Mathematics Class Slides

Bronx Early College Academy

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GQ: How does a function's graph relate to its derivatives?

CCSS: HSF.IF.B.4 Interpret key features of functions and their graphs 12.1

Do Now: Differential calculus

1. Take the 1st & 2nd derivatives of $f(x) = x^3 - 6x^2 + 6x$.
2. Sketch the function.
   Challenge: Identify key features, graphically & algebraically.

Lesson: Function graphs, extrema, the 1st & 2nd derivative tests p. 233, 240
Task: 7Q p. 232 #1-3; 7R p. 234 1, 2; 7S p. 236 1, 3
Assessment: Handout graphing problem #1 (#2 challenge)
Homework: IB function / graphing problem set

How do we organize data using sample space diagrams?

CCSS: HSS.CP.A.1 Probabilities: subsets of a sample space 11.1

Do Now (use a diagram or table to support your answer)

1. If a coin is flipped twice, what is the probability of getting at least one heads?
2. How many ways are there to roll a 5 with two dice?

Lesson: Probability concepts: bias and fairness, random variation, & combinations

Task: Exercises 3F page 82.

Assessment: Two cards are drawn from a deck, without replacement. Are the events independent?

Homework: Handout review problems.

Bias and fairness, random variation, & combinations

When rolling two dice, why aren't all the possible totals equally likely?

Definition:
A fair (p. 67) or unbiased (p. 79) process

In mathematics we usually simplify and assume a random process follows exact, idealized probabilities. For example, we assume heads and tails are equally likely results of a coin toss.

Bias and fairness, random variation, & combinations

When rolling two dice, why aren't all the possible totals equally likely?

Definition:
Experimental or empirical (p. 65) results

In real life, the results of any experiment have a degree of random variation. The observed relative frequencies are estimates of the underlying theoretical probabilities, which grow more accurate with additional trials.

Bias and fairness, random variation, & combinations

When rolling two dice, why aren't all the possible totals equally likely?

Counting events in a sample space (p. 78) or calculating combinations (p. 184)

The six possible results of rolling a single die are equally likely, $\mathrm{P}(x) = \frac{1}{6}$, if we assume the die is fair. Similarly, the probability of any of the 36 (6×6) possible results of rolling two dice are equally likely, $\mathrm{P}(x) = (\frac{1}{6})^2$. However, the probability of a particular total varies according to how many combinations lead to that total. Thus, for example, 7 can be rolled six different ways, so $\mathrm{P}(7) = \frac{6}{36}$, while 2 can only result one way, $\mathrm{P}(2) = \frac{1}{36}$.

Sets, subsets, & proper subsets

Definitions:
A set is an unordered collection of elements.
e.g. {red, white, blue} (do not repeat elements)

Subset: Set A is a subset of set B if and only if all of the elements of A are elements of B.
Written: $A \subseteq B$

Proper subset: $A \subseteq B$ and A is not equal to B. Written: $A \subset B$

The empty set is a subset of all sets. $\{\}$ or $\emptyset$

GQ: How do we organize data using Venn diagrams?

CCSS: HSS.CP.B.6 Probabilities 11.2

Do Now: Set theory

Create a Venn diagram organizing the people in room 414:
A = the set of students in uniform,
B = the set of males in the room

Lesson: Set theory review homework problem packets

Task: Probability problem set

Assessment: Classwork problem set #1

Homework: Set theory handout

GQ: Combinatorics problem

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function

Show the formula and then use your calculator function

1. You have a \$1 bill, a \$5 bill, a \$10 bill, a \$20 bill, a quarter, a dime, a nickel, and a penny. How many different total amounts can you make by choosing six bills and coins?

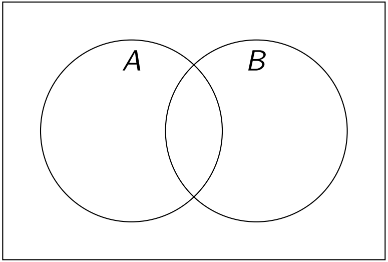
What is the number of the set you are choosing from?
How many are you picking?
Does their order matter?

