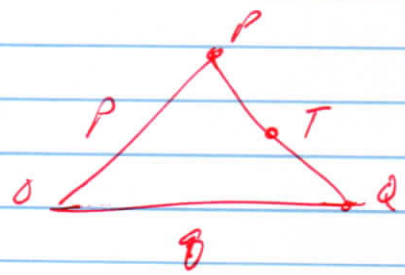
BECA / Hudson / IB math 12.1

Pre-Exam - NOV 5

Solutions

1(a) $\vec{PT} = \frac{1}{2}(\vec{OB} - \vec{OP})$

$= \frac{1}{2}(q - p)$



(a) $\vec{QP} = \vec{OP} - \vec{OQ}$

$= p - q$

(b) $\vec{OT} = \vec{OP} + \frac{1}{2}\vec{PT}$

$= p + \frac{1}{2}(q - p)$

$= \frac{1}{2}p + \frac{1}{2}q$

2(a)(i) $\vec{AB} = \vec{OB} - \vec{OA}$

$= \begin{pmatrix} 6-5 \\ 5-2 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

(ii) $\vec{AC} = \vec{OC} - \vec{OA}$

$= \begin{pmatrix} 7-5 \\ 6-2 \\ (6+1)-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ a \end{pmatrix}$

3(a) $\vec{BC} = \vec{AC} - \vec{AB}$

$= \begin{pmatrix} 4-3 \\ 4-1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

(b) $\vec{BD} = \vec{AD} - \vec{AB}$

$= \vec{BC} - \vec{AB}$

$= \begin{pmatrix} 1-3 \\ 3-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

Bell / Hudson / 12.1 IB math

Pre-Exam Nov 5 Solutions

4(a) $f(2) = a(2)^3 + b(2)^2 + c = 9$

$8a + 4b + c = 9$ (1)

(b) $f(1) = a(1)^3 + b(1)^2 + c = 4$

$a + b + c = 4$ (2)

$f'(x) = 3ax^2 + 2bx$

$f''(1) = 3a(1)^2 + 2b(1) = 0$

$3a + 2b = 0$ (3)

(c) $8a + 4b + c = 9$ (1)

$a + b + c = 4$ (2)

$7a + 3b = 5$ (1-2)

$14a + 6b = 10$ $\times 2$

$9a + 6b = 0$ (3) $\times 3$ } Subtract

$5a = 10$

$a = 2$

$3(2) + 2b = 0$ (3)

$b = -3$

$(2) + (-3) + c = 4$ (2)

$c = 5$

Check

$8(2) + 4(-3) + 5 \stackrel{?}{=} 9$

$16 - 12 + 5 = 9$ ✓

BECA / Hudson / 12.1 IB

Pre Exam Nov 5

Solutions

5(a) $g'(x) = \dfrac{\left(\frac{1}{x}\right)\left(\frac{x}{x^2}\right) - \ln x(2x)}{x^4}$ $\quad g(x) = \dfrac{\ln x}{x^2}$ $\quad u \quad v$

$y' = \dfrac{vu' - v'u}{v^2}$

$= \dfrac{x - 2x\ln x}{x^4}$

$= \dfrac{1 - 2\ln x}{x^3}$

(b) $\dfrac{1 - 2\ln x}{x^3} = 0$

$1 - 2\ln x = 0,$

$\ln x = \frac{1}{2}$

$x = e^{\frac{1}{2}} = \sqrt{e}$

6(a) $f(x) = -10(x - (-4))(x - 6)$

(b) $f(x) = -10(x^2 - 2x - 24)$

$= -10(x^2 - 2x + 1 - 25)$

$= -10(x-1)^2 + 250$

(c) $f(x) = -10x^2 + 20x + 240$

$= 240 + 20x - 10x^2$

(d) $V(t)_{max}$ is at the vertex $(1, 250)$

$t = 1$ second

$V(t) = 0 = -10(x+4)(x-6)$

$t = 6$ seconds (reject negative time)

$a(t) = v'(t) = 20 - 20t$

$v'(6) = 20 - 20(6) = -100 \ ms^{-2}$

/Beck/Hudson/12.1 IB Math

Pre-Exam No. 5 Solutions

7(a)(i) $f'(x) = e^{-x^2}(-2x)$

$= -2xe^{-x^2}$

(ii) $f''(x) = -2e^{-x^2} + -2xe^{-x^2}(-2x)$

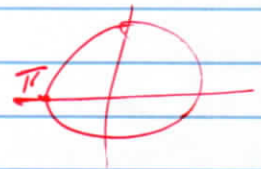
$= 4x^2e^{-x^2} - 2e^{-x^2}$

$= (4x^2 - 2)e^{-x^2}$

8(a) $g'(x) = 2\sin x + 2x\cos x$

(b) $g'(\pi) = 2(0) + 2(\pi)(-1)$

$= -2\pi$



9. (c) $f''(x) = -72x^2 + 18x + 3$

10(a) $f'(x) = \frac{3}{2}x^2 - 2x - 3 = 0$

$3x^2 - 4x - 6 = 0$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-6)}}{2(3)}$

Here is the solution in (3.9)

$= \frac{4 \pm \sqrt{16 + 72}}{6} = \frac{2 \pm \sqrt{22}}{3} \approx -0.897$

Error B $f\left(\frac{2 - \sqrt{22}}{3}\right) = 1.5255239...$

$\simeq 1.53$ $(-0.897, 1.53)$