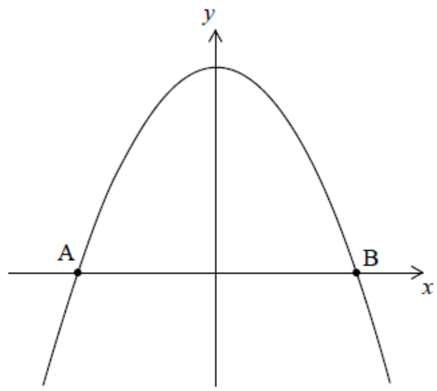
6-5-P1_Calculus-volumes *[189 marks]*

Let
$f(x) = 5 - x^2$. Part of the graph of
$f$is shown in the following diagram.



The graph crosses the
x-axis at the points
A and
$\mathrm{B}.$

1a. Find the
x-coordinate of
A and of
$\mathrm{B}.$ *[3 marks]*

Markscheme

recognizing
$f(x) = 0$ ***(M1)***

eg
$f = 0,\ x^2 = 5$

$x = \pm 2.23606$

$x = \pm\sqrt{5}\ (\text{exact}),\ x = \pm 2.24$ ***A1A1*** ***N3***

[3 marks]

1b. The region enclosed by the graph of
f and the
x-axis is revolved
360° about the
x-axis. *[3 marks]*

Find the volume of the solid formed.

Markscheme

attempt to substitute either limits or the function into formula

involving
f^2 ***(M1)***

eg
$\pi\int\left(5-x^2\right)^2\mathrm{d}x,\ \pi\int_{-2.24}^{2.24}\left(x^4-10x^2+25\right),\ 2\pi\int_0^{\sqrt{5}}f^2$

187.328

volume
$=187$ ***A2*** ***N3***

[3 marks]

Let
$f(x)=(x-1)(x-4)$.

2a. Find the
x-intercepts of the graph of
f. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg
$f(x)=0$, sketch of parabola showing two
x-intercepts

$x=1,\ x=4\ (\text{accept } (1,0),\ (4,0))$ ***A1A1*** ***N3***

[3 marks]

2b. The region enclosed by the graph of
f and the
x-axis is rotated
$360°$ about the
x-axis. *[3 marks]*

Find the volume of the solid formed.

Markscheme

attempt to substitute either limits or the function into formula involving
f^2 ***(M1)***

eg
$\int_1^4(f(x))^2\mathrm{d}x,\ \pi\int((x-1)(x-4))^2$

$\text{volume}=8.1\pi\ (\text{exact}),\ 25.4$ ***A2*** ***N3***

[3 marks]

3a. Find
$\int_4^{10}(x-4)\mathrm{d}x$. *[4 marks]*

Markscheme

correct integration ***A1A1***

e.g.
$\frac{x^2}{2} - 4x$,
$\left[\frac{x^2}{2} - 4x\right]_4^{10}$,
$\frac{(x-4)^2}{2}$

Notes: In the first 2 examples, award ***A1*** for each correct term.

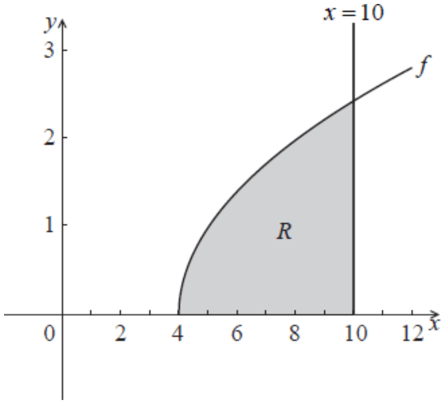
In the third example, award ***A1*** for
$\frac{1}{2}$ and ***A1*** for
$(x-4)^2$.

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

e.g.
$\left(\frac{10^2}{2} - 4(10)\right) - \left(\frac{4^2}{2} - 4(4)\right), 10 - (-8), \frac{1}{2}(6^2 - 0)$

$\int_4^{10}(x-4)\mathrm{d}x = 18$ ***A1*** ***N2***

3b. Part of the graph of $f(x) = \sqrt{x-4}$, for $x \geq 4$, is shown below. The shaded region *R* is enclosed by the graph of f, the line $x = 10$, and the *x*-axis. *[3 marks]*



The region *R* is rotated $360°$ about the *x*-axis. Find the volume of the solid formed.

Markscheme

attempt to substitute either limits or the function into volume formula ***(M1)***

e.g.
$\pi\int_4^{10} f^2\mathrm{d}x, \int_a^b\left(\sqrt{x-4}\right)^2, \pi\int_4^{10}\sqrt{x-4}$

Note: Do not penalise for missing π or d*x*.

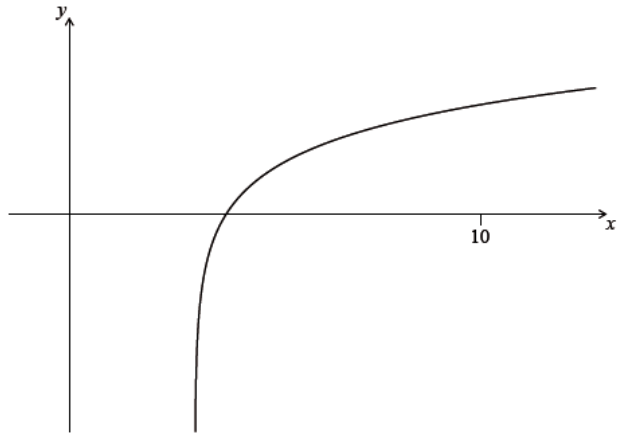
correct substitution (accept absence of d*x* and π) ***(A1)***

e.g.
$\pi\int_4^{10}\left(\sqrt{x-4}\right)^2, \pi\int_4^{10}(x-4)\mathrm{d}x, \int_4^{10}(x-4)\mathrm{d}x$

volume = 18π ***A1*** ***N2***

[3 marks]

Let $f(x) = 2\ln(x-3)$, for $x > 3$. The following diagram shows part of the graph of f.



