1-1_P1_Algebra-sequences *[171 marks]*

In an arithmetic sequence, the first term is 3 and the second term is 7.

1a. Find the common difference. *[2 marks]*

Markscheme

attempt to subtract terms ***(M1)***

eg $d = u_2 - u_1,\ 7 - 3$

$d = 4$ ***A1*** ***N2***

[2 marks]

1b. Find the tenth term. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $u_{10} = 3 + 9(4)$

$u_{10} = 39$ ***A1*** ***N2***

[2 marks]

1c. Find the sum of the first ten terms of the sequence. *[2 marks]*

Markscheme

correct substitution into sum ***(A1)***

eg $S_{10} = 5(3 + 39),\ S_{10} = \frac{10}{2}(2 \times 3 + 9 \times 4)$

$S_{10} = 210$ ***A1*** ***N2***

[2 marks]

In an arithmetic sequence, the first term is 8 and the second term is 5.

2a. Find the common difference. *[2 marks]*

Markscheme

subtracting terms ***(M1)***

eg $5 - 8,\ u_2 - u_1$

$d = -3$ ***A1*** ***N2***

[2 marks]

2b. Find the tenth term. *[2 marks]*

Markscheme

correct substitution into formula ***(A1)***

eg $u_{10} = 8 + (10 - 1)(-3),\ 8 - 27,\ -3(10) + 11$

$u_{10} = -19$ ***A1 N2***

[2 marks]

2c. Find the sum of the first ten terms. *[2 marks]*

Markscheme

correct substitution into formula for sum ***(A1)***

eg $S_{10} = \frac{10}{2}(8 - 19),\ 5\,(2(8) + (10 - 1)(-3))$

$S_{10} = -55$ ***A1 N2***

[2 marks]

In an arithmetic sequence, the first term is 2 and the second term is 5.

3a. Find the common difference. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $d = u_2 - u_1,\ 5 - 2$

$d = 3$ ***A1 N2***

[2 marks]

3b. Find the eighth term. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $u_8 = 2 + 7 \times 3$, listing terms

$u_8 = 23$ ***A1 N2***

[2 marks]

3c. Find the sum of the first eight terms of the sequence. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $S_8 = \frac{8}{2}(2 + 23)$, listing terms, $\frac{8}{2}(2(2) + 7(3))$

$S_8 = 100$ ***A1 N2***

[2 marks]

Total [6 marks]

In an arithmetic sequence,
$u_1 = 2$ and
$u_3 = 8$.

4a. Find *d* . *[2 marks]*

Markscheme

attempt to find *d* ***(M1)***

e.g.
$\frac{u_3 - u_1}{2}$,
$8 = 2 + 2d$

$d = 3$ ***A1*** ***N2***

[2 marks]

4b. Find
u_{20} . *[2 marks]*

Markscheme

correct substitution ***(A1)***

e.g.
$u_{20} = 2 + (20 - 1)3$,
$u_{20} = 3 \times 20 - 1$

$u_{20} = 59$ ***A1*** ***N2***

[2 marks]

4c. Find
S_{20} . *[2 marks]*

Markscheme

correct substitution ***(A1)***

e.g.
$S_{20} = \frac{20}{2}(2 + 59)$,
$S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$

$S_{20} = 610$ ***A1*** ***N2***

[2 marks]

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5. Three consecutive terms of a geometric sequence are $x - 3$, 6 and $x + 2$. *[6 marks]*

Find the possible values of x.

Markscheme

METHOD 1

valid approach ***(M1)***

eg $r = \frac{6}{x-3},\ (x-3) \times r = 6,\ (x-3)r^2 = x+2$

correct equation in terms of x only ***A1***

eg $\frac{6}{x-3} = \frac{x+2}{6},\ (x-3)(x+2) = 6^2,\ 36 = x^2 - x - 6$

correct working ***(A1)***

eg $x^2 - x - 42,\ x^2 - x = 42$

valid attempt to solve **their** quadratic equation ***(M1)***

eg factorizing, formula, completing the square

evidence of correct working ***(A1)***

eg $(x-7)(x+6),\ \frac{1 \pm \sqrt{169}}{2}$

$x = 7,\ x = -6$ ***A1 N4***

METHOD 2 (finding *r* first)

valid approach ***(M1)***

eg $r = \frac{6}{x-3},\ 6r = x+2,\ (x-3)r^2 = x+2$

correct equation in terms of r only ***A1***

eg $\frac{6}{r} + 3 = 6r - 2,\ 6 + 3r = 6r^2 - 2r,\ 6r^2 - 5r - 6 = 0$

evidence of correct working ***(A1)***

eg $(3r+2)(2r-3),\ \frac{5 \pm \sqrt{25+144}}{12}$

$r = -\frac{2}{3},\ r = \frac{3}{2}$ ***A1***

substituting their values of r to find x ***(M1)***

eg $(x-3)\left(\frac{2}{3}\right) = 6,\ x = 6\left(\frac{3}{2}\right) - 2$

$x = 7,\ x = -6$ ***A1 N4***

[6 marks]

In an arithmetic sequence, the third term is 10 and the fifth term is 16.

