

0110Pretest-overall-review [205 marks]

1. This is an example question for the example test. You can delete this question.

[1 mark]

Markscheme

[N/A]

Examiners report

[N/A]

Consider the points A(

5,

2,

1) , B(

6,

5,

3) , and C(

7,

6,

$a + 1$) ,

$a \in \mathbb{R}$.

- 2a. Find

[3 marks]

(i)
 \overrightarrow{AB} ;

(ii)
 \overrightarrow{AC} .

Markscheme

(i) appropriate approach **(M1)**

eg

$\overrightarrow{AO} + \overrightarrow{OB}$,
 $B - A$

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ **A1 N2**

(ii)

$\overrightarrow{AC} = \begin{pmatrix} 2 \\ 4 \\ a \end{pmatrix}$ **A1 N1**

[3 marks]

Examiners report

The majority of candidates successfully found the vectors between the given points in part (a).

Let
 q be the angle between
 \vec{AB} and
 \vec{AC} .

- 2b. Find the value of
 a for which
 $q = \frac{\pi}{2}$.

[4 marks]

Markscheme

valid reasoning (seen anywhere) **R1**

eg scalar product is zero,

$$\cos \frac{\pi}{2} = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$$

correct scalar product of **their**

\vec{AB} and

\vec{AC} (may be seen in part (c)) **(A1)**

eg

$$1(2) + 3(4) + 2(a)$$

correct working for **their**

\vec{AB} and

\vec{AC} **(A1)**

eg

$$2a + 14 ,$$

$$2a = -14$$

$$a = -7 \quad \mathbf{A1} \quad \mathbf{N3}$$

[4 marks]

Examiners report

In part (b), while most candidates correctly found the value of
 a , many unnecessarily worked with the magnitudes of the vectors, sometimes leading to algebra errors.

- 2c. i. Show that

$$\cos q = \frac{2a+14}{\sqrt{14a^2+280}} .$$

[8 marks]

- ii. Hence, find the value of a for which

$$q = 1.2 .$$

Markscheme

correct magnitudes (may be seen in (b)) **(A1)(A1)**

$$\sqrt{1^2 + 3^2 + 2^2} \left(= \sqrt{14} \right),$$

$$\sqrt{2^2 + 4^2 + a^2} \left(= \sqrt{20 + a^2} \right)$$

substitution into formula **(M1)**

eg

$$\cos \theta = \frac{1 \times 2 + 3 \times 4 + 2 \times a}{\sqrt{1^2 + 3^2 + 2^2} \sqrt{2^2 + 4^2 + a^2}},$$

$$\frac{14 + 2a}{\sqrt{14} \sqrt{4 + 16 + a^2}}$$

simplification leading to required answer **A1**

eg

$$\cos \theta = \frac{14 + 2a}{\sqrt{14} \sqrt{20 + a^2}}$$

$$\cos \theta = \frac{2a + 14}{\sqrt{14a^2 + 280}} \quad \mathbf{AG} \quad \mathbf{N0}$$

[4 marks]

correct setup **(A1)**

eg

$$\cos 1.2 = \frac{2a + 14}{\sqrt{14a^2 + 280}}$$

valid attempt to solve **(M1)**

eg sketch,

$$\frac{2a + 14}{\sqrt{14a^2 + 280}} - \cos 1.2 = 0, \text{ attempt to square}$$

$$a = -3.25 \quad \mathbf{A2} \quad \mathbf{N3}$$

[4 marks]

Examiners report

Some candidates showed a minimum of working in part (c)(i); in a “show that” question, candidates need to ensure that their working clearly leads to the answer given. A common error was simplifying the magnitude of vector AC to $\sqrt{20a^2}$ instead of $\sqrt{20 + a^2}$.

In part (c)(ii), a disappointing number of candidates embarked on a usually fruitless quest for an algebraic solution rather than simply solving the resulting equation with their GDC. Many of these candidates showed quite weak algebra manipulation skills, with errors involving the square root occurring in a myriad of ways.

The line L passes through the point $(5, -4, 10)$ and is parallel to the vector

$$\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}.$$

3a. Write down a vector equation for line L .

[2 marks]

Markscheme

any correct equation in the form
 $r = a + tb$ (accept any parameter for t)

where a is

$$\begin{pmatrix} 5 \\ -4 \\ 10 \end{pmatrix}, \text{ and } b \text{ is a scalar multiple of } \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} \quad \mathbf{A2} \quad \mathbf{N2}$$

e.g.

$$r = \begin{pmatrix} 5 \\ -4 \\ 10 \end{pmatrix} + t \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}, r = 5i - 4j + 10k + t(-8i + 4j - 10k)$$

Note: Award **A1** for the form

$a + tb$, **A1** for

$L = a + tb$, **A0** for

$r = b + ta$.

[2 marks]

Examiners report

In part (a), the majority of candidates correctly recognized the equation that contains the position and direction vectors of a line. However, we saw a large number of candidates who continue to write their equations using "

$L =$ ", rather than the mathematically correct "

$r =$ " or "

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = ". r \text{ and}$$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ represent vectors, whereas L is simply the name of the line. For part (b), very few candidates recognized that a

general point on the x -axis will be given by the vector

$$\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}. \text{ Common errors included candidates setting their equation equal to}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \text{ or}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \text{ or even just the number}$$

0.

3b. The line L intersects the x -axis at the point P . Find the x -coordinate of P .

[6 marks]

Markscheme

recognizing that

$$y = 0 \text{ or}$$

$$z = 0 \text{ at } x\text{-intercept (seen anywhere) } \textbf{(R1)}$$

attempt to set up equation for x -intercept (must suggest

$$x \neq 0) \textbf{(M1)}$$

e.g.

$$L = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix},$$

$$5 + 4t = x,$$

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

one correct equation in one variable **(A1)**

e.g.

$$-4 - 2t = 0,$$

$$10 + 5t = 0$$

finding

$$t = -2 \quad \textbf{A1}$$

correct working **(A1)**

e.g.

$$x = 5 + (-2)(4)$$

$$x = -3 \text{ (accept}$$

$$(-3, 0, 0)) \quad \textbf{A1} \quad \textbf{N3}$$

[6 marks]

Examiners report

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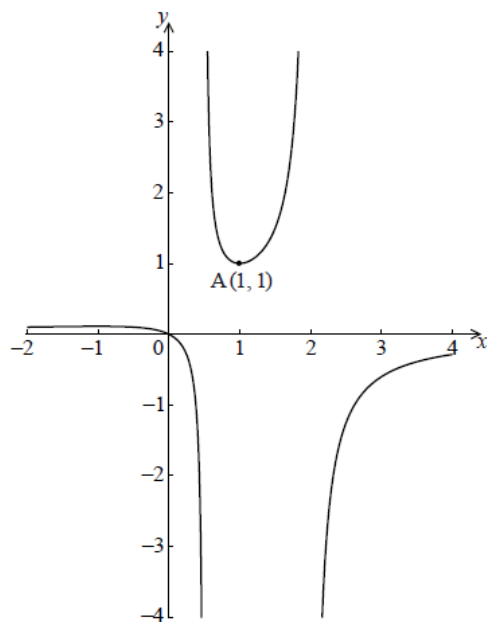
$$\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}. \text{ Common errors included candidates setting their equation equal to}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \text{ or}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \text{ or even just the number}$$

0.