4a. Find the equation of the vertical asymptote to the graph of f. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg horizontal translation 3 units to the right

$x = 3$ (must be an equation) ***A1 N2***

[2 marks]

4b. Find the x-intercept of the graph of f. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $f(x) = 0,\ e^0 = x - 3$

$4,\ x = 4,\ (4,\ 0)$ ***A1 N2***

[2 marks]

4c. The region enclosed by the graph of f, the x-axis and the line $x = 10$ is rotated $360°$ about the x-axis. Find the volume of the solid formed. *[3 marks]*

Markscheme

attempt to substitute either **their correct** limits or the function into formula involving f^2 ***(M1)***

eg $\int_4^{10} f^2,\ \pi\int(2\ln(x-3))^2\mathrm{d}x$

141.537

volume = 142 ***A2 N3***

[3 marks]

Total [7 marks]

Let
$f(x) = x^2$.

5a. Find
$\int_1^2 (f(x))^2\mathrm{d}x$. *[4 marks]*

Markscheme

substituting for
$(f(x))^2$ (may be seen in integral) ***A1***

eg
$\left(x^2\right)^2$, x^4

correct integration,
$\int x^4 \mathrm{d}x = \frac{1}{5}x^5$ ***(A1)***

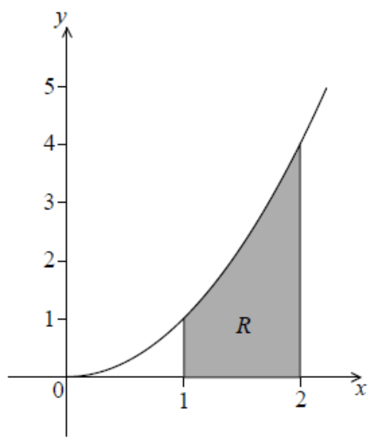
substituting limits into **their integrated** function and subtracting (in any order) ***(M1)***

eg
$\frac{2^5}{5} - \frac{1}{5}$, $\frac{1}{5}(1-4)$

$\int_1^2 (f(x))^2 \mathrm{d}x = \frac{31}{5} (= 6.2)$ ***A1*** ***N2***

[4 marks]

5b. The following diagram shows part of the graph of
f. *[2 marks]*



The shaded region
R is enclosed by the graph of
f, the
x-axis and the lines
$x = 1$ and
$x = 2$.

Find the volume of the solid formed when
R is revolved
360° about the
x-axis.

Markscheme

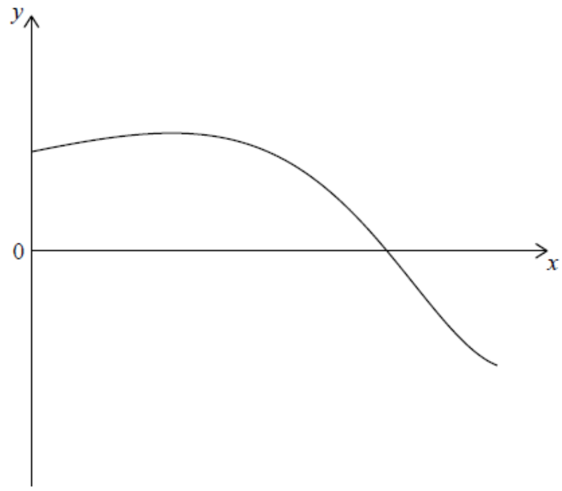
attempt to substitute limits or function into formula involving
f^2 ***(M1)***

eg
$\int_1^2 (f(x))^2 \mathrm{d}x$, $\pi \int x^4 \mathrm{d}x$

$\frac{31}{5}\pi \; (= 6.2\pi)$ ***A1*** ***N2***

[2 marks]

Let $f(x) = \sin(e^x)$ for $0 \le x \le 1.5$. The following diagram shows the graph of f.



6a. Find the *x*-intercept of the graph of f. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $f(x) = 0,\ \ e^x = 180$ or $0\ldots$

1.14472

$x = \ln \pi$ (exact), 1.14 ***A1 N2***

[2 marks]

6b. The region enclosed by the graph of f, the *y*-axis and the *x*-axis is rotated 360° about the *x*-axis. *[3 marks]*

Find the volume of the solid formed.

Markscheme

attempt to substitute either their **limits** or the function into formula involving f^2. ***(M1)***

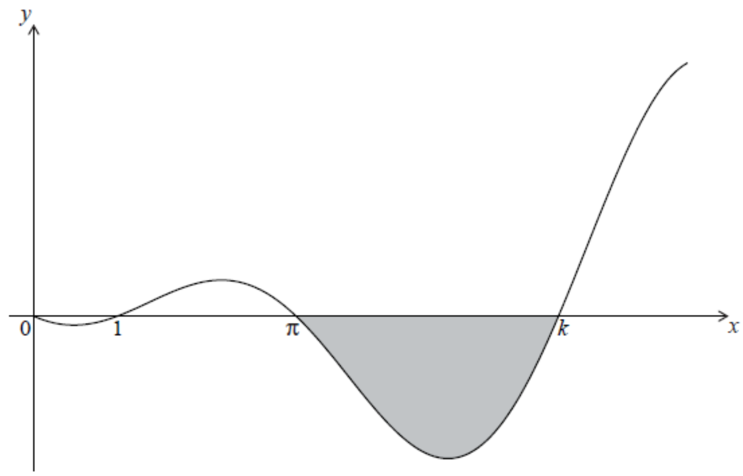
eg $\int_0^{1.14} f^2,\ \pi\int(\sin(e^x))^2 dx,\ 0.795135$

2.49799

volume = 2.50 ***A2 N3***

[3 marks]

The graph of $y = (x-1)\sin x$, for $0 \le x \le \frac{5\pi}{2}$, is shown below.