6a. Find the common difference. *[2 marks]*

Markscheme

attempt to find
d ***(M1)***

eg
$\frac{16-10}{2},\ 10 - 2d = 16 - 4d,\ 2d = 6,\ d = 6$

$d = 3$ ***A1 N2***

[2 marks]

6b. Find the first term. *[2 marks]*

Markscheme

correct approach *(A1)*

eg
$10 = u_1 + 2 \times 3,\ 10 - 3 - 3$

$u_1 = 4$ ***A1*** ***N2***

[2 marks]

6c. Find the sum of the first 20 terms of the sequence. *[3 marks]*

Markscheme

correct substitution into sum or term formula *(A1)*

eg
$\frac{20}{2}(2 \times 4 + 19 \times 3),\ u_{20} = 4 + 19 \times 3$

correct simplification *(A1)*

eg
$8 + 57,\ 4 + 61$

$S_{20} = 650$ ***A1*** ***N2***

[3 marks]

7a. Consider the infinite geometric sequence *[1 mark]*
$3,\ 3(0.9),\ 3(0.9)^2,\ 3(0.9)^3,\ \ldots.$

Write down the 10th term of the sequence. Do not simplify your answer.

Markscheme

$u_{10} = 3(0.9)^9$ ***A1*** ***N1***

[1 mark]

7b. Consider the infinite geometric sequence *[4 marks]*
$3,\ 3(0.9),\ 3(0.9)^2,\ 3(0.9)^3,\ \ldots.$

Find the sum of the infinite sequence.

Markscheme

recognizing
$r = 0.9$ *(A1)*

correct substitution ***A1***

e.g.
$S = \frac{3}{1-0.9}$

$S = \frac{3}{0.1}$ *(A1)*

$S = 30$ ***A1*** ***N3***

[4 marks]

The first three terms of an infinite geometric sequence are 32, 16 and 8.

Write down the value of *r* .

8a. [1 mark]

Markscheme

$r = \frac{16}{32} \left(= \frac{1}{2}\right)$ ***A1 N1***

[1 mark]

8b. Find
u_6 . [2 marks]

Markscheme

correct calculation or listing terms ***(A1)***

e.g.
$32 \times \left(\frac{1}{2}\right)^{6-1}$,
$8 \times \left(\frac{1}{2}\right)^{3}$, 32,
. . . 4, 2, 1

$u_6 = 1$ ***A1 N2***

[2 marks]

8c. Find the sum to infinity of this sequence. [2 marks]

Markscheme

evidence of correct substitution in
S_∞ ***A1***

e.g.
$\frac{32}{1-\frac{1}{2}}$,
$\frac{32}{\frac{1}{2}}$

$S_\infty = 64$ ***A1 N1***

[2 marks]

Consider the following sequence of figures.

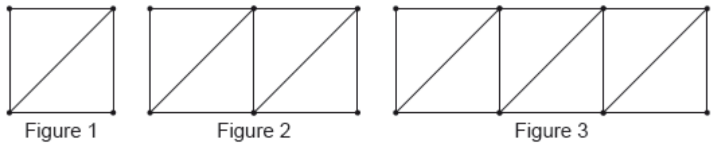


Figure 1 contains 5 line segments.

9a. Given that Figure n contains 801 line segments, show that $n = 200$. [3 marks]

Markscheme

recognizing that it is an arithmetic sequence ***(M1)***

eg $5,\ 5+4,\ 5+4+4,\ \ldots,\ d=4,\ u_n=u_1+(n-1)d,\ 4n+1$

correct equation ***A1***

eg $5+4(n-1)=801$

correct working (do not accept substituting $n=200$) ***A1***

eg $4n-4=796,\ n-1=\frac{796}{4}$

$n=200$ ***AG*** ***N0***

[3 marks]

9b. Find the total number of line segments in the first 200 figures. *[3 marks]*

Markscheme

recognition of sum ***(M1)***

eg $S_{200},\ u_1+u_2+\ldots+u_{200},\ 5+9+13+\ldots+801$

correct working for AP ***(A1)***

eg $\frac{200}{2}(5+801),\ \frac{200}{2}\ (2(5)+199(4))$

$80\,600$ ***A1*** ***N2***

[3 marks]

10a. Consider the arithmetic sequence *[3 marks]*
$2, 5, 8, 11, \ldots$.

Find
u_{101}.

