0327Pre-test_Statistics-free-response *[70 marks]*

The weights of fish in a lake are normally distributed with a mean of 760 g and standard deviation σ. It is known that 78.87% of the fish have weights between 705 g and 815 g.

1a. (i) Write down the probability that a fish weighs more than 760 g. *[4 marks]*

(ii) Find the probability that a fish weighs less than 815 g.

Markscheme

Note: There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

(i) $\mathrm{P}(X > 760) = 0.5\ (\text{exact}),\ [0.499,\ 0.500]$ ***A1 N1***

(ii) evidence of valid approach ***(M1)***

recognising symmetry, $\frac{0.7887}{2},\ 1 - \mathrm{P}(W < 815),\ \frac{21.13}{2} + 78.87\%$

correct working ***(A1)***

eg $0.5 + 0.39435,\ 1 - 0.10565,$

$0.89435\ (\text{exact}),\ 0.894\ [0.894,\ 0.895]$ ***A1 N2***

[4 marks]

1b. (i) Write down the standardized value for 815 g. *[4 marks]*

(ii) Hence or otherwise, find
σ.

Markscheme

(i) 1.24999 ***A1 N1***

$z = 1.25\ [1.24,\ 1.25]$

(ii) evidence of appropriate approach ***(M1)***

eg $\sigma = \frac{x-\mu}{1.25},\ \frac{815-760}{\sigma}$

correct substitution ***(A1)***

eg $1.25 = \frac{815-760}{\sigma},\ \frac{815-760}{1.24999}$

44.0003

$\sigma = 44.0\ [44.0,\ 44.1]\ (\text{g})$ ***A1 N2***

[4 marks]

1c. A fishing contest takes place in the lake. Small fish, called tiddlers, are thrown back into the lake. The maximum weight of a tiddler is 1.5 standard deviations below the mean. *[2 marks]*

Find the maximum weight of a tiddler.

Markscheme

correct working *(A1)*

eg $760 - 1.5 \times 44$

693.999

$694\ [693,\ 694]\ (\mathrm{g})$ ***A1 N2***

[2 marks]

1d. A fish is caught at random. Find the probability that it is a tiddler. *[2 marks]*

Markscheme

0.0668056

$\mathrm{P}(X < 694) = 0.0668\ [0.0668,\ 0.0669]$ ***A2 N2***

[2 marks]

1e. 25% of the fish in the lake are salmon. 10% of the salmon are tiddlers. Given that a fish caught at random is a tiddler, find the probability that it is a salmon. *[2 marks]*

Markscheme

recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P(A|B)},\ \frac{0.025}{0.0668}$

appropriate approach involving conditional probability ***(M1)***

eg $\mathrm{P}(S|T) = \frac{\mathrm{P}(S \text{ and } T)}{\mathrm{P}(T)},$

correct working

eg P (salmon and tiddler) $= 0.25 \times 0.1,\ \frac{0.25 \times 0.1}{0.0668}$ ***(A1)***

