BECA / Huson / IB Math SL Name:
2-3 January 2019
Classwork & homework review. Due Friday at the beginning of class.

1a. *[2 marks]*

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

Find $f^{-1}(x)$.

1b. *[2 marks]*

Show that $\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$.

1c. *[2 marks]*

Let $h(x) = \frac{5}{x+2}$, for $x \geqslant 0$. The graph of *h* has a horizontal asymptote at $y = 0$.

Find the y-intercept of the graph of h.

1d. *[3 marks]*

Hence, sketch the graph of h.

1e. *[1 mark]*

For the graph of h^{-1}, write down the x-intercept;

1f. *[1 mark]*

For the graph of h^{-1}, write down the equation of the vertical asymptote.

1g. *[3 marks]*

Given that $h^{-1}(a) = 3$, find the value of a.

2a. *[3 marks]*

Let $f(x) = \sqrt{x-5}$, for $x \geq 5$.

Find $f^{-1}(2)$.

2b. *[3 marks]*

Let g be a function such that g^{-1} exists for all real numbers. Given that $g(30) = 3$, find $(f \circ g^{-1})(3)$.

3a. *[3 marks]*

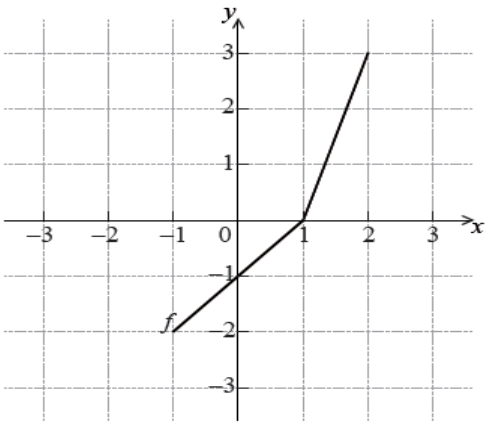
Let $f(x) = 4x - 2$ and $g(x) = -2x^2 + 8$.

Find $f^{-1}(x)$.

3b. *[3 marks]*

Find $(f \circ g)(1)$.

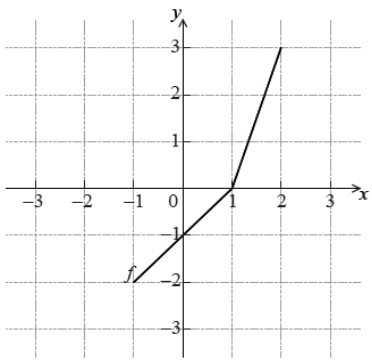
4a. The diagram below shows the graph of a function f, for $-1 \leq x \leq 2$. *[1 mark]*



Write down the value of $f(2)$.

4b. Write down the value of $f^{-1}(-1)$. *[2 marks]*

4c. Sketch the graph of f^{-1} on the grid below. *[3 marks]*



5a. *[6 marks]*

Let f and g be functions such that $g(x) = 2f(x+1) + 5$.

(a) The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k, followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k;

(ii) p;

(iii) q.

(b) Let $h(x) = -g(3x)$. The point A$(6, 5)$ on the graph of g is mapped to the point A' on the graph of h. Find A'.

5b. *[3 marks]*

The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k, followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k;

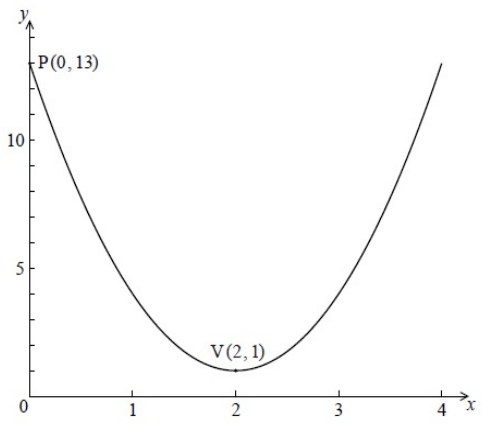
(ii) p;

(iii) q.

5c. Let $h(x) = -g(3x)$. The point A$(6, 5)$ on the graph of g is mapped to the point A' on the graph of h. Find A'. *[3 marks]*

6a. *[4 marks]*

The following diagram shows the graph of a quadratic function *f*, for $0 \leq x \leq 4$.



The graph passes through the point P(0, 13) , and its vertex is the point V(2, 1) .

The function can be written in the form $f(x) = a(x - h)^2 + k$.

(i) Write down the value of *h* and of *k* .

(ii) Show that $a = 3$.

6b. *[3 marks]*

Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$.

6c. *[8 marks]*

Calculate the area enclosed by the graph of *f*, the *x*-axis, and the lines $x = 2$ and $x = 4$.

