7. [Maximum mark: 7]

A particle moves in a straight line. Its velocity $v\,\mathrm{m\,s^{-1}}$ after t seconds is given by

$$v = 6t - 6, \text{ for } 0 \le t \le 2.$$

After p seconds, the particle is $2\,\mathrm{m}$ from its initial position. Find the possible values of p.

6. *[Maximum mark: 8]*

Ramiro and Lautaro are travelling from Buenos Aires to El Moro.

Ramiro travels in a vehicle whose velocity in m s^{-1} is given by $V_R = 40 - t^2$, where t is in seconds.

Lautaro travels in a vehicle whose displacement from Buenos Aires in metres is given by $S_L = 2t^2 + 60$.

When $t = 0$, both vehicles are at the same point.

Find Ramiro's displacement from Buenos Aires when $t = 10$.

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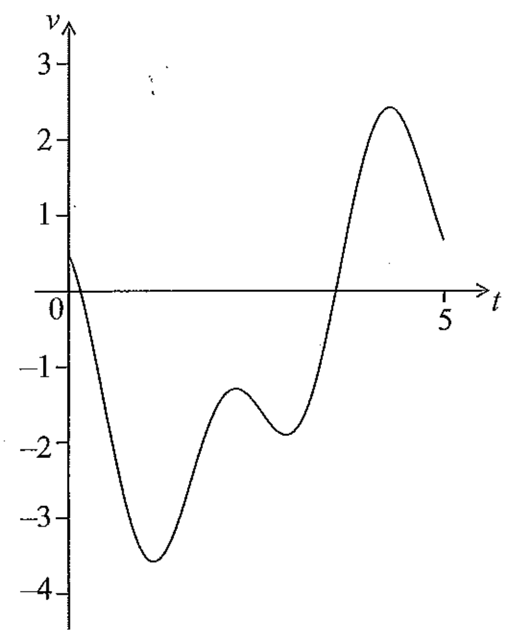
Do **not** write solutions on this page.

9. [Maximum mark: 14]

A particle P moves along a straight line so that its velocity, v ms^{-1}, after t seconds, is given by $v = \cos 3t - 2\sin t - 0.5$, for $0 \le t \le 5$. The initial displacement of P from a fixed point O is 4 metres.

(a) Find the displacement of P from O after 5 seconds. [5]

The following sketch shows the graph of v.



(b) Find when P is first at rest. [2]

(c) Write down the number of times P changes direction. [2]

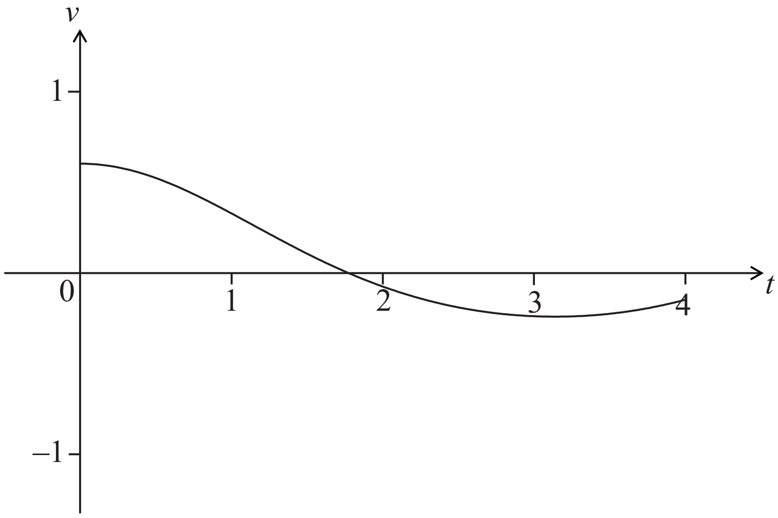
(d) Find the acceleration of P after 3 seconds. [2]

(e) Find the maximum speed of P. [3]

7. *[Maximum mark: 6]*

A particle starts from point A and moves along a straight line. Its velocity, $v\,\mathrm{m\,s^{-1}}$, after t seconds is given by $v(t) = \mathrm{e}^{\frac{1}{2}\cos t} - 1$, for $0 \le t \le 4$. The particle is at rest when $t = \frac{\pi}{2}$.

The following diagram shows the graph of v.



