

# Local Projections and DiD

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## When does Local Projection make sense in a panel?

- Jordà (2005) huge success in macro
  - Goal: estimate impulse response functions in a single time-series
  - Approach: estimate a sequence of OLS regressions of  $y_{t+h}$  on shock and controls in  $t$  for  $h = 0, 1, \dots, H$
- DiD literature highlights issues with
  - Staggered Treatments
  - Heterogeneous Treatment Effects
  - Dynamic Treatment Effects
  - see Paul GP course
- If DiD has all these issues, then LP must have them too!

## De Chaisemartin and d'Haultfoeuille (2022)

- highlights issues with panel local projections
  - applied in complicated DiD treatment settings (continuous treatment intensity, staggered treatment, dynamic effects)
  - "contamination by future"
  - sign-reversal, even with positive treatment effects in every cell estimates may turn negative
- generalize event-study design
- General takeaway: TWFE+LP → misspecified regression when there is variation in treatment timing

## Dube, Girardi, Jorda, and Taylor (2023) Slides

- Goal: estimate treatment effects in a panel with typical DiD issues
- Solution: select observations appropriately/control for previous treatments
- LP is just OLS, so this is super fast compared to other suggested estimators

## Which issues from DiD literature carry over to LP setting with shocks?

- Staggered Treatment  $\rightarrow$  What happens with trend in treatment intensity?
- Heterogeneous Treatment Effects  $\rightarrow$  Bias even with mean zero shocks?
- Dynamic Treatment Effects

## IRF in panel with TWFE +LP

$$y_{it} = \alpha_i + \phi_t + \beta_{0,i}s_{it} + \rho_i y_{it-1} + \varepsilon_{it},$$

- where
  - groups  $i = 1, \dots, I$
  - time periods  $t = 1, \dots, T$
  - outcome of interest  $y_{it}$
  - shock  $s_{it}$  (continuous shock)
  - group specific intercept  $\alpha_i$  and trend  $g_{it}$
  - time specific intercept  $\phi_t$
  - $\varepsilon_{it} \sim N(0, \sigma^2)$  i.i.d.
- Treatment Effects/IRF:

$$\beta_h = \beta_{0,i} \rho_i^{h-1} \quad \forall h \geq 0$$

## IRF in panel with TWFE +LP

- Simulate with
  1. group specific shock mean  $s_{it} \sim N(\mu_i, \sigma_s^2)$
  2. time specific shock mean  $s_{it} \sim N(\mu_t, \sigma_s^2)$
  3. Heterogeneous treatment effects  $\beta_{0,i}$
  4. trend in shock mean  $\mu_{i,t} = \mu_i + \gamma_i t$
- Monte Carlo Version 1 (large T, small I):
  - 10000 repetitions
  - I = 2 groups
  - T = 100 time periods
  - common AR process  $\rho_i = \rho$
- Monte Carlo Version 2 (smallish T, medium I):
  - 10000 repetitions
  - I = 30 groups
  - T = 10 time periods
  - common AR process  $\rho_i = \rho$

## Case 1: Group specific shock mean $T = 100$ , $I = 2$

$$E[s|i] \neq E[s|i']$$

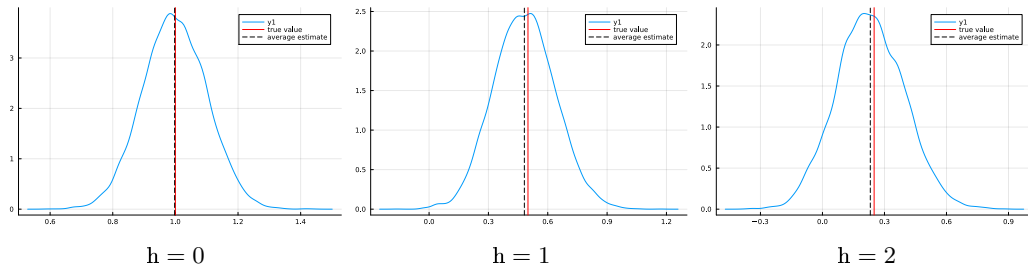


Figure: IRF Case 1

No problem with group specific shock mean (as expected)