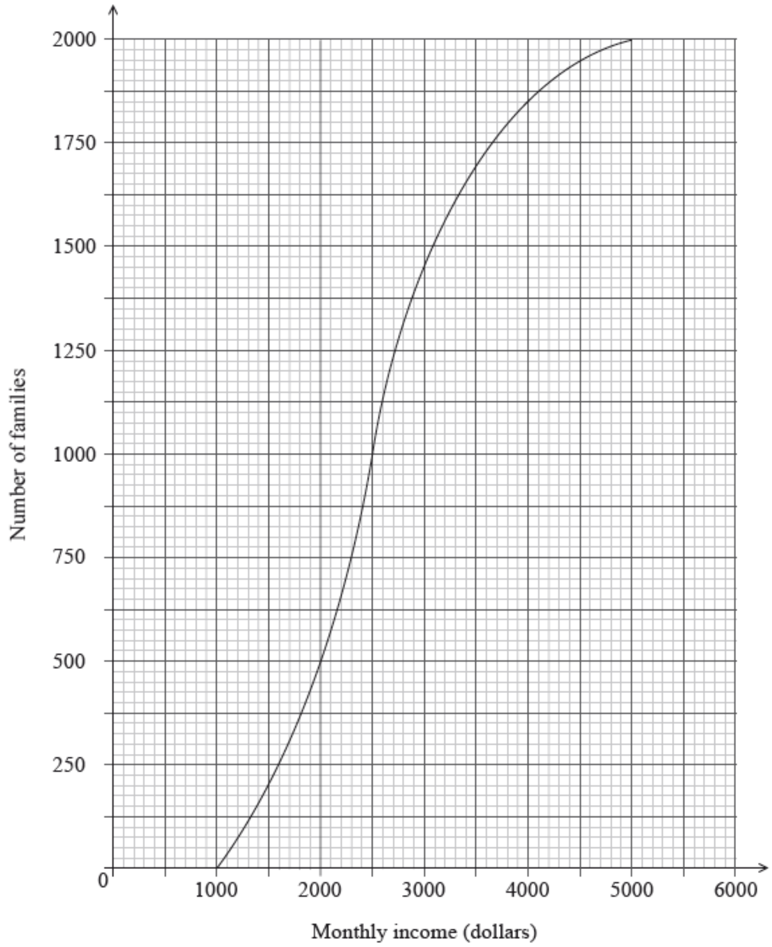
0.374220

$0.374\ [0.374,\ 0.375]$ ***A1 N2***

[4 marks]

Total [16 marks]

The following cumulative frequency graph shows the monthly income, I dollars, of 2000 families.



2a. Find the median monthly income. *[2 marks]*

Markscheme

recognizing that the median is at half the total frequency ***(M1)***

eg $\frac{2000}{2}$

$m = 2500\ (\text{dollars})$ ***A1*** ***N2***

[2 marks]

2b. (i) Write down the number of families who have a monthly income of 2000 dollars or less. *[4 marks]*

(ii) Find the number of families who have a monthly income of more than 4000 dollars.

Markscheme

(i) 500 families have a monthly income less than 2000 ***A1*** ***N1***

(ii) correct cumulative frequency,
1850 ***(A1)***

subtracting **their** cumulative frequency from 2000 ***(M1)***

eg $2000 - 1850$

150 families have a monthly income of more than 4000 dollars ***A1*** ***N2***

Note: If working shown, award ***M1A1A1*** for $128 + 22 = 150$, using the table.

[4 marks]

2c. The 2000 families live in two different types of housing. The following table gives information about the number of families living in each type of housing and their monthly income I. *[2 marks]*

	$1000 < I \le 2000$	$2000 < I \le 4000$	$4000 < I \le 5000$
Apartment	436	765	28
Villa	64	p	122

Find the value of p.

Markscheme

correct calculation ***(A1)***

eg $2000 - (436 + 64 + 765 + 28 + 122),\ 1850 - 500 - 765$ ***(A1)***

$p = 585$ ***A1 N2***

[2 marks]

2d. A family is chosen at random. *[2 marks]*

(i) Find the probability that this family lives in an apartment.

(ii) Find the probability that this family lives in an apartment, given that its monthly income is greater than 4000 dollars.

Markscheme

(i) correct working ***(A1)***

eg $436 + 765 + 28$

0.6145 (exact) ***A1 N2***

$\frac{1229}{2000},\ 0.615\ [0.614,\ 0.615]$

(ii) correct working/probability for number of families ***(A1)***

eg $122 + 28,\ \frac{150}{2000},\ 0.075$

0.186666

$\frac{28}{150}\ \left(= \frac{14}{75}\right),\ 0.187\ [0.186,\ 0.187]$ ***A1 N2***

[4 marks]

2e. Estimate the mean monthly income for families living in a villa. *[2 marks]*

Markscheme

evidence of using correct mid-interval values $(1500, 3000, 4500)$ ***(A1)***

attempt to substitute into $\frac{\sum fx}{\sum f}$ ***(M1)***

eg $\frac{1500 \times 64 + 3000 \times p + 4500 \times 122}{64 + 585 + 122}$

3112.84

$3110\ [3110,\ 3120]\ (\text{dollars})$ ***A1 N2***

[3 marks]

Total [15 marks]

A company produces a large number of water containers. Each container hastwo parts, a bottle and a cap. The bottles and caps are tested to check that they are not defective.

A cap has a probability of 0.012 of being defective. A random sample of 10 caps isselected for inspection.

