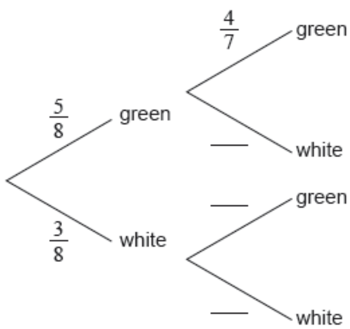
6-15 Test Probability Intro *[89 marks]*

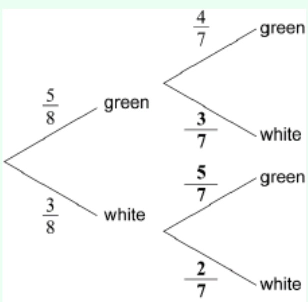
A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.

1a. Complete the following tree diagram. *[3 marks]*



Markscheme

correct probabilities



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** answer.

[3 marks]

1b. Find the probability that exactly one of the selected balls is green. *[3 marks]*

Markscheme

multiplying along branches ***(M1)***

eg $\frac{5}{8} \times \frac{3}{7},\ \frac{3}{8} \times \frac{5}{7},\ \frac{15}{56}$

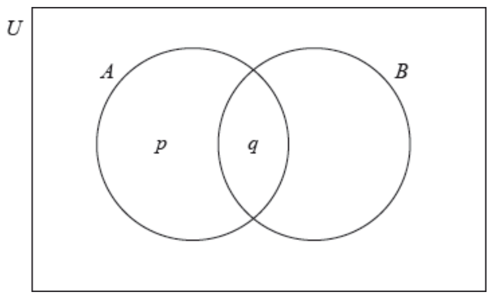
adding probabilities of correct mutually exclusive paths ***(A1)***

eg $\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7},\ \frac{15}{56} + \frac{15}{56}$

$\frac{30}{56}\ \left(= \frac{15}{28}\right)$ ***A1*** ***N2***

[3 marks]

The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.4,\ \mathrm{P}(A \cup B) = 0.8$ and $\mathrm{P}(A \cap B) = 0.1$. The values p and q are probabilities.



2a. (i) Write down the value of q. *[3 marks]*

(ii) Find the value of p.

Markscheme

(i)

$q = 0.1$ ***A1*** ***N1***

(ii) appropriate approach ***(M1)***

eg $\mathrm{P}(A) - q,\ 0.4 - 0.1$

$p = 0.3$ ***A1*** ***N2***

[3 marks]

2b. Find $\mathrm{P}(B)$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B),\ \mathrm{P}(A \cap B) + \mathrm{P}(B \cap A')$

correct values ***(A1)***

eg $0.8 = 0.4 + \mathrm{P}(B) - 0.1,\ 0.1 + 0.4$

$\mathrm{P}(B) = 0.5$ ***A1*** ***N2***

[3 marks]

Two events
A and
B are such that
$\mathrm{P}(A) = 0.2$ and
$\mathrm{P}(A \cup B) = 0.5.$

3a. Given that A and B are mutually exclusive, find $\mathrm{P}(B)$. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $0.5 = 0.2 + \mathrm{P}(B),\ \mathrm{P}(A \cap B) = 0$

$\mathrm{P}(B) = 0.3$ ***A1*** ***N2***

[2 marks]

3b. Alternatively, assuming that A and B are independent, find $\mathrm{P}(B)$. *[4 marks]*

Markscheme

Correct expression for $\mathrm{P}(A \cap B)$ (seen anywhere) ***A1***

eg $\mathrm{P}(A \cap B) = 0.2\mathrm{P}(B),\ 0.2x$

attempt to substitute into correct formula for $\mathrm{P}(A \cup B)$ ***(M1)***

eg $\mathrm{P}(A \cup B) = 0.2 + \mathrm{P}(B) - \mathrm{P}(A \cap B),\ \mathrm{P}(A \cup B) = 0.2 + x - 0.2x$

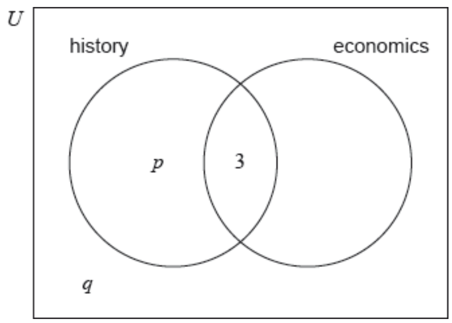
correct working ***(A1)***

eg $0.5 = 0.2 + \mathrm{P}(B) - 0.2\mathrm{P}(B),\ 0.8x = 0.3$

$\mathrm{P}(B) = \frac{3}{8}\ (= 0.375,\ \text{exact})$ ***A1*** ***N3***

[4 marks]

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.