Do Now #1: Phone preferences by gender

Given the frequency table, make a Venn diagram

	Android	iPhone
Boys	15	5
Girls	5	15

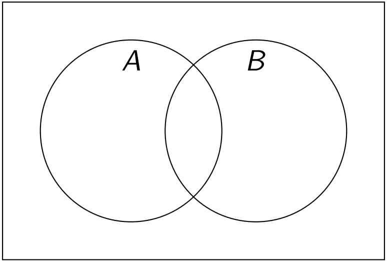
$A = \{\text{prefers Android}\}$ and $B = \{\text{is a boy}\}$



Do Now #2: Independence

Given the situation, make a Venn diagram, frequency table, and tree representing

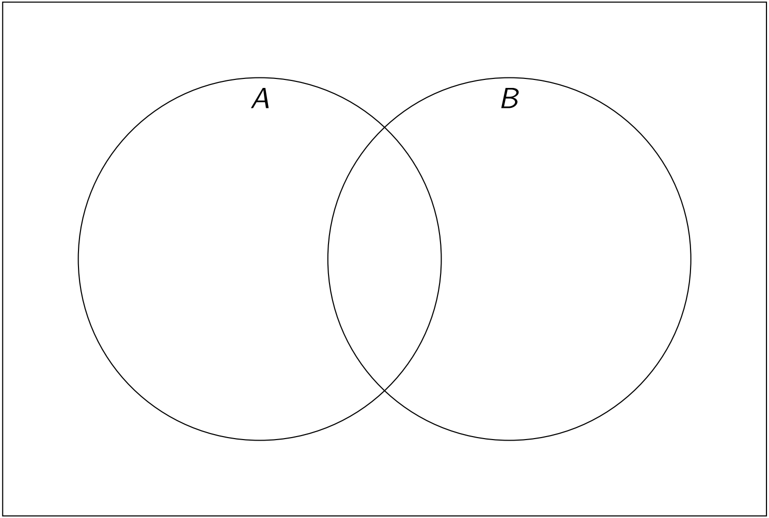
$$\mathrm{P}(A) = 0.6,\ \mathrm{P}(B) = 0.5,\ \mathrm{P}(A \cap B) = 0.3$$



	A	A'
B		
B'		

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

The addition rule



Distributions

Tables and charts used to summarize a problem situation

A frequency distribution displays the number of times each event in the sample space occurs, either in tabular or graphical form.

A probability distribution shows the same data, normalizing the totals to one.

Technical writing

Write a short paper answering the query:
"How many subsets can be picked from a group of four students?"

1. Logical, step-by-step explanation, using an example
2. Precise terminology, succinct: combination, permutation, order (matters), event, sample space, set, subset, with /without replacement, factorial
3. Notation: algebra symbols, tables, trees, grids
4. Summary, big-picture, conceptual idea
5. Audience: student peers

Combinatorics formulas

Combinations, when order doesn't matter

$$ {}_nC_r = \frac{n!}{(n-r)!r!} \qquad \text{"n pick r"} $$

Permutations, when order does matter

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Definition of theoretical probability

The theoretical probability of an event A is $\mathrm{P}(A) = \dfrac{n(A)}{n(U)}$

where $n(A)$ is the number of ways an event can occur

and $n(U)$ is the total number of possible outcomes (p. 65)

Theoretically, in n trials, one would expect the event to occur $n \times \mathrm{P}(A)$ times

Probabilities are between 0 and 1, inclusive. $0 \leq \mathrm{P}(X) \leq 1$

Empirical (experimental) probability

The relative frequency of an event can be used as an estimate of its probability.

$$\mathrm{P}(A) = \frac{\text{number of occurrences of event } A}{\text{total number of trials}}$$

The larger the number of trials the more reliable the estimate of probability.

Independence and mutual exclusivity

Two events are independent if the occurrence of one does not affect the probability of the other.

$$\mathrm{P}(\text{both } A \text{ and } B \text{ occur}) = \mathrm{P}(A) \times \mathrm{P}(B)$$

Two events are mutually exclusive if they never occur together.

