

January 11, 2018

Test: Vectors, calculus, binomial

Solutions

1. (i) $\vec{AB} = \vec{OB} - \vec{OA}$

M1

$$= \begin{pmatrix} 6-5 \\ 5-2 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

A1

(ii) $\vec{AC} = \vec{OC} - \vec{OA}$

$$= \begin{pmatrix} 7-5 \\ 6-2 \\ 4+1-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ a \end{pmatrix}$$

A1

b) $\cos \frac{\pi}{2} = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|}$

R1

$$0 = 1 \cdot 2 + 3 \cdot 4 + 2a$$

A1 A1

$$a = -7$$

A1

c) (i) $\cos \theta = \frac{1 \cdot 2 + 3 \cdot 4 + 2a}{\sqrt{1^2 + 3^2 + 2^2} \times \sqrt{2^2 + 4^2 + a^2}}$

M1

A1 A1

$$= \frac{2a+14}{\sqrt{14} \sqrt{a^2+20}} = \frac{2a+14}{\sqrt{14a^2+280}}$$

A1

(ii)

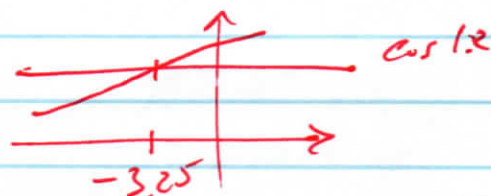
$$\cos 1.2 = \frac{2a+14}{\sqrt{14a^2+280}}$$

A1 M1

$$a = -3.251637...$$

$$\approx -3.25$$

A2



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(2)

Test: Vectors, Calculus

Solutions

$$2.(a) \quad f(2) = a(2^3) + b(2)^2 + c = 9 \quad \text{M1 A1}$$

$$8a + 4b + c = 9$$

$$(b) \quad \cancel{f'(2) = 0} \quad f'(1) = 0 \quad \text{M1 A1}$$

$$f'(x) = 3ax^2 + 2bx \quad \text{A1 A1}$$

$$\cancel{f'(2) = 12a + 4b = 0}$$

$$f'(1) = \underline{3a + 2b = 0} \quad \text{A1} \quad (1)$$

$$f(1) = a(1)^3 + b(1)^2 + c = 4 \quad \text{M1 A1}$$

$$\underline{a + b + c = 4} \quad (2)$$

$$(c) \quad \begin{aligned} 8a + 4b + c &= 9 & (1) \\ 3a + 2b &= 0 & (2) \\ a + b + c &= 4 & (3) \end{aligned}$$

$$7a + 3b = 5 \quad (1-3) \times 2 \quad \text{M1}$$

$$14a + 6b = 10$$

$$\underline{9a + 6b = 0} \quad (2) \times 3$$

$$5a = 10$$

$$a = 2$$

$$\Rightarrow (2) \quad 3(\underline{2}) + 2b = 0$$

$$b = -3$$

$$2 + -3 + c = 4 \Rightarrow c = 5$$

$$a = 2, b = -3, c = 5 \quad \text{A1 A1 A1}$$

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3

Solutions

3. $f'(x) = \frac{1}{4}(2)x = \frac{1}{2}x$

$$f'(4) = \frac{1}{2}(4) = 2$$

L:

$$y - 6 = 2(x - 4)$$

4

4. (a) $P(x=2) + P(x=3) = 0.5$

M1

$$r + 0.2 = 0.5$$

$$r = 0.3$$

A1

(b) $E(x) = 0p + 1q + 2(0.3) + 3(0.2) = 1.4$ ~~A1~~ A1

$$q + 1.2 = 1.4$$

A1

$$q = 0.2$$

A1

$$p + 0.2 + 0.3 + 0.2 = 1.0$$

M1 A1

$$p = 0.3$$

A1

8

5. (a) $p = 0.05$ $n = 240$

$$E(x) = np = 0.05 \cdot 240 = 12$$
 A1 A1

(b) $P(X=15) = \binom{240}{15} 0.95^{(240-15)} \cdot 0.05^{15}$ M1 A1

$$= 0.0732959...$$

$$\approx 0.0733$$

(c) $P(x \geq 10) = 1 - P(x \leq 9)$ A1

$$= 1 - 0.2356711...$$
 M1

$$= 0.7643288...$$

$$\approx 0.764$$

A1

7

Test

Solutions

$$6.(a) \quad r = \frac{4}{1.6} = 2.5 \quad A1$$

$$(b) \quad S_6 = 0.64 \frac{(1 - 2.5^6)}{1 - 2.5} \quad A1$$

$$= 103.74 \quad A1$$

$$(c) \quad S_n = 0.64 \left(\frac{2.5^n - 1}{2.5 - 1} \right) > 75,000 \quad M1 A1$$

$$2.5^n > 75,000 \left(\frac{1.5}{0.64} \right) + 1 = 175,782.25$$

$$n > \log_{2.5} 175,782.25 = 13.1803...$$

$$n = 14 \quad A1$$

(7)

$$7. \quad a = 13.3823... \approx 13.4 \quad M1 A1 A1$$

$$b = 137.4827... \approx 137$$

$$(b) \quad c(6) = 13.38... \cdot 6 + 137.48... \quad A1$$

$$c(7) = 13.38... (7) + 137... = 231.1589... \quad A1$$

$$\approx 231$$

(integer)

$$(c) \quad f(0) = \frac{2000}{1 + 99e^{-k(0)}} = \frac{2000}{100} = 20 \quad A1$$

