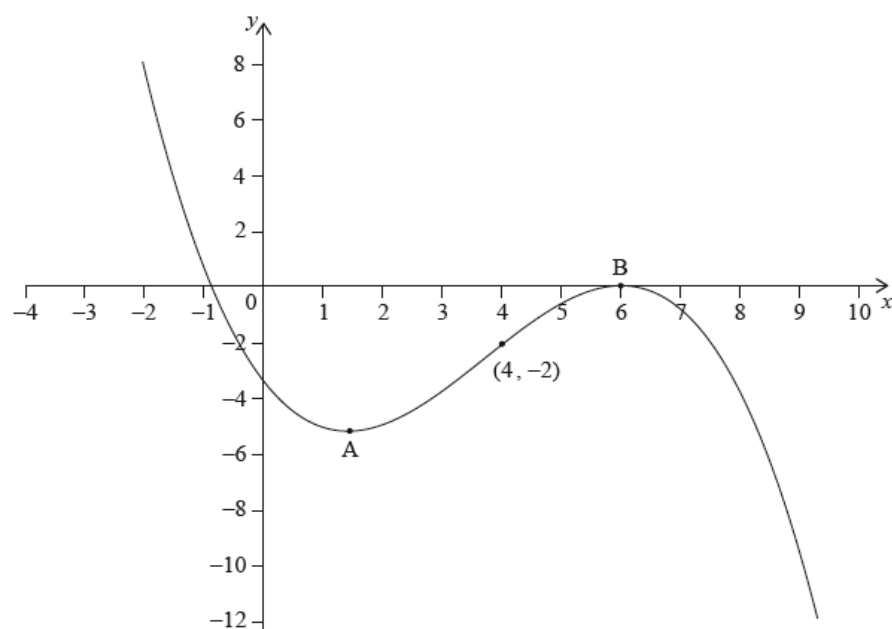


28 February 2018

Homework: Function graphs

1a. The following diagram shows the graph of f' , the derivative of f .



The graph of f' has a local minimum at A, a local maximum at B and passes through $(4, -2)$.

The point $P(4, 3)$ lies on the graph of the function, f .

Write down the gradient of the curve of f at P.

[1 mark]

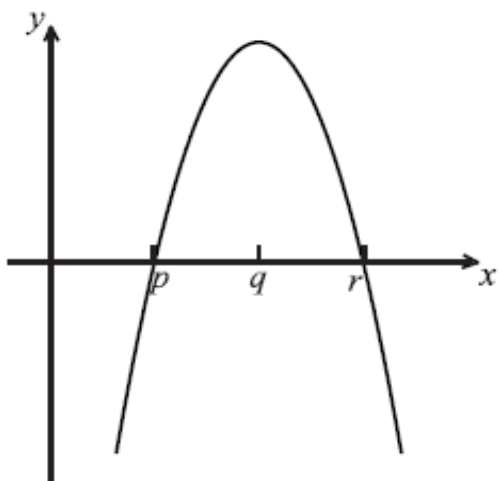
1b. Find the equation of the normal to the curve of f at P.

[3 marks]

1c. Determine the concavity of the graph of f when $4 < x < 5$ and justify your answer.

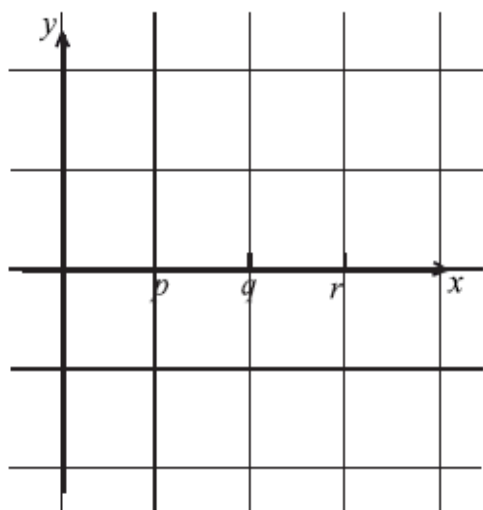
[2 marks]

2a. The diagram below shows part of the graph of the gradient function, $y = f'(x)$.



On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the x-intercept.

[2 marks]



2b. Complete the table, for the graph of $y = f(x)$.

[2 marks]

	x-coordinate
(i) Maximum point on f	
(ii) Inflexion point on f	

2c. Justify your answer to part (b) (ii).