Markscheme

$d=3$ ***(A1)***

evidence of substitution into
$u_n=a+(n-1)d$ ***(M1)***

e.g.
$u_{101}=2+100\times 3$

$u_{101}=302$ ***A1*** ***N3***

[3 marks]

10b. Consider the arithmetic sequence *[3 marks]*
$2, 5, 8, 11, \ldots$.

Find the value of *n* so that
$u_n=152$.

Markscheme

correct approach ***(M1)***

e.g.
$152 = 2 + (n - 1) \times 3$

correct simplification ***(A1)***

e.g.
$150 = (n - 1) \times 3$,
$50 = n - 1$,
$152 = -1 + 3n$

$n = 51$ ***A1*** ***N2***

[3 marks]

11. An arithmetic sequence has the first term $\ln a$ and a common difference $\ln 3$. *[6 marks]*

The 13th term in the sequence is $8 \ln 9$. Find the value of a.

Markscheme

Note: There are many approaches to this question, and the steps may be done in any order. There are 3 relationships they may need to apply at some stage, for the 3rd, 4th and 5th marks. These are

equating bases *eg* recognising 9 is 3^2

log rules: $\ln b + \ln c = \ln(bc),\ \ln b - \ln c = \ln\left(\frac{b}{c}\right)$,

exponent rule: $\ln b^n = n \ln b$.

The exception to the ***FT*** rule applies here, so that if they demonstrate correct application of the 3 relationships, they may be awarded the ***A*** marks, even if they have made a previous error. However all applications of a relationship need to be correct. Once an error has been made, do not award ***A1FT*** for their final answer, even if it follows from their working.

Please check working and award marks in line with the markscheme.

correct substitution into u_{13} formula ***(A1)***

eg $\ln a + (13-1)\ln 3$

set up equation for u_{13} in any form (seen anywhere) ***(M1)***

eg $\ln a + 12\ln 3 = 8\ln 9$

correct application of relationships ***(A1)(A1)(A1)***

$a = 81$ ***A1*** ***N3***

[6 marks]

Examples of application of relationships

Example 1

correct application of exponent rule for logs ***(A1)***

eg $\ln a + \ln 3^{12} = \ln 9^8$

correct application of addition rule for logs ***(A1)***

eg $\ln(a3^{12}) = \ln 9^8$

substituting for 9 or 3 in ln expression in equation ***(A1)***

eg $\ln(a3^{12}) = \ln 3^{16},\ \ln(a9^6) = \ln 9^8$

Example 2

recognising $9 = 3^2$ ***(A1)***

eg $\ln a + 12\ln 3 = 8\ln 3^2,\ \ln a + 12\ln 9^{\frac{1}{2}} = 8\ln 9$

one correct application of exponent rule for logs relating $\ln 9$ to $\ln 3$ ***(A1)***

eg $\ln a + 12\ln 3 = 16\ln 3,\ \ln a + 6\ln 9 = 8\ln 9$

another correct application of exponent rule for logs ***(A1)***

eg $\ln a = \ln 3^4,\ \ln a = \ln 9^2$

An arithmetic sequence has $u_1 = \log_c(p)$ and $u_2 = \log_c(pq)$, where $c > 1$ and $p,\ q > 0$.

12a. Show that $d = \log_c(q)$. *[2 marks]*

Markscheme

valid approach involving addition or subtraction ***M1***
eg $u_2 = \log_c p + d,\ u_1 - u_2$

correct application of log law ***A1***
eg $\log_c(pq) = \log_c p + \log_c q,\ \log_c\left(\frac{pq}{p}\right)$

$d = \log_c q$ ***AG N0***

[2 marks]

12b. Let $p = c^2$ and $q = c^3$. Find the value of $\sum_{n=1}^{20} u_n$. *[6 marks]*

Markscheme

METHOD 1 (finding u_1 and *d*)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

attempt to find u_1 or *d* using $\log_c c^k = k$ ***(M1)***
eg $\log_c c$, $3\log_c c$, correct value of u_1 or *d*

u_1 = 2, *d* = 3 (seen anywhere) ***(A1)(A1)***

correct working ***(A1)***
eg $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$, $S_{20} = \frac{20}{2}(2 + 59)$, $10(61)$

$\sum_{n=1}^{20} u_n$ = 610 ***A1 N2***

METHOD 2 (expressing *S* in terms of *c*)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

correct expression for *S* in terms of *c* ***(A1)***
eg $10\left(2\log_c c^2 + 19\log_c c^3\right)$

$\log_c c^2 = 2,\ \log_c c^3 = 3$ (seen anywhere) ***(A1)(A1)***

correct working ***(A1)***

eg $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$, $S_{20} = \frac{20}{2}(2 + 59)$, $10(61)$

