Final Exam

Answer in pen. Show work. Graph carefully using pencil.

1. Given a loan or investment there are certain values to substitute into the appropriate formula. Assume

   Principal amount invested, $P_0 = \$1,000$

   Interest rate, $r = 5\% = 0.05$

   Time, $t = 5$ years

   Compounding periods per year, $n = 12$

   For each interest rate convention below, first write the formula, then substitute the values, finally, write down the final balance (use your calculator).
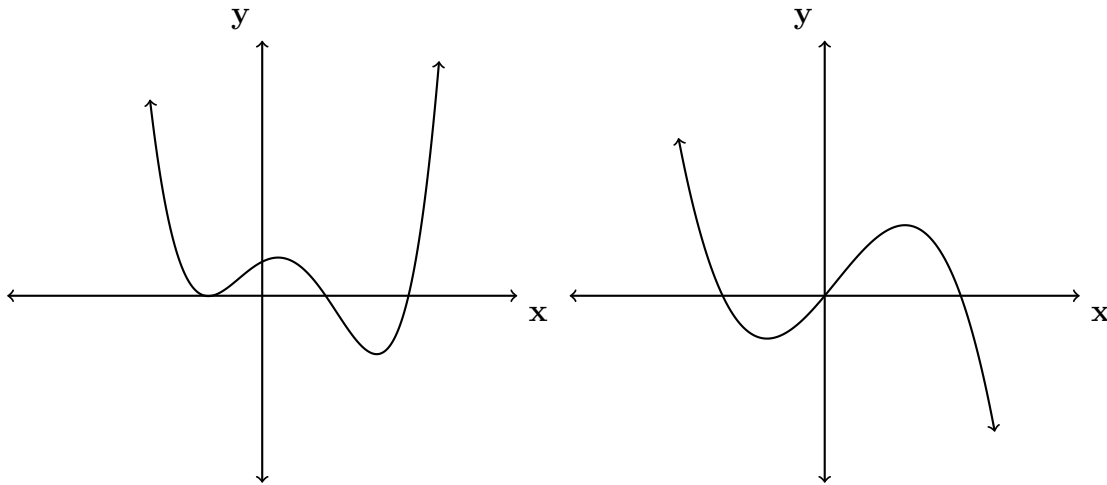
   (a) Continuous interest

   (b) Monthly compounded interest

2. Explain how $\left(2^{\frac{1}{5}}\right)^3$ can be written as the equivalent radical expression $\sqrt[5]{8}$.
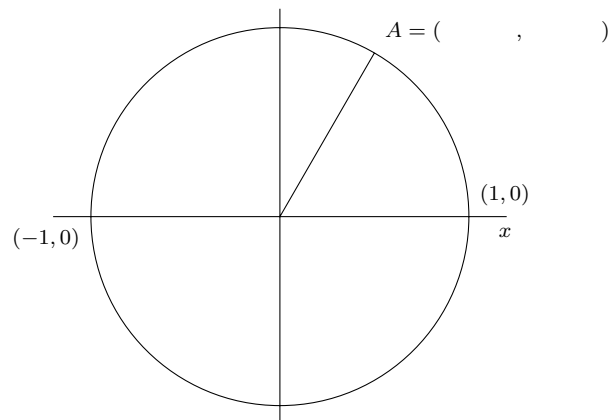
3. For each polynomial graph, state

    (a) its degree,

    (b) how many distinct zeros it has, and

    (c) the sign of its leading coefficient.

