

5.11 Pre-Exam: Graphing and interpreting quadratic functions

Answer this section's problems on lined paper using IB standards. (and no ragged edges)

1. $f(x) = x^2 - 5x$
2. $f(x) = x^2 + 5x + 6$
3. $f(x) = 2x^2 - 15x + 7$
4. $f(x) = \frac{1}{2}x^2 + 4x - 10$

Using the quadratic formula

Find an exact solution by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5. $x^2 + 3x - 5 = 0$

6. $3x^2 + 7x = 2$

Use the discriminant in the following two problems. $D = b^2 - 4ac$

7. Show that the function $f(x) = -2x^2 - 6x + 5$ has two distinct zeros.
8. Solve for k such that the function $g(x) = x^2 - kx + 25$ has a single (double) root.

Completing the square

Rewrite the function in vertex form: $f(x) = a(x - h)^2 + k$. Include the step showing the $(-\frac{b}{2a})^2$ term. State the vertex as an ordered pair and the equation for the axis of symmetry.

9. $f(x) = x^2 + 6x + 4$
10. $f(x) = x^2 - 12x + 20$

The inverse of a function

Derive the inverse of each function. Simplify the expression.

11. $f(x) = 2x + 1$
12. $f(x) = \sqrt{x + 2}$

Function substitution

13. Given $f(x) = 3x^2 - x + 17$. Simplify $f(-3x)$.
14. Given $f(x) = 5 - (x^2 + 4x)$. Simplify $f(\frac{1}{3}x + 1)$.

Function composition

In each exercise, perform the composition $f \circ g$ and simplify.

15. Given $f(x) = x^2 - x$ and $g(x) = 3x - 1$
16. Given $f(x) = 1 - \frac{2x}{x^2 - x}$ and $g(x) = 2x$

Function transformations

Describe how the functions below have transformed from the parent function $f(x) = |x|$.

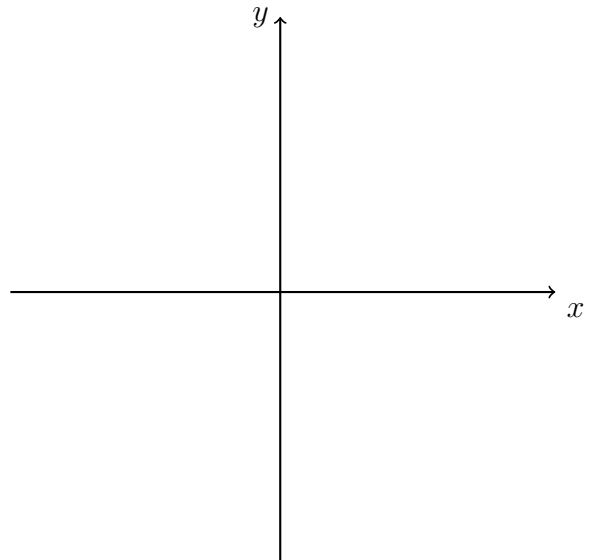
17. $g(x) = |x + 2|$
18. $h(x) = -|x| + 2$

Sketching a quadratic function

Answer in the space provided. (you may also use additional lined paper)

19. Given $f(x) = -(x - 3)^2 + 16$

- (a) Write down the vertex of the function as an ordered pair.
- (b) Write down the equation of the axis of symmetry.
- (c) Expand the function from vertex form to standard form, $ax^2 + bx + c$ where $a, b, c \in \mathbb{R}$.
- (d) Write down the value of $f(0)$. Explain what this represents on the graph.
- (e) Hence factor the function. Write down the roots.
- (f) Sketch the function, labeling the intercepts with values and the vertex as an ordered pair. Show the axis of symmetry as a dotted line and label it with its equation.

