0302HW_Review-problems *[90 marks]*

Let $f(x) = 8x + 3$ and $g(x) = 4x$, for $x \in \mathbb{R}$.

1a. Write down $g(2)$. *[1 mark]*

Markscheme

$g(2) = 8$ ***A1*** ***N1***

[1 mark]

1b. Find $(f \circ g)(x)$. *[2 marks]*

Markscheme

attempt to form composite (in any order) ***(M1)***

eg

$f(4x),\ 4 \times (8x + 3)$

$(f \circ g)(x) = 32x + 3$ ***A1*** ***N2***

[2 marks]

1c. Find $f^{-1}(x)$. *[2 marks]*

Markscheme

interchanging x and y (may be seen at any time) ***(M1)***

eg $x = 8y + 3$

$f^{-1}(x) = \frac{x-3}{8}$ $\left(\text{accept } \frac{x-3}{8},\ y = \frac{x-3}{8}\right)$ ***A1*** ***N2***

[2 marks]

In an arithmetic sequence $u_{10} = 8,\ u_{11} = 6.5$.

2a. Write down the value of the common difference. *[1 mark]*

Markscheme

$d = -1.5$ ***A1*** ***N1***

[1 mark]

2b. Find the first term. *[3 marks]*

Markscheme

METHOD 1

valid approach ***(M1)***

eg $u_{10} = u_1 + 9d,\ 8 = u_1 - 9(-1.5)$

correct working ***(A1)***

eg $8 = u_1 + 9d,\ 6.5 = u_1 + 10d,\ u_1 = 8 - 9(-1.5)$

$u_1 = 21.5$ ***A1 N2***

METHOD 2

attempt to list 3 or more terms in either direction ***(M1)***

eg $9.5,\ 11,\ 12.5,\ \ldots;\ 5,\ 3.5,\ 2,\ \ldots\ \ldots$

correct list of 4 or more terms in **correct** direction ***(A1)***

eg $9.5,\ 11,\ 12.5,\ 14$

$u_1 = 21.5$ ***A1 N2***

[3 marks]

2c. Find the sum of the first 50 terms of the sequence. *[2 marks]*

Markscheme

correct expression ***(A1)***

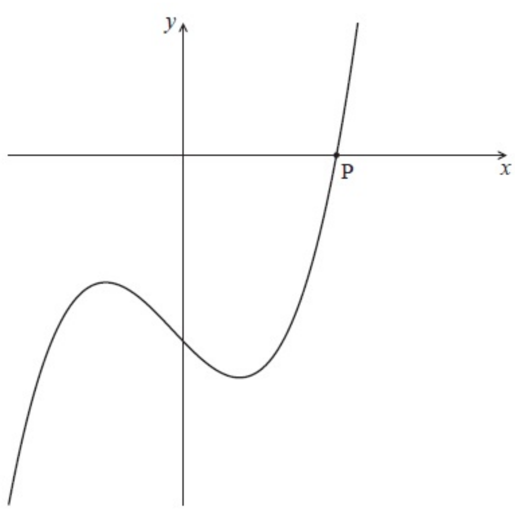
eg $\frac{50}{2}(2(21.5) + 49(-1.5))$, $\frac{50}{2}(21.5 - 52)$, $\sum_{k=1}^{50} 21.5 + (k-1)(-1.5)$

$\text{sum} = -762.5 \quad (\text{exact})$ ***A1 N2***

[2 marks]

Total [6 marks]

Let
$f(x) = x^3 - 2x - 4$. The following diagram shows part of the curve of *f*.



The curve crosses the *x*-axis at the point P.

