

29 November 2017

Exam: Vector algebra and differential calculus (cumulative review)

1a. Consider the points  $A(5, 2, 1)$ ,  $B(6, 5, 3)$ , and  $C(7, 6, a)$ ,  $a \in \mathbb{R}$ . Find

(i)  $\overrightarrow{AB}$ ;

[3 marks]

(ii)  $\overrightarrow{AC}$ .

1b. Find the value of  $a$  for which  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$  are perpendicular.

[4 marks]

2a. Let  $g(x) = \frac{\ln x}{x^2}$ , for  $x > 0$ .

Use the quotient rule to show that  $g'(x) = \frac{1-2\ln x}{x^3}$ .

[4 marks]

2b. The graph of  $g$  has a maximum point at A. Find the x-coordinate of A.

[3 marks]

3a. Let  $g(x) = 2x \sin x$ .

Find  $g'(x)$ .

[4 marks]

3b. Find the gradient of the graph of  $g$  at  $x = \pi$ .

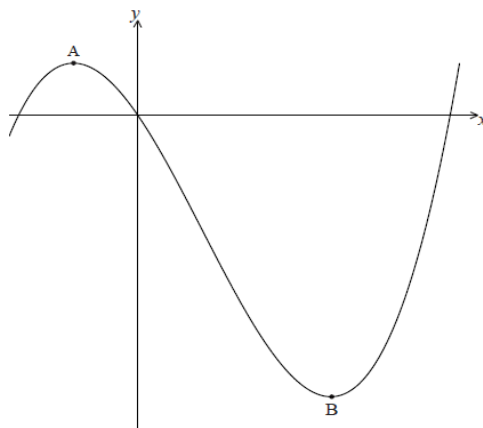
[3 marks]

4. Let  $f'(x) = -24x^3 + 9x^2 + 3x + 1$ .

[3 marks]

There are two points of inflexion on the graph of  $f$ . Write down the x-coordinates of these points.

5a. Let  $f(x) = \frac{1}{3}x^3 - x^2 - 3x$ . Part of the graph of  $f$  is shown below.



There is a maximum point at A and a minimum point at  $B(3, -9)$ .

Find the coordinates of A.

[8 marks]

5b. Write down the coordinates of

[6 marks]

(i) the image of B after reflection in the  $y$ -axis;

(ii) the image of B after translation by the vector  $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ ;

(iii) the image of B after reflection in the  $x$ -axis followed by a horizontal stretch with scale factor  $\frac{1}{2}$ .

6a. Let  $f(x) = \frac{\cos x}{\sin x}$ , for  $\sin x \neq 0$ .

Use the quotient rule to show that  $f'(x) = \frac{-1}{\sin^2 x}$ .

[5 marks]

6b. Find  $f''(x)$ .

[3 marks]

7a. In an arithmetic sequence,  $u_1 = 3$  and  $u_3 = 11$ .

Find  $d$ .

[2 marks]

7b. Find  $u_{20}$ .

[2 marks]

7c. Find  $S_{20}$ .

[2 marks]

8a. The first three terms of an infinite geometric sequence are 27, 9 and 3.

Write down the value of  $r$ .

[1 mark]

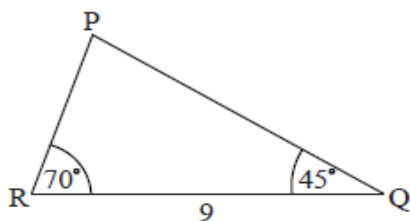
8b. Find  $u_6$ .

[2 marks]

8c. Find the sum to infinity of this sequence.

[2 marks]

9a. The following diagram shows  $\triangle PQR$ , where  $RQ = 9$  cm,  $\hat{P}RQ = 70^\circ$  and  $\hat{P}QR = 45^\circ$ .



*diagram  
not to scale*

Find  $\hat{R}PQ$ .

[1 mark]

9b. Find  $PR$ .

[3 marks]

9c. Find the area of  $\triangle PQR$ .

[2 marks]