Statistical Analysis with Missing Data
Problem 1.6
Bootstrap,Jackknife

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1 Problem 1.6

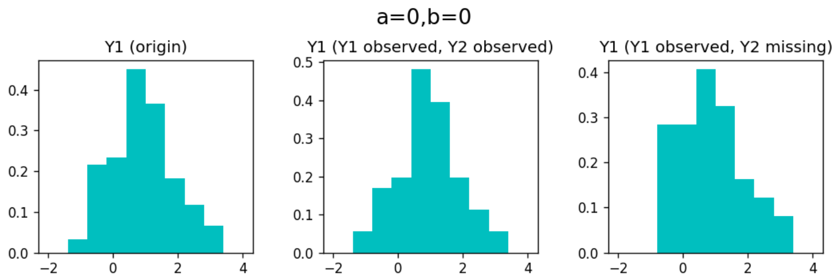
100 trivariate normal observations $\{(y_{i1}, y_{i2}, u_i), i = 1, ..., 100\}$ **on** (Y_1, Y_2, U) **as follows:**

$$\begin{aligned} y_{i1} &= 1 + z_{i1} \\ y_{i2} &= 2 * z_{i1} + z_{i2} \\ u_i &= a * (y_{i1} - 1) + b * (y_{i2} - 5) + z_{i3} \end{aligned} \tag{1}$$

where $\{(z_{i1}, z_{i2}, z_{i3}), i = 1, ..., 100\}$ **are independent standard normal deviates. The latent variable U determines missingness of** Y_2 **as follows:**

$$y_{i2} \; is \; missing \; if \; u_i \; < \; 0 \tag{2}$$

Label Description
origin : original generated Data
obs : Y_1 **and** Y_2 **both observed**
mis : Y_1 **observed and** Y_2 **missing**



Case 1: a=0,b=0

Y1(origin) : mean =8.882104e-01, std=9.724232e-01

Y1(obs) : mean =9.310380e-01, std=9.661219e-01

Y1(mis) : mean =8.265806e-01, std=9.781347e-01

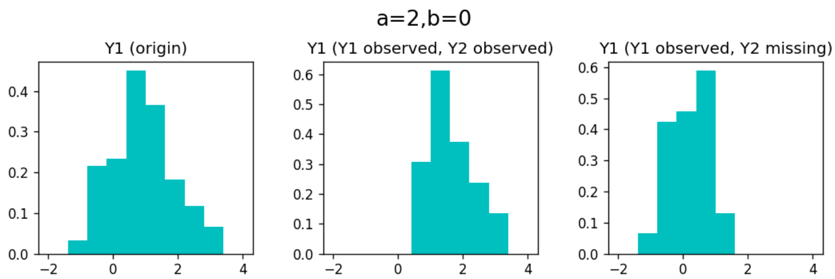
origin Vs mis : t =3.3875e-01, pvalue =7.3531e-01

obs Vs mis : t =5.2375e-01, pvalue =6.0164e-01

Conclusion : There is no Significant differences.

$f(M|Y_1, Y_2, U) = F(M|Z_3)$,$Z_3$ **is independent of** Y_2,Y_1**.**

The machenism is MCAR.



Case 2: a=2,b=0

Y1(origin) : mean =8.882104e-01, std=9.724232e-01

Y1(obs) : mean =1.635554e+00, std=6.775540e-01

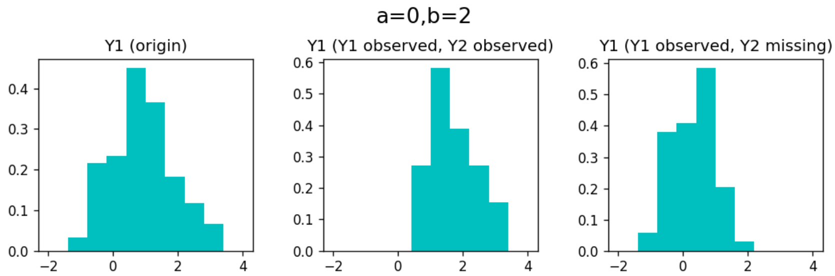
Y1(mis) : mean =1.701749e-01, std=6.007170e-01

origin Vs mis : t =4.7925e+00, pvalue =3.9468e-06

obs Vs mis : t =1.1339e+01, pvalue =1.5452e-19

Conclusion : There is a significant difference.

f(M|Y1,Y2,U) = F(M|Y1,Z3),The machenism is MAR.



Case 3: a=0,b=2

Y1(origin) : mean =8.882104e-01, std=9.724232e-01

Y1(obs) : mean =1.701187e+00, std=6.813406e-01

Y1(mis) : mean =2.749121e-01, std=6.588037e-01

origin Vs mis : t =4.2122e+00, pvalue =4.2763e-05

obs Vs mis : t =1.0455e+01, pvalue =1.2476e-17

Conclusion : There is a significant difference.

f(M|Y1,Y2,U) = F(M|Y2,Z3),the machanism is NMAR.

2 Bootstrap,Jackknife

$\{X_i, i = 1, ..., 100\}$ **is a random sample from a** $N(7, 4)$ **population. Three methods(Moment,Bootstrap,Jackknife) are applied to estimate the population mean** μ **and variance** σ^2**.**

Method	μ(estimate)	variance
Moment	7.11961603	0.04063306
Bootstrap	7.11606507	0.04967192
Jackknife	7.11961603	0.04104350

Table 1: estimates of μ

Method	σ^2(estimate)	variance
Moment	4.06330648	0.26774111
Bootstrap	4.06615074	0.32251543
Jackknife	4.10434998	0.27593671

Table 2: estimates of σ^2

It appears that the traditional moment method makes the best estimation of both μ **and** σ^2 **while another two methods also make similar estimates.Overall,There is no significant difference among the estimates made by these three methods.**