0423HW_prob+stats *[79 marks]*

1. Let $f'(x) = 3x^2 + 2$. Given that $f(2) = 5$, find $f(x)$. *[6 marks]*

Markscheme

evidence of anti-differentiation ***(M1)***

e.g.
$\int f'(x)$,
$\int (3x^2 + 2)\mathrm{d}x$

$f(x) = x^3 + 2x + c$ (seen anywhere, including the answer) ***A1A1***

attempt to substitute (2, 5) ***(M1)***

e.g.
$f(2) = (2)^3 + 2(2)$,
$5 = 8 + 4 + c$

finding the value of *c* ***(A1)***

e.g.
$5 = 12 + c$,
$c = -7$

$f(x) = x^3 + 2x - 7$ ***A1*** ***N5***

[6 marks]

2. The random variable X has the following probability distribution. *[6 marks]*

x	1	2	3
$\mathrm{P}(X = x)$	s	0.3	q

Given that $\mathrm{E}(X) = 1.7$, find *q* .

Markscheme

correct substitution into
$\mathrm{E}(X) = \sum px$ (seen anywhere) ***A1***

e.g.
$1s + 2 \times 0.3 + 3q = 1.7$,
$s + 3q = 1.1$

recognizing
$\sum p = 1$ (seen anywhere) ***(M1)***

correct substitution into
$\sum p = 1$ ***A1***

e.g.
$s + 0.3 + q = 1$

attempt to solve simultaneous equations ***(M1)***

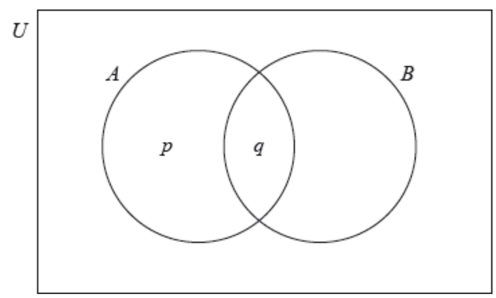
correct working ***(A1)***

e.g.
$0.3 + 2q = 0.7$,
$2s = 1$

$q = 0.2$ ***A1*** ***N4***

[6 marks]

The following Venn diagram shows the events A and B, where
$\mathrm{P}(A) = 0.4$, $\mathrm{P}(A \cup B) = 0.8$ and
$\mathrm{P}(A \cap B) = 0.1$. The values p and q are probabilities.



3a. (i) Write down the value of q. *[3 marks]*

(ii) Find the value of p.

Markscheme

(i)
$q = 0.1$ ***A1*** ***N1***

(ii) appropriate approach ***(M1)***

eg $\mathrm{P}(A) - q$, $0.4 - 0.1$

$p = 0.3$ ***A1*** ***N2***

[3 marks]

3b. Find $\mathrm{P}(B)$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B),\ \mathrm{P}(A \cap B) + \mathrm{P}(B \cap A')$

correct values ***(A1)***

eg $0.8 = 0.4 + \mathrm{P}(B) - 0.1,\ 0.1 + 0.4$

$\mathrm{P}(B) = 0.5$ ***A1*** ***N2***

[3 marks]

There are 10 items in a data set. The sum of the items is 60.

4a. Find the mean. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $\frac{60}{10}$

$\text{mean} = 6$ ***A1*** ***N2***

The variance of this data set is 3. Each value in the set is multiplied by 4.

4b. (i) Write down the value of the new mean. *[3 marks]*

(ii) Find the value of the new variance.

Markscheme

(i) new mean $= 24$ ***A1*** ***N1***

(ii) valid approach ***(M1)***

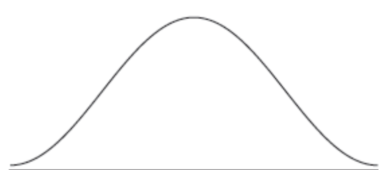
eg $\text{variance} \times (4)^2,\ 3 \times 16$, new standard deviation $= 4\sqrt{3}$

new variance $= 48$ ***A1*** ***N2***

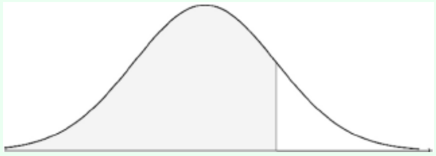
[3 marks]

A random variable X is distributed normally with a mean of 20 and standard deviation of 4.

