Test : Calculus I - Differentiation Formulas

1. If $f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$ then $f'(x)$ is

$$f(x) = (x^{-1} + x^{-2})(3x^3 + 27)$$
$$f(x) = 3x^2 + 3x + \frac{27}{x^2} + \frac{27}{x}$$
$$f'(x) = 6x + 3 + \frac{(27)(-2)}{x^3} + \frac{(27)(-1)}{x^2}$$
$$f'(x) = 3 + 6x - 27x^{-2} - 54x^{-3}$$

2. If $y = (2x^7 - x^2)\left(\frac{x-1}{x+1}\right)$ then $y'(1)$ is

$$y' = (2x^7 - x^2)\left[\frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}\right] + (14x^6 - 2x)\left(\frac{x-1}{x+1}\right)$$
$$y' = (2x^7 - x^2)\left[\frac{x+1-x+1}{(x+1)^2}\right] + (14x^6 - 2x)\left(\frac{x-1}{x+1}\right)$$
$$y' = 2x^7 - x^2\left[\frac{2}{(x+1)^2}\right] + (14x^6 - 2x)\left(\frac{x-1}{x+1}\right)$$
$$y'(1) = \left[2(1)^7 - (1)^2\right]\left[\frac{2}{(1+1)^2}\right] + \left[14(1)^6 - 2(1)\right]\left(\frac{1-1}{1+1}\right)$$
$$y'(1) = (2-1)\left(\frac{2}{4}\right) + (14-2)(0) = \frac{1}{2}$$

3. If $f(x) = (x^2+1)\sec x$ then $f'(x)$ is

$\frac{dy}{dx} \sec x = \sec x \tan x$

$$f(x) = (x^2+1)\sec x \tan x + (2x)\sec x$$

4. If $f(x) = \frac{1}{\cot x}$ then $f'(x)$ is

$\frac{1}{\cot x} = \tan x$

$\frac{dy}{dx} \tan x =$

$$f(x) = \frac{1}{\cot x} = \tan x$$
$$f'(x) = \sec^2 x$$

5. If $y = \tan x$ then $\frac{d^2y}{dx^2} =$

$\frac{dy}{dx} \tan = \sec^2 x$

$$\frac{dy}{dx} = \sec^2 x$$
$$\frac{d^2y}{dx^2} = 2\sec^2 x \tan x$$

6. If $f(x) = \sin^3 x$ then $f'(x)$ is

$f'(x) = 3\sin^2 x \cos x$ $\qquad \frac{dy}{dx}\sin x = \cos x$

7. If $f(x) = \cos^3\left(\frac{x}{x+1}\right)$ then $f'(x)$ is

$f'(x) = 3\cos^2\left(\frac{x}{x+1}\right)\left[-\sin\left(\frac{x}{x+1}\right)\right]\frac{x+1(1)-x}{(x+1)^2}$ $\qquad \frac{dy}{dx}\cos x = -\sin x$

$= 3\cos^2\left(\frac{x}{x+1}\right)\left[-\sin\left(\frac{x}{x+1}\right)\right]\frac{1}{(x+1)^2}$ $\qquad \frac{dy}{dx}\cos(f(x)) = -f'(x)\sin f(x)$

$= -\frac{3}{(x+1)^2}\cos^2 \sin\left(\frac{x}{x+1}\right)$

8. If $f(x) = \left[x^4 - \sec(4x^2-2)\right]^{-4}$ then $f'(x)$ is $\qquad \frac{dy}{dx}\sec x = (\sec x)(\tan x)$

$f'(x) = -4\left[x^4 - \sec(4x^2-2)\right]^{-5}\left[4x^3 - \sec(4x^2-2)\tan(4x^2-2)(8x)\right]$

$= -4\left[x^4 - \sec(4x^2-2)\right]^{-5} 4x\left[x^3 - \sec(4x^2-2)\tan(4x^2-2)\,2\right]$

$= -16x\left[x^4 - \sec(4x^2-2)\right]^{-5}\left[x^3 - 2\sec(4x^2-2)\tan(4x^2-2)\right]$

$\cancel{9.}$ If $y = \frac{\sin x}{\sec(3x+1)}$ then y' is $\qquad \frac{dy}{dx}\sin = \cos x$

$\qquad \frac{dy}{dx}\sec x = \sec x \tan x$

$y' = \frac{\cos x}{\sec(3x+1)\tan(3x+1)(3)}$

10. If $y = \left(\frac{1+x^2}{1-x^2}\right)^{17}$ then y' is $\qquad 2x - 2x^3$ $\quad + 2x + 2x^3$

$y' = 17\left(\frac{1+x^2}{1-x^2}\right)^{16}\left[\frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2}\right]$

