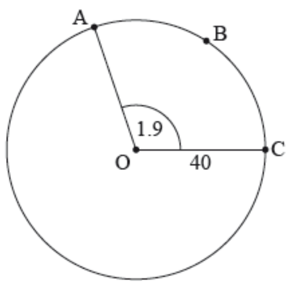
3-1+2+3_Trig-mild *[103 marks]*

The following diagram shows a circle with centre O and radius 40 cm.

diagram not to scale



The points A, B and C are on the circumference of the circle and $\mathrm{A\hat{O}C} = 1.9 \text{ radians}.$

1a. Find the length of arc ABC. *[2 marks]*

Markscheme

correct substitution into arc length formula ***(A1)***

eg $(40)(1.9)$

$\text{arc length} = 76 \text{ (cm)}$ ***A1 N2***

[2 marks]

1b. Find the perimeter of sector OABC. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $\text{arc} + 2r,\ 76 + 40 + 40$

$\text{perimeter} = 156 \text{ (cm)}$ ***A1 N2***

[2 marks]

1c. Find the area of sector OABC. *[2 marks]*

Markscheme

correct substitution into area formula *(A1)*

eg $\frac{1}{2}(1.9)(40)^2$

$\text{area} = 1520\ (\text{cm}^2)$ ***A1*** ***N2***

[2 marks]

The following diagram shows a circle with centre O and radius 3 cm.

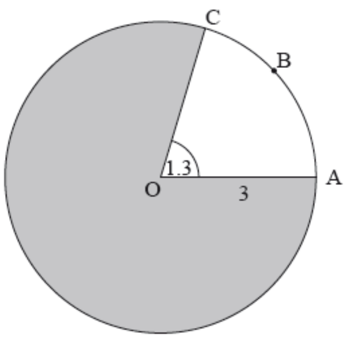


diagram not to scale

Points A, B, and C lie on the circle, and $\mathrm{A\hat{O}C} = 1.3\ \text{radians}$.

2a. Find the length of arc ABC. *[2 marks]*

Markscheme

correct substitution *(A1)*

eg $l = 1.3 \times 3$

l = 3.9 (cm) ***A1*** ***N2***

[2 marks]

2b. Find the area of the shaded region. *[4 marks]*

Markscheme

METHOD 1

valid approach ***(M1)***

eg finding reflex angle, $2\pi - \mathrm{C\hat{O}A}$

correct angle ***(A1)***

eg $2\pi - 1.3,\ 4.98318$

correct substitution ***(A1)***

eg $\frac{1}{2}(2\pi - 1.3)3^2$

22.4243

$\text{area} = 9\pi - 5.85\ (\text{exact}),\ 22.4\ \ (\text{cm}^2)$ ***A1 N3***

METHOD 2

correct area of small sector ***(A1)***

eg $\frac{1}{2}(1.3)3^2,\ 5.85$

valid approach ***(M1)***

eg circle – small sector, $\pi r^2 - \frac{1}{2}\theta r^2$

correct substitution ***(A1)***

eg $\pi(3^2) - \frac{1}{2}(1.3)3^2$

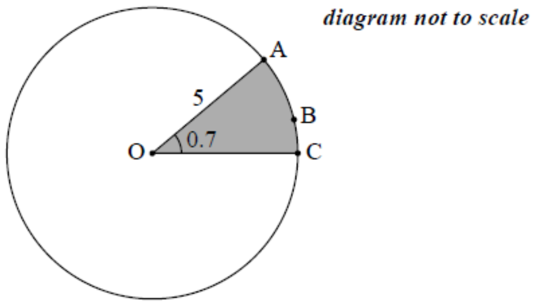
22.4243

$\text{area} = 9\pi - 5.85\ (\text{exact}),\ 22.4\ \ (\text{cm}^2)$ ***A1 N3***

[4 marks]

Total [6 marks]

The following diagram shows a circle with centre
O and radius
$5\ \mathrm{cm}$.



The points
A,
rmB and
rmC lie on the circumference of the circle, and
$\mathrm{A\hat{O}C} = 0.7$ radians.

