ARC vs Modified Risk Aversion model comparison for Shock/Reward Task

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Abstract

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The prospect theory inspired RIsk Aversion model is a well-established model for explaining how people decide whether or not to gamble when presented with the opportunity to potentially earn money by risking some of their own money. We intended to see if this Risk Aversion model can be modified to explain cognitive behaviour in an experimental setting where people risk non-financial losses (in this case a shock) in order to potentially gain money. We then aimed to compare how this modified RA model compares with a Aversion-Risk Conflict model created by the authors specifically to explain the behaviour o this dataset. Various model comparison techniques including LOOIC and PPC were employed to compare how the various ARC and modified RA models compare in their predictive performance. Results find that while the ARC model gave a better predictive account of the experimental data, a Risk Aversion model modified to account for a constant dislike for pain also provided a reasonable account of the experimental data. Parameter recovery also verified the validity of both the Risk Aversion model and Aversion-Risk Conflict models. Parameter recovery failed to recover the Tau in the const_noTau model, indicating that the const_noTau model may be the superior of the two, although this is debatable.

Introduction

The experiment

- Dataset taken from DARPA sponsored research for an upcoming research paper entitled - *The Neural Basis of Decision Conflict: A Model Based Analysis (Zorowitz et al. 2019)*
- Experiment is based on a research paradigm designed by Sierra-Mercado & Colleagues (2014)
- Participants choose between two conditions
  - No risk :
    - Guaranteed 1 cent reward
  - Risk:
    - Three reward levels (low, medium, high), with rewards ranging from .01$ -> 1$
    - Three risk levels (low, medium, high) determining how likely a participant is to receive a shock
      - Highly annoying but not painful
    - Participants must decide whether the risk of shock is worth the reward
- 35 participants completed a maximum of 108 trials each

Dataset

	Trial	RiskType	RewardType	ResponseType	Reward	Shock	SubjID
0	1	1	2	1	0.53	0	1
1	2	2	1	1	0.13	0	1
2	3	2	3	1	0.91	1	1
3	4	3	1	0	0.15	1	1
4	5	1	2	1	0.61	0	1
5	6	3	1	1	0.31	1	1
6	7	3	3	1	0.79	1	1
7	8	1	1	0	0.1	0	1
8	9	3	2	1	0.58	1	1
9	10	3	2	1	0.36	1	1
10	11	1	3	1	0.82	0	1
11	12	3	1	0	0.22	1	1
12	13	3	3	1	0.87	0	1
13	14	1	2	1	0.45	0	1
14	15	2	3	1	0.7	0	1
15	16	1	1	1	0.27	0	1

Models

Aversion-Risk Conflict model

- Model seeks to resolve the conflict between desire for reward and eagerness to avoid a negative outcome when decision making
- Four factors contribute to how likely a participant is to take a risk on a trial :
  - Baseline level of risk taking (Intercept)
  - Impact of medium level risk of shock
  - Impact of high level risk of shock
  - How much the potential reward deviates from the average reward
- Two Models:
  - Aversion-Risk Conflict Model
  - HIerarchical Aversion-Risk Conflict Model

Aversion Risk Conflict Model

$$p(gamble\ \ acceptance)$$
$$exp([\xi_1,\ \xi_2,\ \xi_3.\ \xi_4]\ \cdot\ [\beta_1,\ \beta_2,\ \beta_3,\ \beta_4])$$

Where:

ξ_1 = 1 (Intercept)

ξ_2 = 1 if medium risk of shock (else 0)

ξ_3 = 1 if high risk of shock (else 0)

ξ_4 = standard deviation of potential Reward from mean reward

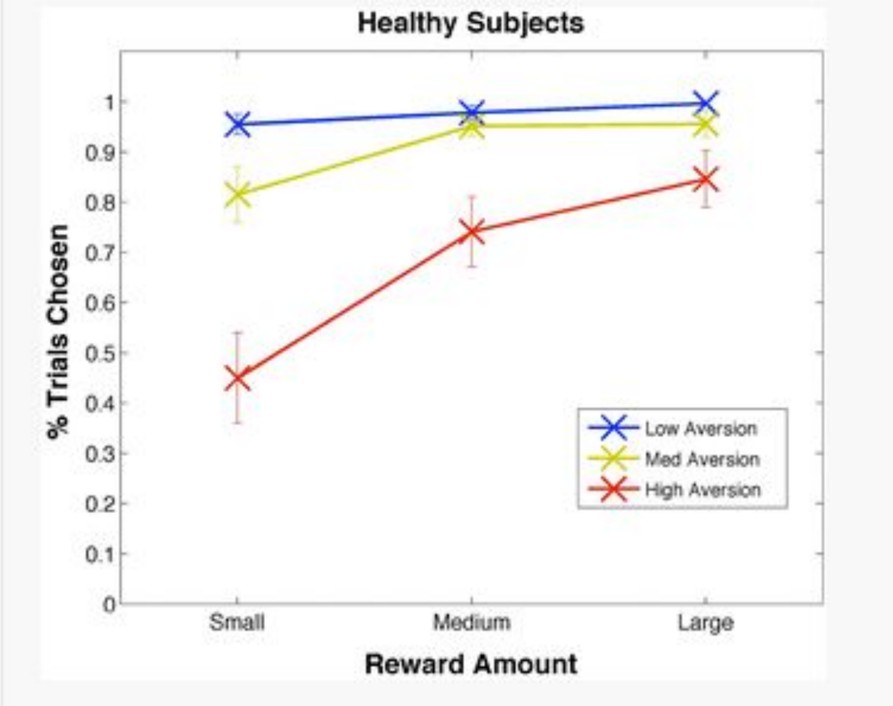
β_1 = participant inclination to gamble

β_2 = participant aversion to medium risk shock

β_3 = participant aversion to high risk shock

β_4 = participant value of potential reward money

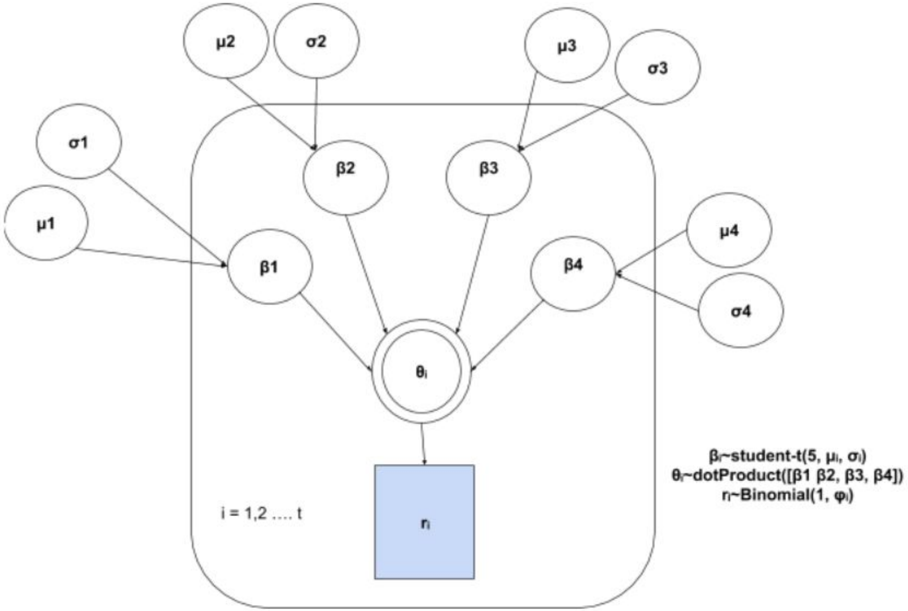
Aversion-Risk Conflict model



Previous research (Sierra- Mercado et al. 2015) which inspired the current model found:

- Low risk of punishment (in this case a puff of air to the eye) did not deter users from gambling
- Medium risk of punishment deterred participants for gambling for small rewards
- High risk of punishment deterred participants for from risking all types of

Aversion-Risk Conflict model



Modified RA models

Model seeks to evaluate monetary value against some pain measurement.
This is evaluated against the baseline (safe bet)

- Safe bet: Subjective value of safe gain
  - 1 cent
- Gamble value: Subjective value of gain - Subjective dislike for pain-risk
  - 0~1$ - Risk for Pain
  - ***Models vary in measure for Pain***
- Gambling money vs pain
  - The original model was multiplicative, multiplying a participants gambled amount by their subject level of loss aversiveness
  - We decided to modify the RA model to be additive, as multiplying value of money vs aversion to shocks seemed counterintuitive to us
- Parameters
  - Risk = ρ
  - Pain Aversion[3] = Λ1 Λ2 Λ3
  - Inverse Temperature = μ
  - (Pain Retention) = ς (only used in exp and pow models)

