BECA / Huson / 12.1 IB Math SL   Name:
11 January 2018

Test: Vectors, calculus, binomial distributions, review

1a. Consider the points A(5, 2, 1), B(6, 5, 3), and C(7, 6, a+1), $a \in \mathbb{R}$.

Find

(i) $\overrightarrow{\mathrm{AB}}$;

(ii) $\overrightarrow{\mathrm{AC}}$. *[3 marks]*

1b. Let q be the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$.

Find the value of a for which $q = \frac{\pi}{2}$. *[4 marks]*

1c. i. Show that $\cos q = \frac{2a+14}{\sqrt{14a^2+280}}$.

ii. Hence, find the value of a for which $q = 1.2$. *[8 marks]*

2a. Let $f(x) = ax^3 + bx^2 + c$, where *a*, *b* and *c* are real numbers. The graph of *f* passes through the point (2, 9) .

Show that $8a + 4b + c = 9$. *[2 marks]*

2b. The graph of *f* has a local minimum at $(1, 4)$.

Find two other equations in *a* , *b* and *c* , giving your answers in a similar form to part (a). *[7 marks]*

2c. Find the value of *a*, of *b* and of *c* . *[4 marks]*

3. Let $f(x) = \frac{1}{4}x^2 + 2$. The line *L* is the tangent to the curve of *f* at (4, 6) .

Find the equation of *L*. *[4 marks]*

4a. The random variable *X* has the following probability distribution, with $\mathrm{P}(X > 1) = 0.5$.

x	0	1	2	3
$\mathrm{P}(X = x)$	p	q	r	0.2

Find the value of *r* . *[2 marks]*

4b. Given that $\mathrm{E}(X) = 1.4$, find the value of *p* and of *q* . *[6 marks]*

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5a. A box holds 240 eggs. The probability that an egg is brown is 0.05.

Find the expected number of brown eggs in the box. *[2 marks]*

5b. Find the probability that there are 15 brown eggs in the box. *[2 marks]*

5c. Find the probability that there are at least 10 brown eggs in the box. *[3 marks]*

6a. The first three terms of a geometric sequence are $u_1 = 0.64,\ u_2 = 1.6$, and $u_3 = 4$.

Find the value of r. *[2 marks]*

6b. Find the value of S_6. *[2 marks]*

6c. Find the least value of n such that $S_n > 75\,000$. *[3 marks]*

7a. An environmental group records the numbers of coyotes and foxes in a wildlife reserve after t years, starting on 1 January 1995.

Let c be the number of coyotes in the reserve after t years. The following table shows the number of coyotes after t years.

number of years (t)	0	2	10	15	19
number of coyotes (c)	115	197	265	320	406

The relationship between the variables can be modelled by the regression equation $c = at + b$.

Find the value of a and of b. *[3 marks]*

7b. Use the regression equation to estimate the number of coyotes in the reserve when $t = 7$. *[3 marks]*

7c. Let f be the number of foxes in the reserve after t years. The number of foxes can be modelled by the equation $f = \frac{2000}{1+99\mathrm{e}^{-kt}}$, where k is a constant.

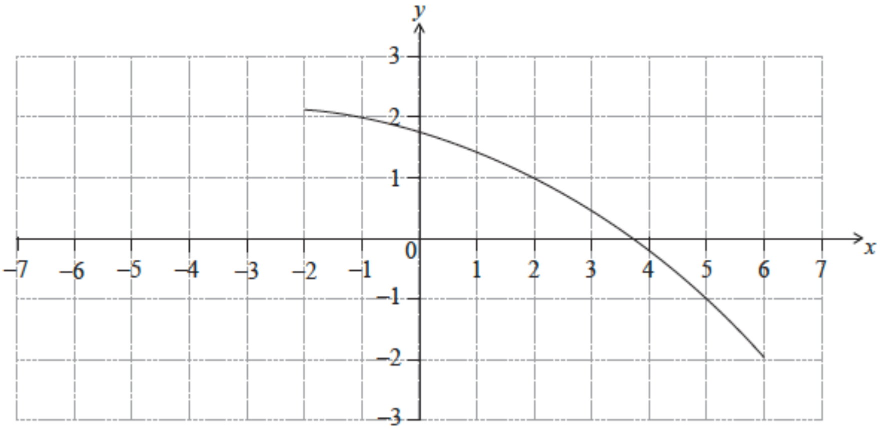
Find the number of foxes in the reserve on 1 January 1995. *[3 marks]*

7d. During which year were the number of coyotes the same as the number of foxes? *[4 marks]*

8a. Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n. *[2 marks]*

8b. Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. *[4 marks]*

9a. The following diagram shows the graph of a function f.



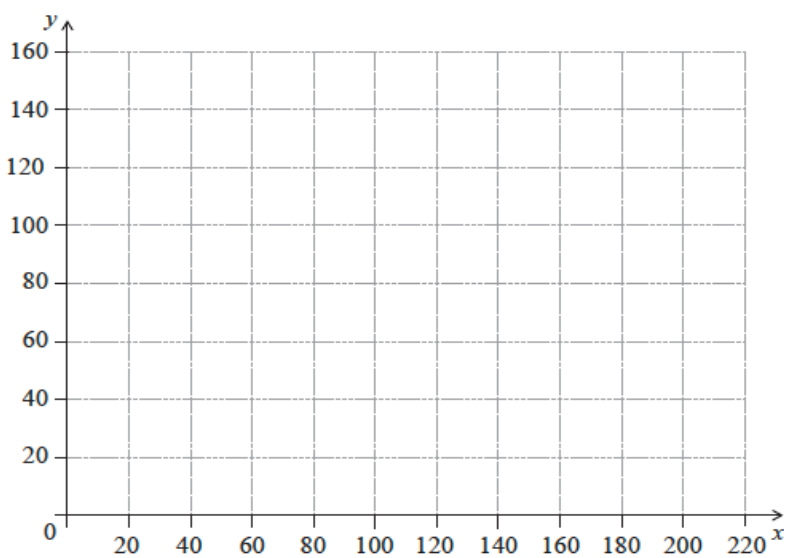
Find $f^{-1}(-1)$. *[2 marks]*

9b. Find $(f \circ f)(-1)$. *[3 marks]*

9c. On the same diagram, sketch the graph of $y = f(-x)$. *[2 marks]*

10a. Let $G(x) = 95\mathrm{e}^{(-0.02x)} + 40$, for $20 \le x \le 200$.

On the following grid, sketch the graph of G. *[3 marks]*



10b. Robin and Pat are planning a wedding banquet. The cost per guest, *G* dollars, is modelled by the function $G(n) = 95\mathrm{e}^{(-0.02n)} + 40$, for $20 \le n \le 200$, where n is the number of guests.

Calculate the **total** cost for 45 guests. *[3 marks]*