3a. Find the length of the arc ABC. *[2 marks]*

Markscheme

correct substitution into arc length formula ***(A1)***

eg 0.7×5

arc length $= 3.5$ (cm) ***A1*** ***N2***

[2 marks]

3b. Find the perimeter of the shaded sector. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $3.5 + 5 + 5,\ \mathrm{arc} + 2r$

perimeter $= 13.5$ (cm) ***A1*** ***N2***

[2 marks]

3c. Find the area of the shaded sector. *[2 marks]*

Markscheme

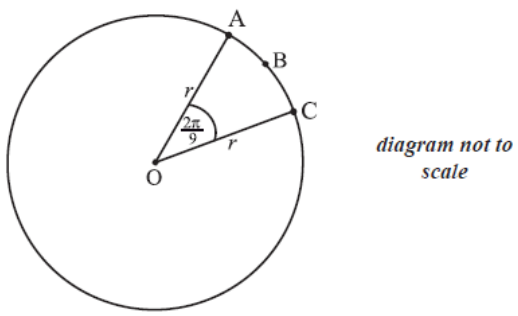
correct substitution into area formula *(A1)*

eg $\frac{1}{2}(0.7)(5)^2$

$\text{area} = 8.75\ (\text{cm}^2)$ ***A1*** ***N2***

[2 marks]

The diagram below shows a circle centre O, with radius *r*. The length of arc ABC is 3π cm and $\text{A}\hat{\text{O}}\text{C} = \frac{2\pi}{9}$.



4a. Find the value of *r*. *[2 marks]*

Markscheme

evidence of appropriate approach ***M1***

e.g. $3\pi = r\frac{2\pi}{9}$

$r = 13.5$ (cm) ***A1*** ***N1***

[2 marks]

4b. Find the perimeter of sector OABC. *[2 marks]*

Markscheme

adding two radii plus 3π ***(M1)***

$\text{perimeter} = 27 + 3\pi$ (cm) $(= 36.4)$ ***A1*** ***N2***

[2 marks]

4c. Find the area of sector OABC. *[2 marks]*

Markscheme

evidence of appropriate approach ***M1***

e.g. $\frac{1}{2} \times 13.5^2 \times \frac{2\pi}{9}$

area $= 20.25\pi\ (\mathrm{cm}^2)\ (= 63.6)$ ***A1*** ***N1***

[2 marks]

The circle shown has centre O and radius 3.9 cm.

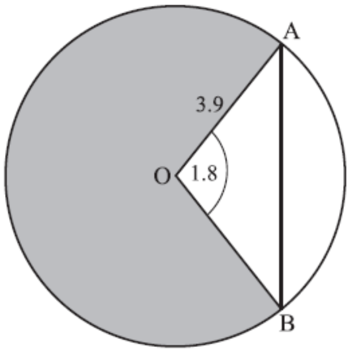


diagram not to scale

Points A and B lie on the circle and angle AOB is 1.8 radians.

5a. Find AB. *[3 marks]*

Markscheme

METHOD 1

choosing cosine rule ***(M1)***

substituting correctly ***A1***

e.g. $\text{AB} = \sqrt{3.9^2 + 3.9^2 - 2(3.9)(3.9)\cos 1.8}$

$\text{AB} = 6.11$ (cm) ***A1*** ***N2***

METHOD 2

evidence of approach involving right-angled triangles ***(M1)***

substituting correctly ***A1***

e.g. $\sin 0.9 = \frac{x}{3.9}$, $\frac{1}{2}\text{AB} = 3.9\sin 0.9$

$\text{AB} = 6.11$ (cm) ***A1*** ***N2***

METHOD 3

choosing the sine rule ***(M1)***

substituting correctly ***A1***

e.g. $\frac{\sin 0.670\ldots}{3.9} = \frac{\sin 1.8}{\text{AB}}$

$\text{AB} = 6.11$ (cm) ***A1*** ***N2***

[3 marks]