$\sum_{n=1}^{20} u_n$ = 610 ***A1 N2***

METHOD 3 (expressing *S* in terms of *c*)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

correct expression for *S* in terms of *c* ***(A1)***
eg $10\left(2\log_c c^2 + 19\log_c c^3\right)$

correct application of log law ***(A1)***
eg $2\log_c c^2 = \log_c c^4$, $19\log_c c^3 = \log_c c^{57}$, $10\left(\log_c \left(c^2\right)^2 + \log_c \left(c^3\right)^{19}\right)$, $10\left(\log_c c^4 + \log_c c^{57}\right)$, $10\left(\log_c c^{61}\right)$

correct application of definition of log ***(A1)***
eg $\log_c c^{61} = 61$, $\log_c c^4 = 4$, $\log_c c^{57} = 57$

correct working ***(A1)***
eg $S_{20} = \frac{20}{2}(4 + 57)$, $10(61)$

$\sum_{n=1}^{20} u_n$ = 610 ***A1 N2***

[6 marks]

The first three terms of a geometric sequence are $\ln x^{16}$, $\ln x^8$, $\ln x^4$, for $x > 0$.

13a. Find the common ratio. *[3 marks]*

Markscheme

correct use $\log x^n = n \log x$ ***A1***

eg $16 \ln x$

valid approach to find r ***(M1)***

eg $\frac{u_{n+1}}{u_n}$, $\frac{\ln x^8}{\ln x^{16}}$, $\frac{4 \ln x}{8 \ln x}$, $\ln x^4 = \ln x^{16} \times r^2$

$r = \frac{1}{2}$ ***A1*** ***N2***

[3 marks]

13b. Solve $\sum_{k=1}^{\infty} 2^{5-k} \ln x = 64$. *[5 marks]*

Markscheme

recognizing a sum (finite or infinite) ***(M1)***

eg $2^4 \ln x + 2^3 \ln x$, $\frac{a}{1-r}$, S_∞, $16 \ln x + \ldots$

valid approach (seen anywhere) ***(M1)***

eg recognizing GP is the same as part (a), using **their** r value from part (a), $r = \frac{1}{2}$

correct substitution into infinite sum (only if $|r|$ is a constant and less than 1) ***A1***

eg $\frac{2^4 \ln x}{1-\frac{1}{2}}$, $\frac{\ln x^{16}}{\frac{1}{2}}$, $32 \ln x$

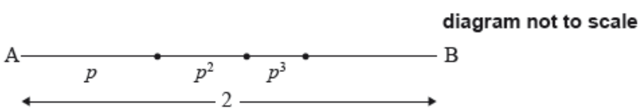
correct working ***(A1)***

eg $\ln x = 2$

$x = \mathrm{e}^2$ ***A1*** ***N3***

[5 marks]

14a. The following diagram shows [AB], with length 2 cm. The line is divided into an infinite number of line segments. The diagram shows the first three segments. *[5 marks]*



The length of the line segments are p cm, p^2 cm, p^3 cm, $\ldots$, where $0 < p < 1$.

Show that $p = \frac{2}{3}$.

Markscheme

infinite sum of segments is 2 (seen anywhere) ***(A1)***

eg $p + p^2 + p^3 + \ldots = 2,\ \frac{u_1}{1-r} = 2$

recognizing GP ***(M1)***

eg ratio is $p,\ \frac{u_1}{1-r},\ u_n = u_1 \times r^{n-1},\ \frac{u_1(r^n-1)}{r-1}$

correct substitution into S_∞ formula (may be seen in equation) ***A1***

eg $\frac{p}{1-p}$

correct equation ***(A1)***

eg $\frac{p}{1-p} = 2,\ p = 2 - 2p$

correct working leading to answer ***A1***

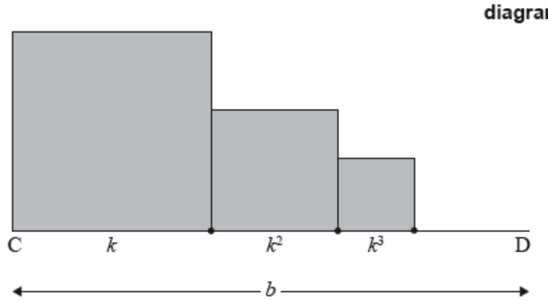
eg $3p = 2,\ 2 - 3p = 0$

$p = \frac{2}{3}\,(\mathrm{cm})$ ***AG*** ***N0***

[5 marks]

14b. The following diagram shows [CD], with length $b\ \mathrm{cm}$, where $b > 1$. Squares with side lengths $k\ \mathrm{cm},\ k^2\ \mathrm{cm},\ k^3\ \mathrm{cm},\ \ldots$, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares. *[9 marks]*

diagram not to scale



The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b.

