0425Calculus-review *[192 marks]*

1. Let $f'(x) = 6x^2 - 5$. Given that $f(2) = -3$, find $f(x)$. *[6 marks]*

Markscheme

evidence of antidifferentiation ***(M1)***

eg $f = \int f'$

correct integration (accept absence of C) ***(A1)(A1)***

$f(x) = \frac{6x^3}{3} - 5x + C,\ 2x^3 - 5x$

attempt to substitute $(2,\ -3)$ into **their** integrated expression (must have C) ***M1***

eg $2(2)^3 - 5(2) + C = -3,\ 16 - 10 + C = -3$

Note: Award ***M0*** if substituted into original or differentiated function.

correct working to find C ***(A1)***

eg $16 - 10 + C = -3,\ 6 + C = -3,\ C = -9$

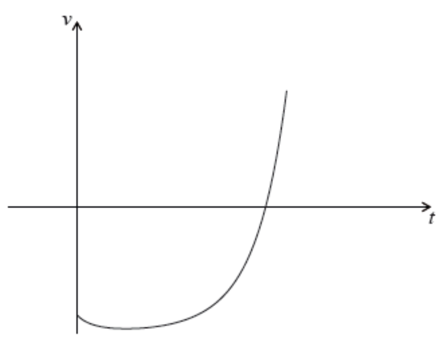
$f(x) = 2x^3 - 5x - 9$ ***A1 N4***

[6 marks]

The velocity $v\ \mathrm{ms}^{-1}$ of a particle after t seconds is given by

$v(t) = (0.3t + 0.1)^t - 4$, for $0 \le t \le 5$

The following diagram shows the graph of v.



2a. Find the value of t when the particle is at rest. *[3 marks]*

Markscheme

recognizing particle at rest when $v = 0$ ***(M1)***

eg $(0.3t + 0.1)^t - 4 = 0$, x-intercept on graph of v

$t = 4.27631$

$t = 4.28\ (\text{seconds})$ ***A2 N3***

[3 marks]

2b. Find the value of t when the acceleration of the particle is 0. *[3 marks]*

Markscheme

valid approach to find t when a is 0 ***(M1)***

eg $v'(t) = 0$, v minimum

$t = 1.19236$

$t = 1.19$ (seconds) ***A2 N3***

[3 marks]

Total [6 marks]

3. Let $f(x) = \frac{\ln(4x)}{x}$ for $0 < x \leq 5$. *[7 marks]*

Points $\mathrm{P}(0.25,\ 0)$ and Q are on the curve of f. The tangent to the curve of f at P is perpendicular to the tangent at Q. Find the coordinates of Q.

Markscheme

recognizing that the gradient of tangent is the derivative ***(M1)***

eg f'

finding the gradient of f at P ***(A1)***

eg $f'(0.25) = 16$

evidence of taking negative reciprocal of **their** gradient at P ***(M1)***

eg $\frac{-1}{m}$, $-\frac{1}{f'(0.25)}$

equating derivatives ***M1***

eg $f'(x) = \frac{-1}{16}$, $f' = -\frac{1}{m}$, $\frac{x\left(\frac{1}{x}\right) - \ln(4x)}{x^2} = 16$

finding the x-coordinate of Q, $x = 0.700750$

$x = 0.701$ ***A1 N3***

attempt to substitute **their** x into f to find the y-coordinate of Q ***(M1)***

eg $f(0.7)$

$y = 1.47083$

$y = 1.47$ ***A1 N2***

[7 marks]

Let $f(x) = -x^4 + 2x^3 - 1$, for $0 \leq x \leq 2$.

4a. Sketch the graph of f on the following grid. *[3 marks]*

Markscheme

A1A1A1 N3

Note: Award ***A1*** for both endpoints in circles,

A1 for approximately correct shape (concave up to concave down).

Only if this ***A1*** for shape is awarded, award ***A1*** for maximum point in circle.

4b. Solve $f(x) = 0$. *[2 marks]*

Markscheme

$x = 1 \quad x = 1.83928$

$x = 1\ (\text{exact}) \quad x = 1.84\ [1.83,\ 1.84]$ ***A1A1*** ***N2***

[2 marks]

4c. The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. *[3 marks]*

Find the volume of the solid formed.

Markscheme

attempt to substitute either (***FT***) limits or function into formula with f^2 ***(M1)***

eg $V = \pi \int_1^{1.84} f^2,\ \int \left(-x^4 + 2x^3 - 1\right)^2 \mathrm{d}x$

0.636581

$V = 0.637\ [0.636,\ 0.637]$ ***A2*** ***N3***

[3 marks]

Total [8 marks]

Let
$f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.

5a. Find $f'(x)$. *[2 marks]*

Markscheme

expressing f as $x^{\frac{4}{3}}$ ***(M1)***

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} \left(= \frac{4}{3}\sqrt[3]{x}\right)$ ***A1*** ***N2***

[2 marks]

5b. Find $\int f(x)\mathrm{d}x$. *[4 marks]*

Markscheme

attempt to integrate $\sqrt[3]{x^4}$ ***(M1)***

eg $\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}$

$\int f(x)\mathrm{d}x = \frac{3}{7}x^{\frac{7}{3}} - \frac{x}{2} + c$ ***A1A1A1*** ***N4***

[4 marks]

Consider
$f(x) = x^2 \sin x$.

