7.0 Limits and derivatives (12.1 IB SL)

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September 7, 2017

How do we use limits?

- CCSS: Limits
- Do Now: Prior learning problems
- Lesson: Introduction to limits
- Homework:

Prior learning

- Factoring polynomials
- Expanding powers of binomials
- Rational exponents

Limits of sequences

Consider this sequence. If it went on forever, what would it's value be?

Element	Value
u_1	$\frac{1}{3}$
u_2	$\frac{4}{9}$
u_3	$\frac{13}{27}$
u_4	$\frac{40}{81}$
$\vdots$	$\vdots$

Limits of sequences

Notation:

$$\lim_{n \to \infty} u_n = L$$

"The limit as n approaches infinity of u sub n equals L."

Element	Value
u_1	$\frac{1}{3}$
u_2	$\frac{4}{9}$
u_3	$\frac{13}{27}$
u_4	$\frac{40}{81}$
$\vdots$	$\vdots$

Convergence and divergence

Definition

A Convergent sequence approaches a fixed value (real number).
Example: $\frac{1}{3}, \frac{4}{9}, \frac{13}{27}, \frac{40}{81}, \ldots$ (approaches $\frac{1}{2}$)

A Divergent sequence does not converge.
Example: 1, 1, 2, 3, 5, 8, 13, ...

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Exercise 7A

Limit of a function

Definition

A function, $f(x)$, is said to have a limit for a specific input value, c, if as x gets sufficiently close to c (from either side), $f(x)$ gets close to a value, L.

Notation:

$$\lim_{x \to c} f(x) = L$$

Limit of a function

Example

Find the limit or state that it does not exist.

(a) $\lim_{x \to 2} x^2$

(b) $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

(c) $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} 1 & \text{for } x \geq 0 \\ -1 & \text{for } x \leq 0 \end{cases}$

Limit of a function

Example 2(b) pg. 198

Find the limit or state that it does not exist.

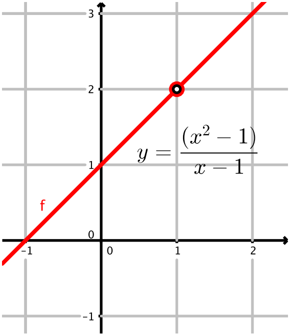
$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

Three approaches: graphing, algebra, data table

Limit of a function

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

Graphing:



Note the discontinuity.

Limit of a function

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

Algebra:

$$f(x) = \frac{x^2 - 1}{x - 1} = \frac{(x+1)(x-1)}{x - 1} = x + 1$$

$$x \neq 1$$

Limit of a function

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

Data table:

0.9	0.99	0.999	1.001	1.01	1.1
1.9	1.99	1.999	2.001	2.01	2.1

Continuity

Definition

A function, $f(x)$, is said to be continuous if for all values, c, in its domain, $f(c)$ exists and, $\lim_{x \to c} f(x) = f(c)$

Informally, if the function can be drawn without lifting your pencil, it is continuous.