Markscheme

recognizing infinite geometric series with squares ***(M1)***

eg $k^2 + k^4 + k^6 + \ldots,\ \frac{k^2}{1-k^2}$

correct substitution into $S_\infty = \frac{9}{16}$ (must substitute into formula) ***(A2)***

eg $\frac{k^2}{1-k^2} = \frac{9}{16}$

correct working ***(A1)***

eg $16k^2 = 9 - 9k^2,\ 25k^2 = 9,\ k^2 = \frac{9}{25}$

$k = \frac{3}{5}$ (seen anywhere) ***A1***

valid approach with segments and CD (may be seen earlier) ***(M1)***

eg $r = k,\ S_\infty = b$

correct expression for b in terms of k (may be seen earlier) ***(A1)***

eg $b = \frac{k}{1-k},\ b = \sum_{n=1}^{\infty} k^n,\ b = k + k^2 + k^3 + \ldots$

substituting **their** value of k into **their** formula for b ***(M1)***

eg $\frac{\frac{3}{5}}{1-\frac{3}{5}},\ \frac{\left(\frac{3}{5}\right)}{\left(\frac{2}{5}\right)}$

$b = \frac{3}{2}$ ***A1 N3***

[9 marks]

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

15a. Find an expression for r in terms of θ. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $\frac{u_2}{u_1},\ \frac{u_1}{u_2}$

$r = \frac{12\sin^2\theta}{18} \left(= \frac{2\sin^2\theta}{3}\right)$ ***A1 N2***

[2 marks]

15b. Find the possible values of r. *[3 marks]*

Markscheme

recognizing that $\sin\theta$ is bounded ***(M1)***

eg $0 \leq \sin^2\theta \leq 1,\ -1 \leq \sin\theta \leq 1,\ -1 < \sin\theta < 1$

$0 < r \leq \frac{2}{3}$ ***A2 N3***

Note: If working shown, award ***M1A1*** for correct values with incorrect inequality sign(s).
If no working shown, award ***N1*** for correct values with incorrect inequality sign(s).

[3 marks]

15c. Show that the sum of the infinite sequence is $\frac{54}{2+\cos(2\theta)}$. *[4 marks]*

Markscheme

correct substitution into formula for infinite sum ***A1***

eg $\frac{18}{1-\frac{2\sin^2\theta}{3}}$

evidence of choosing an appropriate rule for cos 2 θ (seen anywhere) ***(M1)***

eg $\cos 2\theta = 1 - 2\sin^2\theta$

correct substitution of identity/working (seen anywhere) ***(A1)***

eg $\frac{18}{1-\frac{2}{3}\left(\frac{1-\cos 2\theta}{2}\right)}$, $\frac{54}{3-2\left(\frac{1-\cos 2\theta}{2}\right)}$, $\frac{18}{\frac{3-2\sin^2\theta}{3}}$

correct working that clearly leads to the given answer ***A1***

eg $\frac{18\times 3}{2+(1-2\sin^2\theta)}$, $\frac{54}{3-(1-\cos 2\theta)}$

$\frac{54}{2+\cos(2\theta)}$ ***AG N0***

[4 marks]

15d. Find the values of θ which give the greatest value of the sum. *[6 marks]*

Markscheme

METHOD 1 (using differentiation)

recognizing $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta} = 0$ (seen anywhere) ***(M1)***

finding any correct expression for $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta}$ ***(A1)***

eg $\frac{0-54\times(-2\sin 2\theta)}{(2+\cos 2\theta)^2}$, $-54(2+\cos 2\theta)^{-2}\,(-2\sin 2\theta)$

correct working ***(A1)***

eg $\sin 2\theta = 0$

any correct value for $\sin^{-1}(0)$ (seen anywhere) ***(A1)***

eg $0, \pi, \ldots$, sketch of sine curve with *x*-intercept(s) marked both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta = \pi, 3\pi$ (accept values in degrees)

both correct answers $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ ***A1 N4***

Note: Award ***A0*** if either or both correct answers are given in degrees.
Award ***A0*** if additional values are given.

METHOD 2 (using denominator)

recognizing when S_∞ is greatest ***(M1)***

eg $2 + \cos 2\theta$ is a minimum, $1-r$ is smallest
correct working ***(A1)***

eg minimum value of $2 + \cos 2\theta$ is 1, minimum $r = \frac{2}{3}$

correct working ***(A1)***

eg $\cos 2\theta = -1$, $\frac{2}{3}\sin^2\theta = \frac{2}{3}$, $\sin^2\theta = 1$

EITHER (using $\cos 2\theta$)

any correct value for $\cos^{-1}(-1)$ (seen anywhere) ***(A1)***

eg $\pi, 3\pi, \ldots$ (accept values in degrees), sketch of cosine curve with *x*-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta = \pi, 3\pi$ (accept values in degrees)

OR (using $\sin\theta$)

$\sin\theta = \pm 1$ (A1)

$\sin^{-1}(1) = \frac{\pi}{2}$ (accept values in degrees) (seen anywhere) ***A1***

THEN

both correct answers $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ ***A1 N4***

Note: Award ***A0*** if either or both correct answers are given in degrees.
Award ***A0*** if additional values are given.

