

**Spiral Review: 6-4+5 P1 (No Calculator) Calculus Integration**

**Using the calculator for definite integrals**

1. 16M.2.sl.TZ1.2 [6 marks]

Let  $f(x) = x^2$  and  $g(x) = 3 \ln(x + 1)$ , for  $x > -1$ .

(a) Solve  $f(x) = g(x)$  [3]

(b) Find the area of the region enclosed by the graphs of  $f$  and  $g$ . [3]

2. 16N.2.sl.TZ0.4 [6 marks]

Let  $f(x) = xe^{-x}$  and  $g(x) = -3f(x) + 1$ .

The graphs of  $f$  and  $g$  intersect at  $x = p$  and  $x = q$ , where  $p < q$ .

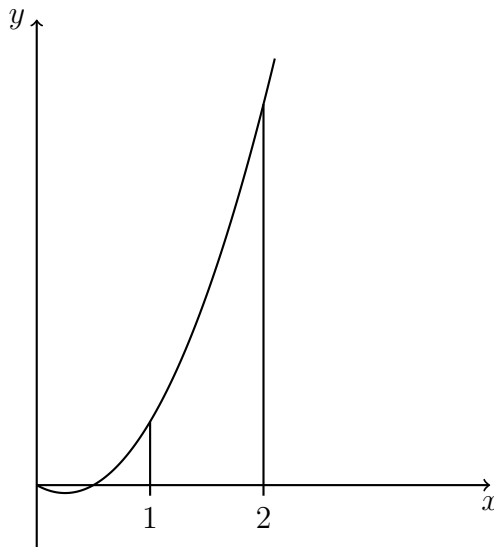
(a) Find the values of  $f$  and  $q$ . [3]

(b) Hence, find the area of the region enclosed by the graphs of  $f$  and  $g$ . [3]

**No Calculator section**

3. 18M.1.sl.TZ2.2 [6 marks]

Let  $f(x) = 6x^2 - 3x$ . The graph of  $f$  is shown in the following diagram.



(a) Find  $f'(x)$ . [2]

(b) Find the area of the region enclosed by the graph of  $f$ , the  $x$ -axis and the lines  $x = 1$  and  $x = 2$ . [4]

4. 15N.1.sl.TZ0.3 [6 marks]

Let  $f'(x) = 6x^2 - 5$ . and  $f(2) = -3$ , find  $f(x)$ . [6]

5. 13N.1.sl.TZ0.4

[6 marks]

Consider a function  $f(x)$  such that  $\int_1^6 f(x) dx = 8$ .

(a) Find  $\int_1^6 2f(x) dx$ .

[2]

(b) Find  $\int_1^6 (f(x) + 2) dx$ .

[4]

6. 18M.2.sl.TZ1.4

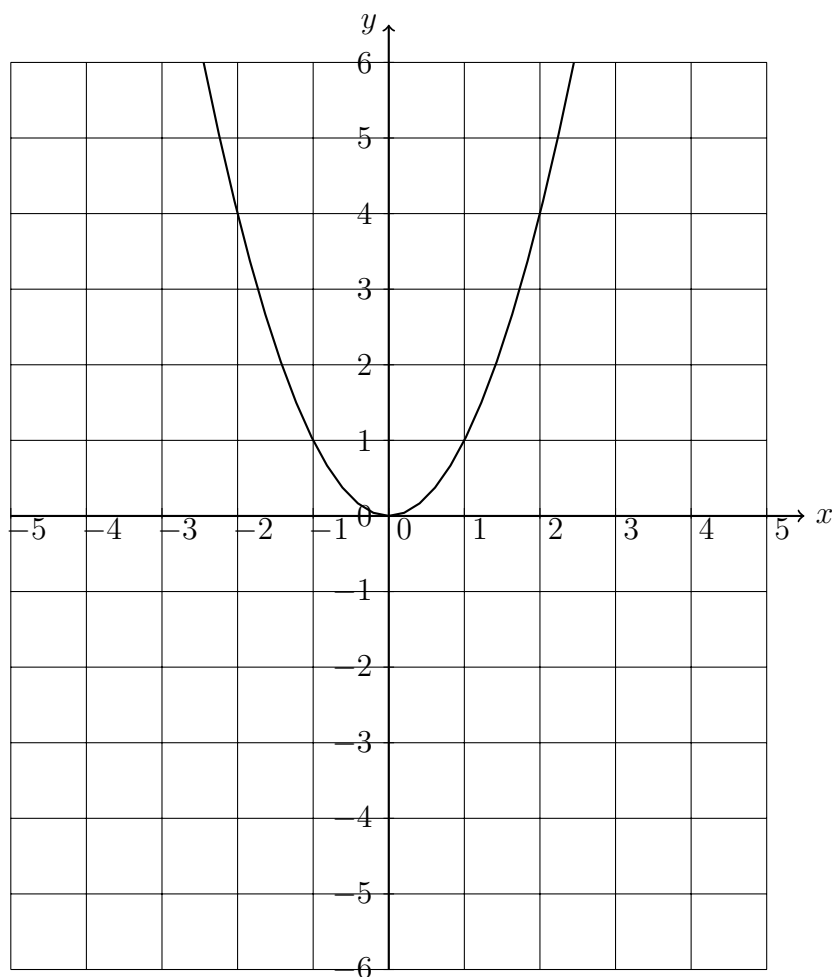
[7 marks]

Let  $g(x) = -(x - 1)^2 + 5$ .