(a) Find the distance travelled by the particle for $0 \le t \le \frac{\pi}{2}$. *[2]*

(b) Explain why the particle passes through A again. *[4]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

A particle moves in a straight line. Its velocity, $v\,\mathrm{ms}^{-1}$, at time t seconds, is given by

$$v = (t^2 - 4)^3, \text{ for } 0 \le t \le 3.$$

(a) Find the velocity of the particle when $t = 1$. *[2]*

(b) Find the value of t for which the particle is at rest. *[3]*

(c) Find the total distance the particle travels during the first three seconds. *[3]*

(d) Show that the acceleration of the particle is given by $a = 6t(t^2 - 4)^2$. *[3]*

(e) Find all possible values of t for which the velocity and acceleration are both positive or both negative. *[4]*

7. *[Maximum mark: 7]*

Let $f(x) = \dfrac{g(x)}{h(x)}$, where $g(2) = 18$, $h(2) = 6$, $g'(2) = 5$, and $h'(2) = 2$. Find the equation of the normal to the graph of f at $x = 2$.

Do **not** write solutions on this page.

9. [Maximum mark: 14]

Let $f(x)=\frac{1}{x-1}+2$, for $1<x<4$.

(a) Write down the equation of the horizontal asymptote of the graph of f. [2]

(b) Find $f'(x)$. [2]

Let $g(x)=a\mathrm{e}^{-x}+b$, for $x\geq 1$. The graphs of f and g have the same horizontal asymptote.

(c) Write down the value of b. [2]

(d) Given that $g'(1)=-\mathrm{e}$, find the value of a. [4]

(e) There is a value of x for which the graphs of f and g have the same gradient. Find this gradient. [4]

5. (a) $t = 5$ ***(A1)***

correct substitution into formula ***(A1)***

eg $210\sin(0.5 \times 5 - 2.6) + 990$, $P(5)$

969.034982…

969 (deer) (must be an integer) ***A1*** ***N3***

[3 marks]

(b) (i) evidence of considering derivative ***(M1)***

eg P'

104.475

104 (deer per month) ***A1*** ***N2***

(ii) (the deer population size is) **increasing** ***A1*** ***N1***

[3 marks]

Total [6 marks]

6. **METHOD 1**

$S_L(0) = 60$ (seen anywhere) ***(A1)***

recognizing need to integrate V_R ***(M1)***

eg $S_R(t) = \int V_R \, dt$

correct expression

eg $40t - \frac{1}{3}t^3 + C$ ***A1A1***

Note: Award ***A1*** for $40t$, and ***A1*** for $-\frac{1}{3}t^3$.

equate displacements to find C ***(R1)***

eg $40(0) - \frac{1}{3}(0)^3 + C = 60$, $S_L(0) = S_R(0)$

$C = 60$ ***A1***

attempt to find displacement ***(M1)***

eg $S_R(10)$, $40(10) - \frac{1}{3}(10)^3 + 60$

126.666

$126\frac{2}{3}$ (exact), 127 (m) ***A1*** ***N5***

continued …

9. (a) substituting $t = 1$ into v ***(M1)***

eg $v(1), \left(1^2 - 4\right)^3$

velocity $= -27 \ \left(\text{ms}^{-1}\right)$ ***A1*** ***N2***

[2 marks]

(b) valid reasoning ***(R1)***

eg $v = 0$, $\left(t^2 - 4\right)^3 = 0$

correct working ***(A1)***

eg $t^2 - 4 = 0$, $t = \pm 2$, sketch

$t = 2$ ***A1*** ***N2***

[3 marks]

(c) correct integral expression for distance ***(A1)***

eg $\int_0^3 |v|$, $\int \left|\left(t^2 - 4\right)^3\right|$, $-\int_0^2 v\,dt + \int_2^3 v\,dt$,

$\int_0^2 \left(4 - t^2\right)^3 \mathrm{d}t + \int_2^3 \left(t^2 - 4\right)^3 \mathrm{d}t$ (do not accept $\int_0^3 v\,\mathrm{d}t$)

86.2571

distance $= 86.3$ (m) ***A2*** ***N3***

[3 marks]

(d) evidence of differentiating velocity ***(M1)***

eg $v'(t)$

$a = 3\left(t^2 - 4\right)^2 (2t)$ ***A2***

$a = 6t\left(t^2 - 4\right)^2$ ***AG*** ***N0***

[3 marks]