Let
 $f(x) = \frac{x}{-2x^2+5x-2}$ for
 $-2 \leq x \leq 4$,
 $x \neq \frac{1}{2}$,
 $x \neq 2$. The graph of
 f is given below.



The graph of
 f has a local minimum at A(
 1,
 1) and a local maximum at B.

- 4a. Use the quotient rule to show that

$$f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}.$$

[6 marks]

Markscheme

correct derivatives **applied** in quotient rule (A1)A1A1

$$1, \\ -4x + 5$$

Note: Award (A1) for 1, A1 for

$-4x$ and A1 for

5, **only** if it is clear candidates are using the quotient rule.

correct substitution into quotient rule A1

e.g.

$$\frac{1 \times (-2x^2 + 5x - 2) - x(-4x + 5)}{(-2x^2 + 5x - 2)^2}, \\ \frac{-2x^2 + 5x - 2 - x(-4x + 5)}{(-2x^2 + 5x - 2)^2}$$

correct working (A1)

e.g.

$$\frac{-2x^2 + 5x - 2 - (-4x^2 + 5x)}{(-2x^2 + 5x - 2)^2}$$

expression clearly leading to the answer A1

e.g.

$$\frac{-2x^2 + 5x - 2 + 4x^2 - 5x}{(-2x^2 + 5x - 2)^2}$$

$$f'(x) = \frac{2x^2 - 2}{(-2x^2 + 5x - 2)^2} \quad \text{AG} \quad \text{NO}$$

[6 marks]

Examiners report

While most candidates answered part (a) correctly, there were some who did not show quite enough work for a "show that" question. A very small number of candidates did not follow the instruction to use the quotient rule.

4b. Hence find the coordinates of B.

[7 marks]

Markscheme

evidence of attempting to solve

$$f'(x) = 0 \quad (M1)$$

e.g.

$$2x^2 - 2 = 0$$

evidence of correct working **A1**

e.g.

$$x^2 = 1, \frac{\pm\sqrt{16}}{4}, 2(x-1)(x+1)$$

correct solution to quadratic **(A1)**

e.g.

$$x = \pm 1$$

correct x-coordinate

$x = -1$ (may be seen in coordinate form

$$\left(-1, \frac{1}{9}\right) \quad \mathbf{A1} \quad \mathbf{N2}$$

attempt to substitute

-1 into f (do not accept any other value) **(M1)**

e.g.

$$f(-1) = \frac{-1}{-2 \times (-1)^2 + 5 \times (-1) - 2}$$

correct working

e.g.

$$\frac{-1}{-2-5-2} \quad \mathbf{A1}$$

correct y-coordinate

$y = \frac{1}{9}$ (may be seen in coordinate form

$$\left(-1, \frac{1}{9}\right) \quad \mathbf{A1} \quad \mathbf{N2}$$

[7 marks]

Examiners report

In part (b), most candidates knew that they needed to solve the equation

$f'(x) = 0$, and many were successful in answering this question correctly. However, some candidates failed to find both values of x , or made other algebraic errors in their solutions. One common error was to find only one solution for $x^2 = 1$; another was to work with the denominator equal to zero, rather than the numerator.

- 4c. Given that the line $y = k$ does not meet the graph of f , find the possible values of k .

[3 marks]

Markscheme

recognizing values between max and min **(R1)**

$$\frac{1}{9} < k < 1 \quad \mathbf{A2} \quad \mathbf{N3}$$

[3 marks]

Examiners report

In part (c), a significant number of candidates seemed to think that the line

$y = k$ was a vertical line, and attempted to find the vertical asymptotes. Others tried looking for a horizontal asymptote. Fortunately, there were still a good number of intuitive candidates who recognized the link with the graph and with part (b), and realized that the horizontal line must pass through the space between the given local minimum and the local maximum they had found in part (b).

Let
 $f(x) = \cos(e^x)$, for
 $-2 \leq x \leq 2$.

- 5a. Find
 $f'(x)$.

[2 marks]

Markscheme

$$f'(x) = -e^x \sin(e^x) \quad \mathbf{A1A1} \quad \mathbf{N2}$$

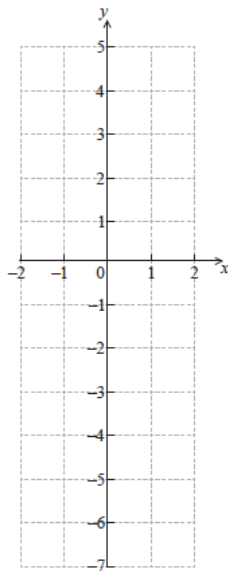
[2 marks]

Examiners report

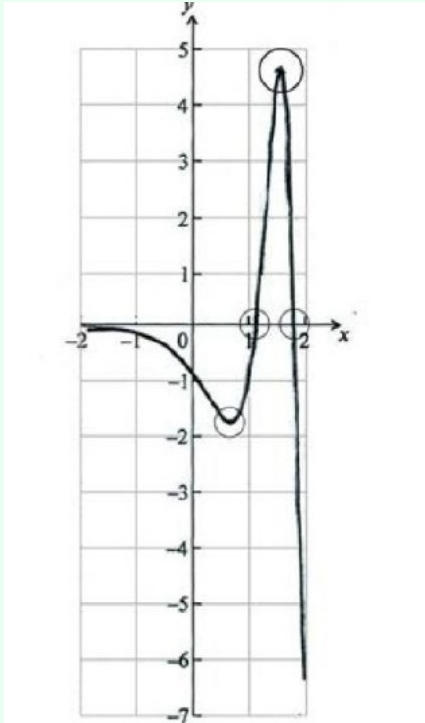
Many students failed in applying the chain rule to find the correct derivative, and some inappropriately used the product rule. However, many of those obtained full follow through marks in part (b) for the sketch of the function they found in part (a).

- 5b. On the grid below, sketch the graph of
 $f'(x)$.

[4 marks]



Markscheme



A1A1A1A1 N4

Note: Award **A1** for shape that must have the correct domain (from -2 to $+2$) and correct range (from -6 to 4), **A1** for minimum in circle, **A1** for maximum in circle and **A1** for intercepts in circles.

[4 marks]

Examiners report

Many students failed in applying the chain rule to find the correct derivative, and some inappropriately used the product rule. However, many of those obtained full follow through marks in part (b) for the sketch of the function they found in part (a).

Most candidates sketched an approximately correct shape in the given domain, though there were some that did not realize they had to set their GDC to radians, producing a meaningless sketch.

It is very important to stress to students that although they are asked to produce a sketch, it is still necessary to show its key features such as domain and range, stationary points and intercepts.

