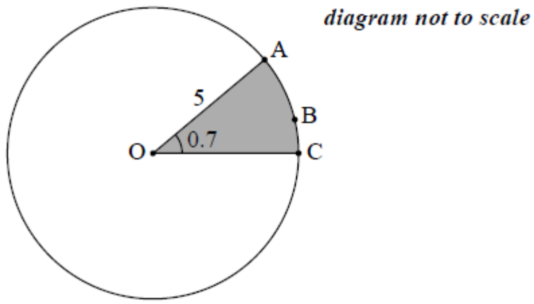
0418HW_Trig-problems2 *[81 marks]*

The following diagram shows a circle with centre
O and radius
$5\,\mathrm{cm}$.



The points
$\mathrm{A},$
rmB and
rmC lie on the circumference of the circle, and
$\mathrm{A\hat{O}C} = 0.7$ radians.

1a. Find the length of the arc
ABC. *[2 marks]*

Markscheme

correct substitution into arc length formula ***(A1)***
eg
0.7×5
arc length
$= 3.5$ (cm) ***A1 N2***
[2 marks]

1b. Find the perimeter of the shaded sector. *[2 marks]*

Markscheme

valid approach ***(M1)***
eg
$3.5 + 5 + 5,\ \mathrm{arc} + 2r$
perimeter
$= 13.5$ (cm) ***A1 N2***
[2 marks]

1c. Find the area of the shaded sector. *[2 marks]*

Markscheme

correct substitution into area formula *(A1)*

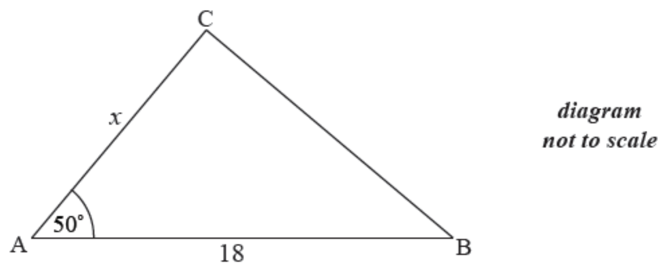
eg

$\frac{1}{2}(0.7)(5)^2$

$\text{area} = 8.75\ (\text{cm}^2)$ ***A1*** ***N2***

[2 marks]

The following diagram shows a triangle ABC.



The area of triangle ABC is 80 cm^2 , AB $= 18$ cm , AC $= x$ cm and $\text{B}\hat{\text{A}}\text{C} = 50^\circ$.

2a. Find x . *[3 marks]*

Markscheme

correct substitution into area formula *(A1)*

eg

$\frac{1}{2}(18x)\sin 50$

setting **their** area expression equal to 80 ***(M1)***

eg

$9x \sin 50 = 80$

$x = 11.6$ ***A1*** ***N2***

[3 marks]

2b. Find BC. *[3 marks]*

Markscheme

evidence of choosing cosine rule ***(M1)***

eg
$c^2 = a^2 + b^2 + 2ab \sin C$

correct substitution into right hand side (may be in terms of x) ***(A1)***

eg
$11.6^2 + 18^2 - 2(11.6)(18) \cos 50$

BC
$= 13.8$ ***A1*** ***N2***

[3 marks]

In triangle ABC, $\mathrm{AB} = 6\,\mathrm{cm}$ and $\mathrm{AC} = 8\,\mathrm{cm}$. The area of the triangle is $16\,\mathrm{cm}^2$.

