

1208CW Vector + Calculus [29 marks]

1. Line L_1 has equation $\mathbf{r}_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$ and line L_2 has equation $\mathbf{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$. [7 marks]

Lines L_1 and L_2 intersect at point A. Find the coordinates of A.

- 2a. The line L passes through the point $(5, -4, 10)$ and is parallel to the vector $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$. [2 marks]

Write down a vector equation for line L .

- 2b. The line L intersects the x -axis at the point P. Find the x -coordinate of P. [6 marks]

- 3a. Consider points $A(1, -2, -1)$, $B(7, -4, 3)$ and $C(1, -2, 3)$. The line L_1 passes through C and is parallel to \overrightarrow{AB} . [2 marks]

Find \overrightarrow{AB} .

- 3b. Hence, write down a vector equation for L_1 . [2 marks]

- 3c. Consider points $A(1, -2, -1)$, $B(7, -4, 3)$ and $C(1, -2, 3)$. The line L_1 passes through C and is parallel to \overrightarrow{AB} . [3 marks]

A second line, L_2 , is given by $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s \begin{pmatrix} 3 \\ -3 \\ p \end{pmatrix}$.

Given that L_1 is perpendicular to L_2 , show that $p = -6$.

- 3d. Consider points $A(1, -2, -1)$, $B(7, -4, 3)$ and $C(1, -2, 3)$. The line L_1 passes through C and is parallel to \overrightarrow{AB} . [7 marks]

A second line, L_2 , is given by $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s \begin{pmatrix} 3 \\ -3 \\ p \end{pmatrix}$.

The line L_1 intersects the line L_2 at point Q. Find the x -coordinate of Q.