Let

$f(x) = ax^3 + bx^2 + c$, where a , b and c are real numbers. The graph of f passes through the point $(2, 9)$.

- 6a. Show that
 $8a + 4b + c = 9$.

[2 marks]

Markscheme

attempt to substitute coordinates in f (M1)

e.g.

$$f(2) = 9$$

correct substitution A1

e.g.

$$a \times 2^3 + b \times 2^2 + c = 9$$

$$8a + 4b + c = 9 \quad \text{AG} \quad \text{N0}$$

[2 marks]

Examiners report

Part (a) was generally well done, with a few candidates failing to show a detailed substitution. Some substituted 2 in place of x , but didn't make it clear that they had substituted 2 in y as well.

- 6b. The graph of f has a local minimum at $(1, 4)$.

[7 marks]

Find two other equations in a , b and c , giving your answers in a similar form to part (a).

Markscheme

recognizing that

$(1, 4)$ is on the graph of f (M1)

e.g.

$$f(1) = 4$$

correct equation A1

e.g.

$$a + b + c = 4$$

recognizing that

$f' = 0$ at minimum (seen anywhere) (M1)

e.g.

$$f'(1) = 0$$

$$f'(x) = 3ax^2 + 2bx \text{ (seen anywhere)} \quad \text{A1A1}$$

correct substitution into derivative (A1)

e.g.

$$3a \times 1^2 + 2b \times 1 = 0$$

correct simplified equation A1

e.g.

$$3a + 2b = 0$$

[7 marks]

Examiners report

A great majority could find the two equations in part (b). However there were a significant number of candidates who failed to identify that the gradient of the tangent is zero at a minimum point, thus getting the incorrect equation $3a + 2b = 4$.

- 6c. Find the value of a , of b and of c .

[4 marks]

Markscheme

valid method for solving system of equations **(M1)**

e.g. inverse of a matrix, substitution

$$a = 2 ,$$

$$b = -3 ,$$

$$c = 5 \quad \mathbf{A1A1A1} \quad \mathbf{N4}$$

[4 marks]

Examiners report

A considerable number of candidates only had 2 equations, so that they either had a hard time trying to come up with a third equation (incorrectly combining some of the information given in the question) to solve part (c) or they completely failed to solve it.

Despite obtaining three correct equations many used long elimination methods that caused algebraic errors. Pages of calculations leading nowhere were seen.

Those who used matrix methods were almost completely successful.

Let

$f(x) = \frac{1}{4}x^2 + 2$. The line L is the tangent to the curve off at $(4, 6)$.

7a. Find the equation of L .

[4 marks]

Markscheme

finding

$$f'(x) = \frac{1}{2}x \quad \mathbf{A1}$$

attempt to find

$$f'(4) \quad \mathbf{(M1)}$$

correct value

$$f'(4) = 2 \quad \mathbf{A1}$$

correct equation in any form **A1 N2**

e.g.

$$y - 6 = 2(x - 4) ,$$

$$y = 2x - 2$$

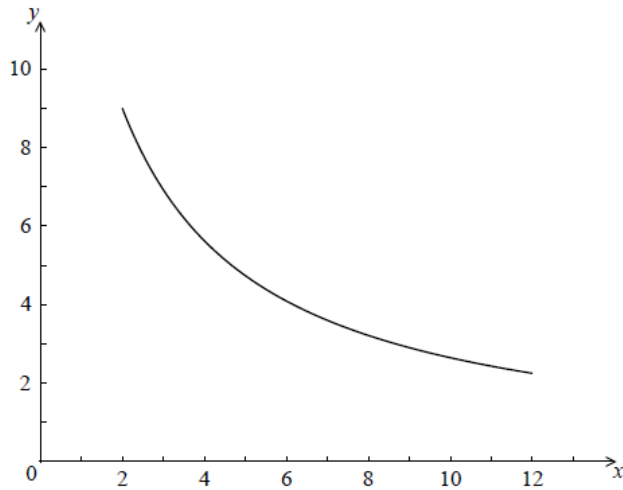
[4 marks]

Examiners report

While most candidates answered part (a) correctly, finding the equation of the tangent, there were some who did not consider the value of their derivative when

$x = 4$.

Let
 $g(x) = \frac{90}{3x+4}$, for
 $2 \leq x \leq 12$. The following diagram shows the graph of g .



- 7b. Find the area of the region enclosed by the curve of g , the x -axis, and the lines $x = 2$ and $x = 12$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$.

[6 marks]

Markscheme

$$\text{area} = \int_2^{12} \frac{90}{3x+4} dx$$

correct integral **A1A1**

e.g.

$$30 \ln(3x + 4)$$

substituting limits and subtracting **(M1)**

e.g.

$$30 \ln(3 \times 12 + 4) - 30 \ln(3 \times 2 + 4),$$

$$30 \ln 40 - 30 \ln 10$$

correct working **(A1)**

e.g.

$$30(\ln 40 - \ln 10)$$

correct application of

$$\ln b - \ln a \quad \textbf{(A1)}$$

e.g.

$$30 \ln \frac{40}{10}$$

$$\text{area} = 30 \ln 4 \quad \textbf{A1} \quad \textbf{N4}$$

[6 marks]

Examiners report

In part (b), most candidates knew that they needed to integrate to find the area, but errors in integration, and misapplication of the rules of logarithms kept many from finding the correct area.

- 7c. The graph of g is reflected in the x -axis to give the graph of h . The area of the region enclosed by the lines L , $x = 2$, $x = 12$ and the x -axis is 120 cm^2 . [3 marks]

Find the area enclosed by the lines L , $x = 2$, $x = 12$ and the graph of h .

Markscheme

valid approach (M1)

e.g. sketch, area $h = \text{area } g, 120 + \text{their answer from (b)}$

area $= 120 + 30 \ln 4$ A2 N3

[3 marks]

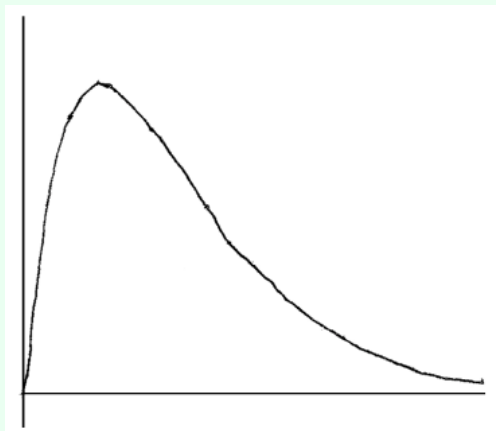
Examiners report

In part (c), it was clear that a significant number of candidates understood the idea of the reflected function, and some recognized that the integral was the negative of the integral from part (b), but only a few recognized the relationship between the areas. Many thought the area between h and the x -axis was 120.

Let
 $f(x) = \frac{20x}{e^{0.3x}}$, for
 $0 \leq x \leq 20$.