Original RA Prospect Model

$$u(x^+) = x^\rho \quad [1]$$

$$u(x^-) = -\lambda \times (-x)^\rho \quad [2]$$

$$p(gamble\ \ acceptance) = (1 + exp\{-\mu(u(gamble)\ -u(guaranteed))\})^{-1} \quad [3]$$

Where:

[1] = estimated gains

[2] = estimated losses

ρ = the loss aversion coefficient

Λ = the curvature of the utility function

μ = the logit sensitivity

Modified RA model

$$u(x^+) = x^\rho \qquad [1]$$

$$u(x^-) = x_T^\rho - (pain\ \ mesure) \qquad [2]$$

$$p(gamble\ \ acceptance) = (1 + exp\{-\tau(u(gamble) - u(guaranteed))\})^{-1} \qquad [3]$$

Pain Measures:

- **const** = $log(\lambda_k + 1)$
- **linear** = $\lambda_k \times number\_of\_shocks$
- **log** = $log(\lambda_k \times number\_of\_shocks + 1)$
- **pow** = $\lambda_k \times number\_of\_shocks^\varsigma$
- **exp** = $\lambda_k \times exp(number\_of\_shocks \times \varsigma)$

Where:

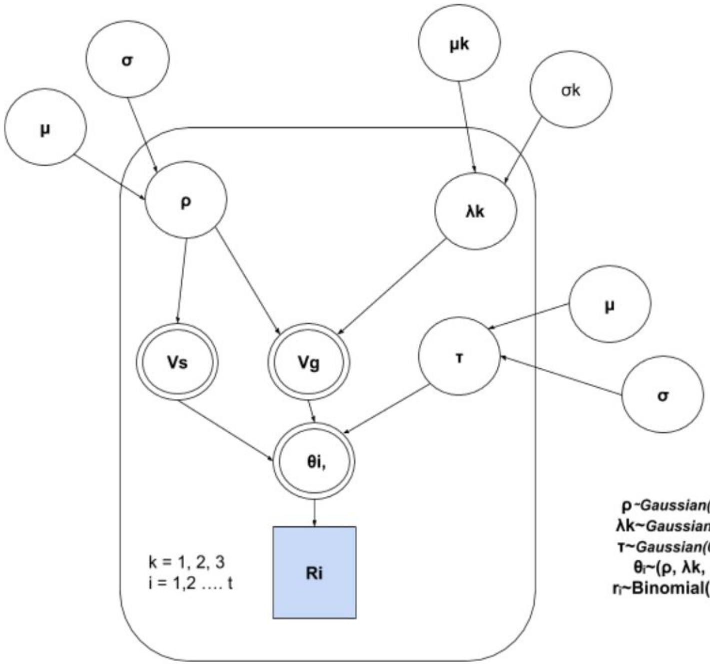
X in [1] equals guaranteed reward 0.01

X_T in [2] equals max reward for reward levels 1, 2 & 3 (0,33, 0.66 & 0.99)

λ_k equals PainAversion measure corresponding to shock likelihood 1, 2 & 3

ς equals PainRetention measure corresponding to how much an individual dislikes based On previous shocks

Modified RA models



ρ ~*Gaussian(0, 5)*
λk~*Gaussian(0, 5)*
τ~*Gaussian(0, 20)*
θ_i~(ρ, λk, τ)
r_i~Binomial(1, φ_i)

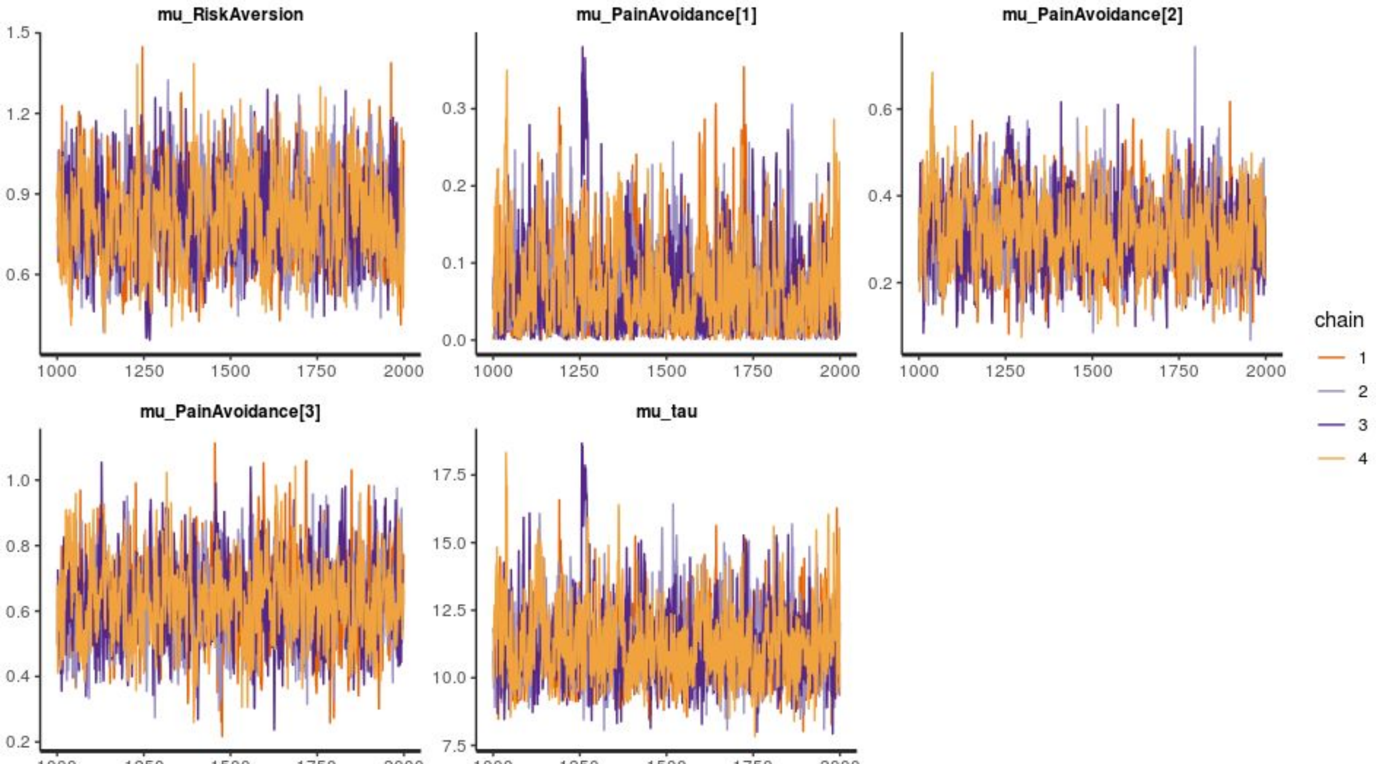
Modified RA models

11 designed model variants

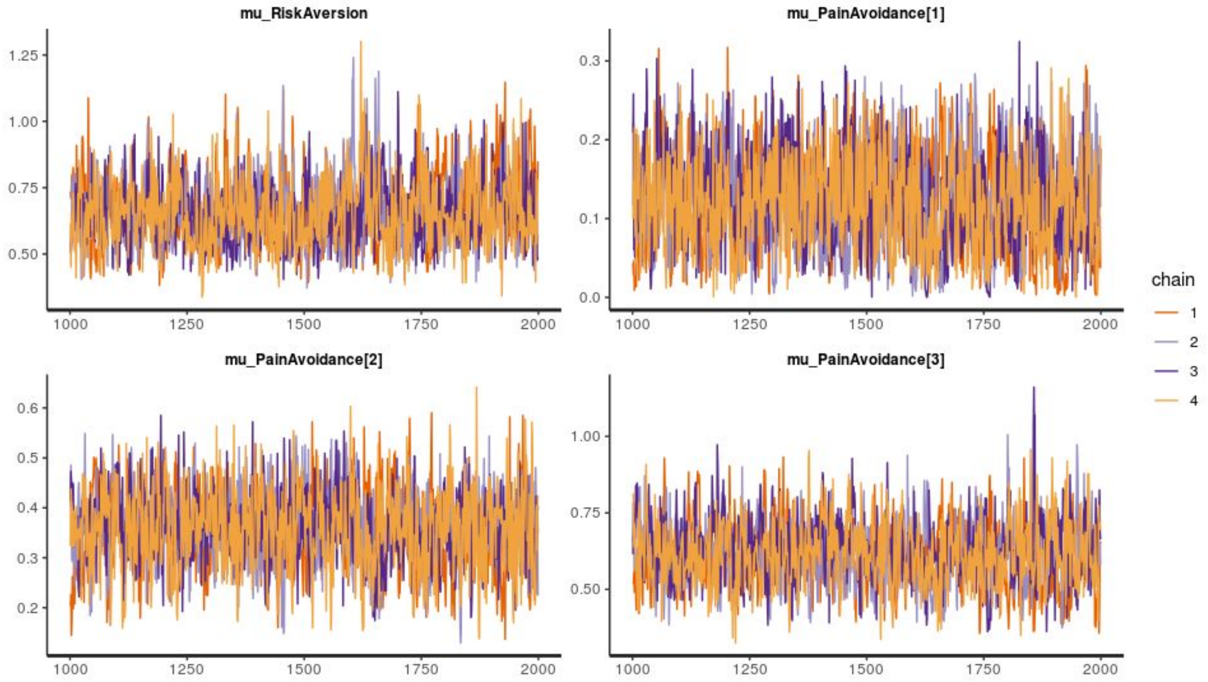
- Exponential Individual/ Hierarchical
- Power Individual/ Hierarchical
- Log Individual/ Hierarchical
- Linear Individual/ Hierarchical
- Constant Individual/ Hierarchical/ Hierarchical No Tau

