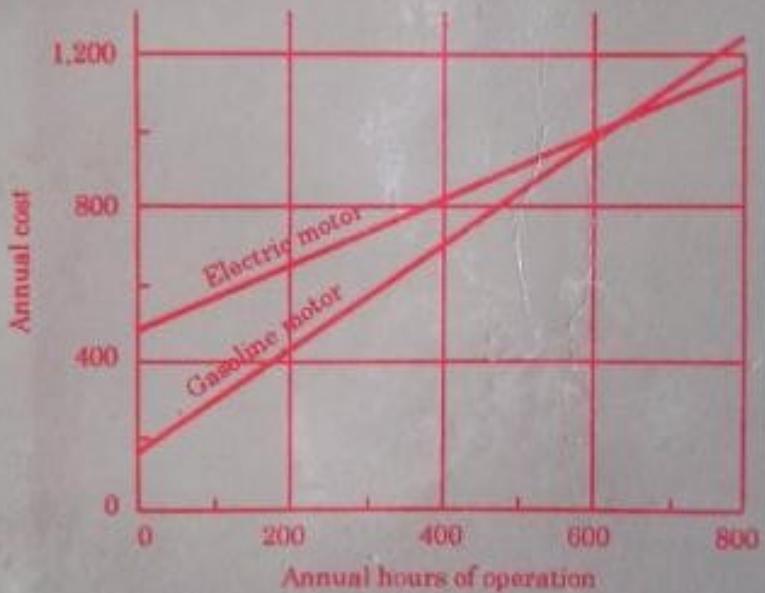


ENGINEERING ECONOMY

THIRD EDITION



HIPOLITO B. STA. MARIA

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PREFACE

Engineering Economy is one of the most important and useful subjects in the engineering curricula, so much so, that it is a subject common to all engineering disciplines. The field of engineering economy cannot be covered thoroughly in a 3-unit, one-semester course. The purpose of the book is to give the student a sound understanding of the basic concepts of the subject and some insights into approaches that can be used for making sound economic decisions. More than 90% of the young engineers, during their first five years after graduation, are required to make economy studies of proposed projects. It is the aim of this book to prepare and equip them with skills in engineering economy so as to better face the challenges of their professions. For those who will pursue further study after graduation they will find the subject of great importance.

Numerous illustrative problems, arranged in the order of increasing difficulty are presented. The best way to study and learn any technical subject is for the student to apply the principles in solving problems. To achieve this, problems with their answers are provided at the end of each chapter, except in Chapter 1.

This book is filled with problem sets mostly taken from examinations I have given in almost 40 years I have taught the subject at the Faculty of Engineering of the University of Santo Tomas.

The author experienced immense joy and satisfaction in teaching engineering economy and in writing the book and hopes that the readers would experience the same joy and satisfaction in reading and learning from it.

The Author

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Chapter 1

The Economic Environment

Engineering economy is the analysis and evaluation of the factors that will affect the economic success of engineering projects to the end that a recommendation can be made which will insure the best use of capital.

Consumer and Producer Goods and Services

Consumer goods and services are those products or services that are directly used by people to satisfy their wants.

Producer goods and services are used to produce consumer goods and services or other producer goods.

Necessities and Luxuries

Necessities are those products or services that are required to support human life and activities, that will be purchased in somewhat the same quantity even though the price varies considerably.

Luxuries are those products or services that are desired by humans and will be purchased if money is available after the required necessities have been obtained.

Demand

Demand is the quantity of a certain commodity that is bought at a certain price at a given place and time.

Elastic demand occurs when a decrease in selling price result in a greater than proportionate increase in sales.

Inelastic demand occurs when a decrease in the selling price produces a less than proportionate increase in sales.

Unitary elasticity of demand occurs when the mathematical product of volume and price is constant.

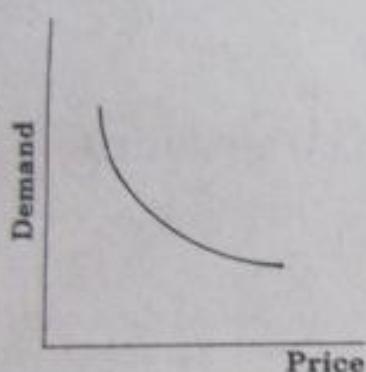


Figure 1-1. General price-demand relationship

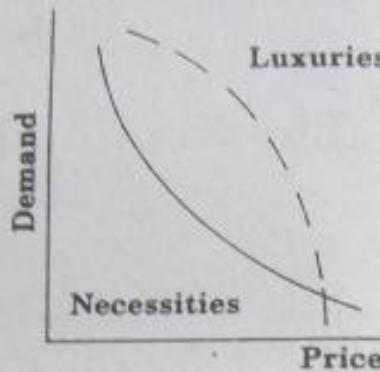


Figure 1-2. Price-demand relationship for luxuries and necessities

Competition, Monopoly and Oligopoly

Perfect competition occurs in a situation where a commodity or service is supplied by a number of vendors and there is nothing to prevent additional vendors entering the market.

Monopoly is the opposite of perfect competition. A perfect monopoly exists when a unique product or service is available from a single vendor and that vendor can prevent the entry of all others into the market.

Oligopoly exists when there are so few suppliers of a product or service that action by one will almost inevitably result in similar action by the others.

The Law of Supply and Demand

Supply is the quantity of a certain commodity that is offered for sale at a certain price at a given place and time.

The law of supply and demand may be stated as follows:

"Under conditions of perfect competition the price at which a given product will be supplied and purchased is the price that will result in the supply and the demand being equal."

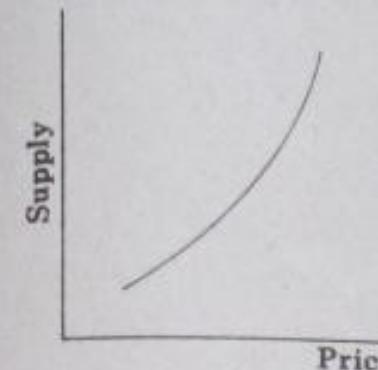


Figure 1-3. General price-supply relationship

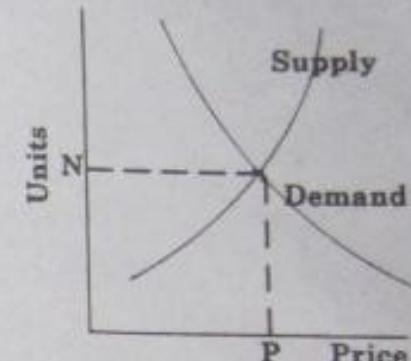


Figure 1-4. Price-supply-demand relationship.

The Law of Diminishing Returns

"When the use of one of the factors of production is limited, either in increasing cost or by absolute quantity, a point will be reached beyond which an increase in the variable factors will result in a less than proportionate increase in output."

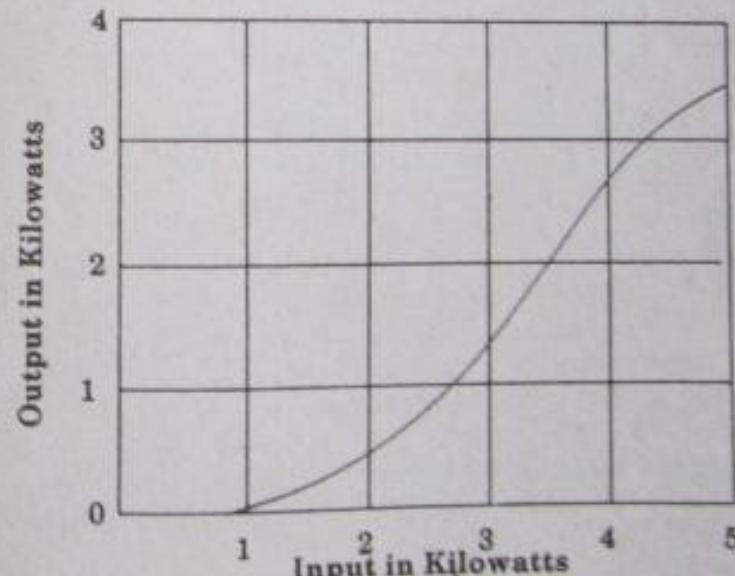


Figure 1-5. Performance curve of an electric motor.

The effect of the law of diminishing returns on the performance of an electric motor is illustrated in Fig. 1-5. For the early increase in input through input of 4.0 kw, the actual increase in output is greater than proportional; beyond this point the output is less than proportional. In this case the fixed input factor is the electric motor.

Chapter 2

Interest and Money-Time Relationships

Interest is the amount of money paid for the use of borrowed capital or the income produced by money which has been loaned.

Simple Interest

Simple interest is calculated using the principal only, ignoring any interest that had been accrued in preceding periods. In practice, simple interest is paid on short-term loans in which the time of the loan is measured in days.

$$I = Pni \quad (2-1)$$

$$F = P + I = P + Pni$$

$$F = P(1 + ni) \quad (2-2)$$

where: I = interest

P = principal or present worth

n = number of interest periods

i = rate of interest per interest period

F = accumulated amount or future worth

(a) Ordinary simple interest is computed on the basis of 12 months of 30 days each or 360 days a year.

1 interest period = 360 days

(b) Exact simple interest is based on the exact number of days in a year, 365 days for an ordinary year and 366 days for a leap year.

$$1 \text{ interest period} = 365 \text{ or } 366 \text{ days}$$

(2-1) Determine the ordinary simple interest on P700 for 8 months and 15 days if the rate of interest is 15%.

Solution

$$\text{Number of days} = (8)(30) + 15 = 255 \text{ days}$$

$$I = Pni = P700 \times \frac{255}{360} \times 0.15 = P74.38$$

(2-2) Determine the exact simple interest on P500 for the period from January 10 to October 28, 1996 at 16% interest

Solution

| | | |
|------------|---|------------------------------------|
| Jan. 10-31 | = | 21 (excluding Jan. 10) |
| February | = | 29 |
| March | = | 31 |
| April | = | 30 |
| May | = | 31 |
| June | = | 30 |
| July | = | 31 |
| August | = | 31 |
| September | = | 30 |
| October | = | 28 (including Oct. 28) 292 days |

$$\text{Exact simple interest} = P500 \times \frac{292}{366} \times 0.16 = P63.83$$

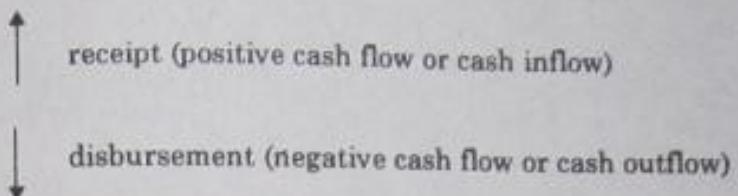
(2-3) What will be the future worth of money after 14 months, if a sum of P10,000 is invested today at a simple interest rate of 12% per year?

Solution

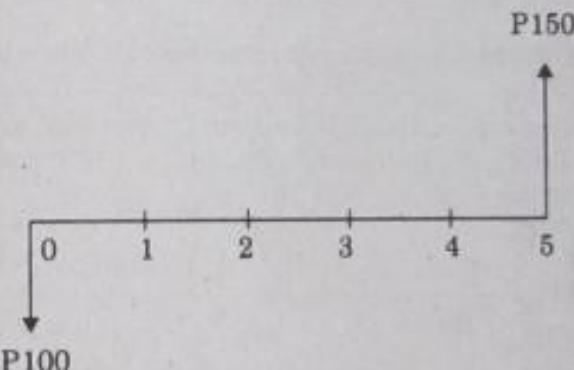
$$F = P(1 + ni) = P10,000[1 + (\frac{14}{12})(0.12)] = P11,400$$

Cash-Flow Diagrams

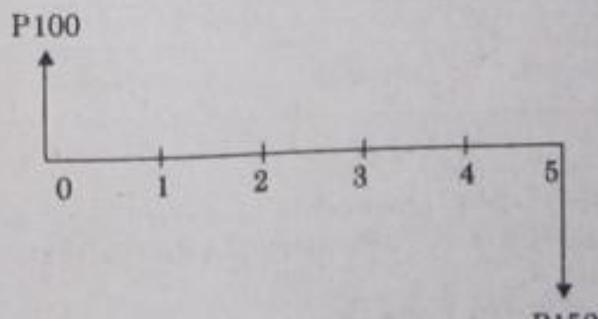
A cash-flow diagram is simply a graphical representation of cash flows drawn on a time scale. Cash-flow diagram for economic analysis problems is analogous to that of free body diagram for mechanics problems.



A loan of P100 at simple interest of 10% will become P150 after 5 years.



Cash flow diagram on the viewpoint of the lender



Cash flow diagram on the viewpoint of the borrower

Compound Interest

In calculations of compound interest, the interest for an interest period is calculated on the principal plus total amount of interest accumulated in previous periods. Thus compound interest means "interest on top of interest."

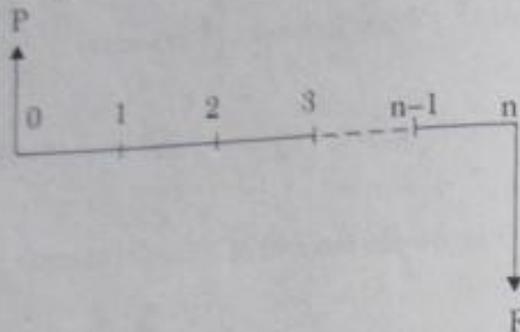


Figure 2-1. Compound Interest (Borrower's Viewpoint)

| Interest Period | Principal at Beginning of Period | Interest Earned During Period | Amount at End of Period |
|-----------------|----------------------------------|-------------------------------|-----------------------------------|
| 1 | P | P_i | $P + P_i = P(1+i)$ |
| 2 | $P(1+i)$ | $P(1+i)i$ | $P(1+i) + P(1+i)i = P(1+i)^2$ |
| 3 | $P(1+i)^2$ | $P(1+i)^2i$ | $P(1+i)^2 + P(1+i)^2i = P(1+i)^3$ |
| ... | ... | ... | ... |
| n | $P(1+i)^{n-1}$ | $P(1+i)^{n-1}i$ | $P(1+i)^n$ |

$$F = P(1+i)^n \quad (2-3)$$

The quantity $(1+i)^n$ is commonly called the "single payment compound amount factor" and is designated by the functional symbol $F/P, i\%, n$. Thus,

$$F = P(F/P, i\%, n) \quad (2-4)$$

The symbol $F/P, i\%, n$ is read as "F given P at i per cent in n interest periods." From Equation (2-3),

$$P = F(1+i)^{-n} \quad (2-5)$$

The quantity $(1+i)^{-n}$ is called the "single payment present worth factor" and is designated by the functional symbol $P/F, i\%, n$. Thus,

$$P = F(P/F, i\%, n) \quad (2-6)$$

The symbol $P/F, i\%, n$ is read as "P given F at i per cent in n interest periods."

Rates of Interest

(a) Nominal rate of interest

The nominal rate of interest specifies the rate of interest and a number of interest periods in one year.

$$i = \frac{r}{m} \quad (2-7)$$

where: i = rate of interest per interest period

r = nominal interest rate

m = number of compounding periods per year

If the nominal rate of interest is 10% compounded quarterly, then $i = 10\%/4 = 2.5\%$, the rate of interest per interest period

(b) Effective rate of interest

Effective rate of interest is the actual or exact rate of interest on the principal during one year. If P1.00 is invested at a nominal rate of 15% compounded quarterly, after one year this will become,

$$P1 \left(1 + \frac{0.15}{4} \right)^4 = P1.1586$$

The actual interest earned is P0.1586, therefore, the rate of interest after one year is 15.86%. Hence,

$$\text{Effective rate} = F_1 - 1 = (1+i)^m - 1 \quad (2-8)$$

where: F_1 = the amount P1.00 will be after one year

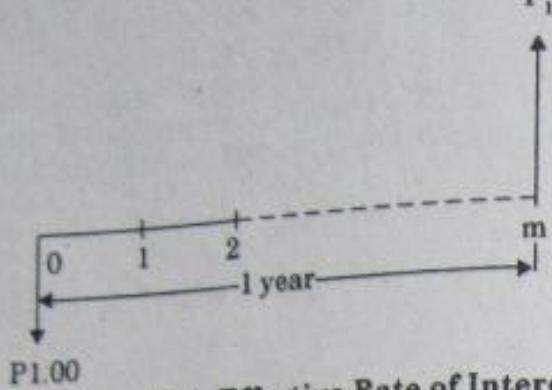


Figure 2-2. Effective Rate of Interest

(2-3) Find the nominal rate which if converted quarterly could be used instead of 12% compounded monthly. What is the corresponding effective rate?

Solution

Let r = the unknown nominal rate

For two or more nominal rates to be equivalent, their corresponding effective rates must be equal.

Nominal rate

Effective rate

$r\%$ compounded quarterly

$$\left(1 + \frac{r}{4}\right)^4 - 1$$

12% compounded monthly

$$\left(1 + \frac{0.12}{12}\right)^{12} - 1$$

$$\left(1 + \frac{r}{4}\right)^4 - 1 = 1(1 + 0.01)^{12} - 1$$

$$1 + \frac{r}{4} = (1.01)^3 = 1.0303$$

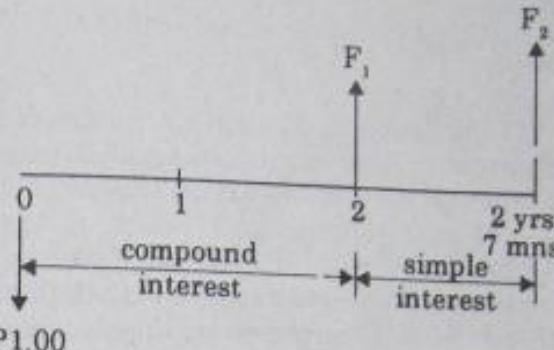
$$r = 0.1212 \text{ or } 12.12\% \text{ compounded quarterly}$$

(2-4) Find the amount at the end of two years and seven months if P1000 is invested at 8% compounded quarterly using simple interest for anytime less than a year interest period.

Solution

$$\text{For compound interest: } i = \frac{8\%}{4} = 2\%, n = (2)(4) = 8$$

$$\text{For simple interest: } i = 8\%, n = \frac{7}{12}$$

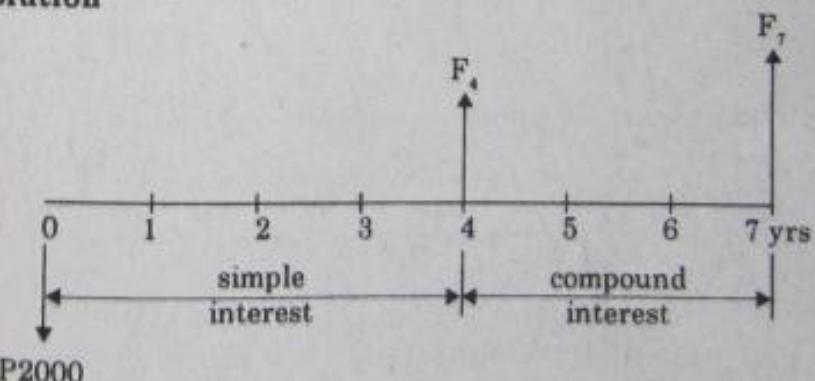


$$F_1 = P(1 + i)^n = P1000(1 + 0.02)^8 = P1171.66$$

$$F_2 = F_1(1 + ni) = P1171.66 \left[1 + \frac{7}{12}(0.08)\right] = P1,226.34$$

(2-5) A P2000 loan was originally made at 8% simple interest for 4 years. At the end of this period the loan was extended for 3 years, without the interest being paid, but the new interest rate was made 10% compounded semiannually. How much should the borrower pay at the end of 7 years?

Solution



Solution

$$F_4 = P(1 + m) = P2,000 [1 + (4)(0.08)] = P2,640$$

$$F_7 = F_4(1 + i)^n = P2,640 (1 + 0.05)^6 = P3,537.86$$

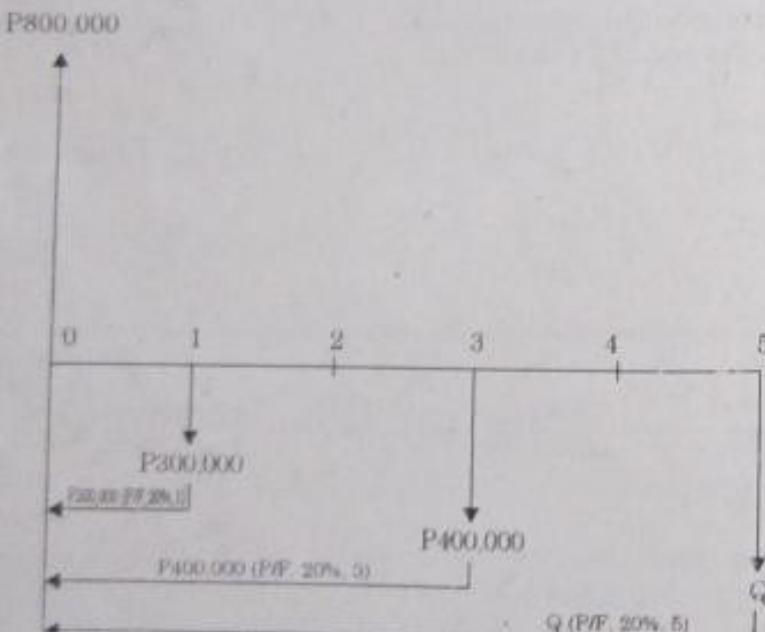
Equation of Value

An equation of value is obtained by setting the sum of the values on a certain comparison or focal date of one set of obligations equal to the sum of the values on the same date of another set of obligations.

(2-6) A man bought a lot worth P1,000,000 if paid in cash. On the installment basis, he paid a down payment of P200,000; P300,000 at the end of one year; P400,000 at the end of three years and a final payment at the end of five years. What was the final payment if interest was 20%?

Solution

Let Q = the final payment



Using today as the focal date, the equation of value is

$$P800,000 = P300,000 (P/F, 20\%, 1) + P400,000 (P/F, 20\%, 3) + Q(P/F, 20\%, 5)$$

$$P800,000 = P300,000 (1.20)^{-1} + P400,000 (1.20)^{-3} + Q(1.20)^{-5}$$

$$P800,000 = P300,000 (0.8333) + P400,000 (0.5787) + Q(0.4019)$$

$$Q = P792,560$$

Continuous Compounding and Discrete Payments

In discrete compounding, the interest is compounded at the end of each finite – length period, such as a month, a quarter or a year.

In continuous compounding, it is assumed that cash payments occur once per year, but the compounding is continuous throughout the year.

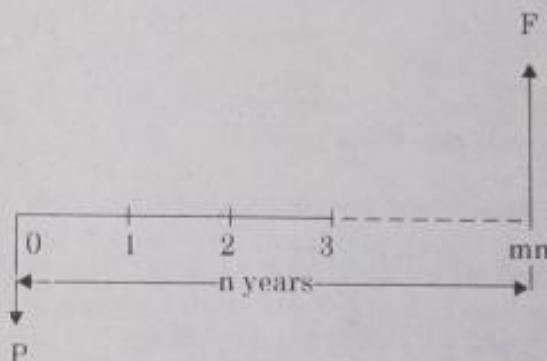


Figure 2-3. Continuous Compounding (Lender's Viewpoint)

r = nominal rate of interest per year

$\frac{r}{m}$ = rate of interest per period

m = number of interest periods per year

mn = number of interest periods in n years

$$F = P \left(1 + \frac{r}{m}\right)^{mn} \quad (2-9)$$

Let $\frac{m}{r} = k$, then $m = rk$, as m increases so must k

$$\left(1 + \frac{r}{m}\right)^{mn} = \left(1 + \frac{1}{k}\right)^{rkn} = \left[\left(1 + \frac{1}{k}\right)^k\right]^m$$

The limit of $(1 + \frac{r}{m})^k$ as k approaches infinite is e

$$\left[\left(1 + \frac{1}{k}\right)^k\right]^m = e^m$$

$$\text{Thus, } F = Pe^m \quad (2-10)$$

$$P = Fe^{-m} \quad (2-11)$$

(2-7) Compare the accumulated amounts after 5 years of P1,000 invested at the rate of 10% per year compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, (e) daily, and (f) continuously.

Solution

Using the formula, $F = P(1 + i)^n$

$$(a) F = P1,000(1 + 0.10)^5 = P1,610.51$$

$$(b) F = P1,000\left(1 + \frac{0.10}{2}\right)^{10} = P1,628.89$$

$$(c) F = P1,000\left(1 + \frac{0.10}{4}\right)^{20} = P1,638.62$$

$$(d) F = P1,000\left(1 + \frac{0.10}{12}\right)^{60} = P1,645.31$$

$$(e) F = P1,000\left(1 + \frac{0.10}{365}\right)^{1825} = P1,648.61$$

$$(f) F = Pe^m = P1,000(e)^{0.10 \times 5} = P1,648.72$$

Discount

Discount on a negotiable paper is the difference between the present worth (the amount received for the paper in cash) and the worth of the paper at some time in the future (the face value of the paper or principal). Discount is interest paid in advance.

Discount = Future Worth - Present Worth

The rate of discount is the discount on one unit of principal for one unit of time.

$$(1 + i)^{-1}$$

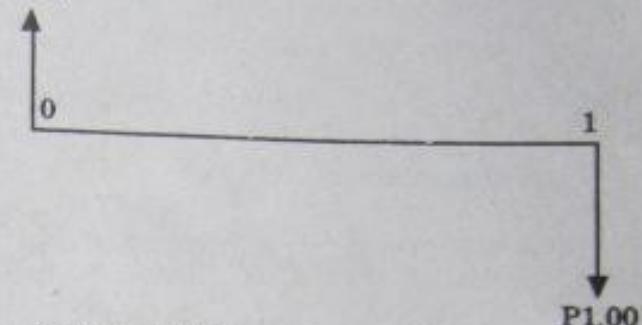


Figure 2-4 Rate of Discount

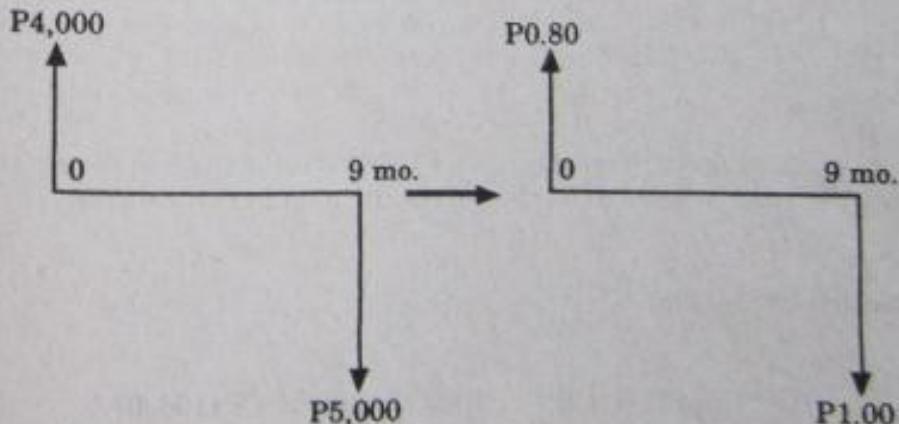
$$d = 1 - (1 + i)^{-1} \quad (2-12)$$

$$i = \frac{d}{1-d} \quad (2-13)$$

where: d = rate of discount for the period involved
 i = rate of interest for the same period

(2-8) A man borrowed P5,000 from a bank and agreed to pay the loan at the end of 9 months. The bank discounted the loan and gave him P4,000 in cash. (a) What was the rate of discount? (b) What was the rate of interest? (c) What was the rate of interest for one year?

Solution



$$(a) d = \frac{\text{discount}}{\text{principal}} = \frac{P1,000}{P5,000} = 0.20 \text{ or } 20\%$$

Another solution, using equation (2-12),

$$d = 1 - 0.80 = 0.20 \text{ or } 20\%$$

$$(b) i = \frac{d}{1-d} = \frac{0.20}{1-0.20} = 0.25 \text{ or } 25\%$$

Another solution,

$$i = \frac{\text{interest}}{\text{present worth}} = \frac{P1,000}{P4,000} = 0.25 \text{ or } 25\%$$

$$(c) i = \frac{I}{Pn} = \frac{P1,000}{(P4,000) \left(\frac{9}{12}\right)} = 0.3333 \text{ or } 33.33\%$$

Inflation

Inflation is the increase in the prices for goods and services from one year to another, thus decreasing the purchasing power of money.

$$FC = PC (1 + f)^n \quad (2-14)$$

where PC = present cost of a commodity

FC = future cost of the same commodity

f = annual inflation rate

n = number of years

(2-9) An item presently costs P1000. If inflation is at the rate of 8% per year, what will be the cost of the item in two years?

Solution

$$FC = PC (1 + f)^n = P1000 (1 + 0.08)^2 = P1166.40$$

In an inflationary economy, the buying power of money decreases as costs increase. Thus,

$$F = \frac{P}{(1 + f)^n} \quad (2-15)$$

where F is the future worth, measured in today's pesos, of a present amount P .

(2-10) An economy is experiencing inflation at an annual rate of 8%. If this continues, what will P1000 be worth two years from now, in terms of today's pesos?

Solution

$$F = \frac{P}{(1 + f)^n} = \frac{P1000}{(1 + 0.08)^2} = P857.34$$

If interest is being compounded at the same time that inflation is occurring, the future worth will be

$$F = \frac{P (1 + i)^n}{(1 + f)^n} = P \left(\frac{1+i}{1+f} \right)^n \quad (2-16)$$

(2-11) A man invested P10,000 at an interest rate of 10% compounded annually. What will be the final amount of his investment, in terms of today's pesos, after five years, if inflation remains the same at the rate of 8% per year?

Solution

$$F = P \left(\frac{1+i}{1+f} \right)^n = P10,000 \left(\frac{1+0.10}{1+0.08} \right)^5 = P10,960.86$$

ANNUITIES

An annuity is a series of equal payments occurring at equal periods of time.

Symbols and Their Meaning

- P = value or sum of money at present
- F = value or sum of money at some future time
- A = a series of periodic, equal amounts of money
- n = number of interest periods
- i = interest rate per interest period

Ordinary Annuity

An ordinary annuity is one where the payments are made at the end of each period.

Finding P when A is Given

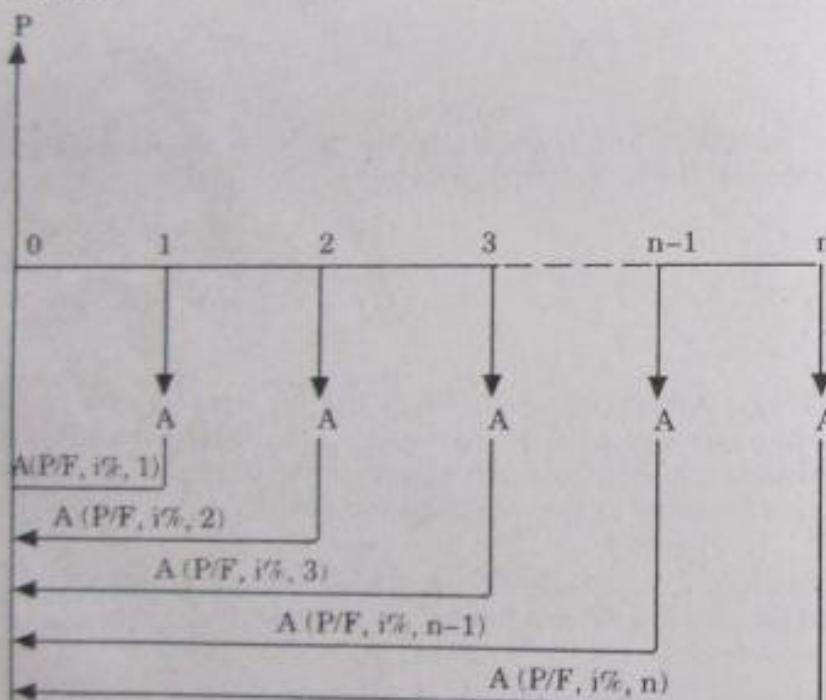


Figure 2-5. Cash flow diagram to find P given A

$$P = A(P/F, i\%, 1) + A(P/F, i\%, 2) + A(P/F, i\%, 3) + \dots + A(P/F, i\%, n-1) + A(P/F, i\%, n)$$

$$P = A(1+i)^{-1} + A(1+i)^{-2} + A(1+i)^{-3} + \dots + A(1+i)^{-(n-1)} + A(1+i)^{-n}$$

Multiplying this equation by $(1+i)$ results in

$$P + Pi = A + A(1+i)^{-1} + A(1+i)^{-2} + \dots + A(1+i)^{-(n-1)} + A(1+i)^{-n}$$

Subtracting the first equation from the second gives

$$\begin{aligned} P + Pi &= A + \cancel{A(1+i)^{-1}} + \cancel{A(1+i)^{-2}} + \dots + \cancel{A(1+i)^{-(n-1)}} \\ -P &= -A \cancel{(1+i)^{-1}} - A \cancel{(1+i)^{-2}} + \dots + A \cancel{(1+i)^{-(n-1)}} - A(1+i)^{-n} \\ Pi &= A \end{aligned}$$

Solving for P gives

$$P = A \left[\frac{1 - (1+i)^{-n}}{i} \right] = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \quad (2-17)$$

The quantity in brackets is called the "uniform series present worth factor" and is designated by the functional symbol $P/A, i\%, n$, read as "P given A at i per cent in n interest periods." Hence Equation (2-17) can be expressed as

$$P = A(P/A, i\%, n) \quad (2-18)$$

Finding F When A is Given

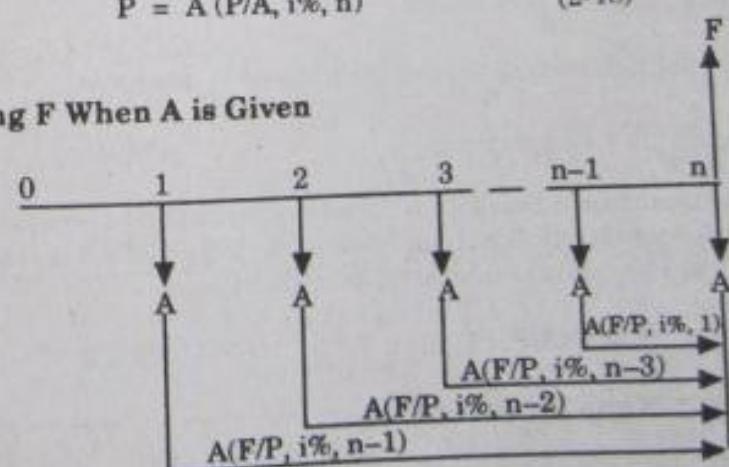


Figure 2-6. Cash flow diagram to find F given A

$$F = A + A(F/A, i\%, 1) + \dots + A(F/P, i\%, n-3) + A(F/P, i\%, n-2) \\ + A(F/P, i\%, n-1)$$

$$F = A + A(1+i) + \dots + A(1+i)^{n-3} + A(1+i)^{n-2} + A(1+i)^{n-1}$$

Multiplying this equation by $(1+i)$ results in

$$F + Fi = A(1+i) + A(1+i)^2 + \dots + A(1+i)^{n-2} + A(1+i)^{n-1} + A(1+i)^n$$

Subtracting the first equation from the second gives

$$\begin{aligned} F + Fi &= \cancel{A(1+i)} + \dots + \cancel{A(1+i)^{n-1}} + A(1+i)^n + A(1+i)^n, \\ -F &= -A - \cancel{A(1+i)} + \dots \cancel{A(1+i)^{n-2}} - \cancel{A(1+i)^{n-1}} \\ \hline Fi &= -A + A(1+i)^n \end{aligned}$$

Solving for F gives

$$F = A \left[\frac{(1+i)^n - 1}{i} \right] \quad (2-19)$$

The quantity in brackets is called the "uniform series compound amount factor" and is designated by the functional symbol $F/A, i\%, n$, read as "F given A at i per cent in n interest periods." Equation (2-16) can now be written as

$$F = A(F/A, i\%, n) \quad (2-20)$$

Finding A When P is Given

Taking Equation (2-14) and solving for A, we have

$$A = P \left[\frac{i}{1 - (1+i)^{-n}} \right] \quad (2-21)$$

The quantity in brackets is called the "capital recovery factor." It will be denoted by the functional symbol $A/P, i\%, n$ which is read as "A given P at i per cent in n interest periods." Hence

$$A = P(A/P, i\%, n) \quad (2-22)$$

Finding A When F is Given

Taking Equation (2-19) and solving for A, we have

$$A = F \left[\frac{i}{(1+i)^n - 1} \right] \quad (2-23)$$

The quantity in brackets is called the "sinking fund factor." It will be denoted by the functional symbol $A/F, i\%, n$ which is read as "A given F at i per cent in n interest periods." Hence

$$A = F(A/F, i\%, n) \quad (2-24)$$

Relation between $A/P, i\%, n$ and $A/F, i\%, n$

$$\frac{i}{(1+i)^n - 1} + i = \frac{i + i(1+i)^n - i}{i(1+i)^n - 1} = \frac{(1+i)^n}{(1+i)^n - 1}$$

$$\frac{i}{(1+i)^n - 1} + i = \frac{i}{1 - (1+i)^{-n}}$$

$$A/F, i\%, n + i = A/P, i\%, n \quad (2-25)$$

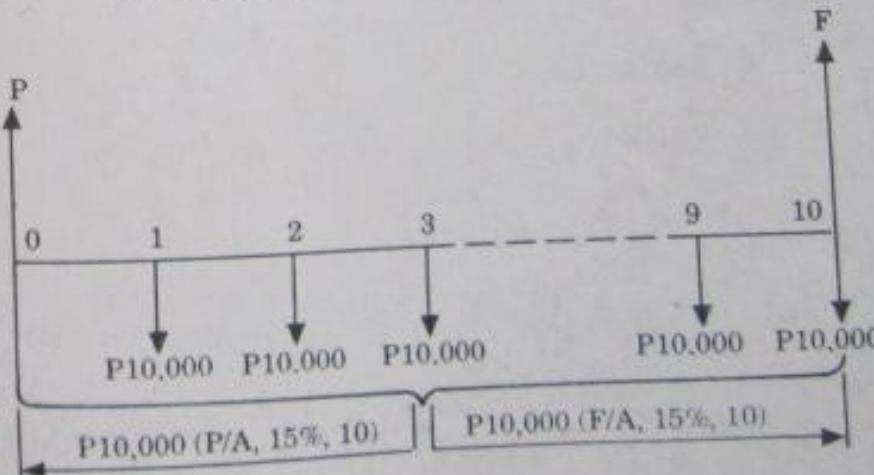
Thus,

sinking fund factor + i = capital recovery factor

(2-12) What are the present worth and the accumulated amount of a 10-year annuity paying P10,000 at the end of each year, with interest at 15% compounded annually?