(a) Write down the coordinates of the vertex of the graph of  $g$ .

[1]

(b) Let  $f(x) = x^2$ . The following diagram shows part of the graph of  $f$ .



The graph of  $g$  intersects the graph of  $f$  at  $x = -1$  and  $x = 2$ .

On the grid above, sketch the graph of  $g$  for  $-2 \leq x \leq 4$ .

[3]

(c) Find the area of the region enclosed by the graphs of  $f$  and  $g$ .

[3]

7. 14M.1.sl.TZ2.5

[6 marks]

The graph of a function  $h$  passes through the point  $(\frac{\pi}{12}, 5)$ .

Given that  $h'(x) = 4 \cos 2x$ , find  $h(x)$ .

[6]

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8. 17M.1.sl.TZ2.5 [6 marks]

Let  $f'(x) = \frac{3x^2}{(x^3 + 1)^5}$ . Given that  $f(0) = 1$ , find  $f(x)$ . [6]

9. 17M.1.sl.TZ1.5a [7 marks]

(a) Find  $\int xe^{x^2-1} dx$ . [4]

(b) Find  $f(x)$ , given that  $f'(x) = xe^{x^2-1}$  and  $f(-1) = 3$  [3]

**Spicy: Extended response**

10. 18M.1.sl.TZ1.8 [13 marks]

A function  $f(x)$  has derivative  $f'(x) = 3x^2 + 18x$ . The graph of  $f$  has an  $x$ -intercept at  $x = -1$ .

(a) Find  $f(x)$ . [6]

(b) The graph of  $f$  has a point of inflexion at  $x = p$ . Find  $p$ . [4]

(c) Find the values of  $x$  for which the graph of  $f$  is concave-down. [3]

11. 16M.1.sl.TZ1.10 [15 marks]

Let  $f(x) = \sqrt{4x + 5}$ , for  $x \geq -1.25$ .

(a) Find  $f'(1)$ . [4]

(b) Consider another function  $g$ . Let  $R$  be a point on the graph of  $g$ . The  $x$ -coordinate of  $R$  is 1. The equation of the tangent to the graph at  $R$  is  $y = 3x + 6$ .

Write down  $g'(1)$ . [2]

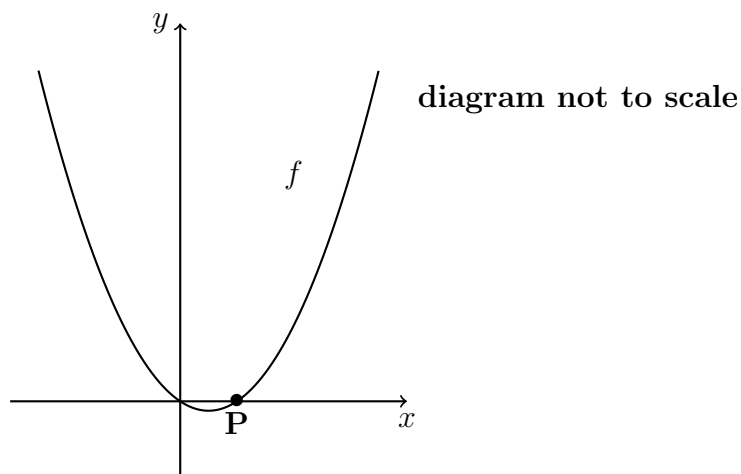
(c) Find  $g(1)$ . [2]

(d) Let  $h(x) = f(x) \times g(x)$ . Find the equation of the tangent to the graph of  $h$  at the point where  $x = 1$ . [7]

12. (#19) 17N.1.sl.TZ0.8

[16 marks]

Let  $f(x) = x^2 - x$ , for  $x \in \mathbb{R}$ . The following diagram shows part of the graph of  $f$ .

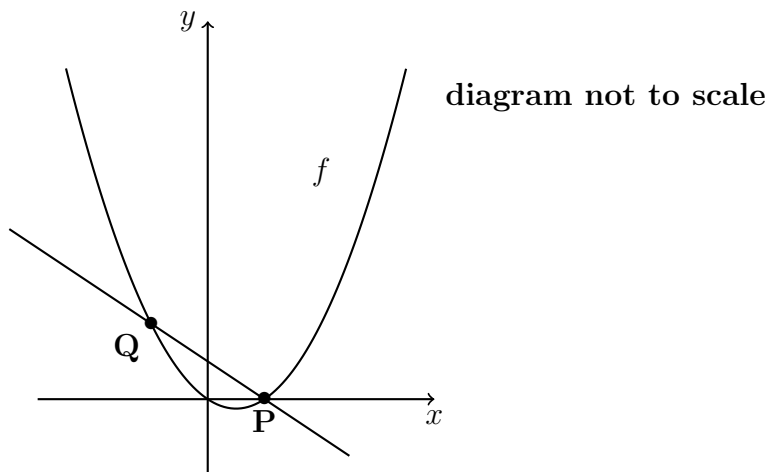


The graph of  $f$  crosses the  $x$ -axis at the origin and at the point  $P(1, 0)$ .