Method (Modelling)

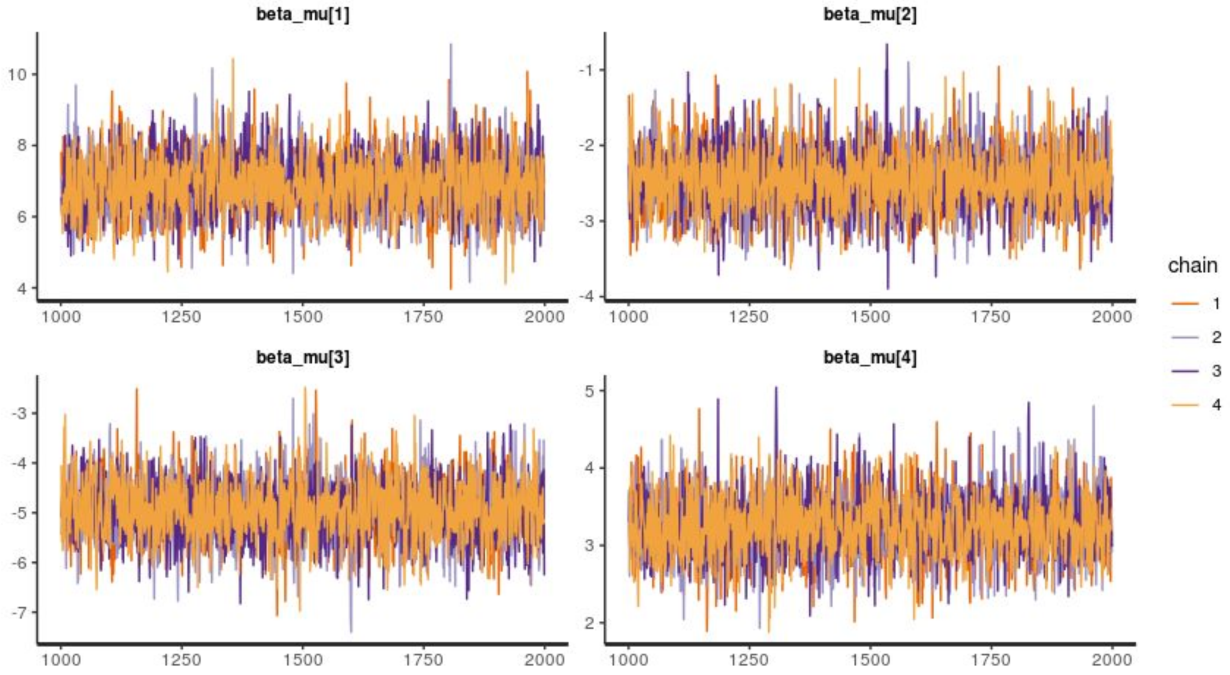
Traceplots- Constant Hierarchical



Traceplots- Constant Hierarchical no Tau



Traceplots- ARC-Hierarchical

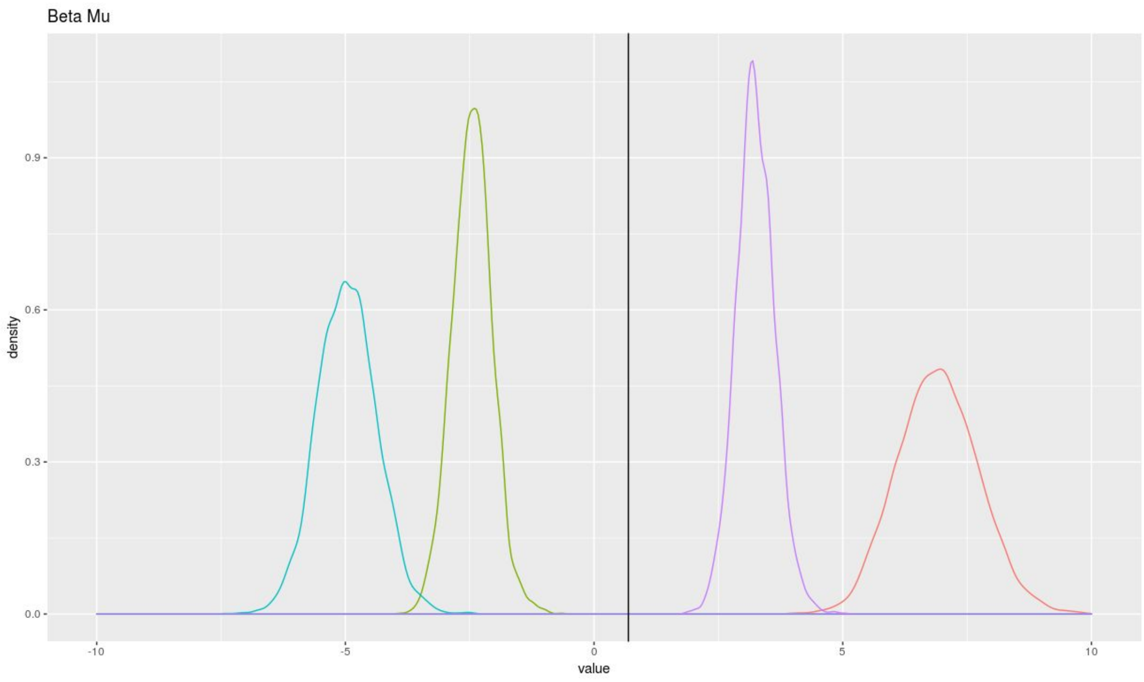


Modeling

- 13 models were attempted
  - 11 RA modified models
  - 2 ARC models
- Each model was run with 4 chains, 2000 iterations with 1000 warm-up iterations
- 3 Model did not converge
  - Exponential Individual
  - Linear Individual
  - Power Individual
- Exponential and Power models appeared to converge towards the constant model (this will be illustrated in the posterior predictions)

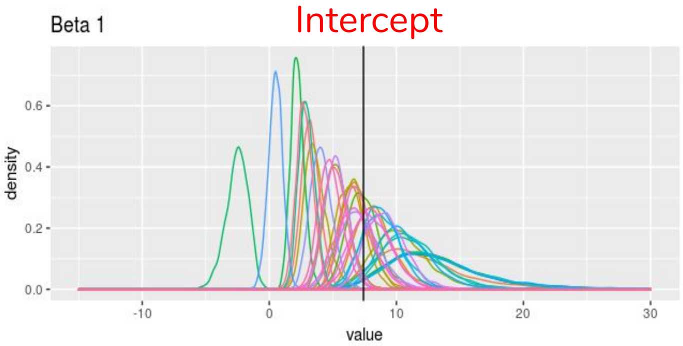
Results
Posterior Plots - ARC

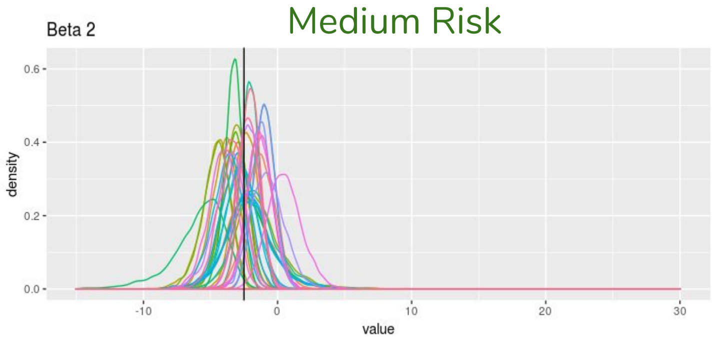
Posterior Plots-(ARC-Hierarchical)