Solution

$$A = P10,000 \quad n = 10 \quad i = 15\%$$



$$P = A(P/A, i\%, n) = P10,000(P/A, 15\%, 10)$$

$$= P10,000 \left[\frac{1 - (1.15)^{-10}}{0.15} \right] = P50,188$$

$$F = A(F/A, i\%, n) = P10,000(F/P, 15\%, 10)$$

$$= P10,000 \left[\frac{(1.15)^{10} - 1}{0.15} \right] = P203,037$$

(2-13) What is the present worth of P500 deposited at the end of every three months for 6 years if the interest rate is 12% compounded semiannually?

Solution

Solving for the interest rate per quarter,

$$(1 + i)^4 - 1 = \left(1 + \frac{0.12}{2}\right)^2 - 1$$

$$1 + i = (1.06)^{0.5}$$

$$i = 0.0296 \text{ or } 2.96\% \text{ per quarter}$$

$$P = A(P/A, 2.96\%, 24)$$

$$= P500 \left[\frac{1 - (1 + 0.0296)^{-24}}{0.0296} \right]$$

$$= P500(17.0087)$$

$$= P8,504$$

(2-14) A businessman needs P50,000 for his operations. One financial institution is willing to lend him the money for one year at 12.5% interest per annum (discounted). Another lender is charging 14%, with the principal and interest payable at the end of one year. A third financier is willing to lend him P50,000 payable in 12 equal monthly installments of P4,600. Which offer is best for him?

Solution

Compare the effective rate of each offer and select the one with the lowest effective rate.

First Offer:

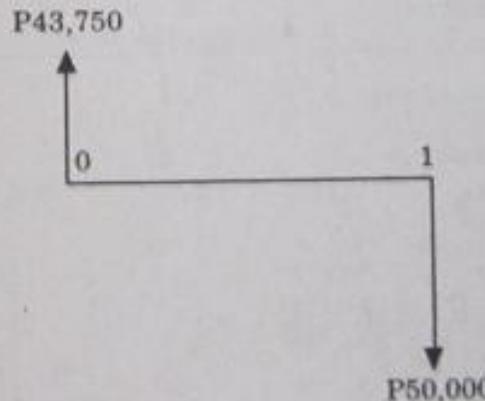
Rate of discount, $d = 12.5\%$

$$\text{Rate of interest, } i = \frac{d}{1 - d} = \frac{0.125}{1 - 0.125} = 14.29\% \text{ per year}$$

Effective rate = 14.29%

Another solution:

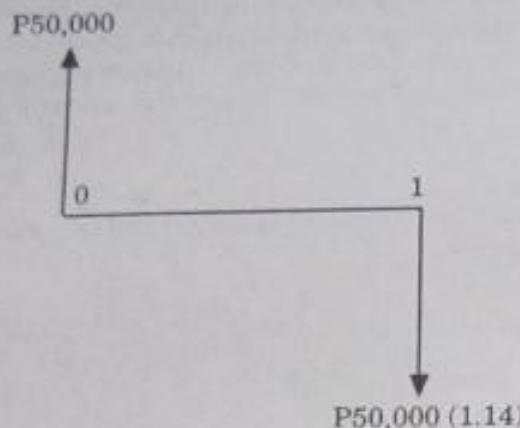
$$\text{Amount received} = P50,000(0.875) = P43,750$$



$$\text{Rate of interest} = \frac{P50,000 - P43,750}{P43,750} = 14.29\% \text{ per year}$$

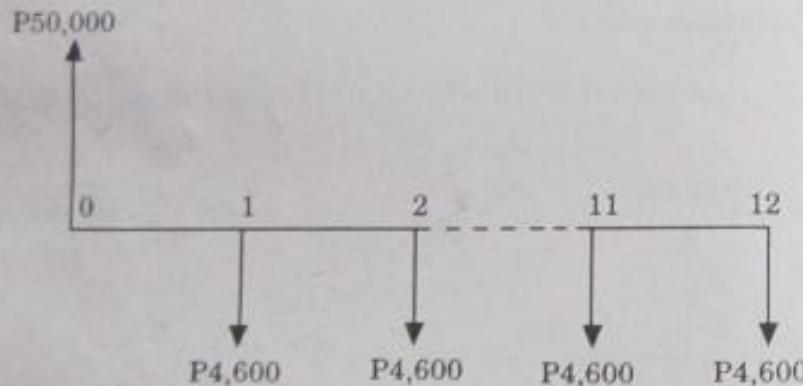
$$\text{Effective rate} = 14.29\%$$

Second Offer:



$$\text{Effective rate} = 14\%$$

Third Offer:



$$P/A, i\%, 12 = \frac{P}{A} = \frac{P50,000}{P4,600}$$

$$\frac{1 - (1+i)^{-12}}{i} = 10.8696$$

$$\text{Try } i = 1\%$$

$$\frac{1 - (1+0.01)^{-12}}{0.01} = 11.2551$$

Try $i = 2\%$

$$\frac{1 - (1+0.02)^{-12}}{0.02} = 10.5753$$

| | | | | |
|----|-----------------|---------|--------|--------|
| 1% | $\frac{x}{1\%}$ | 11.2551 | 0.3855 | 0.6798 |
| i% | | 10.8696 | | |
| 2% | | 10.5753 | | |

$$\frac{x}{1\%} = \frac{0.3855}{0.6798}$$

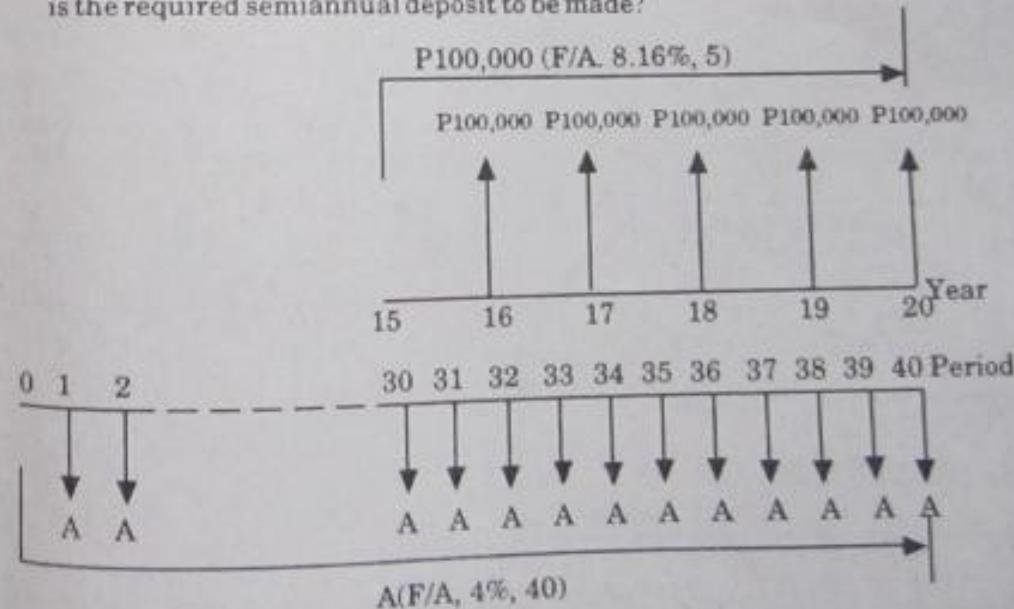
$$x = 0.57$$

$$i = 1\% + 0.57\% = 1.57\% \text{ per month}$$

$$\text{Effective rate} = (1 + 0.0157)^{12} - 1 = 20.26\%$$

The second offer is the best

(2-15) A chemical engineer wishes to set up a special fund by making uniform semiannual end-of-period deposits for 20 years. The fund is to provide P100,000 at the end of each of the last five years of the 20-year period. If interest is 8% compounded semiannually, what is the required semiannual deposit to be made?



For the deposits, $i = \frac{8\%}{2} = 4\%$

For the withdrawals, $i = (1 + 0.04)^t - 1 = 0.0816$ or 8.16%

Using 20 years from today as the focal date, the equation of value is

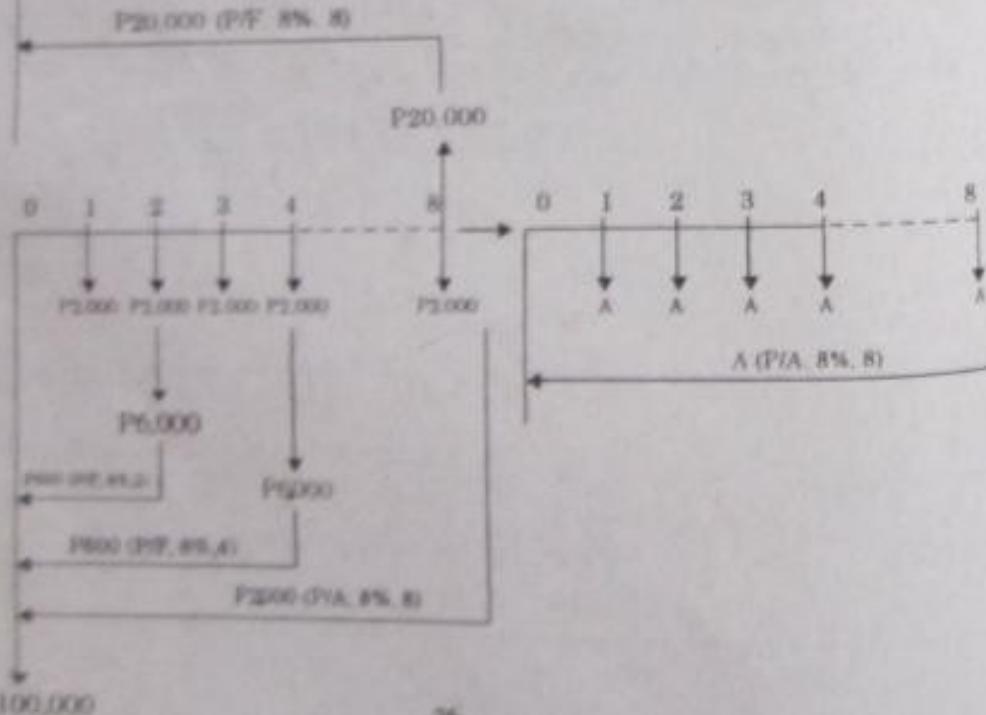
$$A(F/A, 4\%, 40) = P100,000(F/A, 8.16\%, 5)$$

$$A(95.0255) = P100,000(5.8853)$$

$$A = P6,193.39$$

(2-16) Using a compound interest of 8%, find the equivalent uniform annual cost for a proposed machine that has a first cost of P100,000 an estimated salvage value of P20,000 and an estimated economic life of 8 years. Annual maintenance will amount to P2,000 a year and periodic overhaul costing P6,000 each will occur at the end of the second and fourth year.

Solution

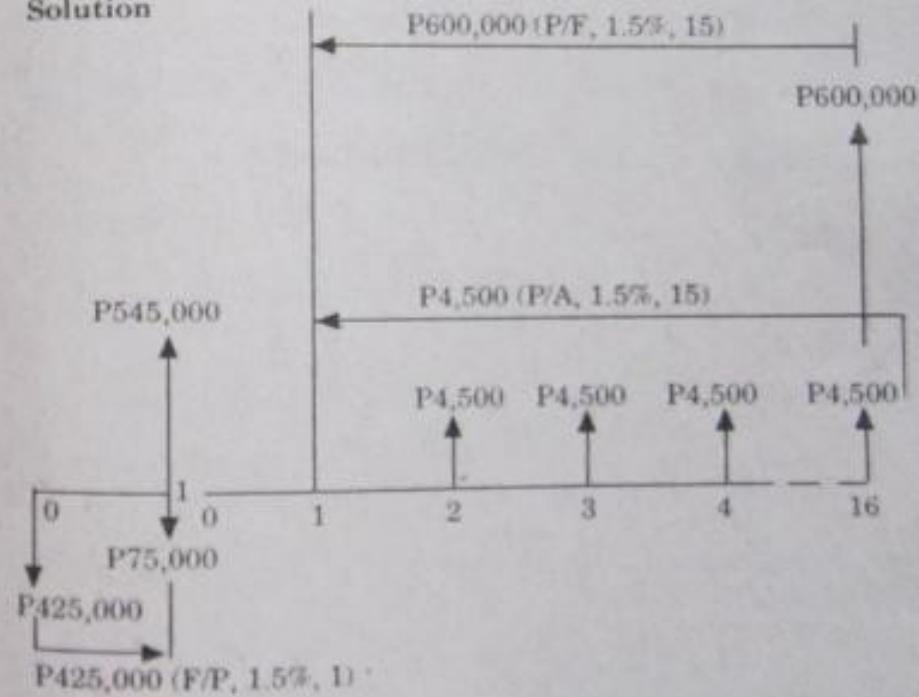


Let A = the equivalent uniform annual cost
Using today as the focal date, the equation of value is

$$\begin{aligned} A(P/A, 8\%, 8) &= P100,000 + P2000(P/A, 8\%, 8) + P6000(P/F, 8\%, 2) \\ &\quad + P6000(P/F, 8\%, 4) - P20,000(P/F, 8\%, 8) \\ A(5.7466) &= P100,000 + P2000(5.7466) + P6000(0.8573) \\ &\quad + P6000(0.7350) - P20,000(0.5403) \\ A &= P19,183 \end{aligned}$$

(2-17) A man purchased a house for P425,000. In the first month that he owned the house, he spent P75,000 on repairs and remodelling. Immediately after the house was remodeled, he was offered P545,000 to sell the house. After some consideration, he decided to keep the house and have it rented for P4,500 per month starting two months after the purchase. He collected rent for 15 months and then sold the house for P600,000. If the interest rate was 1.5% per month, how much extra money did he make or lose by not selling the house immediately after it was remodeled?

Solution



The only focal date that can be used is one month after purchase.
 Profit if the house was sold immediately after remodelling
 $(1 \text{ month after purchase}) = P545,000 - P75,000 - P425,000 (F/P, 1.5\%, 1) = P38,625$

Profit if the house was sold later (16 months after purchase)

$$= P600,000 (P/F, 1.5\%, 15) + P4,500 (P/A, 1.5\%, 15) - P75,000 - P425,000 (F/P, 1.5\%, 1)$$

$$= P600,000 (0.7999) + P4,500 (13.3432) - P75,000 - P425,000 (1.015)$$

$$= P33,610$$

Loss by not selling immediately after remodelling $= P38,625 - P33,610$
 $= P5,015$

Another solution:

Loss by not selling immediately after remodelling

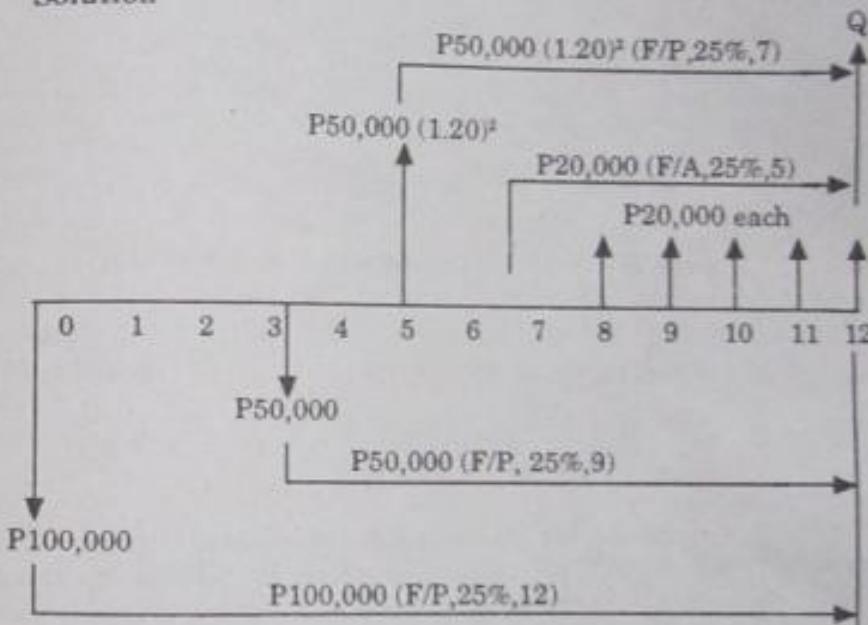
$$= P545,000 - P4,500 (P/A, 1.5\%, 15) - P600,000 (P/F, 1.5\%, 15)$$

$$= P545,000 - P4,500 (13.3432) - P600,000 (0.7999)$$

$$= P5,015$$

(2-18) Today, you invest P100,000 into a fund that pays 25% interest compounded annually. Three years later, you borrow P50,000 from a bank at 20% annual interest and invest in the fund. Two years later, you withdraw enough money from the fund to repay the bank loan and all interest due on it. Three years from this withdrawal you start taking P20,000 per year out of the fund. After five withdrawals, you withdraw the balance in the fund. How much was withdrawn?

Solution



Let Q = the amount withdrawn after 12 years
 Using 12 years from today as the focal date, the equation of value is

$$Q + P20,000(F/A, 25\%, 5) + P50,000(1.20)^2(F/P, 25\%, 7) = P100,000(F/P, 25\%, 12) + P50,000(F/P, 25\%, 9)$$

$$Q + P20,000(8.2070) + P50,000(1.20)^2(4.7684) = P100,000(14.5519) + P50,000(7.4506)$$

$$Q = P1,320,255$$

Deferred Annuity

A deferred annuity is one where the first payment is made several periods after the beginning of the annuity.

Finding P when A is Given

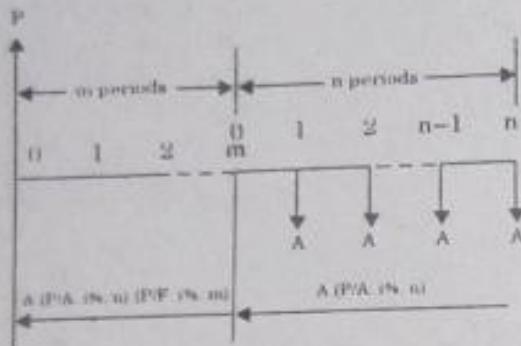


Figure 2-7. Cash flow diagram given A to find P

$$P = A (P/A, i\%, n) (P/F, i\%, m) \quad (2-26)$$

$$P = A \left[\frac{1 - (1+i)^{-n}}{i} \right] (1+i)^{-m} \quad (2-27)$$

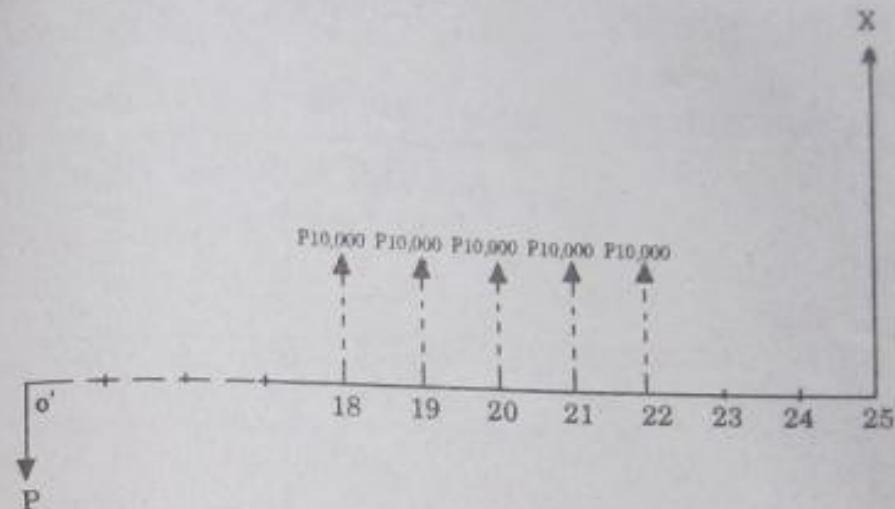
(2-19) On the day his grandson was born, a man deposited to a trust company a sufficient amount of money so that the boy could receive five annual payments of P10,000 each for his college tuition fees, starting with his 18th birthday. Interest at the rate of 12% per annum was to be paid on all amounts on deposit. There was also a provision that the grandson could elect to withdraw no annual payments and receive a single lump amount on his 25th birthday. The grandson chose this option.

- (a) How much did the boy receive as the single payment?
- (b) How much did the grandfather deposit?

Solution

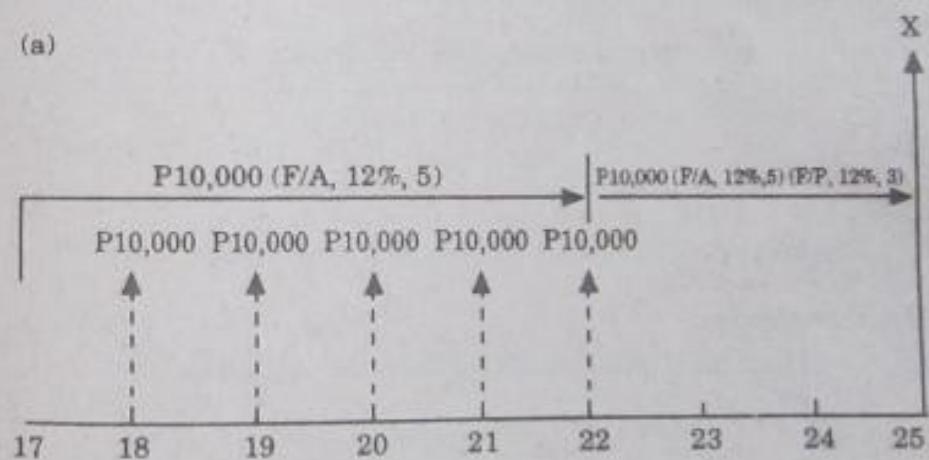
Let P = the amount deposited

X = the amount withdrawn



The P10,000 suppose withdrawals are represented by broken lines, since they did not actually occur. Three separate cash flow diagrams can be drawn.

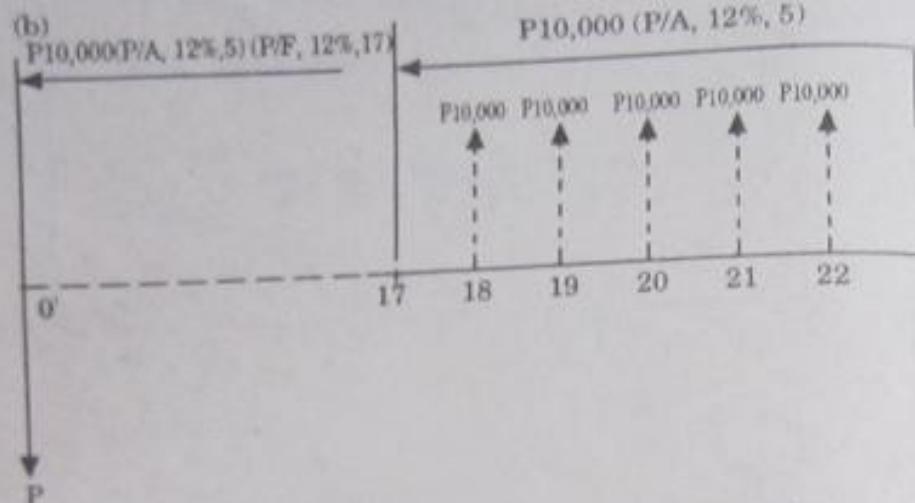
(a)



X and the P10,000 suppose withdrawals are equivalent. Using 25 years of age as the focal date, the equation of value is

$$\begin{aligned} X &= P10,000 (F/A, 12\%, 5) (F/P, 12\%, 3) \\ &= P10,000 (6.3528) (1.4049) \\ &= P89,250 \end{aligned}$$

The other good focal dates are 17 and 22 years from today.



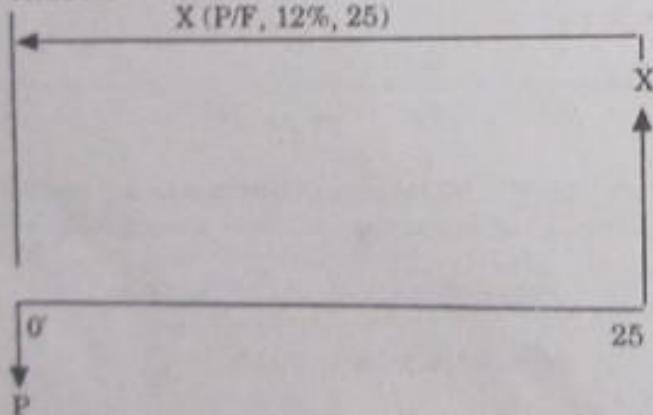
P and the P10,000 suppose withdrawals are equivalent.

Using today as the focal date, the equation of value is

$$\begin{aligned} P &= P10,000 (P/A, 12\%, 5) (P/F, 12\%, 17) \\ &= P10,000 (3.6047) (0.14565) \\ &= P5,250 \end{aligned}$$

The other good focal dates are 17 and 22 years from today.

Another solution:



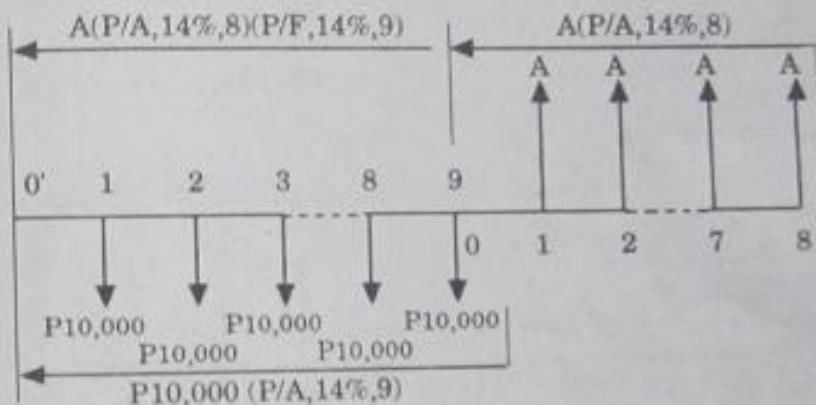
This was what actually happened. P was deposited today and X was withdrawn 25 years later.

Using today as the focal date, the equation of value is

$$\begin{aligned} P &= X (P/F, 12\%, 25) \\ &= P89,250 (0.058821) \\ &= P5,250 \end{aligned}$$

(2-20) If P10,000 is deposited each year for 9 years, how much annuity can a person get annually from the bank every year for 8 years starting 1 year after the 9th deposit is made. Cost of money is 14%.

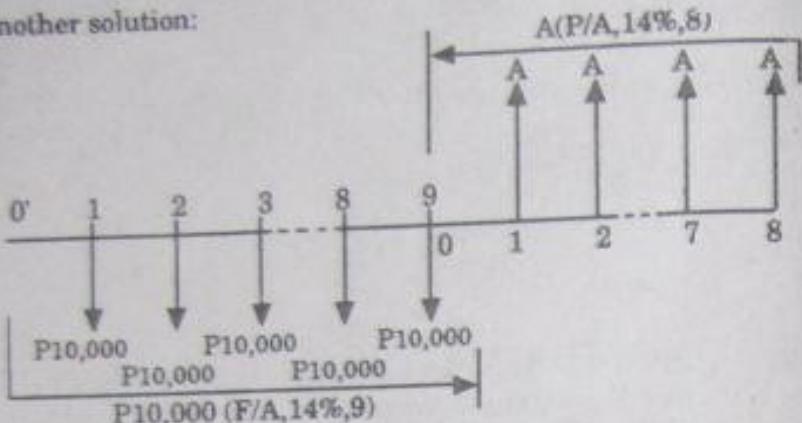
Solution



Using today as the focal date, the equation of value is

$$\begin{aligned} A(P/A, 14\%, 8)(P/F, 14\%, 9) &= P10,000 (P/A, 14\%, 9) \\ A(4.63886)(0.30751) &= P10,000 (4.94637) \\ A &= P34,675 \end{aligned}$$

Another solution:



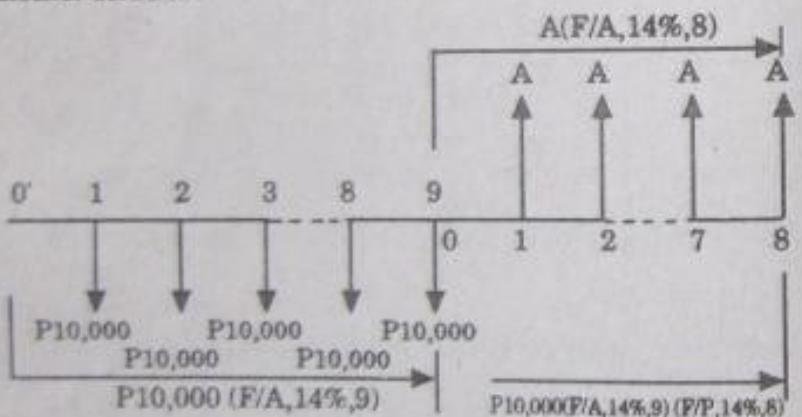
Using 9 years from today as the focal date, the equation of value is

$$A(P/A, 14\%, 8) = P10,000(F/A, 14\%, 9)$$

$$A(4.63886) = P10,000(16.08535)$$

$$A = P34,675$$

Another solution:



Using 17 years from today as the focal date, the equation of value is

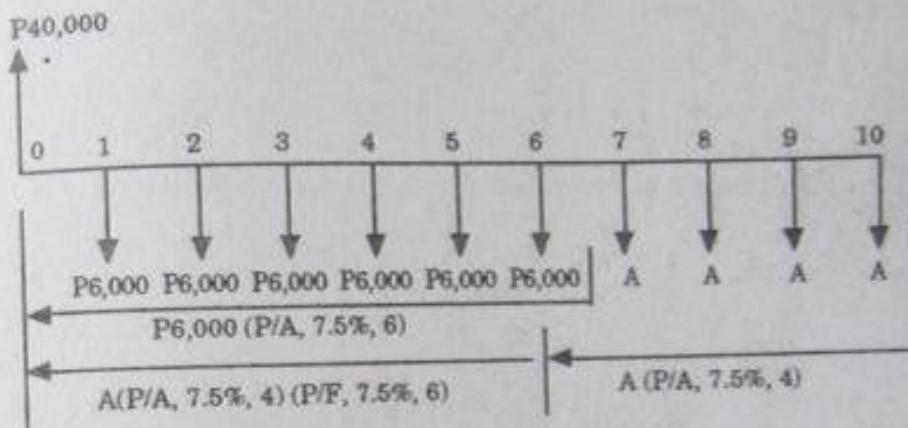
$$A(F/A, 14\%, 8) = P10,000(F/A, 14\%, 9)(F/P, 14\%, 8)$$

$$A(13.23276) = P10,000(16.08535)(2.85259)$$

$$A = P34,675$$

(2-21) A debt of P40,000, whose interest rate is 15% compounded semiannually, is to be discharged by a series of 10 semiannual payments, the first payment to be made 6 months after consummation of the loan. The first 6 payments will be P6,000 each, while the remaining 4 payments will be equal and of such amount that the final payment will liquidate the debt. What is the amount of the last 4 payments?

Solution



Using today as the focal date, the equation of value is

$$P40,000 = P6,000(P/A, 7.5\%, 6) + A(P/A, 7.5\%, 4)(P/F, 7.5\%, 6)$$

$$P40,000 = P6,000(4.6938) + A(3.3493)(0.6480)$$

$$A = P5,454$$

Annuity Due

An annuity due is one where the payments are made at the beginning of each period.