7a. *[5 marks]*

Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

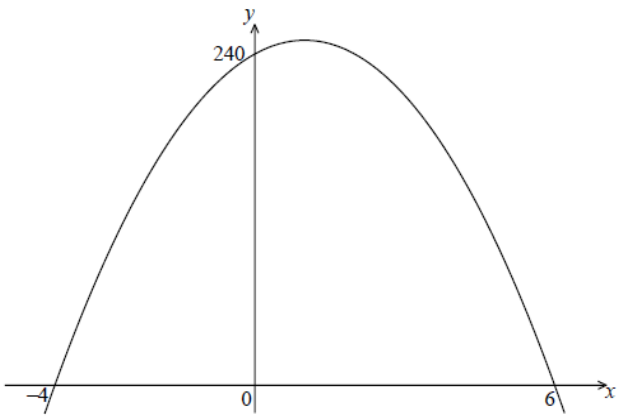
Find the value of *k* .

7b. *[2 marks]*

The line $y = p$ intersects the graph of *f*. Find all possible values of *p* .

8a. *[2 marks]*

The following diagram shows part of the graph of a quadratic function f.



The x-intercepts are at $(-4, 0)$ and $(6, 0)$, and the y-intercept is at $(0, 240)$.

Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$.

8b. *[4 marks]*

Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$.

8c. *[2 marks]*

Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$.

8d. *[7 marks]*

A particle moves along a straight line so that its velocity, v ms^{-1}, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \leq t \leq 6$.

(i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

9a. Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

Show that $f(x) = \log_3 2x$. *[2 marks]*

9b. Find the value of $f(0.5)$ and of $f(4.5)$. *[3 marks]*

9c. *[6 marks]*

The function *f* can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$.

(i) Write down the value of *a* and of *b*.

(ii) Hence on graph paper, **sketch** the graph of *f*, for $-5 \leq x \leq 5$, $-5 \leq y \leq 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

9d. *[1 mark]*

Write down the value of $f^{-1}(0)$.

9e. *[4 marks]*

The point A lies on the graph of *f*. At A, $x = 4.5$.

On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A.

10a. *[2 marks]*

Let $f(x) = 3x^2$. The graph of *f* is translated 1 unit to the right and 2 units down. The graph of *g* is the image of the graph of *f* after this translation.

Write down the coordinates of the vertex of the graph of *g*.

10b. Express *g* in the form $g(x) = 3(x - p)^2 + q$. *[2 marks]*

10c. The graph of *h* is the reflection of the graph of *g* in the *x*-axis.

Write down the coordinates of the vertex of the graph of *h*. *[2 marks]*

11a. *[2 marks]*

Let $f(x) = log_3 \sqrt{x}$, for $x > 0$.

Show that $f^{-1}(x) = 3^{2x}$.

11b. *[1 mark]*

Write down the range of f^{-1}.

11c. *[4 marks]*

Let $g(x) = \log_3 x$, for $x > 0$.

Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

12. *[7 marks]*

Solve $\log_2 x + \log_2 (x - 2) = 3$, for $x > 2$.

13a. *[3 marks]*

Let $f(x) = k\log_2 x$.

Given that $f^{-1}(1) = 8$, find the value of k.

13b. *[4 marks]*

Find $f^{-1}\left(\frac{2}{3}\right)$.

14a. *[3 marks]*

Let $f(x) = \mathrm{e}^{x+3}$.

(i) Show that $f^{-1}(x) = \ln x - 3$.

(ii) Write down the domain of f^{-1}.

14b. *[4 marks]*

Solve the equation $f^{-1}(x) = \ln \frac{1}{x}$.

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15a. *[1 mark]*

Find $\log_2 32$.

15b. *[4 marks]*

Given that $\log_2\left(\frac{32^x}{8^y}\right)$ can be written as $px + qy$, find the value of *p* and of *q*.

16a. Expand $(x-2)^4$ and simplify your result. *[3 marks]*

16b. Find the term in x^3 in $(3x+4)(x-2)^4$. *[3 marks]*

17a. Let $f(x) = 3(x+1)^2 - 12$.

Show that $f(x) = 3x^2 + 6x - 9$. *[2 marks]*

17b. *[8 marks]*

For the graph of *f*

(i) write down the coordinates of the vertex;

(ii) write down the **equation** of the axis of symmetry;

(iii) write down the *y*-intercept;

(iv) find both *x*-intercepts.

17c. Hence sketch the graph of *f*. *[2 marks]*

17d. *[3 marks]*

Let $g(x) = x^2$. The graph of *f* may be obtained from the graph of *g* by the two transformations:

a stretch of scale factor *t* in the *y*-direction

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of *t*.