6a. Find
$f'(x)$. *[4 marks]*

Markscheme

evidence of choosing product rule ***(M1)***

eg
$uv' + vu'$

correct derivatives (must be seen in the product rule)
$\cos x$,
$2x$ ***(A1)(A1)***

$f'(x) = x^2 \cos x + 2x \sin x$ ***A1 N4***

[4 marks]

6b. Find the gradient of the curve of
f at
$x = \frac{\pi}{2}$. *[3 marks]*

Markscheme

substituting
$\frac{\pi}{2}$ into **their**
$f'(x)$ ***(M1)***

eg
$f'\left(\frac{\pi}{2}\right)$,
$\left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right) \sin\left(\frac{\pi}{2}\right)$

correct values for **both**
$\sin \frac{\pi}{2}$ and
$\cos \frac{\pi}{2}$ seen in
$f'(x)$ ***(A1)***

eg
$0 + 2\left(\frac{\pi}{2}\right) \times 1$

$f'\left(\frac{\pi}{2}\right) = \pi$ ***A1 N2***

[3 marks]

---

7. A rocket moving in a straight line has velocity
v km s^{-1} and displacement
s km at time
t seconds. The velocity
v is given by
$v(t) = 6\mathrm{e}^{2t} + t$. When
$t = 0$,
$s = 10$. *[7 marks]*

Find an expression for the displacement of the rocket in terms of
t .

Markscheme

evidence of anti-differentiation ***(M1)***

eg
$\int (6e^{2t} + t)$

$s = 3e^{2t} + \frac{t^2}{2} + C$ ***A2A1***

Note: Award ***A2*** for
$3e^{2t}$, ***A1*** for
$\frac{t^2}{2}$.

attempt to substitute (
0,
10) into **their** integrated expression (even if
C is missing) ***(M1)***

correct working ***(A1)***

eg
$10 = 3 + C$,
$C = 7$

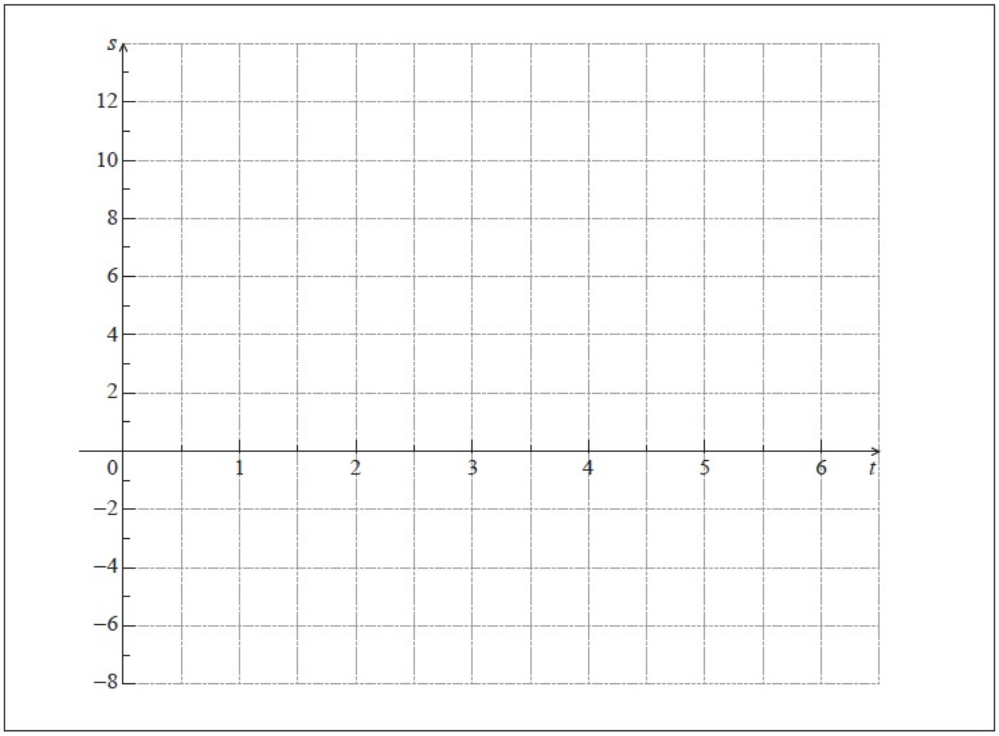
$s = 3e^{2t} + \frac{t^2}{2} + 7$ ***A1*** ***N6***

Note: Exception to the ***FT*** rule. If working shown, allow full ***FT*** on incorrect integration which must involve a power of e.

