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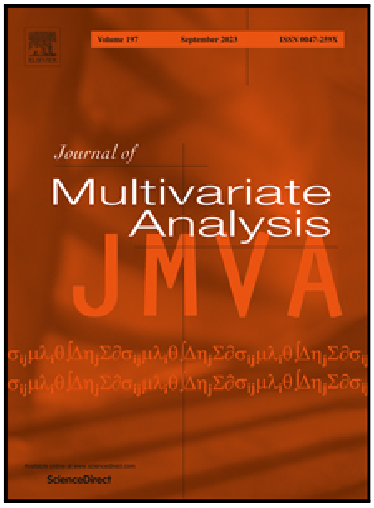

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Christian Genest, Ostap Okhrin, Taras Bodnar

PII: S0047-259X(23)00124-0
DOI: https://doi.org/10.1016/j.jmva.2023.105278
Reference: YJMVA 105278

To appear in: *Journal of Multivariate Analysis*

Please cite this article as: Christian Genest, Ostap Okhrin, Taras Bodnar, Copula modeling from Abe Sklar to the present day, *Journal of Multivariate Analysis* (2023), doi: https://doi.org/10.1016/j.jmva.2023.105278

Copula modeling from Abe Sklar to the present day

Christian Genest[a,*], Ostap Okhrin[b], Taras Bodnar[c]

[a]Department of Mathematics and Statistics, McGill University,
805, rue Sherbrooke ouest, Montréal (Québec) Canada H3A 0B9
[b]Chair of Econometrics and Statistics, especially Transportation, Technische Universität Dresden, D-01062 Dresden, Germany
Center for Scalable Data Analytics and Artificial Intelligence (ScaDS.AI) Dresden/Leipzig, Germany
[c]Department of Mathematics, Stockholm University, SE-10691 Stockholm, Sweden

Abstract

This paper provides a structured overview of the contents of the Special Issue of the *Journal of Multivariate Analysis* on "Copula Modeling from Abe Sklar to the present day," along with a brief history of the development of the field.

Keywords: Copula models, Dependence modeling
2020 MSC: 62H05, 62H10, 62H12, 62H15, 62H20, 62G32

1. Copula modeling from Sklar to the present day

In statistics, the word "copula" refers to a multivariate cumulative distribution function with uniform margins on the unit interval. The term first appeared in a 3-page note by Abe Sklar [216] published in 1959 in the *Publications de l'Institut de statistique de l'Université de Paris*. His paper highlighted the fact that any multivariate distribution can be expressed as a function of its margins and a (possibly non-unique) copula. The thought had occurred to others before him, certainly Hoeffding [123, 124] and possibly Fréchet [70], but by naming the concept, Sklar attracted a great deal of attention to it. Copulas were a recurring theme throughout his scientific career, which is summarized in a tribute to his life [76] published after his death in 2020. This Special Issue is dedicated to his memory.

In a review paper written for the first conference on probability distributions with given margins, held in Rome in April 1990, Schweizer [207] observed that until the beginning of the 1980s, most research about copulas was motivated by the development of probabilistic metric spaces. Schweizer and Sklar were two of the main contributors to this theory, an account of which is given in their book [208]. In that context, copulas arose naturally in the study of families of binary operations on the space of probability distribution functions.

Surprisingly, perhaps, it is not until the mid 1970s that copulas started to emerge in a statistical modeling context. Among the first contributions were the papers by Schweizer and Wolff [209] and Kimeldorf and Sampson [136], although the latter authors speak of "uniform representations" rather than copulas. The role of copulas in the study of stochastic orderings and nonparametric measures of association gradually transpired and is clearly documented in the seminal book by Nelsen [167], which also includes a compendium of the (mostly bivariate) parametric families of copulas known at the time. An expanded and more modern treatment of the mathematical principles of copula theory was provided in 2016 by Durante and Sempi [57], whose meticulously documented text also draws connections with related concepts such as quasi-copulas and semi-copulas whose study has taken on a life of its own.

