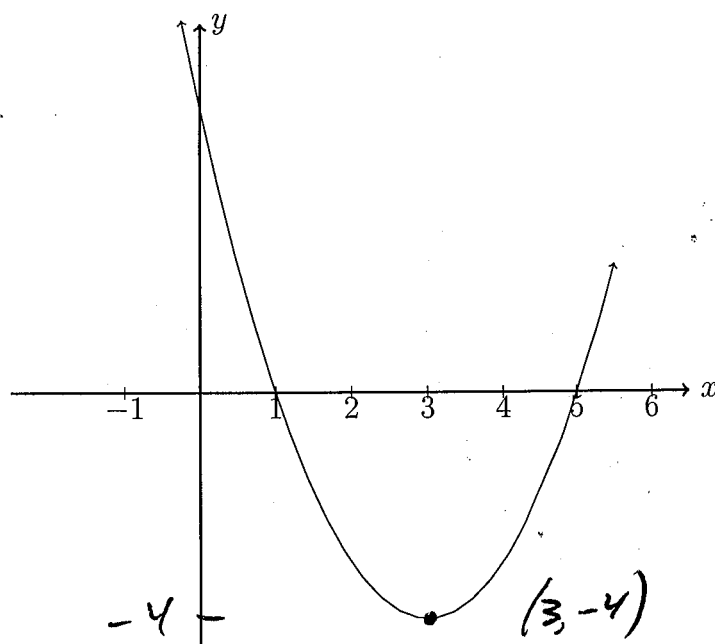


Name: *Soul James***5.11 Exam: Quadratic functions and their graphs (no calculator)**

1. A quadratic function f is shown with x -intercepts of 1 and 5, and vertex $(3, -4)$.



The function f can be written in the form $f(x) = (x - h)^2 + k$.

- (a) Write down h and k .

[2]

The function can also be written in the form $f(x) = \cancel{x}(x - a)(x - b)$

- (b) Write down the value of a and b .

[2]

- (c) Find the y -intercept.

[2]

Working:

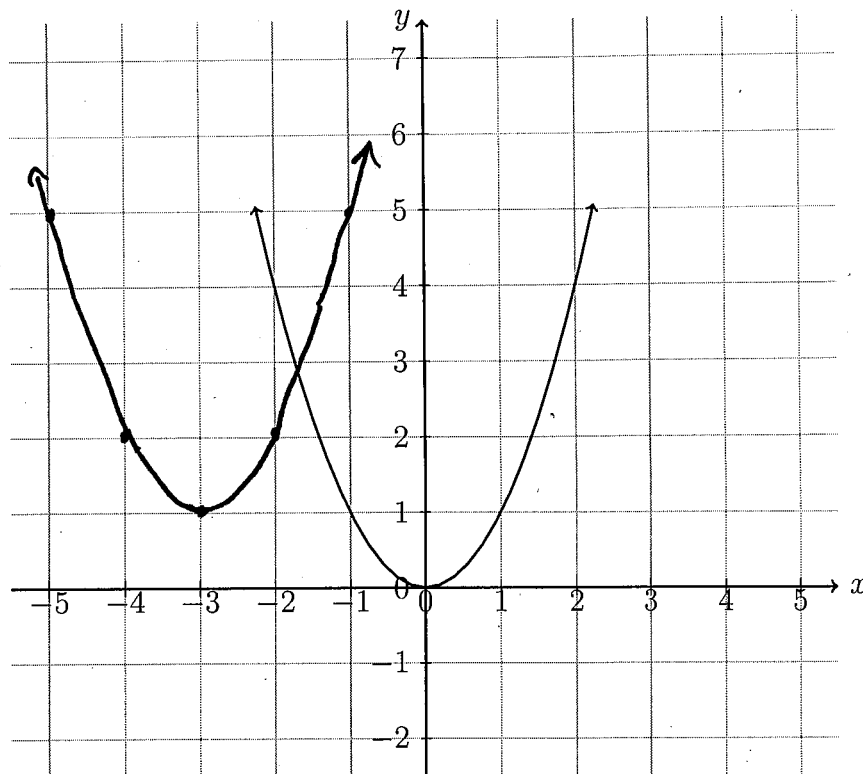
$$\begin{aligned}
 c) \quad f(x) &= (x-3)^2 - 4 \\
 &= x^2 - 6x + 9 - 4 \\
 &= x^2 - 6x + 5 \\
 &= (x-5)(x-1)
 \end{aligned}$$

$$f(0) = (-5)(-1) = 5$$

Answers:

- (a) $h = 3$ $k = -4$
- (b) $1, 5$
- (c) 5

2. The diagram below shows part of the graph of the function $f(x) = x^2$.



- (a) $g(x)$ is the image of f after a translation left 3 and up 1. Draw g . [2]
 (b) g can be written in the form $g(x) = (x - h)^2 + k$. Write down h and k . [2]
 (c) Expand g to standard form, $g(x) = ax^2 + bx + c$. [2]

Working:

$$\begin{aligned} \text{c) } g(x) &= (x+3)^2 + 1 \\ &= x^2 + 6x + 9 + 1 \\ &= x^2 + 6x + 10 \end{aligned}$$

Answers:

~~(a)~~

(b) $h = -3, k = 1$

(c) $g(x) = x^2 + 6x + 10$

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3. Let $f(x) = x^2 + 2x + 1$ and $g(x) = x + 1$.(a) Write down $f(0)$. [1](b) Find $(f - g)(x)$. [1](c) Find $(f \div g)(x)$ in simplest form, $x \neq 0$. [2](d) Write down $g^{-1}(4)$. [2](e) Find $g^{-1}(x)$. [2](f) Find $(f \circ g)(x)$. [2]

Working:

$$b) \quad x^2 + 2x + 1 - (x + 1) = x^2 + x$$

$$c) \quad \frac{f}{g} = \frac{x^2 + 2x + 1}{x + 1} = \frac{(x + 1)^2}{x + 1} = x + 1$$

$$d) \quad f^{-1}: x = y + 1$$

$$y = x - 1$$

$$f) \quad f(g(x)) = (x + 1)^2 + 2(x + 1) + 1$$

$$= x^2 + 2x + 1 + 2x + 2 + 1$$

$$= x^2 + 4x + 4$$

Answers:

(a) 1

(b) $x^2 + x$ (c) $x + 1$

(d) 3

(e) $x - 1$ (f) $x^2 + 4x + 4$

4. Let $f(x) = x^2 - 6x + 7$. f can be written in the form $f(x) = (x - h)^2 + k$.

(a) ~~Write down~~ ^{Find} the value of h and of k .

[2]

(b) Write down the equation of the axis of symmetry.

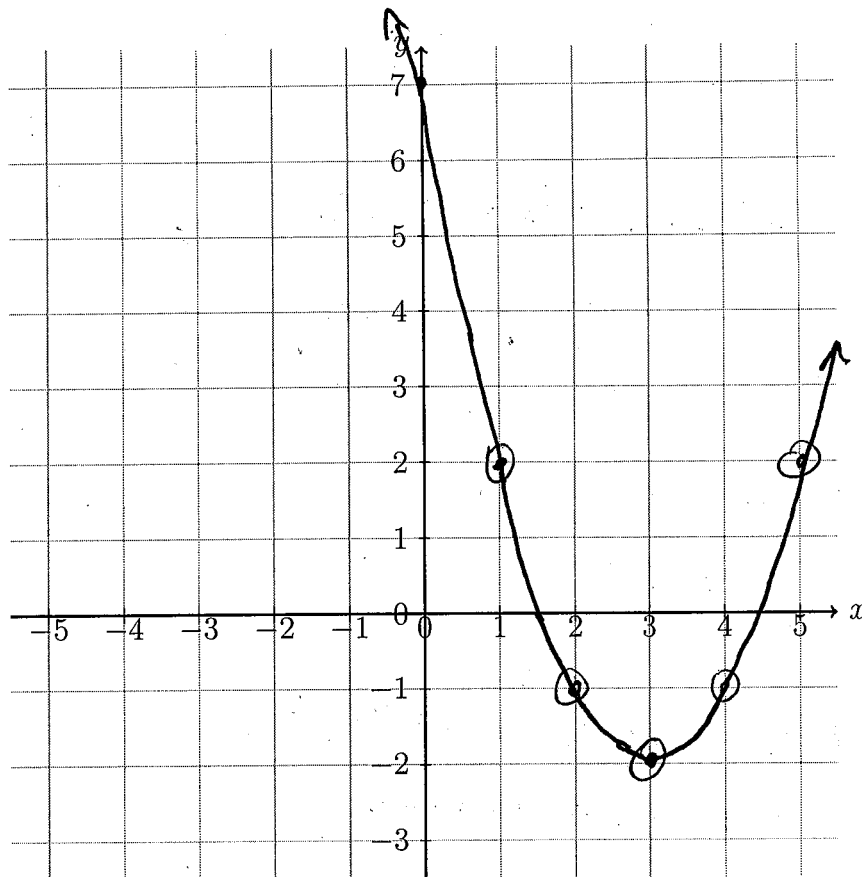
[1]