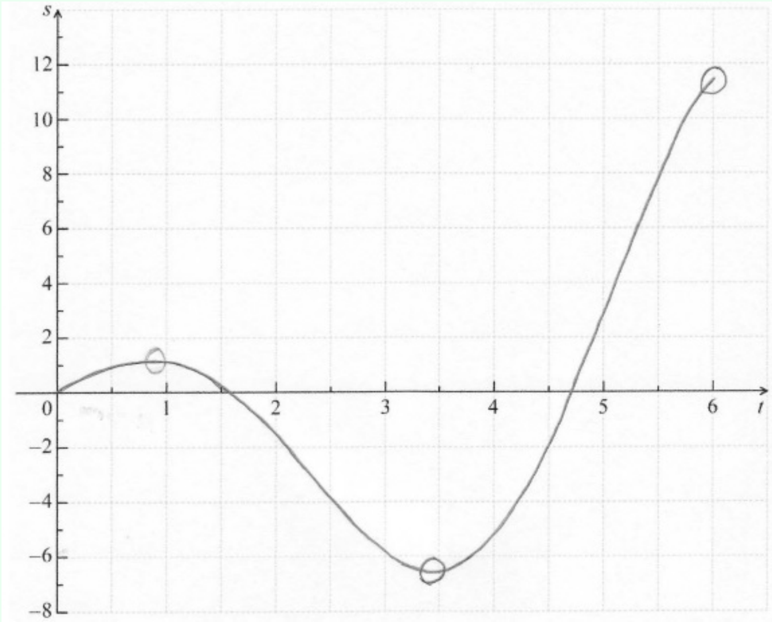
[7 marks]

A particle's displacement, in metres, is given by
$s(t) = 2t\cos t$, for
$0 \le t \le 6$, where t is the time in seconds.

8a. On the grid below, sketch the graph of
s . *[4 marks]*



Markscheme



A1A1A1A1 ***N4***

Note: Award ***A1*** for approximately correct shape (do not accept line segments).

Only if this ***A1*** is awarded, award the following:

A1 for maximum and minimum within circles,

A1 for *x*-intercepts between 1 and 2 **and** between 4 and 5,

A1 for left endpoint at
$(0, 0)$ and right endpoint within circle.

[4 marks]

8b. Find the maximum velocity of the particle. *[3 marks]*

Markscheme

appropriate approach ***(M1)***

e.g. recognizing that
$v = s'$, finding derivative,
$a = s''$

valid method to find maximum ***(M1)***

e.g. sketch of
v ,
$v'(t) = 0$,
$t = 5.08698\ldots$

$v = 10.20025\ldots$

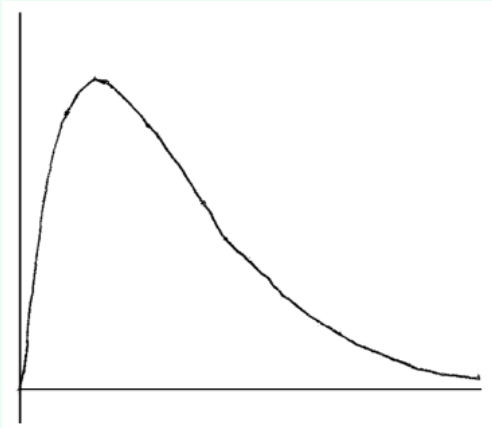
$v = 10.2$
$[10.2, 10.3]$ ***A1*** ***N2***

[3 marks]

Let
$f(x) = \frac{20x}{e^{0.3x}}$, for
$0 \le x \le 20$.

9a. Sketch the graph of *f* . *[3 marks]*

Markscheme



A1A1A1 N3

Note: Award ***A1*** for approximately correct shape with inflexion/change of curvature, ***A1*** for maximum skewed to the left, ***A1*** for asymptotic behaviour to the right.

[3 marks]

9b. (i) Write down the *x*-coordinate of the maximum point on the graph of *f* . *[3 marks]*

(ii) Write down the interval where *f* is increasing.

Markscheme

(i)
$x = 3.33$ ***A1 N1***

(ii) correct interval, with right end point
$3\frac{1}{3}$ ***A1A1 N2***

e.g.
$0 < x \le 3.33$,
$0 \le x < 3\frac{1}{3}$

Note: Accept any inequalities in the right direction.

[3 marks]

9c. Show that
$f'(x) = \frac{20-6x}{e^{0.3x}}$. *[5 marks]*

Markscheme

valid approach *(M1)*

e.g. quotient rule, product rule

2 correct derivatives (must be seen in product or quotient rule) *(A1)(A1)*

e.g.
20 ,
$0.3\mathrm{e}^{0.3x}$ or
$-0.3\mathrm{e}^{-0.3x}$

correct substitution into product or quotient rule *A1*

e.g.
$\frac{20\mathrm{e}^{0.3x}-20x(0.3)\mathrm{e}^{0.3x}}{(\mathrm{e}^{0.3x})^2}$,
$20\mathrm{e}^{-0.3x}+20x(-0.3)\mathrm{e}^{-0.3x}$

correct working *A1*

e.g.
$\frac{20\mathrm{e}^{0.3x}-6x\mathrm{e}^{0.3x}}{\mathrm{e}^{0.6x}}$,
$\frac{\mathrm{e}^{0.3x}(20-20x(0.3))}{(\mathrm{e}^{0.3x})^2}$,
$\mathrm{e}^{-0.3x}(20+20x(-0.3))$