## Case 1: Group specific shock mean and $T = 10$ , $I = 30$

$$E[s|i] \neq E[s|i']$$

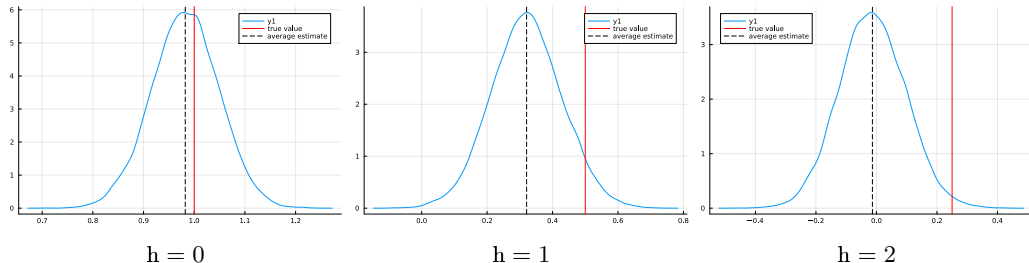


Figure: IRF Case 1

Nickel Bias! (AR process...)

## Case 2: Time specific shock mean (common across groups)

$$E[s|t] = E[s|t, i] = g_t$$

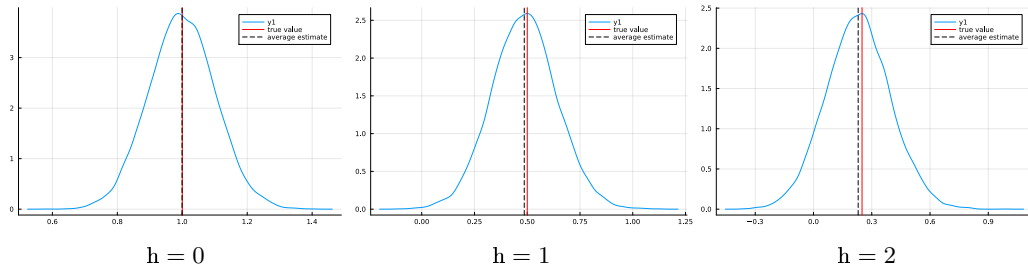


Figure: IRF Case 2

No problem with time specific shock mean (as expected)

Case 3: Does TWFE+LP estimate "average treatment effect" with heterogeneous treatment effects?

$$\beta_{0,i} \neq \beta_{0,i'}$$

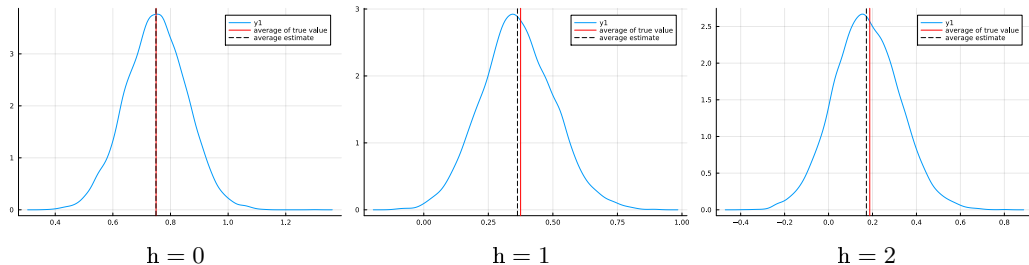


Figure: IRF Case 3

No problem with heterogeneous treatment effects by itself

## Case 4 a): Trend in treatment intensity by group

- TWFE-DiD had issues with staggered treatment  $\rightarrow$  trend in treatment intensity by group

