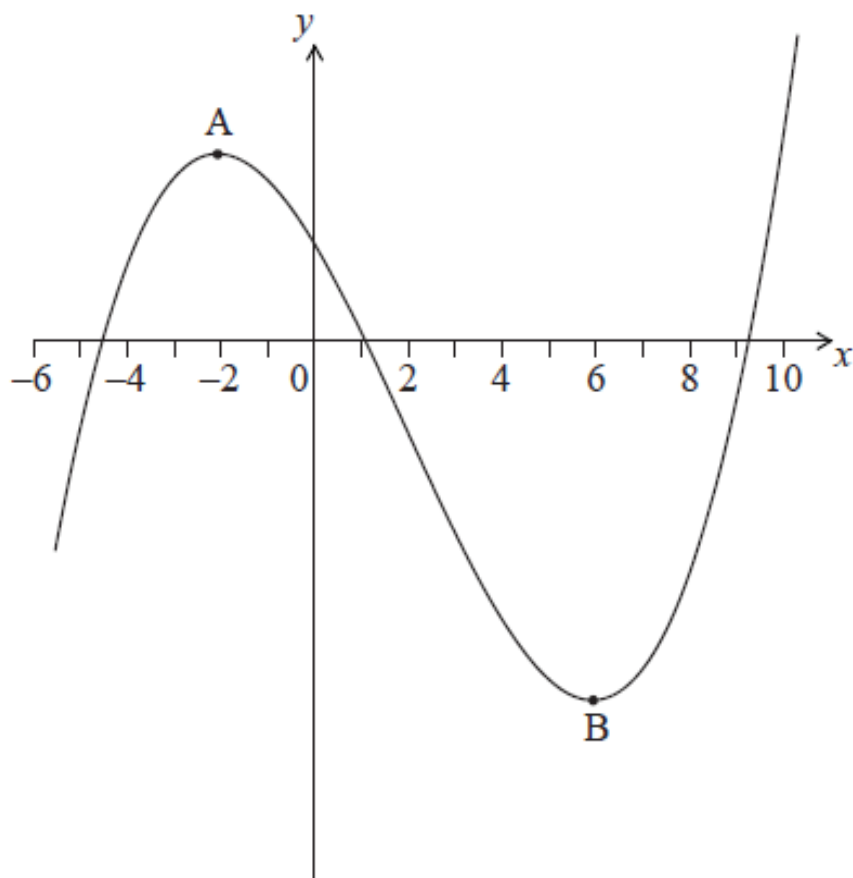**Calculus review: Function graphs**

Answer the first four problems in the space provided.

1. The following diagram shows part of the graph of $y = f(x)$.
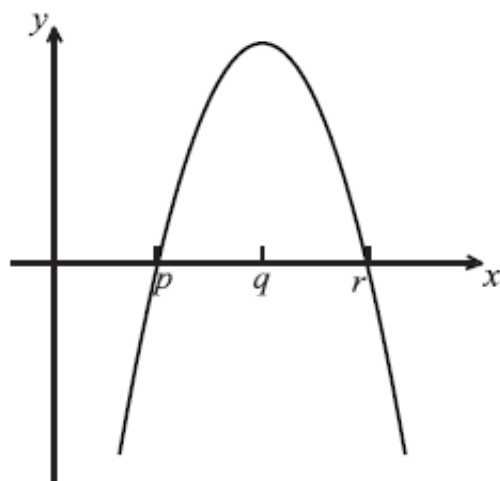

The graph has a local maximum at A, where $x = -2$, and a local minimum at B, where $x = 6$.

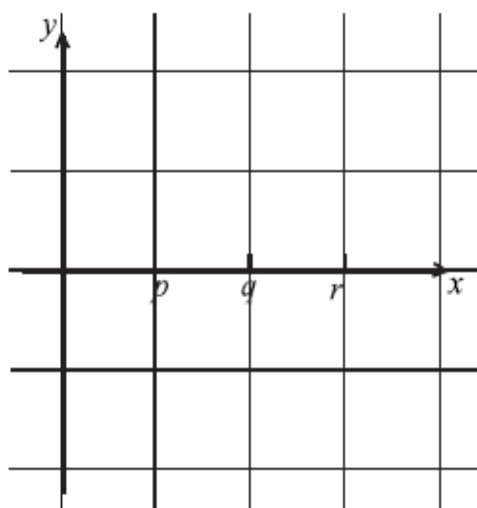On the graph above, sketch the graph of $y = f'(x)$.   *[4 marks]*

2a. The diagram below shows part of the graph of the gradient function, $y = f'(x)$.