It was Deheuvels [50, 51] who, in the late 1970s, opened the door to copula inference by introducing an empirical analog of what he called the "dependence function." In hindsight, Rüschendorf's work [196] was a precursor. Deheuvels used this connection to propose a test of independence later investigated in [101], and this inspired a whole slew of testing procedures along similar lines, as discussed below. Such developments, and indeed copula inference in general, rely heavily on the large-sample behavior of the empirical copula process, which was gradually refined;

*Corresponding author
Email addresses: christian.genest@mcgill.ca (Christian Genest), ostap.okhrin@tu-dresden.de (Ostap Okhrin), taras.bodnar@math.su.se (Taras Bodnar)

see, e.g., [27, 66, 73, 210]. It was recently extended to the multilinear empirical copula process [93, 94, 164], which is key to nonparametric inference for copula models with discontinuous margins [95, 165].

That rank-based methods could also be used to carry out margin-free inference for parametric copula models came to the fore in the late 1980s. Much of the early work along these lines is due to the "Québec school," where rank-based estimation [75, 104] was first considered within the context of bivariate Archimedean copulas, then extended to parametric families of multivariate copulas [81, 213]. Then came diagnostic tools, tests of independence, symmetry, radial symmetry, etc., as well as goodness-of-fit tests for copula models; see, e.g., [13, 77, 91, 92, 100, 103, 109]. This group also investigated connections between extreme-value analysis and copulas; see, e.g., [29, 54, 82, 108].

In 1997, the publication of Joe's book [130] on multivariate models and dependence concepts was instrumental in the recognition of copula modeling as a global strategy for multivariate data analysis. Among many other things, this very dense and influential textbook proposed various ways of constructing multivariate distributions, expanded the list of multivariate (not just bivariate) copula families, and drew attention to tail dependence as well as to the interplay between copulas and extreme-value theory, which was still in its infancy [29]. With the publication, at the turn of the century, of Nelsen's book [167] and widely read introductory articles by Frees and Valdez [71] and Embrechts et al. [59] on dependence modeling with copulas in insurance and risk management, the world was ready for a copula wave, whose meteoric rise, related in [80], also brought its share of criticisms [161, 198]; see also [190].

By the mid 2000s, the proliferation of massive, multivariate data sets gave a new sense of urgency to the need for complex models, and the copula approach grew quickly in popularity, due to its flexibility. This trend is perhaps best exemplified by the books of Cherubini et al. [37, 38], McNeil et al. [157], Trivedi and Zimmer [220], Salvadori et al. [200] and Rüschendorf [197], which stimulated the use of copulas in finance, quantitative risk management, economics, and environmental science. In hydrology, the review paper "Everything you always wanted to know about copula modeling but were afraid to ask" [78] also sparked interest. Other early examples of copula modeling include climate research [205], econometrics [184], engineering [135], genetics [151], and transportation research [17].

The advent of high-dimensional dependence modeling through pair-copula constructions, which gradually surfaced in the late 2000s, marked a major turning point in the development of the field. In an early survey on this topic, Czado [46] credits Joe [129] for the original idea, and Bedford and Cooke [11] for the expression of this construction in a sequence of nested trees. This rich approach, which was expanded in the book by Kurowicka and Cooke [147], spread like wildfire once Aas et al. [1] paved the way to inference in 2009. Czado's practical guide to copula modeling through vines [47], and her review paper with Nagler [48], give an excellent overview of this subject. Other alternative hierarchical approaches have since been proposed, such as hierarchical (nested) Archimedean copulas [156, 177, 202], factor copula models [144, 176], hierarchical Kendall copulas [23], and aggregation based on shocks [55] or sums [6, 43]. Claudia Czado and Harry Joe were pioneers and remain leaders in this area [98, 106].

In parallel, Andrew Patton's introduction of a conditional version of Sklar's theorem [184, 185] led to the development of dynamic copula models [34, 35, 111] in the econometric and financial literature; early contributions surveyed by Patton [186] in 2012 include [9, 67, 217], to which one should add the paper by Rémillard [194], whose publication brought him an award after being significantly delayed. Books which promote some of these techniques in finance are [38, 193]. Copula-based regression modeling, initiated by Song [218] and for which an early survey was written by Kolev and Paiva [143], continues to be an object of intense study. In addition to Irène Gijbels' prolific work on this topic with her collaborators [107], representative recent contributions include [44, 171–173, 224].