The graph has x-intercepts at 0, 1, π and k.

7a. Find k. *[2 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. $y = 0$, $\sin x = 0$

$2\pi = 6.283185\ldots$

$k = 6.28$ ***A1 N2***

[2 marks]

7b. The shaded region is rotated $360°$ about the *x*-axis. Let *V* be the volume of the solid formed. *[3 marks]*

Write down an expression for *V*.

Markscheme

attempt to substitute either limits or the function into formula ***(M1)***

(accept absence of $\mathrm{d}x$)

e.g. $V = \pi \int_\pi^k (f(x))^2 \mathrm{d}x$, $\pi \int ((x-1)\sin x)^2$, $\pi \int_\pi^{6.28\ldots} y^2 \mathrm{d}x$

correct expression ***A2 N3***

e.g. $\pi \int_\pi^{6.28} (x-1)^2 \sin^2 x \mathrm{d}x$, $\pi \int_\pi^{2\pi} ((x-1)\sin x)^2 \mathrm{d}x$

[3 marks]

7c. The shaded region is rotated 360° about the *x*-axis. Let *V* be the volume of the solid formed. *[2 marks]*

Find *V* .

Markscheme

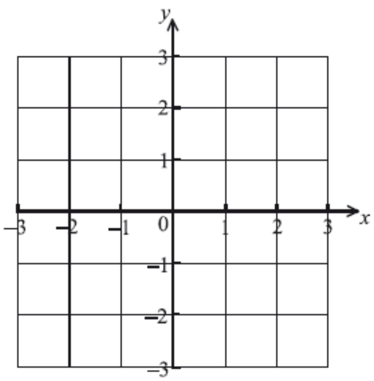
$V = 69.60192562\ldots$

$V = 69.6$ ***A2*** ***N2***

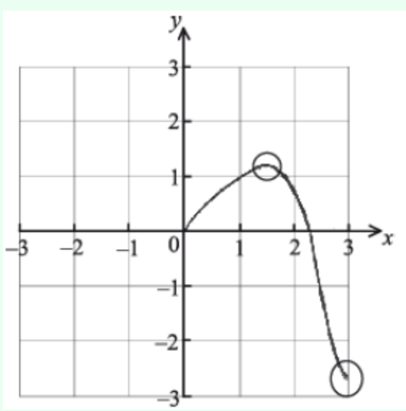
[2 marks]

Let $f(x) = x\cos(x - \sin x)$, $0 \le x \le 3$.

8a. Sketch the graph of *f* on the following set of axes. *[3 marks]*



Markscheme



A1A2 ***N3***

Notes: Award ***A1*** for correct domain, $0 \le x \le 3$. Award ***A2*** for approximately correct shape, with local maximum in circle 1 and right endpoint in circle 2.

[3 marks]

8b. The graph of *f* intersects the *x*-axis when $x = a$, $a \ne 0$. Write down the value of *a*. *[1 mark]*

Markscheme

$a = 2.31$ ***A1 N1***

[1 mark]

8c. The graph of *f* is revolved $360°$ about the *x*-axis from $x = 0$ to $x = a$. Find the volume of the solid formed. *[4 marks]*

Markscheme

evidence of using $V = \pi \int [f(x)]^2 \mathrm{d}x$ ***(M1)***

fully correct integral expression ***A2***

e.g. $V = \pi \int_0^{2.31} [x \cos(x - \sin x)]^2 \mathrm{d}x$, $V = \pi \int_0^{2.31} [f(x)]^2 \mathrm{d}x$ ***A1 N2***

$V = 5.90$

[4 marks]

Let $f(x) = -x^4 + 2x^3 - 1$, for $0 \leq x \leq 2$.

9a. Sketch the graph of f on the following grid. *[3 marks]*

Markscheme

A1A1A1 N3

Note: Award ***A1*** for both endpoints in circles,

A1 for approximately correct shape (concave up to concave down).

Only if this ***A1*** for shape is awarded, award ***A1*** for maximum point in circle.

9b. Solve $f(x) = 0$. *[2 marks]*

Markscheme

$x = 1 \quad x = 1.83928$

$x = 1 \text{ (exact)} \quad x = 1.84\ [1.83,\ 1.84]$ ***A1A1 N2***

[2 marks]

9c. The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. *[3 marks]*

Find the volume of the solid formed.

Markscheme

attempt to substitute either (***FT***) limits or function into formula with f^2 ***(M1)***

eg $V = \pi \int_1^{1.84} f^2,\ \int (-x^4 + 2x^3 - 1)^2 \mathrm{d}x$

0.636581

$V = 0.637\ [0.636,\ 0.637]$ ***A2 N3***

[3 marks]

Total [8 marks]

Let $f(x) = \frac{1}{\sqrt{2x-1}}$, for $x > \frac{1}{2}$.