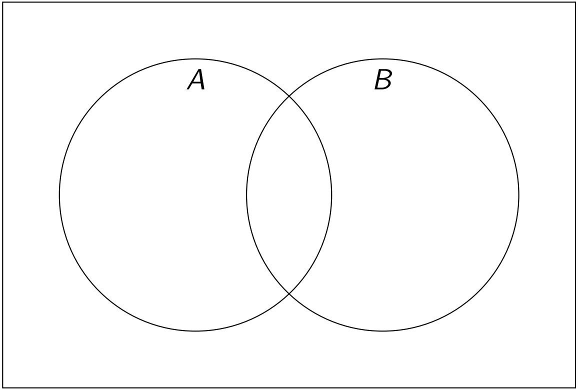
$$\mathrm{P}(\text{both } A \text{ and } B \text{ occur}) = 0 \qquad \text{and}$$

$$\mathrm{P}(\text{either } A \text{ or } B \text{ occur}) = \mathrm{P}(A) + \mathrm{P}(B)$$

Venn diagrams

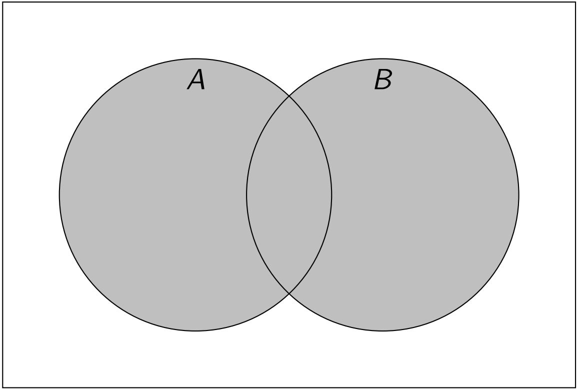
For organizing compound events

When two events can occur, and perhaps both, or neither.



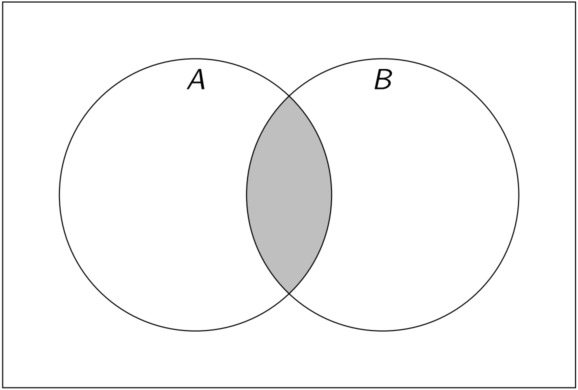
The union of sets: $A \cup B$

That A happens, or B happens, or both



The intersection of sets: $A \cap B$

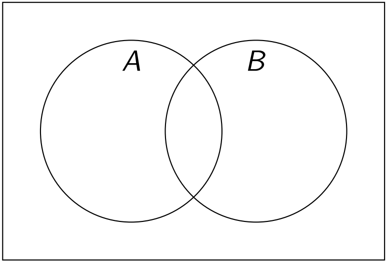
That both A and B happen



The addition rule

That A or B or both occur

When two events can occur, and perhaps both



$$P(\text{either } A \text{ or } B \text{ occur}) = P(A) + P(B) - P(\text{both } A \text{ and } B \text{ occur})$$

Vocabulary for probability & statistics

event, experiment, random

probability, P(A), values [0,1]

theoretical, empirical, subjective

sample space, U; frequency, trials

n(U) = number of possibilities

P(A) = n(A)/n(U); expected = n * P

Interpreting a displacement vs time graph

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function

Consider the function $f(x) = -x^2 + 2x + 3$

1. Factor f and state its zeros.
2. Restate f in vertex form. Write down the vertex as an ordered pair.
3. Over what intervals is the function increasing, decreasing, and neither?
4. If $f(x)$ represents the height of a diver over the domain $0 \leq x \leq 3$, interpret $f(0)$, the vertex, and $f(3)$
5. What does the "slope" of the curve represent?