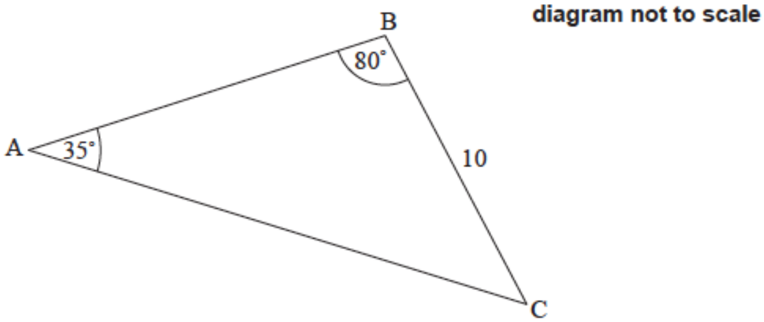
Trig rules *[61 marks]*

The following diagram shows triangle ABC.



$$\mathrm{BC} = 10 \text{ cm}, \mathrm{A\hat{B}C} = 80^\circ \text{ and } \mathrm{B\hat{A}C} = 35^\circ.$$

1a. Find AC. *[3 marks]*

Markscheme

evidence of choosing sine rule ***(M1)***

eg $\frac{\mathrm{AC}}{\sin(\mathrm{A\hat{B}C})} = \frac{\mathrm{BC}}{\sin(\mathrm{B\hat{A}C})}$

correct substitution ***(A1)***

eg $\frac{\mathrm{AC}}{\sin 80^\circ} = \frac{10}{\sin 35^\circ}$

$\mathrm{AC} = 17.1695$

$\mathrm{AC} = 17.2 \text{ (cm)}$ ***A1*** ***N2***

[3 marks]

1b. Find the area of triangle ABC. *[3 marks]*

Markscheme

$\mathrm{A\hat{C}B} = 65°$ (seen anywhere) ***(A1)***

correct substitution ***(A1)***

eg $\frac{1}{2} \times 10 \times 17.1695 \times \sin 65°$

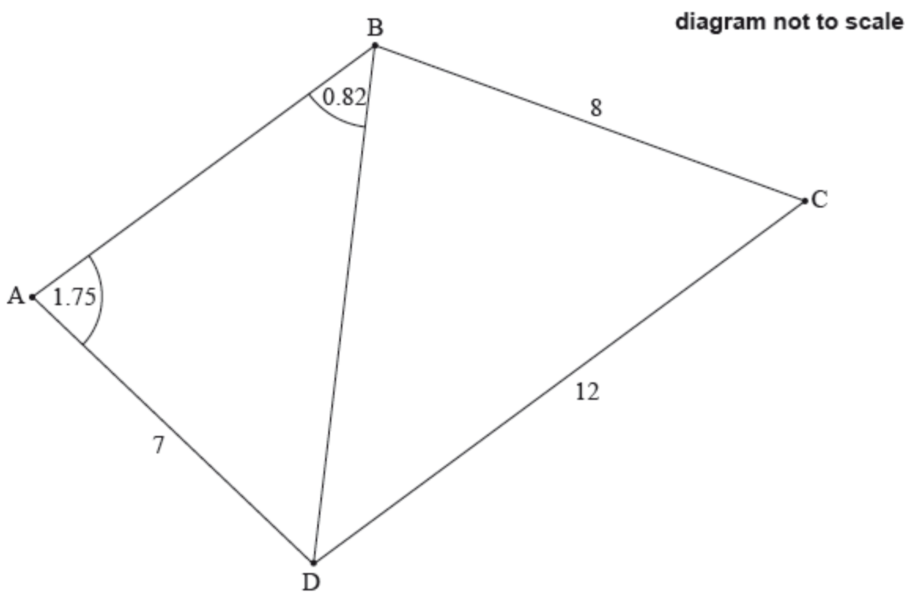
$\text{area} = 77.8047$

$\text{area} = 77.8\ (\text{cm}^2)$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

The following diagram shows a quadrilateral ABCD.



