2.10 Final exam: Measure, trigonometry

All answers should be exact or rounded to three significant figures.

1. A cylinder has a radius of 3.1 cm and height of 11.4 cm.

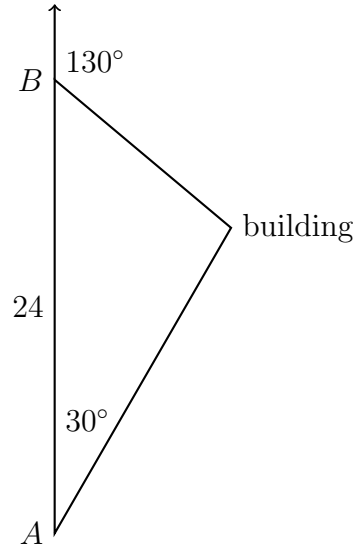
   (a) Calculate the volume of the cylinder. (2 marks)

   (b) Find the total surface area of the cylinder. (2 marks)

2. From the top of a cliff 110 m high a sailor sees two ships in the distance. One ship lies at an angle of depression of 38° and the other at angle of depression of 35°. Given that the ships and the sailor lie in the same vertical plane, find the distance between the two ships. (6 marks)

3. A car is driving due north. The driver spots a tall building in the distance at a bearing of 30 degrees. The car continues to drive north for 24 km, at which point the building has a bearing of 130 degrees.

   What is the distance between the driver's second location and the building? (credit to Randy for this problem) (4 marks)

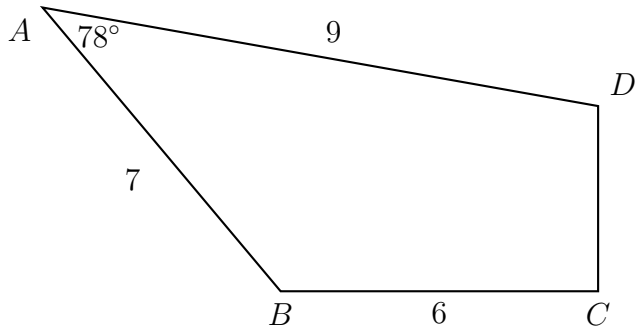


4. A triangular field has boundaries of length 120 m, 145 m, and 155 m.

   (a) Find the size of the smallest interior angle of the field. (3 marks)

   (b) Hence find the area of the field. (3 marks)

5. The following diagram shows quadrilateral $ABCD$ (not drawn to scale).



$AB = 8.0$, $BC = 5.1$, $CD = 4.2$, $AD = 8.3$, and $B\hat{C}D = 78°$

(a) Find the perimeter of the quadrilateral. [5 marks]

(b) Find the area of the quadrilateral. [3 marks]

6. For parallel resistors in an electrical circuit, the total resistance R_{TOT} is given by the formula

$$\frac{1}{R_{TOT}} = \frac{1}{R_1} + \frac{1}{R_2}$$

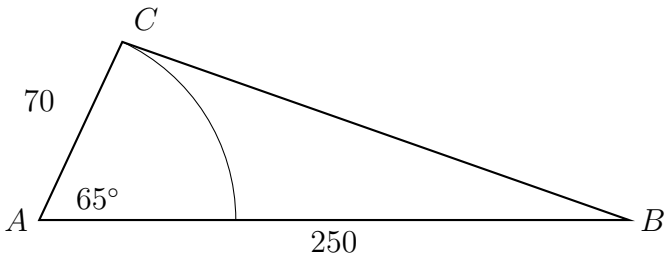
Given that the resistances of two resistors are $R_1 = 5.4\Omega$ and $R_2 = 2.7\Omega$, each measured to the nearest ohm.

(a) Find the upper and lower bounds of the total resistance. (6 marks)

(b) Given that the actual resistance of the circuit is 1.8 ohms, find the range of percentage errors that could be obtained for R_{TOT}. (4 marks)

7. A triangle field is shown with two sides measuring 70 m and 250 m. The measured sides meet at 65° at point A. A horse is tied to a stake at point A with a 70 m line so that it can graze within the sector marked.

Find the area of the field that the horse cannot reach. (6 marks)

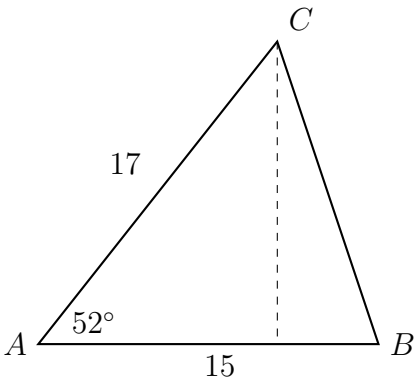


8. In right triangle ABC, hypotenuse $\overline{AB}$ has a length of 26 cm, and side $\overline{BC}$ has a length of 17.6 cm. What is the measure of angle B?