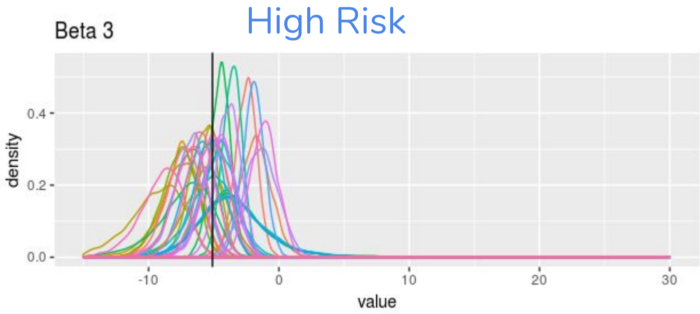


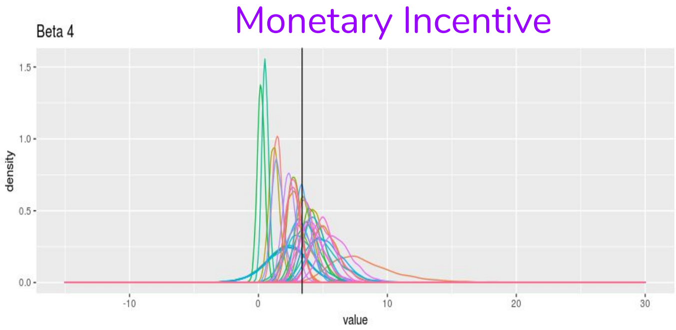
From left to right:
High Risk
Medium Risk
Monetary Incentive
Intercept

Posterior Plots-(ARC-Hierarchical)









Posterior Plots-(ARC-Hierarchical)

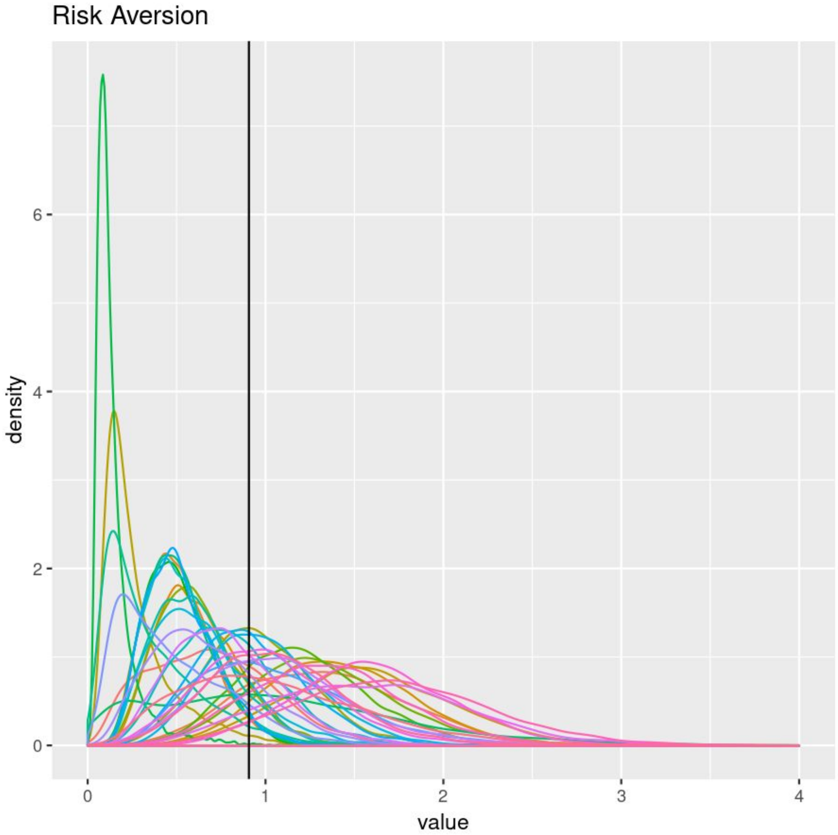
- Posterior plots of the hyperparameters clearly illustrate how this ARC model works
  - Intercept
    - Participants begin with a generally strong desire to gamble
  - Monetary Incentive
    - Monetary incentives, depending on how many positive/negative deviations they are from the average reward may or not have a strong impact on participants likelihood to gamle
  - Shock Likelihood
    - High risk of shock deters participants from gambling significantly more than low risk of shock

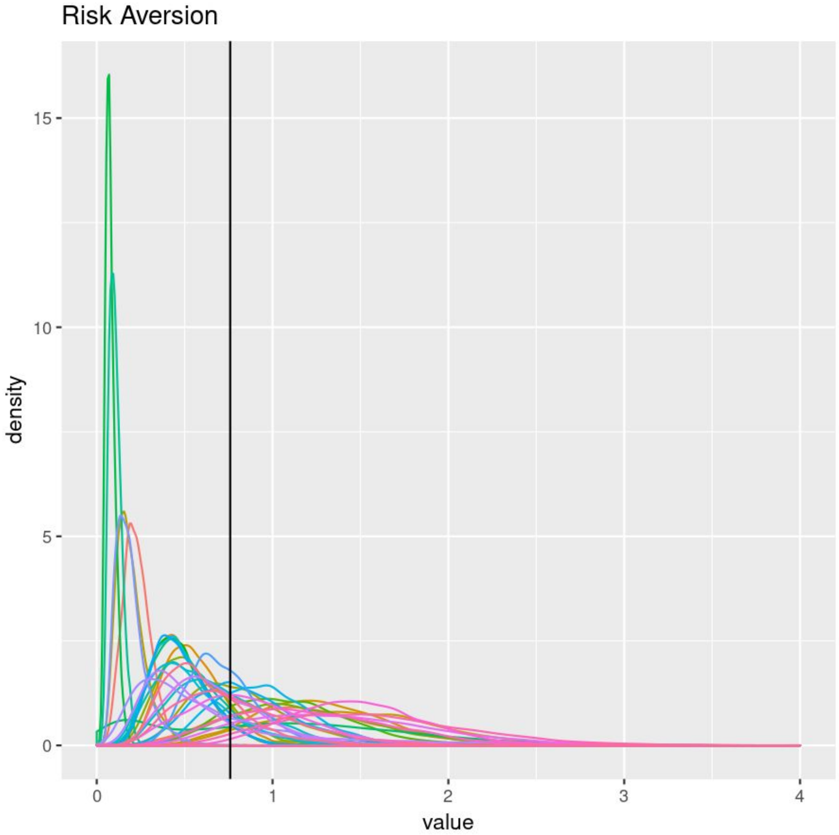
Results
Posterior Plots - modified RA

Constant hierarchical models

RiskAversion

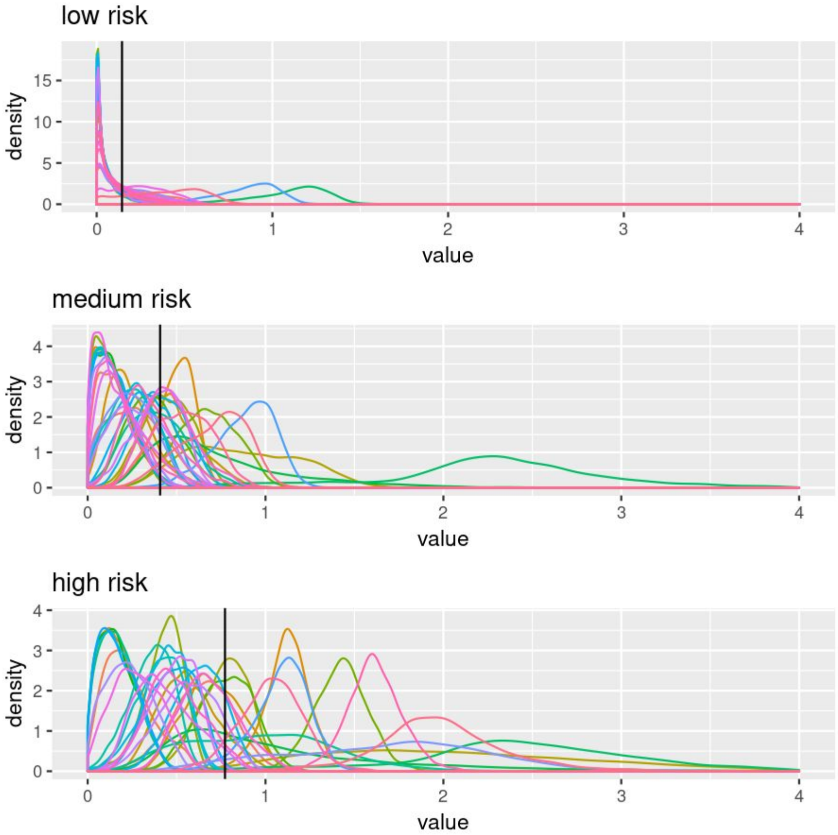
Const (H) | Const (H, no Tau)