3a. Find the probability that exactly one cap in the sample will be defective. *[2 marks]*

Markscheme

Note: There may be slight differences in answers, depending on whether candidates usetables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with **their** working and check carefully for ***FT***.

evidence of recognizing binomial (seen anywhere in the question) ***(M1)***

e.g.
${}_nC_r p^r q^{n-r}$,
$\mathrm{B}(n, p)$,
${}^{10}C_1(0.012)^1(0.988)^9$

$p = 0.108$ ***A1*** ***N2***

[2 marks]

3b. The sample of caps passes inspection if at most one cap is defective. Find theprobability that the sample passes inspection. *[2 marks]*

Markscheme

valid approach ***(M1)***

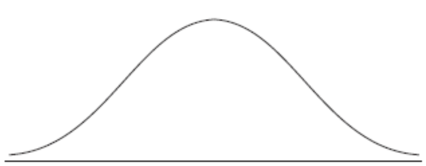
e.g.
$\mathrm{P}(X \le 1)$,
$0.88627\ldots + 0.10764\ldots$

$p = 0.994$ ***A1*** ***N2***

[2 marks]

3c. The heights of the bottles are normally distributed with a mean of 22 cm and a standard deviation of 0.3 cm. *[5 marks]*

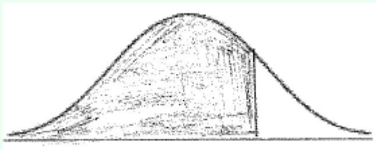
(i) Copy and complete the following diagram, shading the region representingwhere the heights are less than 22.63 cm.



(ii) Find the probability that the height of a bottle is less than 22.63 cm.

Markscheme

(i)



A1A1 ***N2***

Note: Award ***A1*** for vertical line to right of mean, ***A1*** for shading to left of **their** vertical line.

(ii) valid approach ***(M1)***

e.g.
$\mathrm{P}(X < 22.63)$

working to find standardized value ***(A1)***

e.g.
$\frac{22.63-22}{0.3}$, 2.1

$p = 0.982$ ***A1*** ***N3***

[5 marks]

3d. (i) A bottle is accepted if its height lies between $21.37\ \mathrm{cm}$ and $22.63\ \mathrm{cm}$. Find the probability that a bottle selected at random is accepted. *[5 marks]*

(ii) A sample of 10 bottles passes inspection if all of the bottles in the sampleare accepted. Find the probability that the sample passes inspection.

Markscheme

valid approach ***(M1)***

e.g.
$\mathrm{P}(21.37 < X < 22.63)$,
$\mathrm{P}(-2.1 < z < 2.1)$

correct working ***(A1)***

e.g.
$0.982 - (1 - 0.982)$

$p = 0.964$ ***A1*** ***N3***

(ii) correct working ***(A1)***

e.g.
$X \sim \mathrm{B}(10, 0.964)$,
$(0.964)^{10}$

$p = 0.695$ (accept 0.694 from tables) ***A1*** ***N2***

[5 marks]

3e. The bottles and caps are manufactured separately. A sample of 10 bottles anda sample of 10 caps are randomly selected for testing. Find the probability that both samples pass inspection. *[2 marks]*

Markscheme

valid approach ***(M1)***

e.g.
$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$,
$(0.994) \times (0.964)^{10}$

$p = 0.691$ (accept
0.690 from tables) ***A1*** ***N2***

[2 marks]

The weights of players in a sports league are normally distributed with a mean of
$76.6\ \mathrm{kg}$, (correct to three significant figures). It is known that
80% of the players have weights between
$68\ \mathrm{kg}$ and
$82\ \mathrm{kg}$. The probability that a player weighs less than
$68\ \mathrm{kg}$ is 0.05.

4a. Find the probability that a player weighs more than
$82\ \mathrm{kg}$. *[2 marks]*

Markscheme

evidence of appropriate approach ***(M1)***

e.g.
$1 - 0.85$, diagram showing values in a normal curve

$\mathrm{P}(w \geq 82) = 0.15$ ***A1*** ***N2***

[2 marks]

4b. (i) Write down the standardized value, *z*, for
$68\ \mathrm{kg}$. *[4 marks]*

(ii) Hence, find the standard deviation of weights.

Markscheme

(i)
$z = -1.64$ ***A1*** ***N1***

(ii) evidence of appropriate approach ***(M1)***

e.g.
$-1.64 = \frac{x-\mu}{\sigma}$,
$\frac{68-76.6}{\sigma}$

correct substitution ***A1***

e.g.
$-1.64 = \frac{68-76.6}{\sigma}$

$\sigma = 5.23$ ***A1*** ***N1***

[4 marks]

4c. To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean. *[5 marks]*

(i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament.

Markscheme

(i)

$68.8 \leq \text{weight} \leq 84.4$ ***A1A1A1*** ***N3***

Note: Award ***A1*** for 68.8, ***A1*** for 84.4, ***A1*** for giving answer as an interval.

