0515HW-Challenge-mixed *[65 marks]*

1. The equation $x^2 - 3x + k^2 = 4$ has two distinct real roots. Find the possible values of k. *[6 marks]*

Markscheme

evidence of rearranged quadratic equation (may be seen in working) ***A1***

e.g.
$x^2 - 3x + k^2 - 4 = 0$,
$k^2 - 4$

evidence of discriminant (must be seen explicitly, not in quadratic formula) ***(M1)***

e.g.
$b^2 - 4ac$,
$\Delta = (-3)^2 - 4(1)(k^2 - 4)$

recognizing that discriminant is greater than zero (seen anywhere, including answer) ***R1***

e.g.
$b^2 - 4ac > 0$,
$9 + 16 - 4k^2 > 0$

correct working (accept equality) ***A1***

e.g.
$25 - 4k^2 > 0$,
$4k^2 < 25$,
$k^2 = \frac{25}{4}$

both correct values (even if inequality never seen) ***(A1)***

e.g.
$\pm\sqrt{\frac{25}{4}}$,
± 2.5

correct interval ***A1*** ***N3***

e.g.
$-\frac{5}{2} < k < \frac{5}{2}$,
$-2.5 < k < 2.5$

Note: Do not award the final mark for unfinished values, or for incorrect or reversed inequalities, including
$\leq$,
$k > -2.5$,
$k < 2.5$.

Special cases:

If working shown, and candidates attempt to rearrange the quadratic equation to equal zero, but find an incorrect value of *c*, award ***A1M1R1A0A0A0***.

If working shown, and candidates do not rearrange the quadratic equation to equal zero, but find
$c = k^2$ or
$c = \pm 4$, award ***A0M1R1A0A0A0***.

[6 marks]

2a. At a large school, students are required to learn at least one language, Spanish or French. It is known that 75% of the students learn Spanish, and 40% learn French. *[2 marks]*

Find the percentage of students who learn **both** Spanish and French.

Markscheme

valid approach *(M1)*

e.g. Venn diagram with intersection, union formula,

$\mathrm{P}(S \cap F) = 0.75 + 0.40 - 1$

15 (accept
15%) ***A1*** ***N2***

[2 marks]

2b. At a large school, students are required to learn at least one language, Spanishor French. It is known that 75% of the students learn Spanish, and 40% learn French. *[2 marks]*

Find the percentage of students who learn Spanish, but not French.

Markscheme

valid approach involving subtraction *(M1)*

e.g. Venn diagram,
$75 - 15$

60 (accept
60%) ***A1*** ***N2***

[2 marks]

2c. At a large school, students are required to learn at least one language, Spanishor French. It is known that 75% of the students learn Spanish, and 40% learn French. *[5 marks]*

At this school,
52% of the students are girls, and
85% of the girls learn Spanish.

A student is chosen at random. Let *G* be the event that the student is a girl, andlet *S* be the event that the student learns Spanish.

(i) Find
$\mathrm{P}(G \cap S)$.

(ii) Show that *G* and *S* are **not** independent.

Markscheme

(i) valid approach ***(M1)***

e.g. tree diagram, multiplying probabilities,
$\mathrm{P}(S|G) \times \mathrm{P}(G)$

correct calculation ***(A1)***

e.g.
0.52×0.85

$\mathrm{P}(G \cap S) = 0.442$ (exact) ***A1*** ***N3***

(ii) valid reasoning, with words, symbols or numbers (seen anywhere) ***R1***

e.g.
$\mathrm{P}(G) \times \mathrm{P}(S) \neq \mathrm{P}(G \cap S)$,
$\mathrm{P}(S|G) \neq \mathrm{P}(S)$, not equal,

one correct value ***A1***

e.g.
$\mathrm{P}(G) \times \mathrm{P}(S) = 0.39$,
$\mathrm{P}(S|G) = 0.85$,
$0.39 \neq 0.442$

G and *S* are not independent ***AG*** ***N0***

[5 marks]

2d. At a large school, students are required to learn at least one language, Spanishor French. It is known that 75% of the students learn Spanish, and 40% learn French. *[6 marks]*

At this school,
52% of the students are girls, and
85% of the girls learn Spanish.

A boy is chosen at random. Find the probability that he learns Spanish.