4. Given a circle with radius of one, centered on the origin. An angle with measure $60°$ is placed in standard position, with point A, the intersection of the circle and angle leg.
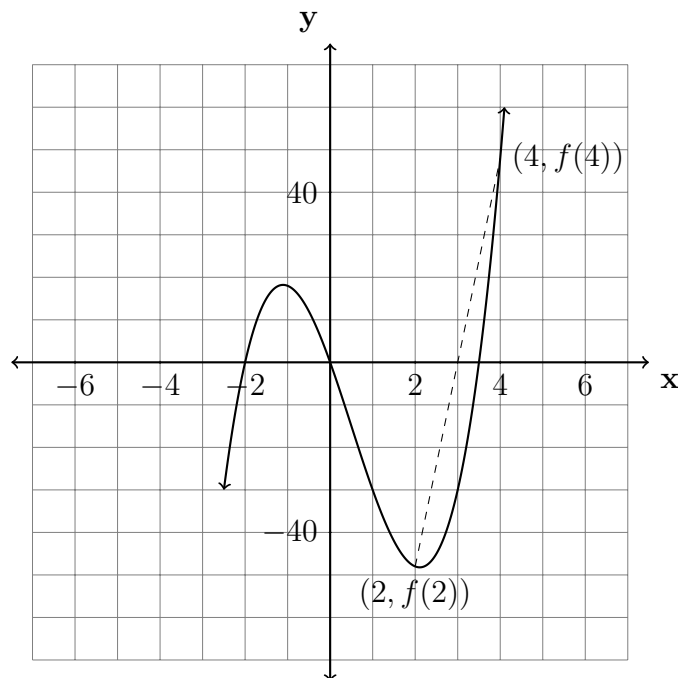

    (a) On the graph, write down the exact values of the ordered pair, A. ("exact" means a fraction, radical, or exact decimal value, not a rounded number)

    (b) Write down the exact value of $\sin 60°$

    (c) Write down the exact value of $\cos 60°$

5. Given i is the imaginary unit, simplify $(3 - zi)^2$ to the form $a + bi$.

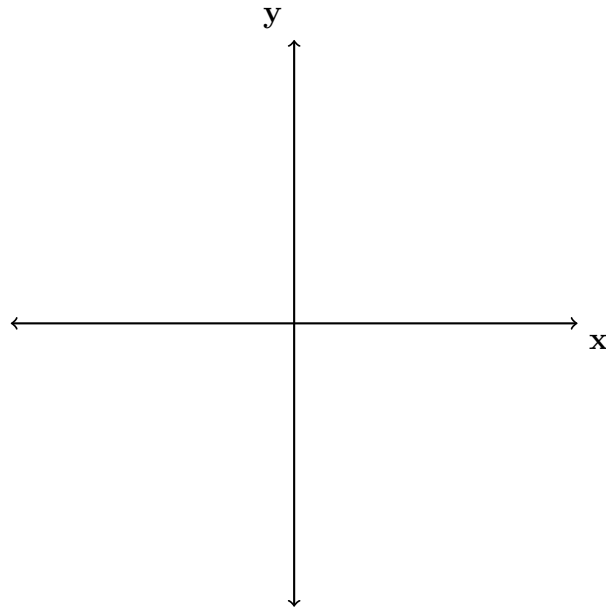6. The polynomial $f(x) = 2x^3 - 3x^2 - 14x$ is shown on the graph below.

(a) The function can be written in factored form as $f(x) = (2x + k)(x + 2)x$. Find k.

(b) What is the *average rate of change* of the function between $x = 2, 4$.

7. Sketch a graph of a cubic polynomial with the following characteristics:

- three positive, real zeros
- as $x \to +\infty$, $f(x) \to +\infty$
- as $x \to -\infty$, $f(x) \to -\infty$