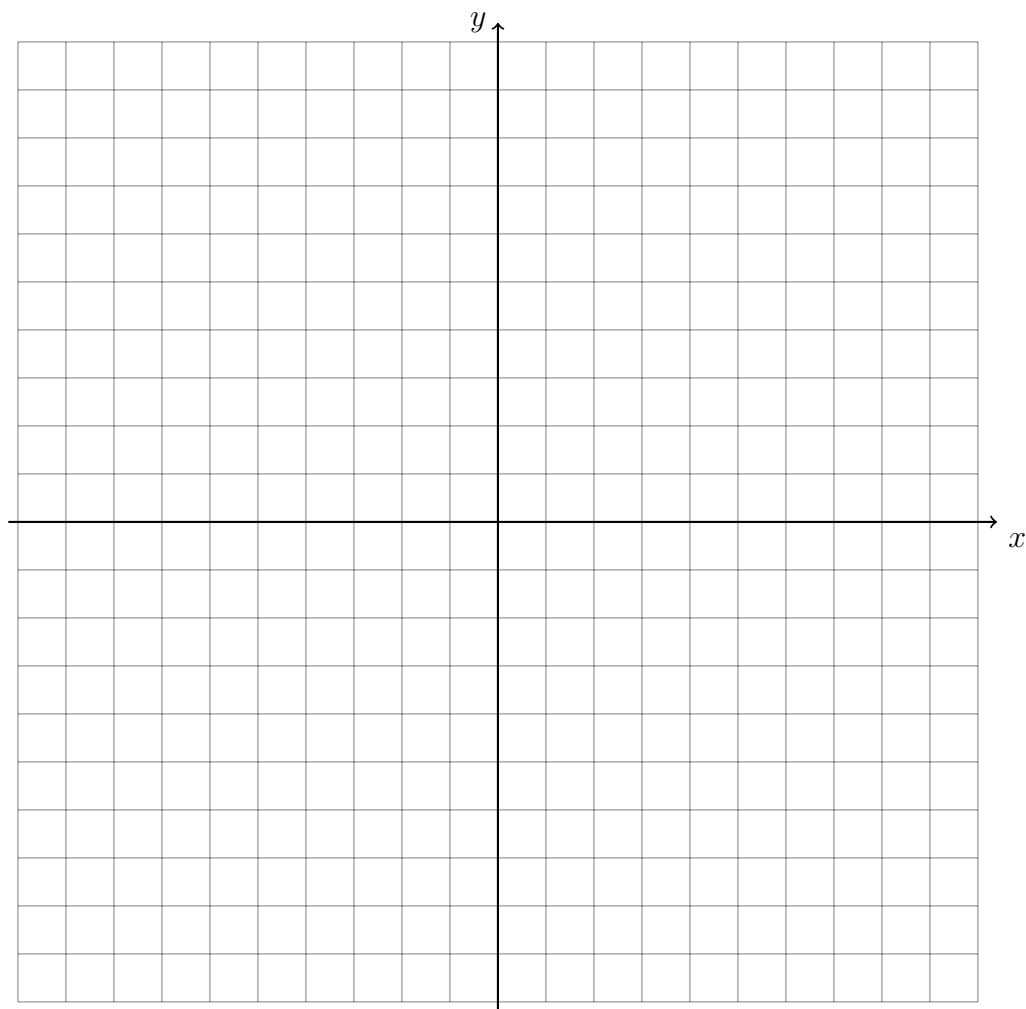


- (g) Write down the domain and range of the function.

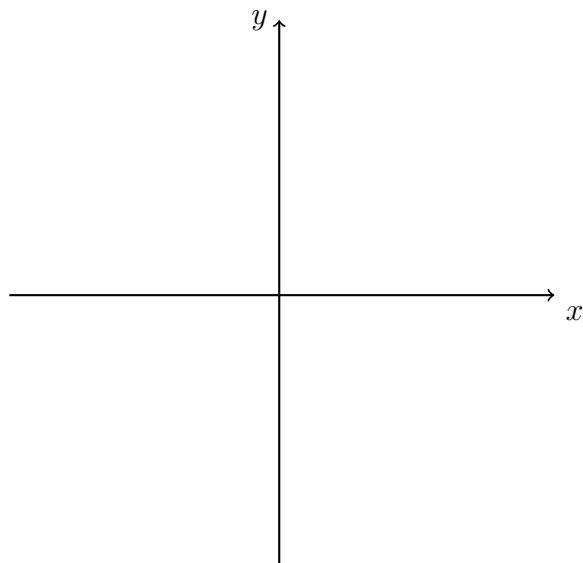
Graphing quadratics

Answer on lined paper. Graph the function on the grid shown below.

20. Given the function $f(x) = -x^2 - x + 6$.
- (a) Write down the y -intercept.
 - (b) State whether the parabola opens upward or downward. Explain how you know this from the function expressed in standard form.
 - (c) Express the function in factored form. Hence state the solutions to $f(x) = 0$.
 - (d) Show that the axis of symmetry of the parabola is $x = -\frac{1}{2}$.
 - (e) Hence state the vertex as an ordered pair.
 - (f) Graph the function. Mark the vertex as an ordered pair and label each intercept with its value. Plot the axis of symmetry as a dotted line and label it with its equation.
 - (g) Write down the domain and range of the function.



21. (a) Graph the parent function $f(x) = x^2$. Mark the point $P(3, f(3))$ on the graph
- (b) The function $g(x)$ is the function f after being translated to the right 5 and down 4. Graph g .
- (c) Mark the point on the function g , Q , that represents the point P after the translation.



Model situations with quadratic functions

22. The path of a diver is given by

$$f(x) = -5x^2 + 12x + 9$$

where y is the height (in meters) and x is time in seconds.

- (a) On the grid below, graph the function over the domain where $x \geq 0$ and the range where $f(x) \geq 0$. Use a horizontal scale of 5 squares equals one second and vertical scale of 1 square equals one meter. Label the intercepts and vertex.
- (b) What is the maximum height of the diver? Label the point on the graph with the word “max.”
- (c) What is the time when the diver enters the water? Label the point on the graph representing this with the word “splash.”