$f'(x)=\frac{20-6x}{\mathrm{e}^{0.3x}}$ *AG* *N0*

[5 marks]

9d. Find the interval where the rate of change of f is increasing. *[4 marks]*

Markscheme

consideration of
f' or
f'' *(M1)*

valid reasoning *R1*

e.g. sketch of
f' ,
f'' is positive,
$f''=0$, reference to minimum of
f'

correct value
$6.6666666\ldots$
$\left(6\frac{2}{3}\right)$ *(A1)*

correct interval, with **both** endpoints *A1* *N3*

e.g.
$6.67 < x \le 20$,
$6\frac{2}{3} \le x < 20$

[4 marks]

The velocity v ms^{-1} of a particle at time t seconds, is given by
$v=2t+\cos 2t$, for
$0 \le t \le 2$.

10a. Write down the velocity of the particle when
$t=0$. *[1 mark]*

Markscheme

$v = 1$ ***A1*** ***N1***

[1 mark]

10b. When $t = k$, the acceleration is zero. *[8 marks]*

(i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$.

Markscheme

(i)
$\frac{\mathrm{d}}{\mathrm{d}t}(2t) = 2$ ***A1***

$\frac{\mathrm{d}}{\mathrm{d}t}(\cos 2t) = -2\sin 2t$ ***A1A1***

Note: Award ***A1*** for coefficient 2 and ***A1*** for $-\sin 2t$.

evidence of considering acceleration = 0 ***(M1)***

e.g. $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, $2 - 2\sin 2t = 0$

correct manipulation ***A1***

e.g. $\sin 2k = 1$, $\sin 2t = 1$

$2k = \frac{\pi}{2}$ (accept $2t = \frac{\pi}{2}$) ***A1***

$k = \frac{\pi}{4}$ ***AG*** ***N0***

(ii) attempt to substitute $t = \frac{\pi}{4}$ into *v* ***(M1)***

e.g. $2\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{4}\right)$

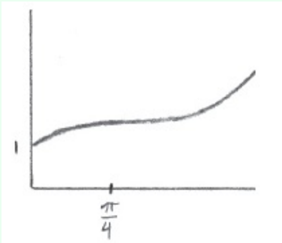
$v = \frac{\pi}{2}$ ***A1*** ***N2***

[8 marks]

10c. When $t < \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ and when $t > \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$. *[4 marks]*

Sketch a graph of *v* against *t* .

Markscheme



A1A1A2 N4

Notes: Award ***A1*** for *y*-intercept at $(0, 1)$, ***A1*** for curve having zero gradient at $t = \frac{\pi}{4}$, ***A2*** for shape that is concave down to the left of $\frac{\pi}{4}$ and concave up to the right of $\frac{\pi}{4}$. If a correct curve is drawn without indicating $t = \frac{\pi}{4}$, do not award the second ***A1*** for the zero gradient, but award the final ***A2*** if appropriate. Sketch need not be drawn to scale. Only essential features need to be clear.

[4 marks]

10d. Let *d* be the distance travelled by the particle for $0 \le t \le 1$. *[3 marks]*

(i) Write down an expression for *d* .

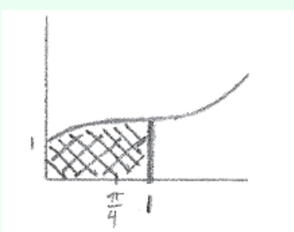
(ii) Represent *d* on your sketch.

Markscheme

(i) correct expression ***A2***

e.g.
$\int_0^1 (2t + \cos 2t)\mathrm{d}t$,
$\left[t^2 + \frac{\sin 2t}{2}\right]_0^1$,
$1 + \frac{\sin 2}{2}$,
$\int_0^1 v\mathrm{d}t$

(ii)

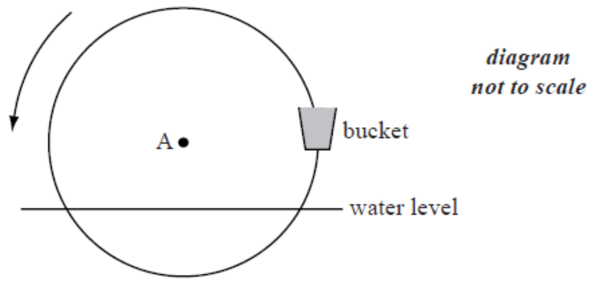


A1

Note: The line at $t = 1$ needs to be clearly after $t = \frac{\pi}{4}$.

[3 marks]

The following diagram shows a waterwheel with a bucket. The wheel rotates at aconstant rate in an anticlockwise (counter-clockwise) direction.



The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metresabove the water level. After *t* seconds, the height of the bucket above the water level is given by
$h = a\sin bt + 2$.