8. Given: $f(x) = 3x^2 - x + 2$ and $g(x) = 3x + 7$

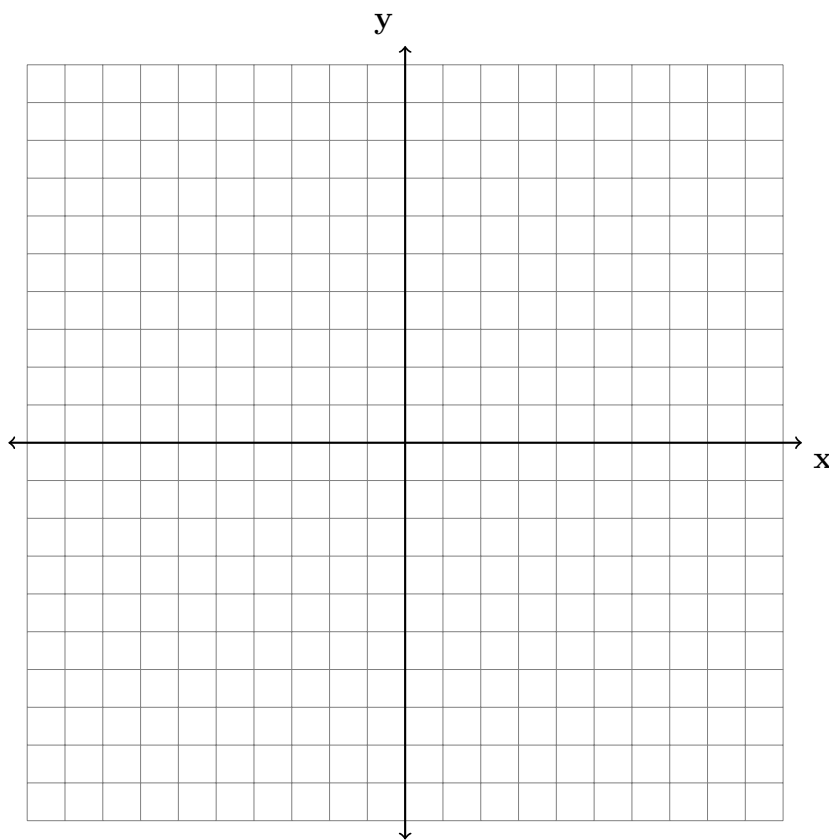   Express $2 \bullet g(x) - f(x)$ as a polynomial in standard form.

9. Two events A and B are such that $P(A) = 0.2$ and $P(A \cup B) = 0.5$. Given that A and B are mutually exclusive, find $P(B)$.

10. Find algebraically the zeros for $g(x) = x^3 + x^2 - 4x - 4$.

On the set of axes below, graph $y = g(x)$.

11. Express $2yi^3(xi - 3)$ as a complex number in the form $a + bi$, $a, b \in \mathbb{R}$.

12. Write $\sqrt[4]{x} \cdot \sqrt{x}$ as a single term with a rational exponent.

13. Given events A and B, such that $P(A) = 0.6, P(B) = 0.5$, and $P(A \cup B) = 0.8$, determine whether A and B are dependent or independent. Justify your answer.

14. The function below models the average price of gas in a small town since January 1st.

$$G(t) = -0.006t^4 + 0.0823t^3 - 0.6t^2 + 1.16t + 3.2, \text{ where } 0 \le t \le 10.$$

If $G(t)$ is the average price of gas in dollars and t represents the time in months since January 1st, the absolute maximum $G(t)$ reaches over the given domain is about what value, to the nearest cent?

15. Researchers in a local area found that the population of rabbits with an initial popu-
lation of 25 grew continuously at the rate of 4% per month. The fox population had
an initial value of 35 and grew continuously at the rate of 2% per month.

   (a) Write an expression representing the population of rabbits as a function of time
   t in months.

   (b) Find the number of rabbits after 7 months, rounded to an integer.

   (c) Find, to the *nearest tenth of a month*, how long it takes for the population of
   rabbits and foxes to be equal.

Final Exam

16. Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below.

x	$q(x)$
-2	-8
-1	0
0	0
1	-2
2	0

Determine which function has the *smaller* minimum value for the domain $[2, 2]$. Justify your answer.

17. A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

18. Jim is looking to buy a vacation home for \$165,000 near his favorite southern beach. The formula to compute a mortgage payment, M, is $M = P \cdot \dfrac{r(1+r)^N}{(1+r)^N - 1}$ where P is the principal amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.5% for a 15-year mortgage.

With no down payment, determine Jim's mortgage payment, rounded to the nearest dollar.

Algebraically determine and state the down payment, rounded to the *nearest dollar*, that Jim needs to make in order for his mortgage payment to be \$1250.

