

# Binomial Expansion problems [31 marks]

Consider the expansion of

$$(x + 3)^{10}.$$

- 1a. Write down the number of terms in this expansion.

[1 mark]

## Markscheme

11 terms **A1 N1**

[1 mark]

## Examiners report

[N/A]

- 1b. Find the term containing

[4 marks]

$$x^3.$$

## Markscheme

evidence of binomial expansion **(M1)**

eg

$$\binom{n}{r}$$

$a^{n-r}b^r$ , attempt to expand

evidence of choosing correct term **(A1)**

eg

8<sup>th</sup> term,  $r = 7$ ,

$$\binom{10}{7},$$

$$(x)^3(3)^7$$

correct working **(A1)**

eg

$$\binom{10}{7}$$

$$(x)^3(3)^7,$$

$$\binom{10}{3}$$

$$(x)^3(3)^7,$$

$$262440x^3 \text{ (accept } 262000x^3) \quad \mathbf{A1 \quad N3}$$

[4 marks]

## Examiners report

[N/A]

Consider the expansion of  $(3x^2 + 2)^9$ .

- 2a. Write down the number of terms in the expansion.

[1 mark]

## Markscheme

10 terms **A1 N1**

[1 mark]

## Examiners report

Many candidates were familiar with the binomial expansion, although some expanded entirely which at times led to careless errors. Others attempted to use Pascal's Triangle. Common errors included misidentifying the binomial coefficient corresponding to this term and not squaring the 3 in  $(3x^2)$ .

- 2b. Find the term in  $x^4$ .

[5 marks]

## Markscheme

evidence of binomial expansion **(M1)**

e.g.

$$a^9b^0 + \binom{9}{1}a^8b + \binom{9}{2}a^7b^2 + \dots,$$

$$\binom{9}{r} (a)^{n-r}(b)^r, \text{ Pascal's triangle}$$

evidence of correct term **(A1)**

e.g. 8th term,

$$r = 7,$$

$$\binom{9}{7},$$

$$(3x^2)^2 2^7$$

correct expression of complete term **(A1)**

e.g.

$$\binom{9}{7} (3x^2)^2 (2)^7,$$

$${}^9_2C (3x^2)^2 (2)^7,$$

$$36 \times 9 \times 128$$

$$41472x^4 \text{ (accept}$$

$$41500x^4) \quad \mathbf{A1 \quad N2}$$

[4 marks]

## Examiners report

Many candidates were familiar with the binomial expansion, although some expanded entirely which at times led to careless errors. Others attempted to use Pascal's triangle. Common errors included misidentifying the binomial coefficient corresponding to this term and not squaring the 3 in  $(3x^2)$ .

- 3a. Expand  $(2 + x)^4$  and simplify your result.

[3 marks]

## Markscheme

evidence of expanding **M1**

e.g.

$$2^4 + 4(2^3)x + 6(2^2)x^2 + 4(2)x^3 + x^4,$$
$$(4 + 4x + x^2)(4 + 4x + x^2)$$

$$(2 + x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4 \quad \mathbf{A2} \quad \mathbf{N2}$$

**[3 marks]**

## Examiners report

Surprisingly few candidates employed the binomial theorem, choosing instead to expand by repeated use of the distributive property. This earned full marks if done correctly, but often proved prone to error.

- 3b. Hence, find the term in  $x^2$  in

**[3 marks]**

$$(2 + x)^4 \left(1 + \frac{1}{x^2}\right).$$

## Markscheme

finding coefficients 24 and 1 **(A1)(A1)**

term is

$$25x^2 \quad \mathbf{A1} \quad \mathbf{N3}$$

**[3 marks]**

## Examiners report

Candidates often expanded the entire expression in part (b). Few recognized that only two distributions are required to answer the question. Some gave the coefficient as the final answer.

