1-2-P1-Algebra-logs *[129 marks]*

Find the value of each of the following, giving your answer as an integer.

1a. $\log_6 36$ *[2 marks]*

Markscheme

correct approach ***(A1)***

eg
$6^x = 36,\ 6^2$

2 ***A1 N2***

[2 marks]

1b. $\log_6 4 + \log_6 9$ *[2 marks]*

Markscheme

correct simplification ***(A1)***

eg
$\log_6 36,\ \log(4 \times 9)$

2 ***A1 N2***

[2 marks]

1c. $\log_6 2 - \log_6 12$ *[3 marks]*

Markscheme

correct simplification ***(A1)***

eg
$\log_6 \frac{2}{12},\ \log(2 \div 12)$

correct working ***(A1)***

eg
$\log_6 \frac{1}{6},\ 6^{-1} = \frac{1}{6}, 6^x = \frac{1}{6}$

-1 ***A1 N2***

[3 marks]

Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y.

2a. $\ln\left(\frac{5}{3}\right)$. *[2 marks]*

Markscheme

correct approach *(A1)*

eg $\ln 5 - \ln 3$

$\ln\left(\frac{5}{3}\right) = y - x$ ***A1 N2***

[2 marks]

2b. $\ln 45$. *[4 marks]*

Markscheme

recognizing factors of 45 (may be seen in log expansion) ***(M1)***

eg $\ln(9 \times 5),\ 3 \times 3 \times 5,\ \log 3^2 \times \log 5$

correct application of $\log(ab) = \log a + \log b$ ***(A1)***

eg $\ln 9 + \ln 5,\ \ln 3 + \ln 3 + \ln 5,\ \ln 3^2 + \ln 5$

correct working ***(A1)***

eg $2\ln 3 + \ln 5,\ x + x + y$

$\ln 45 = 2x + y$ ***A1 N3***

[4 marks]

Let
$\log_3 p = 6$ and
$\log_3 q = 7$.

3a. Find
$\log_3 p^2$. *[2 marks]*

Markscheme

METHOD 1

evidence of correct formula ***(M1)***

eg
$\log u^n = n \log u$,
$2\log_3 p$

$\log_3(p^2) = 12$ ***A1 N2***

METHOD 2

valid method using
$p = 3^6$ ***(M1)***

eg
$\log_3 (3^6)^2$,
$\log 3^{12}$,
$12\log_3 3$

$\log_3(p^2) = 12$ ***A1 N2***

[2 marks]

3b. Find
$\log_3\left(\frac{p}{q}\right)$. *[2 marks]*

Markscheme

METHOD 1

evidence of correct formula ***(M1)***

eg
$\log\left(\frac{p}{q}\right) = \log p - \log q$,
$6 - 7$

$\log_3\left(\frac{p}{q}\right) = -1$ ***A1 N2***

METHOD 2

valid method using
$p = 3^6$ and
$q = 3^7$ ***(M1)***

eg
$\log_3\left(\frac{3^6}{3^7}\right)$,
$\log 3^{-1}$,
$-\log_3 3$

$\log_3\left(\frac{p}{q}\right) = -1$ ***A1 N2***

[2 marks]

3c. Find
$\log_3(9p)$. *[3 marks]*

Markscheme

METHOD 1

evidence of correct formula ***(M1)***

eg
$\log_3 uv = \log_3 u + \log_3 v$,
$\log 9 + \log p$

$\log_3 9 = 2$ (may be seen in expression) ***A1***

eg
$2 + \log p$

$\log_3(9p) = 8$ ***A1 N2***

METHOD 2

valid method using
$p = 3^6$ ***(M1)***

eg
$\log_3(9 \times 3^6)$,
$\log_3(3^2 \times 3^6)$

correct working ***A1***

eg
$\log_3 9 + \log_3 3^6$,
$\log_3 3^8$

$\log_3(9p) = 8$ ***A1 N2***

[3 marks]

Total [7 marks]

4a. Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n. *[2 marks]*

Markscheme

$m = 3,\ n = 4$ ***A1A1*** ***N2***

[2 marks]

