Homework: Integration of solids from rotation (100 points)
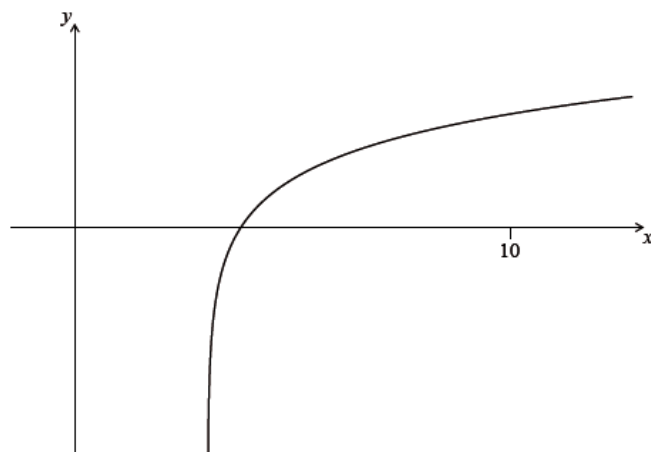
Select your problems. The first six are 6-point, volume-integration problems. Problem #7 is a free response solid-of-rotation problem, and the final three are mixed free response questions.

1a. Let $f(x) = 2\ln(x - 3)$, for $x > 3$. The following diagram shows part of the graph of f.

Find the equation of the vertical asymptote to the graph of f.   *[2 marks]*

1b. Find the x-intercept of the graph of f.   *[2 marks]*

1c. The region enclosed by the graph of f, the x-axis and the line $x = 10$ is rotated $360°$ about the x-axis. Find the volume of the solid formed.   *[3 marks]*

2a. Let $f(x) = -x^4 + 2x^3 - 1$, for $0 \le x \le 2$.

Sketch the graph of f.   *[3 marks]*

2b. Solve $f(x) = 0$.   *[2 marks]*

2c. The region enclosed by the graph of f and the x-axis is rotated $360°$ about the x-axis.

Find the volume of the solid formed.   *[3 marks]*
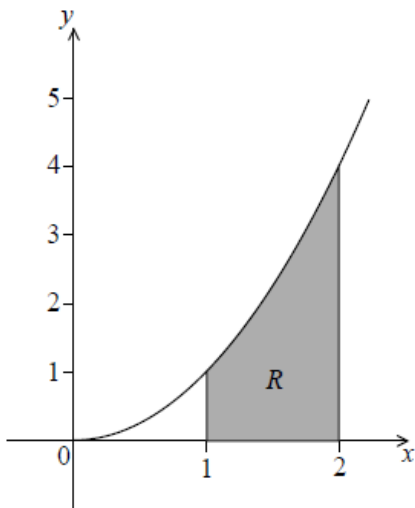
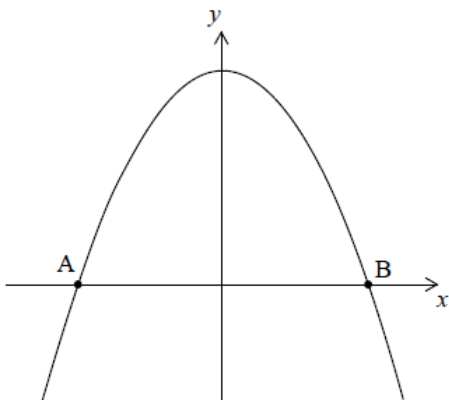**3a.** Let $f(x) = x^2$.   [no calculator on this problem]

Find $\int_1^2 (f(x))^2 dx$.   *[4 marks]*

3b. The following diagram shows part of the graph of f.

The shaded region R is enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$.

Find the volume of the solid formed when R is revolved $360°$ about the x-axis.   *[2 marks]*

4a. Let $f(x) = 5 - x^2$. Part of the graph of f is shown in the following diagram.

The graph crosses the x-axis at the points A and B.

Find the x-coordinate of A and of B.   *[3 marks]*
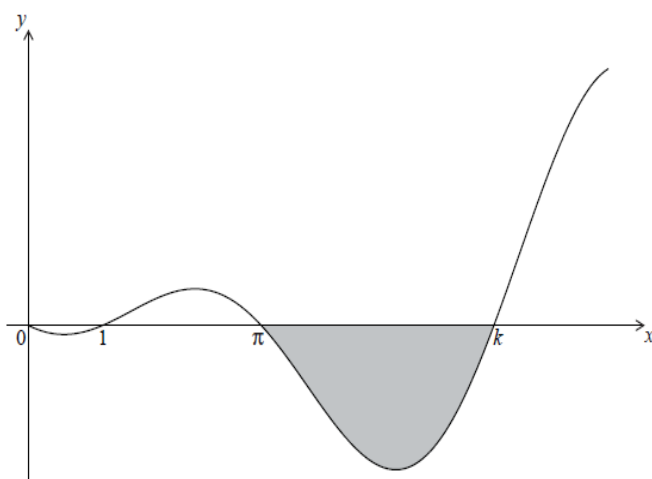
4b. The region enclosed by the graph of f and the x-axis is revolved $360°$ about the x-axis.

Find the volume of the solid formed.   *[3 marks]*

5a. The graph of $y = (x-1)\sin x$, for $0 \le x \le \frac{5\pi}{2}$, is shown below.

The graph has x-intercepts at $0, 1, \pi$ and k.