$= 17\left(\frac{1+x^2}{1-x^2}\right)^{16}\left[\frac{2x - 2x^3 - (-2x + 2x^3)}{(1-x^2)^2}\right]$

$= 17\left(\frac{1+x^2}{1-x^2}\right)^{16}\left[\frac{4x}{(1-x^2)^2}\right]$ $\qquad 17 \times 4 = 68$

$\left[\frac{68x(1+x^2)^{16}}{(1-x^2)^{16}}\right]\left(\frac{1}{(1-x^2)^2}\right) = \frac{68x(1+x^2)^{16}}{(1-x^2)^{18}}$

Test: Calculus I - Derivatives of Logarithmic and Exponential Functions

1. If $y = \ln\left(\frac{(x^2+1)^5}{\sqrt{1-x}}\right)$ then y' is

$y = [\ln(x^2+1)^5] + \left(\frac{1}{\ln\sqrt{1-x}}\right) = [\ln(x^2+1)^5] - \ln\sqrt{1-x}^{-1}$

$= [5\ln(x^2+1)] - [\tfrac{1}{2}\ln(1-x)]$

$y' = \frac{5(2x)}{x^2+1} - \frac{1}{2}\left(\frac{-1}{1-x}\right)$

$= \frac{10x}{x^2+1} + \frac{1}{-2(x+1)} = \frac{10x}{x^2+1} - \frac{1}{2x+2}$

2. If $y = \ln(\ln(x))$ then y' is

$\frac{dy}{dx}\ln x = \frac{f'(x)}{f(x)} = \ln\left(\frac{1}{\ln(x)}\right)$ $\quad \frac{1}{1/\ln(x)} = \ln x$

3. If $y = \frac{x\ln x}{1+\ln x}$ then y' is

$y' = \frac{[(1+\ln x) + (x\frac{1}{x})](1+\ln x) - (x\ln x)(\frac{1}{x})}{(1+\ln x)^2}$ $\quad (1+\ln)(1+\ln) = \ln^2 + 2\ln +$

$y' = \frac{(1+\ln x)^2 - \ln x}{(1+\ln x)^2} = \frac{1-\ln x^2}{(1+\ln x)^2}$

4. If $y = \frac{1+\ln t}{t}$ then y' is

$y' = \frac{(1/t)t - (1+\ln t)(1)}{t^2}$

$= \frac{1-1-\ln t}{t^2} = \frac{-\ln t}{t^2}$

5. If $y = t\sqrt{\ln t}$ then y' is $\quad \sqrt{\ln t} = \ln t^{1/2}$, or $\frac{dy}{dx}\sqrt{\ln t} = \frac{1}{2\sqrt{\ln t}}$

$y' = (1)(\sqrt{\ln t}) + (t)\left(\frac{1}{2\sqrt{\ln t}}\right)$

$= \sqrt{\ln t} + \frac{t}{2\sqrt{\ln t}}$

6. If $y = \ln\frac{10}{x}$ then y' is $\qquad \frac{d}{dx}\ln x = \frac{1}{x}$

$$y = \ln 10x^{-1}$$

$$y' = \left(\frac{x}{10}\right)\left[(-1)(10)x^{-2}\right]$$

$$= \left(\frac{x}{10}\right)\left(\frac{-10}{x^2}\right)$$

$$= -\frac{1}{x}$$

7. If $f(x) = x^{1-x}$ then $f'(x)$ is

$$\ln f(x) = (1-x)(\ln x)$$

$$f'(x) = \left(\frac{1}{f(x)}\right)[f'(x)] = (-1)(\ln x) + (1-x)\left(\frac{1}{x}\right)$$

$$\left(\frac{1}{f(x)}\right)[f'(x)] = -\ln x + \frac{1-x}{x}$$

$$\cancel{\left(\frac{1}{f(x)}\right)}\ f'(x) = \left(-\ln x - 1 + \frac{1}{x}\right) f(x)$$