11a. Show that $a = 4$. *[2 marks]*

Markscheme

METHOD 1

evidence of recognizing the amplitude is the radius ***(M1)***

e.g. amplitude is half the diameter

$a = \frac{8}{2}$ ***A1***

$a = 4$ ***AG*** ***N0***

METHOD 2

evidence of recognizing the maximum height ***(M1)***

e.g.
$h = 6$,
$a\sin bt + 2 = 6$

correct reasoning

e.g.
$a\sin bt = 4$ and
$\sin bt$ has amplitude of 1 ***A1***

$a = 4$ ***AG*** ***N0***

[2 marks]

11b. The wheel turns at a rate of one rotation every 30 seconds. *[2 marks]*

Show that
$b = \frac{\pi}{15}$.

Markscheme

METHOD 1

period = 30 ***(A1)***

$b = \frac{2\pi}{30}$ ***A1***

$b = \frac{\pi}{15}$ ***AG N0***

METHOD 2

correct equation ***(A1)***

e.g.
$2 = 4\sin 30b + 2$,
$\sin 30b = 0$

$30b = 2\pi$ ***A1***

$b = \frac{\pi}{15}$ ***AG N0***

[2 marks]

11c. In the first rotation, there are two values of *t* when the bucket is **descending** at a rate of $0.5\ \mathrm{ms}^{-1}$. *[6 marks]*

Find these values of *t* .

Markscheme

recognizing
$h'(t) = -0.5$ (seen anywhere) ***R1***

attempting to solve ***(M1)***

e.g. sketch of
h' , finding
h'

correct work involving
h' ***A2***

e.g. sketch of
h' showing intersection,
$-0.5 = \frac{4\pi}{15}\cos\left(\frac{\pi}{15}t\right)$

$t = 10.6$,
$t = 19.4$ ***A1A1 N3***

[6 marks]

11d. In the first rotation, there are two values of *t* when the bucket is **descending** at a rate of $0.5\ \mathrm{ms}^{-1}$. *[4 marks]*

Determine whether the bucket is underwater at the second value of *t* .

Markscheme

METHOD 1

valid reasoning for **their** conclusion (seen anywhere) ***R1***

e.g.
$h(t) < 0$ so underwater;
$h(t) > 0$ so not underwater

evidence of substituting into *h* ***(M1)***

e.g.
$h(19.4)$,
$4\sin\frac{19.4\pi}{15}+2$

correct calculation ***A1***

e.g.
$h(19.4) = -1.19$

correct statement ***A1*** ***N0***

e.g. the bucket is underwater, yes

METHOD 2

valid reasoning for **their** conclusion (seen anywhere) ***R1***

e.g.
$h(t) < 0$ so underwater;
$h(t) > 0$ so not underwater

evidence of valid approach ***(M1)***

e.g. solving
$h(t) = 0$, graph showing region below *x*-axis

correct roots ***A1***

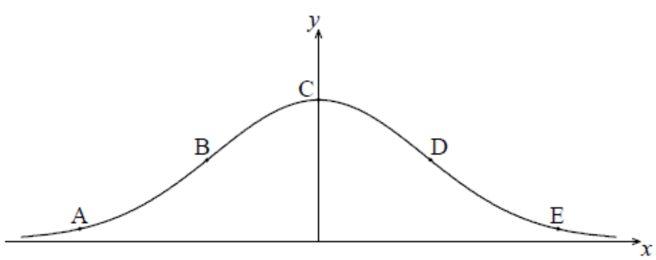
e.g.
17.5,
27.5

correct statement ***A1*** ***N0***

e.g. the bucket is underwater, yes

[4 marks]

The following diagram shows the graph of
$f(x) = \mathrm{e}^{-x^2}$.



The points A, B, C, D and E lie on the graph of *f* . Two of these are points of inflexion.

12a. Identify the **two** points of inflexion. *[2 marks]*

Markscheme

B, D ***A1A1*** ***N2***

[2 marks]

12b. (i) Find $f'(x)$. *[5 marks]*

(ii) Show that $f''(x) = (4x^2 - 2)e^{-x^2}$.

Markscheme

(i)
$f'(x) = -2xe^{-x^2}$ ***A1A1 N2***

Note: Award ***A1*** for e^{-x^2} and ***A1*** for $-2x$.

(ii) finding the derivative of $-2x$, i.e. -2 ***(A1)***

evidence of choosing the product rule ***(M1)***

e.g.
$-2e^{-x^2}$
$-2x \times -2xe^{-x^2}$

$-2e^{-x^2} + 4x^2e^{-x^2}$ ***A1***

$f''(x) = (4x^2 - 2)e^{-x^2}$ ***AG N0***

[5 marks]

12c. Find the *x*-coordinate of each point of inflexion. *[4 marks]*

Markscheme

valid reasoning ***R1***

e.g.
$f''(x) = 0$

attempting to solve the equation ***(M1)***

e.g.
$(4x^2 - 2) = 0$, sketch of $f''(x)$

$p = 0.707$ $\left(= \frac{1}{\sqrt{2}}\right)$,
$q = -0.707$ $\left(= -\frac{1}{\sqrt{2}}\right)$ ***A1A1 N3***