3a. Write down the *x*-coordinate of P. *[1 mark]*

Markscheme

$x = 2$ (accept $(2, 0)$) ***A1*** ***N1***

[1 mark]

3b. Write down the gradient of the curve at P. *[2 marks]*

Markscheme

evidence of finding gradient of *f* at $x = 2$ ***(M1)***

e.g. $f'(2)$

the gradient is 10 ***A1*** ***N2***

[2 marks]

3c. Find the equation of the normal to the curve at P, giving your equation in the form $y = ax + b$. *[3 marks]*

Markscheme

evidence of negative reciprocal of gradient ***(M1)***

e.g. $\frac{-1}{f'(x)}$, $-\frac{1}{10}$

evidence of correct substitution into equation of a line ***(A1)***

e.g. $y - 0 = \frac{-1}{10}(x - 2)$, $0 = -0.1(2) + b$

$y = -\frac{1}{10}x + \frac{2}{10}$ (accept $a = -0.1$, $b = 0.2$) ***A1*** ***N2***

[3 marks]

The following figures consist of rows and columns of squares. The figures form a continuing pattern.

Figure 1 has two rows and one column. Figure 2 has three rows and two columns.

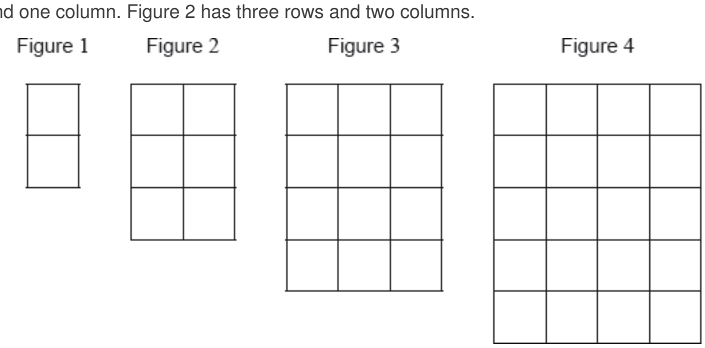


Figure 5 has p rows and q columns.

4a. Write down the value of p; *[1 mark]*

Markscheme

$p = 6$ ***A1*** ***N1***

[1 mark]

4b. Write down the value of q. *[1 mark]*

Markscheme

$q = 5$ ***A1*** ***N1***

[1 mark]

Each small square has an area of $1\ \mathrm{cm}^2$. Let A_n be the total area of Figure n. The following table gives the first five values of A_n.

n	1	2	3	4	5
$A_n\ (\mathrm{cm}^2)$	2	6	12	20	k

4c. Find the value of k. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $p \times q,\ 5 \times 6$

$k = 30$ ***A1*** ***N2***

[2 marks]

4d. Find an expression for A_n in terms of n. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg rows $= n + 1$, columns $= n$

$A(n) = n(n+1)\ (= n^2 + n)\ (\mathrm{cm}^2)$ ***A1 N2***

[2 marks]

Consider the following frequency table.

x	Frequency
2	8
4	15
7	21
10	28
11	3

5a. Write down the mode. *[1 mark]*

Markscheme

$\text{mode} = 10$ ***A1*** ***N1***

[1 mark]

5b. Find the value of the range. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $x_{\text{max}} - x_{\text{min}}$, interval 2 to 11

$\text{range} = 9$ ***A1*** ***N2***

[2 marks]

5c. Find the mean. *[2 marks]*

Markscheme

7.14666

$\text{mean} = 7.15$ ***A2*** ***N2***

[2 marks]

5d. Find the variance. *[2 marks]*

Markscheme

recognizing that variance is $(\text{sd})^2$ ***(M1)***

eg $\text{var} = \sigma^2,\ 2.90605^2,\ 2.92562^2$

$\sigma^2 = 8.44515$

$\sigma^2 = 8.45$ ***A1*** ***N2***

[2 marks]

Let $f(x) = x^2 + x - 6$.

6a. Write down the y-intercept of the graph of f. *[1 mark]*

Markscheme

y-intercept is $-6,\ (0,\ -6),\ y = -6$ ***A1***

[1 mark]

6b. Solve $f(x) = 0$. *[3 marks]*

Markscheme

valid attempt to solve ***(M1)***

eg $(x-2)(x+3)=0,\ x=\frac{-1\pm\sqrt{1+24}}{2}$, one correct answer

$x=2,\ x=-3$ ***A1A1*** ***N3***

[3 marks]

Let $f(x)=3\sin(\pi x)$.

