5-27Integration-solids *[41 marks]*

1. Let $f'(x) = 6x^2 - 5$. Given that $f(2) = -3$, find $f(x)$. *[6 marks]*

Markscheme

evidence of antidifferentiation ***(M1)***

eg $f = \int f'$

correct integration (accept absence of C) ***(A1)(A1)***

$f(x) = \frac{6x^3}{3} - 5x + C,\ 2x^3 - 5x$

attempt to substitute $(2,\ -3)$ into **their** integrated expression (must have C) ***M1***

eg $2(2)^3 - 5(2) + C = -3,\ 16 - 10 + C = -3$

Note: Award ***M0*** if substituted into original or differentiated function.

correct working to find C ***(A1)***

eg $16 - 10 + C = -3,\ 6 + C = -3,\ C = -9$

$f(x) = 2x^3 - 5x - 9$ ***A1 N4***

[6 marks]

Let
$f(x) = x^2$.

2a. Find $\int_1^2 (f(x))^2 \mathrm{d}x$. *[4 marks]*

Markscheme

substituting for $(f(x))^2$ (may be seen in integral) ***A1***

eg $(x^2)^2,\ x^4$

correct integration, $\int x^4 \mathrm{d}x = \frac{1}{5}x^5$ ***(A1)***

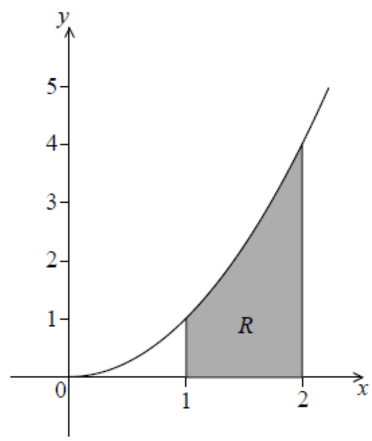
substituting limits into **their integrated** function and subtracting (in any order) ***(M1)***

eg $\frac{2^5}{5} - \frac{1}{5},\ \frac{1}{5}(1 - 4)$

$\int_1^2 (f(x))^2 \mathrm{d}x = \frac{31}{5} (= 6.2)$ ***A1 N2***

[4 marks]

2b. The following diagram shows part of the graph of f. *[2 marks]*



The shaded region R is enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$.

Find the volume of the solid formed when R is revolved 360° about the x-axis.

Markscheme

attempt to substitute limits or function into formula involving f^2 ***(M1)***

eg $\int_1^2 (f(x))^2 \mathrm{d}x, \pi \int x^4 \mathrm{d}x$

$\frac{31}{5}\pi \ (= 6.2\pi)$ ***A1 N2***

[2 marks]

3a. Find $\int \frac{1}{2x+3} \mathrm{d}x$. *[2 marks]*

Markscheme

$\int \frac{1}{2x+3} \mathrm{d}x = \frac{1}{2}\ln(2x+3) + C$ (accept $\frac{1}{2}\ln|(2x+3)| + C$) ***A1A1 N2***

[2 marks]

3b. Given that $\int_0^3 \frac{1}{2x+3} \mathrm{d}x = \ln\sqrt{P}$, find the value of *P*. *[4 marks]*

Markscheme

$\int_0^3 \frac{1}{2x+3} dx = \left[\frac{1}{2}\ln(2x+3)\right]_0^3$

evidence of substitution of limits ***(M1)***

e.g.$\frac{1}{2}\ln 9 - \frac{1}{2}\ln 3$

evidence of correctly using $\ln a - \ln b = \ln \frac{a}{b}$ (seen anywhere) ***(A1)***

e.g. $\frac{1}{2}\ln 3$

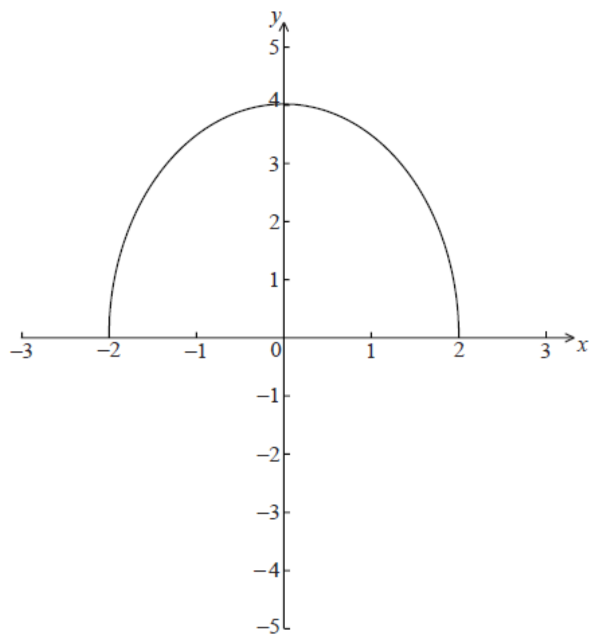
evidence of correctly using $a \ln b = \ln b^a$ (seen anywhere) ***(A1)***

e.g. $\ln \sqrt{\frac{9}{3}}$

$P = 3$ (accept $\ln \sqrt{3}$) ***A1*** ***N2***

[4 marks]

The graph of
$f(x) = \sqrt{16 - 4x^2}$, for
$-2 \le x \le 2$, is shown below.