Find P when A is Given

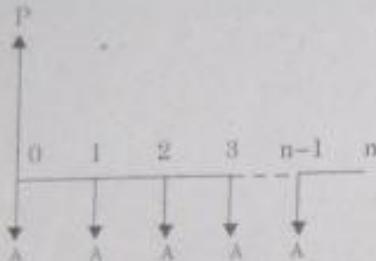


Figure 2-8. Cash flow diagram given A to find P.

$$P = A + A (P/A, i\%, n-1) \quad (2-28)$$

$$P = A (1 + P/A, i\%, n-1) \quad (2-29)$$

Finding F When A is Given

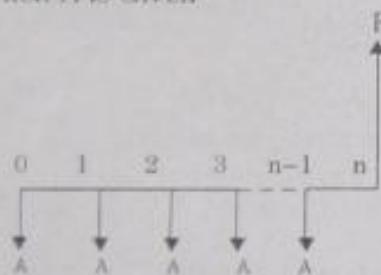


Figure 2-9. Cash flow diagram given A to find F.

$$F = A (F/A, i\%, n + 1) - A \quad (2-30)$$

$$F = A [(F/A, i\%, n + 1) - 1] \quad (2-31)$$

(2-22) A man bought an equipment costing P60,000 payable in 12 quarterly payments, each installment payable at the beginning of each period. The rate of interest is 24% compounded quarterly. What is the amount of each payment?

Solution

$$\begin{aligned} P &= P60,000 \quad n = 12 \quad i = 24\% = 8\% \\ P &= A (1 + P/A, 2\%, n - 1) \quad 3 \\ P60,000 &= A (1 + P/A, 8\%, 11) \\ P60,000 &= A (1 + 7.1390) \\ A &= P7,371.91 \end{aligned}$$

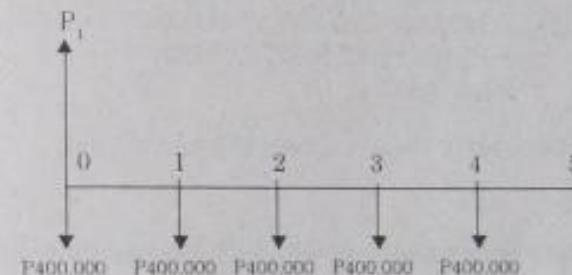
(2-23) A certain property is being sold and the owner received two bids.

The first bidder offered to pay P400,000 each year for 5 years, each payment is to be made at the beginning of each year. The second bidder offered to pay P240,000 first year, P360,000 the second year and P540,000 each year for the next 3 years, all payments will be made at the beginning of each year.

If money is worth 20% compounded annually, which bid should the owner of the property accept?

Solution

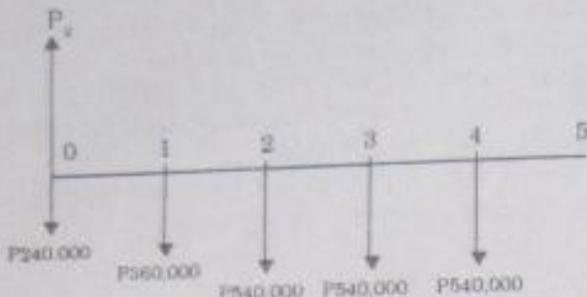
First bid:



Let P_1 = present worth of the first bid

$$\begin{aligned} P_1 &= A (1 + P/A, 20\%, 4) \\ &= P400,000 (1 + P/A, 20\%, 4) \\ &= P400,000 (1 + 2.5887) \\ &= P1,435,480 \end{aligned}$$

Second bid:



$$\begin{aligned} \text{Let } P_2 &= \text{present worth of the second bid} \\ P_2 &= P240,000 + P360,000 (P/F, 20\%, 1) + P540,000 \\ &\quad (P/A, 20\%, 3) (P/F, 20\%, 1) \\ &= P240,000 + 360,000 (0.8333) \\ &\quad + P540,000 (2.1065) (0.8333) \\ &= P1,487.875 \end{aligned}$$

The owner of property should accept the second bid.

Perpetuity

A perpetuity is an annuity in which the payments continue indefinitely.

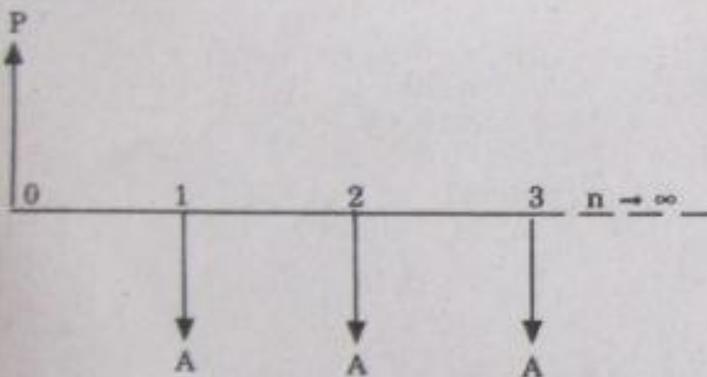


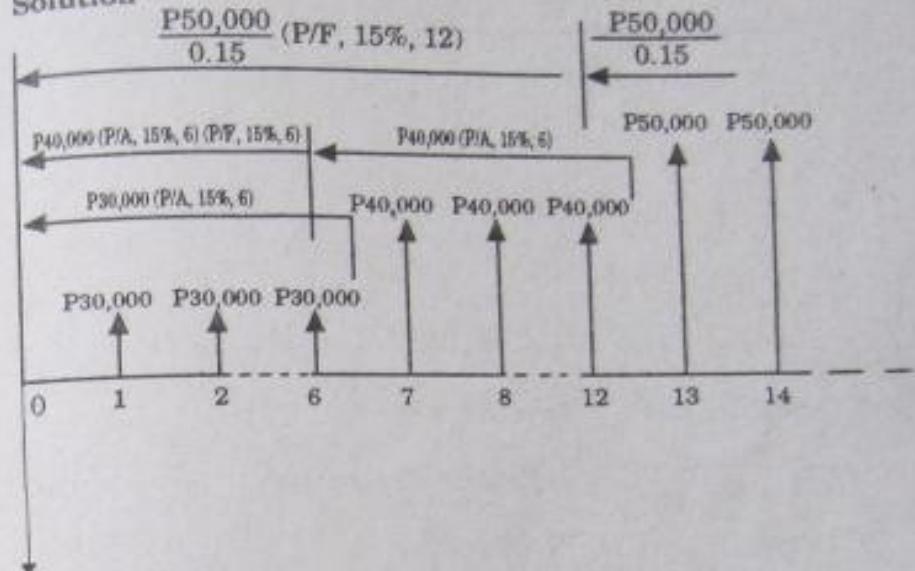
Figure 2-10. Cash flow diagram to find P given A

$$P = A \left[\frac{1 - (1 + i)^{-n}}{i} \right] = A \left[\frac{1 - (1 + i)^{-\infty}}{i} \right]$$

$$P = \frac{A}{i} \quad (2-32)$$

(2-24) What amount of money invested today at 15% interest can provide the following scholarships: P30,000 at the end of each year for 6 years; P40,000 for the next 6 years and P50,000 thereafter?

Solution



p. Using today as the focal date, the equation of value is

$$\begin{aligned} P &= P30,000 (P/A, 15\%, 6) + P40,000 (P/A, 15\%, 6) (P/F, 15\%, 6) \\ &\quad + \frac{P50,000}{0.15} (P/F, 15\%, 12) \\ &= P30,000 (3.7845) + P40,000 (3.7845) (0.4323) + \frac{P50,000 (0.1869)}{0.15} \\ &= P241,277 \end{aligned}$$

Capitalized cost

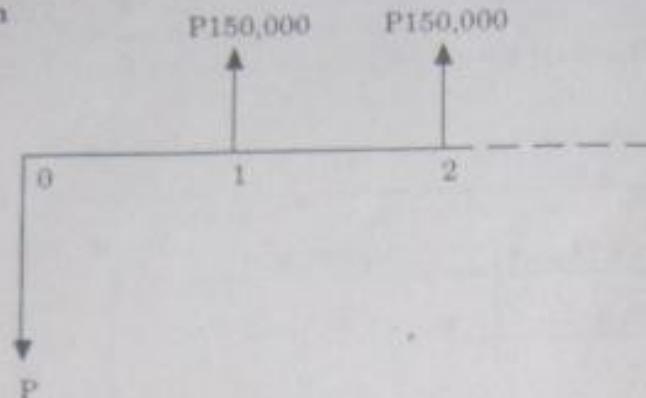
One of the most important applications of perpetuity is in capitalized cost. The capitalized cost of any property is the sum of the first cost and the present worth of all costs of replacement, operation and maintenance for a long time or forever.

Case 1. No replacement, only maintenance and/or operation every period.

Capitalized cost = First cost + Present worth of perpetual operation and/or maintenance.

(2-25) Determine the capitalized cost of a structure that requires an initial investment of P1,500,000 and an annual maintenance of P150,000. Interest is 15%.

Solution



$$P = \frac{A}{i} = \frac{P150,000}{0.15} = P1,000,000$$

Capitalized cost = First cost + P

$$\begin{aligned} &= P1,500,000 + P1,000,000 \\ &= P2,500,000 \end{aligned}$$

Case 2. Replacement only, no maintenance and or operation.

Capitalized cost = First cost + Present worth of perpetual replacement

Let S = amount needed to replace a property every k periods

X = amount of principal invested at rate i% the interest on which will amount to S every k periods

X_i = interest on X every period, the periodic deposit towards the accumulation of S

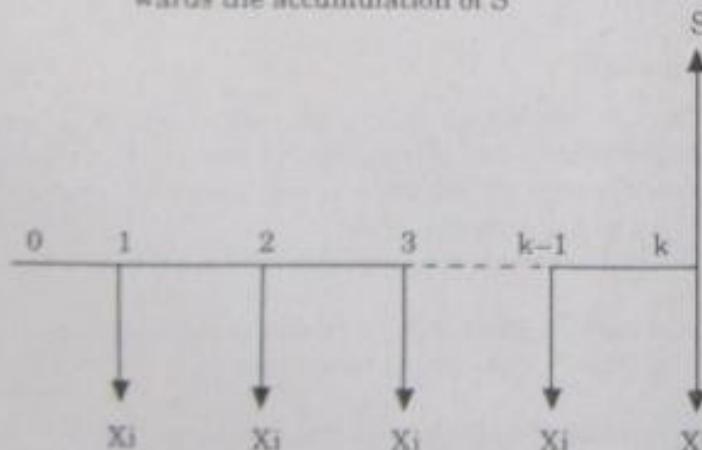


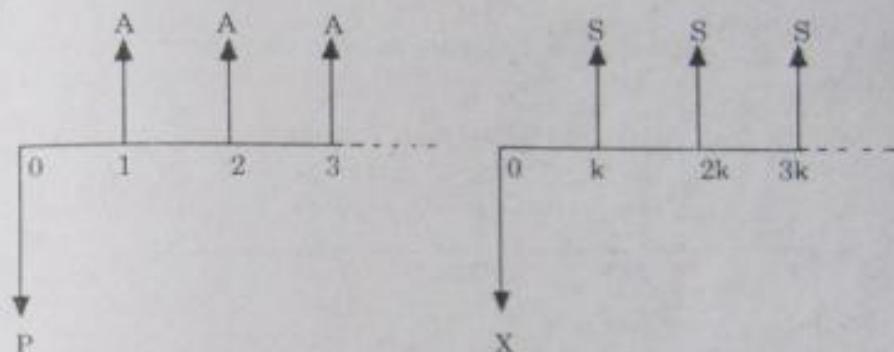
Figure 2-11. Cash flow diagram to find X given S

$$S = X_i (F/A, i\%, k)$$

$$X = \frac{S}{i} \left[\frac{1}{F/A, i\%, k} \right] = \frac{S}{i} \left[\frac{i}{(1+i)^k - 1} \right]$$

$$X = \frac{S}{(1+i)^k - 1} \quad (2-33)$$

Difference between P and X in a perpetuity.



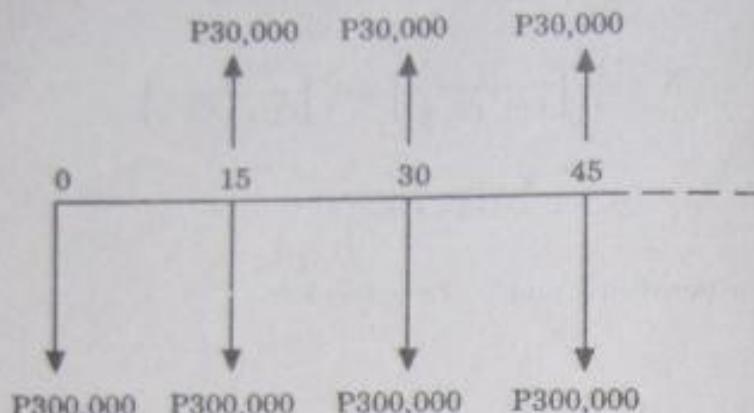
$$P = \frac{A}{i}$$

$$X = \frac{S}{(1+i)^k - 1}$$

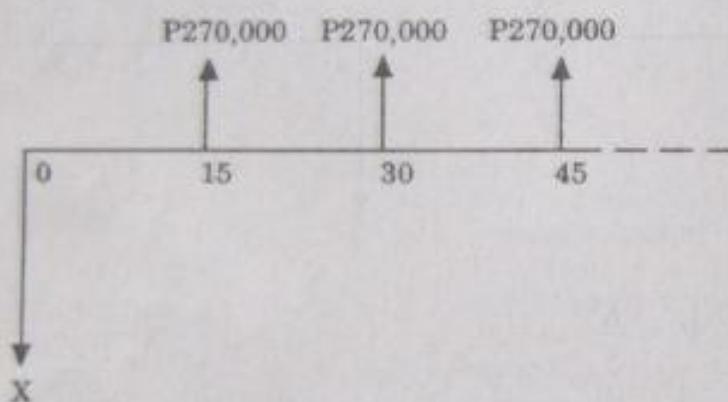
P is the amount invested now at $i\%$ per period whose interest at the end of every period forever is A while X is the amount invested now at $i\%$ per period whose interest at the end of every k periods forever is S. If $k = 1$, then, $X = P$.

(2-26) A new engine was installed by a textile plant at a cost of P300,000 and projected to have a useful life of 15 years. At the end of its useful life, it is estimated to have a salvage value of P30,000. Determine its capitalized cost if interest is 18% compounded annually.

Solution



Cash flow diagram for the engine



$$X = \frac{S}{(1+i)^k - 1} = \frac{P270,000}{(1+0.18)^{15} - 1} = P24,604$$

$$\begin{aligned}\text{Capitalized cost} &= \text{First cost} + X = P300,000 + P24,604 \\ &= P324,604\end{aligned}$$

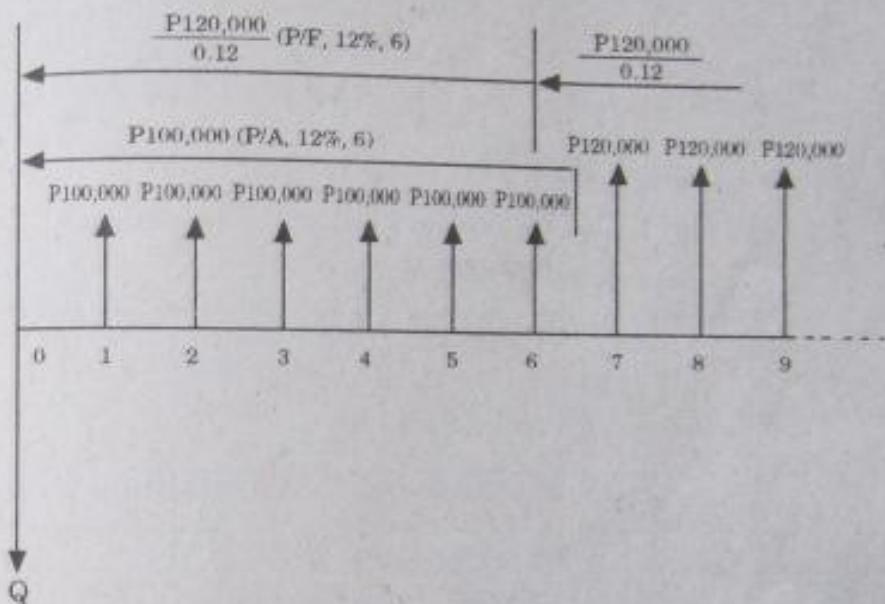
Case 3. Replacement, maintenance and/or operation every period

Capitalized cost = First cost + Present worth of cost of perpetual operation and/or maintenance + Present worth of cost of perpetual replacement

(2-27) Determine the capitalized cost of a research laboratory which requires P5,000,000 for original construction; P100,000 at the end of every year for the first 6 years and then P120,000 each year thereafter for operating expenses, and P500,000 every 5 years for replacement of equipment with interest at 12% per annum?

Solution

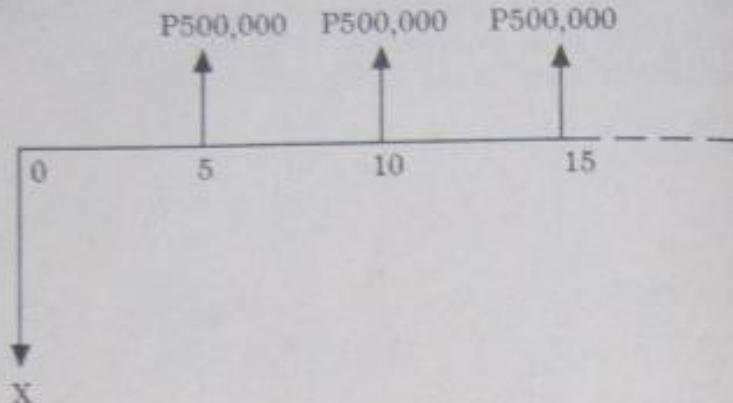
Operation:



Let Q = the present worth of cost of perpetual operation

$$\begin{aligned}Q &= P100,000(P/A, 12\%, 6) + \frac{P120,000}{0.12}(P/F, 12\%, 6) \\ &= P100,000(4.1114) + \frac{P120,000}{0.12}(0.5066) \\ &= P917,740 \\ &\quad \text{711,853\$}\end{aligned}$$

Replacement:



Let X = the present worth of cost of perpetual replacement

$$X = \frac{S}{(1+i)^k - 1} = \frac{P500,000}{(1+0.12)^5 - 1} = P655,910$$

Capitalized cost = First cost + $Q + X$

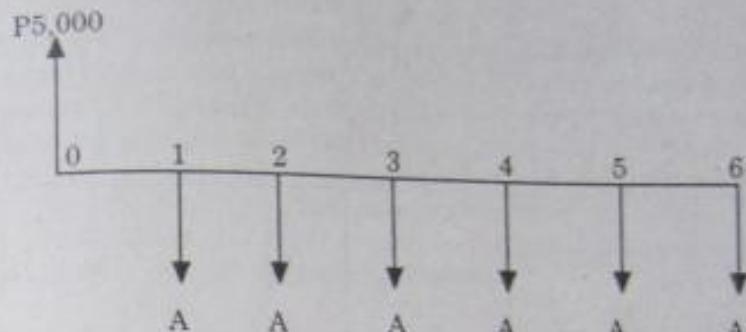
$$\begin{aligned} &= P5,000,000 + P917,740 + P655,910 \\ &= P6,753,650 \\ &= 6,753,650 \text{ PhP} \end{aligned}$$

Amortization

Amortization is any method of repaying a debt, the principal and interest included, usually by a series of equal payments at equal interval of time.

(2-28) A debt of P5,000 with interest at 12% compounded semiannually is to be amortized by equal semiannual payments over the next 3 years, the first due in 6 months. Find the semiannual payment and construct an amortization schedule.

Solution



$$A = \frac{P}{P/A, 6\%, 6} = \frac{P5,000}{4.9173} = P1,016.82$$

Amortization Schedule

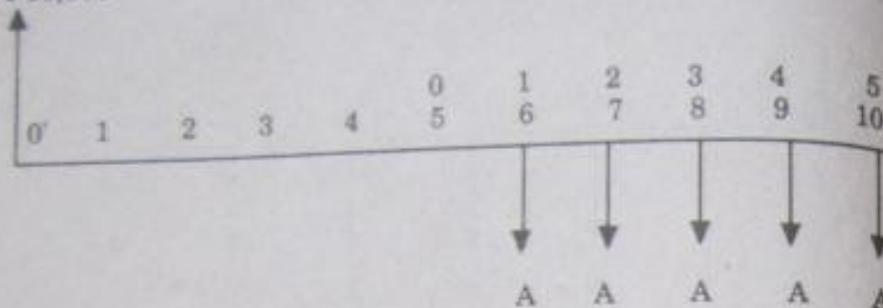
| Period | Outstanding principal at beginning of period | Interest due at end of period | Payment | Principal repaid at end of period |
|--------|--|-------------------------------|-----------|-----------------------------------|
| 1 | P5,000.00 | P 300.00 | P1,016.82 | P 716.82 |
| 2 | 4,283.18 | 256.99 | 1,016.82 | 759.83 |
| 3 | 3,523.35 | 211.40 | 1,016.82 | 805.42 |
| 4 | 2,717.93 | 163.08 | 1,016.82 | 853.74 |
| 5 | 1,864.19 | 111.85 | 1,016.82 | 904.97 |
| 6 | 959.22 | 57.55 | 1,016.82 | 959.27 |

TOTALS P1,100.87 P6,100.92 P5,000.05

(2-29) A debt of P10,000 with interest at the rate of 20% compounded semiannually is to be amortized by 5 equal payments at the end of each 6 months, the first payment is to be made after 3 years. Find the semiannual payment and construct an amortization schedule.

Solution

P10,000



$$\begin{aligned} P &= A(P/A, 10\%, 5)(P/F, 10\%, 5) \\ A &= P(A/P, 10\%, 5)(F/P, 10\%, 5) \\ &= P10,000(0.2638)(1.6105) \\ &= P4,248.50 \end{aligned}$$

Amortization Schedule

| Period | Outstanding principal at beginning of period | Interest due at end of period | Payment | Principal repaid at end of period |
|--------|--|-------------------------------|------------|-----------------------------------|
| 1 | P10,000.00 | P 1,000.00 | | |
| 2 | 11,000.00 | 1,100.00 | | |
| 3 | 12,100.00 | 1,210.00 | | |
| 4 | 13,310.00 | 1,331.00 | | |
| 5 | 14,641.00 | 1,464.10 | | |
| 6 | 16,105.10 | 1,610.51 | P 4,248.50 | P 2,637.99 |
| 7 | 13,467.11 | 1,346.71 | 4,248.50 | 2,901.79 |
| 8 | 10,565.32 | 1,056.53 | 4,248.50 | 3,191.97 |
| 9 | 7,373.35 | 737.34 | 4,248.50 | 3,511.16 |
| 10 | 3,862.19 | 386.22 | 4,248.50 | 3,862.28 |

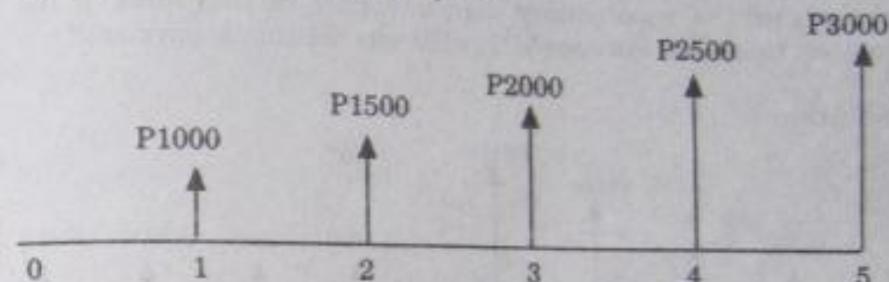
TOTALS

P11,242.41 P21,242.50 P16,105.19

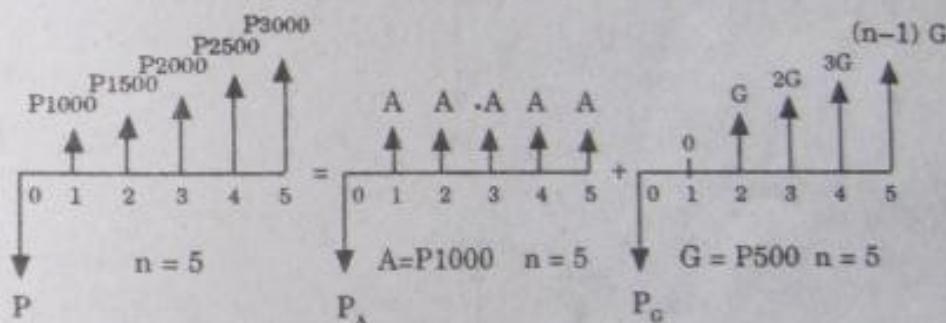
Uniform Arithmetic Gradient

In certain cases, economic analysis problems involve receipts or disbursements that increase or decrease by a uniform amount each period. For example, maintenance and repair expenses on specific equipment or property may increase by a relatively constant amount each period. This is known as a uniform arithmetic gradient.

Suppose that the maintenance expense on a certain machine is P1000 at the end of the first year and increasing at a constant rate of P500 each year for the next four years.



This cash flow may be resolved into two components



G is known as the uniform gradient amount

$$P = P_A + P_G \quad (2-34)$$

$$P_A = A(P/A, i\%, n) \quad (2-35)$$

$$P_G = G(1+i)^{-1} + 2G(1+i)^{-2} + 3G(1+i)^{-3} + \dots + (n-1)G(1+i)^{(n-1)} \quad (2-36)$$

$$P_G = \frac{G}{i} \left[\frac{(1+i)^n - 1}{i} - n \right] \left[\frac{1}{(1+i)^n} \right] \quad (2-36)$$

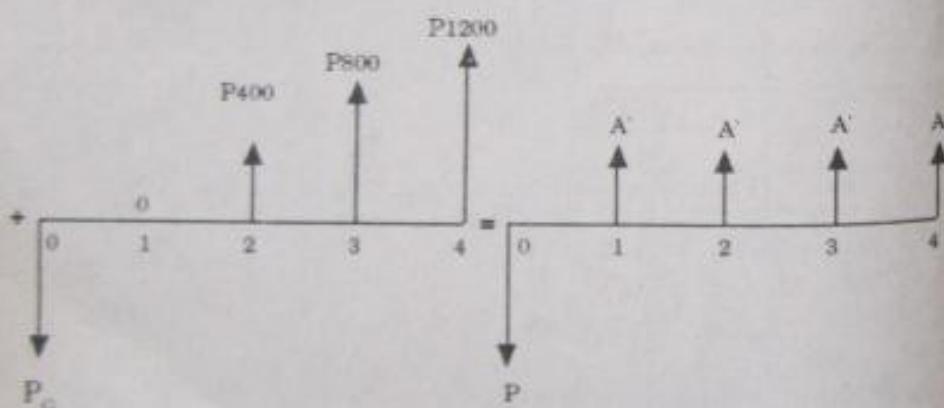
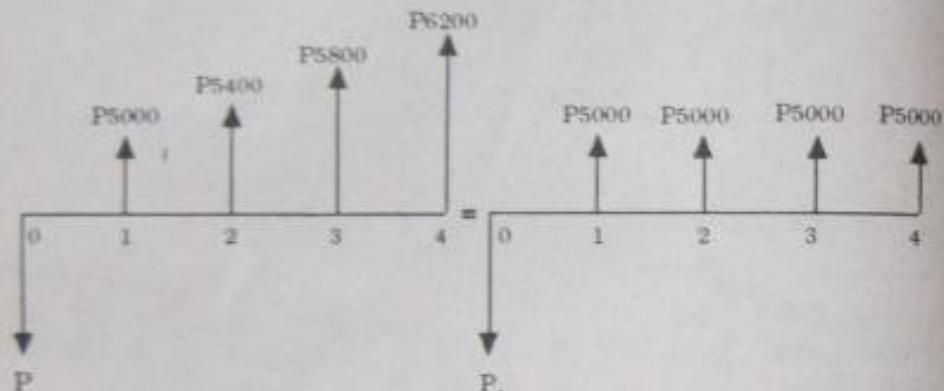
$$P_G = G(P/G, i\%, n) \quad (2-37)$$

$P/G, i\%, n = \frac{1}{i} \left[\frac{(1+i)^n - 1}{i} \cdot n \right] \left[\frac{1}{(1+i)^n} \right]$, this factor is called the gradient to present worth conversion factor. Thus,

$$P = A(P/A, i\%, n) + G(P/G, i\%, n) \quad (2-38)$$

(2-30) A loan was to be amortized by a group of four end-of-year payments forming an ascending arithmetic progression. The initial payment was to be P5,000 and the difference between successive payments was to be P400. But the loan was renegotiated to provide for the payment of equal rather than uniformly varying sums. If the interest rate of the loan was 15%, what was the annual payment?

Solution



Let A' = the annual payment

$$A = P5000 \quad G = P400 \quad n = 4 \quad i = 15\%$$

$$P/A, 15\%, 4 = \frac{1 - (1.15)^4}{0.15} = 2.8550$$

$$P/G, 15\%, 4 = \frac{1}{0.15} \left[\frac{(1.15)^4 - 1}{0.15} - 4 \right] \left[\frac{1}{(1.15)^4} \right] = 3.7865$$

$$\begin{aligned} P &= A(P/A, 15\%, 4) + G(P/G, 15\%, 4) \\ &= (P5000)(2.8550) + P400(3.7865) \\ &= P15,789.60 \end{aligned}$$

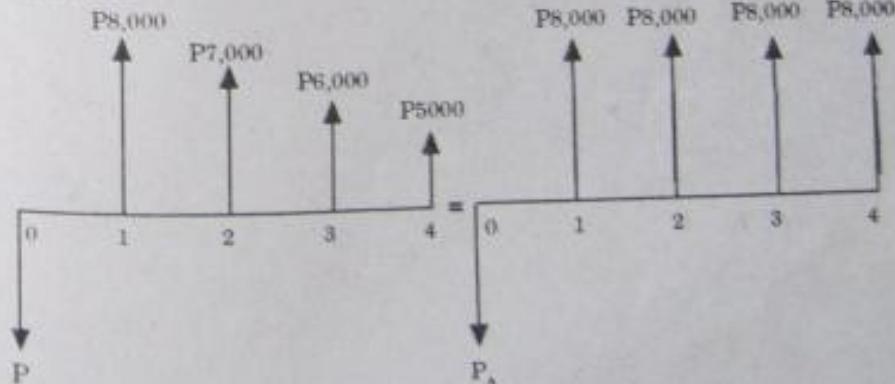
$$A(P/A, 15\%, 4) = P15,789.60$$

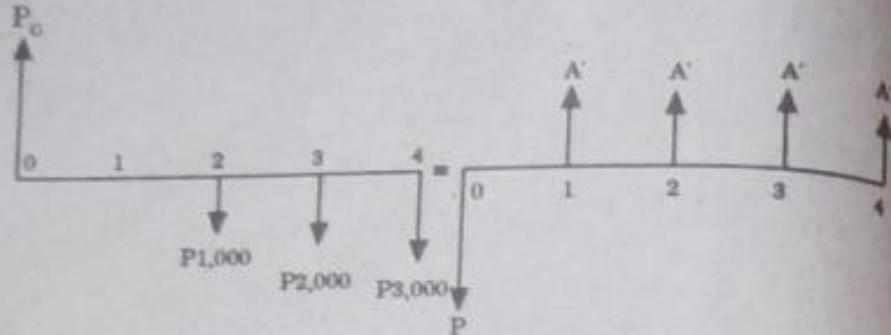
$$A = P5530.51$$

(2-31) Find the equivalent annual payment of the following obligations at 20% interest.

| End of Year | Payment |
|-------------|---------|
| 1 | P8,000 |
| 2 | 7,000 |
| 3 | 6,000 |
| 4 | 5,000 |

Solution





Let A' = the equivalent annual payment

$$A = P8,000 \quad G = P1,000 \quad n = 4 \quad i = 20\%$$

$$P/A, 20\%, 4 = \frac{1 - (1.20)^{-4}}{0.20} = 2.5887$$

$$P/G, 20\%, 4 = \frac{1}{0.20} \left[\frac{(1.20)^4 - 1}{0.20} - 4 \right] = 3.2986$$

$$P = P_A - P_G$$

$$= A(P/A, 20\%, 4) - G(P/G, 20\%, 4)$$

$$= P8,000(2.5887) - (P1,000)(3.2986)$$

$$= P17,411$$

$$A'(P/A, 20\%, 4) = P17,411$$

$$A' = P6,725.77$$

PROBLEMS

1. What is the annual rate of interest if P265 is earned in four months on an investment of P15,000?
Ans. 5.3%

2. A loan of P2,000 is made for a period of 13 months, from January 1 to January 31 the following year, at a simple interest rate of 20%. What future amount is due at the end of the loan period?
Ans. P2,433.33

3. If you borrow money from your friend with simple interest of 12%, find the present worth of P20,000, which is due at the end of nine months.

Ans. P18,348.62

4. Determine the exact simple interest on P5,000 for the period from Jan. 15 to Nov. 28, 1992, if the rate of interest is 22%.

Ans. P955.74

5. A man wishes his son to receive P200,000 ten years from now. What amount should he invest if it will earn interest of 10% compounded annually during the first 5 years and 12% compounded quarterly during the next 5 years?

Ans. P68,758.67

6. By the conditions of a will, the sum of P25,000 is left to a girl to be held in trust by her guardian until it amounts to P45,000. When will the girl receive the money if the fund is invested at 8% compounded quarterly?

Ans. 7.42 years

7. At a certain interest rate compounded semiannually, P5,000 will amount to P20,000 after 10 years. What is the amount at the end of 15 years?

Ans. P40,029.72

8. Jones Corporation borrowed P9,000 from Brown Corporation on Jan. 1, 1978 and P12,000 on Jan. 1, 1980. Jones Corporation made a partial payment of P7,000 on Jan. 1, 1981. It was agreed that the balance of the loan would be amortized by two payments, one of Jan. 1, 1982 and the other on Jan. 1, 1983; the second being

50% larger than the first. If the interest rate is 12%, what is the amount of each payment?

Ans. P9,136.91; P13,705.36

9. A woman borrowed P3,000 to be paid after $1\frac{1}{2}$ years with interest at 12% compounded semiannually and P5,000 to be paid after 3 years at 12% compounded monthly. What single payment must she pay after $3\frac{1}{2}$ years at an interest rate of 16% compounded quarterly to settle the two obligations?

Ans. P12,627.59

10. Mr. J. de la Cruz borrowed money from a bank. He received from the bank P1,342 and promise to repay P1,500 at the end of 9 months. Determine the simple interest rate and the corresponding discount rate or often referred to as the "Banker's discount."

Ans. 11.7%; 10.53%

11. A man deposits P50,000 in a bank account at 6% compounded monthly for 5 years. If the inflation rate of 6.5% per year continues for this period, will this effectively protect the purchasing power of the original principal?

Ans. No; P49,225

12. What is the future worth of P600 deposited at the end of every month for 4 years if the interest rate is 12% compounded quarterly?

Ans. P36,641

13. A young woman, 22 years old, has just graduated from college. She accepts a good job and desires to establish her own retirement fund. At the end of each year thereafter she plans to deposit P2,000 in a fund at 15% annual interest. How old will she be when the fund has an accumulated value of P1,000,000?