While copula modeling was initially developed for continuous and complete data, it was eventually recognized as a powerful tool in more complex settings. In survival analysis, where data are typically incomplete, the early work of Oakes on frailty models [174, 175] spurred extensive research as related in the book by Emura et al. [60]. Copula-based modeling with missing data has also been considered in [53, 119, 120], among others, and the use of copulas eventually extended to medical research; for recent examples, see [45, 154]. Finally, as already recognized in Joe's first book [130], copula models are valid for discrete data and have been used with success, e.g., in [183]. Although non-identifiability issues were flagged early [85] and pose technical challenges [65, 74], inference procedures can be developed using the empirical multilinear copula process [88, 93, 94, 168].

Since 2010, research on copula modeling has boomed, and it would be difficult to present an authoritative, balanced, and scholarly survey of recent developments for lack of hindsight. That copulas prove invaluable whenever dependence needs to be modeled is now widely recognized, and countless applications can be found, from multiple testing theory [169, 170, 219] to cybersecurity risk modeling [187] and the detection of Martian sand dunes [30].

Today, copula modeling is a mature field and a thriving area of research, and this Special Issue gives an eloquent proof of its vitality. A structured overview of its contents is provided in the following section. For introductions to the subject from various angles, readers can refer to the books by Nelsen [167], Trivedi and Zimmer [220], Mai and Scherer [155], Joe [130, 131], Durante and Sempi [57], Czado [47], and Hofert et al. [127]; the latter also provides an introduction to the comprehensive R package `copula` [126]. Extended reviews, such as those of Patton [186], Okhrin et al. [181] or Czado and Nagler [48] can also provide valuable insights into specific advances.

2. New contributions to copula modeling in this Special Issue

This Special issue contains 17 new contributions to the literature on copula modeling. They can be divided somewhat arbitrarily into seven broad themes, with the understanding that the categories are not mutually exclusive and that some papers actually touch on several of these topics.

2.1. New classes of copula models

Having access to a vast repertoire of copula models is a prelude to the proper study of complex dependencies. At present, the best known and most commonly used analytic families of copulas include Archimedean [84, 167], elliptical [79, 130], extreme-value [90, 110], Archimax [31], and reciprocal Archimedean copulas [96], which correspond to multivariate max-id copulas with L_1-norm symmetric exponent measure. This Special Issue features two papers which introduce new classes of copula models that can capture specific data features.

In the first paper [19], Blier-Wong, Cossette, Legros, and Marceau propose an approach to build a new class of generalized Farlie–Gumbel–Morgenstern [64, 117, 162] copulas which is suitable for high-dimensional modeling. Elements of this class capture various types of association and asymmetry but also yield exact expressions for dependence coefficients and risk measures which are handy in practice. The authors show how to construct, and sample from, these models using mixtures of power functions based on multivariate Bernoulli and Coxian-2 distributions. They also investigate the properties of these copulas and discuss applications. Their findings extend earlier work of Blier-Wong et al. [20] by relaxing the assumption of symmetry of Bernoulli random variables.

In the second paper, Bernard, Müller, and Oesting [14] investigate L_p-norm spherical copulas in arbitrary dimension. Their work, which builds on that of Gupta and Song [118], is motivated by a suspicion that these distributions lead to tight bounds for a generalized mean difference. The authors give the exact conditions under which these copulas exist and are uniquely defined. They also provide explicit formulas for the density, the correlation coefficient, and the distribution of the radial part of elements of this class of copulas. Moreover, they explore the relation between L_∞-norm spherical distributions and copulas. In their paper, simulation techniques and inference for this class of models are also briefly addressed.

2.2. Copula factor models

In traditional multivariate analysis, factor models have long been used to simplify the dependence structure of a random vector when its components are thought to be explained by a few latent variables called factor scores. These models have found applications in various fields, notably finance [15, 28, 61, 62, 112]. A copula extension of this strategy was proposed about 10 years ago by Krupskii and Joe [144, 145]. Copula factor models are especially valuable for scenarios where variables exhibit stronger tail dependencies than what Gaussian models can capture. However, much work remains to be done to understand their nature and develop efficient inference methods for this broad class of models. This Special Issue includes two papers which pertain to this theme.