[6 marks]

The first two terms of an infinite geometric sequence, in order, are

$$2\log_2 x,\ \log_2 x, \text{ where } x > 0.$$

16a. Find r. *[2 marks]*

Markscheme

evidence of dividing terms (in any order) ***(M1)***

eg $\frac{\mu_2}{\mu_1}$, $\frac{2\log_2 x}{\log_2 x}$

$r = \frac{1}{2}$ ***A1 N2***

[2 marks]

16b. Show that the sum of the infinite sequence is $4\log_2 x$. *[2 marks]*

Markscheme

correct substitution ***(A1)***

eg $\frac{2\log_2 x}{1-\frac{1}{2}}$

correct working ***A1***

eg $\frac{2\log_2 x}{\frac{1}{2}}$

$S_\infty = 4\log_2 x$ ***AG N0***

[2 marks]

The first three terms of an arithmetic sequence, in order, are

$$\log_2 x,\ \log_2\left(\frac{x}{2}\right),\ \log_2\left(\frac{x}{4}\right), \text{ where } x > 0.$$

16c. Find d, giving your answer as an integer. *[4 marks]*

Markscheme

evidence of subtracting two terms (in any order) ***(M1)***

eg $u_3 - u_2$, $\log_2 x - \log_2 \frac{x}{2}$

correct application of the properties of logs ***(A1)***

eg $\log_2\left(\frac{\frac{x}{2}}{x}\right)$, $\log_2\left(\frac{x}{2} \times \frac{1}{x}\right)$, $(\log_2 x - \log_2 2) - \log_2 x$

correct working ***(A1)***

eg $\log_2 \frac{1}{2}$, $-\log_2 2$

$d = -1$ ***A1 N3***

[4 marks]

Let S_{12} be the sum of the first 12 terms of the arithmetic sequence.

16d. Show that $S_{12} = 12\log_2 x - 66$. *[2 marks]*

Markscheme

correct substitution into the formula for the sum of an arithmetic sequence ***(A1)***

eg $\frac{12}{2}(2\log_2 x + (12-1)(-1))$

correct working ***A1***

eg $6(2\log_2 x - 11),\ \frac{12}{2}(2\log_2 x - 11)$

$12\log_2 x - 66$ ***AG N0***

[2 marks]

16e. Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x, giving your answer in the form 2^p, where $p \in \mathbb{Q}$. *[3 marks]*

Markscheme

correct equation ***(A1)***

eg $12\log_2 x - 66 = 2\log_2 x$

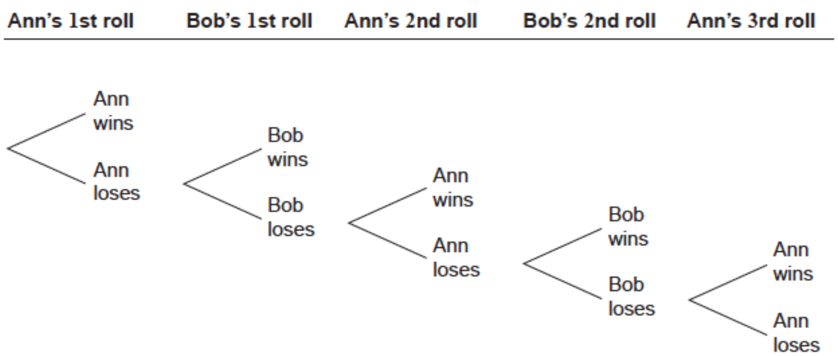
correct working ***(A1)***

eg $10\log_2 x = 66,\ \log_2 x = 6.6,\ 2^{66} = x^{10},\ \log_2\left(\frac{x^{12}}{x^2}\right) = 66$

$x = 2^{6.6}$ (accept $p = \frac{66}{10}$) ***A1 N2***

[3 marks]

Ann and Bob play a game where they each have an eight-sided die. Ann's die has three green faces and five red faces; Bob's die has four green faces and four red faces. They take turns rolling their own die and note what colour faces up. The first player to roll green wins. Ann rolls first. Part of a tree diagram of the game is shown below.