(ii) evidence of appropriate approach ***(M1)***

e.g.

$\text{P}(-1.5 \leq z \leq 1.5)$,

$\text{P}(68.76 < y < 84.44)$

$\text{P}(\text{qualify}) = 0.866$ ***A1*** ***N2***

[5 marks]

4d. Of the players in the league, 25% are women. Of the women, 70% take part in the tournament. *[4 marks]*

Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman.

Markscheme

recognizing conditional probability ***(M1)***

e.g.

$\text{P}(A|B) = \frac{\text{P}(A \cap B)}{\text{P}(B)}$

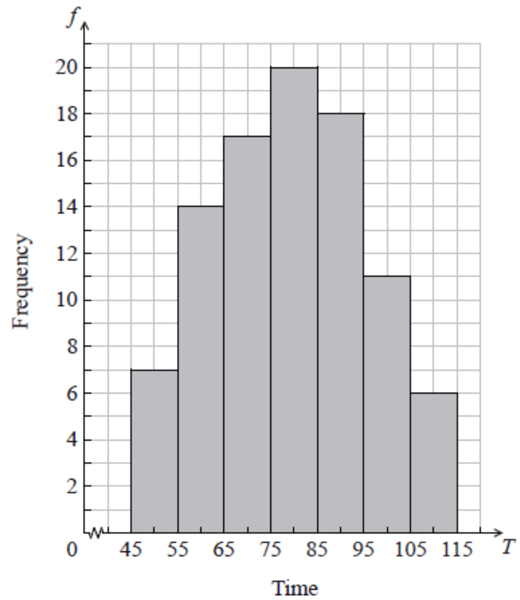
$\text{P}(\text{woman and qualify}) = 0.25 \times 0.7$ ***(A1)***

$\text{P}(\text{woman}|\text{qualify}) = \frac{0.25 \times 0.7}{0.866}$ ***A1***

$\text{P}(\text{woman}|\text{qualify}) = 0.202$ ***A1***

[4 marks]

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for T .

Time	$45 \le T < 55$	$55 \le T < 65$	$65 \le T < 75$	$75 \le T < 85$	$85 \le T < 95$	$95 \le T < 105$	$105 \le T < 115$
Frequency	7	14	p	20	18	q	6

5a. (i) Write down the value of p and of q . *[3 marks]*

(ii) Write down the median class.

Markscheme

(i)

$p = 17$,

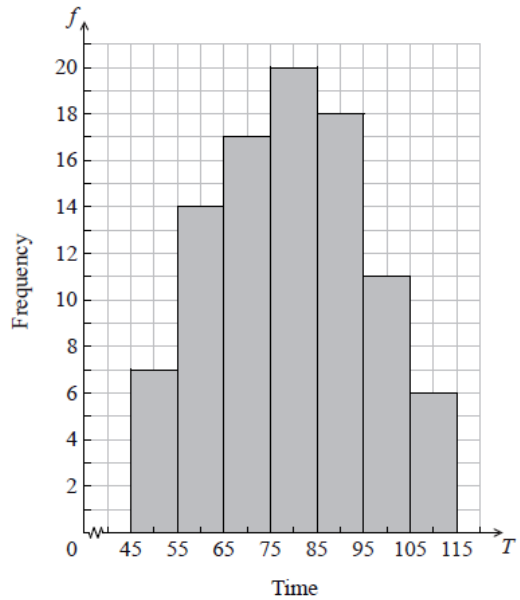
$q = 11$ ***A1A1*** ***N2***

(ii)

$75 \le T < 85$ ***A1*** ***N1***

[3 marks]

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for T .

Time	$45 \le T < 55$	$55 \le T < 65$	$65 \le T < 75$	$75 \le T < 85$	$85 \le T < 95$	$95 \le T < 105$	$105 \le T < 115$
Frequency	7	14	p	20	18	q	6

5b. A child is selected at random. Find the probability that the child takes less than95 seconds to solve the puzzle. *[2 marks]*