$$f'(x) = \left(-\ln x - 1 + \frac{1}{x}\right)\left(x^{1-x}\right)$$

$$\cancel{f'(x) =}$$

$\cancel{8}$ If $f(x) = e^{3x-1}$ then $f'(x) =$

$$f'(x) = (3x-1)\left(e^{3x-2}\right)$$

9. If $f(x) = \frac{1-x}{e^x}$ then $f'(x)$ is

$$f'(x) = \frac{-1e^x - (1-x)(e^x)}{(e^x)^2}$$

$$= \frac{-1e^x - (e^x - xe^x)}{e^x} = \frac{1-(1-x)}{e^x}$$

$$= \frac{x-2}{e^x}$$

10. If $f(x) = e^{\sqrt{x}} + e^{-\sqrt{x}}$ then $f'(x)$ is

$f(x) = e^{x^{1/2}} + e^{-x^{1/2}}$

$f'(x) = \frac{1}{2}x^{-1/2}e^{x^{1/2}} + -\frac{1}{2}x^{1/2}e^{-x^{1/2}}$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{x}}\right)e^{\sqrt{x}} - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{x}}\right)e^{-\sqrt{x}}$

$= \frac{e^{\sqrt{x}}}{2\sqrt{x}} - \frac{e^{-\sqrt{x}}}{2\sqrt{x}} = \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$

11. If $f(x) = \sin(2e^x)$ then $f'(x)$ is

$\frac{dy}{dx} \sin x = \cos x$

$\frac{dy}{dx} \sin(f(x)) = f'(x)\cos x$

$f'(x) = f'(x)\cos(2e^x)$

$= 2e^x \cos(2e^x)$

12. If $f(x) = x(3^{-5x})$ then $f'(x)$ is

$f'(x) = (1)(3^{-5x}) + (x)(\ln 3)(-5x)(3^{-5x})$

$= \cancel{3^{-5x}} + -5x^2(\ln 3)(\cancel{3^{-5x}})$

$3^{-5x}(1 - 5x^2\ln 3)$

Test: Calculus I - Section 3.7 - Indeterminant Forms and L'Hospital's Rule

1. $\lim_{\theta \to 0} \frac{\tan\theta}{\theta}$

$= \frac{\sec^2\theta}{1} = \sec^2\theta = 0$

~~2.~~ $\lim_{x \to \infty} \frac{x^{100}}{e^x}$

$= \frac{100x^{99}}{e^x} = \frac{(99)100x^{98}}{e^x} = \frac{(98)(99)(100x^{97})}{e^x} = \infty$

3. $\lim_{x \to 0} \frac{\sin^{-1} 2x}{x}$

$= \frac{2}{\sqrt{1-2x^2}} / 1$

$= \frac{2}{\sqrt{0}} = 2$

4. $\lim_{x \to \pi/2^-} \sec 3x \cos 5x$

$= \left(\frac{1}{\cos 3x}\right)(\cos 5x) = \frac{\cos 5x}{\cos 3x} = \frac{-5\sin 5x}{-3\sin 3x}$

$\hookrightarrow \frac{-5\sin 5(\frac{\pi}{2})}{-3\sin 3(\frac{\pi}{2})} = \frac{-5}{3}$

$= \frac{-25\cos 5x}{-9\cos 9x} = \frac{25}{9} = \frac{5}{3}$

5. $\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{bx}$

$\ln f(x) = bx\left[\ln\left(1 + \frac{a}{x}\right)\right]$

$= \ln\left(1 + \frac{a}{x}\right) = \frac{\left[1/\left(1 + \frac{a}{x}\right)\right]\left(1 + \frac{a}{x}\right)}{-bx^{-2}b}$

$=$

6. $\lim_{x \to 1} (2-x)^{\tan(\pi/2)x}$

$\ln f(x) = \ln(2-x)\left[\tan\left(\frac{\pi}{2}x\right)\right]$

$(2-x)^{\tan\left(\frac{\pi}{2}x\right)}$ (ln (

7. $\lim_{x \to 0} \left(\csc x - \frac{1}{x}\right)$

$y = \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$

$= \frac{x - \sin(x)}{x \sin(x)}$

$= \lim_{x \to 0} \left(\frac{1 - \cos(x)}{\sin x + x\cos(x)}\right)$

$= \lim_{x \to 0} \left(\frac{\sin x}{2\cos x - x\sin x}\right) = \frac{\sin(0)}{2\cos(0) - 0\sin 0} = 0$

8. $\lim_{x \to \infty} \frac{\ln(\ln x)}{\sqrt{x}}$

$= \frac{\frac{1}{x\ln x}}{\frac{1}{2\sqrt{x}}} = \frac{2}{\sqrt{x}\ln x} = \frac{2}{\underset{\infty}{\sqrt{\infty}}\ \underset{\infty}{\ln \infty}} = \frac{2}{\infty}$