7a. Write down the amplitude of f. *[1 mark]*

Markscheme

amplitude is 3 ***A1*** ***N1***

Let $f(x)=3\sin(\pi x)$.

7b. Find the period of f. *[2 marks]*

Markscheme

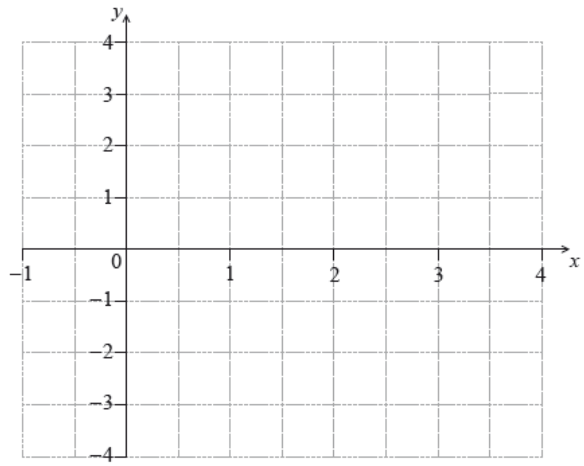
valid approach ***(M1)***

eg $\text{period}=\frac{2\pi}{\pi},\ \frac{360}{\pi}$

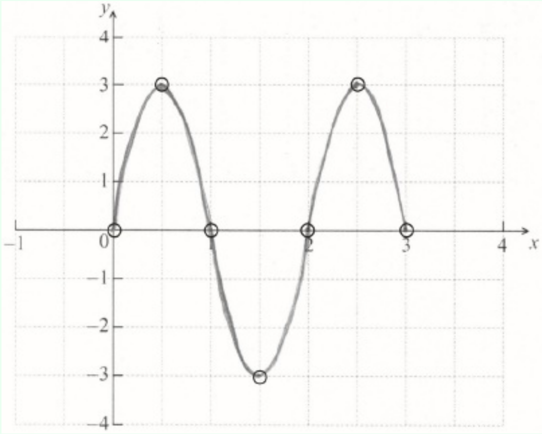
period is 2 ***A1*** ***N2***

Let $f(x)=3\sin(\pi x)$.

7c. On the following grid, sketch the graph of $y=f(x)$, for $0\leq x\leq 3$. *[4 marks]*



Markscheme



A1

A1A1A1 ***N4***

Note: Award ***A1*** for sine curve starting at $(0,\ 0)$ and correct period.

Only if this ***A1*** is awarded, award the following for points in circles:

A1 for correct *x*-intercepts;

A1 for correct max and min points;

A1 for correct domain.

Let
A and
B be independent events, where
$\mathrm{P}(A) = 0.3$ and
$\mathrm{P}(B) = 0.6.$

8a. Find
$\mathrm{P}(A \cap B).$

[2 marks]

Markscheme

correct substitution ***(A1)***

eg

0.3×0.6

$\mathrm{P}(A \cap B) = 0.18$ ***A1*** ***N2***

[2 marks]

8b. Find
$\mathrm{P}(A \cup B).$

[2 marks]

Markscheme

correct substitution ***(A1)***

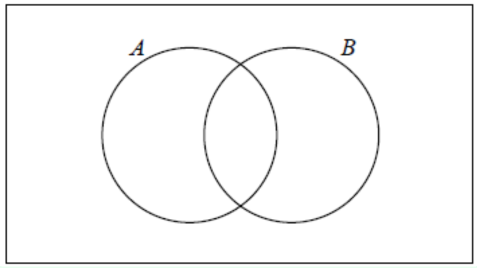
eg

$\mathrm{P}(A \cup B) = 0.3 + 0.6 - 0.18$

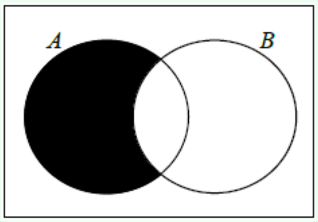
$\mathrm{P}(A \cup B) = 0.72$ ***A1*** ***N2***