4. The third term in the expansion of  $(2x + p)^6$  is  $60x^4$ . Find the possible values of  $p$ .

**[7 marks]**

## Markscheme

attempt to expand binomial **(M1)**

e.g.

$$(2x)^6 p^0 + \binom{6}{1} (2x)^5 (p)^1 + \dots,$$

$$\binom{n}{r} (2x)^r (p)^{n-r}$$

one correct calculation for term in  $x^4$  in the expansion for power 6 **(A1)**

e.g.  $15$ ,  
 $16x^4$

correct expression for term in  $x^4$  **(A1)**

e.g.

$$\binom{6}{2} (2x)^4 (p)^2,$$

$$15 \cdot 2^4 p^2$$

**Notes:** Accept sloppy notation e.g. omission of brackets around  $2x$ .

Accept absence of  $x$  in middle factor.

correct term **(A1)**

e.g.

$$240p^2x^4 \text{ (accept absence of } x^4 \text{)}$$

setting up equation with their coefficient equal to 60 **M1**

e.g.

$$\binom{6}{2} (2)^4 (p)^2 = 60,$$

$$240p^2x^4 = 60x^4,$$

$$p^2 = \frac{60}{240}$$

$$p = \pm \frac{1}{2} (p = \pm 0.5) \quad \mathbf{A1A1} \quad \mathbf{N3}$$

**[7 marks]**

## Examiners report

This question proved challenging for many students. Most candidates recognized the need to expand a binomial but many executed this task incorrectly by selecting the wrong term, omitting brackets, or ignoring the binomial coefficient. Other candidates did not recognize that there were two values for  $p$  when solving their quadratic equation.

5. In the expansion of  $(3x + 1)^n$ , the coefficient of the term in  $x^2$  is  $135n$ , where  $n \in \mathbb{Z}^+$ . Find  $n$ .

**[7 marks]**

## Markscheme

**Note:** Accept sloppy notation (such as missing brackets, or binomial coefficient which includes  $x^2$ ).

evidence of valid binomial expansion with binomial coefficients **(M1)**

$$\text{eg } \binom{n}{r} (3x)^r (1)^{n-r}, (3x)^n + n(3x)^{n-1} + \binom{n}{2} (3x)^{n-2} + \dots, \binom{n}{r} (1)^{n-r} (3x)^r$$

attempt to identify correct term **(M1)**

$$\text{eg } \binom{n}{n-2}, (3x)^2, n-r=2$$

setting **correct** coefficient or term equal to  $135n$  (may be seen later) **A1**

$$\text{eg } 9 \binom{n}{2} = 135n, \binom{n}{n-2} (3x)^2 = 135n, \frac{9n(n-1)}{2} = 135nx^2$$

correct working for binomial coefficient (using  ${}_nC_r$  formula) **(A1)**

$$\text{eg } \frac{n(n-1)(n-2)(n-3)\dots}{2 \times 1 \times (n-2)(n-3)(n-4)\dots}, \frac{n(n-1)}{2}$$

**EITHER**

evidence of correct working (with linear equation in  $n$ ) **(A1)**

$$\text{eg } \frac{9(n-1)}{2} = 135, \frac{9(n-1)}{2} x^2 = 135x^2$$

correct simplification **(A1)**

$$\text{eg } n-1 = \frac{135 \times 2}{9}, \frac{(n-1)}{2} = 15$$

$$n = 31 \quad \mathbf{A1} \quad \mathbf{N2}$$

**OR**

evidence of correct working (with quadratic equation in  $n$ ) **(A1)**

$$\text{eg } 9n^2 - 279n = 0, n^2 - n = 30n, (9n^2 - 9n)x^2 = 270nx^2$$

evidence of solving **(A1)**

$$\text{eg } 9n(n-31) = 0, 9n^2 = 279n$$

$$n = 31 \quad \mathbf{A1} \quad \mathbf{N2}$$

**Note:** Award **A0** for additional answers.

**[7 marks]**

## Examiners report

[N/A]