Test: Calculus II - Integration Quiz 01

1. $\int_{-2}^{0} (2x+5)\,dx$

$y = x^2 + 5x$

$\hookrightarrow [0^2 + 5(0)] - [\overset{4}{-2^2} + \overset{-10}{5(-2)}] = 6$

~~2.~~ $\int_{0}^{\pi} \sin x \, dx$

$-\cos x$

$\hookrightarrow [-\overset{1}{\cos\pi}] - [-\overset{-1}{\cos 0}] = -1 - 1 = 0$

3. $\int_{0}^{\pi} \frac{1}{2}(\cos x + |\cos x|)\,dx$

$= \cos x + \cos x^{\frac{1}{2}}$

$\hookrightarrow [\cos 0 + \cos 0]^{1/2} - [\cos\pi + \cos\pi]^{1/2}$

$= \sqrt{1+1} - \sqrt{1+1} = \emptyset$

4. $\int \frac{9r^2\,dr}{\sqrt{1-r^3}}$

$(9r^2 dr)(\sqrt{1-r^3}^{\,-1}) \rightarrow (1-r^3)^{-1/2}$

$= \frac{1}{2}r^3 \quad r^2 \quad [2(1-r^3)^{1/2}]$

$= \frac{1}{2}r^3 \quad r^2 \; \cancel{2}(1-r^3)^{\frac{1}{2}}$

$= r^3 \quad r^2 (1-r^3)^{\frac{1}{2}}$

$2 \quad (1-r^3)^{1/2} \quad + C$

5. $\int \frac{dx}{\sqrt{5x+8}}$ $\frac{1}{\sqrt{5x+8}} = (5x+8)^{-1/2}$

$u = 5x+8$ $du = 5dx$ $dx = \frac{1}{5}du$

$= \int \left(\frac{1}{\sqrt{u}}\right)\left(\frac{1}{5}du\right)$

$= \int \frac{1}{5\sqrt{u}}du = \frac{1}{5}\int \frac{1}{\sqrt{u}}du$

$= \frac{1}{5}\int u^{-1/2}du = \frac{1}{5}\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \cancel{\frac{1}{5}}\frac{(5x+8)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$= \frac{2}{5}(5x+8) + C$

6. $\int \tan^7\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)dx$

$u = \tan\left(\frac{x}{2}\right)$ $du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$ $dx = 2\cos^2\frac{x}{2}du$

$= \int u^7 \sec^2\left(\frac{x}{2}\right)\left(2\cos^2\frac{x}{2}du\right)$

$= \int 2u^7du = 2\int u^7du = 2\frac{u^{7+1}}{7+1}$

$= 2\frac{\tan^{7+1}\left(\frac{x}{2}\right)}{8} + C$

7. $\int xe^{2x}dx$

$u = x$ $v = e^{2x} = \frac{e^{2x}}{2}$

$= x \cdot \frac{e^{2x}}{2} - \int (1)\frac{e^{2x}}{2}dx$

$= \frac{1}{2}e^{2x} - \int \frac{e^{2x}}{2}dx = \frac{e^{2x}}{4}$

$= \frac{1}{2}e^{2x} - \frac{e^{2x}}{4} + C$

8. $\int \frac{x^2}{x+1}\, dx$

$u = x+1 \quad du = 1dx \quad dx = 1du \quad x = u-1$

$= \int \frac{x^2}{u}\, du$

$= \int \frac{(u-1)^2}{u}\, du$

$= \int (u + \frac{1}{u} - 2\, du) = \int u\, du + \int \frac{1}{u}\, du - \int 2du$

$\frac{u^2}{2} \qquad \ln u \qquad 2u$

$= \frac{u^2}{2} + \ln u - 2u$

$= \frac{x+1^2}{2} + \ln(x+1) - 2x+1 = \frac{x^2}{2} - x + (\ln x+1) + C$

9. $\int \frac{1}{\sqrt{16-x^2}}\, dx$

10. $\int_2^\infty x^{-3/2}\, dx$

$= \frac{x^{-3/2+1}}{-3/2+1}$

$= (x^{-3/2+1})(\frac{-2}{3}) = \frac{-2}{\sqrt{x}} + C$

$\lim_{x \to 2} = \frac{-2}{\sqrt{2}} = -\sqrt{2} = 1.4142$

$\lim_{x \to \infty} = \frac{-2}{\sqrt{\infty}} = 0$

$= 0 - (-\sqrt{2}) = \sqrt{2} = 1.4142$