4a. Find the value of p; *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $p + 3 = 13,\ 13 - 3$

$p = 10$ ***A1*** ***N2***

[2 marks]

4b. Find the value of q. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $p + 3 + 5 + q = 20,\ 10 - 10 - 8$

$q = 2$ ***A1*** ***N2***

[2 marks]

4c. A girl is selected at random. Find the probability that she takes economics but not history. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $20 - p - q - 3,\ 1 - \frac{15}{20},\ n(E \cap H') = 5$

$\frac{5}{20}$ $\left(\frac{1}{4}\right)$ ***A1*** ***N2***

[2 marks]

A box contains six red marbles and two blue marbles. Anna selects a marble fromthe box. She replaces the marble and then selects a second marble.

5a. Write down the probability that the first marble Anna selects is red. *[1 mark]*

Markscheme

Note: In this question, method marks may be awarded for selecting without replacement, as noted in the examples.

$\mathrm{P}(R) = \frac{6}{8}\left(= \frac{3}{4}\right)$ ***A1*** ***N1***

[1 mark]

5b. Find the probability that Anna selects two red marbles. *[2 marks]*

Markscheme

attempt to find $\mathrm{P}(\mathrm{Red}) \times \mathrm{P}(\mathrm{Red})$ ***(M1)***

e.g. $\mathrm{P}(R) \times \mathrm{P}(R)$, $\frac{3}{4} \times \frac{3}{4}$, $\frac{6}{8} \times \frac{5}{7}$

$\mathrm{P}(2R) = \frac{36}{64}\left(= \frac{9}{16}\right)$ ***A1*** ***N2***

[2 marks]

5c. Find the probability that one marble is red and one marble is blue. *[3 marks]*

Markscheme

METHOD 1

attempt to find $\mathrm{P(Red)} \times \mathrm{P(Blue)}$ ***(M1)***

e.g. $\mathrm{P}(R) \times \mathrm{P}(B)$, $\frac{6}{8} \times \frac{2}{8}$, $\frac{6}{8} \times \frac{2}{7}$

recognizing two ways to get one red, one blue ***(M1)***

e.g. $\mathrm{P}(RB) + \mathrm{P}(BR)$, $2\left(\frac{12}{64}\right)$, $\frac{6}{8} \times \frac{2}{7} + \frac{2}{8} \times \frac{6}{7}$

$\mathrm{P}(1R, 1B) = \frac{24}{64} \left(= \frac{3}{8}\right)$ ***A1 N2***

[3 marks]

METHOD 2

recognizing that $\mathrm{P}(1R, 1B)$ is $1 - \mathrm{P}(2B) - \mathrm{P}(2R)$ ***(M1)***

attempt to find $\mathrm{P}(2R)$ and $\mathrm{P}(2B)$ ***(M1)***

e.g. $\mathrm{P}(2R) = \frac{3}{4} \times \frac{3}{4}$, $\frac{6}{8} \times \frac{5}{7}$; $\mathrm{P}(2B) = \frac{1}{4} \times \frac{1}{4}$, $\frac{2}{8} \times \frac{1}{7}$

$\mathrm{P}(1R, 1B) = \frac{24}{64} \left(= \frac{3}{8}\right)$ ***A1 N2***

[3 marks]

Let
A and
B be independent events, where
$\mathrm{P}(A) = 0.3$ and
$\mathrm{P}(B) = 0.6$.