Triangle area sine formula

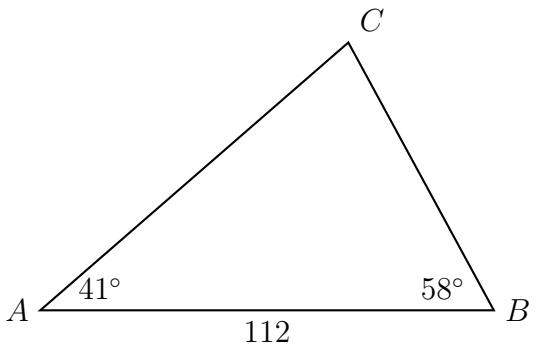
9. Find the area of triangle ABC, with $AB = 15$, $AC = 17$, $m\angle A = 52°$.

   Hint: To use the area formula $A = \frac{1}{2}bh$ first find the altitude using sine and the hypotenuse $AC = 17$.



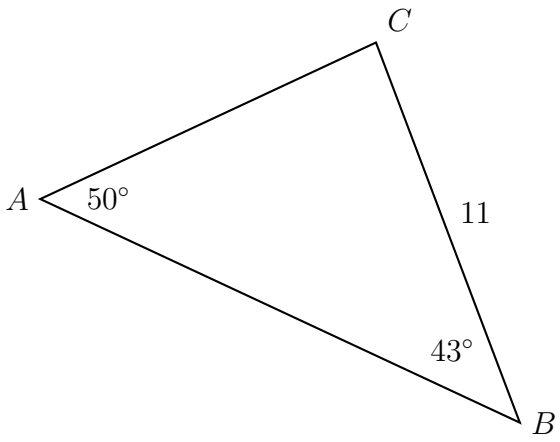
Law of cosines

10. Solve the given triangle (determine the values of all lengths and angles)



Law of sines

11. The following diagram shows triangle ABC (not drawn to scale).

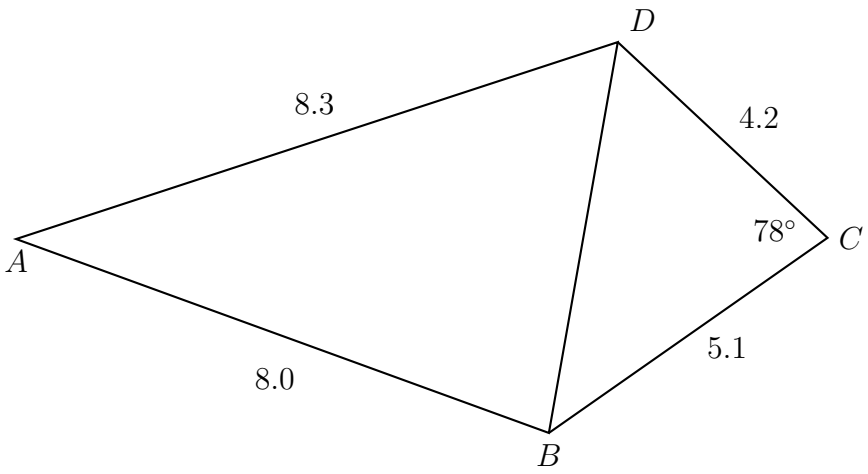


$BC = 11$, $C\hat{A}B = 50°$, and $A\hat{B}C = 43°$

(a) Find AC. [3 marks]

(b) Find the area of triangle ABC. [3 marks]

12. The following diagram shows quadrilateral $ABCD$ (not drawn to scale).



$AB = 8.0$, $BC = 5.1$, $CD = 4.2$, $AD = 8.3$, and $B\hat{C}D = 78°$

(a) Find BD. [3 marks]

(b) Find $A\hat{B}D$. [3 marks]

Precision application

13. BMI is a measure of a healthy personal weight,

$$BMI = \frac{w}{h^2}$$

where
w is a person's weight in kilograms, and
h is height in meters

(a) Given a height of 160 cm and weight of 54 kg, find the BMI [3 marks]

(b) These measurements are not exact. Assuming the height is between 159-161 cm and weight 53-55 kg, find the bounds of the BMI. [4 marks]

Sine ambiguous case

14. Triangle ABC has an area of 25, with $AB = 7$ and $AC = 8$.

    (a) Find the two possible measures for $\hat{A}$. [4 marks]

    (b) Given that $\hat{A}$ is obtuse, find BC. [3 marks]

Solid geometry

15. Find the slant height of a pyramid with square base 4 meters on a side and height of 4 m. [3 marks]

16. Find the volume of a spherical balloon 36 meters in diameter. [3 marks]

17. A cone has a height of 24 cm and volume of $220.5\pi\,\text{cm}^3$. Find its radius. [3 marks]