3a. Find the two possible values for $\hat{A}$. *[4 marks]*

Markscheme

correct substitution into area formula ***(A1)***

eg
$\frac{1}{2}(6)(8) \sin A = 16,\ \ \sin A = \frac{16}{24}$

correct working ***(A1)***

eg
$A = \arcsin\left(\frac{2}{3}\right)$

$A = 0.729727656\ldots, 2.41186499\ldots;$
$(41.8103149°, 138.1896851°)$
$A = 0.730;$
2.41 ***A1A1*** ***N3***

(accept degrees *ie*
$41.8°;$
$138°$)

[4 marks]

3b. Given that $\hat{A}$ is obtuse, find BC. *[3 marks]*

Markscheme

evidence of choosing cosine rule ***(M1)***

eg

$\mathrm{BC}^2 = \mathrm{AB}^2 + \mathrm{AC}^2 - 2(\mathrm{AB})(\mathrm{AC})\cos A,\ a^2 + b^2 - 2ab\cos C$

correct substitution into RHS (angle must be obtuse) ***(A1)***

eg

$\mathrm{BC}^2 = 6^2 + 8^2 - 2(6)(8)\cos 2.41,\ 6^2 + 8^2 - 2(6)(8)\cos 138^\circ,$

$\mathrm{BC} = \sqrt{171.55}$

$\mathrm{BC} = 13.09786$

$\mathrm{BC} = 13.1\ \mathrm{cm}$ ***A1*** ***N2***

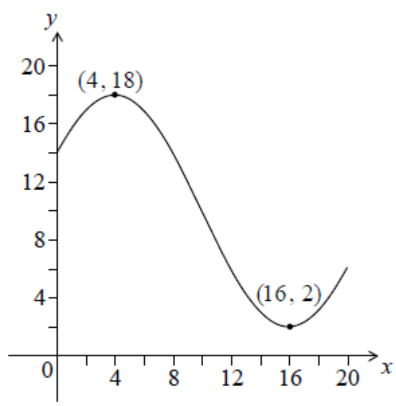
[3 marks]

Let

$f(x) = p\cos(q(x + r)) + 10$, for

$0 \leqslant x \leqslant 20$. The following diagram shows the graph of

f.



The graph has a maximum at

$(4, 18)$ and a minimum at

$(16, 2)$.

4a. Write down the value of

r. *[2 marks]*

Markscheme

$r = -4$ ***A2*** ***N2***

Note: Award ***A1*** for

$r = 4$.

[2 marks]

4b. Find

p. *[2 marks]*

Markscheme

evidence of valid approach *(M1)*

eg
$\frac{\max y \text{ value} -- y \text{ value}}{2}$, distance from
$y = 10$

$p = 8$ ***A1*** ***N2***

[2 marks]

4c. Find *[2 marks]*
q.

Markscheme

valid approach *(M1)*

eg period is
$24,$
$\frac{360}{24}$, substitute a point into **their**
$f(x)$

$q = \frac{2\pi}{24}\left(\frac{\pi}{12}, \text{ exact}\right),$

0.262 (do not accept degrees) ***A1*** ***N2***

[2 marks]

4d. Solve *[2 marks]*
$f(x) = 7.$

Markscheme

valid approach *(M1)*

eg line on graph at
$y = 7,\ 8\cos\left(\frac{2\pi}{24}(x-4)\right) + 10 = 7$

$x = 11.46828$

$x = 11.5$ (accept
$(11.5, 7)$) ***A1*** ***N2***

[2 marks]

Note: Do not award the final ***A1*** if additional values are given. If an incorrect value of
q leads to multiple solutions, award the final ***A1*** only if **all** solutions within the domain are given.

Let
$f(x) = \sin\left(x + \frac{\pi}{4}\right) + k$. The graph of *f* passes through the point
$\left(\frac{\pi}{4}, 6\right)$.

5a. Find the value of *[3 marks]*
k.

Markscheme

METHOD 1

attempt to substitute both coordinates (in any order) into

f ***(M1)***

eg

$f\left(\frac{\pi}{4}\right)=6,\ \frac{\pi}{4}=\sin\left(6+\frac{\pi}{4}\right)+k$

correct working ***(A1)***

eg

$\sin\frac{\pi}{2}=1,\ 1+k=6$

$k=5$ ***A1*** ***N2***

[3 marks]

METHOD 2

recognizing shift of

$\frac{\pi}{4}$ left means maximum at

6 ***R1)***

recognizing

k is difference of maximum and amplitude ***(A1)***

eg

$6-1$

$k=5$ ***A1*** ***N2***

[3 marks]

5b. Find the minimum value of *[2 marks]*

$f(x)$.