[4 marks]

12d. Use the second derivative to show that one of these points is a point of inflexion. *[4 marks]*

Markscheme

evidence of using second derivative to test values on either side of POI ***M1***

e.g. finding values, reference to graph of f'' , sign table

correct working ***A1A1***

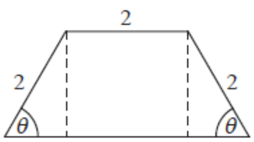
e.g. finding any two correct values either side of POI,

checking sign of f'' on either side of POI

reference to sign change of $f''(x)$ ***R1*** ***N0***

[4 marks]

The diagram below shows a plan for a window in the shape of a trapezium.



Three sides of the window are 2 m long. The angle between the sloping sides of thewindow and the base is θ , where $0 < \theta < \frac{\pi}{2}$.

13a. Show that the area of the window is given by $y = 4\sin\theta + 2\sin 2\theta$. *[5 marks]*

Markscheme

evidence of finding height, *h* ***(A1)***

e.g. $\sin\theta = \frac{h}{2}$, $2\sin\theta$

evidence of finding base of triangle, *b* ***(A1)***

e.g. $\cos\theta = \frac{b}{2}$, $2\cos\theta$

attempt to substitute valid values into a formula for the area of the window ***(M1)***

e.g. two triangles plus rectangle, trapezium area formula

correct expression (must be in terms of θ) ***A1***

e.g. $2\left(\frac{1}{2} \times 2\cos\theta \times 2\sin\theta\right) + 2 \times 2\sin\theta$, $\frac{1}{2}(2\sin\theta)(2 + 2 + 4\cos\theta)$

attempt to replace $2\sin\theta\cos\theta$ by $\sin 2\theta$ ***M1***

e.g. $4\sin\theta + 2(2\sin\theta\cos\theta)$

$y = 4\sin\theta + 2\sin 2\theta$ ***AG*** ***N0***

[5 marks]

13b. Zoe wants a window to have an area of $5\ \mathrm{m}^2$. Find the two possible values of θ . *[4 marks]*

Markscheme

correct equation ***A1***

e.g.
$y = 5$,
$4\sin\theta + 2\sin 2\theta = 5$

evidence of attempt to solve ***(M1)***

e.g. a sketch,
$4\sin\theta + 2\sin\theta - 5 = 0$

$\theta = 0.856$
(49.0°) ,
$\theta = 1.25$
(71.4°) ***A1A1*** ***N3***

[4 marks]

13c. John wants two windows which have the same area *A* but different values of θ . *[7 marks]*

Find all possible values for *A* .

Markscheme

recognition that lower area value occurs at
$\theta = \frac{\pi}{2}$ ***(M1)***

finding value of area at
$\theta = \frac{\pi}{2}$ ***(M1)***

e.g.
$4\sin\left(\frac{\pi}{2}\right) + 2\sin\left(2 \times \frac{\pi}{2}\right)$, draw square

$A = 4$ ***(A1)***

recognition that maximum value of *y* is needed ***(M1)***

$A = 5.19615\ldots$ ***(A1)***

$4 < A < 5.20$ (accept
$4 < A < 5.19$) ***A2*** ***N5***

[7 marks]

A particle moves in a straight line. Its velocity,
$v\ \mathrm{ms}^{-1}$, at time
t seconds, is given by

$$v = \left(t^2 - 4\right)^3,\ \text{for } 0 \leqslant t \leqslant 3.$$

14a. Find the velocity of the particle when
$t = 1.$ *[2 marks]*

Markscheme

substituting

$t = 1$ into

v ***(M1)***

eg

$v(1),\ \left(1^2 - 4\right)^3$

velocity

$= -27\ \left(\mathrm{ms}^{-1}\right)$ ***A1 N2***

[2 marks]

14b. Find the value of t for which the particle is at rest. *[3 marks]*

Markscheme

valid reasoning ***(R1)***

eg

$v = 0,\ \left(t^2 - 4\right)^3 = 0$

correct working ***(A1)***

eg

$t^2 - 4 = 0,\ t = \pm 2$, sketch

$t = 2$ ***A1 N2***

[3 marks]

14c. Find the total distance the particle travels during the first three seconds. *[3 marks]*

Markscheme

correct integral expression for distance ***(A1)***

eg

$\int_0^3 |v|,\ \int \left|\left(t^2 - 4\right)^3\right|,\ -\int_0^2 v\mathrm{d}t + \int_2^3 v\mathrm{d}t,$

$\int_0^2 \left(4 - t^2\right)^3 \mathrm{d}t + \int_2^3 \left(t^2 - 4\right)^3 \mathrm{d}t$ (do not accept $\int_0^3 v\mathrm{d}t$)

86.2571

$\text{distance} = 86.3\ (\mathrm{m})$ ***A2 N3***

[3 marks]

14d. Show that the acceleration of the particle is given by $a = 6t(t^2 - 4)^2$. *[3 marks]*