(a) Show that  $f'(1) = 1$ . [3]

(b) The line  $L$  is the normal to the graph of  $f$  at  $P$ .  
Find the equation of  $L$  in the form  $y = ax + b$ . [3]

(c) The line  $L$  intersects the graph of  $f$  at another point  $Q$ , as shown in the following diagram.



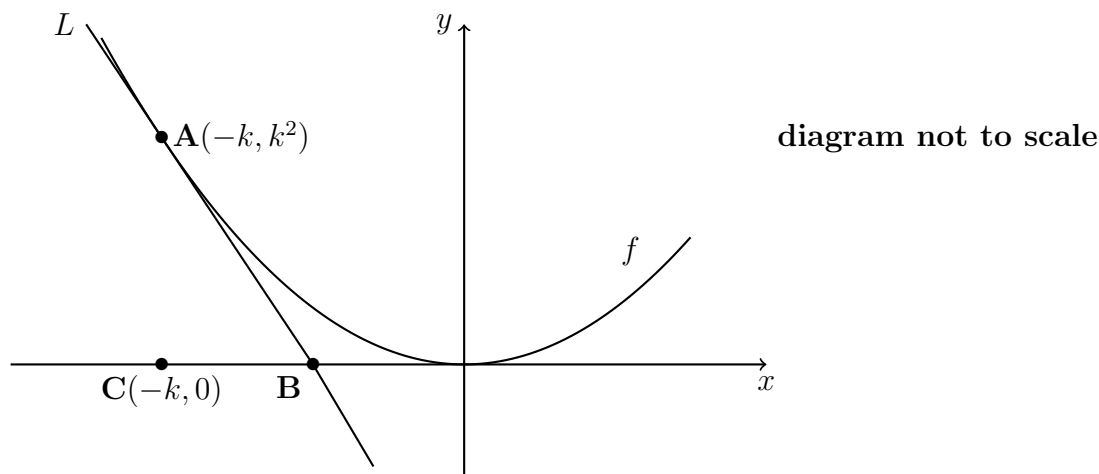
Find the  $x$ -coordinate of  $Q$ . [4]

(d) Find the area of the region enclosed by the graph of  $f$  and the line  $L$ . [6]

13. (#23) 17M.1.sl.TZ2.10

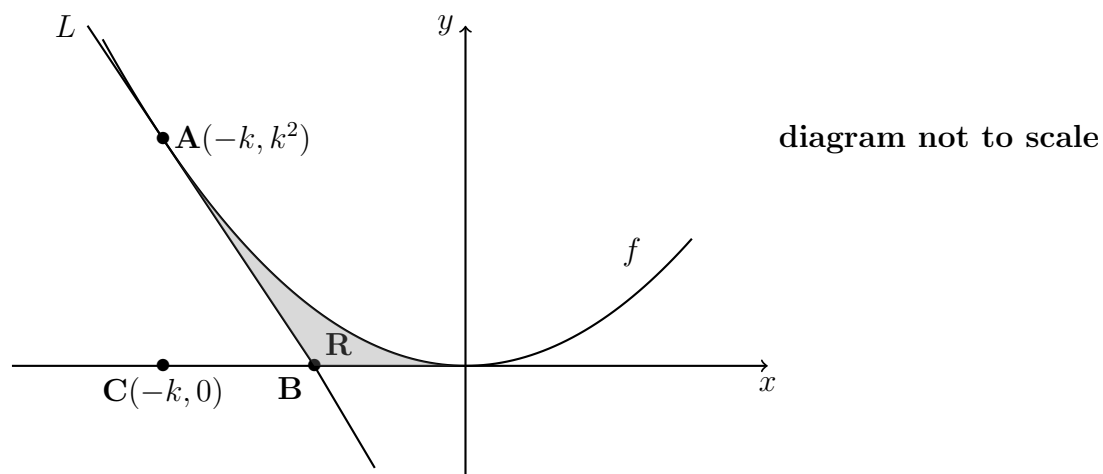
[17 marks]

Let  $f(x) = x^2$ . The following diagram shows part of the graph of  $f$ .



The line  $L$  is the tangent to the graph of  $f$  at the point  $A(-k, k^2)$ , and intersects the  $x$ -axis at point  $B$ . The point  $C$  is  $(-k, 0)$ .

- Write down  $f'(x)$ . [1]
- Find the gradient of  $L$ . [2]
- Show that the  $x$ -coordinate of  $B$  is  $-\frac{k}{2}$ . [5]
- Find the area of triangle  $ABC$ , giving your answer in terms of  $k$ . [2]
- The region  $R$  is enclosed by  $L$ , the graph of  $f$ , and the  $x$ -axis. This is shown in the following diagram.



Given that the area of triangle  $ABC$  is  $p$  times the area of  $R$ , find the value of  $p$ . [7]