Markscheme

METHOD 1

48% are boys (seen anywhere) ***A1***

e.g.
$\mathrm{P}(B) = 0.48$

appropriate approach ***(M1)***

e.g.
$\mathrm{P}(\text{girl and Spanish}) + \mathrm{P}(\text{boy and Spanish}) = \mathrm{P}(\text{Spanish})$

correct approach to find P(boy and Spanish) ***(A1)***

e.g.
$\mathrm{P}(B \cap S)= \mathrm{P}(S) - \mathrm{P}(G \cap S)$,
$\mathrm{P}(B \cap S)= \mathrm{P}(S|B) \times \mathrm{P}(B)$, 0.308

correct substitution ***(A1)***

e.g.
$0.442 + 0.48x = 0.75$,
$0.48x = 0.308$

correct manipulation ***(A1)***

e.g.
$\mathrm{P}(S|B) = \frac{0.308}{0.48}$

$\mathrm{P}(\text{Spanish}|\text{boy}) = 0.641666\ldots$,
$0.641\bar{6}$

$\mathrm{P}(\text{Spanish}|\text{boy}) = 0.642$
$[0.641, 0.642]$ ***A1 N3***

[6 marks]

METHOD 2

48% are boys (seen anywhere) ***A1***

e.g. 0.48 used in tree diagram

appropriate approach ***(M1)***

e.g. tree diagram

correctly labelled branches on tree diagram ***(A1)***

e.g. first branches are boy/girl, second branches are Spanish/not Spanish

correct substitution ***(A1)***

e.g.
$0.442 + 0.48x = 0.75$

correct manipulation ***(A1)***

e.g.
$0.48x = 0.308$,
$\mathrm{P}(S|B) = \frac{0.308}{0.48}$

$\mathrm{P}(\text{Spanish}|\text{boy}) = 0.641666\ldots$,
$0.641\bar{6}$

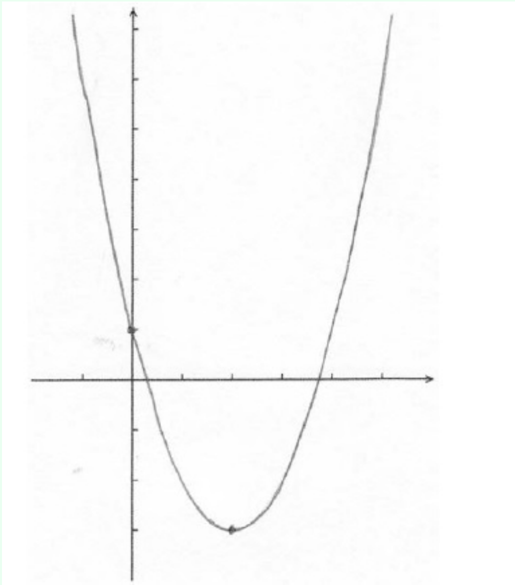
$\mathrm{P}(\text{Spanish}|\text{boy}) = 0.642$
$[0.641, 0.642]$

[6 marks]

Consider the function
$f(x) = x^2 - 4x + 1$.

3a. Sketch the graph of *f* , for
$-1 \le x \le 5$. *[4 marks]*

Markscheme



A1A1A1A1 ***N4***

Note: The shape **must** be an approximately correct upwards parabola.

Only if the shape is approximately correct, award the following:

A1 for vertex
$x \approx 2$, ***A1*** for *x*-intercepts between 0 and 1, and 3 and 4,***A1*** for correct *y*-intercept
$(0, 1)$, ***A1*** for correct domain
$[-1, 5]$.

Scale not required on the axes, but approximate positions need to be clear.

[4 marks]

3b. This function can also be written as
$f(x) = (x - p)^2 - 3$.

[1 mark]

Write down the value of *p* .

Markscheme

$p = 2$ ***A1*** ***N1***

[1 mark]

3c. The graph of *g* is obtained by reflecting the graph of *f* in the *x*-axis, followed by a translation of
$\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

[4 marks]

Show that
$g(x) = -x^2 + 4x + 5$.

Markscheme

correct vertical reflection, correct vertical translation ***(A1)(A1)***

e.g.
$-f(x)$,
$-((x-2)^2-3)$,
$-y$,
$-f(x)+6$,
$y+6$

transformations in correct order ***(A1)***

e.g.
$-(x^2-4x+1)+6$,
$-((x-2)^2-3)+6$

simplification which clearly leads to given answer ***A1***

e.g.
$-x^2+4x-1+6$,
$-(x^2-4x+4-3)+6$

$g(x)=-x^2+4x+5$ ***AG N0***

Note: If working shown, award ***A1A1A0A0*** if transformations correct, but done in reverse order, e.g. $-(x^2-4x+1+6)$.