4. The region enclosed by the curve of *f* and the *x*-axis is rotated 360° about the *x*-axis. *[6 marks]*
Find the volume of the solid formed.

Markscheme

attempt to set up integral expression ***M1***

e.g. $\pi\int\sqrt{16-4x^2}^2\mathrm{d}x$, $2\pi\int_0^2(16-4x^2)$, $\int\sqrt{16-4x^2}^2\mathrm{d}x$

$\int 16\mathrm{d}x = 16x$, $\int 4x^2\mathrm{d}x = \frac{4x^3}{3}$ (seen anywhere) ***A1A1***

evidence of substituting limits into the integrand ***(M1)***

e.g. $\left(32-\frac{32}{3}\right)-\left(-32+\frac{32}{3}\right)$, $64-\frac{64}{3}$

volume $= \frac{128\pi}{3}$ ***A2*** ***N3***

[6 marks]

Let
$f(x)=\sqrt{x}$. Line *L* is the normal to the graph of *f* at the point (4, 2) .

5a. Show that the equation of L is $y=-4x+18$. *[4 marks]*

Markscheme

finding derivative ***(A1)***

e.g. $f'(x)=\frac{1}{2}x^{\frac{1}{2}}$, $\frac{1}{2\sqrt{x}}$

correct value of derivative or its negative reciprocal (seen anywhere) ***A1***

e.g. $\frac{1}{2\sqrt{4}}$, $\frac{1}{4}$

gradient of normal = $\frac{1}{\text{gradient of tangent}}$ (seen anywhere) ***A1***

e.g. $-\frac{1}{f'(4)}=-4$, $-2\sqrt{x}$

substituting into equation of line (for normal) ***M1***

e.g. $y-2=-4(x-4)$

$y=-4x+18$ ***AG*** ***N0***

[4 marks]

5b. Point A is the x-intercept of L . Find the x-coordinate of A. *[2 marks]*

Markscheme

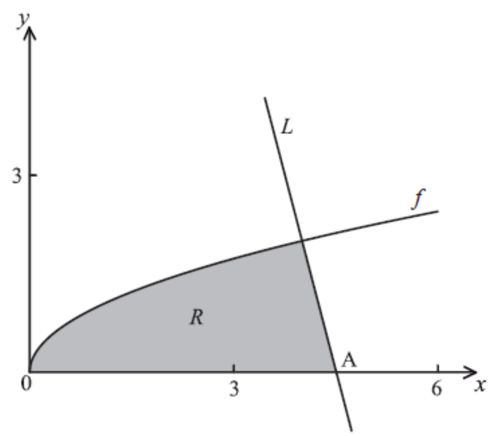
recognition that $y = 0$ at A ***(M1)***

e.g. $-4x + 18 = 0$

$x = \frac{18}{4}\left(= \frac{9}{2}\right)$ ***A1 N2***

[2 marks]

In the diagram below, the shaded region *R* is bounded by the *x*-axis, the graph of *f* and the line *L* .



5c. Find an expression for the area of *R* . *[3 marks]*

Markscheme

splitting into two appropriate parts (areas and/or integrals) ***(M1)***

correct expression for area of *R* ***A2 N3***

e.g. area of *R* = $\int_0^4 \sqrt{x}\mathrm{d}x + \int_4^{4.5}(-4x+18)\mathrm{d}x$, $\int_0^4 \sqrt{x}\mathrm{d}x + \frac{1}{2} \times 0.5 \times 2$ (triangle)

Note: Award ***A1*** if d*x* is missing.

[3 marks]

5d. The region *R* is rotated 360° about the *x*-axis. Find the volume of the solid formed, giving your answer in terms of π . *[8 marks]*

Markscheme

correct expression for the volume from $x = 0$ to $x = 4$ ***(A1)***

e.g. $V = \int_0^4 \pi \left[f(x)^2\right] \mathrm{d}x$, $\int_0^4 \pi \sqrt{x}^2 \mathrm{d}x$, $\int_0^4 \pi x \mathrm{d}x$

$V = \left[\frac{1}{2}\pi x^2\right]_0^4$ ***A1***

$V = \pi\left(\frac{1}{2} \times 16 - \frac{1}{2} \times 0\right)$ ***(A1)***

$V = 8\pi$ ***A1***

finding the volume from $x = 4$ to $x = 4.5$

EITHER

recognizing a cone ***(M1)***

e.g. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(2)^2 \times \frac{1}{2}$ ***(A1)***

$= \frac{2\pi}{3}$ ***A1***

total volume is $8\pi + \frac{2}{3}\pi \left(= \frac{26}{3}\pi\right)$ ***A1*** ***N4***

OR

$V = \pi \int_4^{4.5} (-4x + 18)^2 \mathrm{d}x$ ***(M1)***

$= \int_4^{4.5} \pi(16x^2 - 144x + 324)\mathrm{d}x$

$= \pi\left[\frac{16}{3}x^3 - 72x^2 + 324x\right]_4^{4.5}$ ***A1***

$= \frac{2\pi}{3}$ ***A1***

total volume is $8\pi + \frac{2}{3}\pi \left(= \frac{26}{3}\pi\right)$ ***A1*** ***N4***

[8 marks]