0415PreTest-Stats+sequences+review *[109 marks]*

Consider the following frequency table.

x	Frequency
2	8
4	15
7	21
10	28
11	3

1a. Write down the mode. *[1 mark]*

Markscheme

mode $= 10$ *A1*   *N1*

[1 mark]

1b. Find the value of the range. *[2 marks]*

Markscheme

valid approach   *(M1)*

eg $x_{\max} - x_{\min}$, interval 2 to 11

range $= 9$ *A1*   *N2*

[2 marks]

1c. Find the mean. *[2 marks]*

Markscheme

7.14666

mean $= 7.15$ *A2*   *N2*

[2 marks]

1d. Find the variance. *[2 marks]*

Markscheme

recognizing that variance is $(\text{sd})^2$ *(M1)*

eg var $= \sigma^2$, 2.90605^2, 2.92562^2

$\sigma^2 = 8.44515$

$\sigma^2 = 8.45$ *A1*   *N2*

[2 marks]

There are 10 items in a data set. The sum of the items is 60.

2a. Find the mean. *[2 marks]*

Markscheme

correct approach   *(A1)*

eg $\frac{60}{10}$

$\text{mean} = 6$ *A1*   *N2*

The variance of this data set is 3. Each value in the set is multiplied by 4.

2b. (i)   Write down the value of the new mean. *[3 marks]*

(ii)   Find the value of the new variance.

Markscheme

(i)   new mean $= 24$ *A1*   *N1*

(ii)   valid approach   *(M1)*

eg variance $\times (4)^2$, 3×16, new standard deviation $= 4\sqrt{3}$

new variance $= 48$ *A1*   *N2*

[3 marks]

Let $f(x) = e^{0.5x} - 2$.

3a. For the graph of f: *[4 marks]*

(i)   write down the y-intercept;

(ii)   find the x-intercept;

(iii)   write down the equation of the horizontal asymptote.

Markscheme

(i)
$y = -1$ *A1*   *N1*

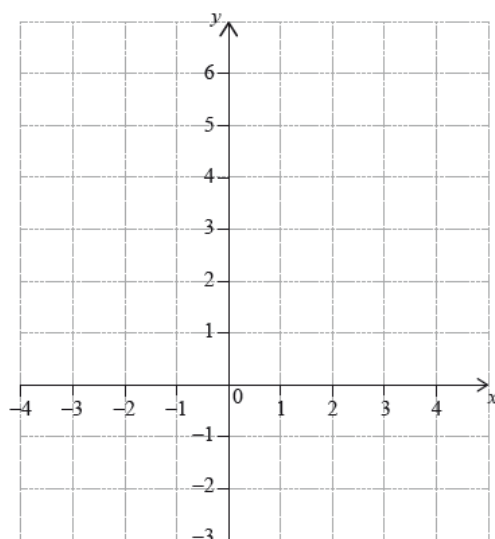(ii)   valid attempt to find x-intercept   *(M1)*

eg $f(x) = 0$

1.38629   *A1*   *N2*

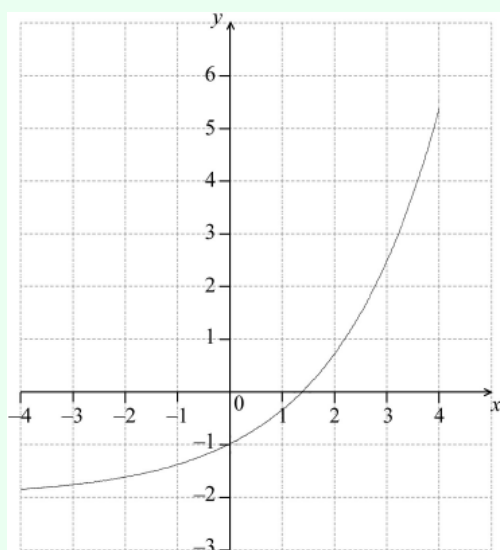$x = 2\ln 2 \ (\text{exact}), \ 1.39$

(iii)
$y = -2$ (must be equation)   *A1*   *N1*

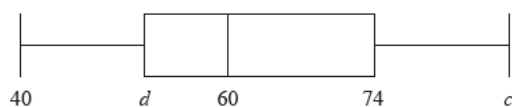3b. On the following grid, sketch the graph of f, for $-4 \leqslant x \leqslant 4$.

[3 marks]

The following box-and-whisker plot represents the examination scores of a group of students.

