Homework: Post-test challenge-problem set

1. The equation $x^2 - 3x + k^2 = 4$ has two distinct real roots. Find the possible values of k.   *[6 marks]*

2a. At a large school, students are required to learn at least one language, Spanish or French. It is known that 75% of the students learn Spanish, and 40% learn French.

Find the percentage of students who learn **both** Spanish and French.   *[2 marks]*

2b. Find the percentage of students who learn Spanish, but not French.   *[2 marks]*

2c. At this school, 52% of the students are girls, and 85% of the girls learn Spanish.

A student is chosen at random. Let G be the event that the student is a girl, and let S be the event that the student learns Spanish.

(i) Find $P(G \cap S)$.

(ii) Show that G and S are **not** independent.   *[5 marks]*

2d. At this school, 52% of the students are girls, and 85% of the girls learn Spanish.

A boy is chosen at random. Find the probability that he learns Spanish.   *[6 marks]*

3a. Consider the function $f(x) = x^2 - 4x + 1$.

Sketch the graph of f, for $-1 \le x \le 5$.   *[4 marks]*

3b. This function can also be written as $f(x) = (x - p)^2 - 3$.

Write down the value of p.   *[1 mark]*

3c. The graph of g is obtained by reflecting the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

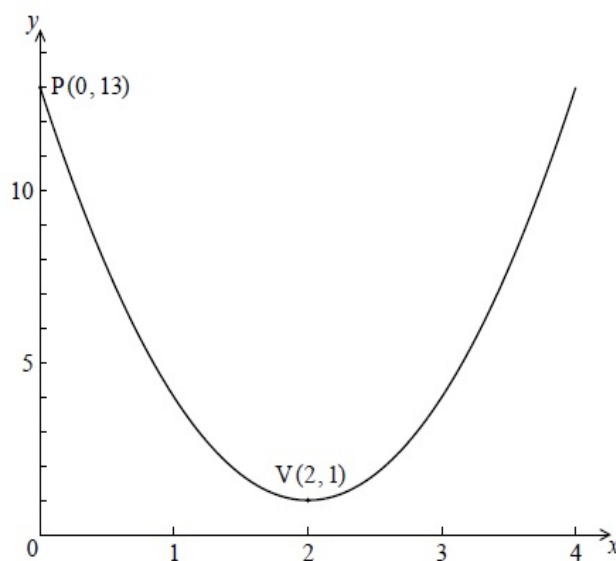Show that $g(x) = -x^2 + 4x + 5$.   *[4 marks]*

3d. The graphs of f and g intersect at two points.

Write down the x-coordinates of these two points.   *[3 marks]*

4a. The following diagram shows the graph of a quadratic function f, for $0 \leq x \leq 4$.

The graph passes through the point P(0, 13) , and its vertex is the point V(2, 1) .

The function can be written in the form $f(x) = a(x - h)^2 + k$.

(i) Write down the value of h and of k .

(ii) Show that $a = 3$.   *[4 marks]*

4b. Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$.   *[3 marks]*

5a. Consider an infinite geometric sequence with $u_1 = 40$ and $r = \frac{1}{2}$.

(i) Find u_4 .

(ii) Find the sum of the infinite sequence.   *[4 marks]*

5b. Consider an arithmetic sequence with n terms, with first term (-36) and eighth term (-8) .

(i) Find the common difference.

(ii) Show that $S_n = 2n^2 - 38n$.   *[5 marks]*

5c. The sum of the infinite geometric sequence is equal to twice the sum of the arithmetic sequence. Find n.

           [5 marks]