6a. Find $\mathrm{P}(A \cap B)$. *[2 marks]*

Markscheme

correct substitution ***(A1)***

eg 0.3×0.6

$\mathrm{P}(A \cap B) = 0.18$ ***A1 N2***

[2 marks]

6b. Find $\mathrm{P}(A \cup B)$. *[2 marks]*

Markscheme

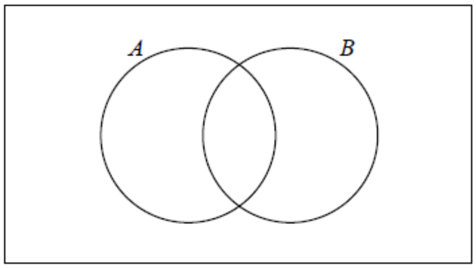
correct substitution *(A1)*

eg $\mathrm{P}(A \cup B) = 0.3 + 0.6 - 0.18$

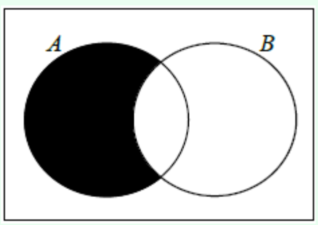
$\mathrm{P}(A \cup B) = 0.72$ ***A1 N2***

[2 marks]

6c. On the following Venn diagram, shade the region that represents $A \cap B'$. *[1 mark]*



Markscheme



A1 N1

6d. Find $\mathrm{P}(A \cap B')$. *[2 marks]*

Markscheme

appropriate approach *(M1)*

eg $0.3 - 0.18,\ \mathrm{P}(A) \times \mathrm{P}(B')$

$\mathrm{P}(A \cap B') = 0.12$ (may be seen in Venn diagram) ***A1 N2***

[2 marks]

Events A and B are independent with $\mathrm{P}(A \cap B) = 0.2$ and $\mathrm{P}(A' \cap B) = 0.6$.

7a. Find $\mathrm{P}(B)$. *[2 marks]*

Markscheme

valid interpretation (may be seen on a Venn diagram) ***(M1)***

eg $\mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B),\ 0.2 + 0.6$

$\mathrm{P}(B) = 0.8$ ***A1*** ***N2***

[2 marks]

7b. Find $\mathrm{P}(A \cup B)$. *[4 marks]*

Markscheme

valid attempt to find $\mathrm{P}(A)$ ***(M1)***

eg $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B),\ 0.8 \times A = 0.2$

correct working for $\mathrm{P}(A)$ ***(A1)***

eg $0.25,\ \frac{0.2}{0.8}$

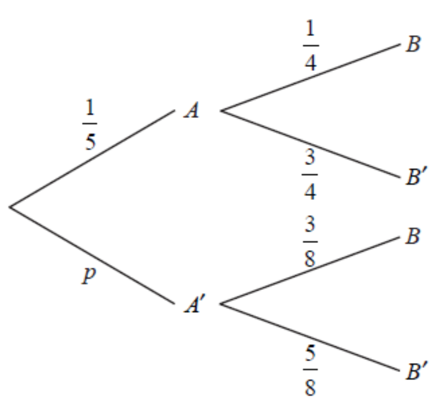
correct working for $\mathrm{P}(A \cup B)$ ***(A1)***

eg $0.25 + 0.8 - 0.2,\ 0.6 + 0.2 + 0.05$

$\mathrm{P}(A \cup B) = 0.85$ ***A1*** ***N3***

[4 marks]

The diagram below shows the probabilities for events A and B, with $\mathrm{P}(A') = p$.



8a. Write down the value of p. *[1 mark]*

Markscheme

$p = \frac{4}{5}$ ***A1*** ***N1***

[1 mark]

8b. Find $\mathrm{P}(B)$. *[3 marks]*

Markscheme

multiplying along the branches ***(M1)***

e.g. $\frac{1}{5} \times \frac{1}{4}$, $\frac{12}{40}$

adding products of probabilities of two mutually exclusive paths ***(M1)***

e.g. $\frac{1}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{8}$, $\frac{1}{20} + \frac{12}{40}$

$\mathrm{P}(B) = \frac{14}{40} \left(= \frac{7}{20}\right)$ ***A1*** ***N2***

[3 marks]

8c. Find $\mathrm{P}(A'|B)$. *[3 marks]*

Markscheme

appropriate approach which must include A' (may be seen on diagram) ***(M1)***

e.g. $\frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)}$ (do not accept $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$)

$\mathrm{P}(A'|B) = \frac{\frac{4}{5} \times \frac{3}{8}}{\frac{7}{20}}$ ***(A1)***

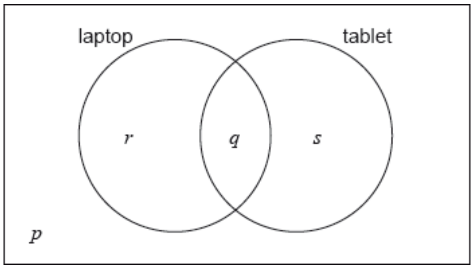
$\mathrm{P}(A'|B) = \frac{12}{14} \left(= \frac{6}{7}\right)$ ***A1*** ***N2***

[3 marks]

In a class of 21 students, 12 own a laptop, 10 own a tablet, and 3 own neither.