[2 marks]

3a. Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

Use the quotient rule to show that $g'(x) = \frac{1-2\ln x}{x^3}$. [4 marks]

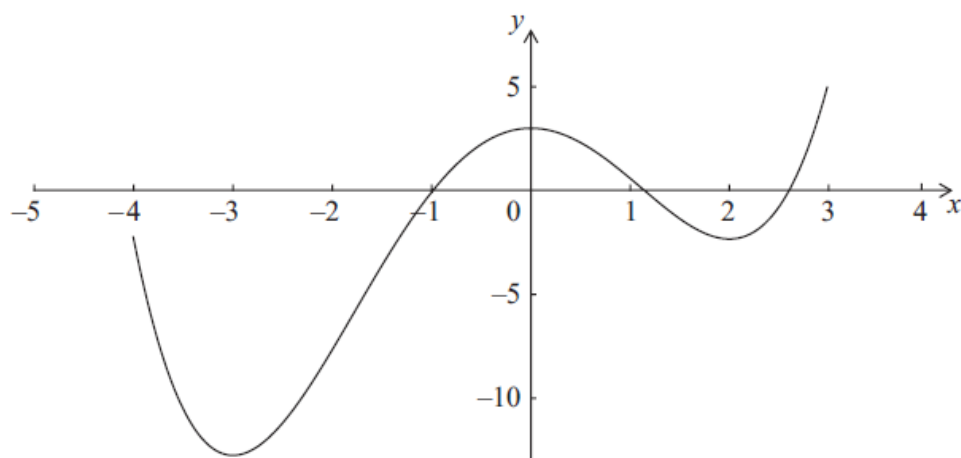
3b. The graph of g has a maximum point at A. Find the x -coordinate of A. [3 marks]

4a. Let $f'(x) = -24x^3 + 9x^2 + 3x + 1$. [3 marks]

There are two points of inflexion on the graph of f . Write down the x -coordinates of these points.

4b. Let $g(x) = f''(x)$. Explain why the graph of g has no points of inflexion. [2 marks]

5a. A function f is defined for $-4 \leq x \leq 3$. The graph of f is given below.



The graph has a local maximum when $x = 0$, and local minima when $x = -3$, $x = 2$.

Write down the x -intercepts of the graph of the **derivative** function, f' . [2 marks]

5b. Write down all values of x for which $f'(x)$ is positive. [2 marks]

5c. At point D on the graph of f , the x -coordinate is -0.5 . Explain why $f''(x) < 0$ at D. [2 marks]

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6a. Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

Find the x -coordinate of P.

[3 marks]

6b. The y -coordinate of P is $\ln 27$.

Find $f(x)$, expressing your answer as a single logarithm.

[8 marks]

6c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates (a, b) .

Find the value of a and of b , where $a, b \in \mathbb{N}$.

7a. Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

Show that $f'(x) = \frac{\ln x}{x}$.

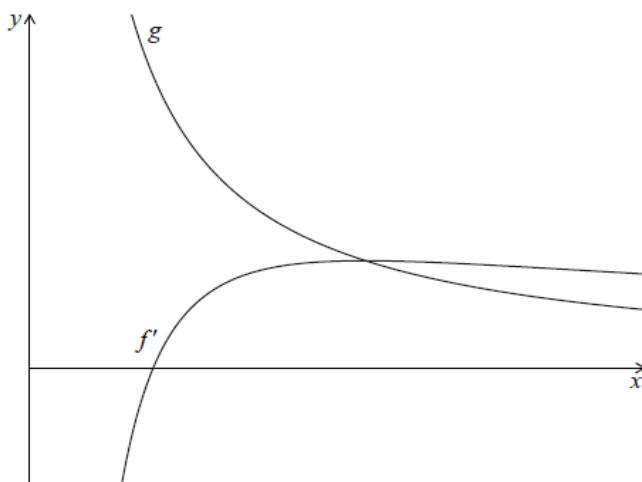
[2 marks]

7b. There is a minimum on the graph of f . Find the x -coordinate of this minimum.

[3 marks]

7c. Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g .

[2 marks]



The graph of f' has an x -intercept at $x = p$.

Write down the value of p .

7d. The graph of g intersects the graph of f' when $x = q$.

Find the value of q .

[3 marks]

7e. Let R be the region enclosed by the graph of f' , the graph of g and the line $x = p$.

Show that the area of R is $\frac{1}{2}$.

[5 marks]