BECA / Huson / 12.1 IB Math SL
26 February 2018

Name:

Homework: Function graphs

1a. A function f has its derivative given by $f'(x) = 3x^2 - 2kx - 9$, where k is a constant.

Find $f''(x)$. *[2 marks]*

1b. The graph of f has a point of inflexion when $x = 1$.

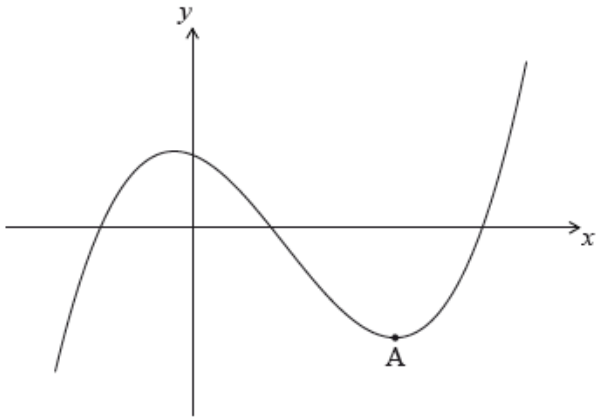
Show that $k = 3$. *[3 marks]*

1c. Find $f'(-2)$. *[2 marks]*

1d. Find the equation of the tangent to the curve of f at $(-2,\ 1)$, giving your answer in the form $y = ax + b$. *[4 marks]*

1e. Given that $f'(-1) = 0$, explain why the graph of f has a local maximum when $x = -1$. *[3 marks]*

2a. The following diagram shows the graph of a function f. There is a local minimum point at A, where $x > 0$.

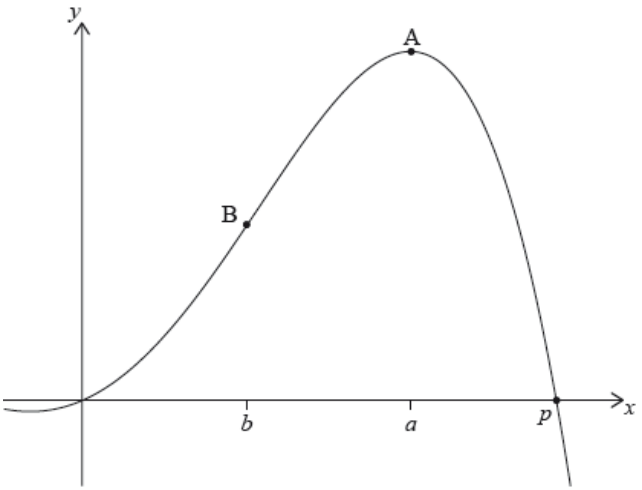


The derivative of f is given by $f'(x) = 3x^2 - 8x - 3$.

Find the x-coordinate of A. *[5 marks]*

2b. The y-intercept of the graph is at $(0, 6)$. Find an expression for $f(x)$. *[6 marks]*

3a. Let $f(x) = -0.5x^4 + 3x^2 + 2x$. The following diagram shows part of the graph of f.



There are x-intercepts at $x = 0$ and at $x = p$. There is a maximum at A where $x = a$, and a point of inflexion at B where $x = b$.

Find the value of p. *[2 marks]*

3b. Write down the coordinates of A. *[2 marks]*

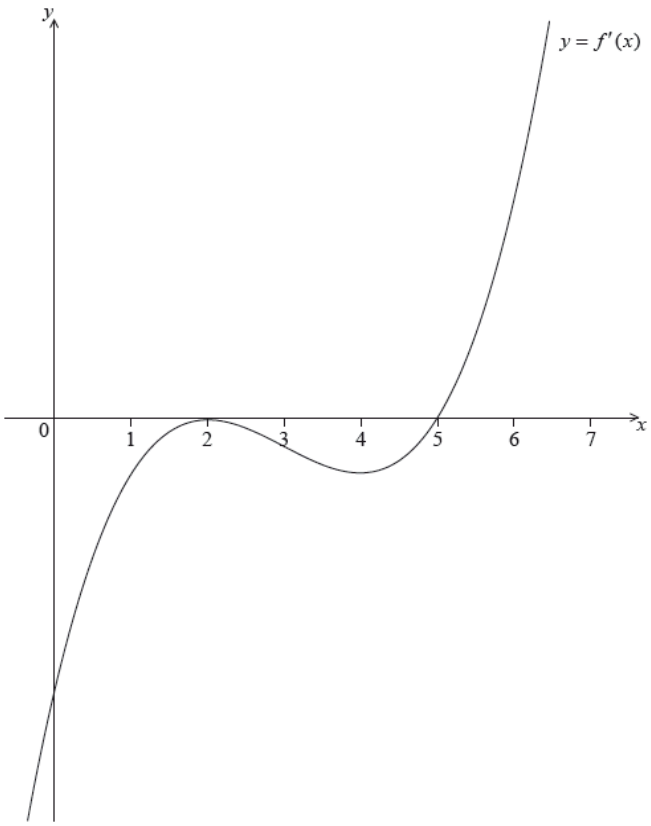
3c. Write down the rate of change of f at A. *[1 mark]*

3d. Find the coordinates of B. *[4 marks]*

3e. Find the rate of change of f at B. *[3 marks]*

3f. Let R be the region enclosed by the graph of f, the x-axis, the line $x = b$ and the line $x = a$. The region R is rotated 360° about the x-axis. Find the volume of the solid formed. *[3 marks]*

4a. Let $y = f(x)$, for $-0.5 \leq x \leq 6.5$. The following diagram shows the graph of f', the derivative of f.



