Trig Unit Circle Practice *[46 marks]*

Let
$f(x) = \cos 2x$ and
$g(x) = 2x^2 - 1$.

1a. Find $f\left(\frac{\pi}{2}\right)$. *[2 marks]*

Markscheme

$f\left(\frac{\pi}{2}\right) = \cos \pi$ ***(A1)***

$= -1$ ***A1*** ***N2***

[2 marks]

1b. Find $(g \circ f)\left(\frac{\pi}{2}\right)$. *[2 marks]*

Markscheme

$(g \circ f)\left(\frac{\pi}{2}\right) = g(-1) \ (= 2(-1)^2 - 1)$ ***(A1)***

$= 1$ ***A1*** ***N2***

[2 marks]

1c. Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k, $k \in \mathbb{Z}$. *[3 marks]*

Markscheme

$(g \circ f)(x) = 2(\cos(2x))^2 - 1 \ (= 2\cos^2(2x) - 1)$ ***A1***

evidence of $2\cos^2\theta - 1 = \cos 2\theta$ (seen anywhere) ***(M1)***

$(g \circ f)(x) = \cos 4x$

$k = 4$ ***A1*** ***N2***

[3 marks]

Let
$p = \sin 40^\circ$ and
$q = \cos 110^\circ$. Give your answers to the following in terms of p and/or q .

2a. Write down an expression for *[2 marks]*

(i) $\sin 140^\circ$;

(ii) $\cos 70^\circ$.

Markscheme

(i) $\sin 140^\circ = p$ ***A1 N1***

(ii) $\cos 70^\circ = -q$ ***A1 N1***

[2 marks]

2b. Find an expression for $\cos 140^\circ$. *[3 marks]*

Markscheme

METHOD 1

evidence of using $\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

e.g. diagram, $\sqrt{1-p^2}$ (seen anywhere)

$\cos 140^\circ = \pm\sqrt{1-p^2}$ ***(A1)***

$\cos 140^\circ = -\sqrt{1-p^2}$ ***A1 N2***

METHOD 2

evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ ***(M1)***

$\cos 140^\circ = 2\cos^2 70 - 1$ ***(A1)***

$\cos 140^\circ = 2(-q)^2 - 1\ (= 2q^2 - 1)$ ***A1 N2***

[3 marks]

2c. Find an expression for $\tan 140^\circ$. *[1 mark]*

Markscheme

METHOD 1

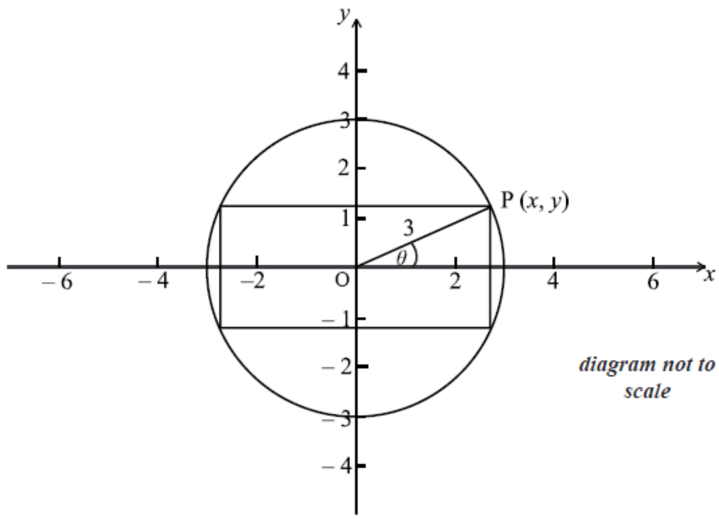
$\tan 140^\circ = \frac{\sin 140^\circ}{\cos 140^\circ} = -\frac{p}{\sqrt{1-p^2}}$ ***A1 N1***

METHOD 2

$\tan 140^\circ = \frac{p}{2q^2-1}$ ***A1 N1***

[1 mark]

A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point P(x , y) is a vertex of the rectangle and also lies on the circle. The anglebetween (OP) and the x-axis is θ radians, where
$0 \le \theta \le \frac{\pi}{2}$.

3a. Write down an expression in terms of θ for *[2 marks]*

(i) x ;

(ii) y .

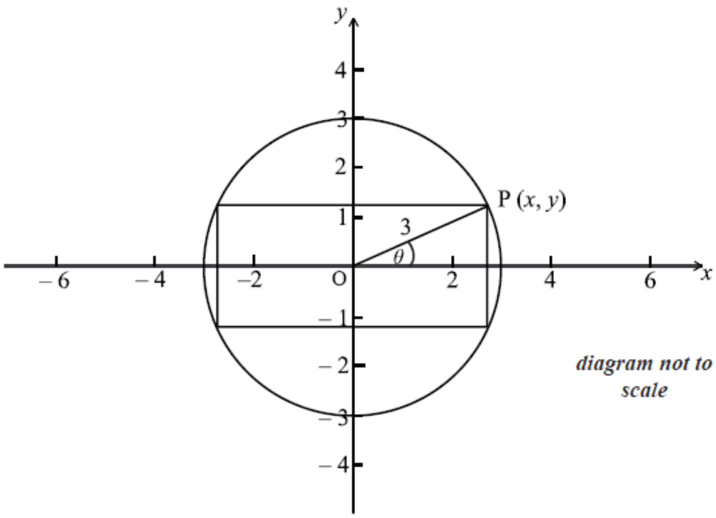
Markscheme

(i) $x = 3\cos\theta$ ***A1 N1***

(ii) $y = 3\sin\theta$ ***A1 N1***

[2 marks]