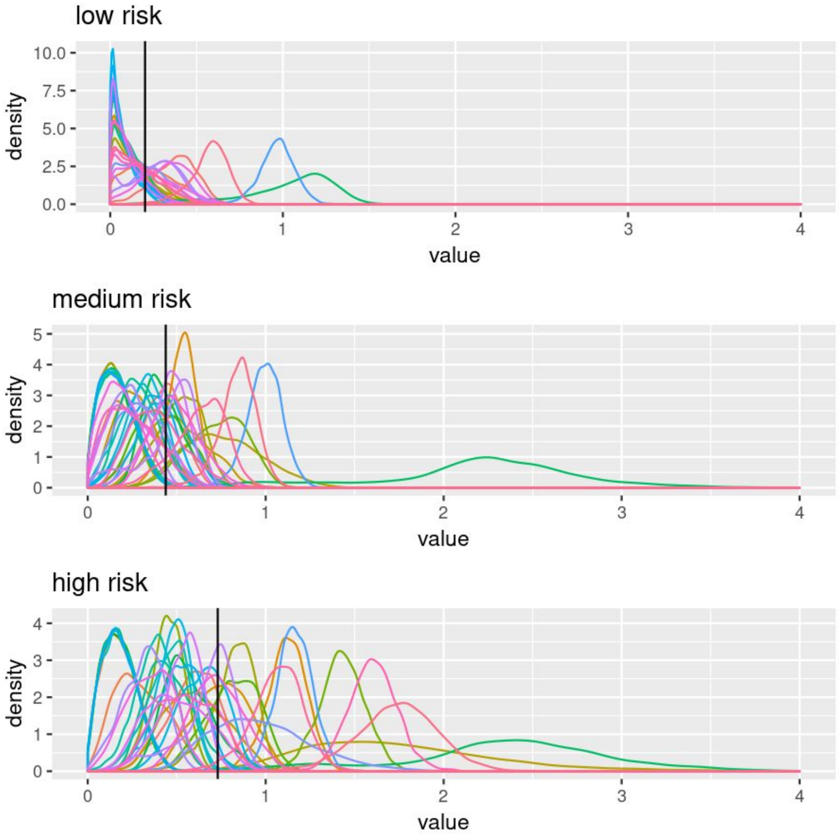




PainAvoidance

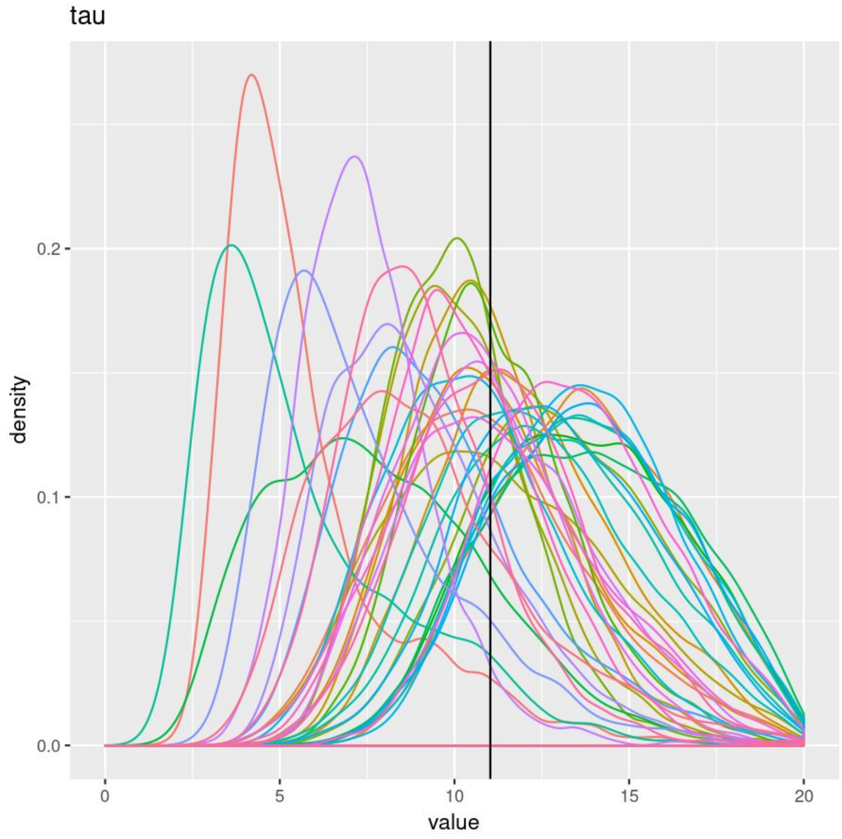
Const (H) | Const (H, no Tau)





Inverse Temperature

Const (H) | Const (H, no Tau)

