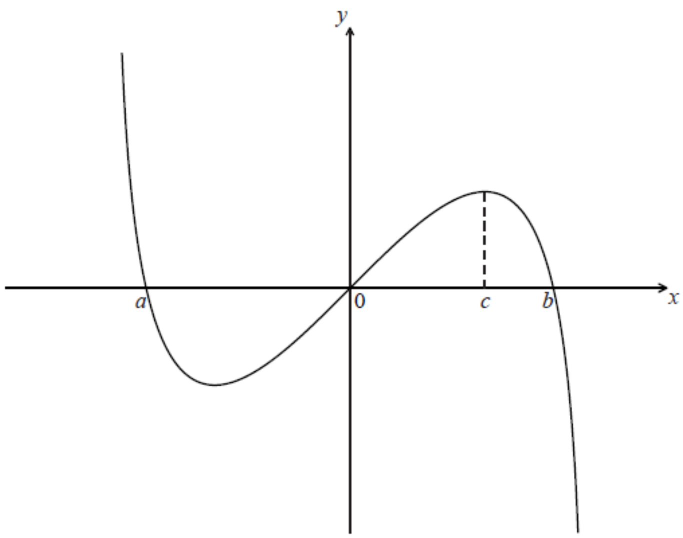
6-5-P2-Exam-extras *[40 marks]*

Let
$f(x) = x\ln(4 - x^2)$, for
$-2 < x < 2$. The graph of *f* is shown below.



The graph of *f* crosses the *x*-axis at
$x = a$,
$x = 0$ and
$x = b$.

1a. Find the value of a and of b . *[3 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. $f(x) = 0$, graph

$a = -1.73$, $b = 1.73$ $(a = -\sqrt{3}, b = \sqrt{3})$ ***A1A1*** ***N3***

[3 marks]

1b. The graph of *f* has a maximum value when $x = c$. *[2 marks]*

Find the value of c .

Markscheme

attempt to find max ***(M1)***

e.g. setting $f'(x) = 0$, graph

$c = 1.15$ (accept (1.15, 1.13)) ***A1*** ***N2***

[2 marks]

1c. The region under the graph of f from $x = 0$ to $x = c$ is rotated $360°$ about the x-axis. Find the volume of the solid formed. *[3 marks]*

Markscheme

attempt to substitute either limits or the function into formula ***M1***

e.g. $V = \pi \int_0^c [f(x)]^2 \mathrm{d}x$, $\pi \int \left[x \ln(4 - x^2)\right]^2$, $\pi \int_0^{1.149...} y^2 \mathrm{d}x$

$V = 2.16$ ***A2*** ***N2***

[3 marks]

1d. Let R be the region enclosed by the curve, the x-axis and the line $x = c$, between $x = a$ and $x = c$. *[4 marks]*

Find the area of R .

Markscheme

valid approach recognizing 2 regions ***(M1)***

e.g. finding 2 areas

correct working ***(A1)***

e.g. $\int_0^{-1.73...} f(x)\mathrm{d}x + \int_0^{1.149...} f(x)\mathrm{d}x$, $-\int_{-1.73...}^{0} f(x)\mathrm{d}x + \int_0^{1.149...} f(x)\mathrm{d}x$

area $= 2.07$ (accept 2.06) ***A2*** ***N3***

[4 marks]

Let
$f(x) = \mathrm{e}^{2x} \cos x$,
$-1 \le x \le 2$.

2a. Show that $f'(x) = \mathrm{e}^{2x}(2\cos x - \sin x)$. *[3 marks]*

Markscheme

correctly finding the derivative of e^{2x}, i.e. $2\mathrm{e}^{2x}$ ***A1***

correctly finding the derivative of $\cos x$, i.e. $-\sin x$ ***A1***

evidence of using the product rule, seen anywhere ***M1***

e.g. $f'(x) = 2\mathrm{e}^{2x}\cos x - \mathrm{e}^{2x}\sin x$

$f'(x) = 2\mathrm{e}^{2x}(2\cos x - \sin x)$ ***AG*** ***N0***

[3 marks]

2b. Let the line *L* be the normal to the curve of *f* at $x = 0$. *[5 marks]*

Find the equation of *L*.

