BECA / Huson / 12.1 IB Math Name:
15 November 2018
Exam: Vector algebra and differential calculus (cumulative review)

1a. Consider the points A$(5, 2, 1)$, B$(6, 5, 3)$, and C$(7, 6, a)$, $a \in \mathbb{R}$. Find

(i) $\overrightarrow{AB}$; *[3 marks]*

(ii) $\overrightarrow{AC}$.

1b. Find the magnitude (length) of $\overrightarrow{AB}$. *[2 marks]*

2a. Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

Use the quotient rule to show that $g'(x) = \frac{1-2\ln x}{x^3}$. *[4 marks]*

2b. The graph of g has a maximum point at A. Find the x-coordinate of A. *[3 marks]*

3a. Let $g(x) = 2x \sin x$.

Find $g'(x)$. *[4 marks]*

3b. Find the gradient of the graph of g at $x = \pi$. *[3 marks]*

3a. The following diagram shows quadrilateral ABCD, with $\overrightarrow{AD} = \overrightarrow{BC}$, $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, and $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

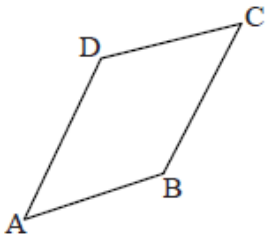
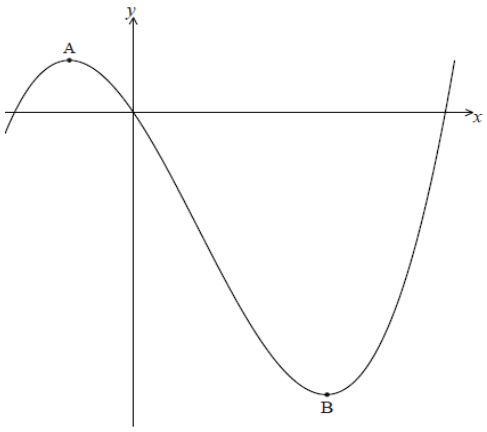


diagram not to scale

Find $\overrightarrow{BC}$. *[2 marks]*

3b. Show that $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$. *[2 marks]*

4a. Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x$. Part of the graph of f is shown below.



There is a maximum point at A and a minimum point at B$(3, -9)$.

Find the coordinates of A. *[8 marks]*

4b. Write down the coordinates of *[6 marks]*

(i) the image of B after reflection in the *y*-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the *x*-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

5a. Let $f(x) = \frac{\cos x}{\sin x}$, for $\sin x \neq 0$.

Use the quotient rule to show that $f'(x) = \frac{-1}{\sin^2 x}$. *[5 marks]*

5b. Find $f''(x)$. *[3 marks]*

6a. In an arithmetic sequence, $u_1 = 3$ and $u_3 = 11$.

Find d. *[2 marks]*

6b. Find u_{20}. *[2 marks]*

6c. Find S_{20}. *[2 marks]*

7a. The first three terms of an infinite geometric sequence are 27, 9 and 3.

Write down the value of r. *[1 mark]*

7b. Find u_6. *[2 marks]*

7c. Find the sum to infinity of this sequence. *[2 marks]*