Examination scores

4a. Write down the median score.

[1 mark]

The range of the scores is 47 marks, and the interquartile range is 22 marks.

4b. Find the value of   [4 marks]

(i) c;

(ii) d.

5. Three consecutive terms of a geometric sequence are $x - 3$, 6 and $x + 2$.   [6 marks]

Find the possible values of x.

6. In a geometric sequence, the fourth term is 8 times the first term. The sum of the first 10 terms is 2557.5. Find the 10th term of this   *[6 marks]*
sequence.

Markscheme

correct equation to find r *(A1)*

eg $u_1r^3 = 8u_1$, $r^3 = 8$

$r = 2$ (seen anywhere) *(A1)*

correct equation to find u_1 *A1*

eg $u_1(2^{10} - 1) = 2557.5$, $u_1 = \frac{2557.5}{r^{10}-1}(r - 1)$

$u_1 = 2.5$ *(A1)*

$u_{10} = 2.5(2)^9$ *(M1)*

1280 *A1* *N4*

[6 marks]

The first three terms of an arithmetic sequence are $u_1 = 0.3$, $u_2 = 1.5$, $u_3 = 2.7$.

7a. Find the common difference. *[2 marks]*

Markscheme

valid approach *(M1)*

eg $1.5 - 0.3$, $1.5 - 2.7$, $2.7 = 0.3 + 2d$

$d = 1.2$ *A1* *N2*

[2 marks]

7b. Find the 30th term of the sequence. *[2 marks]*

Markscheme

correct substitution into term formula *(A1)*

eg $0.3 + 1.2(30 - 1)$, $u_{30} = 0.3 + 29(1.2)$

$u_{30} = 35.1$ *A1* *N2*

[2 marks]

7c. Find the sum of the first 30 terms. *[2 marks]*

Markscheme

correct substitution into sum formula *(A1)*

eg $S_{30} = \frac{30}{2}(0.3 + 35.1)$, $\frac{30}{2}(2(0.3) + 29(1.2))$

$S_{30} = 531$ *A1* *N2*

[2 marks]

The sums of the terms of a sequence follow the pattern

$S_1 = 1 + k, \ S_2 = 5 + 3k, \ S_3 = 12 + 7k, \ S_4 = 22 + 15k, \ \ldots,$ where $k \in \mathbb{Z}$.

8a. Given that [4 marks]

$u_1 = 1 + k$, find

$u_2, \ u_3$ and

u_4.

Markscheme

valid method   *(M1)*

eg

$u_2 = S_2 - S_1, \ 1 + k + u_2 = 5 + 3k$

$u_2 = 4 + 2k, \ u_3 = 7 + 4k, \ u_4 = 10 + 8k$ *A1A1A1*   *N4*

[4 marks]

8b. Find a general expression for [4 marks]

u_n.

Markscheme

correct AP **or** GP   *(A1)*

eg   finding common difference is

3, common ratio is

2

valid approach using arithmetic **and** geometric formulas   *(M1)*

eg

$1 + 3(n - 1)$ **and**

$r^{n-1}k$

$u_n = 3n - 2 + 2^{n-1}k$ *A1A1*   *N4*

Note: Award *A1* for

$3n - 2$, *A1* for

$2^{n-1}k$.

[4 marks]

Let

$f(x) = 3 \ln x$ and

$g(x) = \ln 5x^3$.

9a. Express [4 marks]

$g(x)$ in the form

$f(x) + \ln a$, where

$a \in \mathbb{Z}^+$.

Markscheme

attempt to apply rules of logarithms   *(M1)*

e.g.
$\ln a^b = b \ln a$,
$\ln ab = \ln a + \ln b$

correct application of
$\ln a^b = b \ln a$ (seen anywhere)   *A1*

e.g.
$3 \ln x = \ln x^3$

correct application of
$\ln ab = \ln a + \ln b$ (seen anywhere)   *A1*

e.g.
$\ln 5x^3 = \ln 5 + \ln x^3$

so
$\ln 5x^3 = \ln 5 + 3 \ln x$

$g(x) = f(x) + \ln 5$ (accept
$g(x) = 3 \ln x + \ln 5$)   *A1   N1*

[4 marks]

9b. The graph of *g* is a transformation of the graph of *f*. Give a full geometric description of this transformation.   *[3 marks]*