In the first paper, Fan and Joe [63] extend recent work of Krupskii and Joe [146] on the use of proxies as a way to facilitate the selection and estimation of the bivariate parametric copula families linking the observed variables to the latent variables within a copula factor model. Exploiting the fact that in structural factor models, factor scores are defined as conditional expectations of latent variables given the observed variables, the authors provide weak conditions under which the proxies are consistent for the corresponding latent variables, as the sample size and the number of observed variables linked to each latent variable increase. For the case where the copulas are not known in advance, they propose valid sequential procedures for latent variables estimation, copula family selection, and parameter estimation. Their paper generously lists various avenues for further research and applications.

In the second paper, Ansari and Rüschendorf [5] compare general factor models in terms of the supermodular and directionally convex order. Their work extends and strengthens some of their previous findings [2–4]. It builds upon orthant ordering results for $\star$-products [49, 56], which represent the copula of factor models, and extends these conclusions to factor models with general conditional dependencies. As an illustration, they derive worst-case scenarios in certain classes of factor models, leading to sharp bounds in financial and insurance risk models.

2.3. Capturing dependence

Searching for dependence in a haystack of random variables is a common and challenging problem. Measures of central dependence often used to this end include Pearson's correlation and standard rank-based concordance measures such as Spearman's rho, Kendall's tau, Spearman's footrule, Blomqvist's beta, van der Waerden's coefficient, and many others; see, e.g., [39, 86, 97]. The compatibility and attainability of matrices of such coefficients are the subject of ongoing research as in [125, 139, 159]. Kendall's tau [87, 134] is particularly popular and has recently been used with much success in detecting structure in high-dimensional data [188, 189]. In some applications, however, other distributional characteristics may be more suitable, such as tail dependence coefficients [130, 203] and measures of asymmetry [121, 132]. This Special Issue contains two papers on this theme.

In the first paper, Koike and Hofert [138] compare measures of concordance based on Pearson's linear correlation coefficient between random variables transformed in such a way that they follow the so-called concordance-inducing distribution. Spearman's, Blomqvist's, and van der Waerden's coefficients belong to this broad class of concordance measures whose elements are compared in terms of their best and worst asymptotic variances for some canonical estimators over a certain set of dependence structures. This prompts the authors to propose a simple criterion according to which Blomqvists beta would be the optimal transformed rank correlation. They also find that Kendalls tau, which is not a transformed rank correlation, shares a certain optimal structure with Blomqvists beta.

In the second paper, Fuchs [72] studies an index of the dependence of a random variable on a set of covariates which Azadkia and Chatterjee [7] used to derive a new variable selection algorithm. The latter authors' work, which is based on [32, 52], has attracted a lot of attention recently; for an overview of this literature, see [33]. Fuchs observes that the unconditional version of this index can be expressed in terms of Spearman's footrule of the copula between two conditionally independent copies of the endogenous variable. Consideration is then given to the use of alternative coefficients, notably Spearman's and Gini's gamma, which yield meaningful measures of the dependence between the exogenous and endogenous variables. A consistent estimator of this underlying copula is also proposed, which is related to the graph-based estimator of the index developed in [7].

2.4. Focusing on tail dependence

Modeling tail dependence is crucial in risk management and extreme-value analysis. As the 2008 financial crisis tragically showed [149, 150], failure to understand tail behavior leads to a poor assessment of the simultaneous risk of multiple defaults which is so valuable in credit risk modeling and stress testing. Yet the study of tail dependence has a long history. Sibuya [215] attracted attention to it as early as 1960, Joe [130] gave it prominence in his book, and there were several papers on the subject by the mid-2000s; see, e.g., [69, 115, 203, 204]. Much work has been done on this topic, particularly in finance and in hydrology; examples of recent work regarding attainability and estimation include [58] and [137]. This Special Issue contains two papers that contribute to the understanding and modeling of tail behavior, which can help to ensure preparedness and mitigate risk through forecasting and early warning systems.

In the first paper, Siburg and Strothmann [214] investigate the intricate connection between a local stochastic dominance relation for copulas and a natural ordering for their tail dependence functions. While these two orderings are generally distinct, their study uncovers instances of equivalence, notably within various important copula classes, such as multivariate Archimedean and bivariate lower-extreme value copulas. The authors' findings make it possible to conclude that while a smaller tail dependence function does not necessarily imply a smaller risk of extremal events, such is the case at least for some classes of copulas which they identify.