5b. Find the area of the shaded region. *[4 marks]*

Markscheme

METHOD 1

reflex $\mathrm{A\hat{O}B} = 2\pi - 1.8 \ (= 4.4832)$ ***(A2)***

correct substitution $A = \frac{1}{2}(3.9)^2(4.4832\ldots)$ ***A1***

area =34.1 (cm^2) ***A1*** ***N2***

METHOD 2

finding area of circle $A = \pi(3.9)^2 \ (= 47.78\ldots)$ ***(A1)***

finding area of (minor) sector $A = \frac{1}{2}(3.9)^2(1.8) \ (= 13.68\ldots)$ ***(A1)***

subtracting ***M1***

e.g. $\pi(3.9)^2 - 0.5(3.9)^2(1.8)$, $47.8 - 13.7$

area = 34.1 (cm^2) ***A1*** ***N2***

METHOD 3

finding reflex $\mathrm{A\hat{O}B} = 2\pi - 1.8 \ (= 4.4832)$ ***(A2)***

finding proportion of total area of circle ***A1***

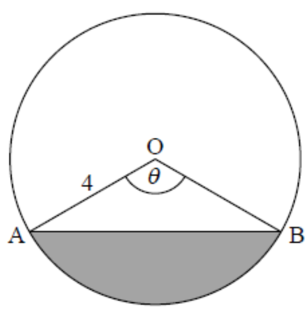
e.g. $\frac{2\pi - 1.8}{2\pi} \times \pi(3.9)^2$, $\frac{\theta}{2\pi} \times \pi r^2$

area = 34.1 (cm^2) ***A1*** ***N2***

[4 marks]

The diagram shows a circle, centre O, with radius 4 cm. Points A and B lie on the circumference of the circle and $\mathrm{A\hat{O}B} = \theta$, where $0 \le \theta \le \pi$.

diagram not to scale



6a. Find the area of the shaded region, in terms of θ. *[3 marks]*

Markscheme

valid approach to find area of segment ***(M1)***

eg area of sector – area of triangle, $\frac{1}{2}r^2(\theta - \sin\theta)$

correct substitution ***(A1)***

eg $\frac{1}{4}(4)^2\theta - \frac{1}{2}(4)^2\sin\theta,\ \frac{1}{2} \times 16[\theta - \sin\theta]$

area = $80 - 8\sin\theta$, $8(\theta - \sin\theta)$ ***A1 N2***

[3 marks]

6b. The area of the shaded region is 12 cm ^{2}. Find the value of θ. *[3 marks]*

Markscheme

setting **their** area expression equal to 12 ***(M1)***

eg $12 = 8(\theta - \sin\theta)$

2.26717

$\theta = 2.27$ (do not accept an answer in degrees) ***A2 N3***

[3 marks]

The following diagram shows a circle, centre O and radius r mm. The circle is divided into five equal sectors.

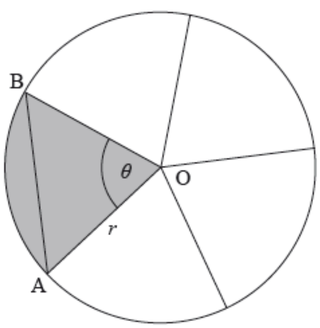


diagram not to scale

One sector is OAB, and $\mathrm{A\hat{O}B} = \theta$.

7a. Write down the **exact** value of θ in radians. *[1 mark]*

Markscheme

$\theta = \frac{2\pi}{5}$ ***A1 N1***

[1 mark]

The area of sector AOB is $20\pi\ \mathrm{mm}^2$.

7b. Find the value of r. *[3 marks]*

Markscheme

correct expression for area ***(A1)***

eg $A = \frac{1}{2}r^2\left(\frac{2\pi}{5}\right),\ \frac{\pi r^2}{5}$

evidence of equating their expression to 20π ***(M1)***

eg $\frac{1}{2}r^2\left(\frac{2\pi}{5}\right) = 20\pi,\ r^2 = 100,\ r = \pm 10$

$r = 10$ ***A1 N2***

[3 marks]

7c. Find AB. *[3 marks]*

Markscheme

METHOD 1

evidence of choosing cosine rule ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc\cos A$

correct substitution of **their** r and θ into RHS ***(A1)***

eg $10^2 + 10^2 - 2 \times 10 \times 10\cos\left(\frac{2\pi}{5}\right)$

11.7557

$\mathrm{AB} = 11.8\ (\mathrm{mm})$ ***A1 N2***

METHOD 2

evidence of choosing sine rule ***(M1)***

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution of **their** r and θ ***(A1)***