10a. Find $\int (f(x))^2 \mathrm{d}x$. *[3 marks]*

Markscheme

correct working ***(A1)***

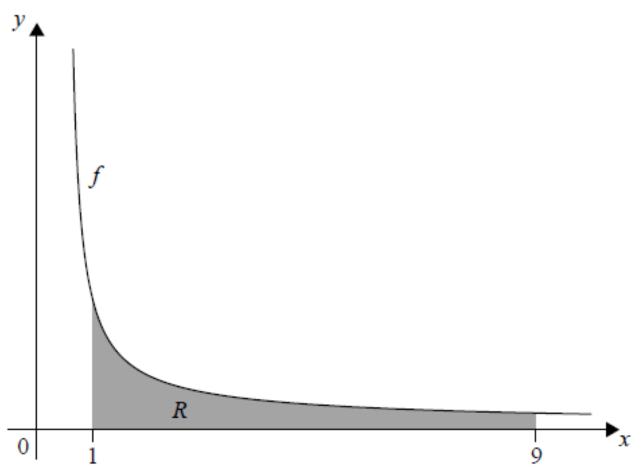
eg $\int \frac{1}{2x-1}\mathrm{d}x,\ \int (2x-1)^{-1},\ \frac{1}{2x-1},\ \int \left(\frac{1}{\sqrt{u}}\right)^2 \frac{\mathrm{d}u}{2}$

$\int (f(x))^2 \mathrm{d}x = \frac{1}{2}\ln(2x-1) + c$ ***A2 N3***

Note: Award ***A1*** for $\frac{1}{2}\ln(2x-1)$.

[3 marks]

10b. Part of the graph of *f* is shown in the following diagram. *[4 marks]*



The shaded region *R* is enclosed by the graph of *f*, the *x*-axis, and the lines *x* = 1 and *x* = 9 . Find the volume of the solid formed when *R* is revolved 360° about the *x*-axis.

Markscheme

attempt to substitute either limits or the function into formula involving f^2 (accept absence of π / dx) ***(M1)***

eg $\int_1^9 y^2 \mathrm{d}x, \pi\int\left(\frac{1}{\sqrt{2x-1}}\right)^2 \mathrm{d}x, \left[\frac{1}{2}\ln(2x-1)\right]_1^9$

substituting limits into **their** integral and subtracting (in any order) ***(M1)***

eg $\frac{\pi}{2}(\ln(17)-\ln(1)), \pi\left(0-\frac{1}{2}\ln(2\times9-1)\right)$

correct working involving calculating a log value or using log law ***(A1)***

eg $\ln(1)=0, \ln\left(\frac{17}{1}\right)$

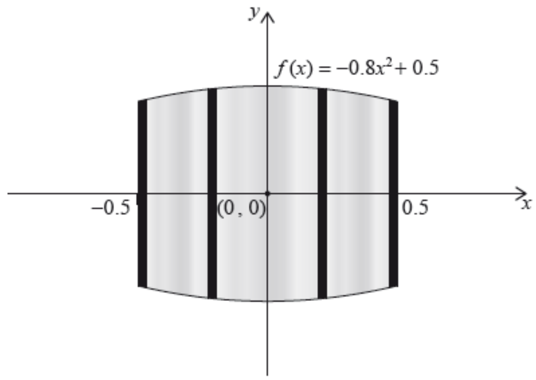
$\frac{\pi}{2}\ln 17 \quad \left(\text{accept } \pi\ln\sqrt{17}\right)$ ***A1 N3***

Note: Full ***FT*** may be awarded as normal, from their incorrect answer in part (a), however, do not award the final two ***A*** marks unless they involve logarithms.

[4 marks]

All lengths in this question are in metres.

Let $f(x)=-0.8x^2+0.5$, for $-0.5\leqslant x\leqslant 0.5$. Mark uses $f(x)$ as a model to create a barrel. The region enclosed by the graph of f, the x-axis, the line $x=-0.5$ and the line $x=0.5$ is rotated 360° about the x-axis. This is shown in the following diagram.



11a. Use the model to find the volume of the barrel. *[3 marks]*

Markscheme

attempt to substitute correct limits or the function into the formula involving

y^2

eg $\pi\int_{-0.5}^{0.5} y^2\mathrm{d}x, \pi\int(-0.8x^2+0.5)^2\mathrm{d}x$

0.601091

volume $=0.601\ (\mathrm{m}^3)$ ***A2 N3***

[3 marks]

11b. The empty barrel is being filled with water. The volume $V\ \mathrm{m}^3$ of water in the barrel after t minutes is given by $V=0.8(1-\mathrm{e}^{-0.1t})$. How long will it take for the barrel to be half-full? *[3 marks]*

Markscheme

attempt to equate half **their** volume to V ***(M1)***

eg

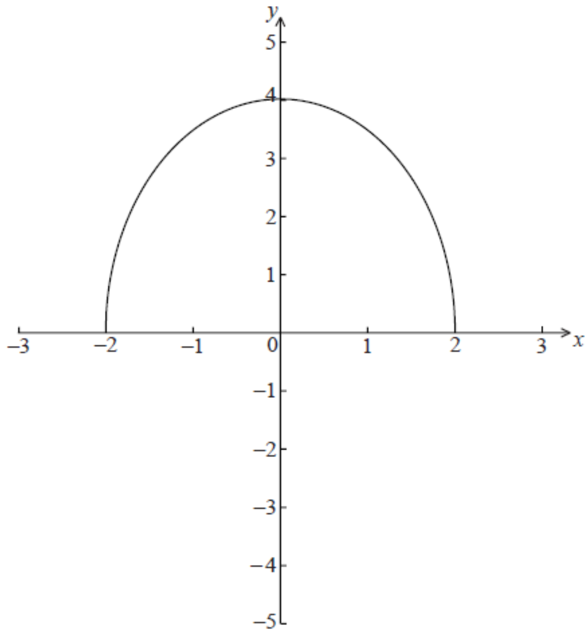
$0.30055 = 0.8(1 - \mathrm{e}^{-0.1t})$, graph

4.71104

4.71 (minutes) ***A2 N3***

[3 marks]

The graph of
$f(x) = \sqrt{16 - 4x^2}$, for
$-2 \le x \le 2$, is shown below.