$\text{AD} = 7\text{ cm},\ \text{BC} = 8\text{ cm},\ \text{CD} = 12\text{ cm},\ \mathrm{D\hat{A}B} = 1.75\text{ radians},\ \mathrm{A\hat{B}D} = 0.82\text{ radians}.$

2a. Find BD. *[3 marks]*

Markscheme

evidence of choosing sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution ***(A1)***

eg $\frac{a}{\sin 1.75} = \frac{7}{\sin 0.82}$

9.42069

$\mathrm{BD} = 9.42\ (\mathrm{cm})$ ***A1*** ***N2***

[3 marks]

2b. Find $\mathrm{D\hat{B}C}$. *[3 marks]*

Markscheme

evidence of choosing cosine rule ***(M1)***

eg $\cos B = \frac{d^2+c^2-b^2}{2dc},\ a^2 = b^2 + c^2 - 2bc\cos B$

correct substitution ***(A1)***

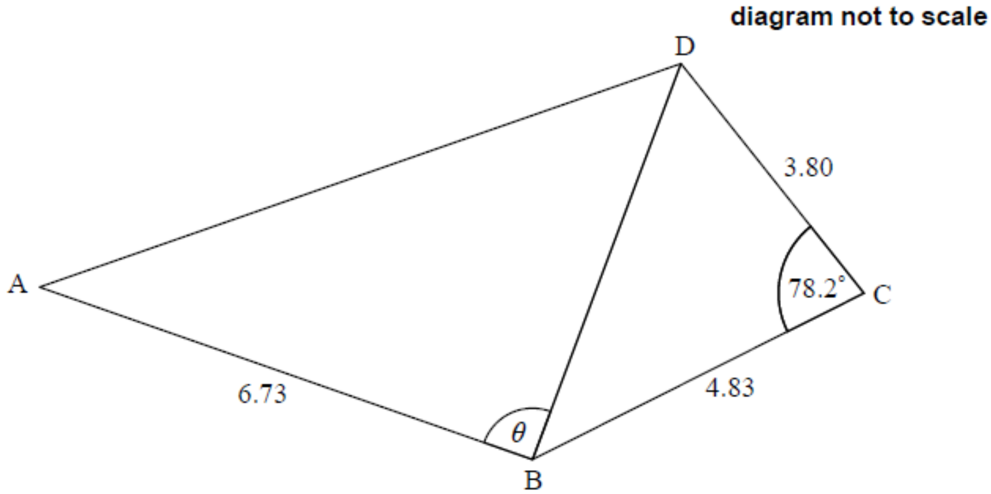
eg $\frac{8^2+9.42069^2-12^2}{2\times8\times9.42069},\ 144 = 64 + \mathrm{BD}^2 - 16\mathrm{BD}\cos B$

1.51271

$\mathrm{D\hat{B}C} = 1.51$ (radians) (accept 86.7°) ***A1*** ***N2***

[3 marks]

The following diagram shows the quadrilateral ABCD.



AB = 6.73 cm, BC = 4.83 cm, $B\hat{C}D$ = 78.2° and CD = 3.80 cm.

3a. Find BD. *[3 marks]*

Markscheme

choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS ***(A1)***

eg $4.83^2 + 3.80^2 - 2 \times 4.83 \times 3.80 \times \cos 78.2$, $30.2622,$

$4.83^2 + 3.80^2 - 2\,(4.83)\,(3.80) \times \cos 1.36$

5.50111

5.50 (cm) ***A1 N2***

[3 marks]

3b. The area of triangle ABD is 18.5 cm^2. Find the possible values of θ. *[4 marks]*

Markscheme

correct substitution for area of triangle ABD ***(A1)***

eg $\frac{1}{2}\times 6.73\times 5.50111\sin\theta$

correct equation ***A1***

eg $\frac{1}{2}\times 6.73\times 5.50111\sin\theta = 18.5$, $\sin\theta = 0.999393$

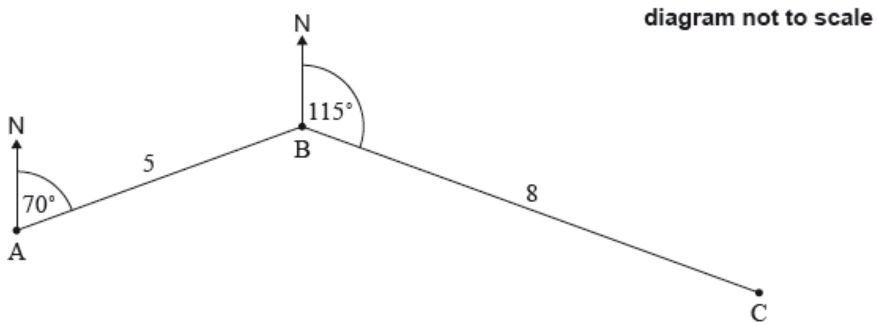
88.0023, 91.9976, 1.53593, 1.60566

θ = 88.0 (degrees) or 1.54 (radians)

θ = 92.0 (degrees) or 1.61 (radians) ***A1A1 N2***

[4 marks]

The following diagram shows three towns A, B and C. Town B is 5 km from Town A, on a bearing of 070°. Town C is 8 km from Town B, on a bearing of 115°.