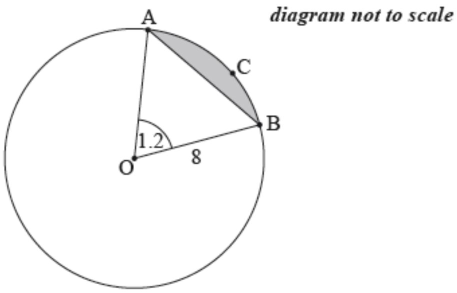
eg $\frac{\sin\frac{2\pi}{5}}{\mathrm{AB}} = \frac{\sin\left(\frac{1}{2}\left(\pi - \frac{2\pi}{5}\right)\right)}{10}$

11.7557

$\mathrm{AB} = 11.8\ (\mathrm{mm})$ ***A1 N2***

[3 marks]

The following diagram shows a circle with centre O and radius 8 cm.



The points A, B and C are on the circumference of the circle, and $\mathrm{A\hat{O}B}$ radians.

8a. Find the length of arc ACB. *[2 marks]*

Markscheme

correct substitution into formula ***(A1)***

eg $l = 1.2 \times 8$

$9.6\ (\mathrm{cm})$ ***A1 N2***

[2 marks]

8b. Find AB. *[3 marks]*

Markscheme

METHOD 1

evidence of choosing cosine rule ***(M1)***

eg $2r^2 - 2 \times r^2 \times \cos(\mathrm{A\hat{O}B})$

correct substitution into right hand side ***(A1)***

eg $8^2 + 8^2 - 2 \times 8 \times 8 \times \cos(1.2)$

9.0342795

$\mathrm{AB} = 9.03\ [9.03,\ 9.04]\ (\mathrm{cm})$ ***A1 N2***

METHOD 2

evidence of choosing sine rule ***(M1)***

eg $\frac{\mathrm{AB}}{\sin(\mathrm{A\hat{O}B})} = \frac{\mathrm{OB}}{\sin(\mathrm{O\hat{A}B})}$

finding angle OAB or OBA (may be seen in substitution) ***(A1)***

eg $\frac{\pi - 1.2}{2}$, 0.970796

$\mathrm{AB} = 9.03\ [9.03,\ 9.04]\ (\mathrm{cm})$ ***A1 N2***

[3 marks]

8c. Hence, find the perimeter of the shaded segment ABC. *[2 marks]*

Markscheme

correct working *(A1)*

eg $P = 9.6 + 9.03$

18.6342

$18.6\ [18.6,\ 18.7]\ (\mathrm{cm})$ ***A1*** ***N2***

[2 marks]

Total [7 marks]

Consider the following circle with centre O and radius 6.8 cm.

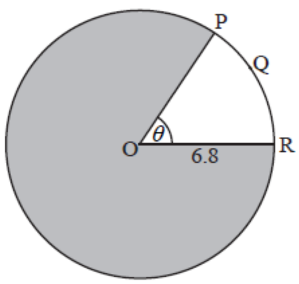


diagram not to scale

The length of the arc PQR is 8.5 cm.

9a. Find the value of θ . *[2 marks]*

Markscheme

correct substitution *(A1)*

e.g. $8.5 = \theta(6.8)$, $\theta = \frac{8.5}{6.8}$

$\theta = 1.25$ (accept $71.6°$) ***A1*** ***N2***

[2 marks]

9b. Find the area of the shaded region. *[4 marks]*

Markscheme

METHOD 1

correct substitution into area formula (seen anywhere) ***(A1)***

e.g. $A = \pi(6.8)^2$, $145.267\ldots$

correct substitution into area formula (seen anywhere) ***(A1)***

e.g. $A = \frac{1}{2}(1.25)(6.8^2)$, 28.9

valid approach ***M1***

e.g. $\pi(6.8)^2 - \frac{1}{2}(1.25)(6.8^2)$; $145.267\ldots - 28.9$; $\pi r^2 - \frac{1}{2}r^2 \sin\theta$