4b. Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. *[4 marks]*

Markscheme

attempt to apply $(2^a)^b = 2^{ab}$ ***(M1)***

eg $6x + 3,\ 4(2x - 3)$

equating **their** powers of 2 (seen anywhere) ***M1***

eg $3(2x + 1) = 8x - 12$

correct working ***A1***

eg $8x - 12 = 6x + 3,\ 2x = 15$

$x = \frac{15}{2}\quad (7.5)$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

5a. Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$. *[3 marks]*

Markscheme

correct application of $\ln a^b = b\ln a$ (seen anywhere) ***(A1)***

eg $\ln 4 = 2\ln 2,\ 3\ln 2 = \ln 2^3,\ 3\log 2 = \log 8$

correct working ***(A1)***

eg $3\ln 2 - 2\ln 2,\ \ln 8 - \ln 4$

$\ln 2\quad (\text{accept } k = 2)$ ***A1*** ***N2***

[3 marks]

5b. Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$. *[3 marks]*

Markscheme

METHOD 1

attempt to substitute **their** answer into the equation *(M1)*

eg $\ln 2 = -\ln x$

correct application of a log rule *(A1)*

eg $\ln \frac{1}{x},\ \ln \frac{1}{2} = \ln x,\ \ln 2 + \ln x = \ln 2x \quad (= 0)$

$x = \frac{1}{2}$ ***A1 N2***

METHOD 2

attempt to rearrange equation, with $3\ln 2$ written as $\ln 2^3$ or $\ln 8$ *(M1)*

eg $\ln x = \ln 4 - \ln 2^3,\ \ln 8 + \ln x = \ln 4,\ \ln 2^3 = \ln 4 - \ln x$

correct working applying $\ln a \pm \ln b$ *(A1)*

eg $\frac{4}{8},\ 8x = 4,\ \ln 2^3 = \ln \frac{4}{x}$

$x = \frac{1}{2}$ ***A1 N2***

[3 marks]

Total [6 marks]

Write down the value of

6a. (i) $\log_3 27$; *[1 mark]*

Markscheme

(i)
$\log_3 27 = 3$ ***A1 N1***

[1 mark]

6b. (ii) $\log_8 \frac{1}{8}$; *[1 mark]*

Markscheme

(ii)
$\log_8 \frac{1}{8} = -1$ ***A1 N1***

[1 mark]

6c. (iii) $\log_{16} 4$. *[1 mark]*

Markscheme

(iii)
$\log_{16} 4 = \frac{1}{2}$ ***A1 N1***

[1 mark]

6d. Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$. *[3 marks]*

Markscheme

correct equation with **their** three values ***(A1)***

eg $\frac{3}{2} = \log_4 x,\ 3 + (-1) - \frac{1}{2} = \log_4 x$

correct working involving powers ***(A1)***

eg $x = 4^{\frac{3}{2}},\ 4^{\frac{3}{2}} = 4^{\log_4 x}$

$x = 8$ ***A1 N2***

[3 marks]

7a. Find $\log_2 32$. *[1 mark]*

Markscheme

5 ***A1 N1***

[1 mark]

7b. Given that $\log_2 \left(\frac{32^x}{8^y}\right)$ can be written as $px + qy$, find the value of *p* and of *q*. *[4 marks]*

Markscheme

METHOD 1

$\log_2 \left(\frac{32^x}{8^y}\right) = \log_2 32^x - \log_2 8^y$ ***(A1)***

$= x\log_2 32 - y\log_2 8$ ***(A1)***

$\log_2 8 = 3$ ***(A1)***

$p = 5$, $q = -3$ (accept $5x - 3y$) ***A1 N3***

METHOD 2

$\frac{32^x}{8^y} = \frac{(2^5)^x}{(2^3)^y}$ ***(A1)***

$= \frac{2^{5x}}{2^{3y}}$ ***(A1)***

$= 2^{5x-3y}$ ***(A1)***

$\log_2 (2^{5x-3y}) = 5x - 3y$

$p = 5$, $q = -3$ (accept $5x - 3y$) ***A1 N3***

[4 marks]

Let
$f(x) = 3 \ln x$ and
$g(x) = \ln 5x^3$.