(e) **METHOD 1**

valid approach ***M1***

eg graphs of v and a

correct working ***(A1)***

eg areas of same sign indicated on graph

$2 < t \leq 3$ (accept $t > 2$) ***A2*** ***N2***

METHOD 2

recognizing that $a \geq 0$ (accept *a is* always positive) (seen anywhere) ***R1***

recognizing that v is positive when $t > 2$ (seen anywhere) ***(R1)***

$2 < t \leq 3$ (accept $t > 2$) ***A2*** ***N2***

[4 marks]

Total [15 marks]

9. (a) **METHOD 1**

recognizing $s = \int v$ *(M1)*

recognizing displacement of P in first 5 seconds (seen anywhere) *A1*
(accept missing $\mathrm{d}t$)

eg $\int_0^5 v\mathrm{d}t$, -3.71591

valid approach to find total displacement *(M1)*

eg $4 + (-3.7159)$, $s = 4 + \int_0^5 v$

0.284086
0.284 (m) *A2* *N3*

METHOD 2

recognizing $s = \int v$ *(M1)*

correct integration *A1*

eg $\frac{1}{3}\sin 3t + 2\cos t - \frac{t}{2} + c$ (do not penalize missing "c")

attempt to find c *(M1)*

eg $4 = \frac{1}{3}\sin(0) + 2\cos(0) - \frac{0}{2} + c$, $4 = \frac{1}{3}\sin 3t + 2\cos t - \frac{t}{2} + c$, $2 + c = 4$

attempt to substitute $t = 5$ into their expression with c *(M1)*

eg $s(5)$, $\frac{1}{3}\sin(15) + 2\cos(5) - \frac{5}{2} + 2$

0.284086
0.284 (m) *A1* *N3*
[5 marks]

(b) recognizing that at rest, $v = 0$ *(M1)*

$t = 0.179900$

$t = 0.180$ (secs) *A1* *N2*
[2 marks]

(c) recognizing when change of direction occurs *(M1)*
eg v crosses t axis

2 (times) *A1* *N2*
[2 marks]

continued…

Question 9 continued

(d) acceleration is v' (seen anywhere) ***(M1)***
eg $v'(3)$

0.743631
$0.744 \ (\text{m s}^{-2})$ ***A1 N2***
[2 marks]

(e) valid approach involving max or min of v ***(M1)***
eg $v' = 0$, $a = 0$, graph

one correct co-ordinate for min ***(A1)***
eg $1.14102, -3.27876$

$3.28 \ (\text{m s}^{-1})$ ***A1 N2***
[3 marks]

Total [14 marks]

10. (a) valid approach (addition or subtraction) ***(M1)***
eg $\text{AO} + \text{OB}$, $\text{B} - \text{A}$

$$\overrightarrow{\text{AB}} = \begin{pmatrix} 9 \\ 6 \\ -3 \end{pmatrix}$$ ***A1 N2***

[2 marks]

(b) **METHOD 1**

valid approach using $\overrightarrow{\text{OC}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ ***(M1)***

eg $\overrightarrow{\text{AC}} = \begin{pmatrix} x+3 \\ y+2 \\ z-2 \end{pmatrix}$, $\overrightarrow{\text{CB}} = \begin{pmatrix} 6-x \\ 4-y \\ -1-z \end{pmatrix}$

correct working ***A1***
eg $\begin{pmatrix} x+3 \\ y+2 \\ z-2 \end{pmatrix} = \begin{pmatrix} 12-2x \\ 8-2y \\ -2-2z \end{pmatrix}$

all three equations ***A1***
eg $x+3 = 12-2x$, $y+2 = 8-2y$, $z-2 = -2-2z$,

$$\overrightarrow{\text{OC}} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$$ ***AG N0***

continued…

7. recognizing need to find $f(2)$ or $f'(2)$ ***(R1)***

$f(2)=\frac{18}{6}$ (seen anywhere) ***(A1)***

correct substitution into the quotient rule ***(A1)***

eg $\frac{6(5)-18(2)}{6^2}$

$f'(2)=-\frac{6}{36}$ ***A1***

gradient of normal is 6 ***(A1)***

attempt to use the point and gradient to find equation of straight line ***(M1)***

eg $y-f(2)=-\frac{1}{f'(2)}(x-2)$

correct equation in any form ***A1*** ***N4***

eg $y-3=6(x-2)$, $y=6x-9$

[7 marks]