Markscheme

evidence of appropriate approach ***(M1)***

eg minimum value of

$\sin x$ is

$-1,\ -1+k,\ f'(x)=0,\ \left(\frac{5\pi}{4},\ 4\right)$

minimum value is

4 ***A1*** ***N2***

[2 marks]

5c. Let *[2 marks]*

$g(x)=\sin x$. The graph of g is translated to the graph of

f by the vector

$\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

p and of

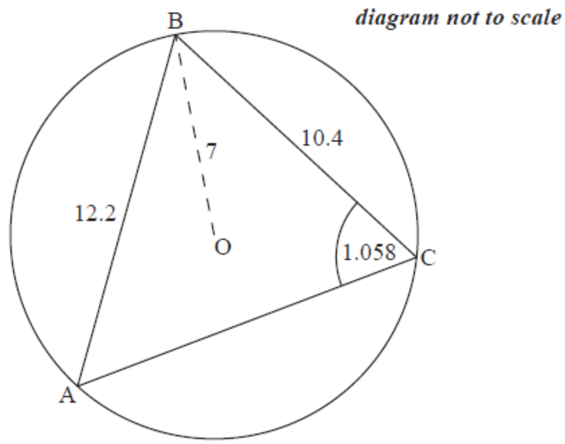
q.

Markscheme

$p=-\frac{\pi}{4},\ q=5\ \left(\text{accept}\begin{pmatrix} -\frac{\pi}{4} \\ 5 \end{pmatrix}\right)$ ***A1A1*** ***N2***

[2 marks]

Consider a circle with centre O and radius 7 cm. Triangle ABC is drawn such that its vertices are on the circumference of the circle.



$\mathrm{AB} = 12.2$ cm,
$\mathrm{BC} = 10.4$ cm and
$\mathrm{A\hat{C}B} = 1.058$ radians.

6a. Find $\mathrm{B\hat{A}C}$. *[3 marks]*

Markscheme

Notes: In this question, there may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

Candidates may have their GDCs in degree mode, leading to incorrect answers. If working shown, award marks in line with the markscheme, with ***FT*** as appropriate.

Ignore missing or incorrect units.

evidence of choosing sine rule ***(M1)***

eg

$\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b}$

correct substitution ***(A1)***

eg

$\frac{\sin \hat{A}}{10.4} = \frac{\sin 1.058}{12.2}$

$\mathrm{B\hat{A}C} = 0.837$ ***A1 N2***

[3 marks]

6b. Find AC. *[5 marks]*

Markscheme

Notes: In this question, there may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.
Candidates may have their GDCs in degree mode, leading to incorrect answers. If working shown, award marks in line with the markscheme, with ***FT*** as appropriate.
Ignore missing or incorrect units.

METHOD 1
evidence of subtracting angles from
π ***(M1)***
eg
$\mathrm{A\hat{B}C} = \pi - A - C$
correct angle (seen anywhere) ***A1***
$\mathrm{A\hat{B}C} = \pi - 1.058 - 0.837,\ 1.246,\ 71.4^\circ$
attempt to substitute into cosine or sine rule ***(M1)***
correct substitution ***(A1)***
eg
$12.2^2 + 10.4^2 - 2 \times 12.2 \times 10.4 \cos 71.4,\ \frac{\mathrm{AC}}{\sin 1.246} = \frac{12.2}{\sin 1.058}$
$\mathrm{AC} = 13.3\ (\mathrm{cm})$ ***A1 N3***
METHOD 2
evidence of choosing cosine rule ***M1***
eg
$a^2 = b^2 + c^2 - 2bc\cos A$
correct substitution ***(A2)***
eg
$12.2^2 = 10.4^2 + b^2 - 2 \times 10.4b\cos 1.058$
$\mathrm{AC} = 13.3\ (\mathrm{cm})$ ***A2 N3***
[5 marks]

6c. Hence or otherwise, find the length of arc *[6 marks]*
ABC.