[2 marks]

8c. On the following Venn diagram, shade the region that represents $A \cap B'$. *[1 mark]*



Markscheme



A1 N1

8d. Find $\mathrm{P}(A \cap B')$. *[2 marks]*

Markscheme

appropriate approach ***(M1)***

eg

$0.3 - 0.18,\ \mathrm{P}(A) \times \mathrm{P}(B')$

$\mathrm{P}(A \cap B') = 0.12$ (may be seen in Venn diagram) ***A1 N2***

[2 marks]

Let $f(x) = \mathrm{e}^{6x}$.

9a. Write down $f'(x)$. *[1 mark]*

Markscheme

$f'(x) = 6\mathrm{e}^{6x}$ ***A1 N1***

[1 mark]

9b. The tangent to the graph of *f* at the point $\mathrm{P}(0, b)$ has gradient *m* . *[4 marks]*

(i) Show that $m = 6$.

(ii) Find *b* .

Markscheme

(i) evidence of valid approach ***(M1)***

e.g.
$f'(0)$,
$6e^{6\times0}$

correct manipulation ***A1***

e.g.
$6e^0$,
6×1

$m=6$ ***AG*** ***N0***

(ii) evidence of finding
$f(0)$ ***(M1)***

e.g.
$y=e^{6(0)}$

$b=1$ ***A1*** ***N2***

[4 marks]

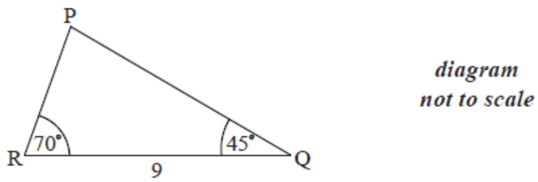
9c. Hence, write down the equation of this tangent. *[1 mark]*

Markscheme

$y=6x+1$ ***A1*** ***N1***

[1 mark]

The following diagram shows
$\Delta\mathrm{PQR}$, where RQ = 9 cm,
$\mathrm{P\hat{R}Q}=70^\circ$ and
$\mathrm{P\hat{Q}R}=45^\circ$.



10a. Find
$\mathrm{R\hat{P}Q}$. *[1 mark]*

Markscheme

$\mathrm{R\hat{P}Q}=65^\circ$ ***A1*** ***N1***

[1 mark]

10b. Find PR . *[3 marks]*

Markscheme

evidence of choosing sine rule ***(M1)***

correct substitution ***A1***

e.g.
$\frac{\mathrm{PR}}{\sin 45^\circ} = \frac{9}{\sin 65^\circ}$

7.021854078

$\mathrm{PR} = 7.02$ ***A1*** ***N2***

[3 marks]

10c. Find the area of $\Delta\mathrm{PQR}$. *[2 marks]*

Markscheme

correct substitution ***(A1)***

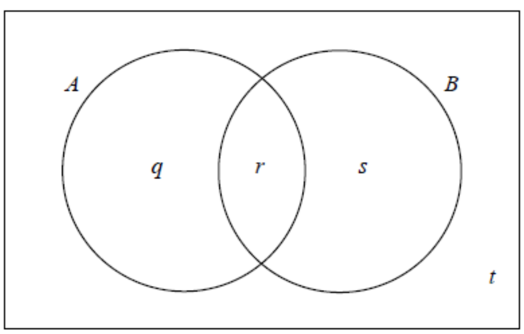
e.g.
$\text{area} = \frac{1}{2} \times 9 \times 7.02\ldots \times \sin 70^\circ$

29.69273008

$\text{area} = 29.7$ ***A1*** ***N2***

[2 marks]

Events *A* and *B* are such that
$\mathrm{P}(A) = 0.3$,
$\mathrm{P}(B) = 0.6$ and
$\mathrm{P}(A \cup B) = 0.7$.