8a. Express
$g(x)$ in the form
$f(x) + \ln a$, where
$a \in \mathbb{Z}^+$. *[4 marks]*

Markscheme

attempt to apply rules of logarithms ***(M1)***

e.g.
$\ln a^b = b \ln a$,
$\ln ab = \ln a + \ln b$

correct application of
$\ln a^b = b \ln a$ (seen anywhere) ***A1***

e.g.
$3 \ln x = \ln x^3$

correct application of
$\ln ab = \ln a + \ln b$ (seen anywhere) ***A1***

e.g.
$\ln 5x^3 = \ln 5 + \ln x^3$

so
$\ln 5x^3 = \ln 5 + 3 \ln x$

$g(x) = f(x) + \ln 5$ (accept
$g(x) = 3 \ln x + \ln 5$) ***A1*** ***N1***

[4 marks]

8b. The graph of *g* is a transformation of the graph of *f* . Give a full geometric description of this transformation. *[3 marks]*

Markscheme

transformation with correct name, direction, and value ***A3***

e.g. translation by
$\begin{pmatrix} 0 \\ \ln 5 \end{pmatrix}$, shift up by
$\ln 5$, vertical translation of
$\ln 5$

[3 marks]

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9. Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$. *[7 marks]*

Markscheme

correct application of $\log a + \log b = \log ab$ ***(A1)***

eg $\log_2(2\sin x\cos x),\ \log 2 + \log(\sin x) + \log(\cos x)$

correct equation without logs ***A1***

eg $2\sin x\cos x = 2^{-1},\ \sin x\cos x = \frac{1}{4},\ \sin 2x = \frac{1}{2}$

recognizing double-angle identity (seen anywhere) ***A1***

eg $\log(\sin 2x),\ 2\sin x\cos x = \sin 2x,\ \sin 2x = \frac{1}{2}$

evaluating $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\ (30°)$ ***(A1)***

correct working ***A1***

eg $x = \frac{\pi}{12} + 2\pi,\ 2x = \frac{25\pi}{6},\ \frac{29\pi}{6},\ 750°,\ 870°,\ x = \frac{\pi}{12}$**and** $x = \frac{5\pi}{12}$, one correct final answer

$x = \frac{25\pi}{12},\ \frac{29\pi}{12}$ (do not accept additional values) ***A2*** ***N0***

[7 marks]

10. Solve $\log_2 x + \log_2(x-2) = 3$, for $x > 2$. *[7 marks]*

Markscheme

recognizing $\log a + \log b = \log ab$ (seen anywhere) ***(A1)***

e.g. $\log_2(x(x-2))$, $x^2 - 2x$

recognizing $\log_a b = x \Leftrightarrow a^x = b$ ***(A1)***

e.g. $2^3 = 8$

correct simplification ***A1***

e.g. $x(x-2) = 2^3$, $x^2 - 2x - 8$

evidence of correct approach to solve ***(M1)***

e.g. factorizing, quadratic formula

correct working ***A1***

e.g. $(x-4)(x+2)$, $\frac{2\pm\sqrt{36}}{2}$

$x = 4$ ***A2*** ***N3***

[7 marks]

Let $f(x) = e^{x+3}$.

11a. (i) Show that $f^{-1}(x) = \ln x - 3$. *[3 marks]*

(ii) Write down the domain of f^{-1} .

Markscheme

(i) interchanging *x* and *y* (seen anywhere) ***M1***

e.g.
$x = \mathrm{e}^{y+3}$

correct manipulation ***A1***

e.g.
$\ln x = y + 3$,
$\ln y = x + 3$

$f^{-1}(x) = \ln x - 3$ ***AG*** ***N0***

(ii)
$x > 0$ ***A1*** ***N1***

[3 marks]

11b. Solve the equation
$f^{-1}(x) = \ln \frac{1}{x}$.

[4 marks]

Markscheme

collecting like terms; using laws of logs ***(A1)(A1)***

e.g.
$\ln x - \ln\left(\frac{1}{x}\right) = 3$,
$\ln x + \ln x = 3$,
$\ln\left(\frac{x}{\frac{1}{x}}\right) = 3$,
$\ln x^2 = 3$

simplify ***(A1)***

e.g.
$\ln x = \frac{3}{2}$,
$x^2 = \mathrm{e}^3$

$x = \mathrm{e}^{\frac{3}{2}} \left(= \sqrt{\mathrm{e}^3}\right)$ ***A1*** ***N2***

[4 marks]

12a. Find the value of
$\log_2 40 - \log_2 5$.

