Q1 Using Matrix operations, describe the solutions for the following family of equations.

$$x + 2y - 3z = 5$$
$$2x + y - 3z = 13$$
$$-x + y = -8$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -3 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 13 \\ -8 \end{bmatrix}$$

$Ax = b$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 1 & -3 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 1 & -3 & 0 & 1 & 0 \\ 0 & 3 & -3 & 1 & 0 & 1 \end{array}\right] \quad R3 = R1 + R3$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 1 & -3 & 0 & 1 & 0 \\ 0 & 1 & -1 & \frac{1}{3} & 0 & \frac{1}{3} \end{array}\right] = \begin{bmatrix} 5 \\ 3 \\ -8 \end{bmatrix} \neq 0$$

$y = x - 8$

$x + 2(x-8) - 3z = 5$

$x + 2x - 16 - 3z = 5$

$3x - 3z = 21$

$3x = 3z + 21$

$x = z + 7$

$y = z + 7 - 8 = z - 1$

$z + 7 + 2(z-1) - 3z = 5$

$z + 7 + 2z - 2 - 3z = 5$

$0 = 0 \quad z = \infty$

$y = 0 - 1 = -1$

$x + 2(-1) - 0 = 5$

$x = 7$

Q3 Solve for AB by hand

$$A = \begin{bmatrix} 4 & -3 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$$

$3 \times 2 \qquad 2 \times 2$

[redacted]

$$R1 = \begin{bmatrix} 4 & -3 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 - 9 \\ -3 + 15 \\ 0 + 3 \end{bmatrix} = \begin{bmatrix} -5 \\ 12 \\ 3 \end{bmatrix}$$

$$R2 = \begin{bmatrix} 4 & -3 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 16 + 6 \\ -12 - 10 \\ 0 - 2 \end{bmatrix} = \begin{bmatrix} 22 \\ -22 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 22 \\ 12 & -22 \\ 3 & -2 \end{bmatrix}$$