17a. Find the probability that Ann wins on her first roll. *[2 marks]*

Markscheme

recognizing Ann rolls green ***(M1)***

eg $\mathrm{P(G)}$

$\frac{3}{8}$ ***A1 N2***

[2 marks]

17b. Find the probability that Ann wins the game. *[7 marks]*

Markscheme

recognize the probability is an infinite sum ***(M1)***

eg Ann wins on her 1^{st} roll or 2^{nd} roll or 3^{rd} roll…, S_∞

recognizing GP ***(M1)***

$u_1 = \frac{3}{8}$ (seen anywhere) ***A1***

$r = \frac{20}{64}$ (seen anywhere) ***A1***

correct substitution into infinite sum of GP ***A1***

eg $\frac{\frac{3}{8}}{1-\frac{5}{16}},\ \frac{3}{8}\left(\frac{1}{1-\left(\frac{5}{8}\times\frac{4}{8}\right)}\right),\ \frac{1}{1-\frac{5}{16}}$

correct working ***(A1)***

eg $\frac{\frac{3}{8}}{\frac{11}{16}},\ \frac{3}{8}\times\frac{16}{11}$

$\mathrm{P}\,(\text{Ann wins}) = \frac{48}{88}\quad\left(=\frac{6}{11}\right)$ ***A1 N1***

[7 marks]

Total [15 marks]

The sums of the terms of a sequence follow the pattern

$S_1 = 1 + k,\ S_2 = 5 + 3k,\ S_3 = 12 + 7k,\ S_4 = 22 + 15k,\ \ldots,$ where $k \in \mathbb{Z}$.

18a. Given that $u_1 = 1 + k$, find u_2, u_3 and u_4. *[4 marks]*

Markscheme

valid method ***(M1)***

eg
$u_2 = S_2 - S_1,\ 1 + k + u_2 = 5 + 3k$

$u_2 = 4 + 2k,\ u_3 = 7 + 4k,\ u_4 = 10 + 8k$ ***A1A1A1 N4***

[4 marks]

18b. Find a general expression for u_n. *[4 marks]*

Markscheme

correct AP **or** GP ***(A1)***

eg finding common difference is
3, common ratio is
2

valid approach using arithmetic **and** geometric formulas ***(M1)***

eg
$1+3(n-1)$ **and**
$r^{n-1}k$

$u_n=3n-2+2^{n-1}k$ ***A1A1*** ***N4***

Note: Award ***A1*** for
$3n-2$, ***A1*** for
$2^{n-1}k$.

[4 marks]

The sides of a square are 16 cm in length. The midpoints of the sides of this square are joined to form a new square and four triangles (diagram 1). The process is repeated twice, as shown in diagrams 2 and 3.

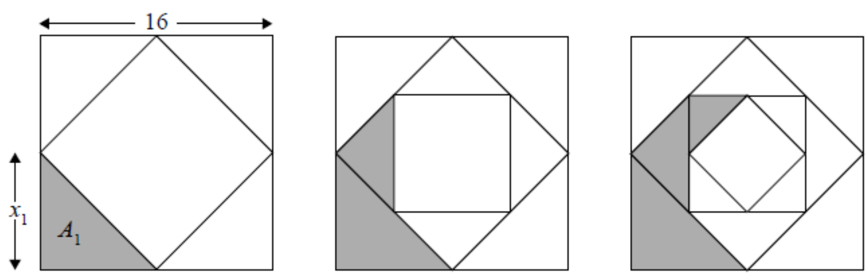


diagram 1 diagram 2 diagram 3

Let
x_n denote the length of one of the equal sides of each new triangle.

Let
A_n denote the area of each new triangle.

19a. The following table gives the values of
x_n and
A_n, for
$1 \leqslant n \leqslant 3$. **Copy** and complete the table. *(Do **not** write on this page.)* *[4 marks]*

n	1	2	3
x_n	8		4
A_n	32	16	

Markscheme

valid method for finding side length ***(M1)***

eg
$8^2+8^2=c^2,\ 45-45-90 \text{ side ratios},\ 8\sqrt{2},\ \frac{1}{2}s^2=16,\ x^2+x^2=8^2$

correct working for area ***(A1)***

eg
$\frac{1}{2}\times 4\times 4$

n 1 2 3
x_n 8
$\sqrt{32}$ 4
A_n 32 16 8
A1A1 ***N2N2***

[4 marks]

19b. The process described above is repeated. Find A_6. *[4 marks]*

Markscheme

METHOD 1

recognize geometric progression for A_n ***(R1)***

eg
$u_n=u_1r^{n-1}$

$r=\frac{1}{2}$ ***(A1)***

correct working ***(A1)***

eg
$32\left(\frac{1}{2}\right)^5;\ 4,\ 2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \ldots$