Markscheme

Notes: In this question, there may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.
Candidates may have their GDCs in degree mode, leading to incorrect answers. If working shown, award marks in line with the markscheme, with ***FT*** as appropriate.
Ignore missing or incorrect units.

METHOD 1
valid approach ***(M1)***
eg
$\cos \mathrm{A\hat{O}C} = \frac{\mathrm{OA}^2 + \mathrm{OC}^2 - \mathrm{AC}^2}{2 \times \mathrm{OA} \times \mathrm{OC}}$,
$\mathrm{A\hat{O}C} = 2 \times \mathrm{A\hat{B}C}$
correct working ***(A1)***
eg
$13.3^2 = 7^2 + 7^2 - 2 \times 7 \times 7\cos \mathrm{A\hat{O}C},\ O = 2 \times 1.246$
$\mathrm{A\hat{O}C} = 2.492\ (142.8^\circ)$ ***(A1)***
EITHER
correct substitution for arc length (seen anywhere) ***A1***

eg

$2.492 = \frac{l}{7},\ l = 17.4,\ 14\pi \times \frac{142.8}{360}$

subtracting arc from circumference ***(M1)***

eg

$2\pi r - l,\ 14\pi = 17.4$

OR

attempt to find

$\mathrm{A\hat{O}C}$ reflex ***(M1)***

eg

$2\pi - 2.492,\ 3.79,\ 360 - 142.8$

correct substitution for arc length (seen anywhere) ***A1***

eg

$l = 7 \times 3.79,\ 14\pi \times \frac{217.2}{360}$

THEN

$\text{arc ABC} = 26.5$ ***A1*** ***N4***

METHOD 2

valid approach to find

$\mathrm{A\hat{O}B}$ or

$\mathrm{B\hat{O}C}$ ***(M1)***

eg choosing cos rule, twice angle at circumference

correct working for finding **one** value,

$\mathrm{A\hat{O}B}$ or

$\mathrm{B\hat{O}C}$ ***(A1)***

eg

$\cos \mathrm{A\hat{O}B} = \frac{7^2+7^2-12.2^2}{2\times7\times7}$,

$\mathrm{A\hat{O}B} = 2.116, \mathrm{B\hat{O}C} = 1.6745$

two correct calculations for arc lengths

eg

$\mathrm{AB} = 7 \times 2 \times 1.058\ (= 14.8135),\ 7 \times 1.6745\ (= 11.7216)$ ***(A1)(A1)***

adding **their** arc lengths (seen anywhere)

eg

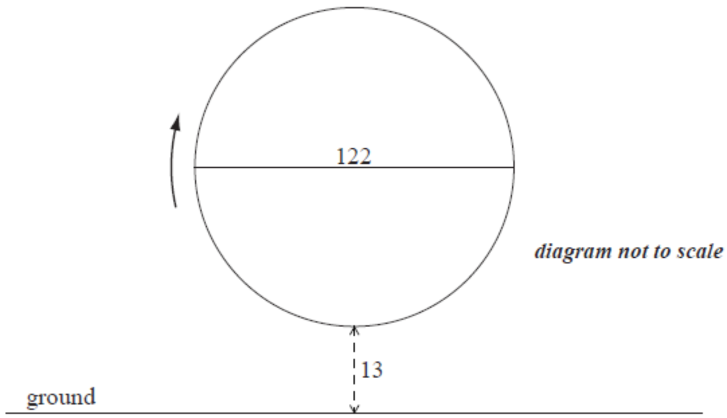
$r\mathrm{A\hat{O}B} + r\mathrm{B\hat{O}C},\ 14.8135 + 11.7216,\ 7(2.116 + 1.6745)$ ***M1***

$\text{arc ABC} = 26.5\ (\text{cm})$ ***A1*** ***N4***

Note: Candidates may work with other interior triangles using a similar method. Check calculations carefully and award marks in line with markscheme.

