7-9 Homework: Mixed Calculus - without calculator   **[58 marks]**

1. 12M.1.sl.TZ1.3   [6 marks]
   Let $f(x) = e^{6x}$.

   (a) Write down $f'(x)$ [1]

   (b) The tangent to the graph of f at the point $P(0, b)$ has gradient m.   [4]

      i. Show that $m = 6$.
      ii. Find b.

   (c) Hence, write down the equation of this tangent.   [1]

2. 09M.1.sl.TZ1.3   [6 marks]
   Let $f(x) = e^x \cos x$. Find the gradient of the normal to the curve of f at $x = \pi$.

3. 13M.1.sl.TZ1.3   [7 marks]
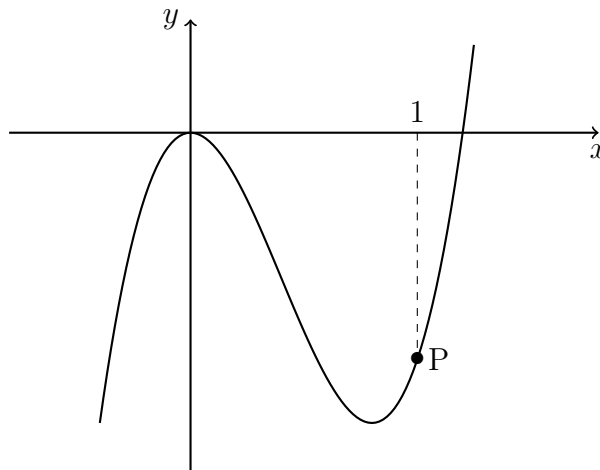   Consider $f(x) = x^2 \sin x$.

   (a) Find $f'(x)$.   [4]

   (b) Find the gradient of the curve of f at $x = \frac{\pi}{2}$.   [3]

4. 12N.1.sl.TZ0.4   [6 marks]
   Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.

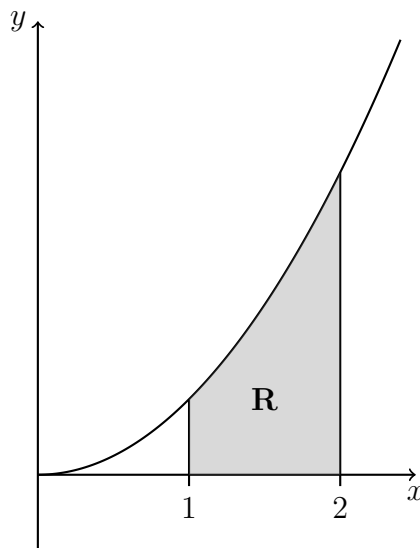The point P lies on the graph of f. At P, $x = 1$.

   (a) Find $f'(x)$.   [2]

   (b) The graph of f has a gradient of 3 at the point P. Find the value of a.   [4]

5. Let $f(x) = x^2$.   [6 marks]

   (a) Find $\int_1^2 (f(x))^2 \, dx$ [4]

   (b) The following diagram shows part of the graph of f.

   The shaded region R is enclosed by the graph of f, the x-axis, and the lines $x = 1$
   and $x = 2$.

   Find the volume of the solid formed when R is revolved $360°$ about the x-axis.
   [2]

6. 13N.1.sl.TZ0.4   [6 marks]
   Consider a function $f(x)$ such that $\int_2^5 f(x) \, dx = 10$.

   (a) Find $\int_2^5 3f(x) \, dx$.   [2]

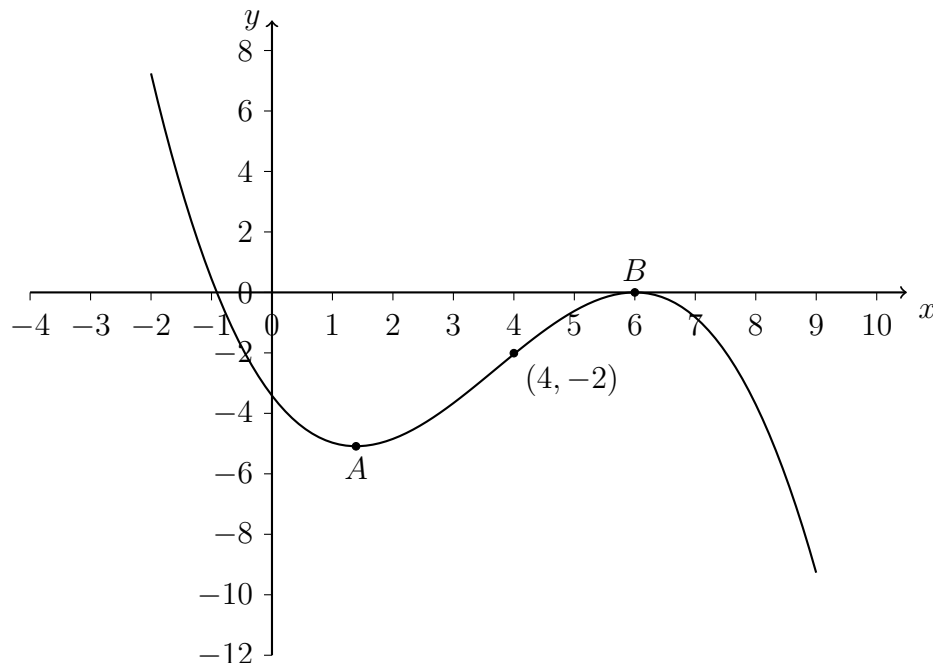   (b) Find $\int_2^5 (f(x) + 12) \, dx$.   [4]

7. 17M.1.sl.TZ1.6   [6 marks]

The following diagram shows the graph of f', the derivative of f.

The graph of f' has a local minimum at A, a local maximum at B and passes through $(4, 2)$. The point $P(4, 3)$ lies on the graph of the function, f.

(a) Write down the gradient of the curve of f at P.   [1]

(b) Find the equation of the normal to the curve of f at P.   [3]

(c) Determine the concavity of the graph of f when $4 < x < 5$ **and** justify your answer.   [2]

8. 16M.1.sl.TZ1.10   [15 marks]

Let $f(x) = \sqrt{4x + 5}$, for $x \geq -1.25$.

(a) Find $f'(1)$.   [4]

(b) Consider another function g. Let R be a point on the graph of g. The x-coordinate of R is 1. The equation of the tangent to the graph at R is $y = 3x + 6$.
Write down $g'(1)$.   [2]

(c) Find $g(1)$.   [2]

(d) Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent to the graph of h at the point where $x = 1$.   [7]