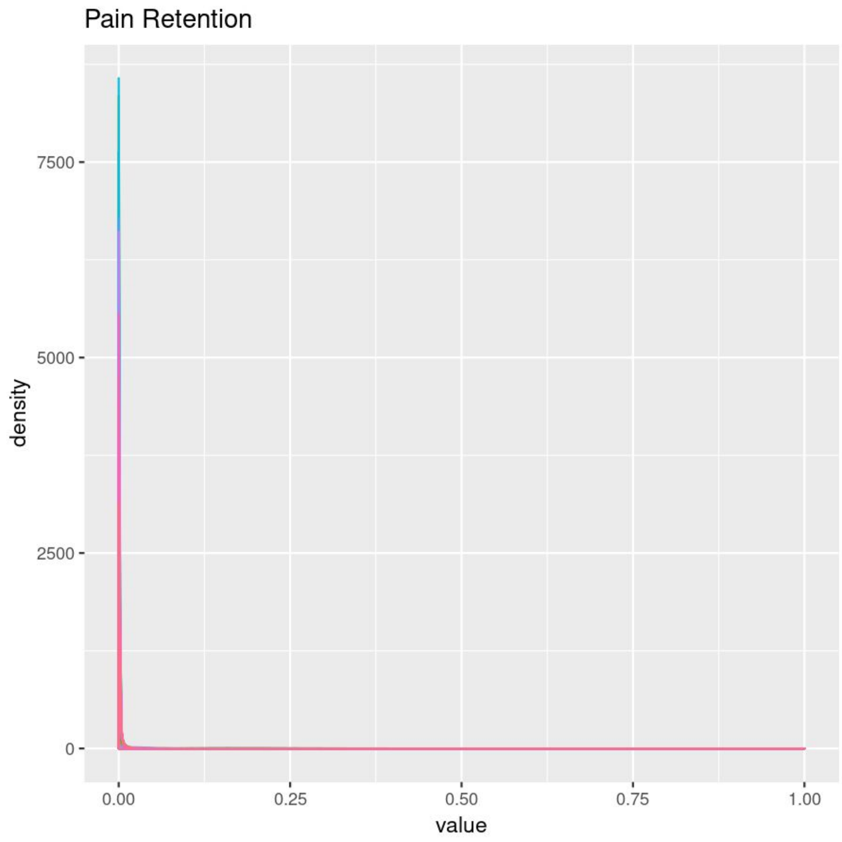


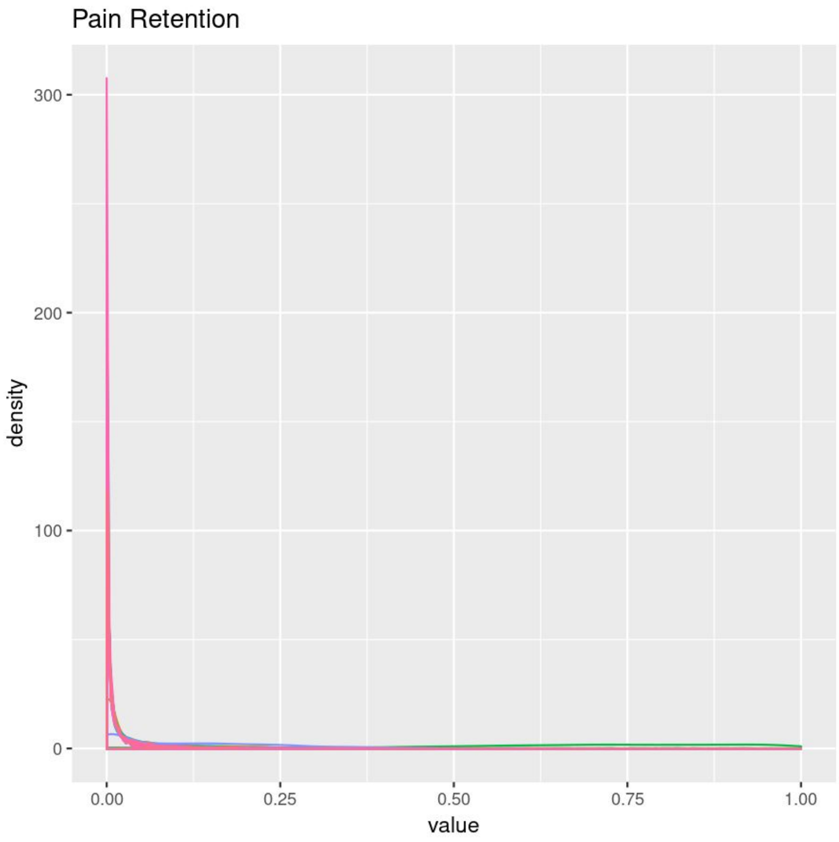
Const (H) | Const (H, no Tau)

- Const_h_noTau is the same as the const_h model, with the Tau parameter fixed to roughly the average of the posterior Tau for all participants (τ=12)
- The posteriors between both const_h and const_h_noTau look very similar between the two models
- This suggests that the added complexity added by the τ parameter may not contribute much to the overall model
    - Indeed PPC checks showed cont_h_noTau performed better than const_h 91.05 % vs 91.03%

Pain Retention

Exp (H) | Pow (H)





Exp (H) | Pow (H)

- ς PainRetention parameters for both exp_h and pow_h models converged to 0 for all participants
- This essentially means that the number_of_shocks the participant received did not change a participants likelihood to gamble
- This meant that both exp_h and pow_h models essentially converged to a model similar to the const_h model, with the ς parameter adding no additional predictive quality to the model

Results
Model Comparisons

Model Comparison Statistics

	ARC	ARC-H	Const	Const-H	Const No-Tau
LOOIC	3152.391	1397.388	15,346.76	1712.046	1715.668
PPC Accuracy	80.92%	93.75%	59.69%	91.03%	91.05%

	Log	Log-H	Pow-H	Exp-H	Linear-H
LOOIC	3031.994	2187.274	1885.05	1706.42	2108.005
PPC Accuracy	85.15%	86.90%	89.10%	91.03%	88.20%

Model Comparison Statistics

LOOIC Score

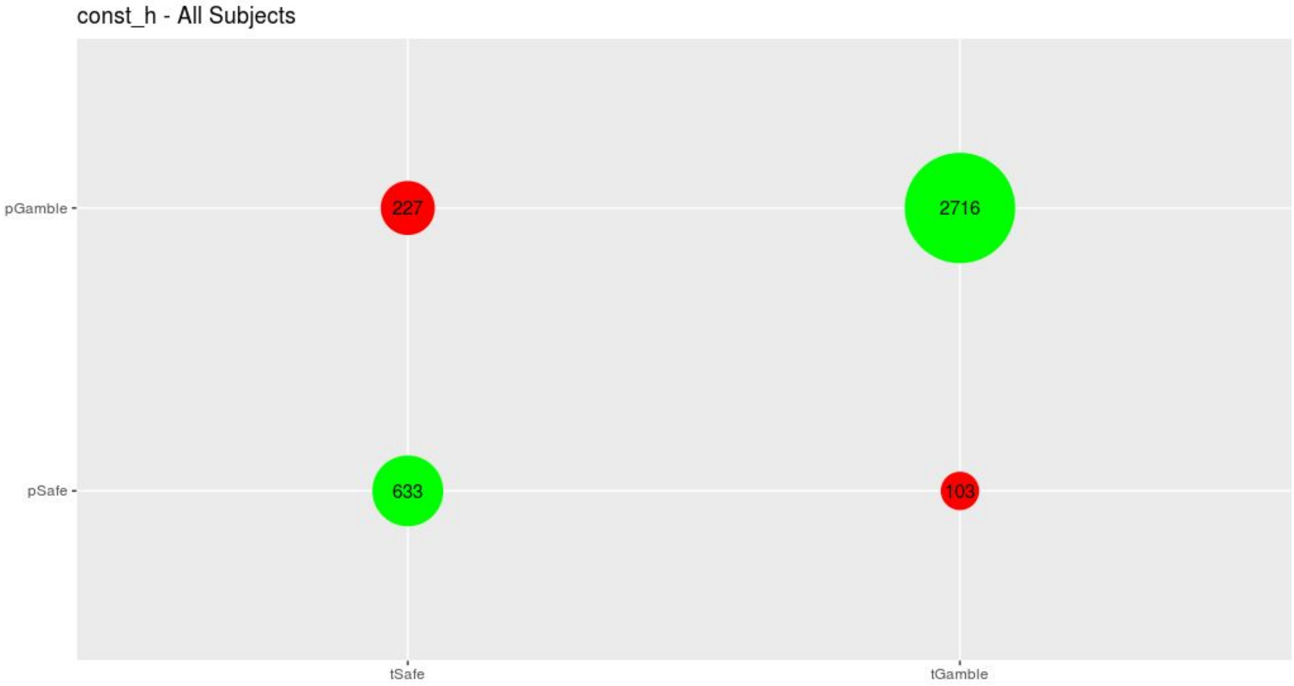
ARC-H < Exp-H < Const-H < Const-H-noTau < Pow- H < Linear-H < Log-H < Log-I < ARC < Const-I

PPC Accuracy

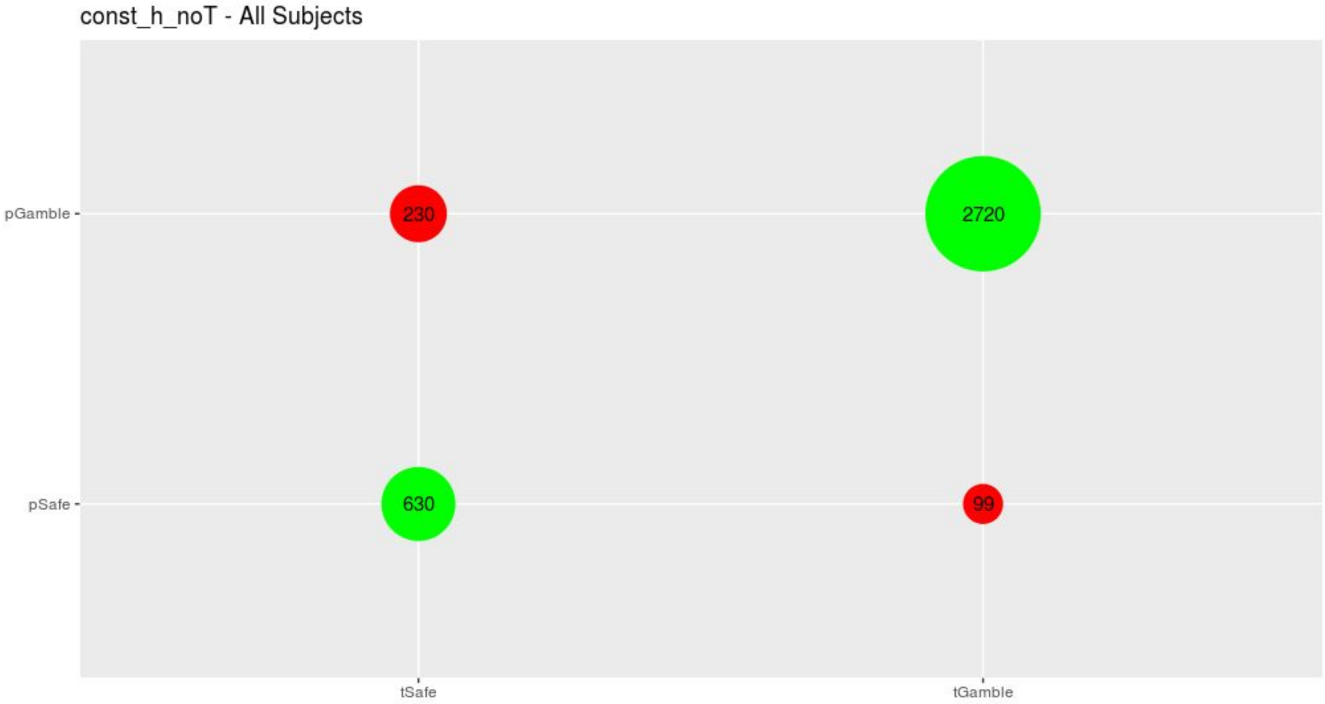
ARC-H > Const-H-noTau > Const-H == Exp-H > Pow- H > Linear-H > Log-H < Log-I < ARC < Const-I