Find k. [2 marks]

5b. The shaded region is rotated $360°$ about the x-axis. Let V be the volume of the solid formed.
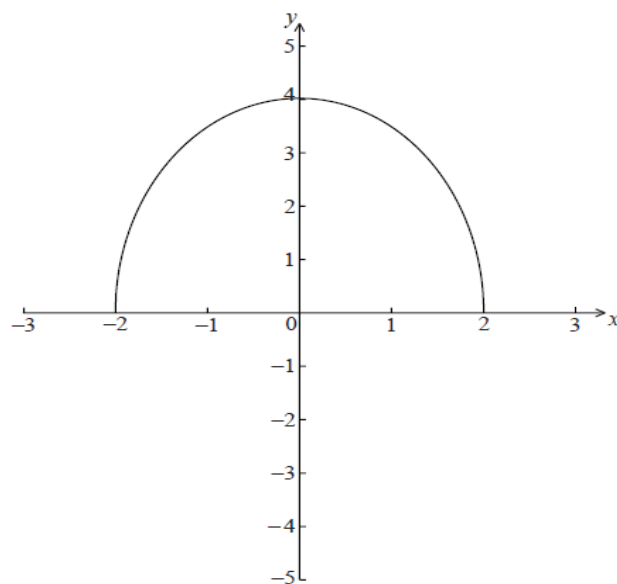
Write down an expression for V. [3 marks]

5c. Find V. [2 marks]

6. The graph of $f(x) = \sqrt{16 - 4x^2}$, for $-2 \le x \le 2$, is shown below. [no calculator on this problem]

The region enclosed by the curve of f and the x-axis is rotated $360°$ about the x-axis.

Find the volume of the solid formed. [6 marks]

7a. Let $f(x) = \sqrt{x}$. Line L is the normal to the graph of f at the point $(4, 2)$.]   [no calculator]
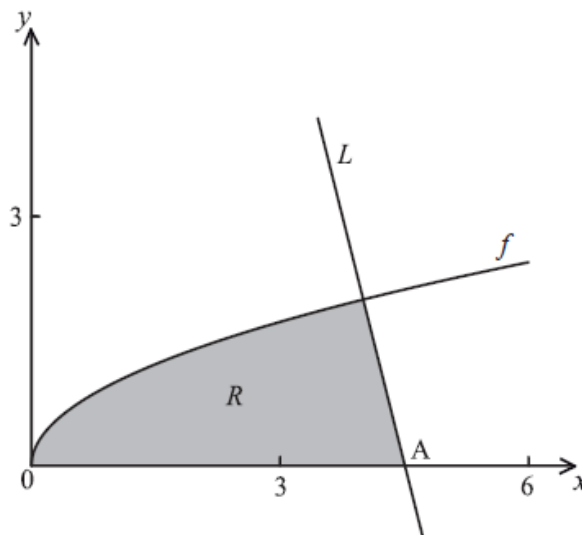
Show that the equation of L is $y = -4x + 18$.   [4 marks

7b. Point A is the x-intercept of L . Find the x-coordinate of A.   [2 marks]

7c. In the diagram below, the shaded region R is bounded by the x-axis, the graph of f and the line L .
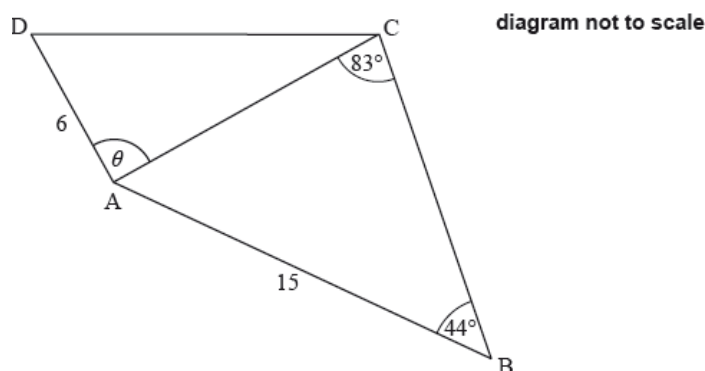

Find an expression for the area of R .   [3 marks]

7d. The region R is rotated $360°$ about the x-axis. Find the volume of the solid formed, giving your answer in terms of π .   [8 marks]

8a. The following diagram shows the quadrilateral $ABCD$.

diagram not to scale

$AD = 6 \text{ cm}, \ AB = 15 \text{ cm}, A\hat{B}C = 44°, A\hat{C}B = 83° \text{ and} D\hat{A}C = \theta$

Find AC.   [3 marks]

8b. Find the area of triangle ABC.   [3 marks]

8c. The area of triangle ACD is half the area of triangle ABC.

Find the possible values of θ.   [5 marks]

8d. Given that θ is obtuse, find CD.   [3 marks]

9a. Let L_x be a family of lines with equation given by $r = \begin{pmatrix} x \\ \frac{2}{x} \end{pmatrix} + t \begin{pmatrix} x^2 \\ -2 \end{pmatrix}$, where $x > 0$.

Write down the equation of L_1.   *[2 marks]*

9b. A line L_a crosses the y-axis at a point P.

Show that P has coordinates $\left(0, \ \frac{4}{a}\right)$.   *[6 marks]*

9c. The line L_a crosses the x-axis at $Q(2a, \ 0)$. Let $d = PQ^2$.

Show that $d = 4a^2 + \frac{16}{a^2}$.   *[2 marks]*

9d. There is a minimum value for d. Find the value of a that gives this minimum value.   *[7 marks]*

10a. The first two terms of a geometric sequence u_n are $u_1 = 4$ and $u_2 = 4.2$.

(i) Find the common ratio.

(ii) Hence or otherwise, find u_5.   *[5 marks]*

10b. Another sequence v_n is defined by $v_n = an^k$, where $a, \ k \in \mathbb{R}$, and $n \in \mathbb{Z}^+$, such that $v_1 = 0.05$ and $v_2 = 0.25$.

(i) Find the value of a.

(ii) Find the value of k.   *[5 marks]*

10c. Find the smallest value of n for which $v_n > u_n$.   *[5 marks]*