12. The region enclosed by the curve of *f* and the *x*-axis is rotated 360° about the *x*-axis. *[6 marks]*

Find the volume of the solid formed.

Markscheme

attempt to set up integral expression ***M1***

e.g.
$\pi\int\sqrt{16-4x^2}^2\mathrm{d}x$,
$2\pi\int_0^2(16-4x^2)$,
$\int\sqrt{16-4x^2}^2\mathrm{d}x$

$\int 16\mathrm{d}x=16x$,
$\int 4x^2\mathrm{d}x=\frac{4x^3}{3}$ (seen anywhere) ***A1A1***

evidence of substituting limits into the integrand ***(M1)***

e.g.
$\left(32-\frac{32}{3}\right)-\left(-32+\frac{32}{3}\right)$,
$64-\frac{64}{3}$

volume
$=\frac{128\pi}{3}$ ***A2*** ***N3***

[6 marks]

13. The graph of $y=\sqrt{x}$ between $x=0$ and $x=a$ is rotated 360° about the *x*-axis. The volume of the solid formed is 32π . Find the value of *a*. *[7 marks]*

Markscheme

attempt to substitute into formula
$V=\int\pi y^2\mathrm{d}x$ ***(M1)***

integral expression ***A1***

e.g.
$\pi\int_0^a(\sqrt{x})^2\mathrm{d}x$,
$\pi\int x$

correct integration ***(A1)***

e.g.
$\int x\mathrm{d}x=\frac{1}{2}x^2$

correct substitution
$V=\pi\left[\frac{1}{2}a^2\right]$ ***(A1)***

equating **their** expression to
32π ***M1***

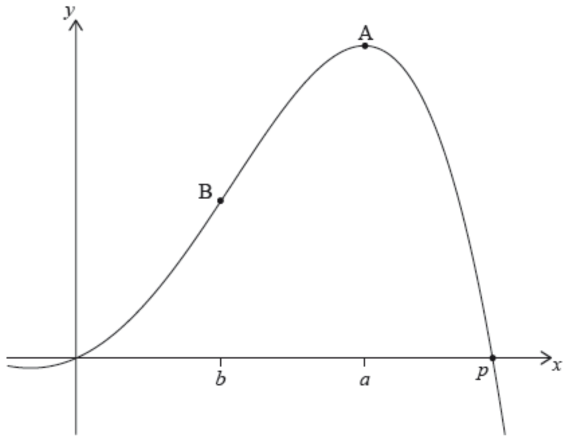
e.g.
$\pi\left[\frac{1}{2}a^2\right]=32\pi$

$a^2=64$

$a=8$ ***A2*** ***N2***

[7 marks]

Let $f(x) = -0.5x^4 + 3x^2 + 2x$. The following diagram shows part of the graph of f.



There are x-intercepts at $x = 0$ and at $x = p$. There is a maximum at A where $x = a$, and a point of inflexion at B where $x = b$.

14a. Find the value of p. *[2 marks]*

Markscheme

evidence of valid approach ***(M1)***

eg $f(x) = 0,\ y = 0$

2.73205

$p = 2.73$ ***A1 N2***

[2 marks]

14b. Write down the coordinates of A. *[2 marks]*

Markscheme

1.87938, 8.11721

$(1.88,\ 8.12)$ ***A2 N2***

[2 marks]

14c. Write down the rate of change of f at A. *[1 mark]*

Markscheme

rate of change is 0 (do not accept decimals) ***A1 N1***

[1 marks]

14d. Find the coordinates of B. *[4 marks]*

Markscheme

METHOD 1 (using GDC)

valid approach ***M1***

eg $f'' = 0$, max/min on f', $x = -1$

sketch of either f' or f'', with max/min or root (respectively) ***(A1)***

$x = 1$ ***A1*** ***N1***

Substituting **their** x value into f ***(M1)***

eg $f(1)$

$y = 4.5$ ***A1*** ***N1***

METHOD 2 (analytical)

$f'' = -6x^2 + 6$ ***A1***

setting $f'' = 0$ ***(M1)***

$x = 1$ ***A1*** ***N1***

substituting **their** x value into f ***(M1)***

eg $f(1)$

$y = 4.5$ ***A1*** ***N1***

[4 marks]

14e. Find the the rate of change of f at B. *[3 marks]*

Markscheme

recognizing rate of change is f' ***(M1)***

eg y', $f'(1)$

rate of change is 6 ***A1*** ***N2***

[3 marks]

14f. Let R be the region enclosed by the graph of f , the x-axis, the line $x = b$ and the line $x = a$. The region R is rotated 360° about the x-axis. Find the volume of the solid formed. *[3 marks]*

Markscheme

attempt to substitute either limits or the function into formula ***(M1)***

involving f^2 (accept absence of π and/or $\mathrm{d}x$)

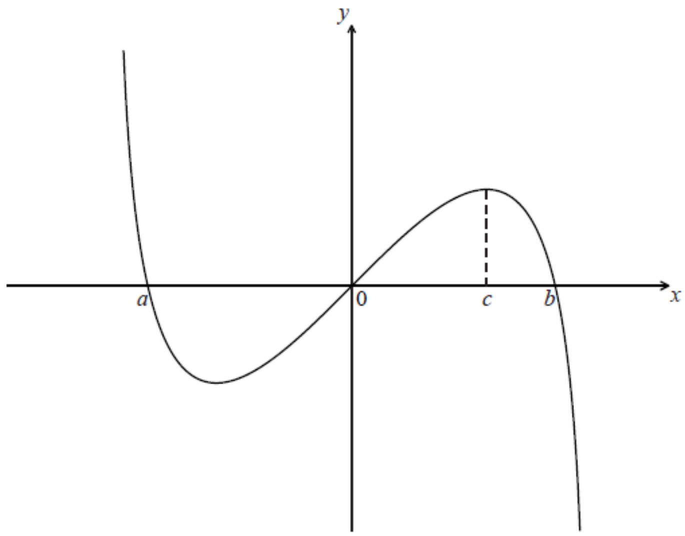
eg $\pi \int \left(-0.5x^4 + 3x^2 + 2x\right)^2 \mathrm{d}x$, $\int_1^{1.88} f^2$

128.890

$\text{volume} = 129$ ***A2*** ***N3***

[3 marks]

Let
$f(x) = x\ln(4 - x^2)$, for
$-2 < x < 2$. The graph of *f* is shown below.



