Local Projections and DiD

Christoph Hedtrich

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When does Local Projection make sense in a panel?

- Jordà (2005) huge success in macro
  - Goal: estimate impulse response functions in a single time-series
  - Approach: estimate a sequence of OLS regressions of y_{t+h} on shock and controls in t for $h = 0, 1, \ldots, H$

- DiD literature highlights issues with
  - Staggered Treatments
  - Heterogeneous Treatment Effects
  - Dynamic Treatment Effects

  $\rightarrow$ see Paul GP course

- If DiD has all these issues, then LP must have them too!

De Chaisemartin and d'Haultfoeuille (2022)

- highlights issues with panel local projections
  - applied in complicated DiD treatment settings (continuous treatment intensity, staggered treatment, dynamic effects)
  - "contamination by future"
  - sign-reversal, even with positive treatment effects in every cell estimates may turn negative

  → generalize event-study design

- General takeaway:TWFE+LP → misspecified regression when there is variation in treatment timing

Dube, Girardi, Jorda, and Taylor (2023) Slides

- Goal: estimate treatment effects in a panel with typical DiD issues
- Solution: select observations appropriately/control for previous treatments
- LP is just OLS, so this is super fast compared to other suggested estimators

Which issues from DiD literature carry over to LP setting with shocks?

- Staggered Treatment $\rightarrow$ What happens with trend in treatment intensity?
- Heterogeneous Treatment Effects $\rightarrow$ Bias even with mean zero shocks?
- Dynamic Treatment Effects

IRF in panel with TWFE +LP

$$y_{it} = \alpha_i + \phi_t + \beta_{0,i} s_{it} + \rho_i y_{it-1} + \varepsilon_{it},$$

- where
  - groups $i = 1, \ldots, I$
  - time periods $t = 1, \ldots, T$
  - outcome of interest y_{it}
  - shock s_{it} (continuous shock)
  - group specific intercept α_i and trend g_{it}
  - time specific intercept ϕ_t
  - $\varepsilon_{it} \sim N(0, \sigma^2)$ i.i.d.

- Treatment Effects/IRF:

$$\beta_h = \beta_{0,i} \rho_i^{h-1} \quad \forall h \geq 0$$

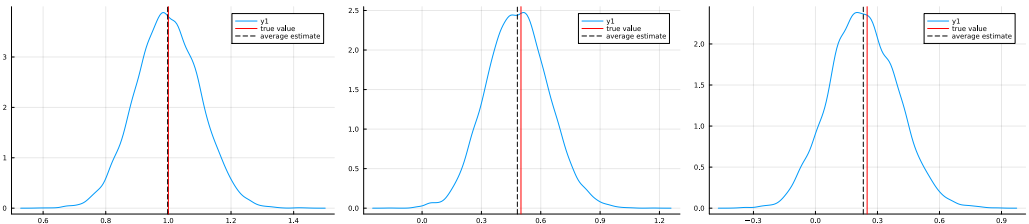
IRF in panel with TWFE +LP

- Simulate with
  1. group specific shock mean $s_{it} \sim N(\mu_i, \sigma_s^2)$
  2. time specific shock mean $s_{it} \sim N(\mu_t, \sigma_s^2)$
  3. Heterogeneous treatment effects $\beta_{0,i}$
  4. trend in shock mean $\mu_{i,t} = \mu_i + \gamma_i t$

- Monte Carlo:
  - 10000 repititions
  - $I = 2$ groups
  - $T = 100$ time periods
  - common AR process $\rho_i = \rho$

Case 1: Group specific shock mean

$$E[s|i] \neq E[s|i']$$



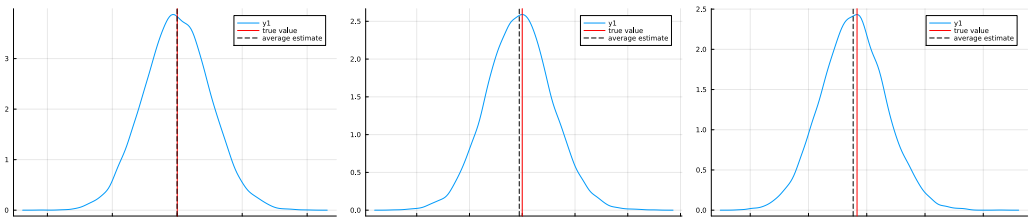
$h = 0$ $h = 1$ $h = 2$

Figure: IRF Case 1

No problem with group specific shock mean (as expected)

Case 2: Time specific shock mean (common across groups)

$$E[s|t] = E[s|t, i] = gt$$



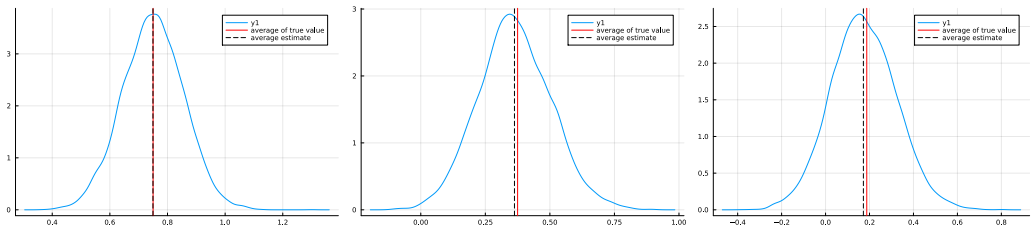
$h = 0$ $h = 1$ $h = 2$

Figure: IRF Case 2

No problem with time specific shock mean (as expected)