A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point P(*x* , *y*) is a vertex of the rectangle and also lies on the circle. The anglebetween (OP) and the *x*-axis is θ radians, where
$0 \le \theta \le \frac{\pi}{2}$.

3b. Let the area of the rectangle be *A*. *[3 marks]*

Show that $A = 18\sin 2\theta$.

Markscheme

finding area ***(M1)***

e.g. $A = 2x \times 2y$, $A = 8 \times \frac{1}{2}bh$

substituting ***A1***

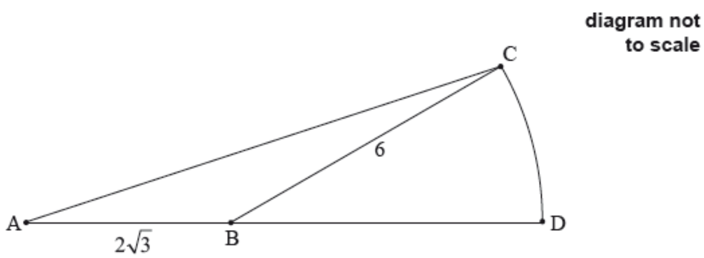
e.g. $A = 4 \times 3\sin\theta \times 3\cos\theta$, $8 \times \frac{1}{2} \times 3\cos\theta \times 3\sin\theta$

$A = 18(2\sin\theta\cos\theta)$ ***A1***

$A = 18\sin 2\theta$ ***AG*** ***N0***

[3 marks]

The following diagram shows a triangle ABC and a sector BDC of a circle with centre B and radius 6 cm. The points A , B and D are on the same line.



$\mathrm{AB} = 2\sqrt{3}\ \mathrm{cm}, \mathrm{BC} = 6\ \mathrm{cm}$, area of triangle ABC $= 3\sqrt{3}\ \mathrm{cm}^2$, $\mathrm{A\hat{B}C}$ is obtuse.

4a. Find $\mathrm{A\hat{B}C}$. *[5 marks]*

Markscheme

METHOD 1

correct substitution into formula for area of triangle ***(A1)***

eg $\frac{1}{2}(6)\left(2\sqrt{3}\right)\sin B,\ 6\sqrt{3}\sin B,\ \frac{1}{2}(6)\left(2\sqrt{3}\right)\sin B = 3\sqrt{3}$

correct working ***(A1)***

eg $6\sqrt{3}\sin B = 3\sqrt{3},\ \sin B = \frac{3\sqrt{3}}{\frac{1}{2}(6)2\sqrt{3}}$

$\sin B = \frac{1}{2}$ ***(A1)***

$\frac{\pi}{6}(30^\circ)$ ***(A1)***

$\mathrm{A\hat{B}C} = \frac{5\pi}{6}(150^\circ)$ ***A1 N3***

METHOD 2

(using height of triangle ABC by drawing perpendicular segment from C to AD)

correct substitution into formula for area of triangle ***(A1)***

eg $\frac{1}{2}\left(2\sqrt{3}\right)(h) = 3\sqrt{3},\ h\sqrt{3}$

correct working ***(A1)***

eg $h\sqrt{3} = 3\sqrt{3}$

height of triangle is 3 ***A1***

$\mathrm{C\hat{B}D} = \frac{\pi}{6}(30^\circ)$ ***(A1)***

$\mathrm{A\hat{B}C} = \frac{5\pi}{6}(150^\circ)$ ***A1 N3***

[5 marks]

4b. Find the exact area of the sector BDC. *[3 marks]*

Markscheme

recognizing supplementary angle ***(M1)***

eg $\mathrm{C\hat{B}D} = \frac{\pi}{6},\ \text{sector} = \frac{1}{2}(180 - \mathrm{A\hat{B}C})(6^2)$

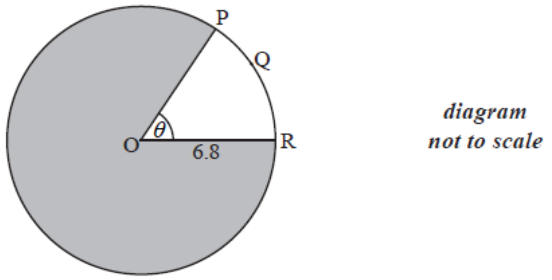
correct substitution into formula for area of sector ***(A1)***

eg $\frac{1}{2}\times\frac{\pi}{6}\times 6^2,\ \pi(6^2)\left(\frac{30}{360}\right)$

$\text{area} = 3\pi\ (\text{cm}^2)$ ***A1 N2***

[3 marks]

Consider the following circle with centre O and radius 6.8 cm.