The graph of *f* crosses the *x*-axis at
$x = a$,
$x = 0$ and
$x = b$.

15a. Find the value of *a* and of *b* . *[3 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g.
$f(x) = 0$, graph

$a = -1.73$,
$b = 1.73$
$(a = -\sqrt{3}, b = \sqrt{3})$ ***A1A1 N3***

[3 marks]

15b. The graph of *f* has a maximum value when
$x = c$. *[2 marks]*

Find the value of *c* .

Markscheme

attempt to find max ***(M1)***

e.g. setting
$f'(x) = 0$, graph

$c = 1.15$ (accept (1.15, 1.13)) ***A1 N2***

[2 marks]

15c. The region under the graph of *f* from
$x = 0$ to
$x = c$ is rotated
360° about the *x*-axis. Find the volume of the solid formed. *[3 marks]*

Markscheme

attempt to substitute either limits or the function into formula ***M1***

e.g.
$V=\pi\int_0^c[f(x)]^2\mathrm{d}x$,
$\pi\int\left[x\ln(4-x^2)\right]^2$,
$\pi\int_0^{1.149\ldots}y^2\mathrm{d}x$

$V=2.16$ ***A2*** ***N2***

[3 marks]

15d. Let *R* be the region enclosed by the curve, the *x*-axis and the line $x=c$, between $x=a$ and $x=c$.

[4 marks]

Find the area of *R* .

Markscheme

valid approach recognizing 2 regions ***(M1)***

e.g. finding 2 areas

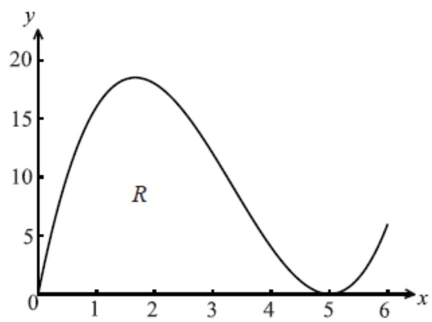
correct working ***(A1)***

e.g.
$\int_0^{-1.73\ldots}f(x)\mathrm{d}x+\int_0^{1.149\ldots}f(x)\mathrm{d}x$,
$-\int_{-1.73\ldots}^0 f(x)\mathrm{d}x+\int_0^{1.149\ldots}f(x)\mathrm{d}x$

area $=2.07$ (accept 2.06) ***A2*** ***N3***

[4 marks]

Let $f(x)=x(x-5)^2$, for $0\le x\le 6$. The following diagram shows the graph of *f* .



Let R be the region enclosed by the *x*-axis and the curve of *f* .

16a. Find the area of *R*.

[3 marks]

Markscheme

finding the limits
$x = 0$,
$x = 5$ ***(A1)***

integral expression ***A1***

e.g.
$\int_0^5 f(x)\mathrm{d}x$

area = 52.1 ***A1*** ***N2***

[3 marks]

16b. Find the volume of the solid formed when *R* is rotated through 360° about the *x*-axis. *[4 marks]*

Markscheme

evidence of using formula
$v = \int \pi y^2 \mathrm{d}x$ ***(M1)***

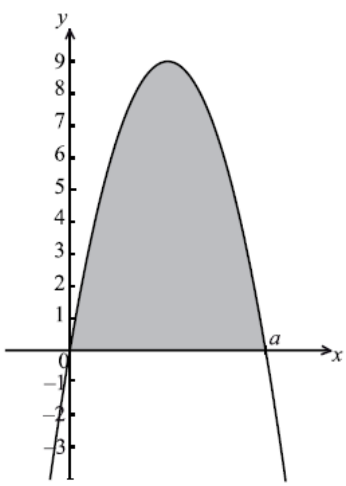
correct expression ***A1***

e.g. volume
$= \pi \int_0^5 x^2(x-5)^4 \mathrm{d}x$

volume = 2340 ***A2*** ***N2***

[4 marks]

16c. The diagram below shows a part of the graph of a quadratic function $g(x) = x(a-x)$. The graph of *g* crosses the *x*-axis when $x = a$. *[7 marks]*



The area of the shaded region is equal to the area of *R*. Find the value of *a*.

Markscheme

area is
$\int_0^a x(a-x)\mathrm{d}x$ ***A1***

$= \left[\frac{ax^2}{2} - \frac{x^3}{3}\right]_0^a$ ***A1A1***

substituting limits ***(M1)***

e.g.
$\frac{a^3}{2} - \frac{a^3}{3}$

setting expression equal to area of *R* ***(M1)***

correct equation ***A1***

e.g.
$\frac{a^3}{2} - \frac{a^3}{3} = 52.1$,
$a^3 = 6 \times 52.1$

$a = 6.79$ ***A1*** ***N3***

[7 marks]

Let
$f : x \mapsto \sin^3 x$.