Markscheme

evidence of valid approach ***(M1)***

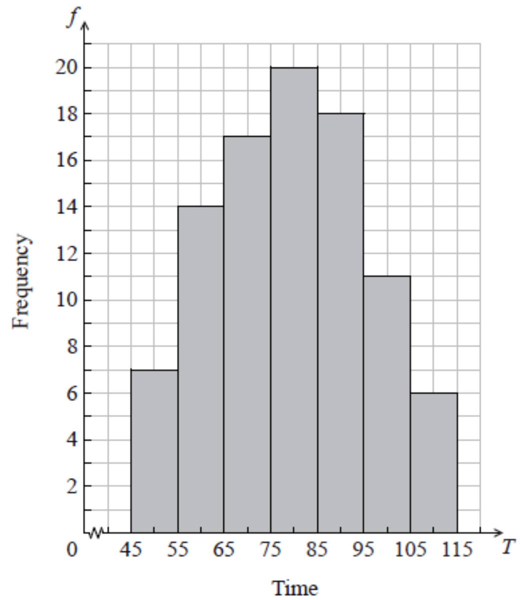
e.g. adding frequencies

$\frac{76}{93} = 0.8172043\ldots$

$\mathrm{P}(T < 95) = \frac{76}{93} = 0.817$ ***A1 N2***

[2 marks]

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



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Frequency	7	14	p	20	18	q	6

5c. Consider the class interval $45 \le T < 55$. *[2 marks]*

(i) Write down the interval width.

(ii) Write down the mid-interval value.

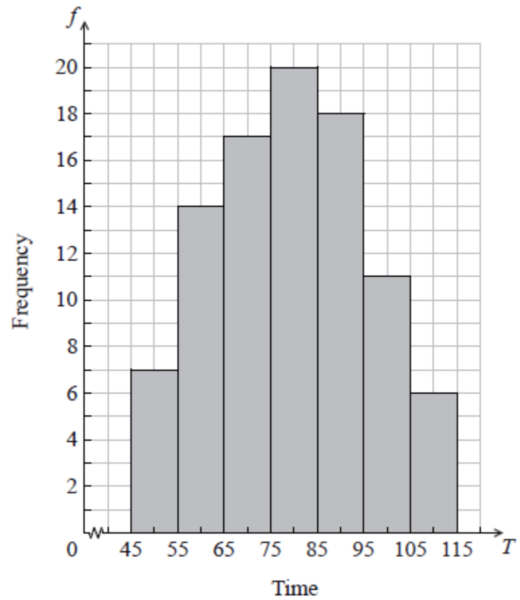
Markscheme

(i) 10 ***A1*** ***N1***

(ii) 50 ***A1*** ***N1***

[2 marks]

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for T .

Time	$45 \le T < 55$	$55 \le T < 65$	$65 \le T < 75$	$75 \le T < 85$	$85 \le T < 95$	$95 \le T < 105$	$105 \le T < 115$
Frequency	7	14	p	20	18	q	6

5d. Hence find an estimate for the *[4 marks]*

(i) mean;

(ii) standard deviation.

Markscheme

(i) evidence of approach using mid-interval values (may be seen in part (ii)) ***(M1)***

79.1397849

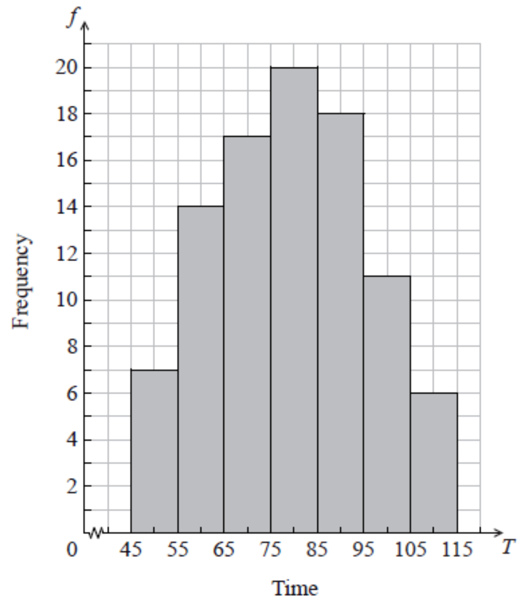
$\overline{x} = 79.1$ ***A2 N3***

(ii)
16.4386061

$\sigma = 16.4$ ***A1 N1***

[4 marks]

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for T.

Time	$45\le T<55$	$55\le T<65$	$65\le T<75$	$75\le T<85$	$85\le T<95$	$95\le T<105$	$105\le T<115$
Frequency	7	14	p	20	18	q	6

5e. John assumes that T is normally distributed and uses this to estimate the probabilitythat a child takes less than 95 seconds to solve the puzzle. *[2 marks]*

Find John's estimate.

Markscheme

e.g. standardizing,
$z = 0.9648\ldots$

0.8326812

$\mathrm{P}(T < 95) = 0.833$ ***A1*** ***N2***

[2 marks]