5a. On the following diagram, shade the region representing $\mathrm{P}(X \leqslant 25)$. *[2 marks]*



Markscheme



A1A1 ***N2***

Note: Award ***A1*** for vertical line clearly to right of mean,

A1 for shading to left of their vertical line.

5b. Write down $\mathrm{P}(X \leqslant 25)$, correct to two decimal places. *[2 marks]*

Markscheme

$\mathrm{P}(X \leqslant 25) = 0.894350$ ***(A1)***

$\mathrm{P}(X \leqslant 25) = 0.89$ (must be 2 d.p.) ***A1*** ***N2***

[2 marks]

5c. Let $\mathrm{P}(X \leqslant c) = 0.7$. Write down the value of c. *[2 marks]*

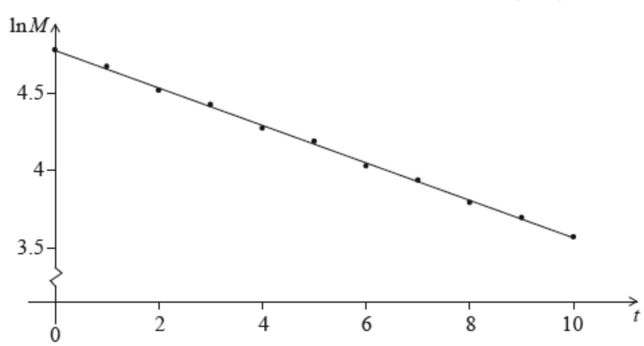
Markscheme

$c = 22.0976$

$c = 22.1$ ***A2*** ***N2***

[2 marks]

The mass M of a decaying substance is measured at one minute intervals. The points $(t,\ \ln M)$ are plotted for $0 \leqslant t \leqslant 10$, where t is in minutes. The line of best fit is drawn. This is shown in the following diagram.



The correlation coefficient for this linear model is $r = -0.998$.

6a. State **two** words that describe the linear correlation between $\ln M$ and t. *[2 marks]*

Markscheme

strong, negative (both required) ***A2*** ***N2***

[2 marks]

6b. The equation of the line of best fit is $\ln M = -0.12t + 4.67$. Given that $M = a \times b^t$, find the value of b. *[4 marks]*

Markscheme

METHOD 1

valid approach ***(M1)***

eg $\mathrm{e}^{\ln M} = \mathrm{e}^{-0.12t+4.67}$

correct use of exponent laws for $\mathrm{e}^{-0.12t+4.67}$ ***(A1)***

eg $\mathrm{e}^{-0.12t} \times \mathrm{e}^{4.67}$

comparing coefficients/terms ***(A1)***

eg $b^t = \mathrm{e}^{-0.12t}$

$b = \mathrm{e}^{-0.12}\ (\text{exact}),\ 0.887$ ***A1*** ***N3***

METHOD 2

valid approach ***(M1)***

eg $\ln(a \times b^t) = -0.12t + 4.67$

correct use of log laws for $\ln(ab^t)$ ***(A1)***

eg $\ln a + t \ln b$

comparing coefficients ***(A1)***

eg $-0.12 = \ln b$

$b = \mathrm{e}^{-0.12}\ (\text{exact}),\ 0.887$ ***A1*** ***N3***

[4 marks]

A competition consists of two independent events, shooting at 100 targets and running for one hour.

The number of targets a contestant hits is the S score. The S scores are normally distributed with mean 65 and standard deviation 10.

7a. A contestant is chosen at random. Find the probability that their S score is less than 50. *[2 marks]*

Markscheme

0.0668072

$\mathrm{P}(S < 50) = 0.0668\ (\text{accept } \mathrm{P}(S \leqslant 49) = 0.0548)$ ***A2*** ***N2***

[2 marks]

The distance in km that a contestant runs in one hour is the R score. The R scores are normally distributed with mean 12 and standard deviation 2.5. The R score is independent of the S score.

Contestants are disqualified if their S score is less than 50 **and** their R score is less than x km.