4a. Find $\mathrm{A\hat{B}C}$. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $70+(180-115),\ 360-(110+115)$

$\mathrm{A\hat{B}C} = 135°$ ***A1 N2***

[2 marks]

4b. Find the distance from Town A to Town C. *[3 marks]*

Markscheme

choosing cosine rule **_(M1)_**

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS **_(A1)_**

eg $5^2 + 8^2 - 2 \times 5 \times 8\cos 135$

12.0651

12.1 (km) **_A1 N2_**

[3 marks]

4c. Use the sine rule to find $\mathrm{A\hat{C}B}$. *[2 marks]*

Markscheme

correct substitution (**must** be into sine rule) **_A1_**

eg $\frac{\sin \mathrm{A\hat{C}B}}{5} = \frac{\sin 135}{\mathrm{AC}}$

17.0398

$\mathrm{A\hat{C}B} = 17.0$ **_A1_** **_N1_**

[2 marks]

In triangle ABC, $\mathrm{AB} = 6\,\mathrm{cm}$ and $\mathrm{AC} = 8\,\mathrm{cm}$. The area of the triangle is $16\,\mathrm{cm}^2$.

5a. Find the two possible values for $\hat{A}$. *[4 marks]*

Markscheme

correct substitution into area formula ***(A1)***

eg $\frac{1}{2}(6)(8)\sin A = 16,\ \sin A = \frac{16}{24}$

correct working ***(A1)***

eg $A = \arcsin\left(\frac{2}{3}\right)$

$A = 0.729727656\ldots, 2.41186499\ldots;\ (41.8103149°, 138.1896851°)$

$A = 0.730;\ 2.41$ ***A1A1*** ***N3***

(accept degrees *ie* $41.8°;\ 138°$)

[4 marks]

5b. Given that $\hat{A}$ is obtuse, find BC. *[3 marks]*

Markscheme

evidence of choosing cosine rule ***(M1)***

eg $\mathrm{BC}^2 = \mathrm{AB}^2 + \mathrm{AC}^2 - 2(\mathrm{AB})(\mathrm{AC})\cos A,\ a^2 + b^2 - 2ab\cos C$

correct substitution into RHS (angle must be obtuse) ***(A1)***

eg $\mathrm{BC}^2 = 6^2 + 8^2 - 2(6)(8)\cos 2.41,\ 6^2 + 8^2 - 2(6)(8)\cos 138°,$

$\mathrm{BC} = \sqrt{171.55}$

$\mathrm{BC} = 13.09786$

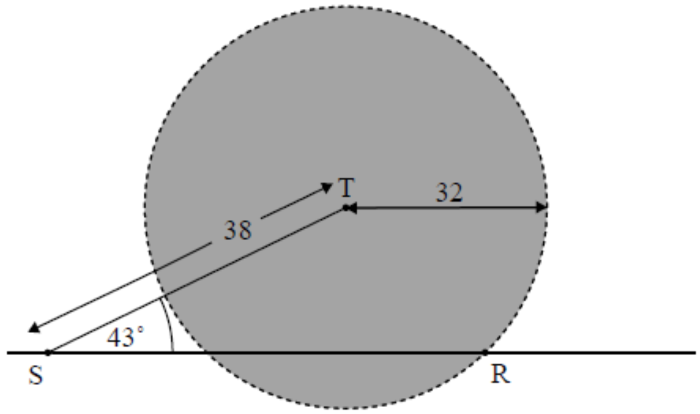
$\mathrm{BC} = 13.1\ \mathrm{cm}$ ***A1*** ***N2***

[3 marks]

A communication tower, T, produces a signal that can reach cellular phones within a radius of 32 km. A straight road passes through the area covered by the tower's signal.