The values *q* , *r* , *s* and *t* represent probabilities.

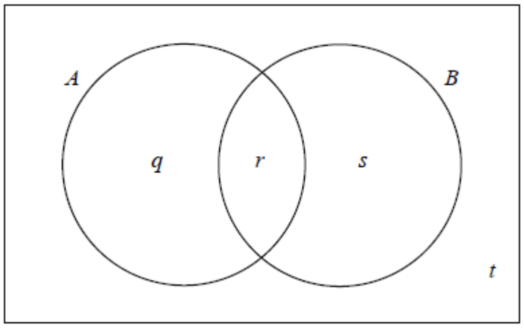
11a. Write down the value of *t* . *[1 mark]*

Markscheme

$t = 0.3$ ***A1*** ***N1***

[1 mark]

Events A and B are such that
$\mathrm{P}(A) = 0.3$,
$\mathrm{P}(B) = 0.6$ and
$\mathrm{P}(A \cup B) = 0.7$.



The values q , r , s and t represent probabilities.

11b. (i) Show that
$r = 0.2$.

[3 marks]

(ii) Write down the value of q and of s .

Markscheme

(i) correct values ***A1***

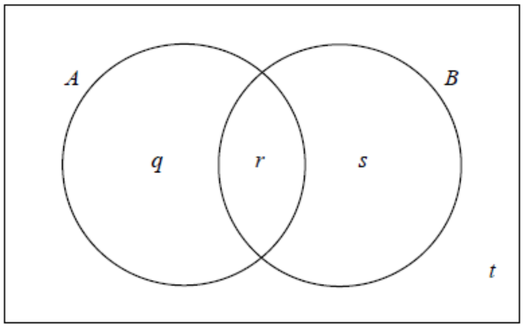
e.g.
$0.3 + 0.6 - 0.7$,
$0.9 - 0.7$

$r = 0.2$ ***AG*** ***N0***

(ii)
$q = 0.1$,
$s = 0.4$ ***A1A1*** ***N2***

[3 marks]

Events A and B are such that
$\mathrm{P}(A) = 0.3$,
$\mathrm{P}(B) = 0.6$ and
$\mathrm{P}(A \cup B) = 0.7$.



The values q , r , s and t represent probabilities.

11c. (i) Write down
$\mathrm{P}(B')$.

[3 marks]

(ii) Find
$\mathrm{P}(A|B')$.

Markscheme

(i)

0.4 ***A1*** ***N1***

(ii)

$\mathrm{P}(A|B') = \frac{1}{4}$ ***A2*** ***N2***

[3 marks]

Let $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$ and $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.

12a. (i) Find $\overrightarrow{\mathrm{AB}}$. *[4 marks]*

(ii) Find $\left|\overrightarrow{\mathrm{AB}}\right|$.

Markscheme

(i) valid approach to find $\overrightarrow{\mathrm{AB}}$

eg $\overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}}$, $\begin{pmatrix} 4-(-1) \\ 1-0 \\ 3-4 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ ***A1*** ***N2***

(ii) valid approach to find $\left|\overrightarrow{\mathrm{AB}}\right|$ ***(M1)***

eg $\sqrt{(5)^2+(1)^2+(-1)^2}$

$\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{27}$ ***A1*** ***N2***

[4 marks]

Let $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$ and $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.