$A_6=1$ ***A1*** ***N3***

METHOD 2

attempt to find x_6 ***(M1)***

eg
$8\left(\frac{1}{\sqrt{2}}\right)^5,\ 2\sqrt{2},\ 2,\ \sqrt{2},\ 1,\ \ldots$

$x_6=\sqrt{2}$ ***(A1)***

correct working ***(A1)***

eg
$\frac{1}{2}\left(\sqrt{2}\right)^2$

$A_6=1$ ***A1*** ***N3***

[4 marks]

19c. Consider an initial square of side length k cm. The process described above is repeated indefinitely. The total area of the shaded regions is k cm^2. Find the value of k. *[7 marks]*

Markscheme

METHOD 1

recognize infinite geometric series ***(R1)***

eg
$S_n = \frac{a}{1-r},\ |r| < 1$

area of first triangle in terms of k ***(A1)***

eg
$\frac{1}{2}\left(\frac{k}{2}\right)^2$

attempt to substitute into sum of infinite geometric series (must have k) ***(M1)***

eg
$\frac{\frac{1}{2}\left(\frac{k}{2}\right)^2}{1-\frac{1}{2}},\ \frac{k}{1-\frac{1}{2}}$

correct equation ***A1***

eg
$\frac{\frac{1}{2}\left(\frac{k}{2}\right)^2}{1-\frac{1}{2}} = k,\ k = \frac{\frac{k^2}{8}}{\frac{1}{2}}$

correct working ***(A1)***

eg
$k^2 = 4k$

valid attempt to solve **their** quadratic ***(M1)***

eg
$k(k-4),\ k = 4 \text{ or } k = 0$

$k = 4$ ***A1 N2***

METHOD 2

recognizing that there are four sets of infinitely shaded regions with equal area ***R1***

area of original square is k^2 ***(A1)***

so total shaded area is $\frac{k^2}{4}$ ***(A1)***

correct equation
$\frac{k^2}{4} = k$ ***A1***

$k^2 = 4k$ ***(A1)***

valid attempt to solve their quadratic ***(M1)***

eg
$k(k-4),\ k = 4 \text{ or } k = 0$

$k = 4$ ***A1 N2***

[7 marks]

The first three terms of a infinite geometric sequence are $m-1,\ 6,\ m+4$, where $m \in \mathbb{Z}$.

20a. Write down an expression for the common ratio, r. *[2 marks]*

Markscheme

correct expression for r ***A1*** ***N1***

eg $r = \frac{6}{m-1},\ \frac{m+4}{6}$

[2 marks]

20b. Hence, show that m satisfies the equation $m^2 + 3m - 40 = 0.$ *[2 marks]*

Markscheme

correct equation ***A1***

eg $\frac{6}{m-1} = \frac{m+4}{6},\ \frac{6}{m+4} = \frac{m-1}{6}$

correct working ***(A1)***

eg $(m+4)(m-1) = 36$

correct working ***A1***

eg $m^2 - m + 4m - 4 = 36,\ m^2 + 3m - 4 = 36$

$m^2 + 3m - 40 = 0$ ***AG*** ***N0***

[2 marks]

20c. Find the two possible values of $m.$ *[3 marks]*

Markscheme

valid attempt to solve ***(M1)***

eg $(m+8)(m-5) = 0,\ m = \frac{-3 \pm \sqrt{9+4\times40}}{2}$

$m = -8,\ m = 5$ ***A1A1*** ***N3***

[3 marks]

20d. Find the possible values of $r.$ *[3 marks]*

Markscheme

attempt to substitute **any** value of m to find r ***(M1)***

eg $\frac{6}{-8-1},\ \frac{5+4}{6}$

$r = \frac{3}{2},\ r = -\frac{2}{3}$ ***A1A1*** ***N3***

[3 marks]

20e. The sequence has a finite sum. *[3 marks]*

State which value of r leads to this sum **and** justify your answer.

Markscheme

$r = -\frac{2}{3}$ (may be seen in justification) ***A1***

valid reason ***R1*** ***N0***

eg
$|r| < 1,\ -1 < \frac{-2}{3} < 1$

Notes: Award ***R1*** for $|r| < 1$ only if ***A1*** awarded.

[2 marks]

20f. The sequence has a finite sum. *[3 marks]*

Calculate the sum of the sequence.

Markscheme

finding the first term of the sequence which has $|r| < 1$ ***(A1)***

eg
$-8 - 1,\ 6 \div \frac{-2}{3}$

$u_1 = -9$ (may be seen in formula) ***(A1)***

correct substitution of u_1 and their r into $\frac{u_1}{1-r}$, as long as $|r| < 1$ ***A1***

eg
$S_\infty = \frac{-9}{1-\left(-\frac{2}{3}\right)},\ \frac{-9}{\frac{5}{3}}$

$S_\infty = -\frac{27}{5}\ (= -5.4)$ ***A1*** ***N3***

[4 marks]