The following diagram shows a line representing the road and a circle representing the area covered by the tower's signal. Point R is on the circumference of the circle and points S and R are on the road. Point S is 38 km from the tower and $R\hat{S}T = 43^\circ$.

diagram not to scale



6a. Let SR = x. Use the cosine rule to show that $x^2 - (76\cos 43^\circ)x + 420 = 0$ *[2 marks]*
.

Markscheme

recognizing TR =32 (seen anywhere, including diagram) ***A1***

correct working ***A1***

eg $32^2 = x^2 + 38^2 - 2(x)(38)\cos 43^\circ$, $1024 = 1444 + x^2 - 76(x)\cos 43^\circ$

$x^2 - (76\cos 43^\circ)x + 420 = 0$ ***AG N0***

[2 marks]

6b. Hence or otherwise, find the total distance along the road where the signal from the tower can reach cellular phones. *[4 marks]*

Markscheme

Note: There are many approaches to this question, depending on which triangle the candidate has used, and whether they used the cosine rule and/or the sine rule. Please check working carefully and award marks in line with the markscheme.

METHOD 1

correct values for x (seen anywhere) ***A1A1***

x = 9.02007, 46.5628

recognizing the need to find difference in values of x ***(M1)***

eg 46.5 − 9.02, $x_1 - x_2$

37.5427

37.5 (km) ***A1 N2***

METHOD 2

correct use of sine rule in ΔSRT

eg $\frac{\sin \mathrm{S\hat{R}T}}{38} = \frac{\sin 43^\circ}{32}$, $\mathrm{S\hat{R}T}$ = 54.0835° (A1)

recognizing isosceles triangle (seen anywhere) ***(M1)***

eg $\hat{T} = 180^\circ - 2 \cdot 54.0835^\circ$, two sides of 32

correct working to find distance ***A1***

eg $\sqrt{32^2 + 32^2 - 2 \cdot 32 \cdot 32 \cos\left(180^\circ - 2 \cdot 54.0835^\circ\right)}$,

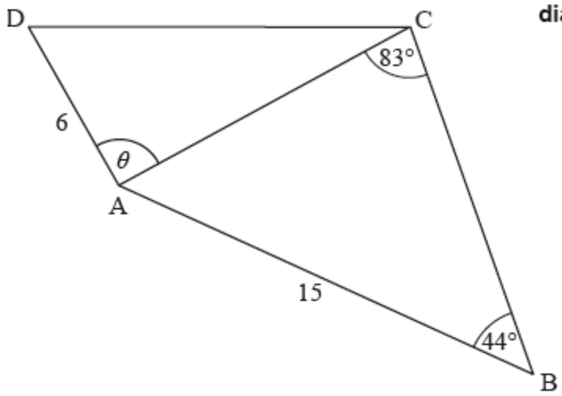
$\frac{\sin 71.8329^\circ}{d} = \frac{\sin 54.0835^\circ}{32}$, $32^2 = 32^2 + x^2 - 2 \cdot 32x \cos(0.944)$

37.5427

37.5 (km) ***A1 N2***

[4 marks]

The following diagram shows the quadrilateral $ABCD$.



$$\mathrm{AD} = 6\text{ cm},\ \mathrm{AB} = 15\text{ cm}, \mathrm{A\hat{B}C} = 44°, \mathrm{A\hat{C}B} = 83° \text{and} \mathrm{D\hat{A}C} = \theta$$

7a. Find AC. *[3 marks]*

Markscheme

evidence of choosing sine rule ***(M1)***

eg $\frac{\mathrm{AC}}{\sin \mathrm{C\hat{B}A}} = \frac{\mathrm{AB}}{\sin \mathrm{A\hat{C}B}}$

correct substitution ***(A1)***

eg $\frac{\mathrm{AC}}{\sin 44°} = \frac{15}{\sin 83°}$

10.4981

$\mathrm{AC} = 10.5\ \ (\mathrm{cm})$ ***A1 N2***

[3 marks]

7b. Find the area of triangle ABC. *[3 marks]*

Markscheme

finding $\mathrm{C\hat{A}B}$ (seen anywhere) ***(A1)***

eg $180° - 44° - 83°, \mathrm{C\hat{A}B} = 53°$

correct substitution for area of triangle ABC ***A1***

eg $\frac{1}{2} \times 15 \times 10.4981 \times \sin 53°$

62.8813

$\text{area} = 62.9 \ \left(\text{cm}^2\right)$ ***A1*** ***N2***

[3 marks]

7c. The area of triangle ACD is half the area of triangle ABC. *[5 marks]*

Find the possible values of θ.