The length of the arc PQR is 8.5 cm.

5a. Find the value of θ . *[2 marks]*

Markscheme

correct substitution *(A1)*

e.g. $8.5 = \theta(6.8)$, $\theta = \frac{8.5}{6.8}$

$\theta = 1.25$ (accept 71.6°) ***A1 N2***

[2 marks]

5b. Find the area of the shaded region. *[4 marks]*

Markscheme

METHOD 1

correct substitution into area formula (seen anywhere) ***(A1)***

e.g. $A = \pi(6.8)^2$, $145.267\dots$

correct substitution into area formula (seen anywhere) ***(A1)***

e.g. $A = \frac{1}{2}(1.25)(6.8^2)$, 28.9

valid approach ***M1***

e.g. $\pi(6.8)^2 - \frac{1}{2}(1.25)(6.8^2)$; $145.267\dots - 28.9$; $\pi r^2 - \frac{1}{2}r^2\sin\theta$

$A = 116\ (\text{cm}^2)$ ***A1*** ***N2***

METHOD 2

attempt to find reflex angle ***(M1)***

e.g. $2\pi - \theta$, $360 - 1.25$

correct reflex angle ***(A1)***

$\text{A}\hat{\text{O}}\text{B} = 2\pi - 1.25\ (= 5.03318\dots)$

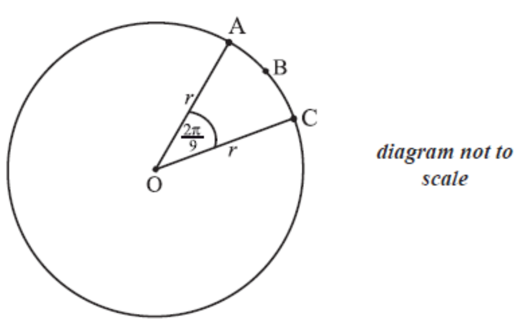
correct substitution into area formula ***A1***

e.g. $A = \frac{1}{2}(5.03318\dots)(6.8^2)$

$A = 116$ (
cm^2) ***A1*** ***N2***

[4 marks]

The diagram below shows a circle centre O, with radius *r*. The length of arc ABC is
3π cm and
$\text{A}\hat{\text{O}}\text{C} = \frac{2\pi}{9}$.



6a. Find the value of *r*. *[2 marks]*

Markscheme

evidence of appropriate approach ***M1***

e.g. $3\pi = r\frac{2\pi}{9}$

$r = 13.5$ (cm) ***A1*** ***N1***

[2 marks]

6b. Find the perimeter of sector OABC. *[2 marks]*

Markscheme

adding two radii plus 3π ***(M1)***

$\text{perimeter} = 27 + 3\pi$ (cm) $(= 36.4)$ ***A1*** ***N2***

[2 marks]

6c. Find the area of sector OABC. *[2 marks]*

Markscheme

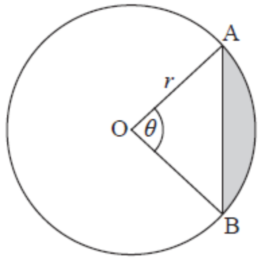
evidence of appropriate approach ***M1***

e.g. $\frac{1}{2} \times 13.5^2 \times \frac{2\pi}{9}$

area $= 20.25\pi\ (\text{cm}^2)\ (= 63.6)$ ***A1*** ***N1***

[2 marks]

A circle centre O and radius
r is shown below. The chord [AB] divides the area of thecircle into two parts. Angle AOB is
θ .



7a. Find an expression for the area of the shaded region. *[3 marks]*

Markscheme

substitution into formula for area of triangle ***A1***

e.g. $\frac{1}{2}r \times r\sin\theta$

evidence of subtraction ***M1***

correct expression ***A1*** ***N2***

e.g. $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$, $\frac{1}{2}r^2(\theta - \sin\theta)$

[3 marks]

7b. The chord [AB] divides the area of the circle in the ratio 1:7. Find the value of θ . *[5 marks]*

Markscheme

evidence of recognizing that shaded area is $\frac{1}{8}$ of area of circle ***M1***

e.g. $\frac{1}{8}$ seen anywhere

setting up correct equation ***A1***

e.g. $\frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{8}\pi r^2$

eliminating 1 variable ***M1***

e.g. $\frac{1}{2}(\theta - \sin\theta) = \frac{1}{8}\pi$, $\theta - \sin\theta = \frac{\pi}{4}$

attempt to solve ***M1***

e.g. a sketch, writing $\sin x - x + \frac{\pi}{4} = 0$

$\theta = 1.77$ (do not accept degrees) ***A1*** ***N1***

[5 marks]