7b. Given that 1% of the contestants are disqualified, find the value of x. *[4 marks]*

Markscheme

valid approach ***(M1)***

Eg $\mathrm{P}(S < 50) \times \mathrm{P}(R < x)$

correct equation (accept any variable) ***A1***

eg $\mathrm{P}(S < 50) \times \mathrm{P}(R < x) = 1\%,\ 0.0668072 \times p = 0.01,\ \mathrm{P}(R < x) = \frac{0.01}{0.0668}$

finding the value of $\mathrm{P}(R < x)$ ***(A1)***

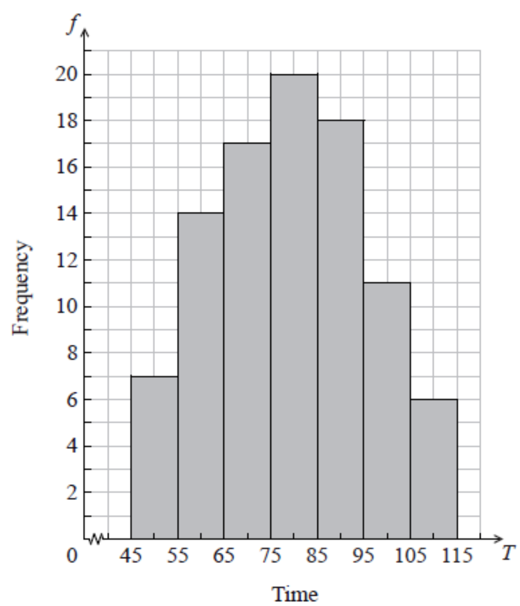
eg $\frac{0.01}{0.0668},\ 0.149684$

9.40553

$x = 9.41$ (accept $x = 9.74$ from 0.0548) ***A1*** ***N3***

[4 marks]

The histogram below shows the time *T* seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for *T* .

Time	$45 \le T < 55$	$55 \le T < 65$	$65 \le T < 75$	$75 \le T < 85$	$85 \le T < 95$	$95 \le T < 105$	$105 \le T < 115$
Frequency	7	14	p	20	18	q	6

8a. (i) Write down the value of p and of q . *[3 marks]*

(ii) Write down the median class.

Markscheme

(i)
$p = 17$,
$q = 11$ ***A1A1*** ***N2***

(ii)
$75 \le T < 85$ ***A1*** ***N1***

[3 marks]

8b. A child is selected at random. Find the probability that the child takes less than95 seconds to solve the puzzle. *[2 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. adding frequencies

$\frac{76}{93} = 0.8172043\ldots$

$\mathrm{P}(T < 95) = \frac{76}{93} = 0.817$ ***A1*** ***N2***

[2 marks]

8c. Consider the class interval $45 \le T < 55$. *[2 marks]*

(i) Write down the interval width.

(ii) Write down the mid-interval value.

Markscheme

(i) 10 ***A1*** ***N1***

(ii) 50 ***A1*** ***N1***

[2 marks]

8d. Hence find an estimate for the *[4 marks]*

(i) mean;

(ii) standard deviation.

Markscheme

(i) evidence of approach using mid-interval values (may be seen in part (ii)) ***(M1)***

79.1397849

$\overline{x} = 79.1$ ***A2*** ***N3***

(ii)
16.4386061

$\sigma = 16.4$ ***A1*** ***N1***

[4 marks]

8e. John assumes that T is normally distributed and uses this to estimate the probabilitythat a child takes less than 95 seconds to solve the puzzle. *[2 marks]*

Find John’s estimate.

Markscheme

e.g. standardizing,
$z = 0.9648\ldots$

0.8326812

$\mathrm{P}(T < 95) = 0.833$ ***A1*** ***N2***

[2 marks]

The weights of players in a sports league are normally distributed with a mean of $76.6\ \mathrm{kg}$, (correct to three significant figures). It is known that 80% of the players have weights between $68\ \mathrm{kg}$ and $82\ \mathrm{kg}$. The probability that a player weighs less than $68\ \mathrm{kg}$ is 0.05.

9a. Find the probability that a player weighs more than $82\ \mathrm{kg}$. *[2 marks]*

Markscheme

evidence of appropriate approach ***(M1)***

e.g.
$1 - 0.85$, diagram showing values in a normal curve

$\mathrm{P}(w \geq 82) = 0.15$ ***A1 N2***

[2 marks]

9b. (i) Write down the standardized value, *z*, for $68\ \mathrm{kg}$. *[4 marks]*

(ii) Hence, find the standard deviation of weights.