- 8a. Sketch the graph of f . [3 marks]

Markscheme



A1A1A1 N3

Note: Award **A1** for approximately correct shape with inflexion/change of curvature, **A1** for maximum skewed to the left, **A1** for asymptotic behaviour to the right.

[3 marks]

Examiners report

Many candidates earned the first four marks of the question in parts (a) and (b) for correctly using their GDC to graph and find the maximum value.

- 8b. (i) Write down the x -coordinate of the maximum point on the graph of f .
(ii) Write down the interval where f is increasing.

[3 marks]

Markscheme

(i)

$$x = 3.33 \quad \mathbf{A1} \quad \mathbf{N1}$$

(ii) correct interval, with right end point

$$3\frac{1}{3} \quad \mathbf{A1A1} \quad \mathbf{N2}$$

e.g.

$$0 < x \leq 3.33,$$

$$0 \leq x < 3\frac{1}{3}$$

Note: Accept any inequalities in the right direction.

[3 marks]

Examiners report

Many candidates earned the first four marks of the question in parts (a) and (b) for correctly using their GDC to graph and find the maximum value.

- 8c. Show that
 $f'(x) = \frac{20-6x}{e^{0.3x}}.$

[5 marks]

Markscheme

valid approach **(M1)**

e.g. quotient rule, product rule

2 correct derivatives (must be seen in product or quotient rule) **(A1)(A1)**

e.g.

$$20,$$

$$0.3e^{0.3x} \text{ or}$$

$$-0.3e^{-0.3x}$$

correct substitution into product or quotient rule **A1**

e.g.

$$\frac{20e^{0.3x} - 20x(0.3)e^{0.3x}}{(e^{0.3x})^2},$$

$$20e^{-0.3x} + 20x(-0.3)e^{-0.3x}$$

correct working **A1**

e.g.

$$\frac{20e^{0.3x} - 6xe^{0.3x}}{e^{0.6x}},$$

$$\frac{e^{0.3x}(20 - 20x(0.3))}{(e^{0.3x})^2},$$

$$e^{-0.3x}(20 - 20x(-0.3))$$

$$f'(x) = \frac{20-6x}{e^{0.3x}} \quad \mathbf{AG} \quad \mathbf{NO}$$

[5 marks]

Examiners report

Most had a valid approach in part (c) using either the quotient or product rule, but many had difficulty applying the chain rule with a function involving e and simplifying.

8d. Find the interval where the rate of change of f is increasing.

[4 marks]

Markscheme

consideration of

f' or
 f'' (M1)

valid reasoning R1

e.g. sketch of

f' ,
 f'' is positive,
 $f'' = 0$, reference to minimum of
 f'

correct value

$6.6666666\ldots$

$\left(6\frac{2}{3}\right)$ (A1)

correct interval, with **both** endpoints A1 N3

e.g.

$6.67 < x \leq 20$,

$6\frac{2}{3} \leq x < 20$

[4 marks]

Examiners report

Part (d) was difficult for most candidates. Although many associated rate of change with derivative, only the best-prepared students had valid reasoning and could find the correct interval with both endpoints.

The following table shows the probability distribution of a discrete random variable X .

x	0	2	5	9
$P(X = x)$	0.3	k	$2k$	0.1

9a. Find the value of k .

[3 marks]

Markscheme

evidence of summing to 1 (M1)

e.g.

$$\sum p = 1, 0.3 + k + 2k + 0.1 = 1$$

correct working (A1)

e.g.

$$0.4 + 3k, 3k = 0.6$$

$$k = 0.2 \quad \text{A1} \quad \text{N2}$$

[3 marks]

Examiners report

Overall, this question was very well done. A few candidates left this question blank, or used methods which would indicate they were unfamiliar with discrete random variables. In part (b), there were a good number of candidates who set up their work correctly, but then had trouble adding or multiplying decimals without a calculator. A common type of error for these candidates was $5(0.4) = 0.2$.

- 9b. Find $E(X)$.

[3 marks]

Markscheme

correct substitution into formula

$$E(X) \quad (A1)$$

e.g.

$$0(0.3) + 2(k) + 5(2k) + 9(0.1), 12k + 0.9$$

correct working

e.g.

$$0(0.3) + 2(0.2) + 5(0.4) + 9(0.1), 0.4 + 2.0 + 0.9 \quad (A1)$$

$$E(X) = 3.3 \quad A1 \quad N2$$

[3 marks]

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The random variable X has the following probability distribution, with $P(X > 1) = 0.5$.

x	0	1	2	3
$P(X = x)$	p	q	r	0.2

- 10a. Find the value of r .

[2 marks]

Markscheme

attempt to substitute

$$P(X > 1) = 0.5 \quad (M1)$$

e.g.

$$r + 0.2 = 0.5$$

$$r = 0.3 \quad A1 \quad N2$$

[2 marks]

Examiners report

The majority of candidates were successful in earning full marks on this question.

- 10b. Given that $E(X) = 1.4$, find the value of p and of q .

[6 marks]

Markscheme

correct substitution into
 $E(X)$ (seen anywhere) **(A1)**

e.g.

$$0 \times p + 1 \times q + 2 \times r + 3 \times 0.2$$

correct equation **A1**

e.g.

$$q + 2 \times 0.3 + 3 \times 0.2 = 1.4,$$

$$q + 1.2 = 1.4$$

$$q = 0.2 \quad \mathbf{A1} \quad \mathbf{N1}$$

evidence of choosing

$$\sum p_i = 1 \quad \mathbf{M1}$$

e.g.

$$p + 0.2 + 0.3 + 0.2 = 1,$$

$$p + q = 0.5$$

correct working **(A1)**

$$p + 0.7 = 1,$$

$$1 - 0.2 - 0.3 - 0.2,$$

$$p + 0.2 = 0.5$$

$$p = 0.3 \quad \mathbf{A1} \quad \mathbf{N2}$$

Note: Exception to the **FT** rule. Award **FT** marks on an incorrect value of q , even if q is an inappropriate value. Do not award the final **A** mark for an inappropriate value of p .

[6 marks]

Examiners report

In part (b), a small number of candidates did not use the correct formula for $E(X)$, even though this formula is given in the formula booklet. There were also a few candidates who incorrectly assumed that

$p = 0$, forgetting that the sum of the probabilities must equal 1. There were a few candidates who left this question blank, which raises concerns about whether they had been exposed to probability distributions during the course.

- 11a. The probability of obtaining “tails” when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining **at least** four tails.

[4 marks]

Markscheme

evidence of recognizing binomial distribution **(M1)**

e.g.

$$X \sim B(10, 0.57),$$

$$p = 0.57,$$

$$q = 0.43$$

EITHER

$$P(X \leq 3) = 2.16 \times 10^{-4} + 0.00286 + 0.01709 + 0.06041 \\ (= 0.08057) \quad \mathbf{(A1)}$$

evidence of using complement **(M1)**

e.g.