The point C is such that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

12b. Show that the coordinates of C are $(-2,\ 1,\ 3)$. *[1 mark]*

Markscheme

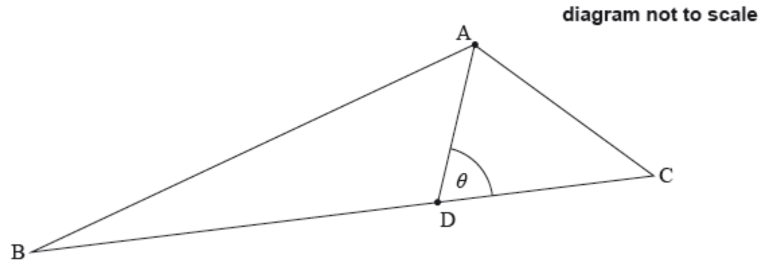
correct approach ***A1***

eg $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$

C has coordinates $(-2,\ 1,\ 3)$ ***AG*** ***N0***

[1 mark]

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle $\mathrm{ADC} = \theta$.



12c. Write down an expression in terms of θ for *[2 marks]*

(i) angle ADB;

(ii) area of triangle ABD.

Markscheme

(i)

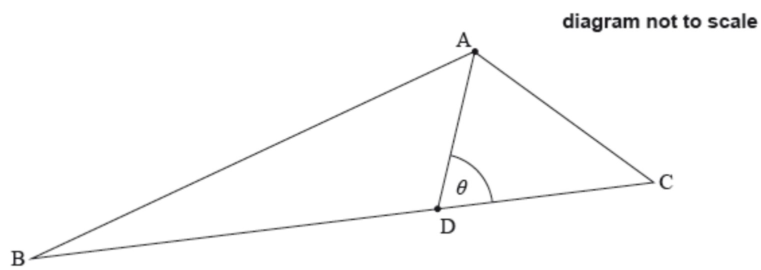
$\mathrm{A\hat{D}B} = \pi - \theta, \hat{\mathrm{D}} = 180 - \theta$ ***A1 N1***

(ii) any correct expression for the area involving θ ***A1 N1***

eg $\text{area} = \frac{1}{2} \times \mathrm{AD} \times \mathrm{BD} \times \sin(180 - \theta),\ \frac{1}{2}ab\sin\theta,\ \frac{1}{2}\left|\overrightarrow{\mathrm{DA}}\right|\left|\overrightarrow{\mathrm{DB}}\right|\sin(\pi - \theta)$

[2 marks]

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle $\mathrm{ADC} = \theta$.



12d. Given that $\frac{\text{area } \Delta\mathrm{ABD}}{\text{area } \Delta\mathrm{ACD}} = 3$, show that $\frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3}{4}$. *[5 marks]*

Markscheme

METHOD 1 (using sine formula for area)

correct expression for the area of triangle ACD (seen anywhere) ***(A1)***

eg $\frac{1}{2}\mathrm{AD} \times \mathrm{DC} \times \sin\theta$

correct equation involving areas ***A1***

eg $\frac{\frac{1}{2}\mathrm{AD}\times\mathrm{BD}\times\sin(\pi-\theta)}{\frac{1}{2}\mathrm{AD}\times\mathrm{DC}\times\sin\theta} = 3$

recognizing that $\sin(\pi - \theta) = \sin\theta$ (seen anywhere) ***(A1)***

$\frac{\mathrm{BD}}{\mathrm{DC}} = 3$ (seen anywhere) ***(A1)***

correct approach using ratio ***A1***

eg $3\overrightarrow{\mathrm{DC}} + \overrightarrow{\mathrm{DC}} = \overrightarrow{\mathrm{BC}}$, $\overrightarrow{\mathrm{BC}} = 4\overrightarrow{\mathrm{DC}}$

correct ratio $\frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3}{4}$ ***AG*** ***N0***