$A = 116\ (\text{cm}^2)$ ***A1*** ***N2***

METHOD 2

attempt to find reflex angle ***(M1)***

e.g. $2\pi - \theta$, $360 - 1.25$

correct reflex angle ***(A1)***

$\text{A}\hat{\text{O}}\text{B} = 2\pi - 1.25\ (= 5.03318\ldots)$

correct substitution into area formula ***A1***

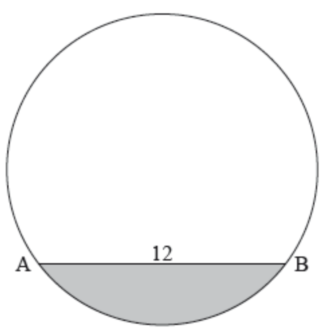
e.g. $A = \frac{1}{2}(5.03318\ldots)(6.8^2)$

$A = 116$ (
cm^2) ***A1*** ***N2***

[4 marks]

10. The following diagram shows the chord [AB] in a circle of radius 8 cm, where $\text{AB} = 12\text{ cm}$. *[7 marks]*

diagram not to scale



Find the area of the shaded segment.

Markscheme

attempt to find the central angle or half central angle ***(M1)***

eg  , cosine rule, right triangle

correct working ***(A1)***

eg $\cos\theta = \frac{8^2+8^2-12^2}{2\bullet 8\bullet 8}$, $\sin^{-1}\left(\frac{6}{8}\right)$, 0.722734, 41.4096°, $\frac{\pi}{2}-\sin^{-1}\left(\frac{6}{8}\right)$

correct angle $\mathrm{A\hat{O}B}$ (seen anywhere)

eg 1.69612, 97.1807°, $2\times\sin^{-1}\left(\frac{6}{8}\right)$ ***(A1)***

correct sector area

eg $\frac{1}{2}(8)(8)(1.70)$, $\frac{97.1807}{360}(64\pi)$, 54.2759 ***(A1)***

area of triangle (seen anywhere) ***(A1)***

eg $\frac{1}{2}(8)(8)\sin 1.70$, $\frac{1}{2}(8)(12)\sin 0.722$, $\frac{1}{2}\times\sqrt{64-36}\times 12$, 31.7490

appropriate approach (seen anywhere) ***(M1)***

eg $A_{\text{triangle}} - A_{\text{sector}}$, their sector-their triangle

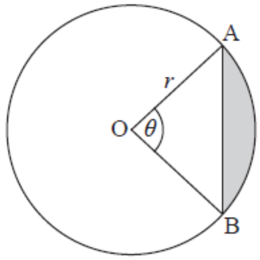
22.5269

area of shaded region $= 22.5\ (\mathrm{cm}^2)$ ***A1*** ***N4***

Note: Award ***M0A0A0A0A1*** then ***M1A0*** (if appropriate) for correct triangle area without any attempt to find an angle in triangle OAB.

[7 marks]

A circle centre O and radius
r is shown below. The chord [AB] divides the area of thecircle into two parts. Angle AOB is
θ .



11a. Find an expression for the area of the shaded region. *[3 marks]*

Markscheme

substitution into formula for area of triangle ***A1***

e.g. $\frac{1}{2}r \times r\sin\theta$

evidence of subtraction ***M1***

correct expression ***A1*** ***N2***

e.g. $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$, $\frac{1}{2}r^2(\theta - \sin\theta)$

[3 marks]

11b. The chord [AB] divides the area of the circle in the ratio 1:7. Find the value of θ . *[5 marks]*

Markscheme

evidence of recognizing that shaded area is $\frac{1}{8}$ of area of circle ***M1***

e.g. $\frac{1}{8}$ seen anywhere

setting up correct equation ***A1***

e.g. $\frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{8}\pi r^2$

eliminating 1 variable ***M1***

e.g. $\frac{1}{2}(\theta - \sin\theta) = \frac{1}{8}\pi$, $\theta - \sin\theta = \frac{\pi}{4}$

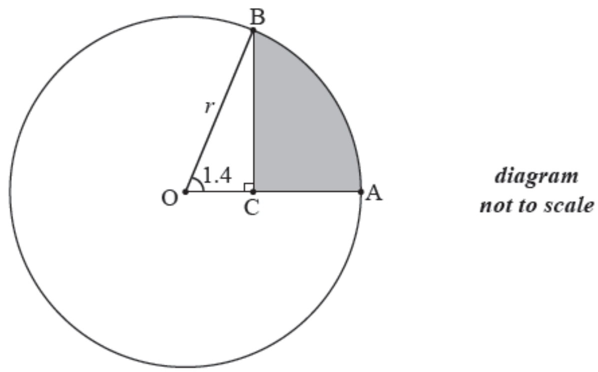
attempt to solve ***M1***

e.g. a sketch, writing $\sin x - x + \frac{\pi}{4} = 0$

$\theta = 1.77$ (do not accept degrees) ***A1*** ***N1***

[5 marks]

The following diagram shows a circle with centre O and radius r cm.