Results indicate that ARC-H, Const-H and Cont-H-noTau were the best models . Exp-H was excluded because it converges to a liner model (PainRetention=0)

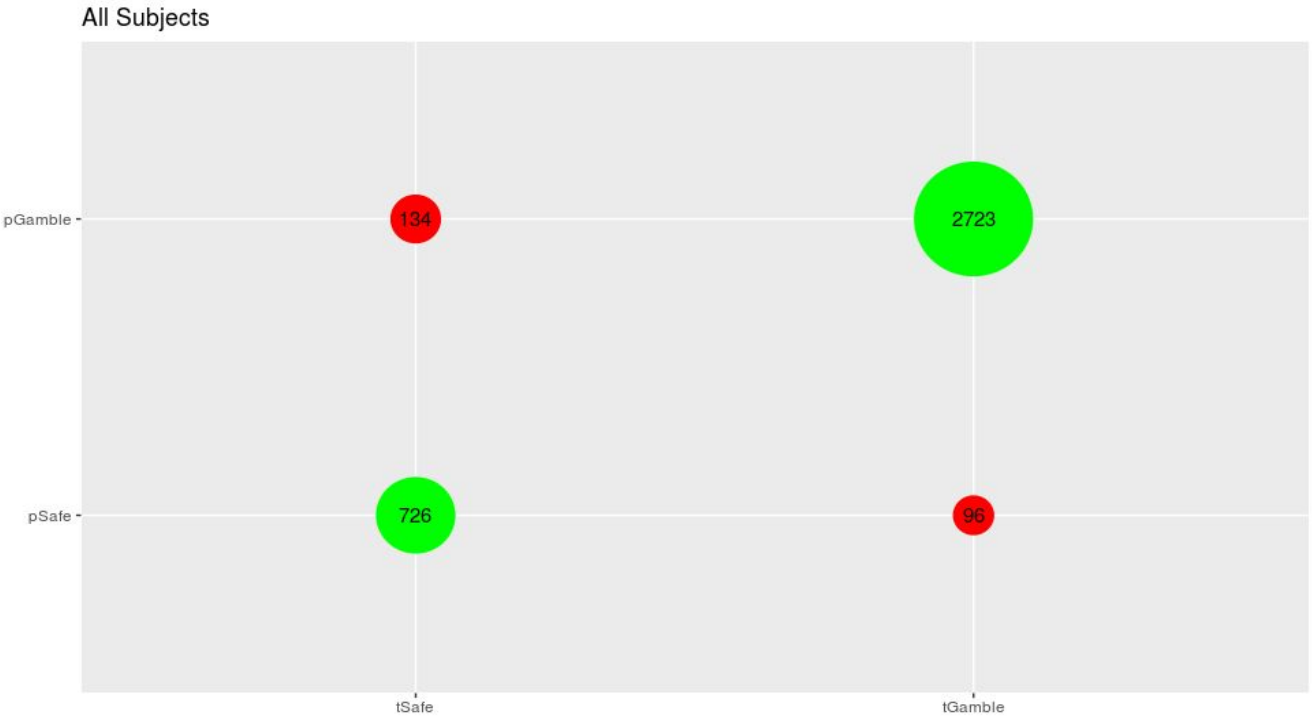
PPC for Hierarchical Constant



PPC for Hierarchical Constant (No Tau)



PPC for Hierarchical ARC



Discussion

Discussion

- 10 of the 13 models converged
- PPC checks ranged from 60% -> 94%
- LOOIC and PPC both point to 4 models as being worthy of further consideration
  - ARC Hierarchical Model
  - Constant-HIerarchical
  - Constant-HIerarchical no-Tau
  - Exponential-Hierarchical
- When examining the posteriors, it appears that exponential model is approximating a version of the constant model (as discussed in posterior section)
- ARC model performed significantly better than all other models when accurately predicting Safe Gamble
  - 84% vs 73% for the next best model

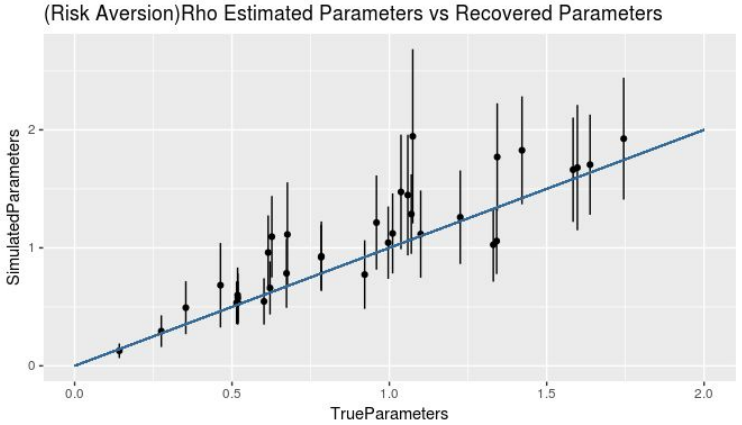
Conclusions

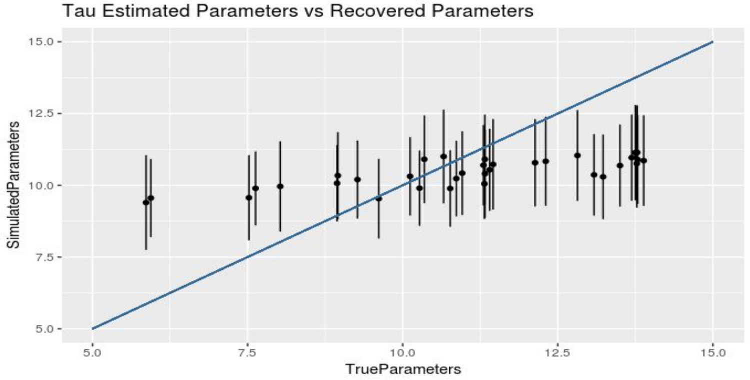
Conclusions

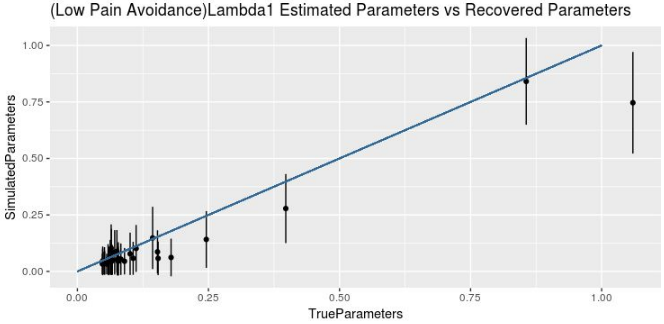
- Main finding:
  - Modified RA models can be used to explain behavioural data in non-monetary risk aversion
- Finding 1:
  - Two fundamentally different models provide reasonable accounts of the data
    - RA model values money in absolute terms from 0
    - ARC model values money relative to the average amount of money received prior
- Finding 2:
  - Hierarchical Modelling performed consistently better across all models
  - This was seen in both ARC and our own modifier RA models
- Finding 3
  - Impact of functional form of model can be substantial
  - Exponential approximated the constant model
- Finding 4
  - Experiencing shock did not discourage future gambles
  - Additional pain parameter