[3 marks]

Markscheme

evidence of correct formula ***(M1)***

eg
$\log a - \log b = \log \frac{a}{b}$,
$\log\left(\frac{40}{5}\right)$,
$\log 8 + \log 5 - \log 5$

Note: Ignore missing or incorrect base.

correct working ***(A1)***

eg
$\log_2 8$,
$2^3 = 8$

$\log_2 40 - \log_2 5 = 3$ ***A1 N2***

[3 marks]

12b. Find the value of $8^{\log_2 5}$. *[4 marks]*

Markscheme

attempt to write 8 as a power of 2 (seen anywhere) ***(M1)***

eg
$(2^3)^{\log_2 5}$,
$2^3 = 8$,
2^a

multiplying powers ***(M1)***

eg
$2^{3\log_2 5}$,
$a\log_2 5$

correct working ***(A1)***

eg
$2^{\log_2 125}$,
$\log_2 5^3$,
$\left(2^{\log_2 5}\right)^3$

$8^{\log_2 5} = 125$ ***A1 N3***

[4 marks]

Let $f(x) = log_3\sqrt{x}$, for $x > 0$.

13a. Show that $f^{-1}(x) = 3^{2x}$. *[2 marks]*

Markscheme

interchanging *x* and *y* (seen anywhere) ***(M1)***

e.g.
$x = \log\sqrt{y}$ (accept any base)

evidence of correct manipulation ***A1***

e.g.
$3^x = \sqrt{y}$,
$3^y = x^{\frac{1}{2}}$,
$x = \frac{1}{2}\log_3 y$,
$2y = \log_3 x$

$f^{-1}(x) = 3^{2x}$ ***AG*** ***N0***

[2 marks]

13b. Write down the range of f^{-1} . *[1 mark]*

Markscheme

$y > 0$,
$f^{-1}(x) > 0$ ***A1*** ***N1***

[1 mark]

13c. Let $g(x) = \log_3 x$, for $x > 0$. *[4 marks]*

Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

Markscheme

METHOD 1

finding
$g(2) = log_3 2$ (seen anywhere) ***A1***

attempt to substitute ***(M1)***

e.g.
$(f^{-1} \circ g)(2) = 3^{2\log_3 2}$

evidence of using log or index rule ***(A1)***

e.g.
$(f^{-1} \circ g)(2) = 3^{\log_3 4}$,
$3^{\log_3 2^2}$

$(f^{-1} \circ g)(2) = 4$ ***A1*** ***N1***

METHOD 2

attempt to form composite (in any order) ***(M1)***

e.g.
$(f^{-1} \circ g)(x) = 3^{2\log_3 x}$

evidence of using log or index rule ***(A1)***

e.g.
$(f^{-1} \circ g)(x) = 3^{\log_3 x^2}$,
$3^{\log_3 x}{}^2$

$(f^{-1} \circ g)(x) = x^2$ ***A1***

$(f^{-1} \circ g)(2) = 4$ ***A1*** ***N1***

[4 marks]

Let
$f(x) = k\log_2 x$.

14a. Given that
$f^{-1}(1) = 8$, find the value of
k . *[3 marks]*

Markscheme

METHOD 1

recognizing that
$f(8) = 1$ ***(M1)***

e.g.
$1 = k\log_2 8$

recognizing that
$\log_2 8 = 3$ ***(A1)***

e.g.
$1 = 3k$

$k = \frac{1}{3}$ ***A1 N2***

METHOD 2

attempt to find the inverse of
$f(x) = k\log_2 x$ ***(M1)***

e.g.
$x = k\log_2 y$,
$y = 2^{\frac{x}{k}}$

substituting 1 and 8 ***(M1)***

e.g.
$1 = k\log_2 8$,
$2^{\frac{1}{k}} = 8$

$k = \frac{1}{\log_2 8}$
$\left(k = \frac{1}{3}\right)$ ***A1 N2***

[3 marks]

