The Number e In Compound Interest

The number e can be defined by looking at compound interest. Compound interest for one year is defined as:

$$FV = PV(1 + r)^t$$

FV = future value
PV = present value (the amount before it is compounded)
r = the interest rate (in decimals)
t = the period (usually measured in years)

For example, if $1 is invested ($PV = 1$) with a 5% interest rate ($r = 0.05$) over a period of 1 year ($t = 1$) then there will be $1.05 at the end of the year. By adding 1 and the interest rate ($1 + 0.05$), then raising it by 1 year (1.05^1), and finally multiplying it by 1 (1.05×1), the final answer $1.05 is reached.

Compound interest for multiple years or multiple compounding intervals is defined differently. It is defined as:

$$FV = PV\left[1 + \left(\frac{r}{n}\right)\right]^{nt}$$

FV = future value
PV = present value (the amount before it is compounded)
r = the interest rate (in decimals)
n = the number of times the interest is compounded per year
t = the period (in years)
nt = the total number of times it is compounded

For example, if 1 dollar is invested with an interest rate of 1 compounded once for one year, then the total amount at the end of the year is $2, as shown in the formula solved below.

$$FV = 1\left[1 + \left(\frac{1}{1}\right)\right]^1$$
$$FV = 1[2]$$
$$FV = 2$$

However, if 1 dollar is invested with an interest rate of 1 compounded twice for one year, the total amount at the end of the year is $2.25, which can be seen below.

$$FV = 1\left[1 + \left(\frac{1}{2}\right)\right]^2$$
$$FV = 1[2.25]$$
$$FV = 2$$

Below is how Leonard investigated the breeding pattern of rabbits and investigated the identities of the Fibonacci sequence. Each box expresses a rabbit. F, A, C, E and C letters stand for female;

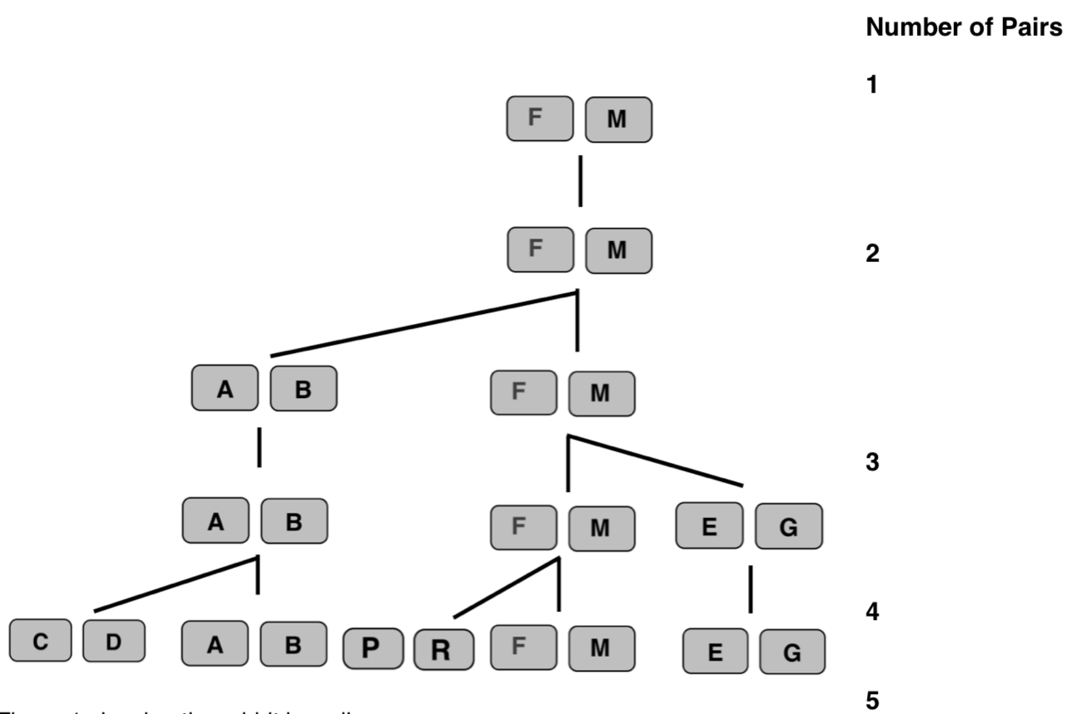

Figure 1 showing the rabbit breeding (Knott Ronald)

M, B, D, G, R letters stand for male rabbits.

As it can be seen from figure 1, Leonard started his exploration with two rabbits. These two rabbits were able to mate within a month so at the end of the month the female produces a pair of rabbits indicated as A and B. Since, while doing this exploration Leonard supposed that these rabbits never die, he assumed that the female will continue to mate and reproduce.[4]

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[4]Clark, Mary Catherine.*The Golden Ratio and Fibonacci Numbers in Nature*.http://www.math.uga.edu/~clint/2008/geomF08/projects/clark.ppt20 Jan. 2015.

The following symbols are used within the SIR model.

- R_0 is the **basic reproduction number**, or the average number of infections that result from one infected individual.
- $1/\varepsilon$ represents the **latent period**, also called the **incubation period**. This is the period between infection and the onset of symptoms. Individuals in this stage are said to be infected but not infectious.
- $1/\gamma$ represents **the infectious period**, or the period when the individual is infectious but not yet recovered or dead.
- β represents the **contact rate** of the disease, meaning the average number of people the infected individual comes into contact with. This can be considered as "how likely someone will get the disease when in contact with someone who is ill" (IB Maths Resources 2014)

D. The SIR Formula

I. *The formula*

The SIR formula is one of the many formulae that exist for the modelling of infectious disease. It is an example of a deterministic model, which is the type of model used in epidemiology when dealing with large populations. Members of the population are separated into sub-categories, represented by letters, which show the different stages of the epidemic. The transition rates between classes are expressed as derivatives.

The SIR formula divides the population into 3 compartments: *S* representing the portion of the population who are susceptible, *I* representing the portion of the population who are infected, and *R* representing those who are recovered, although recovery does not necessarily mean a return to health. In the context of the formula *R* can mean those who die from the infection. These three variables are represented as functions with respect to *t*, meaning time. Therefore, the formula is as follows:

$$S(t) + I(t) + R(t) = N$$

Where N is the total population. Within the model there exists a flow, where members of the population begin at stage *S*, move to stage *I* when infected, and progress to stage *R* from either death or a return to health. This is represented by the flow diagram:

$$S \rightarrow I \rightarrow R$$

Kermack and McKendrick, after devising the formula, created the following equations (IB Maths Resources). As they are differential equations, the represent rates of change:

$$\frac{dS}{dt} = -\frac{\beta SI}{N}$$

This equation represents the rate of change those who are susceptible to the disease with respect to time. Presumably, in a population with the assumptions of the SIR formula (see following section) this number will eventually reach 0 when all susceptible individuals have become infected.

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I$$

This equation represents the rate of change of those who are infected with respect to time. This number will begin very high, as the number of people becoming infected rapidly increases. As the disease spreads, this number will approach 0, the point where no new infections are