17a. (i) Write down the range of the function *f* . *[5 marks]*

(ii) Consider
$f(x) = 1$,
$0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

Markscheme

(i) range of *f* is
$[-1, 1]$,
$(-1 \le f(x) \le 1)$ ***A2*** ***N2***

(ii)
$\sin^3 x \Rightarrow 1 \Rightarrow \sin x = 1$ ***A1***

justification for one solution on
$[0, 2\pi]$ ***R1***

e.g.
$x = \frac{\pi}{2}$, unit circle, sketch of
$\sin x$

1 solution (seen anywhere) ***A1*** ***N1***

[5 marks]

17b. Find
$f'(x)$, giving your answer in the form
$a\sin^p x\cos^q x$ where
$a, p, q \in \mathbb{Z}$. *[2 marks]*

Markscheme

$f'(x) = 3\sin^2 x \cos x$ ***A2*** ***N2***

[2 marks]

17c. Let $g(x) = \sqrt{3}\sin x(\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of *g* is revolved through 2π about the *x*-axis. *[7 marks]*

Markscheme

using $V = \int_a^b \pi y^2 \mathrm{d}x$ ***(M1)***

$V = \int_0^{\frac{\pi}{2}} \pi(\sqrt{3}\sin x \cos^{\frac{1}{2}} x)^2 \mathrm{d}x$ ***(A1)***

$= \pi \int_0^{\frac{\pi}{2}} 3\sin^2 x \cos x \mathrm{d}x$ ***A1***

$V = \pi\left[\sin^3 x\right]_0^{\frac{\pi}{2}}$
$\left(= \pi\left(\sin^3\left(\frac{\pi}{2}\right) - \sin^3 0\right)\right)$ ***A2***

evidence of using $\sin\frac{\pi}{2} = 1$ and $\sin 0 = 0$ ***(A1)***

e.g. $\pi(1-0)$

$V = \pi$ ***A1 N1***

[7 marks]

Let $h(x) = \frac{2x-1}{x+1}$, $x \ne -1$.

18a. Find $h^{-1}(x)$. *[4 marks]*

Markscheme

$y = \frac{2x-1}{x+1}$

interchanging *x* and *y* (seen anywhere) ***M1***

e.g. $x = \frac{2y-1}{y+1}$

correct working ***A1***

e.g. $xy + x = 2y - 1$

collecting terms ***A1***

e.g. $x + 1 = 2y - xy$, $x + 1 = y(2-x)$

$h^{-1}(x) = \frac{x+1}{2-x}$ ***A1 N2***

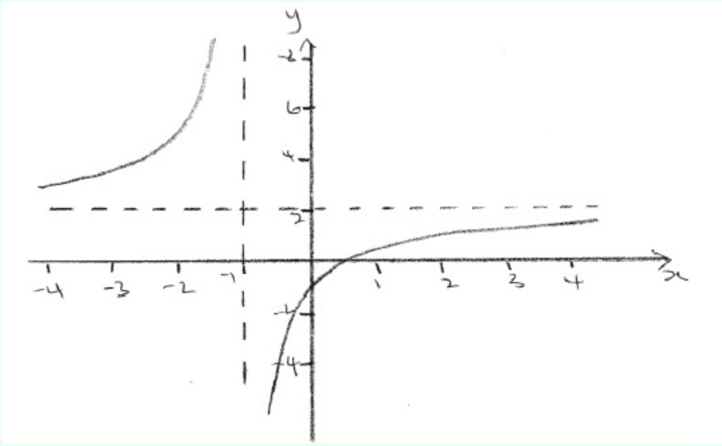
[4 marks]

18b. (i) Sketch the graph of *h* for $-4 \le x \le 4$ and $-5 \le y \le 8$, including any asymptotes. *[7 marks]*

(ii) Write down the equations of the asymptotes.

(iii) Write down the *x*-intercept of the graph of *h* .

Markscheme



A1A1A1A1 N4

Note: Award ***A1*** for approximately correct intercepts, ***A1*** for correct shape, ***A1*** for asymptotes, ***A1*** for approximately correct domain and range.

(ii)
$x = -1$,
$y = 2$ ***A1A1 N2***

(iii)
$\frac{1}{2}$ ***A1 N1***

[7 marks]

18c. Let *R* be the region in the first quadrant enclosed by the graph of h , the *x*-axis and the line $x = 3$. *[5 marks]*

(i) Find the area of *R*.

(ii) Write down an expression for the volume obtained when *R* is revolved through $360°$ about the *x*-axis.

Markscheme

(i)
$\text{area} = 2.06$ ***A2 N2***

(ii) attempt to substitute into volume formula (do not accept $\pi\int_a^b y^2 \mathrm{d}x$) ***M1***

volume
$= \pi\int_{\frac{1}{2}}^{3}\left(\frac{2x-1}{x+1}\right)^2 \mathrm{d}x$ ***A2 N3***

[5 marks]

Let
$f(x) = \sqrt{x}$. Line *L* is the normal to the graph of *f* at the point (4, 2) .

19a. Show that the equation of *L* is $y = -4x + 18$. *[4 marks]*

Markscheme

finding derivative *(A1)*

e.g.
$f'(x) = \frac{1}{2}x^{\frac{1}{2}}, \frac{1}{2\sqrt{x}}$

correct value of derivative or its negative reciprocal (seen anywhere) ***A1***

e.g.
$\frac{1}{2\sqrt{4}}$,
$\frac{1}{4}$

gradient of normal =
$\frac{1}{\text{gradient of tangent}}$ (seen anywhere) ***A1***

e.g.
$-\frac{1}{f'(4)} = -4$,
$-2\sqrt{x}$

substituting into equation of line (for normal) ***M1***

e.g.
$y - 2 = -4(x - 4)$

$y = -4x + 18$ ***AG N0***

[4 marks]

19b. Point A is the *x*-intercept of L . Find the *x*-coordinate of A. *[2 marks]*