In the second paper, Coia, Joe, and Nolde [41] complement this exploration by investigating the comparison of conditional and marginal tail indices. They propose a methodology to break down the conditional tail index into a product of copula-based conditional tail indices and the marginal tail index, offering valuable insights for applications. Their paper also introduces novel copula families designed to accommodate non-constant conditional tail indices, thereby enhancing the flexibility of copula-based modeling approaches. This research heavily builds upon, and extends, the wide strand of literature on vine copula models [10–12, 24, 47].

2.5. Nonparametric copula estimation

In preliminary data analysis and in some applications, parametric copula models may be undesirable or unfeasible. In such cases, practitioners may wish to rely on smooth nonparametric copula estimation instead. Two prominent examples of such constructions are the class of empirical Bernstein copulas [201] and the class of empirical beta copulas [211], but similar approaches were used, e.g., in [25, 26, 42, 68] to estimate the Pickands dependence function characterizing extreme-value copulas. This Special Issue includes three contributions to this general topic.

In the first paper, Kojadinovic and Yia [142] present a new class of smooth nonparametric copula estimators which is sufficiently broad to include the empirical Bernstein and beta copulas. The authors focus on a subclass that depends on a scalar parameter determining the amount of marginal smoothing and a functional parameter controlling the shape of the smoothing region. They find through simulation that two specific data-adaptive members of this class improve upon the empirical beta copula and, with future applications to change-point detection in mind, they determine conditions under which the related sequential empirical copula processes converge.

In the second paper, Purkayastha and Song [191] develop a consistent and powerful estimator of mutual information between random variables. Based on a copula formulation, their algorithm estimates mutual information by leveraging fast Fourier transform-based estimation of the underlying density. Compared to existing methods, a great advantage of their approach is that it does not rely on any parameter tuning. The authors describe an R package called `fastMI` with which they could show that their approach outperforms state-of-the-art estimators with improved estimation accuracy and reduced run-time for large data sets. This work adds to an already wide array of copula packages that includes `copula` [126], `HAC` [179] and `gofCopula` [182], among others.

Coincidentally, the third paper also relates to information theory, as Chen and Sei [36] are concerned with the estimation of the minimum information bivariate copula, i.e., the copula which is closest to being an independent distribution in the sense of Kullback–Leibler divergence while satisfying given constraints on expectation. As the density of such a copula involves a set of normalizing functions that are often difficult to compute, the authors propose the use of a conditional Kullback–Leibler score, which circumvents this problem. They show that the proposed score is strictly proper in the space of copula densities and that, as a result, the estimator derived from it is consistent. Furthermore, the score is convex with respect to the parameters and can be easily optimized by the gradient methods.

2.6. Tests of independence and goodness-of-fit

As mentioned in Section 1, the first tests of independence explicitly based on copulas are due to Deheuvels [50, 51], although the idea actually goes back much further, considering that classical nonparametric tests of independence are themselves copula-based. The literature on this topic is too vast to summarize, but key ingredients in its development are the probability integral transform (PIT), its multivariate version leading to the Kendall distribution and process [8, 105], the theory of U-statistics [148], and empirical process theory [221]. By contrast, goodness-of-fit testing for copula models is a relatively new theme, with the first omnibus tests going back to [223]; see [103] for an early critical review. An essential tool to the implementation of such tests is the parametric bootstrap; see, e.g., [102, 141]. Two paper in this Special Issue directly relate to testing.

In the first paper, Quessy and Lemaire-Paquette [192] propose a new approach for the construction of tests of multivariate independence and goodness-of-fit tests for copulas. While testing procedures based on the distribution of the multivariate PIT had already been considered, e.g., in [99], the authors base their tests on the corresponding weighted empirical characteristic function. Tests based on empirical characteristic functions have a long and successful history, as related by Meintanis [160]. This approach is thus natural and is shown by the authors to have good sampling properties through asymptotic results and extensive simulations.