[4 marks]

3d. The graph of *g* is obtained by reflecting the graph of *f* in the *x*-axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. *[3 marks]*

The graphs of *f* and *g* intersect at two points.

Write down the *x*-coordinates of these two points.

Markscheme

valid approach ***(M1)***

e.g. sketch,
$f=g$

$-0.449489\ldots$,
$4.449489\ldots$

$(2\pm\sqrt{6})$ (exact),
$-0.449\ [-0.450,\ -0.449]$;
$4.45\ [4.44, 4.45]$ ***A1A1 N3***

[3 marks]

3e. The graph of *g* is obtained by reflecting the graph of *f* in the *x*-axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. *[3 marks]*

Let *R* be the region enclosed by the graphs of *f* and *g* .

Find the area of *R* .

Markscheme

attempt to substitute limits or functions into area formula (accept absence of $\mathrm{d}x$)   *(M1)*

e.g.
$\int_a^b ((-x^2+4x+5)-(x^2-4x+1))\mathrm{d}x$,
$\int_{4.45}^{-0.449} (f-g)$,
$\int (-2x^2+8x+4)\mathrm{d}x$

approach involving subtraction of integrals/areas (accept absence of $\mathrm{d}x$)   *(M1)*

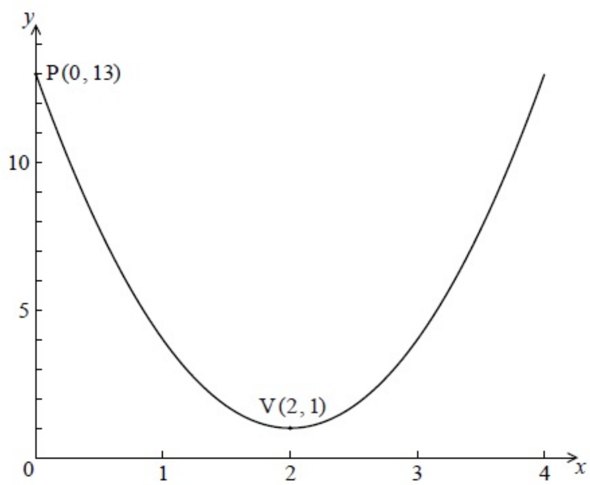
e.g.
$\int_a^b (-x^2+4x+5) - \int_a^b (x^2-4x+1)$,
$\int (f-g)\mathrm{d}x$

$\text{area} = 39.19183\ldots$

$\text{area} = 39.2$
$[39.1, 39.2]$ ***A1   N3***

[3 marks]

The following diagram shows the graph of a quadratic function *f* , for $0 \le x \le 4$.



The graph passes through the point P(0, 13) , and its vertex is the point V(2, 1) .

4a. The function can be written in the form $f(x) = a(x-h)^2 + k$.   *[4 marks]*

(i)   Write down the value of *h* and of *k* .

(ii)   Show that $a = 3$.

Markscheme

(i)
$h = 2$,
$k = 1$ ***A1A1*** ***N2***

(ii) attempt to substitute coordinates of any point (except the vertex)on the graph into *f* ***M1***

e.g.
$13 = a(0 - 2)^2 + 1$

working towards solution ***A1***

e.g.
$13 = 4a + 1$

$a = 3$ ***AG*** ***N0***

[4 marks]

4b. Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$. *[3 marks]*

Markscheme

attempting to expand **their** binomial ***(M1)***

e.g.
$f(x) = 3(x^2 - 2 \times 2x + 4) + 1$,
$(x - 2)^2 = x^2 - 4x + 4$

correct working ***(A1)***

e.g.
$f(x) = 3x^2 - 12x + 12 + 1$

$f(x) = 3x^2 - 12x + 13$ (accept $A = 3$, $B = -12$, $C = 13$) ***A1*** ***N2***

[3 marks]

4c. Calculate the area enclosed by the graph of *f*, the *x*-axis, and the lines $x = 2$ and $x = 4$. *[8 marks]*

Markscheme

METHOD 1

integral expression ***(A1)***

e.g.
$\int_2^4 (3x^2 - 12x + 13)$,
$\int f \mathrm{d}x$

$\text{Area} = [x^3 - 6x^2 + 13x]_2^4$ ***A1A1A1***

Note: Award ***A1*** for x^3, ***A1*** for $-6x^2$, ***A1*** for $13x$.

correct substitution of **correct** limits into **their** expression ***A1A1***

e.g.