Markscheme

transformation with correct name, direction, and value   *A3*

e.g. translation by
$\begin{pmatrix} 0 \\ \ln 5 \end{pmatrix}$, shift up by
$\ln 5$, vertical translation of
$\ln 5$

[3 marks]

10. Solve
$\log_2 x + \log_2(x - 2) = 3$, for
$x > 2$.   *[7 marks]*

Markscheme

recognizing
$\log a + \log b = \log ab$ (seen anywhere)   *(A1)*

e.g.
$\log_2(x(x-2))$,
$x^2 - 2x$

recognizing
$\log_a b = x \Leftrightarrow a^x = b$ *(A1)*

e.g.
$2^3 = 8$

correct simplification   *A1*

e.g.
$x(x-2) = 2^3$,
$x^2 - 2x - 8$

evidence of correct approach to solve   *(M1)*

e.g. factorizing, quadratic formula

correct working   *A1*

e.g.
$(x-4)(x+2)$,
$\frac{2 \pm \sqrt{36}}{2}$

$x = 4$ *A2*   *N3*

[7 marks]

Let
$f(x) = e^{x+3}$.

11a. (i)   Show that
$f^{-1}(x) = \ln x - 3$.

[3 marks]

(ii)   Write down the domain of
f^{-1} .

Markscheme

(i) interchanging x and y (seen anywhere)   *M1*

e.g.
$x = e^{y+3}$

correct manipulation   *A1*

e.g.
$\ln x = y + 3$,
$\ln y = x + 3$

$f^{-1}(x) = \ln x - 3$ *AG*   *N0*

(ii)
$x > 0$ *A1*   *N1*

[3 marks]

11b. Solve the equation
$f^{-1}(x) = \ln \frac{1}{x}$.

[4 marks]

Markscheme

collecting like terms; using laws of logs   *(A1)(A1)*

e.g.

$\ln x - \ln\left(\frac{1}{x}\right) = 3$,

$\ln x + \ln x = 3$,

$\ln\left(\frac{x}{\frac{1}{x}}\right) = 3$,

$\ln x^2 = 3$

simplify   *(A1)*

e.g.

$\ln x = \frac{3}{2}$,

$x^2 = e^3$

$x = e^{\frac{3}{2}}\left(=\sqrt{e^3}\right)$ *A1*   *N2*

[4 marks]

Let $f(x) = (x-5)^3$, for $x \in \mathbb{R}$.

12a. Find $f^{-1}(x)$.   *[3 marks]*

Markscheme

interchanging x and y (seen anywhere)   *(M1)*

eg $x = (y-5)^3$

evidence of correct manipulation   *(A1)*

eg $y - 5 = \sqrt[3]{x}$

$f^{-1}(x) = \sqrt[3]{x} + 5$ $\left(\text{accept } 5 + x^{\frac{1}{3}},\ y = 5 + \sqrt[3]{x}\ \right)$ *A1*   *N2*

Notes: If working shown, and they do not interchange x and y, award *A1A1M0* for $\sqrt[3]{y} + 5$.

If no working shown, award *N1* for $\sqrt[3]{y} + 5$.

12b. Let   *[3 marks]*
g be a function so that $(f \circ g)(x) = 8x^6$. Find $g(x)$.

The following diagram shows the graph of a function f.
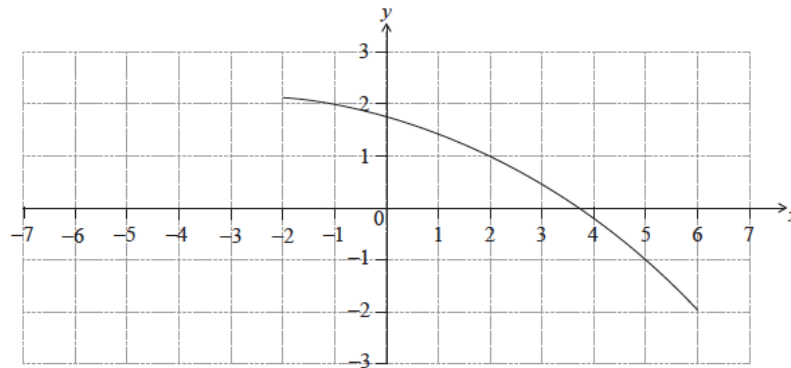
13a. Find $f^{-1}(-1)$.   *[2 marks]*

13b. Find $(f\circ f)(-1)$.   *[3 marks]*

13c. On the same diagram, sketch the graph of $y = f(-x)$. [2 marks]
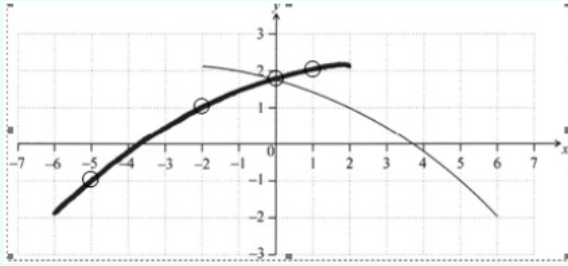
Let $f(x) = 2x + 3$ and $g(x) = x^3$.