14b. Find
$f^{-1}\left(\frac{2}{3}\right)$. *[4 marks]*

Markscheme

METHOD 1

recognizing that
$f(x) = \frac{2}{3}$ ***(M1)***

e.g.
$\frac{2}{3} = \frac{1}{3}\log_2 x$

$\log_2 x = 2$ ***(A1)***

$f^{-1}\left(\frac{2}{3}\right) = 4$ (accept
$x = 4$) ***A2*** ***N3***

METHOD 2

attempt to find inverse of
$f(x) = \frac{1}{3}\log_2 x$ ***(M1)***

e.g. interchanging *x* and *y* , substituting
$k = \frac{1}{3}$ into
$y = 2^{\frac{x}{k}}$

correct inverse ***(A1)***

e.g.
$f^{-1}(x) = 2^{3x}$,
2^{3x}

$f^{-1}\left(\frac{2}{3}\right) = 4$ ***A2*** ***N3***

[4 marks]

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of
f has a maximum point at P.

15a. Find the x-coordinate of P. *[3 marks]*

Markscheme

recognizing $f'(x) = 0$ ***(M1)***

correct working ***(A1)***

eg $6 - 2x = 0$

$x = 3$ ***A1*** ***N2***

[3 marks]

The
y-coordinate of P is $\ln 27$.

15b. Find $f(x)$, expressing your answer as a single logarithm. *[8 marks]*

Markscheme

evidence of integration ***(M1)***

eg $\int f'$, $\int \frac{6-2x}{6x-x^2}\mathrm{d}x$

using substitution ***(A1)***

eg $\int \frac{1}{u}\mathrm{d}u$ where $u = 6x - x^2$

correct integral ***A1***

eg $\ln(u) + c,\ \ln(6x - x^2)$

substituting $(3,\ \ln 27)$ into **their** integrated expression (must have c) ***(M1)***

eg $\ln(6 \times 3 - 3^2) + c = \ln 27,\ \ln(18 - 9) + \ln k = \ln 27$

correct working ***(A1)***

eg $c = \ln 27 - \ln 9$

EITHER

$c = \ln 3$ ***(A1)***

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 3$ ***A1 N4***

OR

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 27 - \ln 9$

correct use of a log law ***(A1)***

eg $f(x) = \ln(6x - x^2) + \ln\left(\frac{27}{9}\right),\ f(x) = \ln\left(27(6x - x^2)\right) - \ln 9$

$f(x) = \ln\left(3(6x - x^2)\right)$ ***A1 N4***

[8 marks]

15c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates $(a,\ b)$.

Find the value of a and of b, where $a,\ b \in \mathbb{N}$.

Markscheme

$a = 3$ ***A1 N1***

correct working ***A1***

eg $\frac{\ln 27}{\ln 3}$

correct use of log law ***(A1)***

eg $\frac{3\ln 3}{\ln 3},\ \log_3 27$

$b = 3$ ***A1 N2***

[4 marks]

The first two terms of an infinite geometric sequence, in order, are

$$2\log_2 x,\ \log_2 x, \text{ where } x > 0.$$

16a. Find r. *[2 marks]*

Markscheme

evidence of dividing terms (in any order) ***(M1)***

eg $\frac{\mu_2}{\mu_1}$, $\frac{2\log_2 x}{\log_2 x}$

$r = \frac{1}{2}$ ***A1 N2***

[2 marks]

16b. Show that the sum of the infinite sequence is $4\log_2 x$. *[2 marks]*

Markscheme

correct substitution ***(A1)***

eg $\frac{2\log_2 x}{1-\frac{1}{2}}$

correct working ***A1***

eg $\frac{2\log_2 x}{\frac{1}{2}}$

$S_\infty = 4\log_2 x$ ***AG N0***

[2 marks]

The first three terms of an arithmetic sequence, in order, are

$$\log_2 x,\ \log_2\left(\frac{x}{2}\right),\ \log_2\left(\frac{x}{4}\right), \text{ where } x > 0.$$

16c. Find d, giving your answer as an integer. *[4 marks]*

Markscheme

evidence of subtracting two terms (in any order) ***(M1)***

eg $u_3 - u_2$, $\log_2 x - \log_2\frac{x}{2}$

correct application of the properties of logs ***(A1)***

eg $\log_2\left(\frac{\frac{x}{2}}{x}\right)$, $\log_2\left(\frac{x}{2} \times \frac{1}{x}\right)$, $(\log_2 x - \log_2 2) - \log_2 x$

correct working ***(A1)***

eg $\log_2\frac{1}{2}$, $-\log_2 2$

$d = -1$ ***A1 N3***

[4 marks]

Let S_{12} be the sum of the first 12 terms of the arithmetic sequence.