Ans. 53 years old

14. Mr. Reyes borrows P600,000 at 12% compounded annually, agreeing to repay the loan in 15 equal annual payments. How much of the original principal is still unpaid after he has made the 8th payment?

Ans. P402,040

15. M purchased a small lot in a subdivision, paying P200,000 down and promising to pay P15,000 every 3 months for the next 10 years. The seller figured interest at 12% compounded quarterly.

- What was the cash price of the lot?
- If M missed the first 12 payments, what must he pay at the time the 13th is due to bring himself up to date?
- After making 8 payments, M wished to discharge his remaining indebtedness by a single payment at the time when the 9th regular payment was due, what must he pay in addition to the regular payment then due?
- If M missed the first 10 payments, what must he pay when the 11th payment is due to discharge his entire indebtedness?

Ans. (a) P546,722; (b) P234,270; (c) P300,006; (d) P479,948

16. A man approaches the ABC Loan Agency for P100,000 to be paid in 24 monthly installments. The agency advertises an interest rate of 1.5% per month. They proceed to calculate the amount of his monthly payment in the following manner.

| | |
|-----------------------|----------|
| Amount requested | P100,000 |
| Credit investigation | 500 |
| Credit risk insurance | 1,000 |
| Total | P101,500 |

$$\text{Interest: } (P101,500)(24)(0.015) = P36,540$$

$$\text{Total owed: } P101,500 + P36,540 = P138,040$$

$$\text{Payment} = \frac{P138,040}{24} = P5,751.67$$

What is the effective rate of interest of the loan?

Ans. 38.83%

17. A new office building was constructed 5 years ago by a consulting engineering firm. At that time the firm obtained the bank loan for P10,000,000 with a 20% annual interest rate, compounded quarterly. The terms of the loan called for equal quarterly payments for a 10-year period with the right of prepayment at any time without penalty.

Due to internal changes in the firm, it is now proposed to refinance the loan through an insurance company. The new loan is planned for a 20-year term with an interest rate of 24% per annum, compounded quarterly. The insurance company has a one time service charge of 5% of the balance. This new loan also calls for equal quarterly payments.

- (a) What is the balance due on the original mortgage (principal) if all payments have been made through a full five years?
(b) What will be the difference between the equal quarterly payments in the existing arrangement and the revised proposal?
Ans. (a) P7,262.733; (b) P120,862

18. An asphalt road requires no upkeep until the end of 2 years when P60,000 will be needed for repairs. After this P90,000 will be needed for repairs at the end of each year for the next 5 years, then P120,00 at the end of each year for the next 5 years.

If money is worth 14% compounded annually, what was the equivalent uniform annual cost for the 12-year period?

Ans. P79,245

19. A man wishes to provide a fund for his retirement such that from his 60th to 70th birthdays he will be able to withdraw equal sums of P18,000 for his yearly expenses. He invests equal amount for his 41st to 59th birthdays in a fund earning 10% compounded annually. How much should each of these amounts be?

Ans. P2,285

20. Juan Reyes inherited P280,000 on his birthday. This he placed in his own pension fund (bank account), which earns interest at 12% compounded annually. He withdraw P450,000 on his 35th birthday to make the initial payment on a house. He drew his annual uniform payment for the pension fund on his 65th birthday and planned to make 20 such withdrawals so as to just exhaust the fund. He died at the age of 71, however, just before withdrawing the amount due on that date. How much is left in the fund to be passed on to the heirs at the time of Juan's death?

Ans. P1,155,980

21. Determine the present worth and the accumulated amount of an annuity consisting of 6 payments of P120,000 each, the payment are made at the beginning of each year. Money is worth 15% compounded annually.

Ans. P52,226.40, P120,801.60

22. Calculate the capitalized cost of a project that has an initial cost of P3,000,000 and an additional investment cost of P1,000,000 at the end of every ten years. The annual operating cost will be P100,000 at the end of every year for the first four years and P160,000

thereafter. In addition, there is expected to be a recurring major rework cost of P300,000 every 13 years. Assume i=15%.

Ans. P4,281,990

23. The will of a wealthy philanthropist left P5,000,000 to establish a perpetual charitable foundation. The foundation trustees decided to spend P1,200,000 to provide facilities immediately and to provide P100,000 of capital replacement at the end of each 5-year period. If the invested funds earned 12% per annum, what would be the year-end amount available in perpetuity from the endowment for charitable purposes?

Ans. P440,259

24. The surface area of a certain plant that requires painting is 8,000 sq. ft. Two kinds of paint are available whose brands are A and B. Paint A costs P1.40 per sq. ft. but needs renewal at the end of every 4 years, while paint B costs P1.80 per sq. ft. If money is worth 12% effective, how often should paint B be renewed so that it will be as economical as Paint A?

Ans. 5.58 years

25. A contract has been signed to lease a building at P200,000 per year with an annual increase of P1,500 for 8 years. Payments are to be made at the end of each year, starting one year from now. The prevailing interest rate is 7%. What lump sum paid today would be equivalent to the 8-year lease-payment plan?

Ans. P147,609

Chapter 3

Depreciation

Depreciation is the decrease in the value of physical property with the passage of time.

Definitions Of Value

Value, in a commercial sense, is the present worth of all future profits that are to be received through ownership of a particular property.

The market value of a property is the amount which a willing buyer will pay to a willing seller for the property where each has equal advantage and is under no compulsion to buy or sell.

The utility or use value of a property is what the property is worth to the owner as an operating unit.

Fair value is the value which is usually determined by a disinterested third party in order to establish a price that is fair to both seller and buyer.

Book value, sometimes called depreciated book value, is the worth of a property as shown on the accounting records of an enterprise.

Salvage, or resale, value is the price that can be obtained from the sale of the property after it has been used.

Scrap value is the amount the property would sell for if disposed off as junk?

Purposes Of Depreciation

1. To provide for the recovery of capital which has been invested in physical property

2. To enable the cost of depreciation to be charged to the cost of producing products or services that results from the use of the property.

Types Of Depreciation

1. Normal depreciation
 - (a) physical
 - (b) functional
2. Depreciation due to changes in price levels
3. Depletion

Physical depreciation is due to the lessening of the physical ability of a property to produce results. Its common causes are wear and deterioration. Functional depreciation is due to the lessening in the demand for the function which the property was designed to render. Its common causes are inadequacy, changes in styles, population centers shift, saturation of markets or more efficient machines are produced.

Depreciation due to changes in price levels is almost impossible to predict and therefore is not considered in economy studies.

Depletion refers to the decrease in the value of a property due to the gradual extraction of its contents.

Physical And Economic Life

Physical life of a property is the length of time during which it is capable of performing the function for which it was designed and manufactured.

Economic life is the length of time during which the property may be operated at a profit.

Requirements Of A Depreciation Method

1. It should be simple.
2. It should recover capital.
3. The book value will be reasonably close to the market value at any time.
4. The method should be accepted by the Bureau of Internal Revenue.

Depreciation Methods

We shall use the following symbols for the different depreciation methods.

- L = useful life of the property in years
 C_o = the original cost
 C_L = the value at the end of the life, the scrap value
 (including gain or loss due to removal)
 d = the annual cost of depreciation
 C_n = the book value at the end of n years
 D_n = depreciation up to age n years

The Straight Line Method

This method assumes that the loss in value is directly proportional to the age of the property.

$$d = \frac{C_o - C_L}{L} \quad (3-1)$$

$$D_n = \frac{n(C_o - C_L)}{L} \quad (3-2)$$

$$C_n = C_o - D_n \quad (3-3)$$

(3-1) An electronic balance costs P90,000 and has an estimated salvage value of P8,000 at the end of its 10 years life time. What would be the book value after three years, using the straight line method in solving for the depreciation?

Solution

$$C_o = P90,000 \quad C_L = P8,000 \quad L = 10 \quad n = 3$$

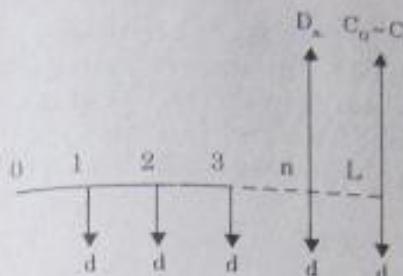
$$d = \frac{C_o - C_L}{L} = \frac{P90,000 - P8,000}{10} = P8,200$$

$$D_3 = (n)(d) = (3)(P8,200) = 24,600$$

$$C_3 = C_o - D_3 = P90,000 - P24,600 = P65,400$$

The Sinking Fund Formula

This method assumes that a sinking fund is established in which funds will accumulate for replacement. The total depreciation that has taken place up to any given time is assumed to be equal to the accumulated amount in the sinking fund at that time.



$$d = \frac{C_o - C_L}{F/A, i\%, L} \quad (3-4)$$

$$D_n = d(F/A, i\%, n) \quad (3-5)$$

$$C_n = C_o - D_n \quad (3-6)$$

(3-2) A broadcasting corporation purchased an equipment for P53,000 and paid P1,500 for freight and delivery charges to the job site. The equipment has a normal life of 10 years with a trade-in value of P5,000 against the purchase of a new equipment at the end of the life.

(a) Determine the annual depreciation cost by the straight line method.

(b) Determine the annual depreciation cost by the sinking fund method. Assume interest at 6-1/2% compounded annually.

Solution

$$C_o = P53,000 + P1,500 = P54,500$$

$$C_L = P5,000$$

$$(a) d = \frac{C_o - C_L}{L} = \frac{P54,500 - P5,000}{10} = P4,950$$

$$(b) d = \frac{C_o - C_L}{F/A, i\%, L} = \frac{P54,500 - P5,000}{F/A, 6.5\%, 10} = \frac{P49,500}{13.3846} = P3,668$$

(3-3) A firm bought an equipment for P56,000. Other expenses including installation amounted to P4,000. The equipment is expected to have a life of 16 years with a salvage value of 10% of the original cost. Determine the book value at the end of 12 years by (a) the straight line method and (b) sinking fund method at 12% interest.

Solution

$$C_o = P56,000 + P4,000 = P60,000$$

$$C_L = P60,000 (0.10) = P6,000$$

$$L = 16 \quad n = 12 \quad i = 12\%$$

(a) Straight line method

$$d = \frac{C_o - C_L}{L} = \frac{P60,000 - P6,000}{16} = P3,375$$

$$D_{12} = (d)(n) = P3,375 (12) = P40,500$$

$$C_{12} = C_o - D_{12} = P60,000 - P40,500 = P19,500$$

(b) Sinking fund method

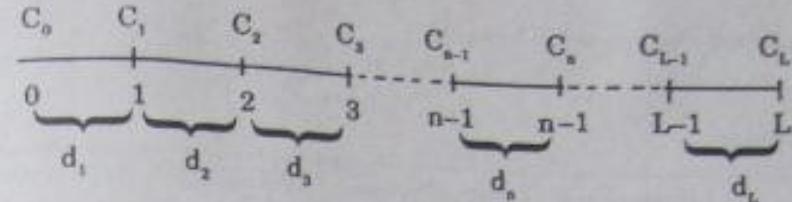
$$d = \frac{C_o - C_L}{F/A, 12\%, 16} = \frac{P60,000 - P6,000}{42.7533} = P1,263$$

$$D_{12} = d(F/A, 12\%, 12) = P1,263 (24.1331) = P30,480$$

$$C_{12} = C_o - D_{12} = P60,000 - P30,480 = P29,520$$

Declining Balance Method

In this method, sometimes called the constant percentage method or the Matheson Formula, it is assumed that the annual cost of depreciation is a fixed percentage of the salvage value at the beginning of the year. The ratio of the depreciation in any year to the book value at the beginning of that year is constant throughout the life of the property and is designated by k , the rate of depreciation.



d_n = depreciation during the nth year

| Year | Book value at beginning of year | Depreciation during the year | Book value at the end of year |
|------|---------------------------------|------------------------------|------------------------------------|
| 1 | C_o | $d_1 = kC_o$ | $C_1 = C_o - d_1 = C_o(1-k)$ |
| 2 | $C_o(1-k)$ | $d_2 = kC_1$ | $C_2 = C_1 - d_2 = C_o(1-k)^2$ |
| 3 | $C_o(1-k)^2$ | $d_3 = kC_2$ | $C_3 = C_2 - d_3 = C_o(1-k)^3$ |
| ... | ... | ... | ... |
| n | $C_o(1-k)^{n-1}$ | $d_n = kC_{n-1}$ | $C_n = C_{n-1} - d_n = C_o(1-k)^n$ |
| ... | ... | ... | ... |
| L | $C_o(1-k)^{L-1}$ | $d_L = kC_{L-1}$ | $C_L = C_{L-1} - d_L = C_o(1-k)^L$ |

$$d_n = C_o(1-k)^{n-1}k \quad (3-7)$$

$$C_n = C_o(1-k)^n = C_o \left[\frac{C_L}{C_o} \right]^{\frac{n}{L}} \quad (3-8)$$

$$C_L = C_o(1-k)^L \quad (3-9)$$

$$k = 1 - \sqrt[n]{\frac{C_n}{C_o}} = 1 - \sqrt[L]{\frac{C_L}{C_o}} \quad (3-10)$$

This method does not apply, if the salvage value is zero, because k will be equal to one and d_1 will be equal to C_o .

(3-4) A certain type of machine loses 10% of its value each year. The machine costs P2,000.00 originally. Make out a schedule showing the yearly depreciation, the total depreciation and the book value at

the end of each year for 5 years.

Solution

| Year | Book value at beginning of year | Depreciation during the year 10% | Total depreciation at end of year | Book value at end of year |
|------|---------------------------------|----------------------------------|-----------------------------------|---------------------------|
| 1 | P2,000.00 | P200.00 | P200.00 | P1,800.00 |
| 2 | 1,800.00 | 180.00 | 380.00 | 1,620.00 |
| 3 | 1,620.00 | 162.00 | 542.00 | 1,458.00 |
| 4 | 1,458.00 | 145.80 | 687.80 | 1,312.20 |
| 5 | 1,312.20 | 131.22 | 819.12 | 1,180.98 |

Double Declining Balance (DDB) Method

This method is very similar to the declining balance method except that the rate of depreciation k is replaced by $2/L$.

$$d_s = C_o \left(1 - \frac{2}{L}\right)^{n-1} - \frac{2}{L} \quad (3-11)$$

$$C_s = C_o \left(1 - \frac{2}{L}\right)^n \quad (3-12)$$

$$C_L = C_o \left(1 - \frac{2}{L}\right)^L \quad (3-13)$$

When the DDB method is used, the salvage value should not be subtracted from the first cost when calculating the depreciation charge.

(3-5) Determine the rate of depreciation, the total depreciation up to the end of the 8th year and the book value at the end of 8 years for an asset that costs P15,000 new and has an estimated scrap value of P2,000 at the end of 10 years by (a) the declining balance method and (b) the double declining balance method.

Solution

$$C_o = P15,000 \quad C_L = P2,000 \quad L = 10 \quad n = 8$$

(a) Declining balance method

$$k = 1 - \sqrt[L]{\frac{C_L}{C_o}} = 1 - \sqrt[10]{\frac{P2,000}{P15,000}} = 0.1825 \text{ or } 18.25\%$$

$$C_s = C_o (1 - k)^n = P15,000 (1 - 0.1825)^8 = P2,992$$

$$D_s = C_o - C_s = P15,000 - P2,992 = P12,008$$

(b) Double declining balance method

$$\text{Rate of depreciation} = \frac{2}{L} = \frac{2}{10} = 0.20 \text{ or } 20\%$$

$$C_s = C_o \left(1 - \frac{2}{L}\right)^n = P15,000 \left(1 - \frac{2}{10}\right)^8 = P2,517$$

$$D_s = C_o - C_s = P15,000 - P2,517 = P12,483$$

(3-6) A plant bought a calciner for P220,000 and used it for 10 years, the life span of the equipment. What is the book value of the calciner after 5 years of use? Assume a scrap value of P20,000 for straight line method; P22,000 for textbook declining balance method and P20,000 for the double declining balance method.

Solution

$$C_o = P220,000 \quad L = 10 \quad n = 5$$

(a) Straight line method

$$C_L = P20,000$$

$$D_s = \frac{n(C_o - C_L)}{L} = \frac{5(P220,000 - P20,000)}{10} = P100,000$$

$$C_s = C_o - D_s = P220,000 - P100,000 = P120,000$$

(b) Declining balance method $C_L = P22,000$

$$C_s = C_o \left(\frac{C_L}{C_o} \right)^{\frac{n}{L}} = P220,000 \left(\frac{P22,000}{P220,000} \right)^{\frac{5}{10}} = P69,570$$

(c) Double declining balance method $C_L = P20,000$

$$C_s = C_o \left(1 - \frac{2}{L} \right)^n = P220,000 \left(1 - \frac{2}{10} \right)^5 = P72,090$$

The Sum-of-the-Years'-Digits (SYD) Method

Let d_n = depreciation charge during the nth year

d_n = (depreciation factor) (total depreciation)

$$d_n = \frac{\text{reverse digit}}{\text{sum of digits}} (C_o - C_L) \quad (3-14)$$

For example, for a property whose life is 5 years.

| Year | Year in Reverse Order | Depreciation factor | Depreciation during the year |
|-------------------------|-----------------------|---------------------|---|
| 1 | 5 | 5/15 | (5/15) (C _o - C _L) |
| 2 | 4 | 4/15 | (4/15) (C _o - C _L) |
| 3 | 3 | 3/15 | (3/15) (C _o - C _L) |
| 4 | 2 | 2/15 | (2/15) (C _o - C _L) |
| 5 | 1 | 1/15 | (1/15) (C _o - C _L) |
| Σ of digits = 15 | | | |

(3-7) A structure costs P12,000 new. It is estimated to have a life of 5 years with a salvage value at the end of life of P1,000. Determine the book value at the end of each year of life.

Solution

$$C_o - C_L = P12,000 - P1,000 = P11,000$$

| Year | Year in Reverse order | Depreciation during the year | Book value at end of year |
|-------------------------|-----------------------|------------------------------|---------------------------|
| 1 | 5 | (5/15) (P11,000) = P3,667 | P8,333 |
| 2 | 4 | (4/15) (P11,000) = 2,933 | 5,400 |
| 3 | 3 | (3/15) (P11,000) = 2,200 | 3,200 |
| 4 | 2 | (2/15) (P11,000) = 1,467 | 1,733 |
| 5 | 1 | (1/15) (P11,000) = 733 | 1,000 |
| Σ of digits = 15 | | | |

(3-8) A consortium of international telecommunication companies contracted for the purchase and installation of a fiber optic cable linking two major cities at a total cost of US\$960 million. This amount includes freight and installation charges estimated at 10% of the above contract price. If the cable shall be depreciated over a period of 15 years with zero salvage value:

(A) Given the sinking fund deposit factor of 0.0430 at 6% interest where n = 15, what is the annual depreciation charge?

(B) What is the depreciation charge during the 8th year using the sum-of-the-years-digits method?

Solution

$$C_o = \$960,000,000 \quad C_L = 0 \quad L = 15$$

$$(A) d = \frac{C_o - C_L}{F/A, 6\%, 15} = (C_o - C_L)(A/F, 6\%, 15) = \$960,000,000(0.0430) \\ = \$41,280,000$$

$$(B) \text{Sum of digits} = \frac{L(L+1)}{2} = \frac{15(15+1)}{2} = 120$$

Reverse digit corresponding to the 8th year of life = 8

$$d_8 = \frac{8}{120} (\$960,000,000) = \$64,000,000$$

The Service-Output Method

This method assumes that the total depreciation that has taken place is directly proportional to the quantity of output of the property up to that time. This method has the advantage of making the unit cost of depreciation constant and giving low depreciation expense during periods of low production.

Let T = total units of output up to the end of life

Q_n = total number of units of output during the n th year

$$\text{Depreciation per unit of output} = \frac{C_o - C_L}{T}$$

$$d_n = \left(\frac{C_o - C_L}{T} \right) (Q_n) \quad (3-15)$$

(3-9) A Television Company purchased machinery for P100,000 on July 1, 1979. It is estimated that it will have a useful life of 10 years; scrap value of P4,000, production of 400,000 units and working hours of 120,000.

The company uses the machinery for 14,000 hours in 1979 and 18,000 hours in 1980. The machinery produces 36,000 units in 1979 and 44,000 units in 1980. Compute the depreciation for 1980 using each method given below:

- (1) Straight line
- (2) Working hours
- (3) Output method

Solution

$$C_o = P100,000 \quad C_L = P4,000 \quad L = 10 \text{ years}$$

$$T = 400,000 \text{ units} \quad H = 120,000 \text{ hours}$$

$$(1) d_{n_0} = \frac{C_o - C_L}{L} = \frac{P100,000 - P4,000}{10} = P9,600$$

$$(2) d_{n_0} = \left(\frac{C_o - C_L}{H} \right) H_{n_0} = \frac{(P100,000 - P4,000)}{120,000} (18,000) \\ = P14,400$$

$$(3) d_{n_0} = \left(\frac{C_o - C_L}{T} \right) Q_{n_0} = \left(\frac{P100,000 - P4,000}{400,000} \right) (44,000) \\ = P10,560$$

What Depreciation Method Should Be Used?

It is worth mentioning what the National Internal Revenue Code says about depreciation specifically Section 29 (f).

(1) General rule. "There shall be allowed as a depreciation deduction a reasonable allowance for the exhaustion, wear and tear (including reasonable allowance for obsolescence) of property used in the trade or business.

(2) Use of certain methods and rates. The term "reasonable allowance" as used in the preceding paragraph shall include (but not limited to) an allowance computed in accordance with regulations prescribed by the Secretary of Finance, under any of the following methods:

(A) The straight line method.

(B) Declining balance method, using a rate not exceeding twice the rate which would have been used had the annual allowance been computed under the method described in paragraph (f) (1).

(C) The sum-of-the-years-digits method, and

(D) Any other method which may be prescribed by the Secretary of Finance upon recommendation of the Commissioner of Internal Revenue.

Notes: Taxpayer may enter into an "agreement in writing specifically dealing with the useful life and rate of depreciation of any property, the rate so agreed upon shall be biding."

(3-10) A machine costs P7,000, last 8 years and has a salvage value at the end of life of P350. Determine the depreciation charge during the 4th year and the book value at the end of 4 years by the (a) straight line method, (b) declining balance method, (c) SYD method, and (d) sinking fund method with interest at 12%.

Solution

$$C_o = P7,000 \quad C_L = P350 \quad L = 8 \quad n = 4$$

(a) Straight line method

$$d = \frac{C_o - C_L}{L} = \frac{P7,000 - P350}{8} = P831$$

$$C_4 = C_o - D_4 = P7,000 - P831(4) = P3,676$$

(b) Declining balance method

$$k = 1 - \sqrt[8]{\frac{C_L}{C_o}} = 1 - \sqrt[8]{\frac{P350}{P7,000}} = 0.3123$$

$$d_4 = C_o (1 - k)^{4-1} k = P7,000 (1 - 0.3123)^3 (0.3123) = P711$$

$$C_4 = C_o (1 - k)^4 = P7,000 (1 - 0.3123)^4 = P1,566$$

(c) SYD method

$$\begin{array}{r} 1 \quad 8 \\ 2 \quad 7 \\ 3 \quad 6 \\ 4 \quad 5 \\ 5 \quad 26 \\ 6 \\ 7 \\ 8 \end{array} \quad \text{Sum of digits} = \frac{8(8+1)}{2} = 36$$

$$d_4 = (5/36)(P7,000 - P350) = P924$$

$$D_4 = (26/36)(P7,000 - P350) = P4,803$$

$$C_4 = C_o - D_4 = P7,000 - P4,803 = P2,197$$

(d) Sinking fund method

$$d = \frac{C_o - C_L}{F/A, 12\%, 8} = \frac{P7,000 - P350}{12.2997} = P541$$

$$D_4 = d(F/A, 12\%, 4) = P541 (4.7793) = P2,586$$

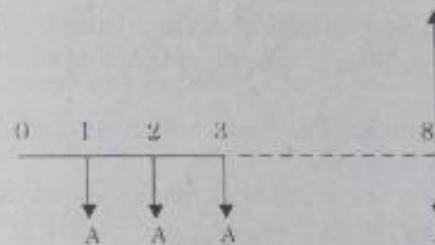
$$C_4 = C_o - D_4 = P7,000 - P2,586 = P4,414$$

(3-11) A machine costs P20,000 and has a salvage value of P2,000 after a useful life of 8 years. Money is worth 12%. If average inflation is 8% per year during this period, what is the annual cost of depreciation to replace the machine after 8 years? What is the annual cost of depreciation if inflation is not considered?

Solution

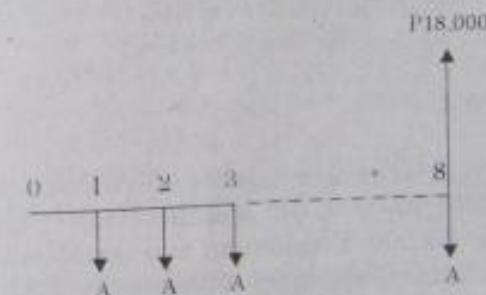
With inflation:

$$\text{Future cost} = PC (1 + i)^n = P20,000 (1 + 0.08)^8 = P37,018$$



$$A = \frac{P37,018}{F/A, 12\%, 8} = 12.2997$$

Without inflation:



$$A = \frac{P18,000}{F/A, 12\%, 8} = \frac{P18,000}{12.2997} = P1,463$$

Valuation

Valuation or appraisal is the process of determining the value of certain property for specific reasons. The person engaged in the task of valuation is called an appraiser.

Intangible Values

In the determination of the value of industrial property or equipment, four intangible items are often encountered.

Goodwill is that element of value which a business has earned through the favorable consideration and patronage of its customers arising from its well-known and well conducted policies and operation.

Franchise is an intangible item of value arising from the exclusive right of a company to provide a specific product or service in a stated region of the country.

Going value is an intangible value which an actually operating concern has due to its operation.

Organization cost is the amount of money spent in organizing a business and arranging for its financing and building.

PROBLEMS

1. A machine shop purchased 10 years ago a milling machine for P60,000. A straight-line depreciation reserve had been provided on a 20 year life of the machine. The owner of the machine shop desires to replace the old milling machine with a modern unit of many advantages costing P100,000. It can sell the old unit for P20,000. How much new capital will be required for the purchase?

Ans. P50,000

2. A tax and duty free importation of a 30 HP sandmill (for paint manufacturing) cost P360,000, CIF Manila. Bank charges, arrastre and brokerage cost P5,000. Foundation and installation costs were P25,000. Other incidental expenses amounted to P20,000. Salvage value of the mill is estimated to be P60,000 after 20 years. Find the appraisal value of the mill, using straight-line depreciation, at the end of (a) 10 years and (b) 15 years.

Ans. (a) P235,000 (b) P147,500

3. On January 1, 1978, the purchasing engineer of a Cement Co. purchased a new machine at a cost of P140,000. Depreciation has been

computed by the straight-line method based on an estimated useful life of five years and residual scrap value of P12,800. On January 2, 1981, extraordinary repairs (which were almost equivalent to a rebuilding of the machinery) were performed at a cost of P30,400. Because of the thorough going nature of these repairs, the normal life of the machinery was extended materially. The revised estimate of useful life was four years from January 1, 1981.

Determine the annual provisions for depreciation for the years 1978 to 1980 and the adjusted provision for depreciation on December 31, 1981. Assume payment in cash for the machine and extraordinary repairs.

Ans. P25,440; P20,320

4. Power to a remote transmitting station is provided by a Diesel-electric generator unit. The original cost of the unit is P65,000. It costs P2,000 to ship the unit to the job site. An additional cost of P3,000 was incurred for installation.

(a) Determine, the annual depreciation cost by the straight line method, if the unit has an expected life of 10 years. The salvage value of the unit at the end of its life was estimated at P5,000.

(b) Determine the annual depreciation cost by the sinking fund method. Assume that the annual charge for depreciation was deposited in a fund drawing compound interest at the rate of 5%

Ans. (a) P6,500 (b) P5,168

5. An industrial plant bought a generator set for P90,000. Other expenses including installation amounted to P10,000. The generator set is to have a life of 17 years with a salvage value at the end of life of P5,000. Determine the depreciation charge during the 13th year and the book value at the end of 13 years by the (a) declining balance method, (b) double declining balance method, (c) sinking fund method at 12% and (d) SYD method.

Ans. (a) P1,949; P10,118 (b) P2,620; P19,649 (c) P1,943; P45,539 (d) P3,105; P11,209

6. A telephone company purchased a microwave radio equipment for P6,000,000. Freight and installation charges amounted to 3% of the purchase price. If the equipment shall be depreciated over a period of 8 years with a salvage value of 5%, determine the following:

(a) Annual depreciation charge using the straight line method.
(b) Depreciation charge during the fifth year using the sum-of-the-years-digits method.

Ans. (a) P733,875 (b) P652,333

Chapter 4

Capital Financing

Equity and Borrowed Capital

Equity capital or ownership funds are those supplied and used by the owners of an enterprise in the expectation that a profit will be earned.

Borrowed funds or capital are those supplied by others on which a fixed rate of interest must be paid and the debt must be repaid at a specified time.

Types of Business Organizations

A. Individual Ownership. The individual ownership or sole proprietorship is the simplest form of business organization, wherein a person uses his or her own capital to establish a business and is the sole owner.

Advantages of the Individual Ownership

1. It is easy to organize.
2. The owner has full control of the enterprise.
3. The owner is entitled to whatever benefits and profits that accrue from the business.
4. It is easy to dissolve.

Disadvantages of the Individual Ownership

1. The amount of equity capital which can be accumulated is limited.

2. The organization ceases upon the death of the owner.
3. It is difficult to obtain borrowed capital, owing to the uncertainty of the life of the organization.
4. The liability of the owner for his debts is unlimited.

B. The Partnership. A partnership is an association of two or more persons for the purpose of engaging in a business for profit.

Advantages of the Partnership

1. More capital may be obtained by the partners pooling their resources together.
2. It is bound by few legal requirements as to its accounts, procedures, tax forms and other items of operation.
3. Dissolution of the partnership may take place at any time by mere agreement of the partners.
4. It provides an easy method whereby two or more persons of differing talents may enter into business, each carrying those burdens that he can best handle.

Disadvantages of the Partnership

1. The amount of capital that can be accumulated is definitely limited.
2. The life of the partnership is determined by the life of the individual partners. When any partner dies, the partnership automatically ends.
3. There may be serious disagreement among the individual partners.
4. Each partner is liable for the debts of the partnership.

C. The Corporation. A corporation is a distinct legal entity, separate from the individuals who own it, and which can engage in almost any type of business transaction in which a real person could occupy himself or herself.

Advantages of the Corporation

1. It enjoys perpetual life without regard to any change in the person of its owners, the stockholders.
2. The stockholders of the corporation are not liable for the debts of the corporation.
3. It is relatively easier to obtain large amounts of money for expansion, due to its perpetual life.
4. The ownership in the corporation is readily transferred.
5. Authority is easily delegated by the hiring of managers.

Disadvantages of the Corporation

1. The activities of a corporation are limited to those stated in its charter.
2. It is relatively complicated in formation and administration.
3. There is a greater degree of governmental control as compared to other types of business organizations.

Capitalization of A Corporation

The capital of a corporation is acquired through the sale of stock. There are two principal types of capital stock: common stock and preferred stock.

Common Stock

Common stock represents ordinary ownership without special guarantees of return. Common stockholders have certain legal rights, among which are the following:

1. Vote at stockholders' meetings.
2. Elect directors and delegates to them power to conduct the affairs of the business.
3. Sell or dissolve the corporation.
4. Make and amend the by laws of the corporation.
5. Subject to government approval, amend, or change the charter or capital structure.
6. Participate in the profits.
7. Inspect the books of the corporation.

Preferred Stock

Preferred stockholders are guaranteed a definite dividend on their stocks. In case the corporation is dissolved, the assets must be used to satisfy the claims of the preferred stockholders before those of the holders of the common stock. Preferred stockholders usually have the right to vote in meetings, but not always.

Financing With Bonds

A bond is a certificate of indebtedness of a corporation usually for a period not less than ten years and guaranteed by a mortgage on certain assets of the corporation or its subsidiaries. Bonds are issued when there is need for more capital such as for expansion of the plant or the services rendered by the corporation.

The face or par value of a bond is the amount stated on the bond.

When the face value has been repaid, the bond is said to have been retired or redeemed. The bond rate is the interest rate quoted on the bond.

Classification of Bonds

1. Registered bonds. The name of the owner of this bond is recorded on the record books of the corporation and interest payments are sent to the owner periodically without any action on his part.
2. Coupon bonds. Coupon bond have coupon attached to the bond for each interest payment that will come due during the life of the bond. The owner of the bond can collect the interest due by surrendering the coupon to the offices of the corporation or at specified banks.

Methods of Bond Retirement

1. The corporation may issue another set of bonds equal to the amount of bonds due for redemption.
2. The corporation may set up a sinking fund into which periodic deposits of equal amount are made. The accumulated amount in the sinking fund is equal to the amount needed to retire the bonds at the time they are due.

A = periodic deposit to the sinking fund

F = accumulated amount, the amount needed to retire the bond

i = rate of interest in the sinking fund

r = bond rate per period

I = interest on the bonds per period

$A + I$ = total periodic expense

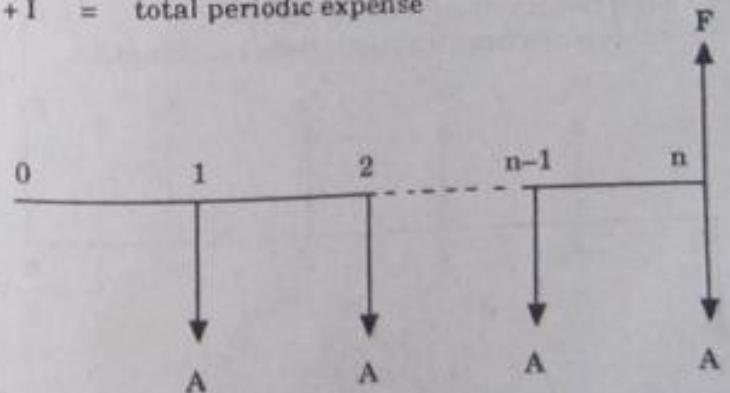


Figure 4-1. Bond retirement by sinking fund

$$A = \frac{F}{F/A, i\%, n} = F(A/F, i\%, n) \quad (4-1)$$

$$I = Fr \quad (4-2)$$

(4-1) A bond issue of P200,000 in 10-year bonds, in P1,000 units, paying 16% nominal interest in semiannual payments, must be retired by the use of a sinking fund that earns 12% compounded semiannually. What is the total semiannual expense?

Solution

$$F = P200,000 \quad r = \frac{16\%}{2} = 8\% \quad i = \frac{12\%}{2} = 6\% \quad n = (10)(2) = 20$$

$$A = \frac{P200,000}{F/A, 6\%, 20} = \frac{P200,000}{36.7856} = P5,437$$

$$I = Fr = (P200,000)(0.08) = P16,000$$

$$\text{Total semiannual expense} = P5,437 + P16,000 = P21,437$$

Bond Value

The value of a bond is the present worth of all future amounts that are expected to be received through ownership of the bond.