Markscheme

evidence of differentiating velocity ***(M1)***

eg

$v'(t)$

$a = 3\left(t^2 - 4\right)^2(2t)$ ***A2***

$a = 6t\left(t^2 - 4\right)^2$ ***AG N0***

[3 marks]

14e. Find all possible values of t for which the velocity and acceleration are both positive orboth negative. *[4 marks]*

Markscheme

METHOD 1

valid approach ***M1***

eg graphs of v and a

correct working ***(A1)***

eg areas of same sign indicated on graph

$2 < t \leqslant 3$ (accept $t > 2$) ***A2 N2***

METHOD 2

recognizing that $a \geqslant 0$ (accept a *is* always positive) (seen anywhere) ***R1***

recognizing that v is positive when $t > 2$ (seen anywhere) ***(R1)***

$2 < t \leqslant 3$ (accept $t > 2$) ***A2 N2***

[4 marks]

The first three terms of a infinite geometric sequence are $m - 1,\ 6, m + 4$, where $m \in \mathbb{Z}$.

15a. Write down an expression for the common ratio, r. *[2 marks]*

Markscheme

correct expression for r ***A1 N1***

eg

$r = \frac{6}{m-1},\ \frac{m+4}{6}$

[2 marks]

15b. Hence, show that m satisfies the equation $m^2 + 3m - 40 = 0$. *[2 marks]*

Markscheme

correct equation ***A1***

eg

$\frac{6}{m-1} = \frac{m+4}{6},\ \frac{6}{m+4} = \frac{m-1}{6}$

correct working ***(A1)***

eg

$(m+4)(m-1) = 36$

correct working ***A1***

eg

$m^2 - m + 4m - 4 = 36,\ m^2 + 3m - 4 = 36$

$m^2 + 3m - 40 = 0$ ***AG*** ***N0***

[2 marks]

15c. Find the two possible values of m. *[3 marks]*

Markscheme

valid attempt to solve ***(M1)***

eg

$(m+8)(m-5) = 0,\ m = \frac{-3 \pm \sqrt{9+4\times 40}}{2}$

$m = -8,\ m = 5$ ***A1A1*** ***N3***

[3 marks]

15d. Find the possible values of r. *[3 marks]*

Markscheme

attempt to substitute **any** value of m to find r ***(M1)***

eg

$\frac{6}{-8-1},\ \frac{5+4}{6}$

$r = \frac{3}{2},\ r = -\frac{2}{3}$ ***A1A1*** ***N3***

[3 marks]

15e. The sequence has a finite sum. *[3 marks]*

State which value of r leads to this sum **and** justify your answer.

Markscheme

$r = -\frac{2}{3}$ (may be seen in justification) ***A1***

valid reason ***R1*** ***N0***

eg

$|r| < 1, \ -1 < \frac{-2}{3} < 1$

Notes: Award ***R1*** for
$|r| < 1$ only if ***A1*** awarded.

[2 marks]

15f. The sequence has a finite sum. *[3 marks]*

Calculate the sum of the sequence.

Markscheme

finding the first term of the sequence which has
$|r| < 1$ ***(A1)***

eg
$-8 - 1, \ 6 \div \frac{-2}{3}$

$u_1 = -9$ (may be seen in formula) ***(A1)***

correct substitution of
u_1 and their
r into
$\frac{u_1}{1-r}$, as long as
$|r| < 1$ ***A1***

eg
$S_\infty = \frac{-9}{1-\left(-\frac{2}{3}\right)}, \ \frac{-9}{\frac{5}{3}}$

$S_\infty = -\frac{27}{5} \ (= -5.4)$ ***A1*** ***N3***

[4 marks]

Consider the lines
L_1 and
L_2 with equations
L_1 :
$r = \begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ and
L_2 :
$r = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$.

The lines intersect at point
P.

16a. Find the coordinates of
P. *[6 marks]*

Markscheme

appropriate approach *(M1)*

eg

$$\begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix},$$

$L_1 = L_2$

any **two** correct equations *A1A1*

eg

$11 + 4s = 1 + 2t,\ 8 + 3s = 1 + t,\ 2 - s = -7 + 11t$

attempt to solve system of equations *(M1)*

eg

$$10 + 4s = 2(7 + 3s), \begin{cases} 4s - 2t = -10 \\ 3s - t = -7 \end{cases}$$

one correct parameter *A1*

eg

$s = -2,\ t = 1$

$\mathrm{P}(3, 2, 4)$ (accept position vector) *A1* *N3*

[6 marks]

16b. Show that the lines are perpendicular. *[5 marks]*

Markscheme

choosing correct direction vectors for

L_1 and

L_2 *(A1)(A1)*

eg

$$\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$$ (or any scalar multiple)

evidence of scalar product (with any vectors) *(M1)*

eg

$a \cdot b,$

$$\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$$

correct substitution *A1*

eg

$4(2) + 3(1) + (-1)(11),\ 8 + 3 - 11$

calculating

$a \cdot b = 0$ *A1*

Note: Do not award the final ***A1*** without evidence of calculation.

vectors are perpendicular *AG* *N0*

[5 marks]

16c. The point $\mathrm{Q}(7, 5, 3)$ lies on L_1. The point R is the reflection of Q in the line L_2.