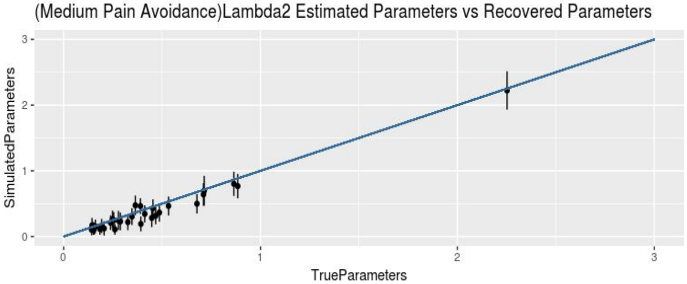
Parameter Recovery

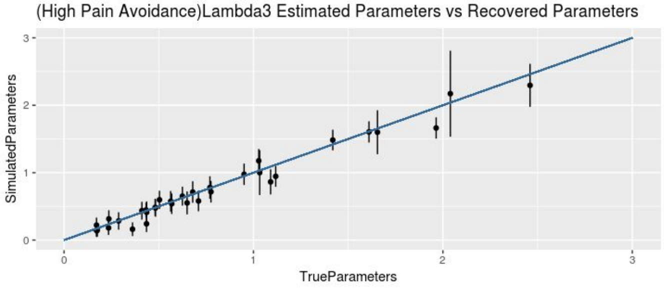
Parameter Recovery for const-h model



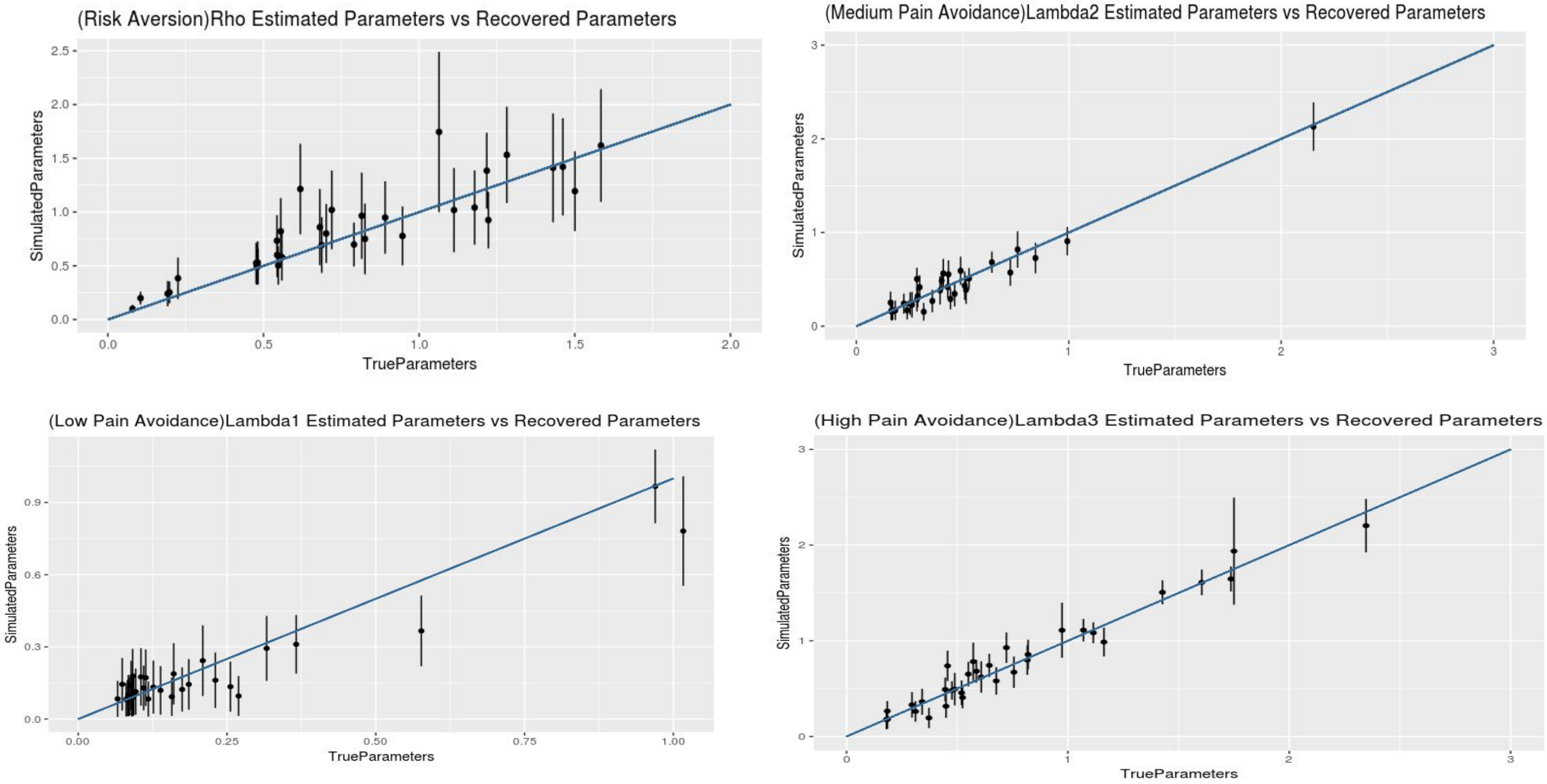




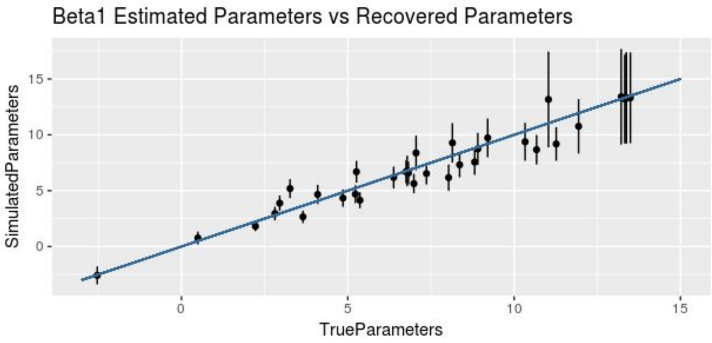


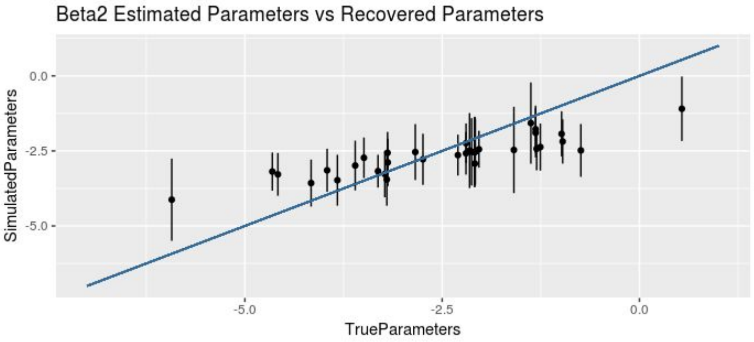


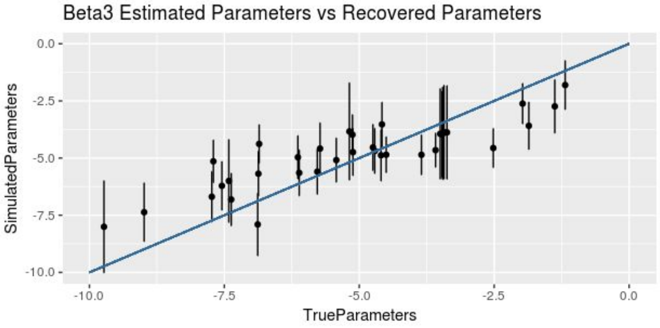
Parameter Recovery for const-h-noTau model

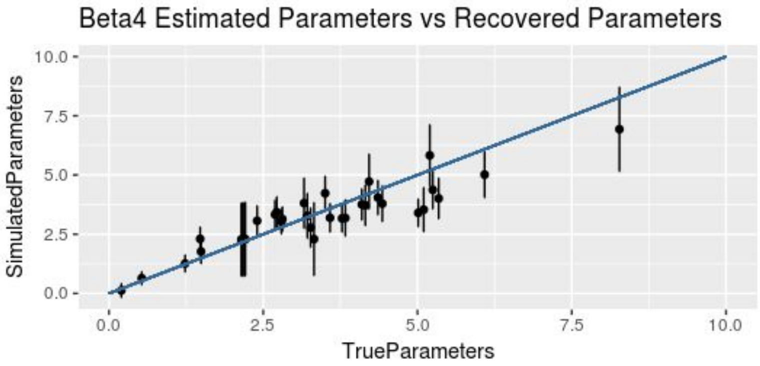


Parameter Recovery for ARC-h model









Parameter Recovery

- Means of posterior distributions were used to generate simulated data for the three best performing models
  - const_h
  - const_h_noTau
  - ARC-h
- Parameters recovered reasonably well for for both ARC-h and const_h_noTau models
- Tau parameters in const_h parameters failed to recover
- This suggests const_h_noTau may be a better model than const_h
  - However inverse temperature Tau parameters are generally difficult to recover

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