$$E[s|t, i] = g_{it}$$

$\rightarrow$  IRF is biased for  $h > 0$

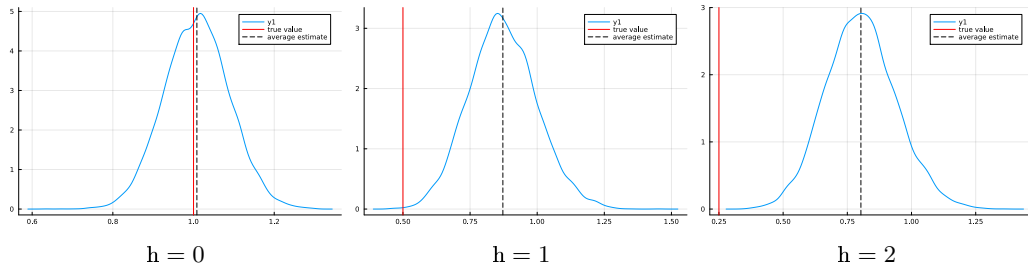


Figure: IRF Case 4 a

## Case 4 b): Trend in treatment intensity by group + Heterogeneous Treatment Effects

- TWFE-DiD had issues with staggered treatment + heterogeneous treatment effects
- Analogue: trend in treatment intensity by group + heterogeneous treatment effects

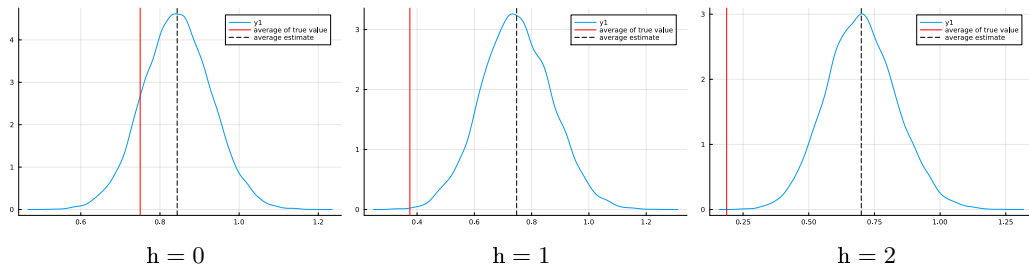


Figure: IRF Case 4 b

## Case 4 b): Trend in treatment intensity by group + Heterogeneous Treatment Effects (ctd.)

- Add a group specific trend:

$$y_{it} = \alpha_i + \phi_t + \beta_{0,i}S_{it} + \rho_i y_{it-1} + g_i t + \varepsilon_{it},$$

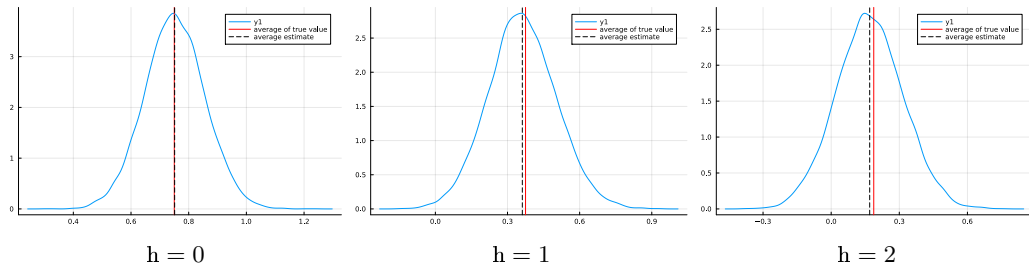


Figure: IRF Case 4 b with linear trend by group

## Local Projection in Panel

See `git@github.com:chrished/localprojection_panel.git` for simulation code

- 1 Case with few treatment times, see Dube et al. (2023) and literature in Paul GP course
- 2 Continuous shocks in (almost) every period:
  - group specific trend in treatment intensity  $\rightarrow$  Biased estimates for  $h > 0$  (similar to staggered treatment)
  - heterogeneous treatment effects  $\rightarrow$  Biased estimates  $\forall h$
  - $\rightarrow$  Need to control for group specific trends!
- What is the correct way? De Chaisemartin and d'Haultfoeuille (2022)?
- Is it okay to simply "de-trend" shocks?
- Other issues that would bias estimates?
- Inference? (small sample estimates of  $\alpha_i$  and  $\phi_t$ , serial correlation, ... )

# References I

- De Chaisemartin, C. and X. d'Haultfoeuille (2022): “Difference-in-differences estimators of intertemporal treatment effects,” Tech. rep., National Bureau of Economic Research.
- Dube, A., D. Girardi, O. Jorda, and A. M. Taylor (2023): “A local projections approach to difference-in-differences event studies,” Tech. rep., National Bureau of Economic Research.
- Jordà, Ò. (2005): “Estimation and inference of impulse responses by local projections,” *American economic review*, 95, 161–182.