Find the coordinates of R. *[6 marks]*

Markscheme

Note: Candidates may take different approaches, which do not necessarily involve vectors.
In particular, most of the working could be done on a diagram. Award marks in line with the markscheme.

METHOD 1

attempt to find $\overrightarrow{\mathrm{QP}}$ or $\overrightarrow{\mathrm{PQ}}$ ***(M1)***

correct working (may be seen on diagram) ***A1***

eg $\overrightarrow{\mathrm{QP}}$ = $\begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix}$, $\overrightarrow{\mathrm{PQ}}$ = $\begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

recognizing R is on L_1 (seen anywhere) ***(R1)***

eg on diagram Q and R are equidistant from P (seen anywhere) ***(R1)***

eg $\overrightarrow{\mathrm{QP}} = \overrightarrow{\mathrm{PR}}$, marked on diagram

correct working ***(A1)***

eg $\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

$\mathrm{R}(-1, -1, 5)$ (accept position vector) ***A1 N3***

METHOD 2

recognizing R is on L_1 (seen anywhere) ***(R1)***

eg on diagram Q and R are equidistant from P (seen anywhere) ***(R1)***

eg P midpoint of QR, marked on diagram valid approach to find **one** coordinate of mid-point ***(M1)***

eg

$x_p = \frac{x_Q + x_R}{2},\ 2y_p = y_Q + y_R,\ \frac{1}{2}(z_Q + z_R)$

one correct substitution ***A1***

eg

$x_R = 3 + (3 - 7),\ 2 = \frac{5 + y_R}{2},\ 4 = \frac{1}{2}(z + 3)$

correct working for one coordinate ***(A1)***

eg

$x_R = 3 - 4,\ 4 - 5 = y_R,\ 8 = (z + 3)$

$\mathrm{R}(-1, -1, 5)$ (accept position vector) ***A1*** ***N3***

[6 marks]

Consider the functions
$f(x)$,
$g(x)$ and
$h(x)$. The following table gives some valuesassociated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	–14	–18
$f'(x)$	1	1
$g'(x)$	–5	–3
$h''(x)$	–6	0

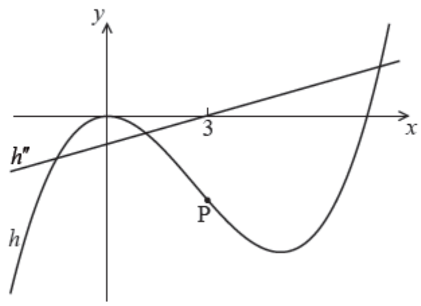
17a. Write down the value of
$g(3)$, of
$f'(3)$, and of
$h''(2)$. *[3 marks]*

Markscheme

$g(3) = -18$,
$f'(3) = 1$,
$h''(2) = -6$ ***A1A1A1*** ***N3***

[3 marks]

The following diagram shows parts of the graphs of h and h'' .



There is a point of inflexion on the graph of h at P, when $x = 3$.

17b. Explain why P is a point of inflexion. *[2 marks]*

Markscheme

$h''(3) = 0$ ***(A1)***

valid reasoning ***R1***

eg h'' changes sign at $x = 3$, change in concavity of h at $x = 3$

so P is a point of inflexion ***AG*** ***N0***

[2 marks]

Given that $h(x) = f(x) \times g(x)$,

17c. find the y-coordinate of P. *[2 marks]*

Markscheme

writing $h(3)$ as a product of $f(3)$ and $g(3)$ ***A1***

eg $f(3) \times g(3)$, $3 \times (-18)$

$h(3) = -54$ ***A1 N1***

[2 marks]

17d. find the equation of the normal to the graph of h at P. *[7 marks]*

Markscheme

recognizing need to find derivative of
h ***(R1)***

eg
h' ,
$h'(3)$

attempt to use the product rule (do **not** accept
$h' = f' \times g'$) ***(M1)***

eg
$h' = fg' + gf'$,
$h'(3) = f(3) \times g'(3) + g(3) \times f'(3)$

correct substitution ***(A1)***

eg
$h'(3) = 3(-3) + (-18) \times 1$

$h'(3) = -27$ ***A1***

attempt to find the gradient of the normal ***(M1)***

eg
$-\frac{1}{m}$,
$-\frac{1}{27}x$

attempt to substitute **their** coordinates and **their** normal gradient into the equation of a line ***(M1)***

eg
$-54 = \frac{1}{27}(3) + b$,
$0 = \frac{1}{27}(3) + b$,
$y + 54 = 27(x - 3)$,
$y - 54 = \frac{1}{27}(x + 3)$

correct equation in any form ***A1*** ***N4***

eg
$y + 54 = \frac{1}{27}(x - 3)$,
$y = \frac{1}{27}x - 54\frac{1}{9}$

[7 marks]