1 – any probability,

$$P(X \geq 4) = 1 - P(X \leq 3)$$

0.919423...

$$P(X \geq 4) = 0.919 \quad \mathbf{A1 \quad N3}$$

OR

summing the probabilities from

$X = 4$ to

$$X = 10 \quad \mathbf{(M1)}$$

correct expression or values **(A1)**

e.g.

$$\sum_{r=4}^{10} \binom{10}{r} (0.57)^r (0.43)^{10-r},$$

$$0.14013 + 0.2229 + \dots + 0.02731 + 0.00362$$

0.919424

$$P(X \geq 4) = 0.919 \quad \mathbf{A1 \quad N3}$$

[4 marks]

Examiners report

This was an accessible problem that created some difficulties for candidates. Most were able to recognize the binomial nature of the problem but were confused by the phrase "at least four tails" which was often interpreted as the complement of four or less. Poor algebraic manipulation also led to unnecessary errors that the calculator approach would have avoided.

- 11b. The probability of obtaining "tails" when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining the fourth tail on the tenth toss. **[3 marks]**

Markscheme

evidence of valid approach (M1)

e.g. three tails in nine tosses,

$$\binom{9}{3} (0.57)^3 (0.43)^6$$

correct calculation

e.g.

$$\binom{9}{3} (0.57)^3 (0.43)^6 \times 0.57,$$

$$0.09834 \times 0.57 \quad (\text{A1})$$

$$0.05605178 \dots$$

$$P(\text{4th tail on 10th toss}) = 0.0561 \quad \text{A1} \quad \text{N2}$$

[3 marks]

Examiners report

This was an accessible problem that created some difficulties for candidates. Most were able to recognize the binomial nature of the problem but were confused by the phrase "at least four tails" which was often interpreted as the complement of four or less. Poor algebraic manipulation also led to unnecessary errors that the calculator approach would have avoided.

- 12a. A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested. [4 marks]
Find the probability that there is at least one defective lamp in the sample.

Markscheme

evidence of recognizing binomial (seen anywhere) (M1)

e.g.

$$B(n, p),$$

$$0.95^{30}$$

finding

$$P(X = 0) = 0.21463876 \quad (\text{A1})$$

appropriate approach (M1)

e.g. complement, summing probabilities

$$0.785361$$

probability is

$$0.785 \quad \text{A1} \quad \text{N3}$$

[4 marks]

Examiners report

Although candidates seemed more confident in attempting binomial probabilities than in previous years, some of them failed to recognize the binomial nature of the question in part (a). Many knew that the complement was required, but often used

$$1 - P(X = 1) \text{ or}$$

$$1 - P(X \leq 1) \text{ instead of}$$

$$1 - P(X = 0).$$

- 12b. A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested. [4 marks]

Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps.

Markscheme

identifying correct outcomes (seen anywhere) **(A1)**

e.g.

$P(X = 1) + P(X = 2)$, 1 or 2 defective,

$0.3389 \dots + 0.2586 \dots$

recognizing conditional probability (seen anywhere) **R1**

e.g.

$P(A|B)$,

$P(X \leq 2 | X \geq 1)$, P(at most 2|at least 1)

appropriate approach involving conditional probability **(M1)**

e.g.

$$\frac{P(X=1)+P(X=2)}{P(X \geq 1)},$$

$$\frac{0.3389 \dots + 0.2586 \dots}{0.785 \dots},$$

$$\frac{1 \text{ or } 2}{0.785}$$

0.760847

probability is

0.761 **A1 N2**

[4 marks]

Examiners report

Part (b) was poorly answered. While some candidates recognized that it was a conditional probability, very few were able to correctly apply the formula, identify the outcomes and follow on to achieve the correct result.

Only a few could find the intersection of the events correctly. Several thought the numerator was a product (i.e. $P(\text{at most } 2) \times P(\text{at least } 1)$), and then cancelled common factors with the denominator. Others realized that $x = 1$ and

$x = 2$ were required but multiplied their probabilities.

This was the most commonly missed out question from Section A.

13. The random variable X has the following probability distribution.

[6 marks]

x	1	2	3
$P(X = x)$	s	0.3	q

Given that

$E(X) = 1.7$, find q .

Markscheme

correct substitution into

$$E(X) = \sum px \text{ (seen anywhere)} \quad \mathbf{A1}$$

e.g.

$$1s + 2 \times 0.3 + 3q = 1.7,$$

$$s + 3q = 1.1$$

recognizing

$$\sum p = 1 \text{ (seen anywhere)} \quad \mathbf{(M1)}$$

correct substitution into

$$\sum p = 1 \quad \mathbf{A1}$$

e.g.

$$s + 0.3 + q = 1$$

attempt to solve simultaneous equations $\mathbf{(M1)}$

correct working $\mathbf{(A1)}$

e.g.

$$0.3 + 2q = 0.7,$$

$$2s = 1$$

$$q = 0.2 \quad \mathbf{A1} \quad \mathbf{N4}$$

[6 marks]

Examiners report

Candidates generally earned either full marks or only one mark on this question. The most common error was where candidates only wrote the equation for

$E(X) = 1.7$, and tried to rearrange that equation to solve for q . The candidates who also knew that the sum of the probabilities must be equal to 1 were very successful in solving the resulting system of equations.

The probability distribution of a discrete random variable X is given by

$$P(X = x) = \frac{x^2}{14}, x \in \{1, 2, k\}, \text{ where } k > 0$$

.

- 14a. Write down
 $P(X = 2)$.

[1 mark]

Markscheme

$$P(X = 2) = \frac{4}{14}$$

$$\left(= \frac{2}{7} \right) \quad \mathbf{A1} \quad \mathbf{N1}$$

[1 mark]

Examiners report

Although many candidates were successful in working with the probability function, students had difficulty following the "show that" instruction of this question. Many substituted

$k = 3$ and worked backwards to show that the sum of probabilities is 1. Some would argue that

$k = 4$ does not work, but were unable to give a complete justification for

$k = 3$. A good number of students seemed unprepared to find an expected value. Many candidates wrote a formula and did not know what to do with it, while others divided

$E(X)$ by 3 or by 6, which confuses the concept of a mean in a probability distribution with the more common understanding.

- 14b. Show that
 $k = 3$.

[4 marks]

Markscheme

$$P(X = 1) = \frac{1}{14} \quad (\mathbf{A1})$$

$$P(X = k) = \frac{k^2}{14} \quad (\mathbf{A1})$$

setting the sum of probabilities
 $= 1 \quad \mathbf{M1}$

e.g.

$$\frac{1}{14} + \frac{4}{14} + \frac{k^2}{14} = 1,$$

$$5 + k^2 = 14$$

$$k^2 = 9 \text{ (accept}$$

$$\frac{k^2}{14} = \frac{9}{14}) \quad \mathbf{A1}$$

$$k = 3 \quad \mathbf{AG} \quad \mathbf{N0}$$

[4 marks]

Examiners report

Although many candidates were successful in working with the probability function, students had difficulty following the "show that" instruction of this question. Many substituted

$k = 3$ and worked backwards to show that the sum of probabilities is 1. Some would argue that

$k = 4$ does not work, but were unable to give a complete justification for

$k = 3$.