Let F = face, or par, value

C = redemption or disposal price (often equal to F)

r = bond rate per period

n = number of periods before redemption

i = investment rate or yield per period

P = value of the bond n periods before redemption

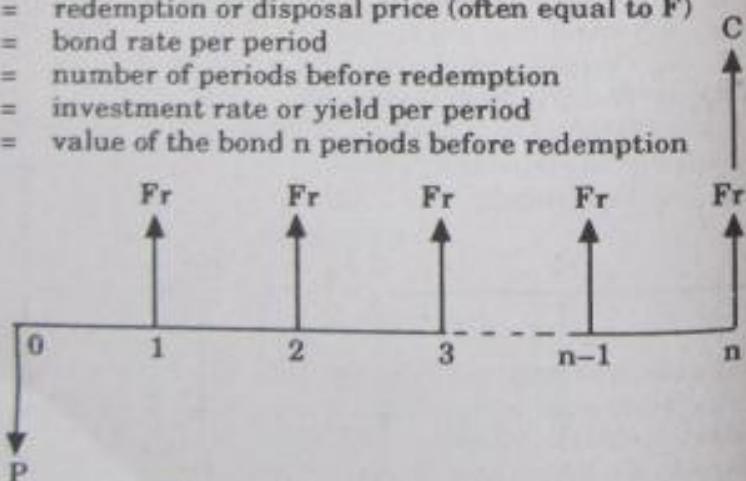


Figure 4-2. Cash flow for a bond investment

$$P = Fr(P/A, i\%, n) + C(P/F, i\%, n) \quad (4-3)$$

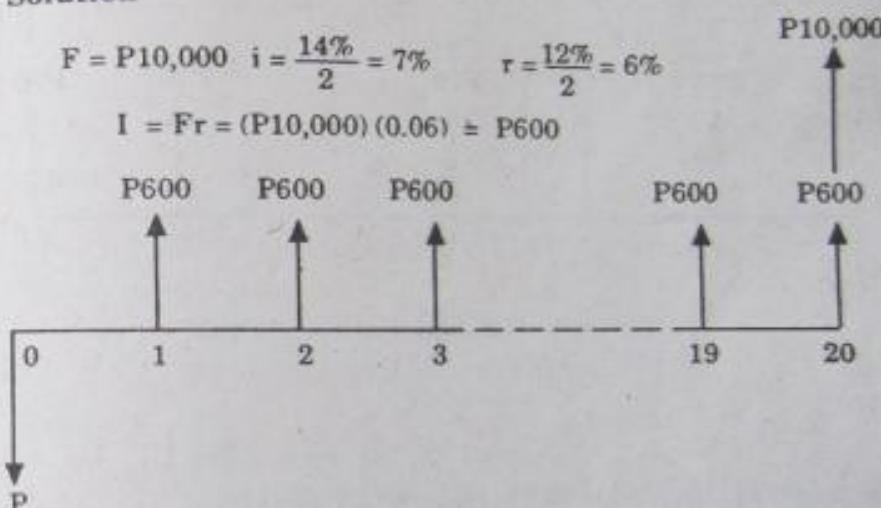
$$P = Fr \left[\frac{1 - (1 + i)^{-n}}{i} \right] + C(1 + i)^{-n} \quad (4-4)$$

(4-2) A man wants to make 14% nominal interest compounded semiannually on a bond investment. How much should the man be willing to pay now for a 12%, P10,000-bond that will mature in 10 years and pays interest semiannually?

Solution

$$F = P10,000 \quad i = \frac{14\%}{2} = 7\% \quad r = \frac{12\%}{2} = 6\%$$

$$I = Fr = (P10,000)(0.06) \approx P600$$



$$P = P600(P/A, 7\%, 20) + P10,000(P/F, 7\%, 20)$$

$$= P600(10.5940) + P10,000(0.2584)$$

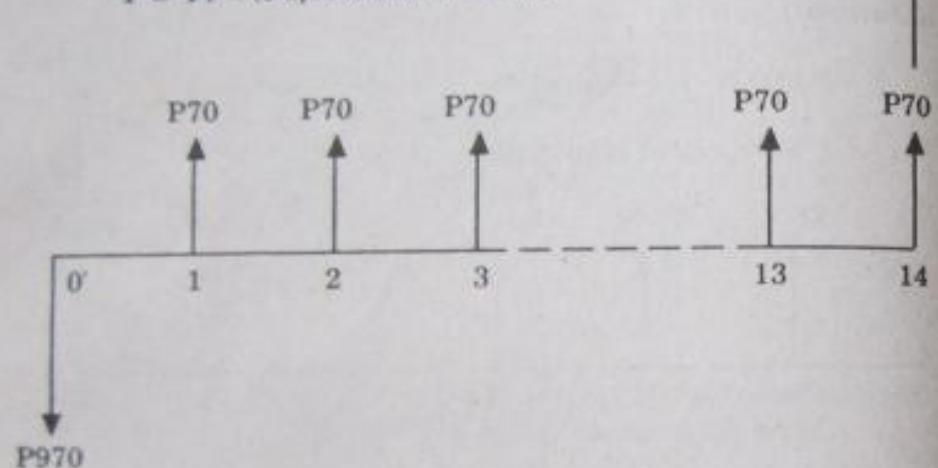
$$= P8,950$$

(4-3) Mr. Romualdo bought a bond having a face value of P1,000 for P970. The bond rate was 14% nominal and interest payments were made to him semiannually for a total of 7 years. At the end of the seventh year, he sold the bond to a friend at a price that resulted a yield of 16% nominal on his investment. What was the selling price?

Solution

$$F = P1,000 \quad i = \frac{16\%}{2} = 8\% \quad r = \frac{14\%}{2} = 7\%$$

$$I = Fr = (P1,000)(0.07) = P70$$



$$P970 = P70(P/A, 8\%, 14) + C(P/F, 8\%, 14)$$

$$P970 = P70(8.2442) + C(0.3405)$$

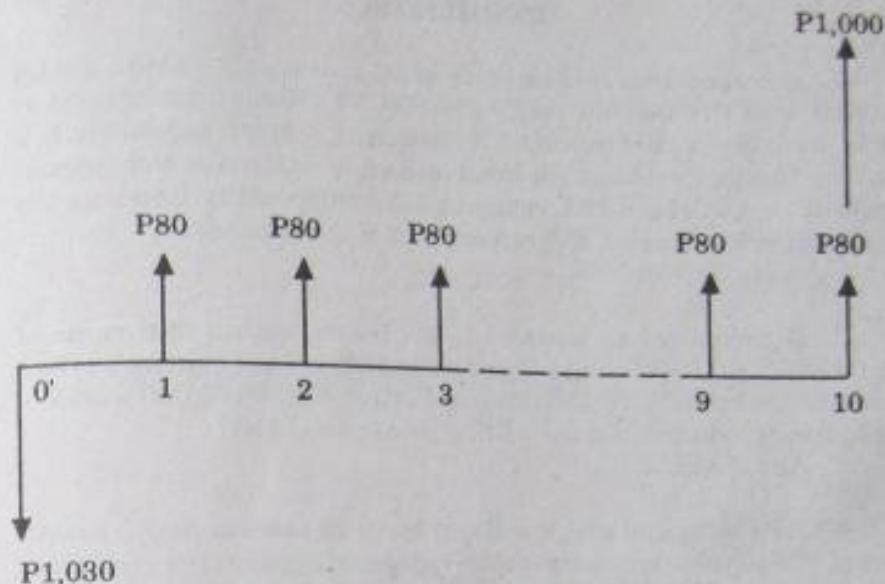
$$C = P1,153.90$$

(4-4) A P1,000-bond which will mature in 10 years and with a bond rate of 8% payable annually is to be redeemed at par at the end of this period. If it is sold at P1,030, determine the yield at this price.

Solution

$$C = P1,000 \quad P = P1,030 \quad r = 8\%$$

$$I = Fr = (P1,000)(0.08) = P80$$



$$P1,030 = P80(P/A, i\%, 10) + P1,000(P/F, i\%, 10)$$

$$\begin{aligned} \text{Try } i &= 8\% \\ &P80(P/A, 8\%, 10) + P1,000(P/F, 8\%, 10) \\ &= P80(6.7101) + P1,000(0.4632) = P1,000.01 \end{aligned}$$

$$\begin{aligned} \text{Try } i &= 7\% \\ &P80(P/A, 7\%, 10) + P1,000(P/F, 7\%, 10) \\ &= P80(7.0236) + P1,000(0.5083) = P1,070.19 \end{aligned}$$

$$1\% \times \left[\begin{array}{cc|c} 8\% & 1,000.01 & \\ i & 1,030.00 & \\ 7\% & 1,070.19 & \end{array} \right] 40.19 \right]^{70.18}$$

$$x = \frac{4.19}{70.18} = 0.57$$

$$i = 7.57\%$$

PROBLEMS

1. A corporation sold an issue of 20-year bonds, having a total face value of P10,000,000 for P9,500,000. The bonds bear interest at 16%, payable semiannually. The company wishes to establish a sinking fund for retiring the bond issue and will make semiannual deposits that will earn 12%, compounded semiannually. Compute the annual cost for interest and redemption of these bonds.

Ans. P1,730,000

2. A company has issued 10-year bonds, with a face value of P1,000,000, in P1,000 units. Interest at 16% is paid quarterly. If an investor desires to earn 20% nominal interest on P100,000 worth of these bonds, what would the selling price have to be?

Ans. P82,836

3. A P1,500-bond which will mature in 10 years and with a bond rate of 15% payable annually is to be redeemed at par at the end of this period. If it is sold now for P1,390, determine the yield at this price.

Ans. 16.56%

Chapter 5

Selections in Present Economy

There are many cases in engineering economy studies where interest is not a factor. These studies are frequently called present economy problems. Such studies usually involve the selection between alternative designs, materials, or methods.

(5-1) An electrical contractor has a job which should be completed in 100 days. At present, he has 80 men on the job and it is estimated that they will finish the work in 130 days. If of the 80 men, 50 are paid P190 a day, 25 at P220 a day, and 5 at P300 a day and if for each day beyond the original 100 days, the contractor has to pay P2,000 liquidated damages:

(a) How many more men should the contractor add so he can complete the work on time?

(b) If the additional men of 5 are paid P220 a day and the rest at P190 a day, would the contractor save money by employing more men and not paying the fine?

Solution

(a) Let x = number of men to be added to complete the job on time

Equating man-days, we have
$$(x + 80)(100) = (80)(130)$$
$$x = 24 \text{ men}$$

(b) 80 men on the job:

| | | | |
|---------|-------------------|---|------------|
| Wages: | (50) (P190) (130) | = | P1,235,000 |
| | (25) (220) (130) | = | 715,000 |
| | (5) (P300) (130) | = | 195,000 |
| Damages | (P2,000) (30) | = | 60,000 |
| | Total expense | | P2,205,000 |

104 men on the job:

| | | | |
|--------|------------------------|---|------------|
| Wages: | (50 + 19) (P190) (100) | = | P1,311,000 |
| | (25 + 5) (220) (100) | = | 660,000 |
| | (5) (P300) (100) | = | 150,000 |
| | Total expense | | P2,121,000 |

$$\begin{aligned}\text{Saving by employing more men} &= \text{P}2,205,000 - \text{P}2,121,000 \\ &= \text{P}84,000\end{aligned}$$

(5-2) The monthly demand for ice cans being manufactured by Mr. Cruz is 3,200 pieces. With a manually operated guillotine, the unit cutting cost is P25.00. An electrically operated hydraulic guillotine was offered to Mr. Cruz at a price of P275,000 and which will cut by 30% the unit cutting cost. Disregarding the cost of money, how many months will Mr. Cruz be able to recover the cost of the machine if he decides to buy now?

Solution

Manually Operated Guillotine

$$\text{Monthly cutting cost} = (3,200) (\text{P}25) = \text{P}80,000$$

Electrically Operated Hydraulic Guillotine

$$\text{Monthly cutting cost} = (3,200) (\text{P}25) (1 - 0.30) = \text{P}56,000$$

$$\text{Savings} = \text{P}80,000 - \text{P}56,000 = \text{P}24,000 \text{ per month}$$

$$\begin{aligned}\text{No. of months to recover cost of machine} &= \frac{\text{P}275,000}{\text{P}24,000} \\ &= 11.5\end{aligned}$$

(5-3) The making of rivets holes in structural steel members can be done by two methods. The first method consists of laying out the position of the holes in the members and using a drill press costing P30,000. The machinist is paid P35 per hour and he can drill 30 holes per hour.

The second method makes use of a multiple-punch machine costing P27,500. The punch operator is paid P30 an hour and he can punch out 4 holes every minute. This method also requires an expense of P1.75 per hole to set the machine.

(a) If all other costs are assumed equal, what is the total cost for each machine for 6,000 holes, assuming the total cost of each machine to be charged to these holes?

(b) For how many holes will the costs be equal?

Solution

(a) Drill Press:

$$\text{Time to drill 6,000 holes} = \frac{6,000}{30} = 200 \text{ hours}$$

$$\begin{array}{rcl} \text{wage of machinist} & = & (\text{P}35) (200) \\ & = & \text{P} 7,000 \\ \text{cost of drill press} & = & \underline{\text{P} 30,000} \\ \text{Total cost} & & \text{P}37,000 \end{array}$$

Multiple Punch Machine:

$$\text{Time to punch 6,000 holes} = \frac{6,000}{(4)(60)} = 25 \text{ hours}$$

| | | | | |
|-----------------------------|---|----------------|---|---------|
| wage of machinist | = | (P30) (25) | = | P 750 |
| cost of setting the machine | = | (1.75) (6,000) | = | 10,500 |
| cost of machine | = | | | 27,500 |
| Total cost | | | | P38,750 |

(b) Let x = number of holes for equal cost

$$\text{cost of drill press} = \frac{x}{30} (\text{P35}) + \text{P30,000} = \frac{7x}{6} + \text{P30,000}$$

$$\begin{aligned}\text{cost for multiple punch machine} &= \frac{x}{(4)(60)} (\text{P30}) + (\text{P1.75})(x) \\ &\quad + \text{P27,500}\end{aligned}$$

$$= \frac{x}{8} + 1.75x + \text{P27,500}$$

$$\text{For equal cost, } \frac{7x}{6} + 30,000 = \frac{x}{8} + 1.75x + 27,500$$

$$x = 3530 \text{ holes}$$

(5-4) The volume of the raw material required for a certain machine part is 2.02 cu. cm. The finished volume is 1.05 cu cm. The time for machining each piece is 45 seconds for steel and 30 seconds for brass. The costs of steel is P32.50 per kg and the value of steel scrap is negligible. The cost of the brass is P60.00 per kg and the value of brass scrap is P25.00 per kg. The wage of the operator is P40.00 per hour and the overhead cost of the machine is P50.00 per hour. The weight of steel and brass are 0.0081 and 0.0088 kg per cu cm, respectively. Which material will you recommend?

Solution

Steel:

$$\begin{aligned}\text{Weight per piece} &= (2.02)(0.0081) = 0.01636 \text{ kg} \\ \text{cost per piece:} &\end{aligned}$$

$$\text{material} = (0.01636)(\text{P32.50}) = \text{P} 0.5317$$

$$\text{wage of operator} = \left(\frac{45}{3,600}\right)(\text{P}40.00) = 0.5000$$

$$\begin{aligned}\text{overhead} &= \left(\frac{45}{3,600}\right)(\text{P}50.00) = 0.6250 \\ \text{Total cost} &= \text{P} 1.6567\end{aligned}$$

Brass:

$$\text{weight per piece} = (2.02)(0.0088) = 0.01778 \text{ kg}$$

cost per piece:

$$\text{material} = (0.01778)(\text{P}60.00) = \text{P} 1.668$$

$$\text{wage of operator} = \left(\frac{30}{3,600}\right)(\text{P}40.00) = 0.3333$$

$$\text{overhead} = \left(\frac{30}{3,600}\right)(\text{P}50.00) = 0.4167$$

$$\begin{aligned}\text{less cost of scrap} &= (2.02 - 1.05)(0.0088)(\text{P}25.00) = 0.2134 \\ \text{Total cost} &= \text{P} 1.6034\end{aligned}$$

Use Steel

(5-5)' High carbon steel or alloy steel can be used for the set of tools on a lathe. The tools must be sharpened periodically. Data for each are as follows:

| | High carbon steel | Alloy steel |
|-------------------------------|-------------------|-------------|
| Output per hour | 60 pcs | 70 pcs |
| Time between tool grinds | 4 h | 6 h |
| Time required to change tools | 1 h | 1 h |

The wage of the lathe operator is P50.00 per hour, based on actual working hours. The tool changer costs P60.00 per hour. Overhead costs for the lathe are P45.00 per hour, including tool-change time. A set of unsharpened high carbon steel costs P500 and can be ground ten times; a set of unsharpened alloy steel costs P850 and can be ground five times. Which type of steel should be used?

Solution

High carbon steel:

$$\begin{aligned}\text{cycle time} &= 4 + 1 = 5 \text{ hours} \\ \text{number of pieces per cycle} &= (4)(60) = 240\end{aligned}$$

Cost per cycle:

$$\text{Operator} = \text{P}50(4) = \text{P}200$$

$$\text{Tool changer} = (\text{P}60)(1) = 60$$

$$\text{Overhead} = (\text{P}45)(5) = 225$$

$$\begin{aligned}\text{Unsharpened tool} &= \frac{\text{P}500}{10} = \frac{50}{\text{P}535} \\ \text{Total} &\end{aligned}$$

$$\text{Cost per piece} = \frac{\text{P}535}{240} = \text{P}2.23$$

Alloy steel:

$$\text{cycle time} = 6 + 1 + 7 \text{ hours}$$

$$\text{number of pieces per cycle} = (6)(70) = 420$$

Cost per cycle:

| | | |
|--------------------------|---|------|
| Operator = P50 (6) | = | P300 |
| Tool changer = (P60) (1) | = | 60 |
| Overhead = (P45) (7) | = | 315 |
| Unsharpened tool = P850 | = | 170 |
| Total | | P845 |

$$\text{Cost per piece} = \frac{\text{P}845}{420} = \text{P}2.02$$

Use alloy steel.

PROBLEMS

1. An industrial engineer has designed two alternative methods for accomplishing a production job. Both methods involve the acquisition of the same working place and other capital equipment to be used for this job only.

Method A calls for a crew consisting of three men each costing P30.00 per hour. This method will result in the production of 10 units per hour of which two will be rejects.

Method B, calls for a crew of two men each costing P35.00 per hour and should result in the production of eight units per hour of which one will be reject.

The cost of the direct material lost in each reject is P20.00. If a certain total number of units is to be produced, find which method is not economical.

Ans. P16.25; P12.86; Method B

2. An executive receives an annual salary of P600,000 and his secretary a salary of P180,000. A certain task can be performed by the executive working alone in 4 hours. If he delegates the task to his secretary it will require him 30 minutes to explain the work and another 45 minutes to check the finished work. Due to the unfamiliarity of the secretary to do the task, it takes her an additional

time of 6 hours after being instructed. Considering salary cost only, determine the cost of performing the task by each method, if the secretary works 2,400 hours a year and the executive 3,000 hours a year.

Ans. P800.00; P737.50

3. A cement grinding mill "A" with a capacity of 50 tons per hour utilizes forged steel grinding balls costing P12,000 per ton, which have a wear rate of 100 grams per ton cement milled. Another cement mill "B" of the same capacity uses high chrome steel grinding balls costing P50,000 per ton with wear rate of 20 grams per ton cement milled. Determine the more economical grinding mill, considering other factors to be the same.

Ans. P60 per hr.; P50 per hr., Mill "B" is more economical

4. A cement kiln with production capacity of 130 tons per day (24 hours) of clinker has at its burning zone about 45 tons of magnesite chrome bricks being replaced periodically, depending on some operational factors and the life of the bricks.

If locally produced bricks costs P25,000 per ton and have a life of 4 months, while certain imported bricks costing P30,000 per ton and have a life of 6 months, determine the more economical bricks and by how much.

Ans. Imported bricks are more economical by P56,250 per month.

5. A manufacturer has been shipping his product (moderately heavy machines), mounted only on skids without complete crating. To avoid crating he must ship in freight cars which contain only his machines. To do this he must pay freight on a car capacity load of 42 tons regardless of whether or not the car is completely full. In the past he actually has shipped only 30 tons in each car. The car load freight rate is P4.10 per hundred pounds. If the machines are crated so that they can be shipped in mixed car lots, along with other merchandise, they can be shipped at a rate of P4.20 per hundred pounds with the freight bill computed only on the actual weight shipped. The cost of crating would be P25.00 per machine and would increase the shipping weight from 1,200 to 1,220 pounds per machine.

Which procedure should be followed? (1 ton = 2,200 lbs.)

Ans. Shipping without crating is cheaper by P404.80

6. A machine used for cutting materials in a factory has the following outputs per hour at various speeds and required periodic tool regrinding at the intervals cited.

| Speed | Output per hour | Tool regrinding |
|-------|-----------------|-----------------|
| A | 200 pieces | Every 8 hours |
| B | 280 pieces | Every 5 hours |

A set of tools costs P1,260 and can be ground twenty times. Each regrinding costs P54.00 and the time needed to regrind and change tools is 1 hour. The machine operator is paid P35.00 per hour, including the time the tool is changed. The tool grinder who also sets the tools to the machine is paid P40.00 per hour. The hourly rate chargeable against the machine is P38.00, regardless of machine speed. Which speed is the most economical?

Ans. Speed B is cheaper by P0.087/pc.

Chapter 6

Basic Methods for Making Economy Studies

BASIC METHODS OR PATTERNS FOR MAKING ECONOMY STUDIES

THE RATE OF RETURN (ROR) METHOD

The rate of return on the capital invested is given by the formula,

$$\text{Rate of return} = \frac{\text{net annual profit}}{\text{capital invested}} \quad (6-1)$$

Rate of return is a measure of the effectiveness of an investment of capital. It is a financial efficiency. When this method is used, it is necessary to decide whether the computed rate of return is sufficient to justify the investment. The advantage of this method is that it is easily understood by management and investors. The applications of the rate of return method is controlled by the following conditions. A single investment of capital at the beginning of the first year of the project life and identical revenue and cost data for each year. The capital invested is the total amount of capital investment required to finance the project, whether equity or borrowed.

THE ANNUAL WORTH (AW) METHOD

In this method, interest on the original investment (sometimes called minimum required profit) is included as a cost. If the excess of annual cash inflows over annual cash outflows is not less than zero the proposed investment is justified – is valid. This method is covered by

the same limitations as the rate of return pattern a single initial investment of capital and uniform revenue and cost throughout the life of the investment.

THE PRESENT WORTH (PW) METHOD

This pattern for economy studies is based on the concept of present worth. If the present worth of the net cash flows is equal to, or greater than, zero, the project is justified economically. The present worth method is flexible and can be used for any type of economy study. It is used extensively in making economy studies in the public works field, where long-lived structures are involved.

THE FUTURE WORTH (FW) METHOD

The future worth method for economy studies is exactly comparable to the present worth method except that all cash inflows and outflows are compounded forward to a reference point in time called the future. If the future worth of the net cash flows is equal to, or greater than, zero, the project is justified economically.

THE PAYBACK (PAYOUT) PERIOD METHOD

The payback period is commonly defined as the length of time required to recover the first cost of an investment from the net cash flow produced by that investment for an interest rate of zero.

$$\text{Payout period (years)} = \frac{\text{investment} - \text{salvage value}}{\text{net annual cash flow}} \quad (6-2)$$

(6-1) An investment of P270,000 can be made in a project that will produce a uniform annual revenue of P185,400 for 5 years and then have a salvage value of 10% of the investment. Out-of-pocket costs for operation and maintenance will be P81,000 per year. Taxes and insurance will be 4% of the first cost per year. The company expects capital to earn not less than 25% before income taxes. Is this a desirable investment? What is the payback period of the investment?

Solution

By the rate of return method

| | |
|--|-----------|
| Annual revenue | P185,400 |
| Annual costs: | |
| Depreciation = $\frac{P270,000 - P27,000}{F/A, 25\%, 5}$ | = P29,609 |
| Operation and maintenance | = 81,000 |
| Taxes and insurance = P270,000 (0.04) | = 10,800 |
| Total annual cost | P121,409 |
| Net annual profit | P 63,991 |
| Rate of return = $\frac{P63,991}{P270,000} \times 100$ | = 23.70% |

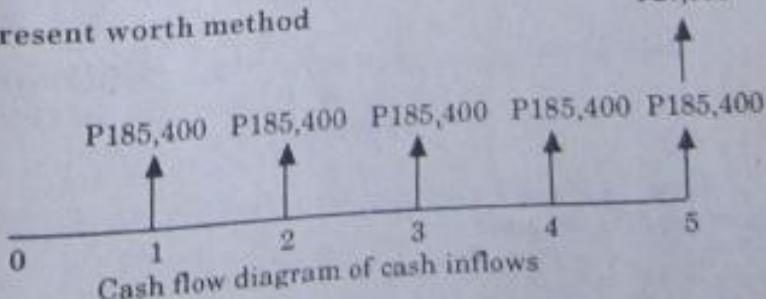
Since the rate of return is less than 25%, the investment is not justified.

By the annual worth method

| | |
|--|-----------|
| Annual revenue | P185,400 |
| Annual costs: | |
| Depreciation = $\frac{P270,000 - P27,000}{F/A, 25\%, 5}$ | = P29,609 |
| Operation and maintenance | = 81,000 |
| Taxes and insurance = P270,000 (0.04) | = 10,800 |
| Interest on capital = P270,000 (0.25) | = 67,500 |
| Total annual cost | P188,909 |
| Excess | - P 3,509 |

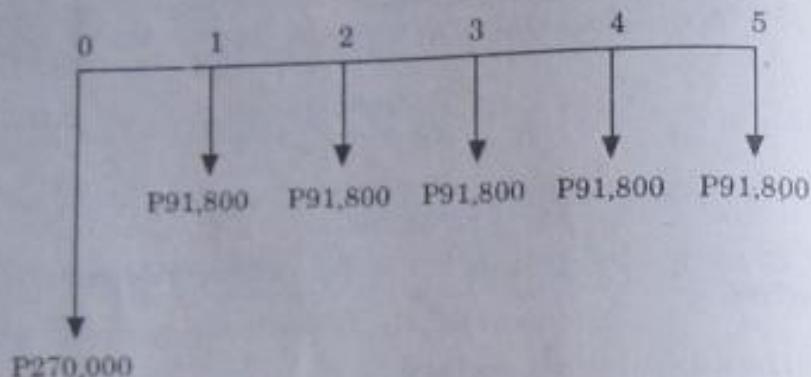
Since the excess of annual cash inflows over annual cash outflows is less than zero (-P3,509), the investment is not justified.

By the present worth method



$$\begin{aligned}
 \text{PW of cash inflows} &= P185,400 (P/A, 25\%, 5) + P27,000 (P/F, 25\%, 5) \\
 &= P185,400 (2.6893) + P27,000 (0.3277) \\
 &= P506,370
 \end{aligned}$$

$$\begin{aligned}
 \text{Annual costs (excluding depreciation)} &= P81,000 + P270,000 (0.04) \\
 &= P91,800
 \end{aligned}$$



Cash flow diagram of cash outflows

$$\text{PW of cash outflows} = P270,000 + P91,800 (P/A, 25\%, 5) = P516,880$$

Since the PW of the net cash flows is less than zero (-P10,510), the investment is not justified.

By the future worth method

Referring to the cash flow diagrams in the solution by the PW method.

$$\begin{aligned}
 \text{FW of cash inflows} &= P27,000 + P185,400 (F/A, 25\%, 5) \\
 &= P27,000 + P185,400 (8.2070) \\
 &= P1,548,580
 \end{aligned}$$

$$\begin{aligned}
 \text{FW of cash outflows} &= P91,800(F/A, 25\%, 5) + P270,000(F/P, 25\%, 5) \\
 &= P91,800(8.2070) + P270,000(3.0518) \\
 &= P1,577,390
 \end{aligned}$$

Since the FW of the net cash flows is less than zero (-P28,810), the investment is not justified.

By the payback period

$$\begin{aligned}
 \text{Total annual cost} &= P81,000 + P270,000 (0.04) = P91,800 \\
 \text{Net annual cash flows} &= P185,400 - P91,800 = P93,600
 \end{aligned}$$

$$\begin{aligned}
 \text{Payback period} &= \frac{\text{investment} - \text{salvage value}}{\text{net annual cash flows}} = \frac{P270,000 - P27,000}{P93,600} \\
 &= 2.6 \text{ years}
 \end{aligned}$$

In computing the total annual cost, depreciation was not included because the method does not consider the time value of money or interest. The use of the payback period for making investment decisions should be avoided as it may produce misleading results.

(6-2) A businessman is considering building a 25-unit apartment in a place near a progressive commercial center. He felt that because of the location of the apartment it will be occupied 90% at all time. He desires a rate of return of 20%. Other pertinent data are the following:

| | |
|---------------------------------|-------------|
| Land investment | P 5,000,000 |
| Building investment | 7,000,000 |
| Study period | 20 years |
| Cost of land after 20 years | 20,000,000 |
| Cost of building after 20 years | 2,000,000 |
| Rent per unit per month | 6,000 |
| Upkeep per unit per year | 500 |
| Property taxes | 1% |
| Insurance | 0.50% |

Is this a good investment?

Solution

Annual income:

$$\text{Rental} = (P6,000)(12)(25)(0.90) = P1,620,000$$

$$\begin{aligned}
 \text{Land} &= \frac{P20,000,000 - P5,000,000}{F/A, 20\%, 20} = \frac{P15,000,000}{186.688} = \underline{\underline{80,350}} \\
 &\text{Total annual income} \quad P1,700,350
 \end{aligned}$$

Annual costs:

| | |
|--|------------|
| Depreciation = $\frac{P7,000,000 - P2,000,000}{F/A, 20\%, 20}$ | = P 26,780 |
| Upkeep = P500 (25) | = 12,500 |
| Taxes = P12,000,000 (0.01) | = 120,000 |
| Insurance = P7,000,000 (0.005) | = 35,000 |
| Total annual cost | P194,280 |
| Net annual profit | P1,506,070 |

$$\text{Rate of return} = \frac{P 1,506,070}{P12,000,000} \times 100 = P12.55\% < 20\%$$

The businessman should not invest.

Another solution

| | |
|--------------------------------------|----------------------------|
| Investment = P5,000,000 + P7,000,000 | = P12,000,000 |
| Amount of investment after 20 years | = P20,000,000 + P2,000,000 |
| | = P22,000,000 |

$$\text{Annual income} = (P6,000) (12) (25) (0.90) = P1,620,000$$

Annual costs:

| | |
|--|------------|
| Depreciation = $\frac{P12,000,000 - P22,000,000}{F/A, 20\%, 20}$ | = P-53,570 |
| Upkeep = P500 (25) | = 12,500 |
| Taxes = P12,000,000 (0.01) | = 120,000 |
| Insurance = P7,000,000 (0.005) | = 35,000 |
| Total annual cost | P113,930 |
| Net annual profit | P1,506,070 |

$$\text{Rate of return} = \frac{P 1,506,070}{P12,000,000} \times 100 = 12.55\% < 20\%$$

The business man should not invest.

(The negative sign for depreciation means that the value of the investment has increased after 20 years.)

(6-3) A man is considering investing P500,000 to open a semi-automatic auto-washing business in a city of 400,000 population. The equipment can wash, on the average, 12 cars per hour, using two men to operate it and to do small amount of hand work. The man plans to hire two men, in addition to himself, and operate the station on an 8-hour basis, 6 days per week, 50 weeks per year. He will pay his employees P25.00 per hour. He expects to charge P25.00 for a car wash. Out-of-pocket miscellaneous cost would be P8,500 per month.

He would pay his employees for 2 week for vacations each year. Because of the length of his lease, he must write off his investment within 5 years. His capital now is earning 15%, and he is employed at a steady job that pays P25,000 per month. He desires a rate of return of at least 20% on his investment.

Would you recommend the investment?

Solution

By the rate of return method

$$\begin{array}{lcl} \text{Annual revenue} = (12) (25) (8) (6) (50) & & = P720,000 \\ \text{Annual costs:} & & \\ \end{array}$$

$$\text{Depreciation } \frac{P500,000}{F/A, 15\%, 5} = P74,160$$

$$\text{Labor} = (2) (48) (50) (P25.00) = 120,000$$

$$\text{Vacation pay} = (2) (2) (48) (25.00) = 4,800$$

$$\text{Miscellaneous} = P8,500 (12) = 102,000$$

$$\text{Owner's salary} = P25,000 (12) = 300,000$$

$$\begin{array}{lcl} \text{Total annual cost} & & P600,960 \\ \text{Net annual profit} & & P119,040 \end{array}$$

$$\text{Rate of return} = \frac{P119,040}{P500,000} \times 100 = 23.81\% > 20\%$$

The man should invest.

By the annual worth method

$$\begin{array}{lcl} \text{Annual revenue} = (12) (P25) (8) (6) (50) & & P720,000 \\ \text{Annual costs:} & & \\ \end{array}$$

$$\text{Depreciation} = \frac{P500,000}{F/A, 15\%, 5} = P74,160$$

$$\text{Labor} = (2) (48) (50) (P25) = 120,000$$

$$\text{Vacation pay} = (2) (2) (48) (P25.00) = 4,800$$

$$\text{Miscellaneous} = P25,000 (12) = 300,000$$

$$\text{Interest on capital} = P500,000 (0.20) = 100,000$$

$$\begin{array}{lcl} \text{Total annual cost} & & P700,960 \\ \text{Excess} & & P19,040 \end{array}$$

Since the excess of annual revenue over annual cost is greater than zero, the investment is justified. The man should invest.

(6-4) A firm is considering purchasing equipment that will reduce

costs by P40,000. The equipment costs P300,000 and has a salvage value of P50,000 and a life of 7 years. The annual maintenance cost is P6,000. While not in use by the firm, the equipment can be rented to others to generate an income of P10,000 per year. If money can be invested for an 8 per cent return, is the firm justified in buying the equipment?

Solution

| | |
|--|---|
| Annual savings: | |
| Reduction in annual cost | = P40,000 |
| Rental | = <u>10,000</u> |
| | Total annual savings |
| | P50,000 |
| Annual costs: | |
| Depreciation = $\frac{P300,000 - P50,000}{F/A, 8\%, 7}$ | = P250,000 / 8.9228 = P28,018 |
| Maintenance | = 6,000 |
| | Total annual cost |
| | P34,018 |
| | Annual net savings |
| | P15,982 |
| Rate of return = $\frac{\text{annual net savings}}{\text{capital}} \times 100$ | = $\frac{P15,982}{P300,000} \times 100$ |
| | = 5.33% < 8% |

The equipment should not be purchased.

(6-5) The MGC company has a contract with a hauler to transport its naptha requirements of 3,600,000 liter per year from a refinery in Batangas to its site in Paco at a cost of P1.05 per liter. It is proposed that the company buys a tanker with a capacity of 18,000 liters to service its requirements at a first cost of P8,000,000 life is 6 years and a salvage value of P800,000. Other expenses are as follows:

- (a) Diesel fuel at P7.95 per liter and the tanker consumes 120 liter per round trip from Paco to Batangas and back.
- (b) Lubricating oil and servicing is P3,200 per month.
- (c) Labor including overtime and fringe benefits for one driver and one helper is P21,000 per month.
- (d) Annual taxes and insurance, 5% of first cost.
- (e) General maintenance per year is P40,000.
- (f) Tires cost P32,000 per set and will be renewed every 150 round trips.