Markscheme

(i)
$z = -1.64$ ***A1 N1***

(ii) evidence of appropriate approach ***(M1)***

e.g.
$-1.64 = \frac{x-\mu}{\sigma}$,
$\frac{68-76.6}{\sigma}$

correct substitution ***A1***

e.g.
$-1.64 = \frac{68-76.6}{\sigma}$

$\sigma = 5.23$ ***A1 N1***

[4 marks]

9c. To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean. *[5 marks]*

(i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament.

Markscheme

(i)

$68.8 \leq \text{weight} \leq 84.4$ ***A1A1A1*** ***N3***

Note: Award ***A1*** for 68.8, ***A1*** for 84.4, ***A1*** for giving answer as an interval.

(ii) evidence of appropriate approach ***(M1)***

e.g.
$\mathrm{P}(-1.5 \leq z \leq 1.5)$,
$\mathrm{P}(68.76 < y < 84.44)$

$\mathrm{P}(\text{qualify}) = 0.866$ ***A1*** ***N2***

[5 marks]

9d. Of the players in the league, 25% are women. Of the women, 70% take part in the tournament. *[4 marks]*

Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman.

Markscheme

recognizing conditional probability ***(M1)***

e.g.
$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

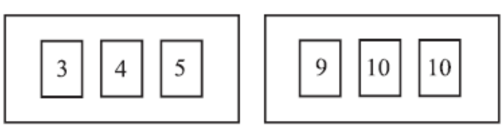
$\mathrm{P}(\text{woman and qualify}) = 0.25 \times 0.7$ ***(A1)***

$\mathrm{P}(\text{woman}|\text{qualify}) = \frac{0.25 \times 0.7}{0.866}$ ***A1***

$\mathrm{P}(\text{woman}|\text{qualify}) = 0.202$ ***A1***

[4 marks]

Two boxes contain numbered cards as shown below.



Two cards are drawn at random, one from each box.

10a. Copy and complete the table below to show all nine equally likely outcomes. *[2 marks]*

3 , 9		
3 , 10		
3 , 10		

Markscheme

$3, 9$	$4, 9$	$5, 9$
$3, 10$	$4, 10$	$5, 10$
$3, 10$	$4, 10$	$5, 10$

A2 N2

[2 marks]

10b. Let *S* be the sum of the numbers on the two cards. *[2 marks]*

Find the probability of each value of *S*.

Markscheme

$\mathrm{P}(12) = \frac{1}{9}$,
$\mathrm{P}(13) = \frac{3}{9}$,
$\mathrm{P}(14) = \frac{3}{9}$,
$\mathrm{P}(15) = \frac{2}{9}$ ***A2 N2***

[2 marks]

10c. Find the expected value of *S*. *[3 marks]*

Markscheme

correct substitution into formula for
$\mathrm{E}(X)$ ***A1***

e.g.
$\mathrm{E}(S) = 12 \times \frac{1}{9} + 13 \times \frac{3}{9} + 14 \times \frac{3}{9} + 15 \times \frac{2}{9}$

$\mathrm{E}(S) = \frac{123}{9}$ ***A2 N2***

[3 marks]

10d. Anna plays a game where she wins $\$50$ if *S* is even and loses $\$30$ if *S* is odd. *[3 marks]*

Anna plays the game 36 times. Find the amount she expects to have at the end of the 36 games.

Markscheme

METHOD 1

correct expression for expected gain E(*A*) for 1 game ***(A1)***

e.g.
$\frac{4}{9} \times 50 - \frac{5}{9} \times 30$

$\mathrm{E}(A) = \frac{50}{9}$

amount at end = expected gain for 1 game
$\times 36$ ***(M1)***

= 200 (dollars) ***A1*** ***N2***

METHOD 2

attempt to find expected number of wins and losses ***(M1)***

e.g.
$\frac{4}{9} \times 36$,
$\frac{5}{9} \times 36$

attempt to find expected gain E(*G*) ***(M1)***

e.g.
$16 \times 50 - 30 \times 20$

$\mathrm{E}(G) = 200$ (dollars) ***A1*** ***N2***

[3 marks]