW require different numerical values to achieve the same effect.

### 11.2 Revision II, July 1998

#### 11.2.1 Intrinsic time constant

*Description:* Intrinsic time constant  $\tau_0$  is calculated as a function of the effective  $\beta$  factor (including vertical field dependent mobility and short-channel effects) instead of maximum mobility according to the KP parameter.

*Consequence:* the NQS time constant has an additional gate voltage dependence, resulting in more conservative (lower) estimation of the NQS time constant at high  $V_G$ , and additional dependence on short-channel effects.

#### 11.2.2 Thermal noise

*Description:* Thermal noise power spectral density  $S_{thermal}$  is calculated as a function of the effective  $\beta$  factor (including vertical field dependent mobility and short-channel effects) instead of maximum mobility according to the KP parameter.

*Consequence:*  $S_{thermal}$  has an additional gate voltage and short-channel effect dependence.

#### 11.2.3 Optional process parameters for calculation of electrical intrinsic parameters

*Description:* The option is introduced to calculate the electrical parameters COX, GAMMA and/or PHI, VTO, KP and UCRIT as a function of the optional parameters TOX, NSUB, VFB, UO, and VMAX, respectively. NSUB and UO have *cm* as length units.

*Consequence:* This accommodates scaling behavior and allows more meaningful statistical circuit simulation due to decorrelation of physical effects. Compatible with former revisions except for default calculation of the parameters mentioned, if the optional parameters are specified.

#### 11.2.4 Optional simplified mobility model

*Description:* The simple mobility model of former model versions, using the parameter THETA, is reinstated as an option.

*Consequence:* Simplifies adaptation from earlier model versions to the current version.

#### 11.2.5 Parameter synonyms

*Description:* The parameters E0 and Q0 can be called by their synonyms EO and QO, respectively.

*Consequence:* Accommodates certain simulators where only alphabetic characters are allowed.

#### 11.2.6 Operating point information

*Description:* The analytical expression for the 'SPICE'-like threshold voltage  $V_{TH}$  in the operating point information is modified to include charge-sharing and reverse short-channel effects. The analytical expression for the saturation voltage  $V_{DSAT}$  in the operating point information is modified such that its value is non-zero in weak inversion.

*Consequence:* Improved information for the designer.

### 11.3 Corrections, March, 1999

Eqn. 45, Eqn. 53, Eqn. 54, Eqn. 58 have been corrected for multiple series device behavior with the parameter NS.