What should the MGC company do if a 15% interest rate on investment is included in the analysis?

Solution

Hauling

$$\text{annual cost} = (P1.05)(3,600,000) = P3,780,000$$

Buying Tanker

Annual cost:

| | | |
|---|-------------|--------------|
| Depreciation = $\frac{P8,000,000 - P800,000}{F/A, 15\%, 6}$ | = P7,200.00 | = P822,575 |
| Fuel = $\frac{3,600,000}{18,000} (120) (P7.95)$ | = 190,800 | |
| Oil = (P3,200) (12) | = 38,400 | |
| Labor = (P21,000) (12) | = 252,000 | |
| Taxes & insurance = (P8,000,000) (0.05) | = 400,000 | |
| Maintenance | = 40,000 | |
| Tires = $\frac{3,600,000}{(18,000) (150)} (P32,000)$ | = 42,667 | |
| Total annual cost | | P1,786,442 |
| Annual savings = P3,780,000 - P1,786,442 | | = P1,993,558 |
| rate of return = $\frac{P1,993,558}{P8,000,000} 100 = 24.92\% > 15\%$ | | |

The company should buy the tanker.

(6-6) A machine that is not equipped with a brake "coasts" 30 seconds after the power is turned off upon completion of each piece, thus preventing removal of the work from the machine. The time per piece, exclusive of this stopping time is 2 minutes. The machine is used to produce 40,000 pieces per year. The operator receives P35.00 per hour and the machine overhead rate is P20.00 per hour.

How much could the company afford to pay for a brake that would reduce the stopping time to 3 seconds, if it would have a life of 5 years? Assume zero salvage value, capital worth 18% and that repairs and maintenance would total not over P300.00 per month.

Solution

$$\text{Annual savings} = \frac{27}{3,600} (P35 + P20)(40,000) = P16,500$$

Let C = the amount that can be invested for the brake

$$\begin{aligned} \text{Annual costs: } & \frac{C}{F/A, 10\%, 5} = \frac{C}{6.1051} = P 0.1638C \\ \text{Depreciation} &= \frac{P1,200,000 - P700,000}{F/A, 20\%, 10} = P 3,600 \\ \text{Repairs and maintenance} &= (P300) (12) = P 3,600 \\ \text{Interest on capital} &= P 1,200,000 (0.025) = P 3,000 \\ \text{Total annual cost} &= P 0.3438C + P 3,600 \end{aligned}$$

Equating annual savings to total annual cost,

$$\begin{aligned} P 0.3438C + P 3,600 &= P 16,500 \\ C &= P 37,522 \end{aligned}$$

In any problem where the unknown quantity is the investment or capital, the best method to use is the annual worth method.

(6-7) A newly-built business property, containing space for a store and two offices, can be purchased for P1,200,000. A prospective buyer estimates that during the next 10 years he can obtain annual rentals of at least P458,460 from the property, and that the annual out-of-pocket disbursements will not exceed P60,000. He believes that he should be able to dispose of the property at the end of 10 years at not less than P700,000. Annual taxes and insurance will total 2.5% of the first cost.

(a) Assume he has sufficient equity capital to purchase the property, and that the average return he is obtaining from his capital is 20%. Would you recommend the investment?

(b) What recommendation would you make if he had to borrow 25% of the required capital, on the basis of a 10-year amortization with interest of 15%?

(c) If the entire capital can be obtained by floating bonds at 15% that will mature in 10 years, what would you recommend? Sinking fund interest is 15%.

Solution

$$\begin{aligned} \text{(a) Annual revenue} &= P 458,460 \\ \text{Annual costs:} & \\ \text{Depreciation} &= \frac{P1,200,000 - P700,000}{F/A, 20\%, 10} = P 19,260 \\ \text{Disbursements} &= 60,000 \\ \text{Taxes & insurance} &= P 1,200,000 (0.025) = 30,000 \\ \text{Total annual cost} &= P 109,260 \\ \text{Net annual profit} &= P 349,200 \end{aligned}$$

$$\text{Rate of return} = \frac{P349,200}{P1,200,000} \times 100 = 29.1 > 20\% \\ \text{The investment is justified.}$$

$$\begin{aligned} \text{(b) Annual revenue} &= P 458,460 \\ \text{Annual costs:} & \\ \text{Depreciation} &= \frac{P900,000 - P700,000}{F/A, 20\%, 10} = P 7,700 \\ \text{Amortization} &= \frac{P1,200,000(0.25)}{P/A, 18\%, 10} = 66,760 \\ \text{Disbursements} &= 60,000 \\ \text{Taxes & insurance} &= P 1,200,000 (0.025) = 30,000 \\ \text{Total annual cost} &= P 164,460 \\ \text{Net annual profit} &= P 294,000 \end{aligned}$$

$$\text{Rate of return} = \frac{P294,000}{P1,200,000} \times 100 = 24.50\% < 20\%$$

The investment is justified.

$$\begin{aligned} \text{(c) Annual revenue} &= P 458,460 \\ \text{Annual costs:} & \\ \text{Sinking fund deposit} &= \frac{P1,200,000 - P700,000}{F/A, 15\%, 10} = P 24,630 \\ \text{Bond interest} &= P 1,200,000 (0.15) = 180,000 \\ \text{Disbursements} &= 60,000 \\ \text{Taxes & insurance} &= P 1,200,000 (0.025) = 30,000 \\ \text{Total annual cost} &= P 294,630 \\ \text{Net annual profit} &= P 163,830 \end{aligned}$$

$$\text{Rate of return} = \frac{P163,830}{P1,200,000} \times 100 = 13.65\% < 20\%$$

The investment is not justified.

(6-8) A proposed project will require the immediate investment of P50,000 and is estimated to have year-end revenues and costs as follows:

| Year | Revenue | Costs |
|------|----------|----------|
| 1 | P 75,000 | P 60,000 |
| 2 | 90,000 | 77,500 |
| 3 | 100,000 | 75,000 |
| 4 | 95,000 | 80,000 |
| 5 | 60,000 | 47,500 |

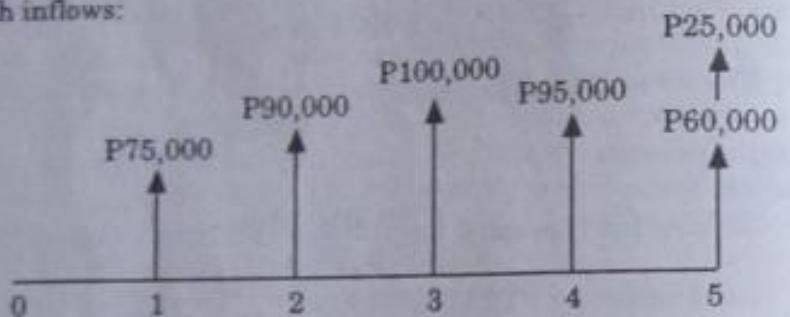
An additional investment of P20,000 will be required at the end of the second year. The project would terminate at the end of the 5th year, and the assets are estimated to have a salvage value of P25,000 at that time.

Is this a good investment?

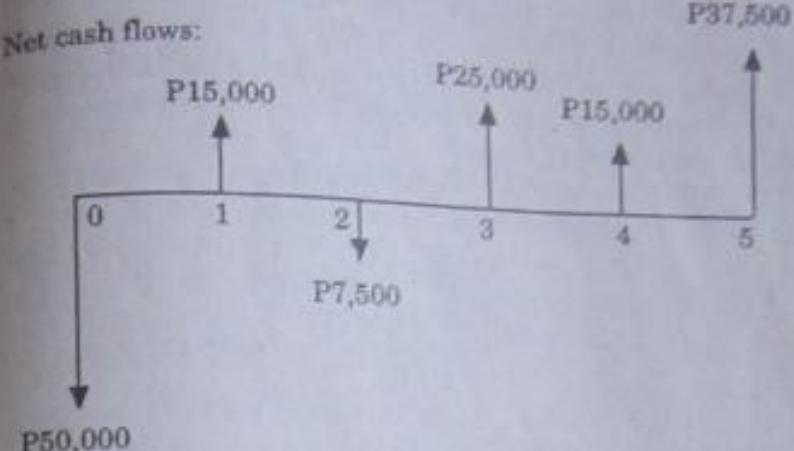
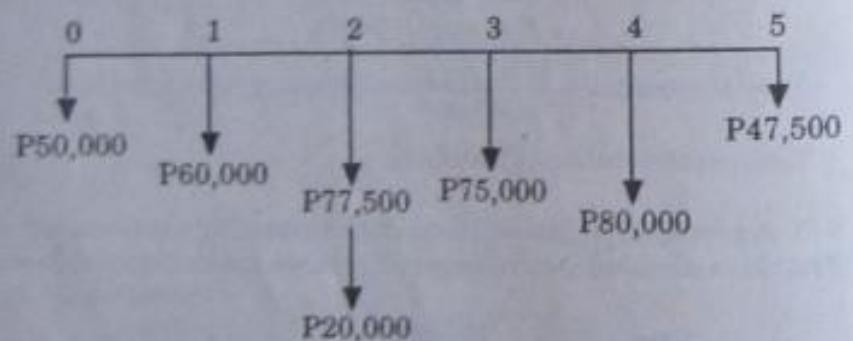
Solution

Solve for the internal rate of return (IRR) and then decide whether the investment is justified or not. The internal rate of return is that rate of return that will exactly reduce the worth of the investment to zero at the end of the life of the investment. Thus, present worth of cash inflows minus present worth of cash outflows must equal to zero.

Cash inflows:



Cash outflows:



$$\begin{aligned} \text{PW of net cash flows} &= -P50,000 + P15,000(1+i)^{-1} \\ &\quad - P7,500(1+i)^{-2} + P25,000(1+i)^{-3} \\ &\quad + P15,000(1+i)^{-4} + P37,500(1+i)^{-5} = 0 \end{aligned}$$

Try $i = 15\%$:

$$\text{PW of net cash flows} = P1,032.75$$

Try $i = 16\%$:

$$\text{PW of net cash flows} = -P486.75$$

$$1520.5 \left[\begin{array}{cc|c} 1032.75 & 15\% & x \\ 0 & i & \\ \hline -486.75 & 16 & \end{array} \right]^{1\%}$$

$$\frac{x}{1} = \frac{1032.75}{1520.50}$$

$$x = 0.68$$

$$i = 15 + 0.68 = 15.68\%$$

The investment is not justified. The internal rate of return of 15.68% is easily obtained in easy investment.

PROBLEMS

1. A young mechanical engineer is considering establishing his own small company. An investment of P400,000 will be required, which will be recovered in 15 years.

It is estimated that sales will be P800,000 per year and that operating expenses will be as follows

| | |
|-----------------|---------------------------------|
| Materials | P160,000 per year |
| Labor | P280,000 per year |
| Overhead | P40,000 + 10% of sales per year |
| Selling expense | P60,000 per year |

The man will give up his regular job paying P216,000 per year and devote full time to the operation of the business; this will result in decreasing labor cost by P40,000 per year, material cost by P28,000 per year and overhead cost by P32,000 per year. If the man expects to earn at least 20% of his capital, should he invest?

Ans. The rate of return is 6.61%, the man should not invest.

2. The ABC company is considering constructing a plant to manufacture a proposed new product. The land costs P15,000,000, the building costs P30,000,000, the equipment costs P12,500,000, and P5,000,000 working capital is required. At the end of 12 years, the land can be sold for P25,000,000, the building for P12,000,000, the equipment for P250,000 and all of the working capital recovered. The annual disbursements for labor, materials, and all other expenses are estimated to cost P23,750,000. If the company requires a minimum return of 25%, what should be the minimum annual sales for 12 years to justify the investment?

Ans. P39,748,560

3. A man formerly employed as chief mechanic of an automobile repair shop has saved P1,000,000 which are now invested in certain securities giving him an annual dividend of 15%. He now plans to invest this amount in his own repair shop. In his present job, he is earning P25,000 a month, but he has to resign to run his own business. He will need the services of the following: 2 mechanics each earning P400 a day, and 8 helpers each earning P200 a day. These men will work on the average 300 days a year. His other expenses are the following:

| | |
|---------------|--------------------|
| Rental | P30,000 a month |
| Miscellaneous | P25,000 a month |
| Sales tax | 3% of gross income |
| Insurance | 2% |

The length of his lease is 5 years. If the average charge for each car repaired by his shop is P1,000. Determine the number of cars he must service in one year so that he will obtain a profit of at least 20% on his investment?

Ans. 2,112

4. A firm is charged P150 per ton for hauling its raw materials by a trucking company. Forty tons per day are hauled for 300 days a year. It is desired to install a railway system which would bring down the cost of hauling to P6.60 per ton. Maintenance cost of this is P12,000 per month. Tax is 1%. Average rate of earning is 20%.

(a) If the company has the cash necessary for the installation would you recommend the change?

(b) If the company has to float P5,000,000 worth of non-callable bonds at 15% that will mature in 10 years to have the capital for the project, would you recommend the change?

Ans. (a) Rate of return is 26.68%, investment is justified.

(b) Rate of return is 10.61% investment is not justified.

5. A food processing plant consumed 600,000 kwh of electric energy annually and pays an average of P2.00 per kwh. A study is being made to generate its own power to supply the plant the energy required, and that the power plant installed would cost P2,000,000. Annual operation and maintenance, P800,000. Other expenses, P100,000 per year. Life of power plant is 15 years; salvage value at the end of life is P200,000; annual taxes and insurance, 6% of first cost; and rate of interest is 15%. Using the sinking fund method for depreciation, determine if the power plant is justifiable.

Ans. Rate of return is 7.11%, the power plant is not a good investment.

6. A fixed capital investment of P10,000,000 is required for a proposed manufacturing plant and an estimated working capital of P2,000,000. Annual depreciation is estimated to be 10% of the fixed capital investment. Determine the rate of return on the total investment and the payout period if the annual profit is P2,500,000.

Ans. 12.50%, 4.8 years

7. A small business purchased now for P50,000 will lose P9,600 each year for the first 4 years. An additional investment of P30,000 in the business will be required at the end of the fourth year. After 15 years the business can be sold for P70,000. What should be the profit each year from the fifth through the fifteenth year to obtain a rate of return of 25%?

Ans. P55,083

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Chapter 7

Comparing Alternatives

In Chapter 6, the different methods for investment of capital were discussed. In that chapter, the problem that was resolved was whether the investment of capital in a certain enterprise was justified or not. However, most engineering and business projects can be accomplished by more than one method or alternative. This chapter will deal with these types of problems.

The fundamental principle on which alternative should be used is stated as follows:

The alternative that requires the minimum investment of capital and will produce satisfactory functional result will always be used unless there are definite reasons why an alternative requiring a larger investment should be adopted.

METHODS OR PATTERNS IN COMPARING ALTERNATIVES

There are several methods for comparing alternatives, but only six patterns will be discussed.

THE RATE OF RETURN ON ADDITIONAL INVESTMENT METHOD

The formula for the rate of return on additional investment is,

$$\text{Rate of return on additional investment} = \frac{\text{annual net savings}}{\text{additional investment}}$$

If the rate of return on additional investment is satisfactory, then, the alternative requiring a bigger investment is more economical and should be chosen.

THE ANNUAL COST (AC) METHOD

To apply this method, the annual cost of the alternatives including interest on investment is determined. The alternative with the least annual cost is chosen. This pattern, like the rate of return on a uniform cost data for each year and a single investment of capital at the beginning of the first year of the project life.

THE EQUIVALENT UNIFORM ANNUAL COST (EUAC) METHOD

In this method, all cash flows (irregular or uniform) must be converted to an equivalent uniform annual cost, that is, a year-end amount which is the same each year. The alternative with the least equivalent uniform annual cost is preferred. When the EUAC method is used, the equivalent uniform annual cost of the alternatives must be calculated for one life cycle only. This method is flexible and can be used for any type of alternative selection problems. The method is a modification of the annual cost pattern.

THE PRESENT WORTH COST (PWC) METHOD

In comparing alternatives by this method, determine the present worth of the net cash outflows for each alternative for the same period of time. The alternative with the least present worth of cost is selected.

THE CAPITALIZED METHOD

The capitalized cost method is a variation of the present worth cost pattern. This method is used for alternatives having long lives. To use the method, determine the capitalized cost of all the alternatives and choose that one with the least capitalized cost.

PAYBACK (PAYOUT) PERIOD METHOD

To use this method, the payback period of each alternative is computed. The alternative with the shortest payback period is adopted. This method is seldom used.

(7-1) A company is considering two types of equipment for its manufacturing plant. Pertinent data are as follows:

| | Type A | Type B |
|------------------------------|----------|----------|
| First cost | P200,000 | P300,000 |
| Annual operating cost | 32,000 | 24,000 |
| Annual labor cost | 50,000 | 32,000 |
| Insurance and property taxes | 3% | 3% |
| | 4% | 4% |
| Payroll taxes | 10 | 10 |
| Estimated life | | |

If the minimum required rate of return is 15%, which equipment should be selected?

Solution

By the rate of return on additional investment method

Type A

Annual costs:

$$\begin{aligned} \text{Depreciation} &= \frac{\text{P}200,000}{F/A, 15\%, 10} = \frac{\text{P}200,000}{20.3037} = \text{P}9,850 \\ \text{Operation} &= 32,000 \\ \text{Labor} &= 50,000 \\ \text{Payroll taxes} &= (\text{P}50,000)(0.04) = 2,000 \\ \text{Taxes & insurance} &= (\text{P}200,000)(0.03) = 6,000 \end{aligned}$$

Total annual cost P99,850

Type B

Annual costs:

$$\begin{aligned} \text{Depreciation} &= \frac{\text{P}300,000}{F/A, 15\%, 10} = \frac{\text{P}300,000}{20.3037} = \text{P}14,776 \\ \text{Operation} &= 24,000 \\ \text{Labor} &= 32,000 \\ \text{Payroll taxes} &= (\text{P}32,000)(0.04) = 1,280 \\ \text{Taxes & insurance} &= (\text{P}300,000)(0.03) = 9,000 \end{aligned}$$

Total annual cost P81,056

$$\text{Annual savings} = \text{P}99,850 - \text{P}81,056 = \text{P}18,794$$

$$\text{Additional investment} = \text{P}300,000 - \text{P}200,000 = \text{P}100,000$$

$$\text{Rate of return on additional investment} = \frac{\text{P}18,794}{\text{P}100,000} \times 100 = 18.79 > 15\%$$

Type B should be selected.

By the annual cost method

Type A

Annual costs:

$$\begin{aligned} \text{Depreciation} &= \frac{\text{P}200,000}{F/A, 15\%, 10} = \frac{\text{P}200,000}{20.3037} = \text{P}9,850 \\ \text{Operation} &= 32,000 \\ \text{Labor} &= 50,000 \\ \text{Payroll taxes} &= (\text{P}50,000)(0.04) = 2,000 \\ \text{Taxes & insurance} &= (\text{P}200,000)(0.03) = 6,000 \\ \text{Interest on capital} &= (\text{P}200,000)(0.15) = 30,000 \\ \text{Total annual cost} &= \text{P}129,850 \end{aligned}$$

Type B

Annual costs:

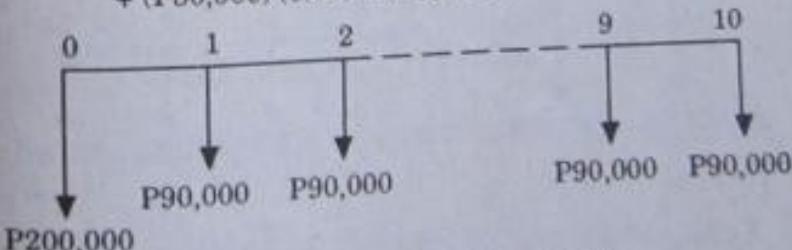
$$\begin{aligned} \text{Depreciation} &= \frac{\text{P}300,000}{F/A, 15\%, 10} = \frac{\text{P}300,000}{20.3037} = \text{P}14,776 \\ \text{Operation} &= 24,000 \\ \text{Labor} &= 32,000 \\ \text{Payroll taxes} &= (\text{P}32,000)(0.04) = 1,280 \\ \text{Taxes & insurance} &= (\text{P}300,000)(0.03) = 9,000 \\ \text{Interest on capital} &= (\text{P}300,000)(0.15) = 45,000 \\ \text{Total annual cost} &= \text{P}126,056 \end{aligned}$$

Since $AC_B < AC_A$, type B should be selected

By the present worth cost method

Type A

$$\begin{aligned} \text{Annual costs (excluding depreciation)} &= \text{P}32,000 + \text{P}50,000 \\ &+ (\text{P}50,000)(0.04) + (\text{P}200,000)(0.03) = \text{P}90,000 \end{aligned}$$

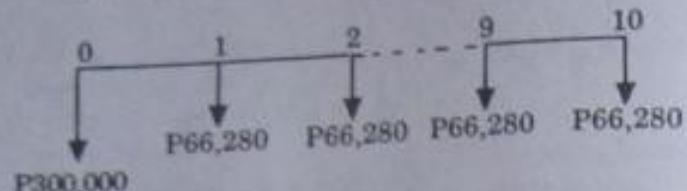


$$PWC_A = \text{P}200,000 + \text{P}90,000 (P/A, 15\%, 10)$$

$$= \text{P}200,000 + (\text{P}90,000)(5.0188) = \text{P}651,692$$

Type B

$$\begin{aligned}\text{Annual costs (excluding depreciation)} &= P24,000 + P32,000 \\ &+ (P32,000)(0.04) + (P300,000)(0.03) = P66,280\end{aligned}$$



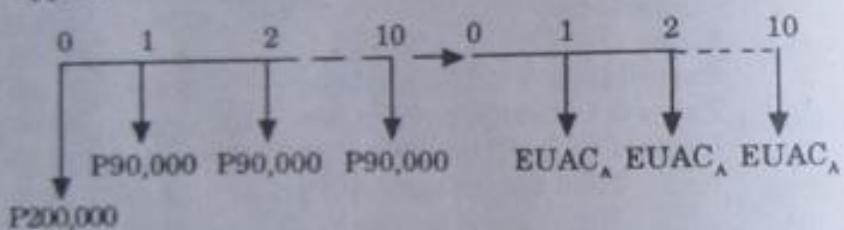
$$PWC_B = P300,000 + P66,280(P/A, 15\%, 10)$$

$$= P300,000 + (P66,280)(5.0188) = P632,646$$

Since $PWC_B < PWC_A$ for the same period of time, type B should be selected.

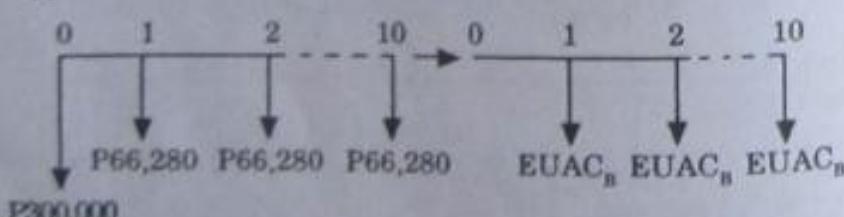
By the equivalent uniform annual cost method

Type A



$$\begin{aligned}EUAC_A &= P200,000(A/P, 15\%, 10) + P90,000 \\ &= (P200,000)(0.1993) + P90,000 = P129,860\end{aligned}$$

Type B



$$\begin{aligned}EUAC_B &= P300,000(A/P, 15\%, 10) + P66,280 \\ &= (P300,000)(0.1993) + P66,280 = P126,070\end{aligned}$$

Since $EUAC_B < EUAC_A$, type B is more economical.

(7-2) Choose from the two machines which is more economical.

| | Machine A | Machine B |
|---------------------|-----------|-----------|
| First cost | P8,000 | P14,000 |
| Salvage value | 0 | 2,000 |
| Annual operation | 3,000 | 2,400 |
| Annual maintenance | 1,200 | 1,000 |
| Taxes and insurance | 3% | 3% |
| Life, years | 10 | 15 |

Money is worth at least 16%.

Solution

By the rate of return on additional investment method

Machine A

$$\text{Annual costs:} \\ \text{Depreciation} = \frac{P8,000}{F/A, 16\%, 10} = \frac{P8,000}{21.3215} = P375$$

$$\begin{aligned}\text{Operation} &= 3,000 \\ \text{Maintenance} &= 1,200 \\ \text{Taxes and insurance} &= (P8,000)(0.03) = 240 \\ \text{Total annual cost} &= P4,815\end{aligned}$$

Machine B

$$\text{Annual costs:} \\ \text{Depreciation} = \frac{P12,000}{F/A, 16\%, 15} = \frac{P12,000}{51.6595} = P232$$

$$\begin{aligned}\text{Operation} &= 2,400 \\ \text{Maintenance} &= 1,000 \\ \text{Taxes and insurance} &= (P14,000)(0.03) = 420 \\ \text{Total annual cost} &= P4,052\end{aligned}$$

$$\begin{aligned}\text{Annual savings} &= P4,815 - P4,052 = P763 \\ \text{Additional investment} &= P14,000 - P8,000 = P6,000\end{aligned}$$

$$\begin{aligned}\text{Rate of return on additional investment} &= \frac{P763}{P6,000} \times 100 \\ &= 12.72\% < 16\%\end{aligned}$$

Machine A is more economical.

By the annual cost method

Machine A

| | |
|--|--|
| Annual costs: | $\frac{P8,000}{F/A,16\%,10} = \frac{P8,000}{21.3215} = P\ 375$ |
| Depreciation | $= 3,000$ |
| Operation | $= 1,200$ |
| Maintenance | $= 240$ |
| Taxes and insurance = $(P8,000)(0.03)$ | $= 240$ |
| Interest on capital = $(P8,000)(0.16)$ | $= 1,280$ |
| Total annual cost | $P6,095$ |

Machine B

| | |
|---|--|
| Annual costs: | $\frac{P12,000}{F/A,16\%,15} = \frac{P12,000}{51.6595} = P\ 232$ |
| Depreciation | $= 2,400$ |
| Operation | $= 1,000$ |
| Maintenance | $= 420$ |
| Taxes and insurance = $(P14,000)(0.03)$ | $= 420$ |
| Interest on capital = $(P14,000)(0.16)$ | $= 2,240$ |
| Total annual cost | $P6,292$ |

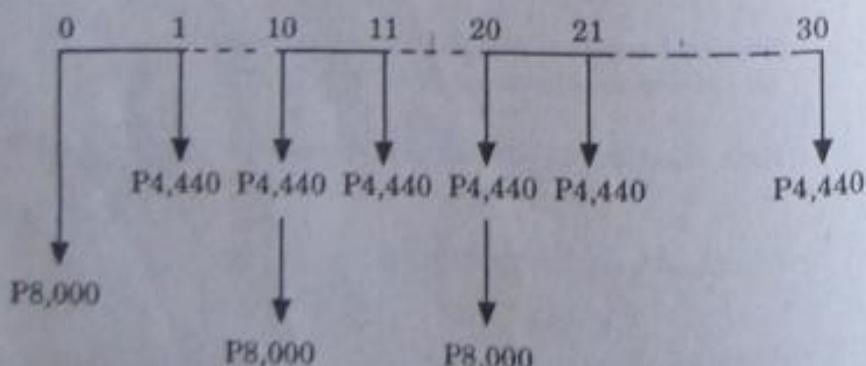
Since $AC_A < AC_B$, Machine A is more economical.

By the present worth cost method

Use 30-year study period, which is the least common multiple of 10 and 15.

Machine A

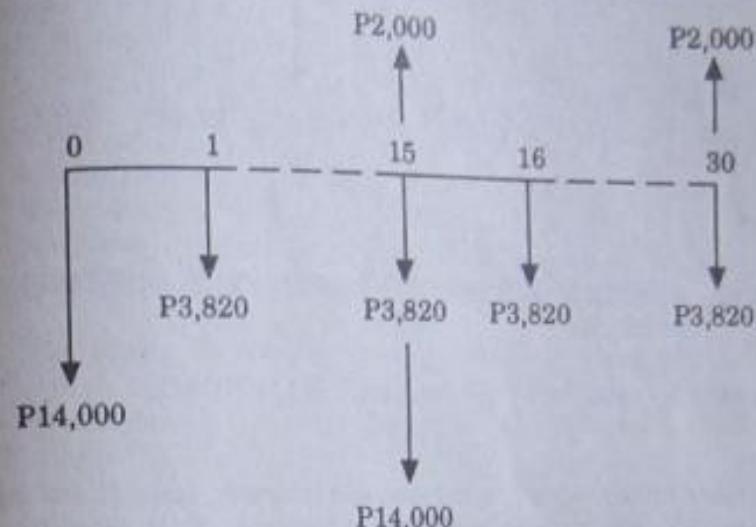
$$\text{Annual costs} = P3,000 + P1,200 + (P8,000)(0.03) = P4,440$$



$$PWC_A = P8,000 + P4,440 (P/A, 16\%, 30) + P8,000 (P/F, 16\%, 10) + P8,000 (P/F, 16\%, 20) = P37,652$$

Machine B

$$\text{Annual costs} = P2,400 + P1,000 + (P14,000)(0.03) = P3,820$$

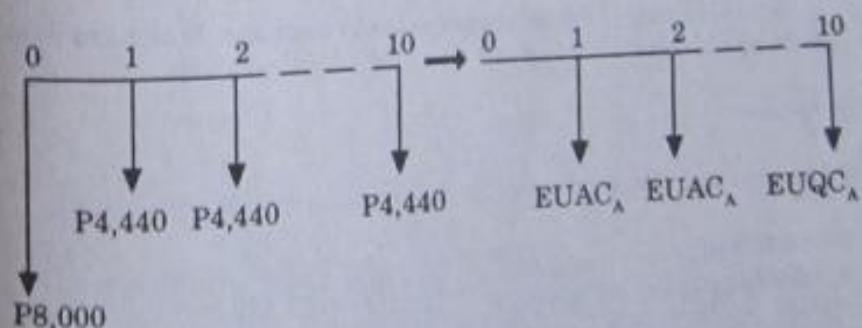


$$PWC_B = P14,000 + P3,820 (P/A, 16\%, 30) + P12,000 (P/F, 16\%, 15) - P2,000 (P/F, 16\%, 30) = P38,869$$

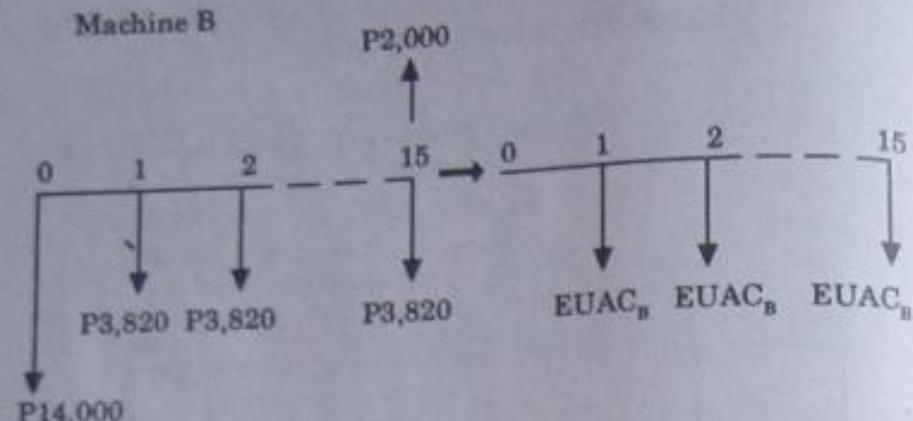
Machine A should be chosen, since $PWC_A < PWC_B$.

By the equivalent uniform annual cost method

Machine A



$$EUAC_A = P8,000 (A/P, 16\%, 10) + P4,440 = P6,095$$



$$EUAC_B = P14,000(A/P, 16\%, 15) + P3,820 - P2,000(A/F, 16\%, 15)$$

$$= P6,292$$

Machine A should be chosen, since $EUAC_A < EUAC_B$

(7-3) A company is going to buy a new machine for manufacturing its product. Four different machines are available. Cost, operating and other expenses are as follows:

| | A | B | C | D |
|----------------------|---------|---------|---------|---------|
| First cost | P24,000 | P30,000 | P49,600 | P52,000 |
| Power per year | 1,300 | 1,360 | 2,400 | 2,020 |
| Labor per year | 11,600 | 9,320 | 4,200 | 2,000 |
| Maintenance per year | 2,800 | 1,900 | 1,300 | 700 |
| Taxes and insurance | 3% | 3% | 3% | 3% |
| Life, years | 5 | 5 | 5 | 5 |

Money is worth 17% before taxes to the company. Which machine should be chosen?

Solution

By the annual cost method

Depreciation:

$$\text{Machine A} = \frac{P24,000}{F/A, 17\%, 5} = \frac{P24,000}{7.0144} = P3,422$$

$$\text{Machine B} = \frac{P30,000}{7.0144} = P4,277$$

$$\text{Machine C} = \frac{P49,600}{7.0144} = P7,071$$

$$\text{Machine D} = \frac{P52,000}{7.0144} = P7,413$$

Comparative Total Annual Cost for Four Machines

| | A | B | C | D |
|---------------------------|--------------|--------------|--------------|--------------|
| Power | P 1,300 | P 1,360 | P 2,400 | P 2,020 |
| Labor | 11,600 | 9,320 | 4,200 | 2,000 |
| Maintenance | 2,800 | 1,900 | 1,300 | 700 |
| Taxes and insurance (3%) | 720 | 900 | 1,488 | 1,560 |
| Depreciation | 3,422 | 4,277 | 7,071 | 7,413 |
| Interest on capital (17%) | <u>4,080</u> | <u>5,100</u> | <u>8,432</u> | <u>8,840</u> |
| Total annual cost | P23,922 | P22,857 | P24,891 | P22,533 |

The economic criterion is to choose that alternative with the minimum annual cost, which is machine D. However, it should be noted that machine B is very close, showing a total annual cost of only P22,857-P22,533=P324 more than machine D.

By the rate of return on additional investment method

| | A | B | C | D |
|--------------------------|---------|---------|---------|---------|
| Power | P 1,300 | P 1,360 | P 2,400 | P 2,020 |
| Labor | 11,600 | 9,320 | 4,200 | 2,000 |
| Maintenance | 2,800 | 1,900 | 1,300 | 700 |
| Taxes and insurance (3%) | 720 | 900 | 1,488 | 1,560 |
| Depreciation | 3,422 | 4,277 | 7,071 | 7,413 |
| Total annual cost | P19,842 | P17,757 | P16,459 | P13,693 |

Comparing Machine A with Machine B.

$$\text{ROR on additional investment on B} = \left(\frac{P19,482 - P17,757}{P30,000 - P24,000} \right) (100)$$

$$= 34.75\% > 17\%$$

Machine B is economical than Machine A.

Comparing Machine B with Machine C.

$$\text{ROR on additional investment on C} = \left(\frac{P17,757 - P16,459}{P49,600 - P30,000} \right) (100)$$

$$= 6.62\% < 17\%$$

Machine B is economical than Machine C.