$(4^3 - 6 \times 4^2 + 13 \times 4) - (2^3 - 6 \times 2^2 + 13 \times 2)$,
$64 - 96 + 52 - (8 - 24 + 26)$

Note: Award ***A1*** for substituting 4, ***A1*** for substituting 2.

correct working ***(A1)***

e.g.
$64 - 96 + 52 - 8 + 24 - 26, 20 - 10$

$\text{Area} = 10$ ***A1*** ***N3***

[8 marks]

METHOD 2

integral expression ***(A1)***

e.g.
$\int_2^4 (3(x-2)^2 + 1)$,
$\int f \mathrm{d}x$

$\text{Area} = [(x-2)^3 + x]_2^4$ ***A2A1***

Note: Award ***A2*** for
$(x-2)^3$, ***A1*** for
x .

correct substitution of **correct** limits into **their** expression ***A1A1***

e.g.
$(4-2)^3 + 4 - [(2-2)^3 + 2]$,
$2^3 + 4 - (0^3 + 2)$,
$2^3 + 4 - 2$

Note: Award ***A1*** for substituting 4, ***A1*** for substituting 2.

correct working ***(A1)***

e.g.
$8 + 4 - 2$

$\text{Area} = 10$ ***A1*** ***N3***

[8 marks]

METHOD 3

recognizing area from 0 to 2 is same as area from 2 to 4 ***(R1)***

e.g. sketch,
$\int_2^4 f = \int_0^2 f$

integral expression ***(A1)***

e.g.
$\int_0^2 (3x^2 - 12x + 13)$,
$\int f \mathrm{d}x$

$\text{Area} = [x^3 - 6x^2 + 13x]_0^2$ ***A1A1A1***

Note: Award ***A1*** for
x^3 , ***A1*** for
$-6x^2$, ***A1*** for
$13x$.

correct substitution of **correct** limits into **their** expression ***A1(A1)***

e.g.
$(2^3 - 6 \times 2^2 + 13 \times 2) - (0^3 - 6 \times 0^2 + 13 \times 0)$,
$8 - 24 + 26$

Note: Award ***A1*** for substituting 2, ***(A1)*** for substituting 0.

$\text{Area} = 10$ ***A1*** ***N3***

[8 marks]

5a. Consider an infinite geometric sequence with $u_1 = 40$ and $r = \frac{1}{2}$. *[4 marks]*

(i) Find u_4 .

(ii) Find the sum of the infinite sequence.

Markscheme

(i) correct approach ***(A1)***

e.g. $u_4 = (40)\frac{1}{2}^{(4-1)}$, listing terms

$u_4 = 5$ ***A1 N2***

(ii) correct substitution into formula for infinite sum ***(A1)***

e.g. $S_\infty = \frac{40}{1-0.5}$, $S_\infty = \frac{40}{0.5}$

$S_\infty = 80$ ***A1 N2***

[4 marks]

5b. Consider an arithmetic sequence with n terms, with first term (-36) and eighth term (-8) . *[5 marks]*

(i) Find the common difference.

(ii) Show that $S_n = 2n^2 - 38n$.

Markscheme

(i) attempt to set up expression for u_8 ***(M1)***

e.g. $-36 + (8-1)d$

correct working ***A1***

e.g. $-8 = -36 + (8-1)d$, $\frac{-8-(-36)}{7}$

$d = 4$ ***A1 N2***

(ii) correct substitution into formula for sum ***(A1)***

e.g. $S_n = \frac{n}{2}(2(-36) + (n-1)4)$

correct working ***A1***

e.g. $S_n = \frac{n}{2}(4n - 76)$, $-36n + 2n^2 - 2n$

$S_n = 2n^2 - 38n$ ***AG N0***

[5 marks]

5c. The sum of the infinite geometric sequence is equal to twice the sum of the arithmetic sequence. Find n . *[5 marks]*

Markscheme

multiplying
S_n (AP) by 2 or dividing S (infinite GP) by 2 ***(M1)***

e.g.
$2S_n$,
$\frac{S_\infty}{2}$, 40

evidence of substituting into
$2S_n = S_\infty$ ***A1***

e.g.
$2n^2 - 38n = 40$,
$4n^2 - 76n - 80$ (
$= 0$)

attempt to solve **their** quadratic (equation) ***(M1)***

e.g. intersection of graphs, formula

$n = 20$ ***A2*** ***N3***

[5 marks]

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