Case 3: Does TWFE+LP estimate "average treatment effect" with heterogeneous treatment effects?

$$\beta_{0,i} \neq \beta_{0,i'}$$



h = 0 h = 1 h = 2

Figure: IRF Case 3

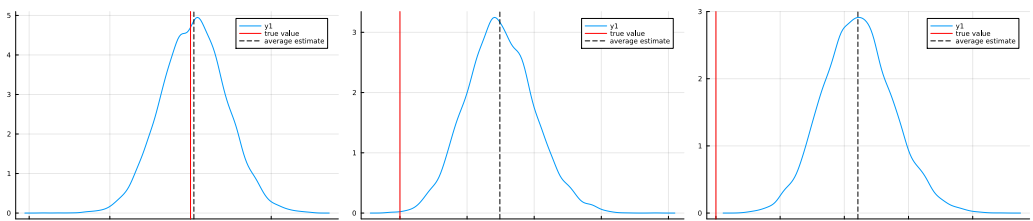
No problem with heterogeneous treatment effects by itself

Case 4 a): Trend in treament intensity by group

- TWFE-DiD had issues with staggered treatment → trend in treatment intensity by group

$$E[s|t, i] = g_i t$$

→ IRF is biased for $h > 0$

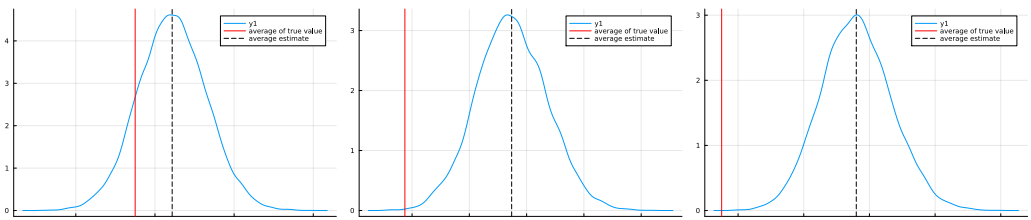


$h = 0$ $h = 1$ $h = 2$

Figure: IRF Case 4 a

Case 4 b): Trend in treament intensity by group + Heterogeneous Treatment Effects

- TWFE-DiD had issues with staggered treatment + heterogeneous treatment effects
- Analogue: trend in treatment intensity by group + heterogeneous treatment effects



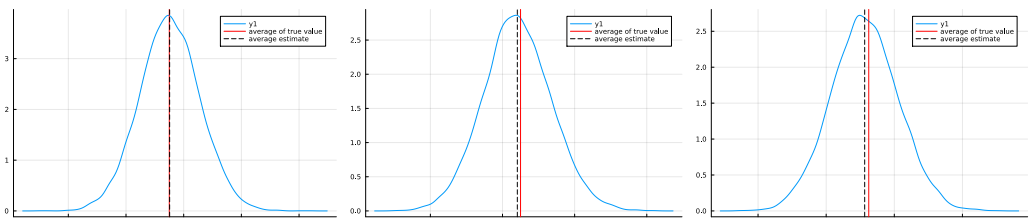
h = 0 h = 1 h = 2

Figure: IRF Case 4 b

Case 4 b): Trend in treament intensity by group + Heterogeneous Treatment Effects (ctd.)

- Add a group specific trend:

$$y_{it} = \alpha_i + \phi_t + \beta_{0,i} s_{it} + \rho_i y_{it-1} + g_i t + \varepsilon_{it},$$



$h = 0$ $h = 1$ $h = 2$

Figure: IRF Case 4 b with linear trend by group

Local Projection in Panel

See `git@github.com:chrished/localprojection_panel.git` for simulation code

1. Case with few treatment times, see Dube et al. (2023) and literature in Paul GP course

2. Continuous shocks in (almost) every period:
   - group specific trend in treament intensity $\rightarrow$ Biased estimates for $h > 0$ (similar to staggered treatment)
   - heterogeneous treatment effects $\rightarrow$ Biased estimates $\forall h$

   $\rightarrow$ Need to control for group specific trends!

- What is the correct way? De Chaisemartin and d'Haultfoeuille (2022)?

- Is it okay to simply "de-trend" shocks?

- Other issues that would bias estimates?

- Inference? (small sample estimates of α_i and ϕ_t, serial correlation, ...)

References I

De Chaisemartin, C. and X. d'Haultfoeuille (2022): "Difference-in-differences estimators of intertemporal treatment effects," Tech. rep., National Bureau of Economic Research.

Dube, A., D. Girardi, O. Jorda, and A. M. Taylor (2023): "A local projections approach to difference-in-differences event studies," Tech. rep., National Bureau of Economic Research.

Jordà, Ò. (2005): "Estimation and inference of impulse responses by local projections," American economic review, 95, 161–182.