[6 marks]

A Ferris wheel with diameter 122 metres rotates clockwise at a constant speed. The wheel completes 2.4 rotations every hour. The bottom of the wheel is 13 metres above the ground.



A seat starts at the bottom of the wheel.

7a. Find the maximum height above the ground of the seat. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg
$13 + \text{diameter}$,
$13 + 122$

maximum height
$= 135$ (m) ***A1 N2***

[2 marks]

After ***t*** minutes, the height h metres above the ground of the seat is given by

$$h = 74 + a\cos bt.$$

7b. (i) Show that the period of h is 25 minutes. *[2 marks]*

(ii) Write down the **exact** value of b.

Markscheme

(i) period $= \frac{60}{2.4}$ ***A1***

period $= 25$ minutes ***AG N0***

(ii)
$b = \frac{2\pi}{25}$
$(= 0.08\pi)$ ***A1 N1***

[2 marks]

7c. Find the value of a . *[3 marks]*

Markscheme

METHOD 1

valid approach ***(M1)***

eg
$\max - 74$,
$|a| = \frac{135-13}{2}$,
$74 - 13$

$|a| = 61$ (accept $a = 61$) ***(A1)***

$a = -61$ ***A1*** ***N2***

METHOD 2

attempt to substitute valid point into equation for ***h*** ***(M1)***

eg
$135 = 74 + a\cos\left(\frac{2\pi\times 12.5}{25}\right)$

correct equation ***(A1)***

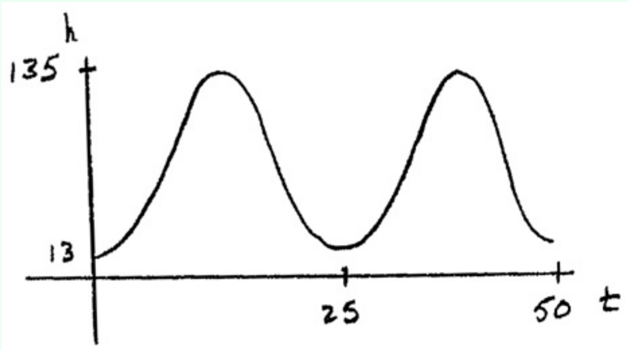
eg
$135 = 74 + a\cos(\pi)$,
$13 = 74 + a$

$a = -61$ ***A1*** ***N2***

[3 marks]

7d. Sketch the graph of h , for $0 \le t \le 50$. *[4 marks]*

Markscheme



A1A1A1A1 ***N4***

Note: Award ***A1*** for approximately correct domain, ***A1*** for approximately correct range,

A1 for approximately correct sinusoidal shape with 2 cycles.

Only if this last ***A1*** awarded, award ***A1*** for max/min in approximately correct positions.

[4 marks]

7e. In one rotation of the wheel, find the probability that a randomly selected seat is at least 105 metres above the ground. *[5 marks]*

Markscheme

setting up inequality (accept equation) ***(M1)***

eg
$h > 105$,
$105 = 74 + a \cos bt$, sketch of graph with line
$y = 105$

any **two** correct values for ***t*** (seen anywhere) ***A1A1***

eg
$t = 8.371\ldots$,
$t = 16.628\ldots$,
$t = 33.371\ldots$,
$t = 41.628\ldots$

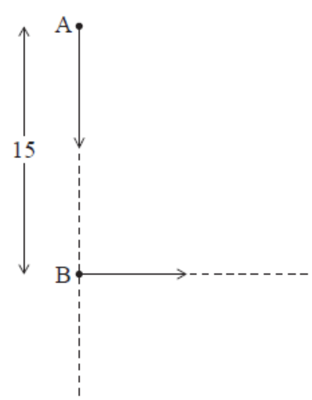
valid approach ***M1***

eg
$\frac{16.628-8.371}{25}$,
$\frac{t_1-t_2}{25}$,
$\frac{2\times 8.257}{50}$,
$\frac{2(12.5-8.371)}{25}$

$p = 0.330$ ***A1*** ***N2***

[5 marks]

The following diagram shows two ships A and B. At noon, ship A was 15 km duenorth of ship B. Ship A was moving south at 15 km h^{-1} and ship B was moving east at11 km h^{-1}.