(c) Find the solutions of $f(x) = 0$.

[2]

(d) Draw the function $f(x)$ on the grid below.

[2]



Working:

$$\begin{aligned} a) f(x) &= x^2 - 6x + 9 - 2 \\ &= (x - 3)^2 - 2 \end{aligned}$$

$$\begin{aligned} c) & \cancel{(x-7)(x+1)} \\ x &= \frac{6 \pm \sqrt{(-6)^2 - 4(7)}}{2} \end{aligned}$$

$$= 3 \pm \sqrt{2}$$

Answers:

(a) $h = 3$ $k = -2$

(b) $x = 3$

(c) $3 \pm \sqrt{2}$

$$\begin{array}{r} 36 \\ 28 \\ \hline 8 \end{array}$$

$$\sqrt{8} = 2\sqrt{2}$$

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5. Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$. The distance between the two zeros of f is 9.

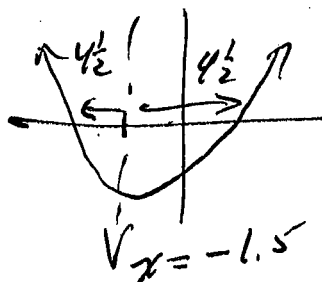
(a) Show that the two zeros are 3 and -6 . [2]

(b) Find the value of q and r . [4]

Working:

$$a) \pm \frac{9}{2} + (-1.5) = -6, 3$$

$$b) f(x) = (x+6)(x-3) \\ = x^2 + 3x - 18$$



Answers:

(b) $q = 3$ $r = -18$

6. Consider the equation $x^2 + (k-2)x = -4$, where k is a real number. Find the values of k for which the equation has two equal real solutions. [7]

Working:

$$\Delta = 0$$

$$\Delta = (k-2)^2 - 4(4) = 0$$

$$= k^2 - 4k + 4 - 16 = 0$$

$$= k^2 - 4k - 12 = 0$$

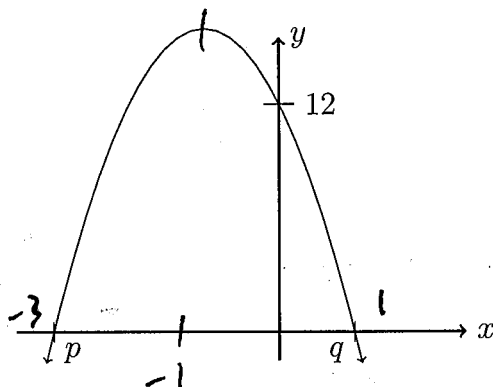
$$(k-6)(k+2) = 0$$

$$k = 6, -2$$

Answers:

$6, -2$

7. Let $f(x) = a(x+3)(x-1)$. The following diagram shows part of the graph of f .



The graph has x -intercepts at $(p, 0)$ and $(q, 0)$, and a y -intercept at $(0, 12)$.

- Write down the value of p and of q . [2]
- Find the value of a . [3]
- Find the equation of the axis of symmetry of the graph of f . [3]
- Find the largest value of f . [3]

The function f can be written in the form $f(x) = (x - h)^2 + k$.

- Write down the value of h and k . [3]

Working:

$$b) f(0) = a(3)(-1) = 12$$

$$a = -4$$

$$c) \frac{-3+1}{2} = -1$$

$$d) f(-1) = -4(-1+3)(-1-1)$$

$$= 16$$

$$e) (-1, 16)$$

Answers:

$$(a) \dots -3, 1$$

$$(b) \dots -4$$

$$(c) \dots x = -1$$

$$(d) \dots 16$$

$$(e) \dots -1, 16$$