METHOD 2 (Geometric approach)

recognising $\Delta\mathrm{ABD}$ and $\Delta\mathrm{ACD}$ have same height ***(A1)***

eg
use of
h for both triangles, $\frac{\frac{1}{2}\mathrm{BD}\times h}{\frac{1}{2}\mathrm{CD}\times h} = 3$

correct approach ***A2***

eg

$\mathrm{BD} = 3x$ and $\mathrm{DC} = x$, $\frac{\mathrm{BD}}{\mathrm{DC}} = 3$

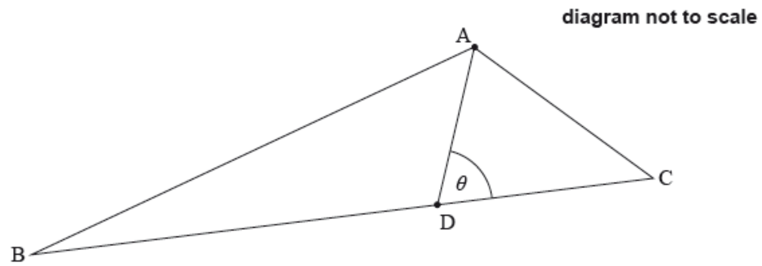
correct working ***A2***

eg $\mathrm{BC} = 4x$, $\mathrm{BD} + \mathrm{DC} = 4\mathrm{DC}$, $\frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3x}{4x}$, $\frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3\mathrm{DC}}{4\mathrm{DC}}$

$\frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3}{4}$ ***AG*** ***N0***

[5 marks]

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle $\mathrm{ADC} = \theta$.



12e. Hence or otherwise, find the coordinates of point D. *[4 marks]*

Markscheme

correct working (seen anywhere) ***(A1)***

eg $\overrightarrow{\mathrm{BD}} = \frac{3}{4}\overrightarrow{\mathrm{BC}}$, $\overrightarrow{\mathrm{OD}} = \overrightarrow{\mathrm{OB}} + \frac{3}{4}\begin{pmatrix}-6\\0\\0\end{pmatrix}$, $\overrightarrow{\mathrm{CD}} = \frac{1}{4}\overrightarrow{\mathrm{CB}}$

valid approach (seen anywhere) ***(M1)***

eg $\overrightarrow{\mathrm{OD}} = \overrightarrow{\mathrm{OB}} + \overrightarrow{\mathrm{BD}}$, $\overrightarrow{\mathrm{BC}} = \begin{pmatrix}-6\\0\\0\end{pmatrix}$

correct working to find x-coordinate ***(A1)***

eg $\begin{pmatrix}4\\1\\3\end{pmatrix} + \frac{3}{4}\begin{pmatrix}-6\\0\\0\end{pmatrix}$, $x = 4 + \frac{3}{4}(-6),\ -2 + \frac{1}{4}(6)$

D is $\left(-\frac{1}{2},\ 1,\ 3\right)$ ***A1 N3***

[4 marks]

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X = x) = \frac{x^2}{14}, x \in \{1, 2, k\}\text{ , where}k > 0$$

.

13a. Write down $\mathrm{P}(X = 2)$. *[1 mark]*

Markscheme

$\mathrm{P}(X = 2) = \frac{4}{14}$
$\left(= \frac{2}{7}\right)$ ***A1 N1***

[1 mark]

13b. Show that $k = 3$. *[4 marks]*

Markscheme

$\mathrm{P}(X = 1) = \frac{1}{14}$ ***(A1)***

$\mathrm{P}(X = k) = \frac{k^2}{14}$ ***(A1)***

setting the sum of probabilities $= 1$ ***M1***

e.g.
$\frac{1}{14} + \frac{4}{14} + \frac{k^2}{14} = 1$,
$5 + k^2 = 14$

$k^2 = 9$ (accept $\frac{k^2}{14} = \frac{9}{14}$) ***A1***

$k = 3$ ***AG N0***

[4 marks]

13c. Find $\mathrm{E}(X)$. *[2 marks]*

Markscheme

correct substitution into $\mathrm{E}(X) = \sum x\mathrm{P}(X = x)$ ***A1***

e.g. $1\left(\frac{1}{14}\right) + 2\left(\frac{4}{14}\right) + 3\left(\frac{9}{14}\right)$

$\mathrm{E}(X) = \frac{36}{14}$ $\left(= \frac{18}{7}\right)$ ***A1 N1***

[2 marks]