The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the x-axis at the point $(5,\ 0)$.

Explain why the graph of f has a local minimum when $x = 5$. *[2 marks]*

4b. Find the set of values of x for which the graph of f is concave down. *[2 marks]*

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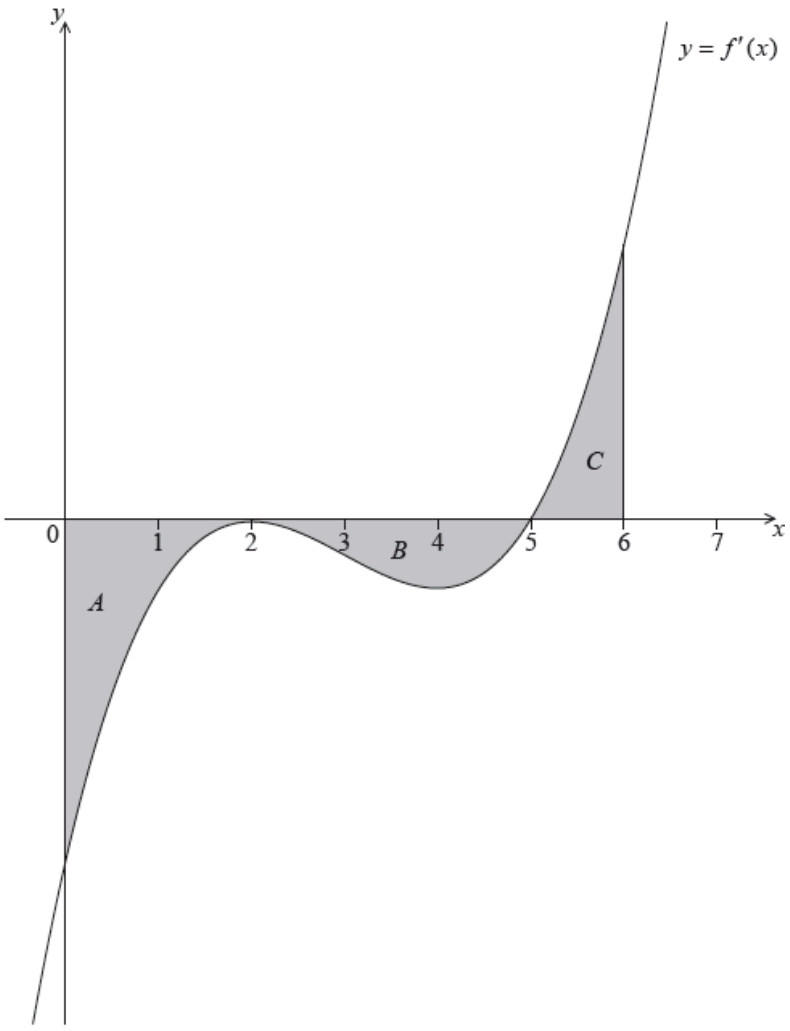
4c. The following diagram shows the shaded regions A, B and C.



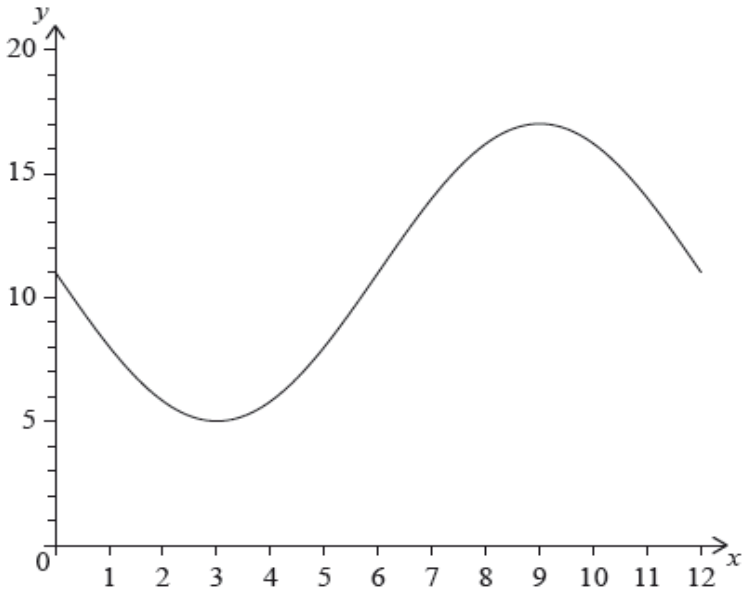
The regions are enclosed by the graph of f', the x-axis, the y-axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Given that $f(0) = 14$, find $f(6)$. *[5 marks]*

4d. Let $g(x) = (f(x))^2$. Given that $f'(6) = 16$, find the equation of the tangent to the graph of g at the point where $x = 6$. *[6 marks]*

5a. The following diagram shows the graph of $f(x) = a\sin bx + c$, for $0 \leqslant x \leqslant 12$.



The graph of f has a minimum point at $(3,\ 5)$ and a maximum point at $(9,\ 17)$.

(i) Find the value of c.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a. *[6 marks]*

5b. The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5,\ 17)$.

(i) Write down the value of k.

(ii) Find $g(x)$. *[3 marks]*

5c. The graph of g changes from concave-up to concave-down when $x = w$. *[6 marks]*

(i) Find w.

(ii) Hence or otherwise, find the maximum positive rate of change of g.