19. The x-value of which function's x-intercept is larger, f or h? Justify your answer.

$$f(x) = \log(x - 4)$$

| x	h(x)
−1	6
0	4
1	2
2	0
3	−2

20. Data collected about jogging from students with two older siblings are shown in the table below.

	Neither Sibling Jogs	One Sibling Jogs	Both Siblings Jogs
Student Does Not Jog	1168	1823	1380
Student Jogs	188	416	400

Using these data, determine whether a student with two older siblings is more likely to jog if one sibling jogs or if *neither* sibling jogs. Justify your answer.
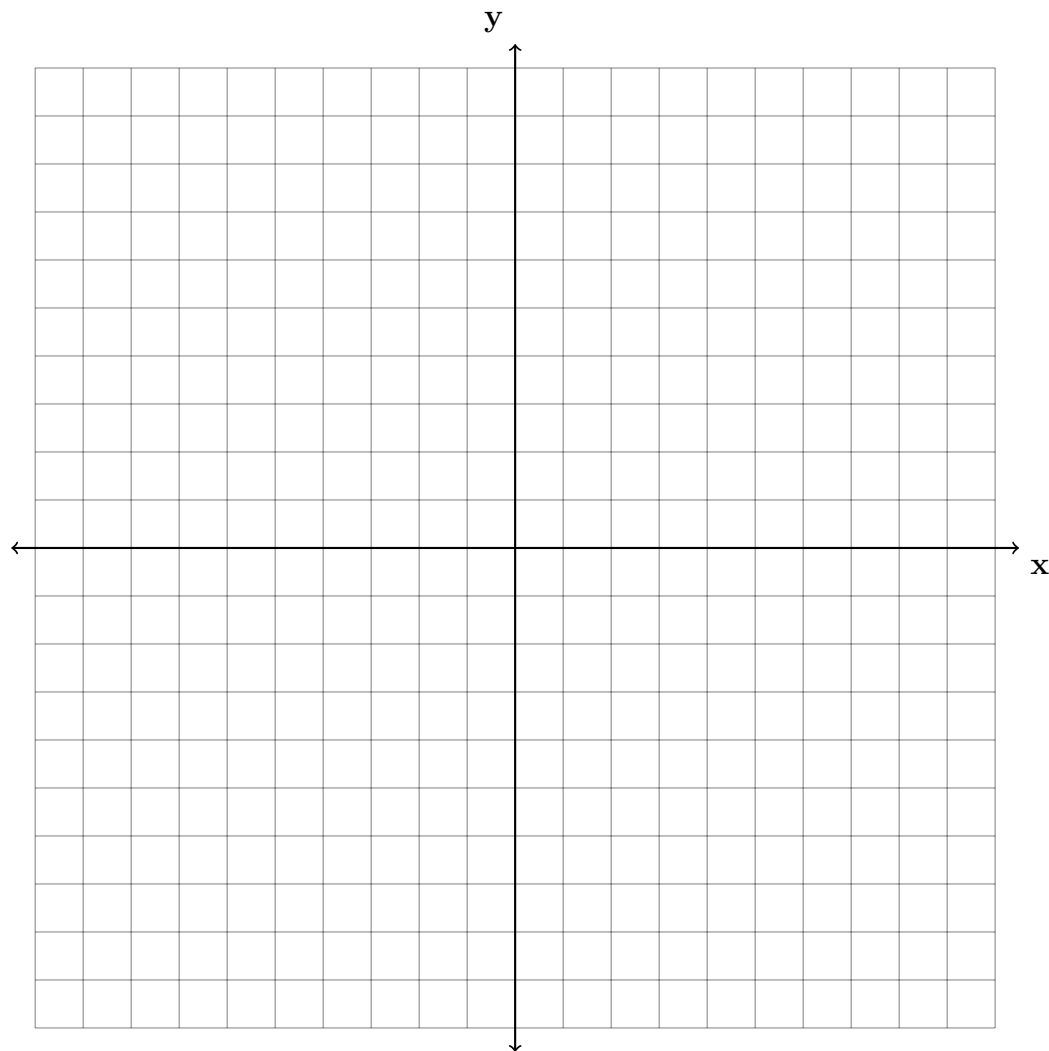
21. On the axes below, graph one cycle of a cosine function with amplitude 3, period $\dfrac{\pi}{2}$, midline $y = -1$, and passing through the point $(0, 2)$.
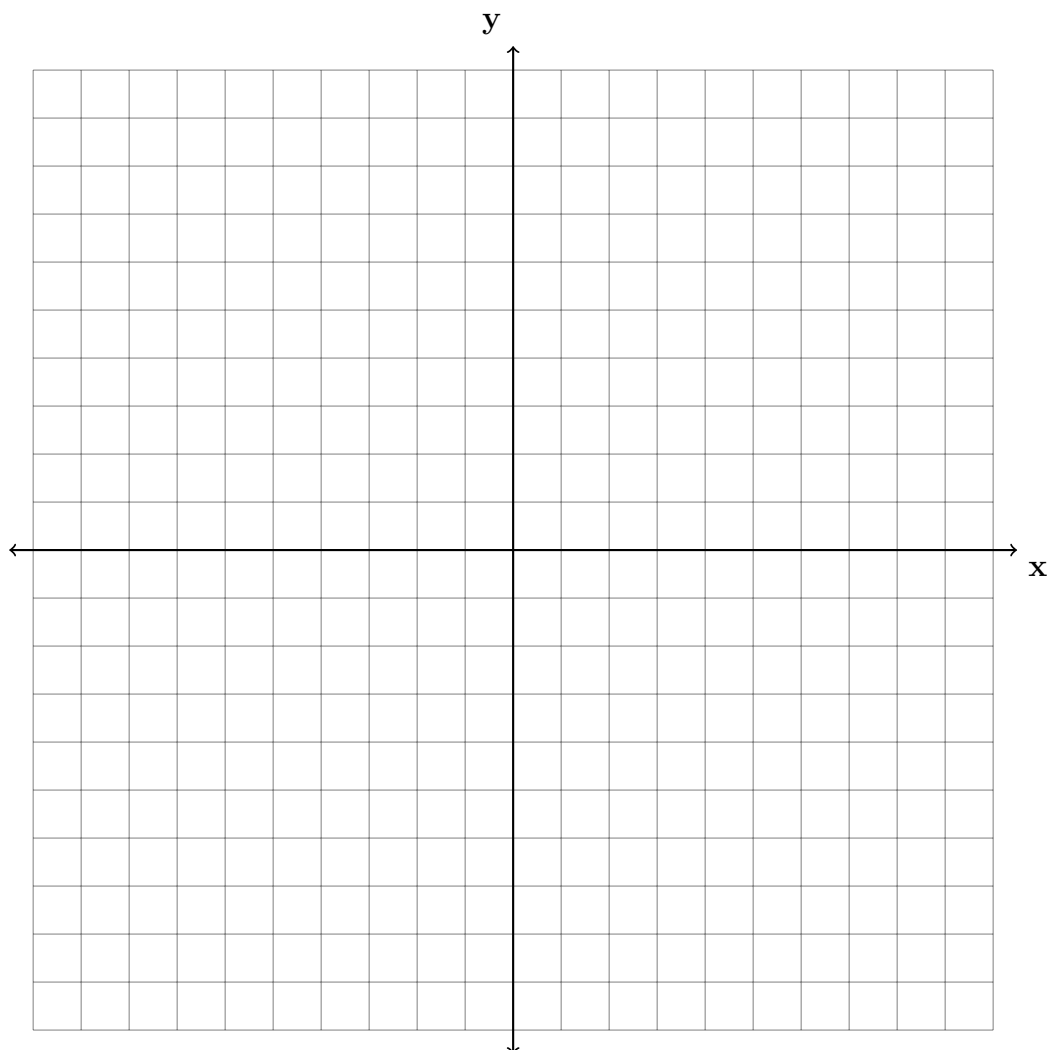
22. Graph $y = \log_2(x+3) - 5$ on the set of axes below. Use an appropriate scale to include both intercepts.

y

x

Describe the behavior of the given function as x approaches -3 and as x approaches positive infinity.