Comparing Machine B with Machine D.

$$\text{ROR on additional investment on D} = \frac{(\text{P}17,757 - \text{P}13,693)}{(\text{P}52,000 - \text{P}30,000)}(100) \\ = 18.47 > 17\%$$

Machine D is economical than Machine B, choose Machine D.

(7-4) An untreated electric wooden pole that will last 10 years under certain soil condition costs P20,000. If a treated pole will last for 20 years, what is the maximum justifiable amount that can be paid for the treated pole, if the maximum return on investment is 20%. Consider annual taxes and insurance amount to be 1% of first cost.

Solution

Let C = maximum amount that can be invested on the treated pole

Untreated Pole

Annual costs:
Depreciation = $\frac{\text{P}20,000}{\text{F/A}, 20\%, 10} = \frac{\text{P}20,000}{25.9587} = \text{P}770$

Taxes and insurance = $(20,000)(0.01) = 200$
Interest on capital = $(\text{P}20,000)(0.20) = 4,000$
Total annual cost = $\text{P}4,970$

Treated Pole

Annual costs:
Depreciation = $\frac{C}{\text{F/A}, 20\%, 20} = \frac{C}{186.688} = 0.005356C$

Taxes and insurance = $(C)(0.01) = 0.01C$
Interest on capital = $(C)(0.20) = 0.20C$
Total annual cost = $\text{P}0.215356C$

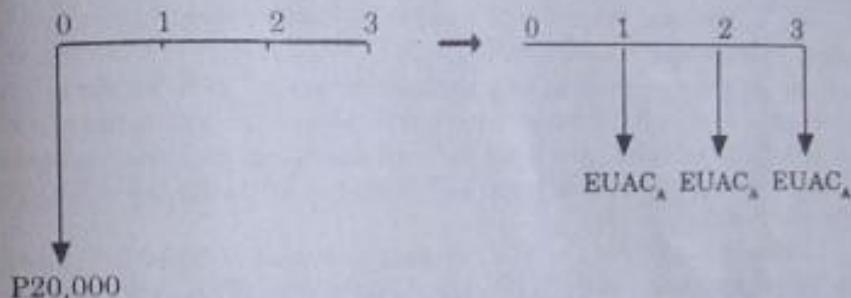
Equating annual costs.

$$\text{P}0.215356C = \text{P}4,970 \\ C = \text{P}23,078$$

(7-5) A company manufacturing acids, upon inspection of the roofing of the plant, found out that it is badly corroded from the acids fumes and would need to be replaced. To try to get some more life out of the roofing, the company consulted a roofing coating contractor who presented the company with two options. The first option is a coating that will cost P20,000 which would extend the life of the roofing for 3 years from date of application, and the second option will cost P30,000 and which would extend the life of the roofing for 5 years from the date of application. At what rate of return are the two investments equal?

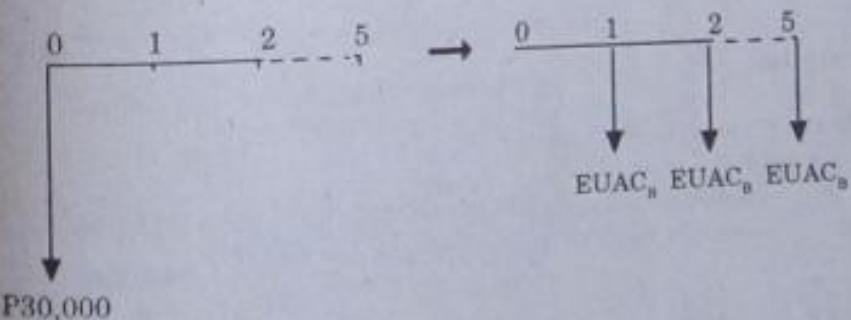
Solution

First option



$$\text{EUAC}_A = \text{P}20,000 (\text{A/P}, i\%, 3)$$

Second option



$$\text{EUAC}_B = \text{P}30,000 (\text{A/P}, i\%, 5)$$

For the two alternatives to be equally economical, $\text{EUAC}_A = \text{EUAC}_B$

$$P200,000(A/P, i\%, 3) = P30,000(A/P, i\%, 5)$$

$$\frac{A/P, i\%, 5}{A/P, i\%, 3} = \frac{P20,000}{P30,000}$$

$$\frac{1-(1+i)^5}{1-(1+i)^3} = 0.6667$$

Try $i = 12\%$

$$\frac{1-(1.12)^5}{1-(1.12)^3} = 0.6667$$

$$0.66662 \approx 0.6667$$

Rate of return = 12%.

(7-6) The engineer of a medium scale industry was instructed to prepare at least two plans which is to be considered by management for the improvement of their operations. Plan "A" calls for an initial investment of P200,000 now with a prospective salvage value of 20% of the first cost 20 years hence. The operation and maintenance disbursements are estimated to be P15,000 a year and taxes will be 2% of first cost.

Plan "B" calls for an immediate investment of P140,000 and a second investment of P160,000 eight years later. The operation and maintenance disbursements will be P9,000 a year for the initial installation, and P8,000 a year for the second installation. At the end of 20 years the salvage value shall be 20% of the investments. Taxes will be 2% of the first cost.

If money is worth 12%, which plan would you recommend?

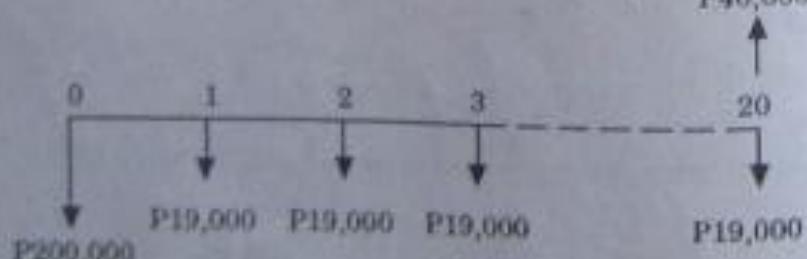
Solution

By the present worth cost method

Plan A:

$$\text{Annual costs} = P15,000 + (P200,000)(0.02) = P19,000$$

$$\text{Salvage value} = (P200,000)(0.20) = P40,000$$



$$\begin{aligned} PWC_A &= P200,000 + (P19,000)(P/A, 12\%, 20) - (P40,000)(P/F, 12\%, 20) \\ &= P337,771 \end{aligned}$$

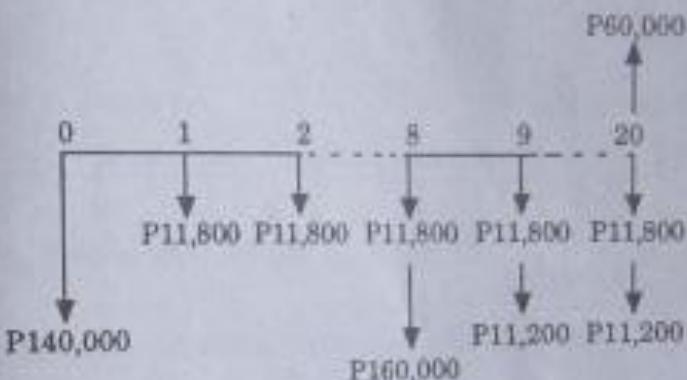
Plan B:

$$\text{Annual costs} = P9,000 + (P140,000)(0.02) = P11,800$$

$$\text{Additional annual costs after 8 years} = P8,000$$

$$+ (P160,000)(0.02) = 11,200$$

$$\text{Salvage value} = (P140,000 + P160,000)(0.20) = P60,000$$



$$\begin{aligned} PWC_B &= P140,000 + P11,800(P/A, 12\%, 20) + P160,000(P/F, 12\%, 8) \\ &\quad + P11,200(P/A, 12\%, 12)(P/F, 12\%, 8) - P60,000(P/F, 12\%, 20) \\ &= P314,564 \end{aligned}$$

Plan B is more economical.

By the equivalent uniform annual cost method

$$\begin{aligned} EUAC_A &= P200,000(A/P, 12\%, 20) + P19,000 \\ &\quad - P40,000(A/F, 12\%, 20) = P45,224 \end{aligned}$$

$$\begin{aligned} EUAC_B &= P140,000(A/P, 12\%, 20) + P11,800 \\ &\quad + P160,000(P/F, 12\%, 8)(A/P, 12\%, 20) \\ &\quad + P11,200(F/A, 12\%, 12)(A/F, 12\%, 20) \\ &\quad - P60,000(A/F, 12\%, 20) \\ &= P34,019 \end{aligned}$$

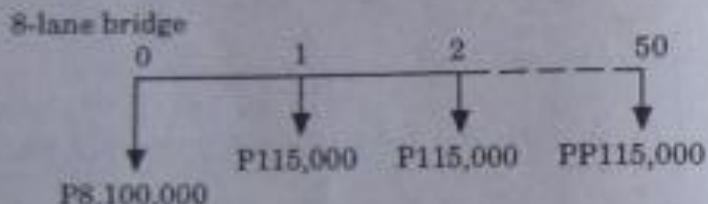
Plan B is more economical

(7-7) The National Government is planning to construct a bridge. At present a 4-lane bridge will be built, but it is expected that at a later date a second 4-lane bridge will be added.

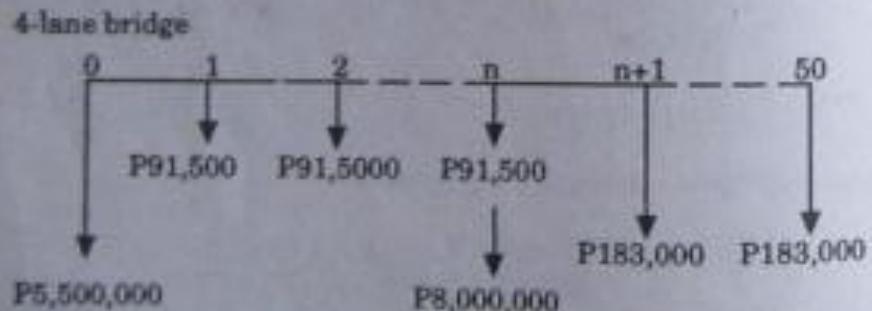
If an 8-lane bridge is built now, it will cost P8,100,000. A single 4-lane bridge can be built now for P5,500,000. It is estimated that a second 4-lane bridge at a later date will cost P8,000,000. Annual upkeep on each 4-lane bridge would be P91,500, and for the 8-lane bridge would be P115,000. Money to build the bridge will cost 12%.

Assume a 50-year functional life for the project and determine the earliest time at which the additional 4 lanes would be required to make it equally economical to build the 8-lane bridge immediately.

Solution



$$PWC_s = P8,100,000 + P115,000(P/A, 12\%, 50) = P9,055,000$$



$$\begin{aligned} PWC_s &= P5,500,000 + P91,500(P/A, 12\%, n) + P8,000,000(P/F, 12\%, n) \\ &\quad + P183,000(P/A, 12\%, (50-n))(P/F, 12\%, n) \end{aligned}$$

When $n = 10$

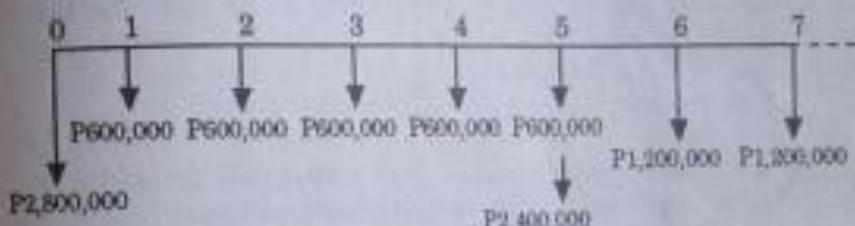
$$\begin{aligned} &= P5,500,000 + P91,500(P/A, 12\%, 10) \\ &\quad + P8,000,000(P/F, 12\%, 10) \\ &\quad + P183,000(P/A, 12\%, 40)(P/F, 12\%, 10) \\ &= P9,079,000 \end{aligned}$$

10 years

(7-8) A plant to provide the company's present needs can be constructed for P2,800,000 with annual operating disbursements of P600,000. It is expected that at the end of 5 years the production requirements could be doubled, which will necessitate the addition of an extension costing P2,400,000. The disbursements after 5 years will likewise double. A plan to provide the entire expected capacity can be constructed for P4,000,000 and its operating disbursements will be P640,000 when operating on half capacity (for the first 5 years) and P900,000 on full capacity. The plants are predicted to have indefinitely long life. The required rate of return is 20%. What would you recommend?

Solution

Deferred Expansion

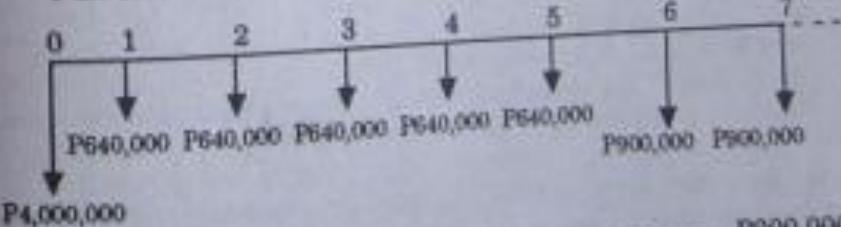


$$\text{Capitalized cost} = P2,800,000 + P600,000(P/A, 20\%, 5) + P2,400,000$$

$$(P/F, 20\%, 5) + \frac{P1,200,000}{0.20} (P/F, 20\%, 5) = P7,970,152$$

$$= 7,970,152$$

Full-size Plant



$$\begin{aligned} \text{Capitalized cost} &= P4,000,000 + P640,000(P/A, 20\%, 5) + \frac{P900,000}{0.20} \\ &\quad (P/F, 20\%, 5) = P7,722,444 \end{aligned}$$

The full-size plant should be constructed.

PROBLEMS

1. An oil company is being offered a special coating for the gasoline underground tank installation in its service stations which will increase the life of the tank from the usual 10 years to 15 years. The cost of the special coating will increase the cost of the P40,000-tank to P58,000. Cost of installation for either of the tanks is P24,000. If salvage value for both is zero, and interest rate is 26%, would you recommend the use of the special coating?

Ans. The special coating should not be used.

2. An electric cooperative is considering the use of a concrete electric pole in the expansion of its power distribution lines. A concrete pole costs P18,000 each and will last 20 years. The company is presently using creosoted wooden poles which cost P12,000 per pole and will last 10 years. If money is worth 12 per cent, which pole should be used? Assume annual taxes amount to 1 per cent of first cost and zero salvage value in both cases.

Ans. The creosoted wooden poles should be used.

3. It is proposed to place a cable on an existing pole line along the shore of a lake to connect two points on opposite sides.

| | Land Route | Submarine Route |
|------------------------------|-------------------|------------------------|
| Length, miles | 10 | 5 |
| First cost of cable per mile | P40,000 | P68,000 |
| Annual maintenance per mile | 950 | 3,500 |
| Interest on investment | 18% | 18% |
| Taxes | 3% | 3% |
| Net salvage value per mile | 12,000 | 22,000 |
| Life, years | 15 | 15 |

Which is more economical?

Ans. The submarine cable is more economical the rate of return on additional investment is only 8.97%.

4. In a cold storage plant, it is desired to determine whether to use insulation two inches thick or three inches in insulating the walls of the cold storage warehouse. Heat absorbed through the walls without insulation would cost P96.00 per year per square meter. A two-inch insulation will cost P30.40 per square meter and will cut out 89% of the loss. A three-inch insulation will cut out 92% of the loss and will cost P65.00 per square meter. Using a life of 15 years for the

insulation with no salvage value and a minimum attractive return of 8%, what thickness of insulation should be used?

Ans. 2-in thick insulation

5. In building their plant, the officers of the International Leather Company had the choice between alternatives.

One alternative is to build in Metro Manila where the plant would cost P2,000,000. Labor would cost annually P120,000 and annual overhead P40,000. Taxes and insurance would total 5% of the first cost of the plant.

The second alternative would be to build in Bulacan a plant costing P2,250,000. Labor would cost annually P100,000 and overhead would be P55,000. Taxes and insurance would be 3% of the first cost. The cost of raw materials would be the same in either plant. If capital must be recovered within 10 years and money is worth at least 20%, which site should the officers of the company choose?

Ans. Plant in Metro Manila

6. A utility company is considering the following plans to provide a certain service required by present demand and the prospective growth of demand for the coming 18 years.

Plan R requires an immediate investment of P500,000 in property that has an estimated life of 18 years and with 20% terminal salvage value. Annual disbursements for operation and maintenance will be P50,000. Annual property taxes will be 2% of first costs.

Plan S requires an immediate investment of P300,000 in property that has an estimated life of 18 years with 20% terminal salvage value. Annual disbursements for its operation and maintenance during the first 6 years will be P40,000. After 6 years, an additional investment of P400,000 will be required in property having an estimated life of 12 years with 40% terminal salvage value. After this additional property is installed, annual disbursements for operation and maintenance of the combined property will be P60,000. Annual property taxes will be 2% of the first cost of property in service at any time. Money is worth 12%. What would you recommend?

Ans. Plant S

Solution

Fixed cost per unit = P0.85 + P0.55 + P1.10 = P2.50

- (a) Annual production 10,000 units
The machine will last for 1 year

$$\begin{array}{rcl} \text{Depreciation} & = & \text{P}30,000 - \text{P}2,000 \\ & = & \text{P}28,000 \\ \text{Housing} & = & 900 \\ \text{Total} & = & \overline{\text{P}28,900} \end{array}$$

$$\text{Total cost per unit} = \text{P}2.50 + \frac{\text{P}28,900}{10,000} = \text{P}5.39$$

- (b) Annual production 2,500 units
The machine will last for 4 years
Annual costs:

$$\begin{array}{rcl} \text{Depreciation} & = & \frac{\text{P}30,000 - \text{P}2,000}{\text{F/A}, 15\%, 4} \\ & = & \text{P}5,607 \\ \text{Housing} & = & 900 \\ \text{Total} & = & \overline{\text{P}6,507} \end{array}$$

$$\text{Total cost per unit} = \text{P}2.50 + \frac{\text{P}6,507}{2,500} = \text{P}5.10$$

- (8-2) A company having a capacity of 1,600 units per year currently is operating at a sales level of only 1,200 units, with a selling price of P720 per unit. The fixed costs of the plant are P365,000 per year, and the variable costs are P416 per unit. It has been estimated that a reduction of P50 per unit in the selling price would increase sales by 300 units per year.

- (a) Would this be a good program to follow?
(b) An alternative being considered is to engage in a modernization plan that would increase the fixed costs by P58,000 per year, but that would reduce the variable costs by P56 per unit. Would this be a better procedure than the price-reduction program?
(c) Can you suggest any other program that might be superior to the foregoing?

Chapter 8

Fixed, Increment, and Sunk Costs

Types Of Costs

Fixed costs are those costs which remain constant, whether or not a given change in operations or policy is adopted.

Variable costs are those costs which vary with output or any change in the activities of an enterprise.

Increment costs are those that arise as the result of a change in operations or policy.

Marginal cost is the additional cost of producing one more unit of a product.

Sunk cost represents money which has been spent or capital which has been invested and which cannot be recovered due to certain reasons.

- (8-1) A machine costing P30,000 is expected to produce 10,000 units of a certain product during its entire life before being replaced. At the end of life, it will have a scrap value of P2,000. The cost of housing the machine will be P900 a year. The power consumption per unit is P0.85 and the maintenance per unit will be P0.55. Labor will cost P1.10 per unit. If depreciation is by sinking fund method at 15%, determine the cost per unit produced if the annual production is (a) 10,000 units and (b) 2,500 units.

Solution

(a) Present revenue = P720 (1,200)

= P 864,000

Present costs:

Fixed

Variable = P416 (1,200)

= P365,000

= 499,200

Total

Loss

864,200
(-) P 200

= P1,005,000

New revenue = P670 (1,500)

Costs:

Fixed

Variable = P416 (1,500)

= 365,000

= 624,000

Total

Profit

989,000
P 16,000

Reducing the price would be a profitable program.

(b) Revenue

= P 864,000

Costs:

Fixed

Variable = P360 (1,200)

= P423,000

= 432,000

Total

Profit

P 855,000
P 9,000

Modernization would be profitable, but it would not be as good a procedure as the price reduction.

(c) Both programs should be combined.

(8-3) The ABC Company has two plants in the same city, each having a capacity of ten units of the same product per month. Fixed costs of Plant I are P30,000 per month and its variable costs are P1,000 X - P10X². Fixed costs of Plant II are P50,000 per month and its variable costs are P1,200 Y - P5Y². X and Y are the number of units produced. At present the sales have been established at 14 units per month with each plant producing 7 units. Should the production volume at the plants be changed, and to what extent?

Solution

Fixed costs at both plants will be unaffected and are therefore neglected.

Plant I Variable costs = P1,000X - P10X²

| X | Variable Costs |
|----|----------------|
| 4 | P 3,840 |
| 5 | 4,750 |
| 6 | 5,640 |
| 7 | 6,510 |
| 8 | 7,360 |
| 9 | 8,190 |
| 10 | 9,000 |

Plant II Variable costs = P1,200 Y - P5Y²

| Y | Variable Costs |
|----|----------------|
| 10 | P7,000 |
| 9 | 7,155 |
| 8 | 7,040 |
| 7 | 6,685 |
| 6 | 6,120 |
| 5 | 5,375 |
| 4 | 4,480 |

| Plant I | Unit per month | Variable Costs | | Total Variable Costs |
|---------|----------------|----------------|----------|----------------------|
| | | Plant I | Plant II | |
| 4 | 10 | P 3,840 | P 7,000 | P10,840 |
| 5 | 9 | 4,750 | 7,155 | 11,905 |
| 6 | 8 | 5,640 | 7,040 | 12,680 |
| 7 | 7 | 6,510 | 6,685 | 13,195 |
| 8 | 6 | 7,360 | 6,120 | 13,480 |
| 9 | 5 | 8,190 | 5,375 | 13,565 |
| 10 | 4 | 9,000 | 4,480 | 13,480 |

Operate Plant II at full capacity.

The manufacturer should produce the new line of products.

(8-4) A manufacturing firm has a waste product that is now being dumped into a river. The waste material could be processed to produce fertilizer. This would require the investment of P320,000 in equipment

and P50,000 per month for other materials. Labor would cost P14,000 per month. Overhead expenses will cost P55,000 per year. Taxes and insurance amount to 3% of the first cost of the equipment per year. Payroll taxes and other benefits amount to 4%. The life of the equipment is 5 years with no salvage value at the end of its life. Money is worth 20% to the firm.

If 3,600 kg of fertilizer are produced per month, which can be sold for P22.00 per kg, should the firm make the investment?

| | |
|--|------------|
| Increment revenue = 3,600 (12) (P22.00) | = P950,400 |
| Increment costs: | |
| Depreciation = $\frac{P320,000}{F/A, 20\%, 5} = \frac{P320,000}{7.4416}$ | = P43,002 |
| Materials = (P53,000) (12) | = 636,000 |
| Labor = (P14,000) (12) | = 168,000 |
| Overhead | = 56,000 |
| Payroll taxes & benefits = (P168,000) (0.04) | = 6,720 |
| Taxes and insurance = (P320,000) (0.03) | = 9,600 |
| Interest on capital = (P320,000) (0.20) | = 64,000 |
| Total annual cost | P983,322 |

The firm should not invest.

(8-5) An old machine was purchased 3 years ago at a cost of P50,000. It was estimated to have a useful life of 8 years, with a salvage value of P5,000. It is now going to be replaced by a new machine costing P70,000 and P30,000 trade-in will be allowed for the old machine. Determine the sunk cost if depreciation has been computed by the (a) straight line method, (b) sinking fund method at 16%, and (c) SYD method.

Solution:

$$C_o = P50,000 \quad C_t = P5,000 \quad L = 8 \quad n = 3$$

(a) Straight line method

$$D_s = \frac{(P50,000 - P5,000)}{8} = P16,875$$

$$C_i = C_o - D_s = P50,000 - P16,875 = P33,125$$

$$\begin{aligned} \text{Sunk cost} &= \text{book value} - \text{resale value} \\ &= P33,125 - P30,000 \\ &= P3,125 \end{aligned}$$

(b) Sinking fund method

$$d = \frac{P50,000 - P5,000}{F/A, 16\%, 8} = \frac{P45,000}{13.2401} = P3,160$$

$$D_s = d(F/A, 16\%, 3) = P3,160 (3.5056) = P11,078$$

$$C_i = C_o - D_s = P50,000 - P11,078 = P38,922$$

$$\begin{aligned} \text{Sunk cost} &= \text{book value} - \text{resale value} \\ &= P38,922 - P30,000 \\ &= P8,922 \end{aligned}$$

(c) SYD method

$$\text{Sum of digits} = 1 + 2 + 3 + \dots + 7 + 8 = \frac{8}{2} (1 + 8) = 36$$

$$D_s = \frac{(8 + 7 + 6)}{36} (P50,000 - P5,000) = P26,250$$

$$C_i = C_o - D_s = P50,000 - P26,250 = P23,750$$

$$\text{Sunk cost} = P23,750 - P30,000 = -P6,250$$

The negative sign means that the amount is not a loss but a gain.

PROBLEMS

(1) The XYZ Company has two plants producing "K Specials." It has the following expected data for the next month's operations. Variable (incremental) costs vary linearly from zero production to maximum-capacity production.

| | Plant A | Plant B |
|---|----------|----------|
| Maximum Capacity, Units | 1,000 | 800 |
| Total Fixed Cost | P750,000 | P480,000 |
| Variable (incremental) Costs Maximum Capacity | P900,000 | P800,000 |

(a) Performance has not been good, so the company expects to receive domestic orders for only 1,200 units next month at a price of P1,400 per unit. How should the production be distributed between the plants for optimum economic operation?

(b) If a friendly foreign power offers to buy 350 additional units at P1,100 per unit, should the company accept the offer? Show the incremental gain or loss.

Ans. (a) Produce 1,000 units at Plant A and 200 units at Plant B.
(b) Incremental gain is P35,000, the company should accept the offer

(2) A company has a new Plant A and an old Plant B in the same metropolitan area, each with a capacity of 12 units of product per month. Fixed expense at A is P40,000 per month and at B is P20,000 per month. Variable expense per month at A is P1,000 \times N², where N = the number of units produced. At B it is P2,000 \times M², where M = the number of units produced. At present the sales have been established at 14 units per month with each plant producing 7 units. Should the interplant load be redistributed? Why? How?

Ans. 9 units and 5 units.

(3) An asset was purchased six years ago at a cost of P70,000. It was estimated to have a useful life of ten years with a salvage value of P300 at the end of the time. It is now of no future use and can be sold for only P800. Determine the sunk cost if depreciation has been computed by:

- (a) The straight-line method
- (b) The sum-of-the-year's digits method

Ans. (a) P2,300 (b) P880

Chapter 9

Replacement Studies

The Four Major Reasons For Replacement

1. Physical impairment

The existing asset is completely or partially worn out and will no longer function satisfactorily without extensive repairs.

2. Inadequacy

The existing asset does not have sufficient capacity to meet the present demands that are placed on it.

3. Obsolescence

This may be caused either by a lessening in the demand for the service rendered by the asset or the availability of more efficient assets which will operate with lower out-of-pocket costs.

4. Rental or lease possibilities

It is possible to rent identical or comparable asset or property, thus freeing capital for other and more profitable use.

Sunk Cost Due to Unamortized Value

Unamortized value of an equipment or property is the difference between its book value and its resale value when replaced. Unamortized value should be considered as a sunk cost or a loss.

Basic Patterns For Replacement Studies

Replacement economy studies may be made by any of the basic procedures or patterns which have been discussed previously. However, in most cases either the rate of return method or the annual cost method is used.

(9-1) An existing factory must be enlarged or replaced to accommodate new production machinery. The structure was built at a cost of P2.6 million. Its present book value, based on straight line depreciation is P700,000 but it has been appraised at P800,000. If the structure is altered, the cost will be P1.6 million and its service life will be extended 8 years with a salvage value of P600,000. A new factory could be purchased or built for P5.0 million. It would have a life of 20 years and a salvage value of P700,000. Annual maintenance of the new building would be P160,000 compared with P100,000 in the enlarged structure. However, the improved layout in the new building would reduce annual production cost by P240,000. All other expenses for the new structure are estimated as being equal. Using an investment rate of 8 per cent, determine which is the more attractive investment for this firm.

Solution

Enlarged Building

| | |
|---|------------|
| Annual costs: | |
| Depreciation = $\frac{P800,000 + P1,600,000 - P600,000}{F/A, 8\%, 8}$ | |
| = $\frac{P1,800,000}{10.6366}$ | = P169,227 |
| Maintenance | = 100,000 |
| Production (excess) | = 240,000 |
| Total annual cost | P509,227 |

New Building

| | |
|---|------------|
| Annual costs: | |
| Depreciation = $\frac{P5,000,000 - P700,000}{F/A, 8\%, 20}$ | |
| = $\frac{P4,300,000}{45.7620}$ | = P 93,964 |
| Maintenance | = 160,000 |
| Total annual cost | P255,964 |

$$\text{Annual savings} = P509,227 - P255,964 = P255,263$$

$$\begin{aligned}\text{Additional investment} &= P5,000,000 - P800,000 - P1,600,000 \\ &= P2,600,000\end{aligned}$$

$$\begin{aligned}\text{Rate of return on additional investment} &= \frac{P255,263}{P2,600,000} \times 100 \\ &= 9.82\%\end{aligned}$$

Construct the new building

(9-2) A decision must be made whether to replace a certain engine with a new one, or to rebore the cylinder of the old engine and thoroughly recondition it. The original cost of the old engine 10 years ago was P70,000; to rebore and recondition it now will cost P28,000, but would extend its useful life for 5 years. A new engine will have a first cost of P62,000 and will have an estimated life of 10 years. It is expected that the annual cost of fuel and lubricants with the reconditioned engine will be about P20,000 and that this cost will be 15% less with the new engine. It is also believed that repairs will be P2,500 a year less with the new engine than with the reconditioned one. Assume that neither engine has any net realizable value when retired. If money is worth 16%, what would you recommend?

Solution

Reconditioned engine

| | |
|---|-----------|
| Annual costs: | |
| Depreciation = $\frac{P28,000}{F/A, 16\%, 5}$ | = P 4,071 |
| Fuel and lubricants | = 20,000 |
| Repairs (excess) | = 2,500 |
| Interest on capital = P28,000 (0.16) | = 4,480 |
| Total annual cost | P31,951 |

New engine

| | |
|--|-----------|
| Annual costs: | |
| Depreciation = $\frac{P62,000}{F/A, 16\%, 10}$ | = P 2,908 |
| Fuel and lubricants = P20,000 (0.85) | = 17,000 |
| Interest on capital = P62,000 (0.16) | = 9,920 |
| Total annual cost | P29,828 |

The old engine should be replaced

(9-3) Four years ago an ore-crushing unit was installed at a mine which cost P81,000. Annual operating costs for this unit are P3,540. This unit was estimated to have a life of 10 years. The quantity of ore to be handled is to be doubled and is expected to continue at this higher rate for at least 10 years. A unit that will handle the same quantity of ore and have the same operating costs as the one now in service can be installed for P75,000. This unit will have a useful life of 6 years.

A unit with double the capacity of the one now in use can be installed for P112,000. Its life is estimated at 6 years and its annual operating costs are estimated at P4,950. The present realizable value of the unit now in use is P26,000. All units under consideration will have an estimated salvage value at retirement age of 12% of the original cost. Interest rate is 20%. Annual taxes and insurance are 2.5% of the original cost.

What would you recommend?

Solution

Augmentation

Annual costs:

Old unit

$$\begin{aligned} \text{Depreciation} &= \frac{P26,000 - (P81,000)(0.12)}{F/A, 20\%, 6} \\ &= \frac{P16,280}{9.9299} = P1,639 \end{aligned}$$

Operation

$$\text{Taxes and insurance} = (P81,000)(0.025) = 2,025$$

New small unit

$$\begin{aligned} \text{Depreciation} &= \frac{P75,000 - (P75,000)(0.12)}{F/A, 20\%, 6} \\ &= \frac{P66,000}{9.9299} = 6,647 \end{aligned}$$

Operation

$$\text{Taxes and insurance} = (P75,000)(0.025) = 1,875$$

$$\text{Total annual cost} = P19,266$$

Replacement

Annual costs:

New big unit

$$\begin{aligned} \text{Depreciation} &= \frac{P112,000 - (P112,000)(0.12)}{F/A, 20\%, 6} \\ &= \frac{P98,560}{9.9299} = P 9,926 \end{aligned}$$

Operation

$$\text{Taxes and insurance} = (P112,000)(0.025) = 4,950$$

$$\text{Total annual cost} = P17,676$$

$$\text{Annual savings} = P19,266 - P17,676 = P1,590$$

$$\text{Additional investment} = P112,000 - P75,000 - P26,000 = P11,000$$

$$\text{ROR on additional investment} = \frac{P1,590}{P11,000} \times 100 = 14.5\%$$

Buy the new small unit to augment the old unit.

(9-4) A car can be purchased for P400,000 when new. There follows a schedule of annual operating expenses for each year and trade-in values at the end of each year. Assume that these amounts would be repeated for future replacements, and that the car will not be kept more than 3 years. If interest on invested capital is 15% before taxes determine at which year's end the car should be replaced so that costs will be minimized.

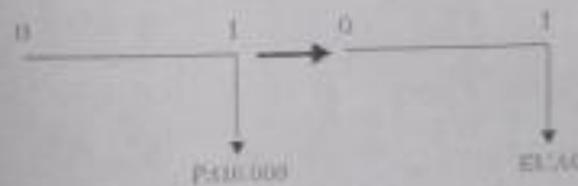
| | Year 1 | Year 2 | Year 3 |
|-------------------------------|---------|---------|---------|
| Operating expenses for year | P34,000 | P38,000 | P41,000 |
| Trade-in value at end of year | 408,000 | 336,000 | 240,000 |

Solution

Cost of keeping each year

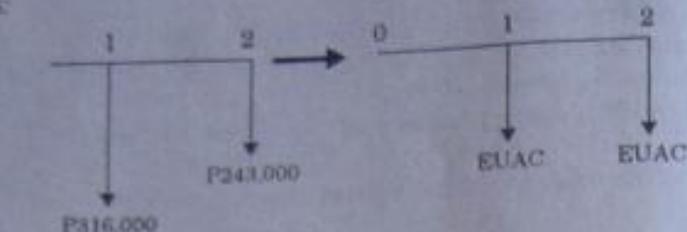
| | 1 | 2 | 3 |
|---------------------------|----------|----------|----------|
| Operation | P34,000 | 38,000 | P41,000 |
| Depreciation | 192,000 | 144,000 | 96,000 |
| Interest on capital (15%) | P90,000 | P61,200 | P50,400 |
| Total | P316,000 | P243,200 | P187,400 |

1 year:



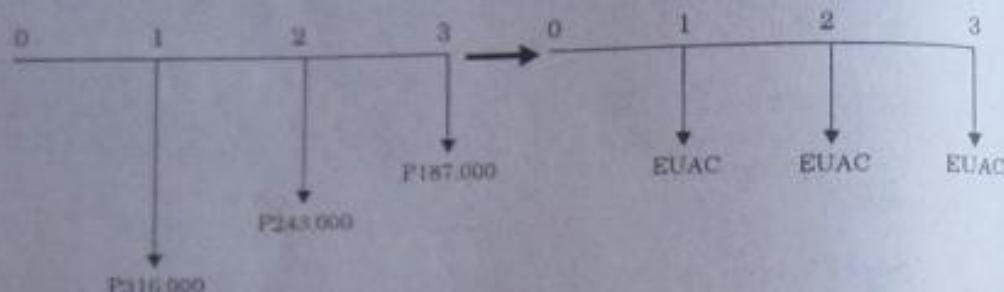
$$\text{EUAC} = P316,000$$

2 years:



$$\begin{aligned} \text{EUAC} &= [P316,000 + P243,000 (P/F, 15\%, 2)] (A/P, 15\%, 2) \\ &= P307,385 \end{aligned}$$

3 years:



$$\begin{aligned} \text{EUAC} &= [P316,000 + P243,000 (P/F, 15\%, 2) + P187,400 \\ &\quad (P/F, 15\%, 3)] (A/P, 15\%, 3) \\ &= P272,851 \end{aligned}$$

Thus it is cheaper to keep the car three years.