- 14c. Find
 $E(X)$.

[2 marks]

Markscheme

correct substitution into

$$E(X) = \sum xP(X = x) \quad \mathbf{A1}$$

e.g.

$$1 \left(\frac{1}{14} \right) + 2 \left(\frac{4}{14} \right) + 3 \left(\frac{9}{14} \right)$$

$$E(X) = \frac{36}{14}$$

$$\left(= \frac{18}{7} \right) \quad \mathbf{A1} \quad \mathbf{N1}$$

[2 marks]

Examiners report

A good number of students seemed unprepared to find an expected value. Many candidates wrote a formula and did not know what to do with it, while others divided

$E(X)$ by 3 or by 6, which confuses the concept of a mean in a probability distribution with the more common understanding.

A box holds 240 eggs. The probability that an egg is brown is 0.05.

- 15a. Find the expected number of brown eggs in the box.

[2 marks]

Markscheme

correct substitution into formula for $E(X)$ **(A1)**

e.g.
 0.05×240

$$E(X) = 12 \quad \mathbf{A1} \quad \mathbf{N2}$$

[2 marks]

Examiners report

Part (a) was answered correctly by most candidates.

- 15b. Find the probability that there are 15 brown eggs in the box.

[2 marks]

Markscheme

evidence of recognizing binomial probability (may be seen in part (a)) **(M1)**

e.g.
 $\binom{240}{15} (0.05)^{15} (0.95)^{225}$,
 $X \sim B(240, 0.05)$

$$P(X = 15) = 0.0733 \quad \mathbf{A1} \quad \mathbf{N2}$$

[2 marks]

Examiners report

In parts (b) and (c), many failed to recognize the binomial nature of this experiment and opted for incorrect techniques in simple probability.

- 15c. Find the probability that there are at least 10 brown eggs in the box.

[3 marks]

Markscheme

$$P(X \leq 9) = 0.236 \quad \mathbf{(A1)}$$

evidence of valid approach **(M1)**

e.g. using complement, summing probabilities

$$P(X \geq 10) = 0.764 \quad \mathbf{A1} \quad \mathbf{N3}$$

[3 marks]

Examiners report

In parts (b) and (c), many failed to recognize the binomial nature of this experiment and opted for incorrect techniques in simple probability. Although several candidates appreciated that (c) involved the idea of a complement, some resorted to elaborate probability addition suggesting they were unaware of the capabilities of their GDC. There was also a great deal of evidence to suggest that candidates did not understand the phrase "at least 10" as several candidates found either

$$1 - P(X \leq 10) ,$$

$$1 - P(X = 10) \text{ or}$$

$$P(X > 10) .$$

Two fair 4-sided dice, one red and one green, are thrown. For each die, the faces are labelled 1, 2, 3, 4. The score for each die is the number which lands face down.

- 16a. List the pairs of scores that give a sum of 6.

[3 marks]

Markscheme

three correct pairs **A1A1A1 N3**

e.g. (2, 4), (3, 3), (4, 2) , *R2G4, R3G3, R4G2*

[3 marks]

Examiners report

All but the weakest candidates managed to score full marks for parts (a) and (b). An occasional error in part (a) was including additional pair(s) or listing (3, 3) twice.

- 16b. The probability distribution for the sum of the scores on the two dice is shown below.

[3 marks]

Sum	2	3	4	5	6	7	8
Probability	p	q	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	r	$\frac{1}{16}$

Find the value of p , of q , and of r .

Markscheme

$$p = \frac{1}{16} ,$$

$$q = \frac{2}{16} ,$$

$$r = \frac{2}{16} \quad \mathbf{A1A1A1 \quad N3}$$

[3 marks]

Examiners report

All but the weakest candidates managed to score full marks for parts (a) and (b).

- 16c. Fred plays a game. He throws two fair 4-sided dice four times. He wins a prize if the sum is 5 on three or more throws.

[6 marks]

Find the probability that Fred wins a prize.

Markscheme

let X be the number of times the sum of the dice is 5

evidence of valid approach **(M1)**

e.g.

$X \sim B(n, p)$, tree diagram, 5 sets of outcomes produce a win

one correct parameter **(A1)**

e.g.

$$n = 4,$$

$$p = 0.25,$$

$$q = 0.75$$

Fred wins prize is

$$P(X \geq 3) \quad \mathbf{(A1)}$$

appropriate approach to find probability **M1**

e.g. complement, summing probabilities, using a CDF function

correct substitution **(A1)**

e.g.

$$1 - 0.949 \dots,$$

$$1 - \frac{243}{256},$$

$$0.046875 + 0.00390625,$$

$$\frac{12}{256} + \frac{1}{256}$$

probability of winning = 0.0508

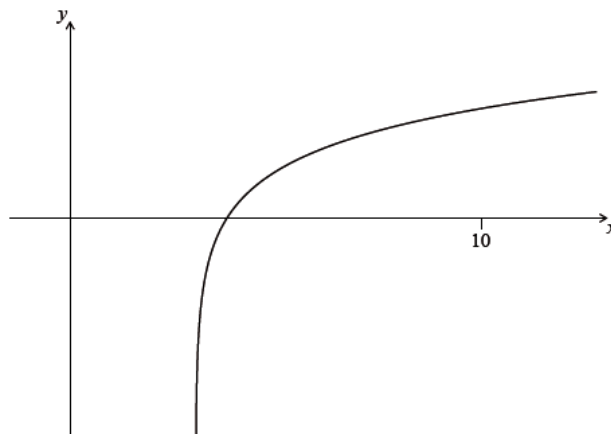
$$\left(\frac{13}{256}\right) \quad \mathbf{A1 \quad N3}$$

[6 marks]

Examiners report

Many candidates found part (c) challenging, as they failed to recognize the binomial probability. Successful candidates generally used either the binomial CDF function or the sum of two binomial probabilities. Some used approaches like multiplying probabilities or tree diagrams, but these were less successful.

Let $f(x) = 2\ln(x - 3)$, for $x > 3$. The following diagram shows part of the graph of f .



17a. Find the equation of the vertical asymptote to the graph of f .

[2 marks]

Markscheme

valid approach (M1)

eg horizontal translation 3 units to the right

$x = 3$ (must be an equation) A1 N2

[2 marks]

Examiners report

[N/A]

- 17b. Find the x -intercept of the graph of f .

[2 marks]

Markscheme

valid approach (M1)

eg $f(x) = 0$, $e^0 = x - 3$

4, $x = 4$, (4, 0) A1 N2

[2 marks]

Examiners report

[N/A]

- 17c. The region enclosed by the graph of f , the x -axis and the line $x = 10$ is rotated 360° about the x -axis. Find the volume of the solid formed.

