

Homework: Calculus review

1. Let $\mathbf{v} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$, for $k > 0$. The angle between \mathbf{v} and \mathbf{w} is $\frac{\pi}{3}$.

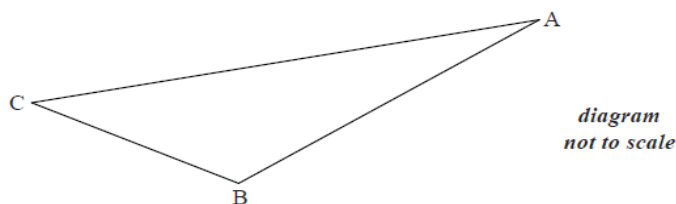
Find the value of k .

[7 marks]

- 2a. The following diagram shows the obtuse-angled triangle ABC such that $\overrightarrow{\mathbf{AB}} = \begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix}$ and

$$\overrightarrow{\mathbf{AC}} = \begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix}.$$

[3 marks]



- (i) Write down $\overrightarrow{\mathbf{BA}}$.

- (ii) Find $\overrightarrow{\mathbf{BC}}$.

- 2b. (i) Find $\cos \hat{\mathbf{ABC}}$.

[7 marks]

- (ii) Hence, find $\sin \hat{\mathbf{ABC}}$.

$$\overrightarrow{\mathbf{CD}} = \begin{pmatrix} -4 \\ 5 \\ p \end{pmatrix}$$

- 2c. The point D is such that $\overrightarrow{\mathbf{CD}} = \begin{pmatrix} -4 \\ 5 \\ p \end{pmatrix}$, where $p > 0$.

[6 marks]

- (i) Given that $|\overrightarrow{\mathbf{CD}}| = \sqrt{50}$, show that $p = 3$.

- (ii) Hence, show that $\overrightarrow{\mathbf{CD}}$ is perpendicular to $\overrightarrow{\mathbf{BC}}$.

3. [6 marks]

Let $f(x) = e^{2x}$. The line L is the tangent to the curve of f at $(1, e^2)$.

Find the equation of L in the form $y = ax + b$.

4a. [2 marks] Let $f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.
Find $f'(x)$.

5a. The population of deer in an enclosed game reserve is modelled by the function

$P(t) = 210 \sin(0.5t - 2.6) + 990$, where t is in months, and $t = 1$ corresponds to 1 January 2014.

Find the number of deer in the reserve on 1 May 2014.

[3 marks]

5b. Find the rate of change of the deer population on 1 May 2014.

[2 marks]

5c. Interpret the answer to part (i) with reference to the deer population size on 1 May 2014.

[1 mark]

6a. [2 marks] Let $f(x) = \frac{3x}{x-q}$, where $x \neq q$.

Write down the equations of the vertical and horizontal asymptotes of the graph of f .

6b. The vertical and horizontal asymptotes to the graph of f intersect at the point $Q(1, 3)$.

Find the value of q .

[2 marks]

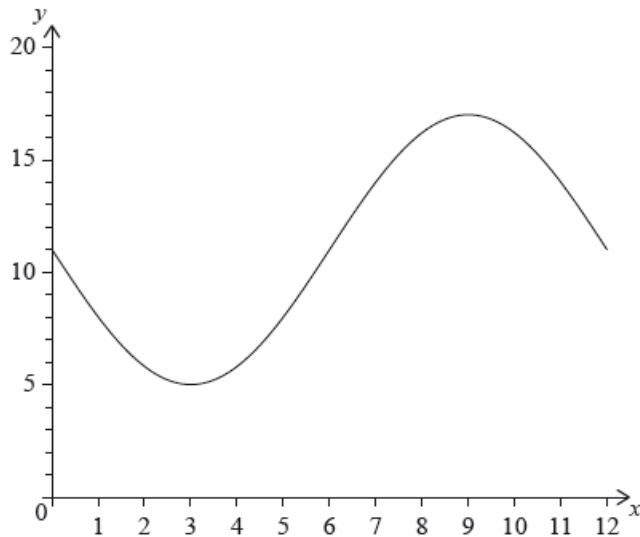
6c. [4 marks]

The point $P(x, y)$ lies on the graph of f . Show that $PQ = \sqrt{(x-1)^2 + \left(\frac{3}{x-1}\right)^2}$.

6d. Hence find the coordinates of the points on the graph of f that are closest to $(1, 3)$.

[6 marks]

7a. The following diagram shows the graph of $f(x) = a \sin bx + c$, for $0 \leq x \leq 12$.



The graph of f has a minimum point at $(3, 5)$ and a maximum point at $(9, 17)$.

[6 marks]

(i) Find the value of c .

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a .

7b. The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5, 17)$. [3 marks]

(i) Write down the value of k .

(ii) Find $g(x)$.

7c. [6 marks] The graph of g changes from concave-up to concave-down when $x = w$.

(i) Find w .

(ii) Hence or otherwise, find the maximum positive rate of change of g .

8a. [2 marks]

Line L_1 passes through points $A(1, -1, 4)$ and $B(2, -2, 5)$.

Find \overrightarrow{AB} .

8b. [2 marks]

Find an equation for L_1 in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

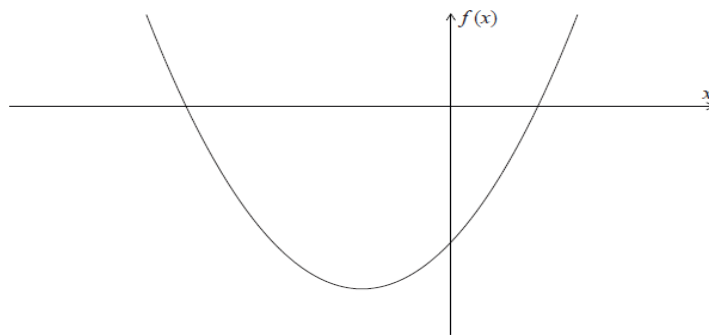
8c. [7 marks]

$$\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

Line L_2 has equation

Find the angle between L_1 and L_2 .

9a. [6 marks] The diagram below shows part of the graph of $f(x) = (x - 1)(x + 3)$.



(a) Write down the x -intercepts of the graph of f .

(b) Find the coordinates of the vertex of the graph of f .

9b. Write down the x -intercepts of the graph of f . [2 marks]

9c. Find the coordinates of the vertex of the graph of f . [4 marks]

10a. [3 marks]

Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \leq x \leq \pi$.

Find $f'(x)$.

10b. [3 marks]

Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g .

Find $g(4)$.

10c. [4 marks]

The function g can be expressed in the form $g(x) = a(x - h)^2 + 3$.

(i) Write down the value of h .

(ii) Find the value of a .

10d. [6 marks]

Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g .