Markscheme

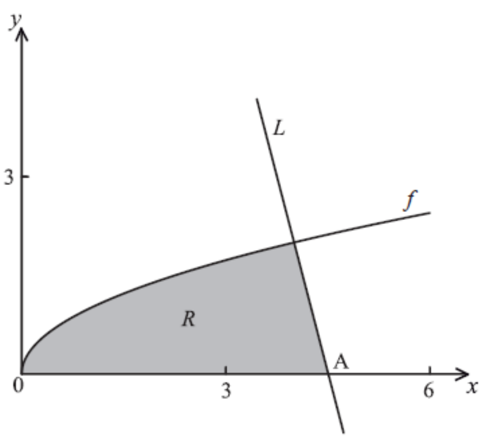
recognition that
$y = 0$ at A ***(M1)***

e.g.
$-4x + 18 = 0$

$x = \frac{18}{4}$
$\left(= \frac{9}{2}\right)$ ***A1 N2***

[2 marks]

In the diagram below, the shaded region R is bounded by the *x*-axis, the graph of *f* and the line L .



19c. Find an expression for the area of R . *[3 marks]*

Markscheme

splitting into two appropriate parts (areas and/or integrals) ***(M1)***

correct expression for area of R ***A2*** ***N3***

e.g. area of R =
$\int_0^4 \sqrt{x}\mathrm{d}x + \int_4^{4.5}(-4x+18)\mathrm{d}x$,
$\int_0^4 \sqrt{x}\mathrm{d}x + \frac{1}{2}\times 0.5 \times 2$ (triangle)

Note: Award ***A1*** if d*x* is missing.

[3 marks]

19d. The region R is rotated 360° about the *x*-axis. Find the volume of the solid formed, giving your answer in terms of π . *[8 marks]*

Markscheme

correct expression for the volume from
$x = 0$ to
$x = 4$ ***(A1)***

e.g.
$V = \int_0^4 \pi\left[f(x)^2\right]\mathrm{d}x$,
$\int_0^4 \pi\sqrt{x}^2\mathrm{d}x$,
$\int_0^4 \pi x\mathrm{d}x$

$V = \left[\frac{1}{2}\pi x^2\right]_0^4$ ***A1***

$V = \pi\left(\frac{1}{2}\times 16 - \frac{1}{2}\times 0\right)$ ***(A1)***

$V = 8\pi$ ***A1***

finding the volume from
$x = 4$ to
$x = 4.5$

EITHER

recognizing a cone ***(M1)***

e.g.
$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(2)^2 \times \frac{1}{2}$ ***(A1)***

$= \frac{2\pi}{3}$ ***A1***

total volume is
$8\pi + \frac{2}{3}\pi$
$\left(= \frac{26}{3}\pi\right)$ ***A1*** ***N4***

OR

$V = \pi\int_4^{4.5}(-4x+18)^2\mathrm{d}x$ ***(M1)***

$= \int_4^{4.5}\pi(16x^2 - 144x + 324)\mathrm{d}x$

$= \pi\left[\frac{16}{3}x^3 - 72x^2 + 324x\right]_4^{4.5}$ ***A1***

$= \frac{2\pi}{3}$ ***A1***

total volume is
$8\pi + \frac{2}{3}\pi$
$\left(= \frac{26}{3}\pi\right)$ ***A1*** ***N4***

[8 marks]

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$\mathrm{P}(A=a)$	$\cos\theta$	$2\cos 2\theta$

20a. Show that $\cos\theta = \frac{3}{4}$. *[6 marks]*

Markscheme

evidence of summing to 1 ***(M1)***

eg $\sum p = 1$

correct equation ***A1***

eg $\cos\theta + 2\cos 2\theta = 1$

correct equation in $\cos\theta$ ***A1***

eg $\cos\theta + 2(2\cos^2\theta - 1) = 1,\ 4\cos^2\theta + \cos\theta - 3 = 0$

evidence of valid approach to solve quadratic ***(M1)***

eg factorizing equation set equal to $0,\ \frac{-1\pm\sqrt{1-4\times4\times(-3)}}{8}$

correct working, clearly leading to required answer ***A1***

eg $(4\cos\theta - 3)(\cos\theta + 1),\ \frac{-1\pm7}{8}$

correct reason for rejecting $\cos\theta \neq -1$ ***R1***

eg $\cos\theta$ is a probability (value must lie between 0 and 1), $\cos\theta > 0$

Note: Award ***R0*** for $\cos\theta \neq -1$ without a reason.

$\cos\theta = \frac{3}{4}$ ***AG N0***

20b. Given that $\tan\theta > 0$, find $\tan\theta$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg sketch of right triangle with sides 3 and 4, $\sin^2 x + \cos^2 x = 1$

correct working

(A1)

eg missing side $= \sqrt{7},\ \frac{\frac{\sqrt{7}}{4}}{\frac{3}{4}}$

$\tan\theta = \frac{\sqrt{7}}{3}$ ***A1 N2***

[3 marks]

20c. Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of $y$between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. *[6 marks]*

Markscheme

attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\pi\int_{\theta}^{\frac{\pi}{4}} f^2,\ \int\left(\frac{1}{\cos x}\right)^2$

correct substitution of both limits and function ***(A1)***

eg $\pi\int_{\theta}^{\frac{\pi}{4}}\left(\frac{1}{\cos x}\right)^2 \mathrm{d}x$

correct integration ***(A1)***

eg $\tan x$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $\tan\frac{\pi}{4} - \tan\theta$

Note: Award ***M0*** if they substitute into original or differentiated function.

$\tan\frac{\pi}{4} = 1$ ***(A1)***

eg $1 - \tan\theta$

$V = \pi - \frac{\pi\sqrt{7}}{3}$ ***A1 N3***

[6 marks]