

# Supplemental Problems — Assignment #3

1. The Cumulative distribution function of the random variable  $X$  is

$$F_X(x) = 1 - e^{-ax}, \quad a > 0, \quad x > 0$$

What is the probability density function?  
 What is the expected value?  
 What is the variance?

$$cdf = \int_0^x 1 - e^{-ax}$$

$$pdf = x - e^{-ax}$$

$$E[X] = \int_0^1 x - e^{-ax} dx$$

$$= x^2 + ae^{-ax}$$

$$Var[X] = E[X^2] - E[X]^2$$

$$= x^3 + ae^{-2ax} - (x^2 + ae^{-ax})^2$$

2. The probability mass function  $f(y) = \frac{1}{a-b} e^{-\frac{y}{b}}$  for a particular random variable  $y \in [0, \infty)$ ,  $b > 0$  derive the mean and standard deviation

$$f(y) = \frac{1}{a-b} e^{-\frac{y}{b}}$$

$$\text{Var}(y) = E[y^2] - E[y]^2$$

$$E[y] = \sum_{y=0}^{\infty} \left[ \frac{y}{a-b} \right] [p(y=1)]$$

$$E[y^2] = \sum_{y=0}^{\infty} \left[ \frac{y^2}{a-b} \right] [p(y=1)]$$

$$\text{Var}(y) = \sum_{y=0}^{\infty} \left[ \frac{y^2}{a-b} \right] [p(y)] - \left[ \sum_{y=0}^{\infty} \left[ \frac{y}{a-b} \right] [p(y)] \right]^2$$

$$\text{std} = \sqrt{\text{Var}(y)}$$