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Test: Exponential and logarithmic functions

13 December 2018 Name:

Answer separately on lined paper unless otherwise instructed.

Part 1: No calculators are allowed on this section.

Simplify. Leave no negative or fractional exponents.

1.
$$15x^2y^2 \div 3x^5y^2$$

2.
$$\sqrt[3]{x^{-9}y^6}$$

$$3. \left(xy^{\frac{1}{4}}\right)^2$$

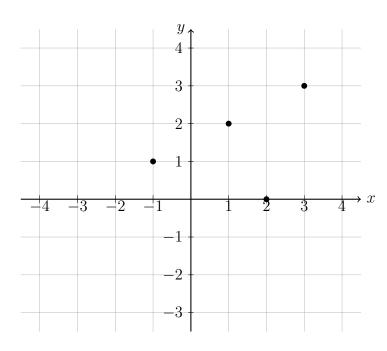
4.
$$\log_5 25$$

5.
$$\log_6 4 + \log_6 9$$

6.
$$\log 200 - \log 2$$

7.
$$(x-3)(x^2+3x+9)$$

8. The diagram below shows the graph of a function f, composed of four points.

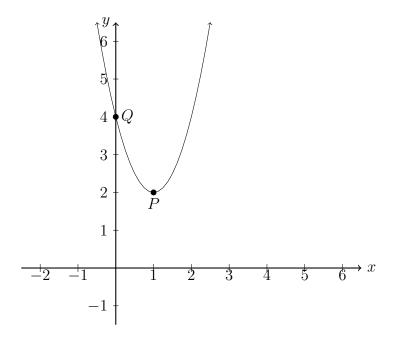


- (a) Write down the value of f(2).
- (b) Write down the domain of f.
- (c) Write down the range of f.
- (d) Write down the value of $f^{-1}(1)$.
- (e) Sketch the inverse of f, f^{-1} , on the grid above.

- 9. Let f(x) = 3x 1 and $g(x) = -2x^2 + 2$
 - (a) Find $f^{-1}(x)$.
 - (b) Find $(f \circ g)(1)$.
- 10. Consider the equation $x^2 + (k-2)x = -4$, where k is a real number. Find the values of k for which the equation has two equal real solutions.

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11. Let f be a quadratic function. Part of the graph of f is shown below. The vertex is at P(1,2) and the y-intercept is at Q(0,4).



- (a) Write down the equation of the axis of symmetry.
- (b) The function f can be written in the form $f(x) = a(x h)^2 + k$. Write down the value of h and of k.
- (c) Find a.

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Part 2: Graphing calculators may be used on this section.

- 12. Let $f(x) = 3x^2 12x + 7$.
 - (a) Write down the coordinates of the vertex.
 - (b) Hence or otherwise, express the function in the form $f(x) = 3(x-h)^2 + k$.
 - (c) Solve the equation f(x) = 0.
- 13. Paulita takes medication. After t minutes, the concentration of medication left in her bloodstream is given by $C(t) = 15(0.5)^{(0.02t)}$, where C is in milligrams per litre.
 - (a) Write down C(0).
 - (b) Find the concentration of medication left in her bloodstream after 25 minutes.
 - (c) At 8:00, when there is no medication in Paulita's bloodstream, she takes her first dose of medication. She can take her medication again when the concentration of medication reaches 3.00 milligrams per litre. What time will Paulita be able to take her medication again?
- 14. A student takes out a loan for \$8,000 to cover college expenses. Interest of 7% per annum is charged, compounded monthly. Use the interest rate formula

$$P(t) = P_0(1 + \frac{r}{n})^{nt}$$

where P(t) is the total interest and principal at time t, P_0 is the initial loan amount, t is the time in years, r = 7% is the interest rate per annum, and n = 12 is the number of compounding periods per year.

- (a) Calculate how much she will owe after five years, rounded to the nearest dollar.
- (b) How long will it take for the outstanding balance to double, to the nearest month?
- (c) Spicy: Use an algebraic method to solve the previous question (the time to double).

- 15. Consider the function $f(x) = x^2 + 2x 2$.
 - (a) Sketch the graph of f, for $-4 \le x \le 2$.
 - (b) This function can also be written in the form $f(x) = (x p)^2 3$. Write down the value of p.
 - (c) The graph of g is obtained by reflecting the graph of f in the x-axis, followed by a translation of (0,2). Show that $g(x) = -x^2 - 2x + 4$.
 - (d) The graphs of f and g intersect at two points. Write down the x-coordinates of these two points.