The following Venn diagram shows the events “own a laptop” and “own a tablet”.

The values p, q, r and s represent numbers of students.



9a. (i) Write down the value of p. *[5 marks]*

(ii) Find the value of q.

(iii) Write down the value of r and of s.

Markscheme

(i)

$p = 3$ ***A1*** ***N1***

(ii) valid approach ***(M1)***

eg $(12 + 10 + 3) - 21,\ 22 - 18$

$q = 4$ ***A1*** ***N2***

(iii)

$r = 8,\ s = 6$ ***A1A1*** ***N2***

A student is selected at random from the class.

9b. (i) Write down the probability that this student owns a laptop. *[4 marks]*

(ii) Find the probability that this student owns a laptop or a tablet but not both.

Markscheme

(i)

$\frac{12}{21}\ \left(= \frac{4}{7}\right)$ ***A2*** ***N2***

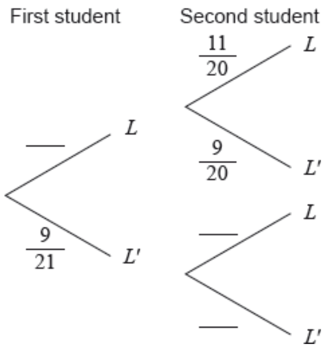
(ii) valid approach ***(M1)***

eg $8 + 6,\ r + s$

$\frac{14}{21}\ \left(= \frac{2}{3}\right)$ ***A1*** ***N2***

Two students are randomly selected from the class. Let L be the event a "student owns a laptop".

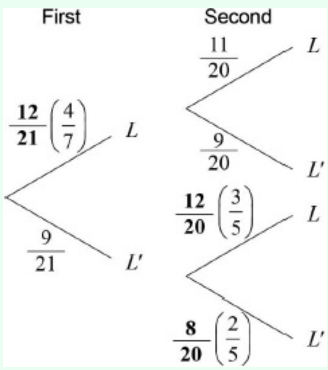
9c. (i) **Copy** and complete the following tree diagram. (Do **not** write on this page.) *[4 marks]*



(ii) Write down the probability that the second student owns a laptop given that the first owns a laptop.

Markscheme

(i)



A1A1A1 N3

(ii)

$\frac{11}{20}$ ***A1 N1***

[4 marks]

10. Celeste wishes to hire a taxicab from a company which has a large number of taxicabs. *[6 marks]*

The taxicabs are randomly assigned by the company.

The probability that a taxicab is yellow is 0.4.

The probability that a taxicab is a Fiat is 0.3.

The probability that a taxicab is yellow or a Fiat is 0.6.

Find the probability that the taxicab hired by Celeste is **not** a yellow Fiat.

Markscheme

recognize need for intersection of Y and F ***(R1)***

eg $\mathrm{P}(Y \cap F), 0.3 \times 0.4$

valid approach to find $\mathrm{P}(Y \cap F)$ ***(M1)***

eg $\mathrm{P}(Y) + \mathrm{P}(F) - \mathrm{P}(Y \cup F)$, Venn diagram

correct working (may be seen in Venn diagram) ***(A1)***

eg $0.4 + 0.3 - 0.6$

$\mathrm{P}(Y \cap F) = 0.1$ ***A1***

recognize need for complement of $Y \cap F$ ***(M1)***

eg $1 - \mathrm{P}(Y \cap F), 1 - 0.1$

$\mathrm{P}\left((Y \cap F)'\right) = 0.9$ ***A1*** ***N3***

[6 marks]

A factory has two machines, A and B. The number of breakdowns of each machine is independent from day to day.