16d. Show that $S_{12} = 12\log_2 x - 66$. *[2 marks]*

Markscheme

correct substitution into the formula for the sum of an arithmetic sequence ***(A1)***

eg $\frac{12}{2}(2\log_2 x + (12-1)(-1))$

correct working ***A1***

eg $6(2\log_2 x - 11),\ \frac{12}{2}(2\log_2 x - 11)$

$12\log_2 x - 66$ ***AG*** ***N0***

[2 marks]

16e. Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x, giving your answer in the form 2^p, where $p \in \mathbb{Q}$. *[3 marks]*

Markscheme

correct equation ***(A1)***

eg $12\log_2 x - 66 = 2\log_2 x$

correct working ***(A1)***

eg $10\log_2 x = 66,\ \log_2 x = 6.6,\ 2^{66} = x^{10},\ \log_2\left(\frac{x^{12}}{x^2}\right) = 66$

$x = 2^{6.6}$ (accept $p = \frac{66}{10}$) ***A1*** ***N2***

[3 marks]

Let $f(x) = \frac{1}{4}x^2 + 2$. The line *L* is the tangent to the curve of *f* at (4, 6) .

17a. Find the equation of *L* . *[4 marks]*

Markscheme

finding $f'(x) = \frac{1}{2}x$ ***A1***

attempt to find $f'(4)$ ***(M1)***

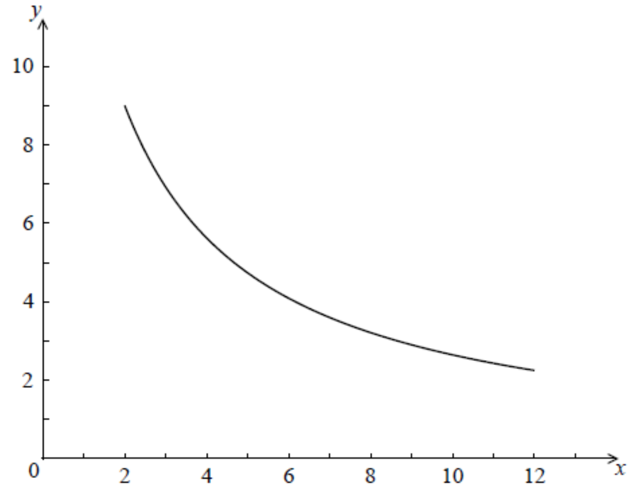
correct value $f'(4) = 2$ ***A1***

correct equation in any form ***A1*** ***N2***

e.g. $y - 6 = 2(x - 4)$, $y = 2x - 2$

[4 marks]

Let
$g(x) = \frac{90}{3x+4}$, for
$2 \le x \le 12$. The following diagram shows the graph of g .



17b. Find the area of the region enclosed by the curve of g , the x-axis, and the lines $x = 2$ and $x = 12$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$. *[6 marks]*

Markscheme

$\text{area} = \int_2^{12} \frac{90}{3x+4} \mathrm{d}x$

correct integral ***A1A1***

e.g.
$30 \ln(3x+4)$

substituting limits and subtracting ***(M1)***

e.g.
$30 \ln(3 \times 12 + 4) - 30 \ln(3 \times 2 + 4)$,
$30 \ln 40 - 30 \ln 10$

correct working ***(A1)***

e.g.
$30(\ln 40 - \ln 10)$

correct application of
$\ln b - \ln a$ ***(A1)***

e.g.
$30 \ln \frac{40}{10}$

$\text{area} = 30 \ln 4$ ***A1*** ***N4***

[6 marks]

17c. The graph of g is reflected in the x-axis to give the graph of h . The area of the region enclosed by the lines L , $x = 2$, $x = 12$ and the x-axis is 120 120 cm^2 . *[3 marks]*

Find the area enclosed by the lines L , $x = 2$, $x = 12$ and the graph of h .

Markscheme

valid approach ***(M1)***

e.g. sketch, area h = area g , 120 + **their** answer from (b)

$\text{area} = 120 + 30\ln 4$ ***A2*** ***N3***

[3 marks]