(d) Error on test. The value of k should be given

(9)

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SOLUTIONS

8. (a) $m = 3$ AI
 $n = 4$ AI

(b) $2^{3(2x+1)} = 2^{4(2x-3)}$ m, m

$$\begin{aligned} 3(2x+1) &= 4(2x-3) \\ 6x+3 &= 8x-12 \quad \text{AI} \\ x &= 7\frac{1}{2} \quad \text{AI} \end{aligned}$$

(6)

9(a) 5 AI $f: (5, -1)$ m

(b) $f(f(-1)) = f(2)$ m, AI
 $= 1$ AI

(c) on exam graph A2 &

(7)

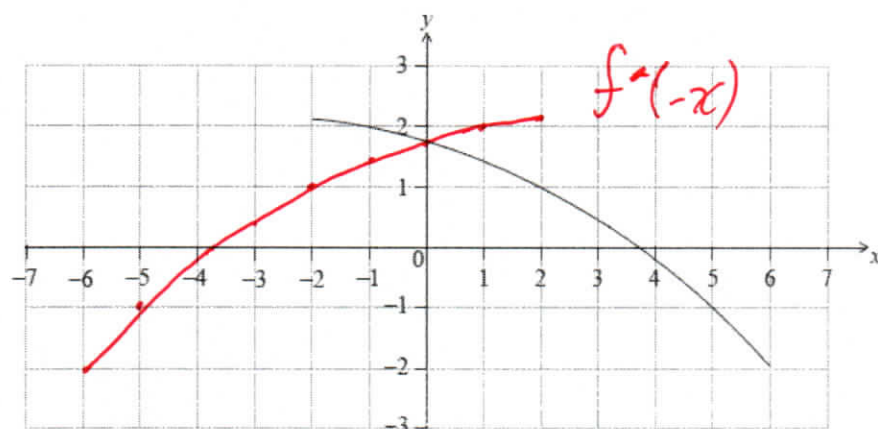
10. (a) on sheet N3

(b) $G(45) = 95e^{(-0.02(45))} + 40$ m
 $= 78.624...$

$$\begin{aligned} G(45) \cdot 45 &= 78.624... \cdot 45 \quad \text{m} \\ &= 3538.08... \\ &\approx \$3540 \quad \text{AI} \end{aligned}$$

(8)

9a. The following diagram shows the graph of a function f .



Find $f^{-1}(-1)$.

[2 marks]

9b. Find $(f \circ f)(-1)$.

[3 marks]

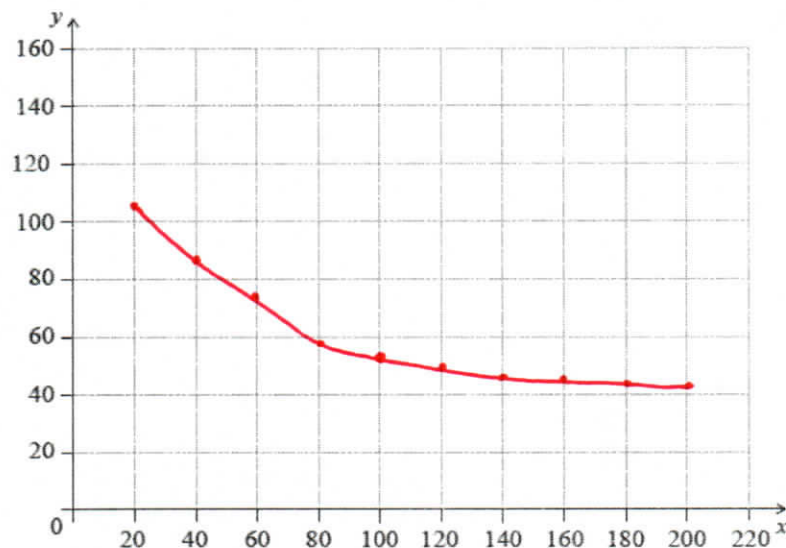
9c. On the same diagram, sketch the graph of $y = f(-x)$.

[2 marks]

10a. Let $G(x) = 95e^{(-0.02x)} + 40$, for $20 \leq x \leq 200$.

On the following grid, sketch the graph of G .

[3 marks]



10b. Robin and Pat are planning a wedding banquet. The cost per guest, G dollars, is modelled by the function $G(n) = 95e^{(-0.02n)} + 40$, for $20 \leq n \leq 200$, where n is the number of guests.

Calculate the **total** cost for 45 guests.

[3 marks]