On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the x-intercept.   *[2 marks]*

2b. Complete the table, for the graph of $y = f(x)$.   *[2 marks]*
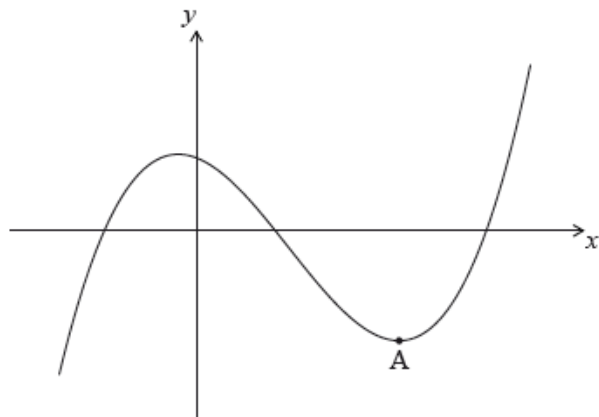
	x-coordinate
(i) Maximum point on f	
(ii) Inflexion point on f	

2c. Justify your answer to part (b) (ii).   *[2 marks]*

3a. The following diagram shows the graph of a function f. There is a local minimum point at A, where $x > 0$.
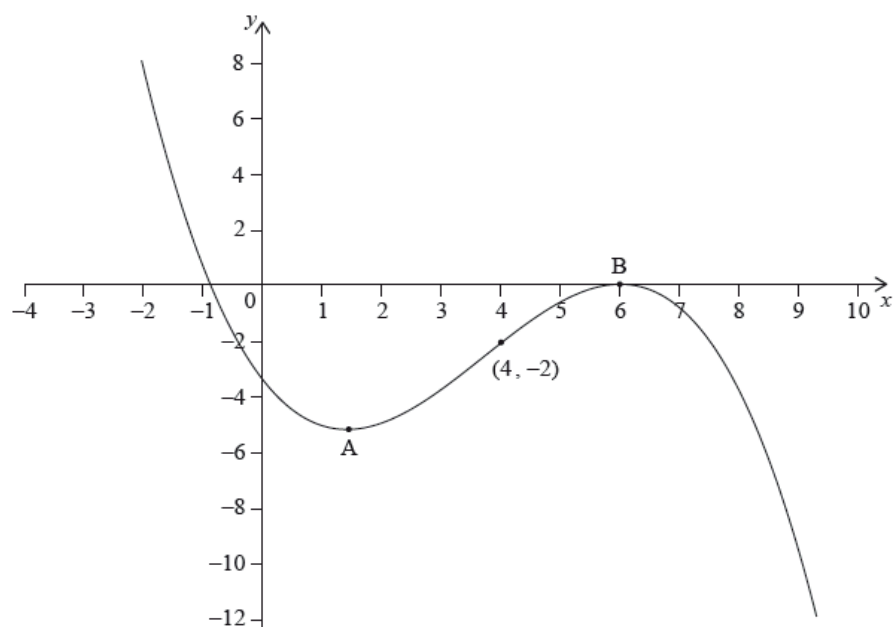

The derivative of f is given by $f'(x) = 3x^2 - 8x - 3$.

Find the x-coordinate of A.   *[5 marks]*

3b. The y-intercept of the graph is at $(0, 6)$. Find an expression for $f(x)$.   *[6 marks]*

4a. The following diagram shows the graph of f', the derivative of f.

The graph of f' has a local minimum at A, a local maximum at B and passes through $(4, \ -2)$.

The point $P(4, \ 3)$ lies on the graph of the function, f.