[3 marks]

Markscheme

attempt to substitute either **their correct** limits or the function into formula involving f^2 (M1)

eg $\int_4^{10} f^2$, $\pi \int (2\ln(x-3))^2 dx$

141.537

volume = 142 A2 N3

[3 marks]

Total [7 marks]

Examiners report

[N/A]

The first three terms of a geometric sequence are $u_1 = 0.64$, $u_2 = 1.6$, and $u_3 = 4$.

- 18a. Find the value of r .

[2 marks]

Markscheme

valid approach **(M1)**

eg $\frac{u_1}{u_2}, \frac{4}{1.6}, 1.6 = r(0.64)$

$r = 2.5 \quad \left(= \frac{5}{2} \right) \quad \mathbf{A1} \quad \mathbf{N2}$

[2 marks]

Examiners report

[N/A]

18b. Find the value of S_6 .

[2 marks]

Markscheme

correct substitution into S_6 **(A1)**

eg $\frac{0.64(2.5^6-1)}{2.5-1}$

$S_6 = 103.74$ (exact), 104 **A1 N2**

[2 marks]

Examiners report

[N/A]

18c. Find the least value of n such that $S_n > 75\,000$.

[3 marks]

Markscheme

METHOD 1 (analytic)

valid approach **(M1)**

eg $\frac{0.64(2.5^n-1)}{2.5-1} > 75\,000, \frac{0.64(2.5^n-1)}{2.5-1} = 75\,000$

correct inequality (accept equation) **(A1)**

eg $n > 13.1803, n = 13.2$

$n = 14 \quad \mathbf{A1} \quad \mathbf{N1}$

METHOD 2 (table of values)

both crossover values **A2**

eg $S_{13} = 63577.8, S_{14} = 158945$

$n = 14 \quad \mathbf{A1} \quad \mathbf{N1}$

[3 marks]

Total [7 marks]

Examiners report

[N/A]

An environmental group records the numbers of coyotes and foxes in a wildlife reserve after t years, starting on 1 January 1995.

Let c be the number of coyotes in the reserve after t years. The following table shows the number of coyotes after t years.

number of years (t)	0	2	10	15	19
number of coyotes (c)	115	197	265	320	406

The relationship between the variables can be modelled by the regression equation $c = at + b$.

- 19a. Find the value of a and of b .

[3 marks]

Markscheme

evidence of setup (M1)

eg correct value for a or b

13.3823, 137.482

$a = 13.4$, $b = 137$ A1A1 N3

[3 marks]

Examiners report

[N/A]

- 19b. Use the regression equation to estimate the number of coyotes in the reserve when $t = 7$.

[3 marks]

Markscheme

correct substitution into **their** regression equation

eg $13.3823 \times 7 + 137.482$ (A1)

correct calculation

231.158 (A1)

231 (coyotes) (must be an integer) A1 N2

[3 marks]

Examiners report

[N/A]

- 19c. Let f be the number of foxes in the reserve after t years. The number of foxes can be modelled by the equation $f = \frac{2000}{1 + 99e^{-kt}}$, where k is a constant. [3 marks]

Find the number of foxes in the reserve on 1 January 1995.

Markscheme

recognizing $t = 0$ (**M1**)

eg $f(0)$

correct substitution into the model

eg $\frac{2000}{1+99e^{-k(0)}}, \frac{2000}{100}$ (**A1**)

20 (foxes) **A1 N2**

[3 marks]

Examiners report

[N/A]

19d. During which year were the number of coyotes the same as the number of foxes?

[4 marks]

Markscheme

valid approach (**M1**)

eg $c = f$, sketch of graphs

correct working (**A1**)

eg $\frac{2000}{1+99e^{-0.237124t}} = 13.382t + 137.482$, sketch of graphs, table of values

$t = 12.0403$ (**A1**)

2007 **A1 N2**

Note: Exception to the **FT** rule. Award **A1FT** on their value of t .

[4 marks]

Total [16 marks]

Examiners report

[N/A]

20a. Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n .

[2 marks]

Markscheme

$m = 3, n = 4$ **A1A1 N2**

[2 marks]

Examiners report

Indices laws were well understood with many candidates solving the equation correctly. Some candidates used logs, which took longer, and errors crept in.

20b. Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$.

[4 marks]

Markscheme

attempt to apply $(2^a)^b = 2^{ab}$ (M1)

eg $6x + 3, 4(2x - 3)$

equating **their** powers of 2 (seen anywhere) M1

eg $3(2x + 1) = 8x - 12$

correct working A1

eg $8x - 12 = 6x + 3, 2x = 15$

$x = \frac{15}{2}$ (7.5) A1 N2

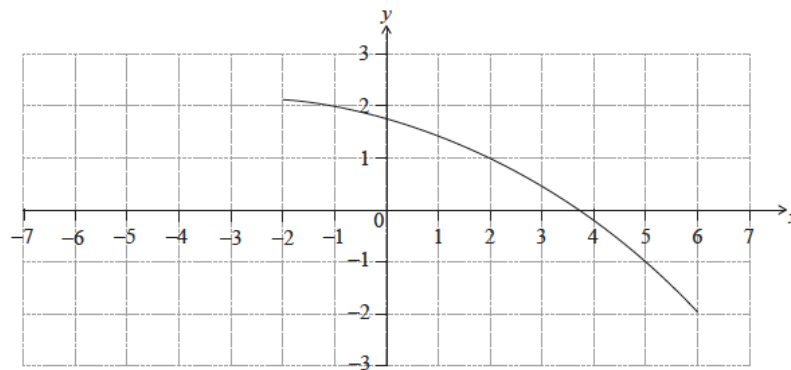
[4 marks]

Total [6 marks]

Examiners report

Indices laws were well understood with many candidates solving the equation correctly. Some candidates used logs, which took longer, and errors crept in.

The following diagram shows the graph of a function f .



21a. Find $f^{-1}(-1)$.

[2 marks]

Markscheme

valid approach (M1)

eg horizontal line on graph at -1 , $f(a) = -1$, $(-1, 5)$

$f^{-1}(-1) = 5$ A1 N2

[2 marks]

Examiners report

Typically candidates were more successful in finding the composite function than the inverse. Some students tried to find the function, rather than read values from the given graph. The sketch of $f(-x)$ was often well done, with the most common error being a reflection in the x -axis.

21b. Find $(f \circ f)(-1)$.

[3 marks]

Markscheme

attempt to find $f(-1)$ (M1)

eg line on graph

$f(-1) = 2$ (A1)

$(f \circ f)(-1) = 1$ A1 N3

[3 marks]

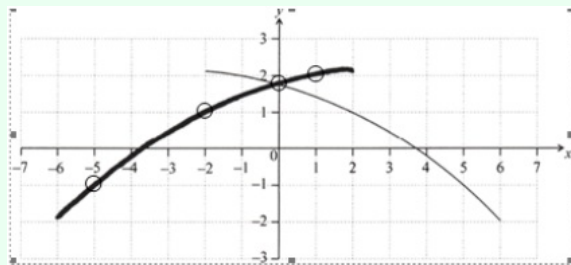
Examiners report

Typically candidates were more successful in finding the composite function than the inverse. Some students tried to find the function, rather than read values from the given graph. The sketch of $f(-x)$ was often well done, with the most common error being a reflection in the x -axis.