PROBLEMS

1. A recapping plant is planning to acquire a new Diesel generating set to replace its present unit which they run during brownouts. The new set would cost P135,000 with a five (5) year-life, and no estimated salvage value. Variable cost would be P150,000 a year.

The present generating set has a book value of P75,000 and a remaining life of 5 years. Its disposal value now is P7,500, but it would be zero after 5 years. Variable operating cost would be P187,500 a year. Money is worth 10%.

Which is profitable, to buy the new generator set or retain the present set? Support your answer by showing your computation.

Ans. Buy the new generating set.

2. A company that sells computers has proposed to a small public utility company that it purchase a small electronic computer for P1,000,000 to replace ten calculating machines and their operators. An annual service maintenance contract for the computer will be provided at a cost of P100,000 per year. One operator will be required at a salary of P96,000 per year and one programmer at a salary of P144,000 per year. The estimated economical life of the computer is 10 years.

The calculating machine costs P7,000 each when new, 5 years ago, and presently can be sold for P2,000 each. They have an estimated life of 8 years and an expected ultimate trade-in value of P1,000 each. Each calculating machine operator receives P84,000 per year. Fringe benefits for all labor cost 8% of annual salary. Annual maintenance costs on the calculating machines have been P500 each. Taxes and insurance on all equipment is 2% of the first cost per year.

If capital costs the company about 25%, would you recommend the computer installation?

Ans. The calculators should be replaced.

3. It is desired to determine the present economic value of an old machine by considering of how it compares with the best modern machine that could replace it. The old machine is expected to require out-of-pocket costs of P85,000 each year for 4 years and then be scrapped for P5,000 residual value. The new machine requires an investment of P40,000 and would have out-of-pocket costs of P79,000 a year for 8 years and then zero salvage value. Invested capital should earn a minimum return of 15% before taxes. Determine the present value of the old machine.

Ans. P11,200

Chapter 10

Break-even Analysis

In engineering economy, many situations are encountered where the cost of two or more alternatives may be affected by a common variable. Break-even point is the value of the variable for which the costs for the alternatives will be equal.

$$C_1 = f_1(x) \quad \text{and} \quad C_2 = f_2(x)$$

where:

- C_1 = certain specified total cost applicable to Alternative 1
- C_2 = certain specified total cost applicable to Alternative 2
- x = a common independent variable affecting Alternative 1 and Alternative 2

The break-even point is where C_1 and C_2 are equal,

$$f_1(x) = f_2(x)$$

which may be solved for x , the break-even point.

BREAK-EVEN CHART

Break-even chart is a graphical representation of break-even analysis. The break-even point is the quantity of production at which the income is equal to total cost. It is the intersection of the income line and the total cost line on the break-even chart.

When two alternatives are to be compared, the break-even point is the intersection of the total cost line for each alternative on the break-even chart.

(10-1) Two machines are being considered for the production of a particular part for which there is a long-term demand. Machine A costs P50,000 and is expected to last 3 years and have a P10,000 salvage value. Machine B costs P75,000 and is expected to last 6 years and have zero salvage value. Machine A can produce a part in 18 seconds; Machine B requires only 12 seconds per part. The out-of-pocket hourly cost of operation is P38 for A and P30 for B. Monthly maintenance costs are P200 for A and P220 for B.

If interest on invested capital is 25%, determine the number of parts per year at which the machines are equally economical. If the expected number of parts per year is greater than this break-even quantity, which machine would be favored?

Algebraic Solution

Let N = number of parts per year for equal costs

By the annual cost method

Machine A

Annual costs:

$$\text{Depreciation} = \frac{P50,000 - P10,000}{F/A, 25\%, 3} = \frac{P40,000}{3.8125} = P10,492$$

$$\text{Maintenance} = (P200)(12) = 2,400$$

$$\text{Operation} = (P38) \left(\frac{18}{3,600} \right) (N) = 0.19N$$

$$\text{Interest on capital} = (P50,000)(0.25) = 12,500$$
$$\text{Total annual cost} = P25,392 + 0.19N$$

Machine B

Annual costs:

$$\text{Depreciation} = \frac{P75,000}{F/A, 25\%, 6} = \frac{P75,000}{11.2588} = P 6,661$$

$$\text{Maintenance} = (P220)(12) = 2,640$$

$$\text{Operation} = (P30) \left(\frac{12}{3,600} \right) (N) = 0.10N$$

$$\text{Interest on capital} = (P75,000)(0.25) = 18,750$$
$$\text{Total annual cost} = P28,051 + 0.10N$$

Equating total annual cost,

$$\begin{aligned} P25,392 + P0.19N &= P28,051 + P0.10N \\ N &= 29,544 \text{ parts} \end{aligned}$$

Machine B will be more economical for number of parts more than 29,544

By the rate of return on additional investment method

Machine A

Annual costs:

$$\text{Depreciation} = \frac{P50,000 - P10,000}{F/A, 25\%, 3} = \frac{P40,000}{3.8125} = P10,492$$

$$\text{Maintenance} = (P200) (12) = 2,400$$

$$\text{Operation} = (P38) \left(\frac{18}{3,600} \right) (N) = 0.19N$$

$$\text{Total annual cost} \quad P12,892 + 0.19N$$

Machine B

Annual costs:

$$\text{Depreciation} = \frac{P75,000}{F/A, 25\%, 6} = \frac{P75,000}{11.2588} = P 6,661$$

$$\text{Maintenance} = (P220) (12) = 2,640$$

$$\text{Operation} = (P30) \left(\frac{12}{3,600} \right) (N) = 0.10N$$

$$\text{Total annual cost} \quad P 9,301 + P0.10N$$

$$\text{Annual savings on Machine B} = (P12,892 + P0.19N)$$

$$- (P9,301 + P0.10N)$$

$$= P3,591 + P0.09N$$

$$\begin{aligned} \text{Additional investment on Machine B} &= P75,000 - P50,000 \\ &= P25,000 \end{aligned}$$

$$\text{Rate of return on additional investment} = \frac{P3,591 + P0.09N}{P25,000} = 0.25$$

$$N = 29,544 \text{ parts}$$

Machine B is more economical for number of parts more than 29,544

GRAPHICAL SOLUTION

Machine A

Fixed costs = P25,392

Variable costs = P0.19N

When N = 50,000 parts

Variable costs = (P0.19) (50,000) = P9,500

Total cost = P25,392 + P9,500 = P34,892

Machine B

Fixed costs = P28,051

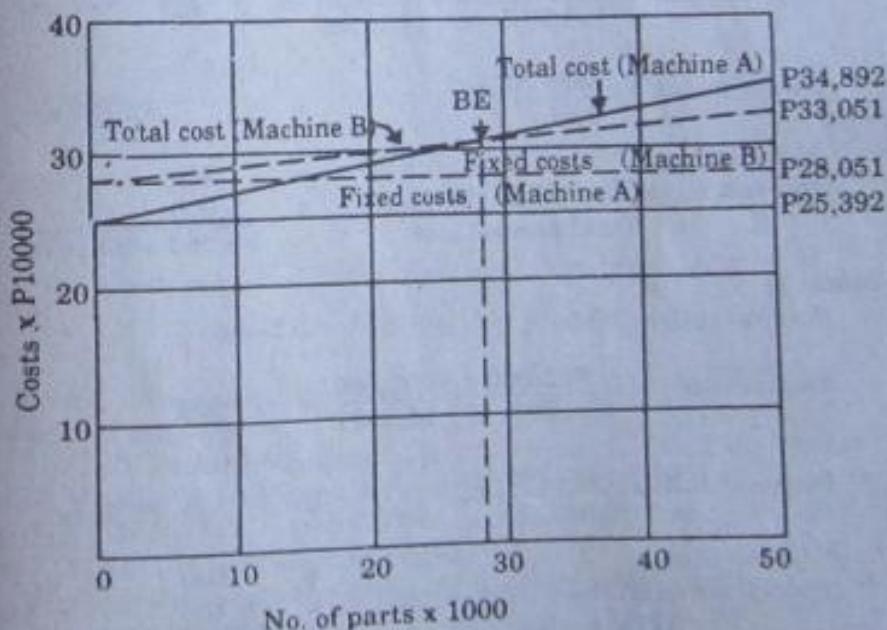
Variable costs = P0.10N

When N = 50,000 parts

Variable costs = (P0.10) (50,000) = P5,000

Total cost = P28,051 + P5,000 = P33,051

Break-Even Chart



Break-even point = 29,500 parts (approximately)

(10-2) Two electric motors are being considered to power an industrial hoist. Each is capable of providing 100 hp. Pertinent data for each motor are as follows:

| | Motor A | Motor B |
|-----------------------|---------|---------|
| Investment | P25,000 | P32,000 |
| Electrical efficiency | 84% | 88% |
| Maintenance per year | 400 | 600 |
| Life, years | 10 | 10 |

Money is worth 20%. If the expected usage of the hoist is 700 hours per year, what would the cost of electrical power have to be before Motor A is favored over Motor B?

Solution

Let x = cost of electrical power for both motors to be equally economical

Motor A

Annual costs:

$$\text{Depreciation} = \frac{\text{P}25,000}{F/A, 20\%, 10} = \frac{\text{P}25,000}{25.9587} = \text{P}963$$

$$\text{Power} = \frac{(100)(0.746)(700)x}{0.84} = 62,167x$$

$$\text{Maintenance} = 400$$

$$\text{Interest on capital} = \text{P}25,000(0.20) = 5,000$$

$$\text{Total annual cost} = \text{P}6,363 + 62,167x$$

Motor B

Annual costs:

$$\text{Depreciation} = \frac{\text{P}32,000}{F/A, 20\%, 10} = \frac{\text{P}32,000}{25.9587} = \text{P}1,232$$

$$\text{Power} = \frac{(100)(0.746)(700)x}{0.88} = 59,341x$$

$$\text{Maintenance} = 600$$

$$\text{Interest on capital} = \text{P}32,000(0.20) = 6,400$$

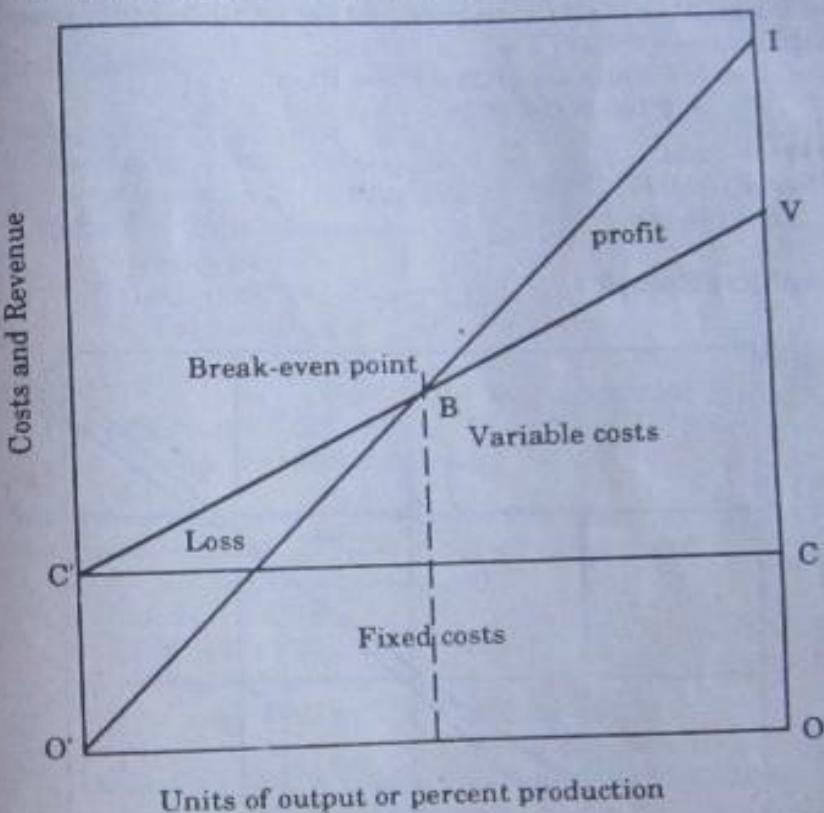
$$\text{Total annual cost} = \text{P}8,232 + 59,341x$$

$$\text{P}6,363 + 62,167x = \text{P}8,232 + 59,341x$$

$$x = \text{P}0.6614 \text{ per kwh}$$

Motor A will be more economical for electrical power cost less than P0.6614 per kwh.

BREAK-EVEN CHART FOR BUSINESS ENTERPRISE



Line C'C represents the fixed costs of production. Area C'CO'C' is the fixed costs region.

Line C'V shows the variation in total variable cost with production. C'V actually represent the sum of all production costs since its starting point is at C'. Area C'VCC' is the variable costs region and area C'VOO'C' is the total cost region.

Line O'I represents the gross income from sales. Area O'IOO' is the income region. Area BIVB is the profit region and area BO'C'B is the loss region.

(10-3) The cost of producing a small transistor radio set consists of P23.00 for labor and P37.00 for materials. The fixed charges in operating the plant are P100,000 per month. The variable cost is P1.00 per set. The radio set can be sold for P75.00 each. Determine how many sets must be produced per month to break-even.

Algebraic Solution

Let x = number of sets to be produced per month to break-even

$$\text{Total income} = P75(x)$$

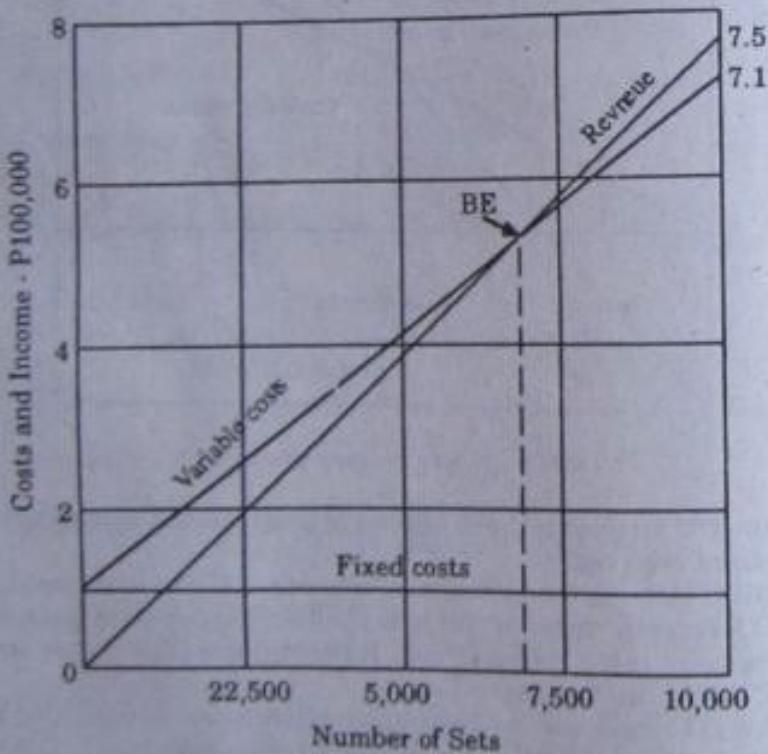
$$\begin{aligned}\text{Total cost} &= P100,000 + (P23 + P37 + P1)x \\ &= P100,000 + P61x\end{aligned}$$

To break-even,

$$P75x = P100,000 + P61x$$

$$x = 7,143 \text{ sets}$$

Graphical Solution



Break-even point = 7,143 sets

(10-4) A company has a production capacity of 500 units per month and its fixed costs are P250,000 a month. The variable costs per unit are P1,150 and each unit can be sold for P2,000. Economy measures are instituted to reduce the fixed costs by 10 per cent and the variable costs by 20 per cent. Determine the old and the new break even points. What are the old and the new profit at 100 per cent capacity?

Algebraic Solution

Income = P2,000 per unit

Old fixed costs = P250,000 per month

Old variable costs = P1,150 per unit

Let x = old break-even point

$$\text{Income} = P2,000x$$

$$\text{Total cost} = P250,000 + P1,150x$$

To break-even

$$\begin{aligned}P2,000x &= P250,000 + P1,150x \\ x &= 294 \text{ units per month}\end{aligned}$$

$$\begin{aligned}\text{Old profit} &= (P2,000)(500) - P250,000 - (P1,150)(500) \\ &= P175,000 \text{ per month}\end{aligned}$$

$$\text{New fixed costs} = (P250,000)(0.90) = P225,000$$

$$\text{New variable costs} = (P1,150)(0.80) = P920 \text{ per unit}$$

Let y = the new break-even point.

$$\text{Income} = P2,000y$$

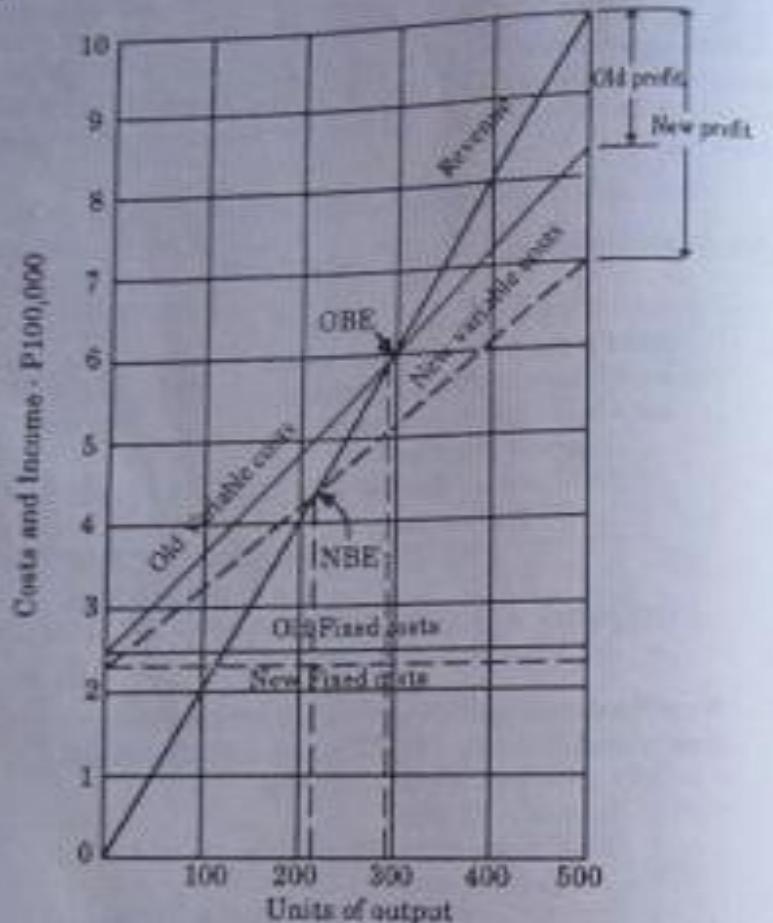
$$\text{Total cost} = P225,000 + P920y$$

To break-even,

$$\begin{aligned}P2,000y &= P225,000 + P920y \\ y &= 208 \text{ units per month}\end{aligned}$$

$$\begin{aligned}\text{New profit} &= (P2,000)(500) - P225,000 - (P920)(500) \\ &= P315,000 \text{ per month}\end{aligned}$$

Graphical Solution



Old break-even point = 294 units per month

Old profit = P175,000 per month

New break-even point = 208 units per month

New profit = P315,000 per month

(10-5) A local factory assembling calculators produces 400 units per month and sells them at P1,800 each. Dividends are 8% on the 8,000 shares with par value of P250 each. The fixed operating cost per month is P25,000. Other costs are P1,000 per unit. Determine the break-even point. If only 200 units were produced per month, determine the profit or loss.

Solution

Income = P1,800 per unit

Fixed costs = P25,000 per month

Variable costs = P1,000 per unit

Dividend = 8% per year

Let x = number of calculators per month to break-even

Income = P1,800x

Total cost = P25,000 + P1,000x

To break-even,

Income = Total cost

P1,800x = P25,000 + P1,000x

$x = 31.25$ say 32 units

$$\text{Dividend} = \frac{(P250)(0.08)(8,000)}{12} = P13,333 \text{ per month}$$

Income = Total cost + Dividend + Profit/loss

For 200 units:

$$(P1,800)(200) = P25,000 + (P1,000)(200) + P13,333 \\ + \text{Profit/loss}$$

$$\text{Profit} = P121,667 \text{ per month}$$

(10-6) A company manufacturing calculators has a capacity of 200 units a month. The variable costs are P1,000 per unit. The average selling price of the calculators is P2,500. Fixed costs of the company amount to P150,000 per month, which include all taxes. The company pays an annual dividend of P12 per share on each of the 30,000 shares of common stocks.

(a) Determine the number of calculators that must be sold each month to break-even and the sales volume corresponding to the unhealthy point.

(b) What is the profit or loss if 150 units were produced and sold a month?

("Unhealthy point" is the sales volume at which the business will be able to pay exactly the desired rate of dividend)

Algebraic Solution

Income = P2,500 per unit
 Fixed costs = P150,000 per month
 Variable costs = P1,000 per unit
 Dividend = P12 per share per year

(a) Let x = the break-even point

$$\begin{aligned} \text{Income} &= \text{P}2,500x \\ \text{Total cost} &= \text{P}150,000 + \text{P}1,000x \end{aligned}$$

To break-even,

$$\begin{aligned} \text{Income} &= \text{Total cost} \\ \text{P}2,500x &= \text{P}150,000 + \text{P}1,000x \\ x &= 100 \text{ units per month} \end{aligned}$$

$$\text{Dividend} = \frac{(\text{P}12)(30,000)}{12} = \text{P}30,000 \text{ per month}$$

Let y = the unhealthy point

At the unhealthy point,

$$\begin{aligned} \text{Income} &= \text{Total cost} + \text{Dividend} \\ \text{P}2,500y &= \text{P}150,000 + \text{P}1,000y + \text{P}30,000 \\ y &= 120 \text{ units per month} \end{aligned}$$

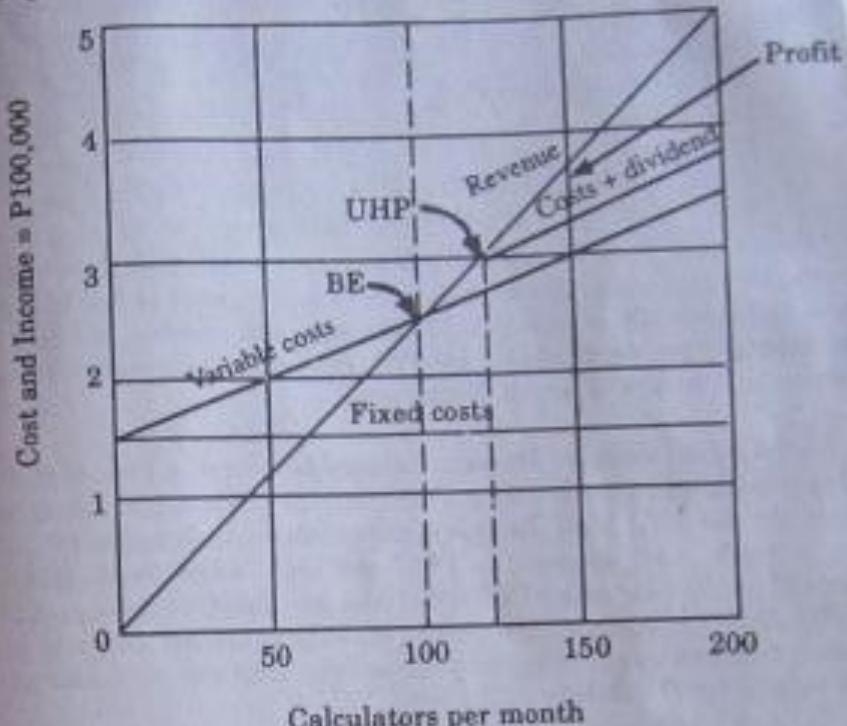
(b) Income = Total cost + Dividend + Profit/loss

For 150 units:

$$\begin{aligned} (\text{P}2,500)(150) &= \text{P}150,000 + (\text{P}1,000)(150) + \text{P}30,000 \\ &\quad + \text{Profit/loss} \end{aligned}$$

$$\text{Profit} = \text{P}45,000 \text{ per month}$$

Graphical Solution



(a) Break-even point = 100 units per month

Unhealthy point = 120 units per month

(b) Profit = P45,000 per month

PROBLEMS

1. A company is considering two alternatives with regards to an equipment which it needs. The alternatives are as follows:

Alternative A: Purchase

| | |
|----------------------|----------|
| Cost of equipment | P700,000 |
| Salvage value | 100,000 |
| Daily operating cost | 500 |
| Economic life, years | 10 |

Alternative B: Rental at P1,500 per day.

At 18% interest, how many days per year must the equipment be in use if Alternative A is to be chosen.

Ans. 152 days

2. Data for two 50-hp motors are as follows:

| | Alpha Motor | Beta Motor |
|---------------------|-------------|------------|
| Original cost | P37,500 | P48,000 |
| Annual maintenance | 1,500 | 750 |
| Life, years | 10 | 10 |
| Efficiency | 87% | 92% |
| Taxes and insurance | 3% | 3% |

Power cost is P2.00 per kwh. If money is worth 20%, how many hours per year would the motors have to be operated at full load for them to be equally economical? If the expected number of hours of operation per year exceeds the break-even point, which motor is more economical? Ans. 444 hours, Beta motor

3. A small shop in Bulacan fabricates portable threshers for palay producers in the locality. The shop can produce each thresher at a labor cost of P1,800. The cost of materials for each unit is P2,500. The variable costs amount to P650 per unit, while fixed charges incurred per annum totals P60,000. If the portable threshers are sold at P7,800 per unit, how many units must be produced and sold per annum to break-even? Support your answer with computations and also by graphical solution.

Ans. 25 units

4. Compute for the number of blocks that an ice plant must be able to sell per month to break-even based on the following data:

| | |
|-------------------------------|--------------|
| Cost of electricity per block | P 20 |
| Tax to be paid per block | 2 |
| Real estate tax | 3,500/month |
| Salaries and wages | 25,000/month |
| Others | 12,000/month |
| Selling price of ice | 55/block |

Ans. 1,238 blocks

5. A local company assembling stereo radio cassette produces 300 units per month at a cost of P800 per unit. Each stereo radio cassette sells for P1,200. If the firm makes a profit of 10% on its 10,000 shares with a par value of P200 per share, and the fixed costs are P20,000.000 per month.

- (a) What is the break-even point?
 (b) How much is the loss or profit if only 100 units are produced in a given month?

Ans. (a) 50 units per month, (b) P3,333 per month

6. A plant produces 300 units of an equipment a month of P3,600 each. A unit sells for P4.00. The company has 10,000 shares of stock at P200 par value whose annual dividend is 20%. The fixed cost of production is P120,000 a month.

- (a) What is the break-even point?
 (b) What is the "unhealthy point"?
 (c) What is the profit or loss if production is 60% of capacity?

Ans. (a) 100 units/month, (b) 128 units/month (c) P62,667 per month

Chapter 11

Benefit/Cost Ratio

The method of selecting alternatives that is most commonly used by government agencies for analyzing the desirability of public projects is the benefit/cost ratio (B/C ratio). The B/C method of analysis is based on the ratio of the benefits to costs associated with a particular project.

$$B/C = \frac{\text{benefits} - \text{disbenefits}}{\text{costs}} \quad (11-1)$$

Benefits are advantages, expressed in terms of pesos which happen to the owner. On the other hand, when the project under consideration involves disadvantages to the owner, these are known as disbenefits. The costs are the anticipated expenditures for construction, operation, maintenance, etc. A B/C ratio greater than or equal to 1.0 indicates that the project under consideration is economically advantageous.

(11-1) A nonprofit educational research organization, is contemplating an investment of P1,500,000 in grants to develop new ways to teach people the rudiments of profession. The grants would extend over a ten-year period and would achieve an estimated savings of P500,000 per year in professors' salaries, student tuition, and other expenses. The program would be an addition to ongoing and planned activities, thus an estimated P100,000 a year would have to be released from other program to support the educational research. a rate of return of 15% is expected. Is this a good program?

Solution

$$\text{Benefit} = \text{P}500,000 \text{ per year}$$

$$\text{Disbenefit} = \text{P}100,000 \text{ per year}$$

$$\text{Cost} = \text{P}1,500,000 (\text{A/P}, 15\%, 10) = \text{P}298,950 \text{ per year}$$

$$B/C = \frac{\text{P}500,000 - \text{P}100,000}{\text{P}298,950} = 1.34$$

The project is justified, since $B/C > 1.00$.

(11-2) The National Government intends to build a dam and hydroelectric project in the Cagayan Valley at a total cost of P455,500,000. The project will be financed by soft foreign loan with a rate of interest of 5% per year. The annual cost for operation, maintenance, distribution facilities and others would total P15,100,000. Annual revenues and benefits are estimated to be P56,500,000.

If the structures are expected to last for 50 years with no salvage value, determine the B/C ratio of the project.

Solution

By the equivalent uniform annual cost method

$$\text{Annual benefit} = \text{P}56,500,000$$

$$\begin{aligned} \text{EUAC} &= \text{P}455,500,000 (\text{A/P}, 5\%, 50) + \text{P}15,100,000 \\ &= \text{P}40,061,400 \end{aligned}$$

$$B/C = \frac{\text{P}56,500,000}{\text{P}40,061,400} = 1.41$$

This is a good project, since $B/C > 1.0$.

By the present worth cost method

$$PW_{\text{benefit}} = \text{P}56,500,000 (\text{P/A}, 5\%, 50) = \text{P}1,031 \times 10^6$$

$$PW_{\text{cost}} = \text{P}455.5 \times 10^6 + \text{P}15.1 \times 10^6 (\text{P/A}, 5\%, 50)$$

$$= \text{P}731.164 \times 10^6$$

$$BC = \frac{\text{P}1,031 \times 10^6}{\text{P}731.164 \times 10^6} = 1.41$$

This is a good project, since $B/C > 1.0$.

Alternative Comparison by Benefit/Cost Analysis.

In computing the benefit/cost ratio by Eq. (11-1) for a given alternative the benefits and costs used in the calculation represent the differences between the alternatives.

(11-3) Two routes are under consideration for a new highway. Route A would be located about five miles from the central business district and would require longer travel distances by local commuter traffic. Route B would pass directly through the down town area and, although its construction cost would be higher, it would reduce the travel time and distance for local commuters. The costs for the two routes are as follows:

| | Route A | Route B |
|-------------------------|--------------|--------------|
| Initial cost | P200,000,000 | P250,000,000 |
| Maintenance per year | 700,000 | 1,100,000 |
| Road-user cost per year | 10,000,000 | 4,000,000 |

If the roads are assumed to last 30 years with no salvage value, which route should be accepted on the basis of a benefit/cost analysis using an interest rate of 15%.

Solution

$$EUAC_A = P200,000,000(A/P, 15\%, 30) + P700,000 = P31.16 \times 10^6$$

$$EUAC_B = P250,000,000(A/P, 15\%, 30) + P1,100,000 = P39.175 \times 10^6$$

$$\text{Incremental annual benefit} = P10 \times 10^6 - P4 \times 10^6 = P6 \times 10^6$$

$$\text{Incremental annual cost} = EUAC_B - EUAC_A$$

$$= P39.175 \times 10^6 - P31.16 \times 10^6$$

$$= P8.015 \times 10^6$$

$$B/C = \frac{\text{incremental benefit}}{\text{incremental cost}} = \frac{P6 \times 10^6}{P8.015 \times 10^6} = 0.7486$$

Route A should be selected for construction.

(11-4) Four alternatives for providing electric power supply to a small town have been identified with the following annual benefits and costs:

| Alternative | Annual benefits | Annual costs |
|-------------|-----------------|--------------|
| A | P1,528,000 | P 780,000 |
| B | 1,398,000 | 664,000 |
| C | 960,000 | 742,000 |
| D | 810,000 | 420,000 |

Select the best alternative using the B/C ratio analysis.

Solution

Comparing alternative A with alternative B.

$$B/C \text{ of } A \text{ over } B = \frac{P1,528,000 - P1,398,000}{P780,000 - P664,000} = 1.12$$

Alternative A is preferred over alternative B.

Comparing alternative A with alternative C.

$$B/C \text{ of } A \text{ over } C = \frac{P1,528,000 - P960,000}{P780,000 - P742,000} = 14.95$$

Alternative A is preferred over alternative C.

Comparing alternative A with alternative D.

$$B/C \text{ of } A \text{ over } D = \frac{P1,528,000 - P810,000}{P780,000 - P420,000} = 1.99$$

Alternative A is preferred over alternative D.

Select alternative A.

PROBLEMS

- The Department of Public Works and Highways (DPWH) is considering the construction of a new highway through a scenic rural area. The road is expected to cost P50 million with annual upkeep estimated at P400,000. The improved accessibility is expected to result in additional income from tourists of P7 million per year. The

road is expected to have a useful life of 25 years. If the rate of interest is 15%, should the road be constructed?

Ans. The road should not be constructed, B/C is 0.86

2. Determine the B/C ratio for the following project.

| | |
|---------------------|----------|
| First cost | P100,000 |
| Project life, years | 5 |
| Salvage value | 10,000 |
| Annual benefits | 60,000 |
| Annual O and M | 22,000 |
| Interest rate, % | 15 |
| Ans. 1.19 | |

3. Data for two alternatives are as follows:

| | Alternative | |
|-----------------------|-------------|---------|
| | A | B |
| Investment | P35,000 | P50,000 |
| Annual benefits | 20,000 | 25,000 |
| Annual O and M | 6,450 | 13,830 |
| Estimated life, years | 4 | 8 |
| Net salvage value | 3,500 | 0 |

Using an interest rate of 20%, which alternative should be chosen?

Ans. Alternative A.

4. There are five alternatives for improvement of a road. Determine which alternative should be chosen if the highway department is willing to invest money as long as there is a B/C ratio of at least 1.00.

| Alternatives | Annual Benefits | Annual cost |
|--------------|-----------------|-------------|
| A | P 900,000 | P1,000,000 |
| B | 1,300,000 | 1,400,000 |
| C | 2,800,000 | 2,100,000 |
| D | 3,300,000 | 2,700,000 |
| E | 4,200,000 | 3,400,000 |

Ans. Alternative E should be the choice.