In the second paper, Nasri and Rémillard [165] construct and study tests of independence for an arbitrary distribution when the data either form a random sample or constitute a stationary and ergodic sequence. Their procedures are derived from copula-based covariances and their multivariate extensions using Möbius transforms [93, 101, 140]. This is in contrast to earlier tests of independence for arbitrary distributions [95, 164], which were based on Cramér–von Mises type statistics or related processes. The authors establish the asymptotic properties of these statistics both under the null and under a sequence of contiguous alternatives. In the spirit of [109], this enables them to find locally most powerful tests within the class of the proposed tests statistics. They also discuss how to combine the proposed tests statistics and assess their finite-sample power through simulations using the R package `MixedIndTests` [166].

2.7. *Statistical inference for Archimedean models*

Archimedean copulas were one of the earliest, non-Gaussian dependence models introduced in the copula literature. Their basic properties were investigated, e.g., in [84, 158, 163], and their estimation was addressed in [89, 104, 128, 178]. Despite their high degree of symmetry, they remain appropriate in a variety of contexts, notably actuarial science [71, 83], finance [122], hydrology [199], and false discovery rate control [21]. Archimedean copulas also play a prominent role in modeling dependent competing risks [22, 195, 222], where their use to model the dependence between survival and censoring times yields insights into the copula-graphic estimator of Zheng and Klein [225]. In high-dimensional contexts, flexible hierarchical copula structures have also been based on Archimedean copulas [113, 114, 177, 202, 212]. This Special Issue features four contributions relating to Archimedean copulas.

In the first paper, Okhrin and Ristig [180] present a new, multi-stage approach for the estimation of parametric hierarchical Archimedean copula structures. Their method, inspired by [177], estimates the parameters and aggregates the structure simultaneously by imposing a non-concave penalty on differences between parameters which coincides with an implicit penalty on the dependence structure. The consistency of the estimator and of the data-driven aggregation procedure is determined, and their small-sample properties are studied by simulation. For practitioners, the proposed methodology is embedded within the R package `HAC` [179].

In the second paper, Kasper [133] derives an explicit formula for the conditional distributions of Archimedean copulas and uses it to prove, among other things, that point-wise convergence of a d-variate Archimedean copula is equivalent to weak convergence of almost all $(d-1)$-dimensional Markov kernels. As detailed in the paper, any estimator of an Archimedean copula generator can then be converted into an estimator of the generator of its conditional copulas. A conditional version of the recent multivariate dependence measure of Griessenberger et al. [116] is also introduced as a tool to investigate the dependence behavior of data from Archimedean copulas given covariate values.

In the third paper, Lo and Wilke [153] consider a dependent competing risks model in which the dependence structure between the risks is embodied in an Archimedean copula, as in [22, 195, 222]. Extending the work of Schwarz et al. [206] and Lo and Wilke [152], the authors show the identifiability of the model when at least one covariate takes at least two values. Estimation is performed by means of a consistent, semi-parametric, two-step procedure. Simulations are used to show the applicability and finite-sample properties of their method, and an application to unemployment duration data confirms the importance of estimating, rather than assuming, risk dependence.

Finally, in the fourth paper, Bevilacqua, Alvarado, and Caamaño-Carrillo [16] propose an alternative to the Gaussian copula model for the analysis of spatial data which may be correlated in time. The random field they propose, which has uniform margins, can be viewed as a spatial generalization of the classical Clayton copula model [40]. They study the second-order properties of the model, provide key analytic expressions, and examine geometric characteristics of the construction in the reflection symmetric case. Moreover, they investigate numerically the use of the weighted pair-wise composite likelihood method for estimation, and illustrate the methodology with the analysis of point-referenced vegetation index data using their associated R package `GeoModels` [18].

3. Valediction

As Guest Editors for this Special Issue, we are grateful to the Editor-in-Chief of the *Journal of Multivariate Analysis*, Dietrich von Rosen, for supporting this project to honor the late Abe Sklar. We also wish to thank all the authors and referees for their collaboration, patience, and understanding in completing this major undertaking. Moreover, we kindly ask for the readers' indulgence if, in spite of our best combined efforts, glaring omissions or involuntary misrepresentation of facts remain; we faced serious time and space constraints in the production of this editorial. Funding in partial support of this work from the Canada Research Chairs Program and the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

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