Points A and B are on the circumference of the circle and $\mathrm{A\hat{O}B} = 1.4$ radians .

The point C is on [OA] such that $\mathrm{B\hat{C}O} = \frac{\pi}{2}$ radians .

12a. Show that $\mathrm{OC} = r\cos 1.4$. *[1 mark]*

Markscheme

use right triangle trigonometry ***A1***

eg $\cos 1.4 = \frac{\mathrm{OC}}{r}$

$\mathrm{OC} = r\cos 1.4$ ***AG*** ***N0***

[1 mark]

12b. The area of the shaded region is 25 cm^2 . Find the value of r . *[7 marks]*

Markscheme

correct value for BC

eg $\mathrm{BC} = r\sin 1.4$, $\sqrt{r^2 - (r\cos 1.4)^2}$ ***(A1)***

area of $\Delta\mathrm{OBC} = \frac{1}{2}r\sin 1.4 \times r\cos 1.4 \left(= \frac{1}{2}r^2 \sin 1.4 \times \cos 1.4\right)$ ***A1***

area of sector $\mathrm{OAB} = \frac{1}{2}r^2 \times 1.4$ ***A1***

attempt to subtract in any order ***(M1)***

eg sector – triangle, $\frac{1}{2}r^2 \sin 1.4 \times \cos 1.4 - 0.7r^2$

correct equation ***A1***

eg $0.7r^2 - \frac{1}{2}r\sin 1.4 \times r\cos 1.4 = 25$

attempt to solve ***their*** equation ***(M1)***

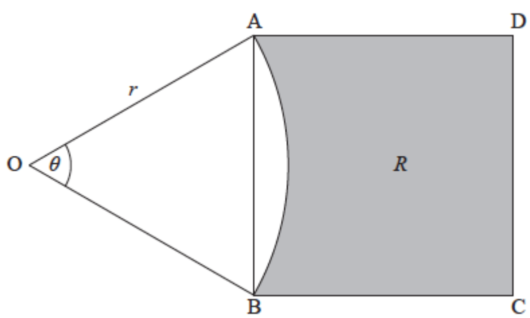
eg sketch, writing as quadratic, $\frac{25}{0.616\ldots}$

$r = 6.37$ ***A1*** ***N4***

[7 marks]

Note: Exception to ***FT*** rule. Award ***A1FT*** for a correct ***FT*** answer from a quadratic equation involving two trigonometric functions.

The following diagram shows a square $ABCD$, and a sector OAB of a circle centre O, radius r. Part of the square is shaded and labelled R.



$$\mathrm{A\hat{O}B} = \theta, \text{ where } 0.5 \leq \theta < \pi.$$

13a. Show that the area of the square $ABCD$ is $2r^2(1 - \cos\theta)$. *[4 marks]*

Markscheme

area of $\mathrm{ABCD} = \mathrm{AB}^2$ (seen anywhere) ***(A1)***

choose cosine rule to find a side of the square ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc\cos\theta$

correct substitution (for triangle AOB) ***A1***

eg $r^2 + r^2 - 2 \times r \times r\cos\theta,\ \mathrm{OA}^2 + \mathrm{OB}^2 - 2 \times \mathrm{OA} \times \mathrm{OB}\cos\theta$

correct working for AB^2 ***A1***

eg $2r^2 - 2r^2\cos\theta$

$\mathrm{area} = 2r^2(1 - \cos\theta)$ ***AG N0***

Note: Award no marks if the only working is $2r^2 - 2r^2\cos\theta$.

[4 marks]

13b. When $\theta = \alpha$, the area of the square $ABCD$ is equal to the area of the sector OAB. *[4 marks]*

(i) Write down the area of the sector when $\theta = \alpha$.

(ii) Hence find α.