14. Find $(f \circ g)(x)$. [2 marks]

15. Let $f(x) = m - \frac{1}{x}$, for $x \neq 0$. The line $y = x - m$ intersects the graph of f in two distinct points. Find the possible values of m. [7 marks]

Markscheme

valid approach   *(M1)*

eg $f = y$, $m - \frac{1}{x} = x - m$

correct working to eliminate denominator   *(A1)*

eg $mx - 1 = x(x - m)$, $mx - 1 = x^2 - mx$

correct quadratic equal to zero   *A1*

eg $x^2 - 2mx + 1 = 0$

correct reasoning   *R1*

eg
  for two solutions, $b^2 - 4ac > 0$

correct substitution into the discriminant formula   *(A1)*

eg $(-2m)^2 - 4$

correct working   *(A1)*

eg

$4m^2 > 4$, $m^2 = 1$, sketch of positive parabola on the x-axis

correct interval   *A1*   *N4*

eg $|m| > 1$, $m < -1$ or
$m > 1$

[7 marks]

Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$.

The distance between the two zeros of f is 9.

16a. Show that the two zeros are 3 and -6.   *[2 marks]*

Markscheme

recognition that the x-coordinate of the vertex is
-1.5 (seen anywhere)   *(M1)*

eg axis of symmetry is -1.5, sketch, $f'(-1.5) = 0$

correct working to find the zeroes   *A1*

eg -1.5 ± 4.5

$x = -6$ and $x = 3$ *AG*   *N0*

[2 marks]

16b. Find the value of q and of r.   *[4 marks]*

Markscheme

METHOD 1 (using factors)

attempt to write factors   *(M1)*

eg $(x - 6)(x + 3)$

correct factors   *A1*

eg $(x - 3)(x + 6)$

$q = 3, r = -18$ *A1A1*   *N3*

METHOD 2 (using derivative or vertex)

valid approach to find q *(M1)*

eg $f'(-1.5) = 0$, $-\frac{q}{2a} = -1.5$

$q = 3$ *A1*

correct substitution   *A1*

eg $3^2 + 3(3) + r = 0$, $(-6)^2 + 3(-6) + r = 0$

$r = -18$ *A1*

$q = 3, r = -18$ *N3*

METHOD 3 (solving simultaneously)

valid approach setting up system of two equations   *(M1)*

eg $9 + 3q + r = 0$, $36 - 6q + r = 0$

one correct value

eg $q = 3, r = -18$ *A1*

correct substitution   *A1*

eg $3^2 + 3(3) + r = 0$, $(-6)^2 + 3(-6) + r = 0$, $3^2 + 3q - 18 = 0$, $36 - 6q - 18 = 0$

second correct value   *A1*

eg $q = 3, r = -18$

$q = 3, r = -18$ *N3*

[4 marks]

Let $f(x) = x^2 + x - 6$.

17a. Write down the y-intercept of the graph of f. *[1 mark]*

Markscheme

y-intercept is -6, $(0, -6)$, $y = -6$ *A1*

[1 mark]

17b. Solve $f(x) = 0$. *[3 marks]*

Markscheme

valid attempt to solve   *(M1)*

eg $(x - 2)(x + 3) = 0$, $x = \frac{-1 \pm \sqrt{1 + 24}}{2}$, one correct answer

$x = 2, x = -3$ *A1A1*   *N3*

[3 marks]

Let $f(x) = p + \frac{9}{x-q}$, for $x \neq q$. The line $x = 3$ is a vertical asymptote to the graph of f.

18a. Write down the value of q. *[1 mark]*

18b. The graph of f has a y-intercept at $(0,\ 4)$. *[4 marks]*

Find the value of p.

18c. The graph of f has a y-intercept at $(0,\ 4)$. *[1 mark]*

Write down the equation of the horizontal asymptote of the graph of f.