Let A be the number of breakdowns of Machine A on any given day. The probability distribution for A can be modelled by the following table.

a	0	1	2	3
$\mathrm{P}(A = a)$	0.55	0.3	0.1	k

11a. Find k. *[2 marks]*

Markscheme

evidence of summing to 1 ***(M1)***

eg $0.55 + 0.3 + 0.1 + k = 1$

$k = 0.05$ (exact) ***A1*** ***N2***

[2 marks]

11b. (i) A day is chosen at random. Write down the probability that Machine A has no breakdowns. *[3 marks]*

(ii) Five days are chosen at random. Find the probability that Machine A has no breakdowns on exactly four of these days.

Markscheme

(i) 0.55 ***A1*** ***N1***

(ii) recognizing binomial probability ***(M1)***

eg $X:\ B(n,\ p),\ \binom{5}{4},\ (0.55)^4(1-0.55),\ \binom{n}{r}p^r q^{n-r}$

$P(X=4)=0.205889$

$P(X=4)=0.206$ ***A1*** ***N2***

[3 marks]

Let B be the number of breakdowns of Machine B on any given day. The probability distribution for B can be modelled by the following table.

b	0	1	2	3
$P(B=b)$	0.7	0.2	0.08	0.02

11c. Find $\mathrm{E}(B)$. *[2 marks]*

Markscheme

correct substitution into formula for $\mathrm{E}(X)$ ***(A1)***

eg $0.2+(2\times 0.08)+(3\times 0.02)$

$\mathrm{E}(B)=0.42\ (\text{exact})$ ***A1*** ***N2***

[2 marks]

On Tuesday, the factory uses both Machine A and Machine B. The variables A and B are independent.

11d. (i) Find the probability that there are exactly two breakdowns on Tuesday. *[8 marks]*

(ii) Given that there are exactly two breakdowns on Tuesday, find the probability that both breakdowns are of Machine A.

Markscheme

(i) valid attempt to find one possible way of having 2 breakdowns ***(M1)***

eg $2A,\ 2B,\ 1A$ and $1B$, tree diagram

one correct calculation for 1 way (seen anywhere) ***(A1)***

eg $0.1 \times 0.7,\ 0.55 \times 0.08,\ 0.3 \times 0.2$

recognizing there are 3 ways of having 2 breakdowns ***(M1)***

eg A twice or B twice or one breakdown each

correct working ***(A1)***

eg $(0.1 \times 0.7) + (0.55 \times 0.08) + (0.3 \times 0.2)$

$\mathrm{P}(2 \text{ breakdowns}) = 0.174 \text{ (exact)}$ ***A1 N3***

(ii) recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A|B),\ \mathrm{P}(2A|2\text{breakdowns})$

correct working ***(A1)***

eg $\frac{0.1 \times 0.7}{0.174}$

$\mathrm{P}(A = 2|\text{two breakdowns}) = 0.402298$

$\mathrm{P}(A = 2|\text{two breakdowns}) = 0.402$ ***A1 N2***

[8 marks]

Let C and D be independent events, with $\mathrm{P}(C) = 2k$ and $\mathrm{P}(D) = 3k^2$, where $0 < k < 0.5$.

12a. Write down an expression for $\mathrm{P}(C \cap D)$ in terms of k. *[2 marks]*

Markscheme

$\mathrm{P}(C \cap D) = 2k \times 3k^2$ ***(A1)***

$\mathrm{P}(C \cap D) = 6k^3$ ***A1 N2***

[2 marks]

12b. Find $\mathrm{P}(C'|D)$. *[3 marks]*

Markscheme

METHOD 1

finding **their** $\mathrm{P}(C' \cap D)$ (seen anywhere) ***(A1)***

eg $0.4 \times 0.27, 0.27 - 0.162, 0.108$

correct substitution into conditional probability formula ***(A1)***

eg $\mathrm{P}(C'|D) = \frac{\mathrm{P}(C' \cap D)}{0.27}, \frac{(1-2k)(3k^2)}{3k^2}$

$\mathrm{P}(C'|D) = 0.4$ ***A1 N2***

METHOD 2

recognizing $\mathrm{P}(C'|D) = \mathrm{P}(C')$ ***A1***

finding **their** $\mathrm{P}(C') = 1 - \mathrm{P}(C)$ (only if first line seen) ***(A1)***

eg $1 - 2k,\ 1 - 0.6$

$\mathrm{P}(C'|D) = 0.4$ ***A1 N2***

[3 marks]

Total [7 marks]