Markscheme

(i) $\frac{1}{2}\alpha r^2$ $\left(\text{accept } 2r^2(1 - \cos\alpha)\right)$ ***A1 N1***

(ii) correct equation in one variable ***(A1)***

eg $2(1 - \cos\alpha) = \frac{1}{2}\alpha$

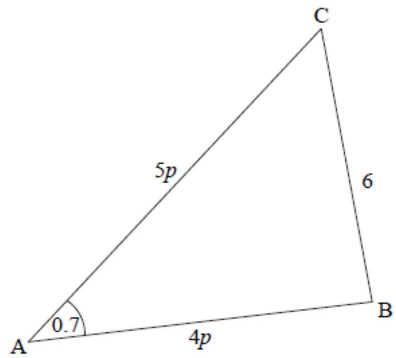
$\alpha = 0.511024$

$\alpha = 0.511$ $(\text{accept } \theta = 0.511)$ ***A2 N2***

Note: Award ***A1*** for $\alpha = 0.511$ and additional answers.

[4 marks]

The following diagram shows a triangle ABC.



$\text{BC} = 6$,
$\text{C}\hat{\text{A}}\text{B} = 0.7$ radians ,
$\text{AB} = 4p$,
$\text{AC} = 5p$, where
$p > 0$.

14a. (i) Show that $p^2(41 - 40\cos 0.7) = 36$. *[4 marks]*

(ii) Find p .

Markscheme

(i) evidence of valid approach ***(M1)***

e.g. choosing cosine rule

correct substitution ***(A1)***

e.g. $6^2 = (5p)^2 + (4p)^2 - 2 \times (4p) \times (5p)\cos 0.7$

simplification ***A1***

e.g. $36 = 25p^2 + 16p^2 - 40p^2\cos 0.7$

$p^2(41 - 40\cos 0.7) = 36$ ***AG N0***

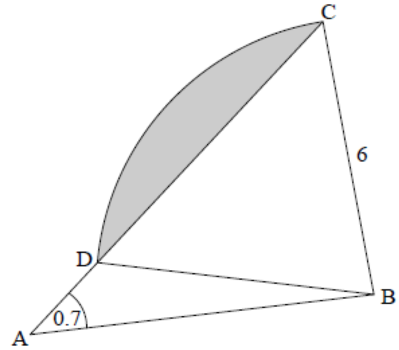
(ii) $1.85995\ldots$

$p = 1.86$ ***A1 N1***

Note: Award ***A0*** for $p = \pm 1.86$, i.e. not rejecting the negative value.

[4 marks]

Consider the circle with centre B that passes through the point C. The circle cuts the line CA at D, and $\mathrm{A\hat{D}B}$ is obtuse. Part of the circle is shown in the following diagram.



14b. Write down the length of BD. *[1 mark]*

Markscheme

$\mathrm{BD} = 6$ ***A1 N1***

[1 mark]

14c. Find $\mathrm{A\hat{D}B}$. *[4 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. choosing sine rule

correct substitution ***A1***

e.g. $\frac{\sin \mathrm{A\hat{D}B}}{4p} = \frac{\sin 0.7}{6}$

acute $\mathrm{A\hat{D}B} = 0.9253166\ldots$ ***(A1)***

$\pi - 0.9253166\ldots = 2.216275\ldots$

$\mathrm{A\hat{D}B} = 2.22$ ***A1 N3***

[4 marks]

14d. (i) Show that $\mathrm{C\hat{B}D} = 1.29$ radians, correct to 2 decimal places. *[6 marks]*

(ii) Hence, find the area of the shaded region.

Markscheme

(i) evidence of valid approach ***(M1)***

e.g. recognize isosceles triangle, base angles equal

$\pi - 2(0.9253\ldots)$ ***A1***

$\mathrm{C\widehat{B}D} = 1.29$ ***AG*** ***N0***

(ii) area of sector BCD ***(A1)***

e.g. $0.5 \times (1.29) \times (6)^2$

area of triangle BCD ***(A1)***

e.g. $0.5 \times (6)^2 \sin 1.29$

evidence of subtraction ***M1***

$5.92496\ldots$

$5.937459\ldots$

$\text{area} = 5.94$ ***A1*** ***N3***

[6 marks]