21c. On the same diagram, sketch the graph of $y = f(-x)$.

[2 marks]

Markscheme



A1A1 N2

Note: The shape **must** be an approximately correct shape (concave down and increasing). **Only** if the shape is approximately correct, award the following for points in circles:

A1 for the y -intercept,

A1 for any **two** of these points $(-5, -1)$, $(-2, 1)$, $(1, 2)$.

[2 marks]

Total [7 marks]

Examiners report

Typically candidates were more successful in finding the composite function than the inverse. Some students tried to find the function, rather than read values from the given graph. The sketch of $f(-x)$ was often well done, with the most common error being a reflection in the x -axis.

$$\text{Let } f(x) = px^2 + (10 - p)x + \frac{5}{4}p - 5.$$

22a. Show that the discriminant of $f(x)$ is $100 - 4p^2$.

[3 marks]

Markscheme

correct substitution into $b^2 - 4ac$ **A1**

eg $(10 - p)^2 - 4(p)\left(\frac{5}{4}p - 5\right)$

correct expansion of each term **A1A1**

eg $100 - 20p + p^2 - 5p^2 + 20p$, $100 - 20p + p^2 - (5p^2 - 20p)$

$100 - 4p^2$ **AG N0**

[3 marks]

Examiners report

Many candidates were able to identify the discriminant correctly and continued with good algebraic manipulation. A commonly seen mistake was identifying the constant as $\frac{5}{4}p$ instead of $\frac{5}{4}p - 5$. Mostly a correct approach to part b) was seen ($\Delta = 0$), with the common error being only one answer given for p , even though the question said values (plural).

22b. Find the values of p so that $f(x) = 0$ has two **equal** roots.

[3 marks]

Markscheme

recognizing discriminant is zero for equal roots **(R1)**

eg $D = 0$, $4p^2 = 100$

correct working **(A1)**

eg $p^2 = 25$, 1 correct value of p

both correct values $p = \pm 5$ **A1 N2**

[3 marks]

Total [6 marks]

Examiners report

Many candidates were able to identify the discriminant correctly and continued with good algebraic manipulation. A commonly seen mistake was identifying the constant as $\frac{5}{4}p$ instead of $\frac{5}{4}p - 5$. Mostly a correct approach to part b) was seen ($\Delta = 0$), with the common error being only one answer given for p , even though the question said values (plural).

Let $f(x) = \frac{2x-6}{1-x}$, for $x \neq 1$.

23. For the graph of f

[5 marks]

- (i) find the x -intercept;
- (ii) write down the equation of the vertical asymptote;
- (iii) find the equation of the horizontal asymptote.

Markscheme

(i) valid approach **(M1)**

eg sketch, $f(x) = 0$, $0 = 2x - 6$

$x = 3$ or $(3, 0)$ **A1 N2**

(ii) $x = 1$ (must be equation) **A1 N1**

(iii) valid approach **(M1)**

eg sketch, $\frac{2x}{-1x}$, inputting large values of x , L'Hopital's rule

$y = -2$ (must be equation) **A1 N2**

[5 marks]

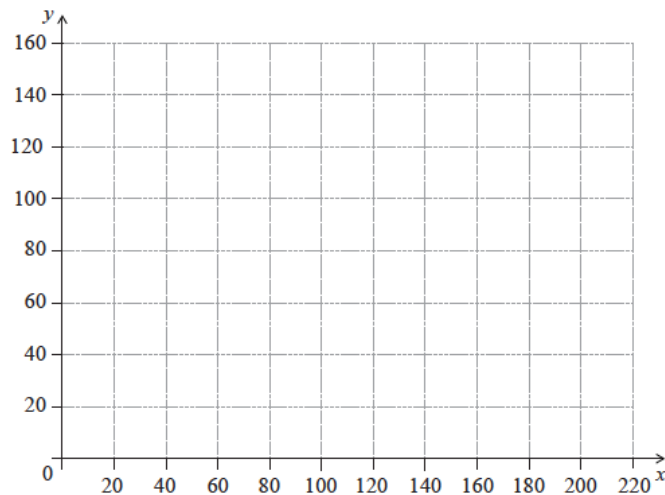
Examiners report

Part (a) was generally well done with candidates using both algebraic and graphical approaches to obtain solutions. There are still some who do not identify their asymptotes using equations.

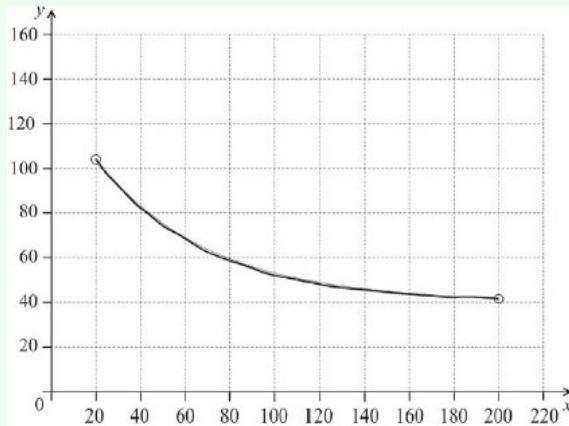
Let $G(x) = 95e^{(-0.02x)} + 40$, for $20 \leq x \leq 200$.

24a. On the following grid, sketch the graph of G .

[3 marks]



Markscheme



A1A1A1 N3

Note: Curve must be approximately correct exponential shape (concave up and decreasing). Only if the shape is approximately correct, award the following:

A1 for left endpoint in circle,

A1 for right endpoint in circle,

A1 for asymptotic to $y = 40$ (must not go below $y = 40$).

[3 marks]

Examiners report

The majority of candidates were able to sketch the shape of the graph accurately, but graph sketching is an area of the syllabus in which candidates continue to lose marks. In this particular question, candidates often did not consider the given domain or failed to accurately show the behaviour of the graph close to the horizontal asymptote as $x \rightarrow \infty$.

- 24b. Robin and Pat are planning a wedding banquet. The cost per guest, G dollars, is modelled by the function $G(n) = 95e^{(-0.02n)} + 40$, for $20 \leq n \leq 200$, where n is the number of guests.

[3 marks]

Calculate the **total** cost for 45 guests.

Markscheme

attempt to find $G(45)$ **(M1)**

eg 78.6241, value read from **their** graph

multiplying cost times number of people **(M1)**

eg 45×78.6241 , $G(45) \times 45$

3538.08

3540 (dollars) **A1 N2**

[3 marks]

Total [6 marks]

Examiners report

In (b), most candidates were able to identify the initial approach by finding $G(45)$, but missed the fact that function defined the cost per guest and not the total cost.