8a. Find the distance between the ships *[5 marks]*

(i) at 13:00;

(ii) at 14:00.

Markscheme

(i) evidence of valid approach ***(M1)***

e.g. ship A where B was, B 11 km away

$\text{distance} = 11$ ***A1*** ***N2***

(ii) evidence of valid approach ***(M1)***

e.g. new diagram, Pythagoras, vectors

$s = \sqrt{15^2 + 22^2}$ ***(A1)***

$\sqrt{709} = 26.62705$

$s = 26.6$ ***A1*** ***N2***

Note: Award ***M0A0A0*** for using the formula given in part (b).

[5 marks]

8b. Let $s(t)$ be the distance between the ships t hours after noon, for $0 \le t \le 4$. *[6 marks]*

Show that $s(t) = \sqrt{346t^2 - 450t + 225}$.

Markscheme

evidence of valid approach ***(M1)***

e.g. a table, diagram, formula $d = r \times t$

distance ship A travels t hours after noon is $15(t-1)$ ***(A2)***

distance ship B travels in t hours after noon is $11t$ ***(A1)***

evidence of valid approach ***M1***

e.g. $s(t) = \sqrt{[15(t-1)]^2 + (11t)^2}$

correct simplification ***A1***

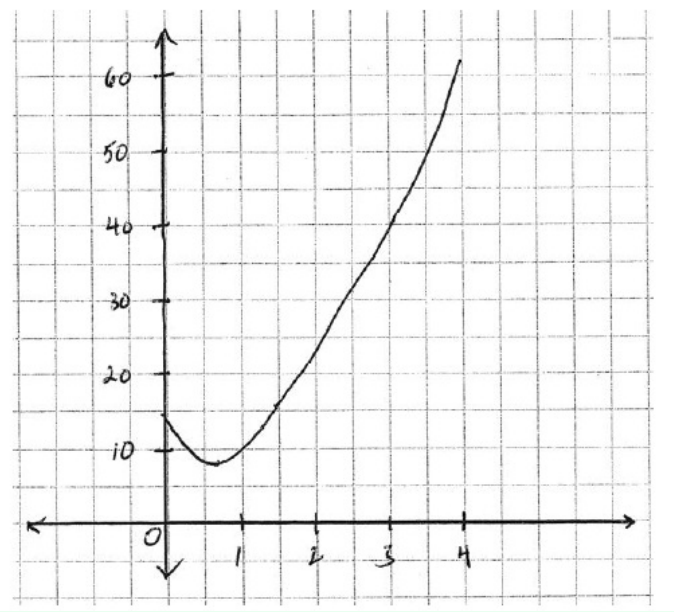
e.g. $\sqrt{225(t^2 - 2t + 1) + 121t^2}$

$s(t) = \sqrt{346t^2 - 450t + 225}$ ***AG*** ***N0***

[6 marks]

8c. Sketch the graph of $s(t)$. *[3 marks]*

Markscheme



A1A1A1 N3

Note: Award ***A1*** for shape, ***A1*** for minimum at approximately $(0.7, 9)$, ***A1*** for domain.

[3 marks]

8d. Due to poor weather, the captain of ship A can only see another ship if they are less than 8 km apart. Explain why the captain cannot see ship B between noon and 16:00. *[3 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g.
$s'(t) = 0$, find minimum of
$s(t)$, graph, reference to "more than 8 km"

$\text{min} = 8.870455\ldots$ (accept 2 or more sf) ***A1***

since
$s_{\text{min}} > 8$, captain cannot see ship B ***R1 N0***

[3 marks]