Markscheme

correct substitution for area of triangle DAC ***(A1)***

eg $\frac{1}{2} \times 6 \times 10.4981 \times \sin \theta$

attempt to equate area of triangle ACD to half the area of triangle ABC ***(M1)***

eg $\text{area ACD} = \frac{1}{2} \times \text{ area ABC}; 2\text{ACD} = \text{ABC}$

correct equation ***A1***

eg $\frac{1}{2} \times 6 \times 10.4981 \times \sin \theta = \frac{1}{2}(62.9),\ 62.9887 \sin \theta = 62.8813,\ \sin \theta = 0.998294$

$86.6531, 93.3468$

$\theta = 86.7°$, $\theta = 93.3°$ ***A1A1*** ***N2***

[5 marks]

7d. Given that θ is obtuse, find CD. *[3 marks]*

Markscheme

Note: Note: If candidates use an acute angle from part (c) in the cosine rule, award ***M1A0A0*** in part (d).

evidence of choosing cosine rule ***(M1)***

eg $\mathrm{CD}^2 = \mathrm{AD}^2 + \mathrm{AC}^2 - 2 \times \mathrm{AD} \times \mathrm{AC} \times \cos\theta$

correct substitution into rhs ***(A1)***

eg $\mathrm{CD}^2 = 6^2 + 10.498^2 - 2(6)(10.498)\cos 93.336°$

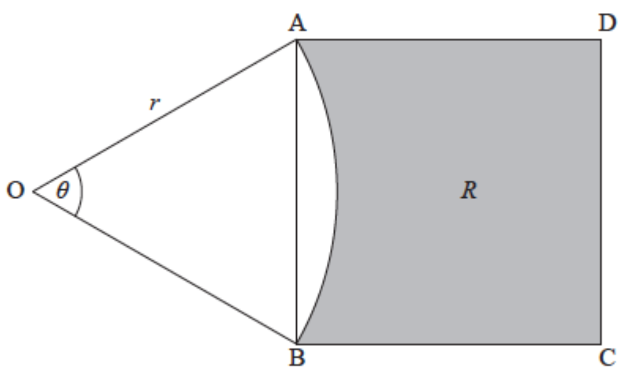
12.3921

$12.4\ \ (\mathrm{cm})$ ***A1*** ***N2***

[3 marks]

Total [14 marks]

The following diagram shows a square $ABCD$, and a sector OAB of a circle centre O, radius r. Part of the square is shaded and labelled R.



$$\mathrm{A\hat{O}B} = \theta, \text{ where } 0.5 \leq \theta < \pi.$$

8a. Show that the area of the square $ABCD$ is $2r^2(1 - \cos\theta)$. *[4 marks]*

Markscheme

area of $\mathrm{ABCD} = \mathrm{AB}^2$ (seen anywhere) ***(A1)***

choose cosine rule to find a side of the square ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc\cos\theta$

correct substitution (for triangle AOB) ***A1***

eg $r^2 + r^2 - 2 \times r \times r\cos\theta,\ \mathrm{OA}^2 + \mathrm{OB}^2 - 2 \times \mathrm{OA} \times \mathrm{OB}\cos\theta$

correct working for AB^2 ***A1***

eg $2r^2 - 2r^2\cos\theta$

$\text{area} = 2r^2(1 - \cos\theta)$ ***AG*** ***N0***

Note: Award no marks if the only working is $2r^2 - 2r^2\cos\theta$.

[4 marks]

8b. When $\theta = \alpha$, the area of the square $ABCD$ is equal to the area of the sector OAB. *[4 marks]*

(i) Write down the area of the sector when $\theta = \alpha$.

(ii) Hence find α.

Markscheme

(i) $\frac{1}{2}\alpha r^2 \quad \left(\text{accept } 2r^2(1 - \cos\alpha)\right)$ ***A1*** ***N1***

(ii) correct equation in one variable ***(A1)***

eg $2(1 - \cos\alpha) = \frac{1}{2}\alpha$

$\alpha = 0.511024$

$\alpha = 0.511 \quad (\text{accept } \theta = 0.511)$ ***A2*** ***N2***

Note: Award ***A1*** for $\alpha = 0.511$ and additional answers.

[4 marks]