Write down the gradient of the curve of f at P.   *[1 mark]*

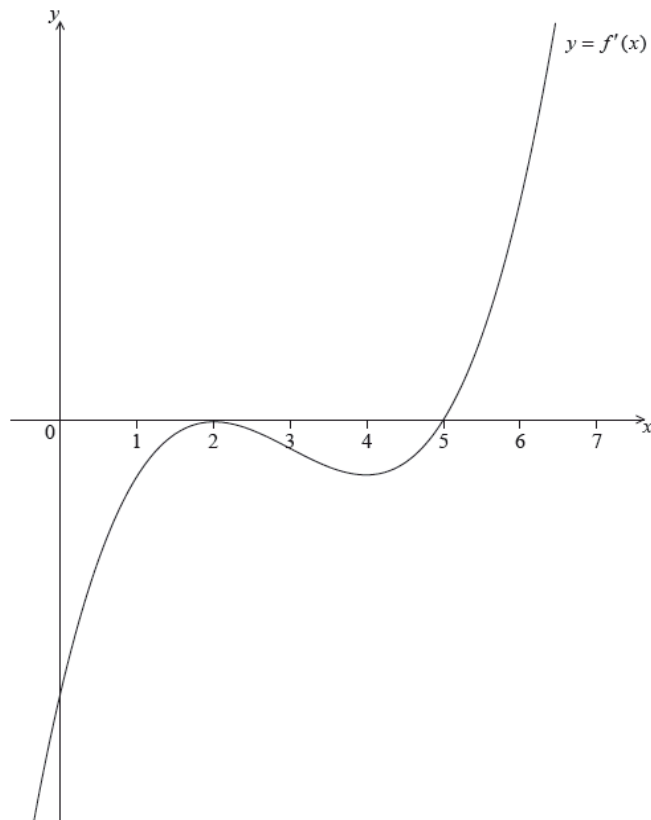**4b.** Find the equation of the normal to the curve of f at P.   *[3 marks]*

4c. Determine the concavity of the graph of f when $4 < x < 5$ **and** justify your answer.   *[2 marks]*

For the remaining problems, answer on lined paper.

5a. Let $y = f(x)$, for $-0.5 \leq x \leq 6.5$. The following diagram shows the graph of f', the derivative of f.

The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the x-axis at the point $(5,\ 0)$.

Explain why the graph of f has a local minimum when $x = 5$.   *[2 marks]*

5b. Find the set of values of x for which the graph of f is concave down.   *[2 marks]*

6a. Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

Use the quotient rule to show that $g'(x) = \frac{1 - 2\ln x}{x^3}$.   *[4 marks]*

6b. The graph of g has a maximum point at A. Find the x-coordinate of A.   *[3 marks]*

7a. Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.
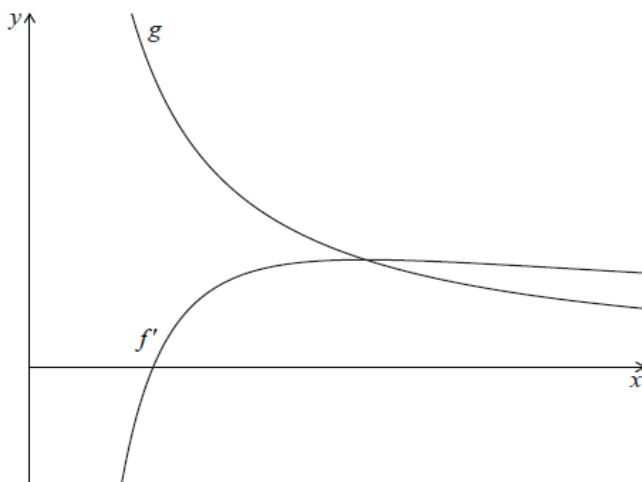
Show that $f'(x) = \frac{\ln x}{x}$.   *[2 marks]*

7b. There is a minimum on the graph of f. Find the x-coordinate of this minimum.   *[3 marks]*

7c. Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g.   *[2 marks]*

The graph of f' has an x-intercept at $x = p$.

Write down the value of p.

7d. The graph of g intersects the graph of f' when $x = q$.

Find the value of q.   *[3 marks]*

7e. Let R be the region enclosed by the graph of f', the graph of g and the line $x = p$.

Show that the area of R is $\frac{1}{2}$.   *[5 marks]*