Markscheme

evidence of finding $f(0) = 1$, seen anywhere ***A1***

attempt to find the gradient of *f* ***(M1)***

e.g. substituting $x = 0$ into $f'(x)$

value of the gradient of *f* ***A1***

e.g. $f'(0) = 2$, equation of tangent is $y = 2x + 1$

gradient of normal $= -\frac{1}{2}$ ***(A1)***

$y - 1 = -\frac{1}{2}x \left(y = -\frac{1}{2}x + 1\right)$ ***A1*** ***N3***

[5 marks]

2c. The graph of *f* and the line *L* intersect at the point (0, 1) and at a second point P. *[6 marks]*

(i) Find the *x*-coordinate of P.

(ii) Find the area of the region **enclosed** by the graph of *f* and the line *L*.

Markscheme

(i) evidence of equating correct functions ***M1***

e.g. $e^{2x}\cos x = -\frac{1}{2}x + 1$, sketch showing intersection of graphs

$x = 1.56$ ***A1*** ***N1***

(ii) evidence of approach involving subtraction of integrals/areas ***(M1)***

e.g. $\int [f(x) - g(x)]\mathrm{d}x$, $\int f(x)\mathrm{d}x - \text{area under trapezium}$

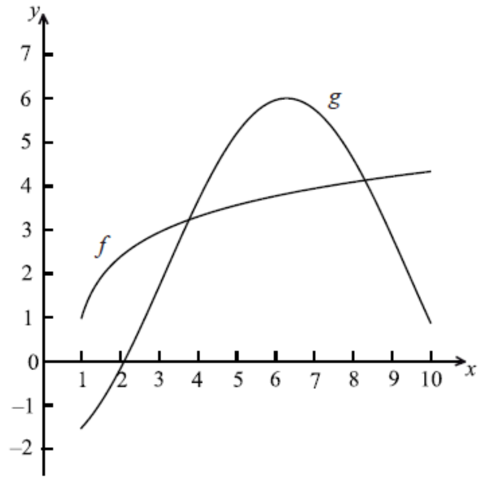
fully correct integral expression ***A2***

e.g. $\int_0^{1.56} \left[e^{2x}\cos x - \left(-\frac{1}{2}x + 1\right)\right]\mathrm{d}x$, $\int_0^{1.56} e^{2x}\cos x\mathrm{d}x - 0.951\ldots$

$\text{area} = 3.28$ ***A1*** ***N2***

[6 marks]

The following diagram shows the graphs of
$f(x) = \ln(3x - 2) + 1$ and
$g(x) = -4\cos(0.5x) + 2$, for
$1 \le x \le 10$.



3a. Let *A* be the area of the region **enclosed** by the curves of *f* and *g*. *[6 marks]*

(i) Find an expression for *A*.

(ii) Calculate the value of *A*.

Markscheme

(i) intersection points $x = 3.77$, $x = 8.30$ (may be seen as the limits) ***(A1)(A1)***

approach involving subtraction and integrals ***(M1)***

fully correct expression ***A2***

e.g. $\int_{3.77}^{8.30}((-4\cos(0.5x)+2)-(\ln(3x-2)+1))\mathrm{d}x$, $\int_{3.77}^{8.30}g(x)\mathrm{d}x - \int_{3.77}^{8.30}f(x)\mathrm{d}x$

(ii) $A = 6.46$ ***A1*** ***N1***

[6 marks]

3b. (i) Find $f'(x)$. *[4 marks]*

(ii) Find $g'(x)$.

Markscheme

(i) $f'(x) = \frac{3}{3x-2}$ ***A1A1*** ***N2***

Note: Award ***A1*** for numerator (3), ***A1*** for denominator $(3x-2)$, but penalize 1 mark for additional terms.

(ii) $g'(x) = 2\sin(0.5x)$ ***A1A1*** ***N2***

Note: Award ***A1*** for 2, ***A1*** for $\sin(0.5x)$, but penalize 1 mark for additional terms.

[4 marks]

3c. There are two values of *x* for which the gradient of *f* is equal to the gradient of *g*. Find both these values of *x*. *[4 marks]*

Markscheme

evidence of using derivatives for gradients ***(M1)***

correct approach ***(A1)***

e.g. $f'(x) = g'(x)$, points of intersection

$x = 1.43$, $x = 6.10$ ***A1A1*** ***N2N2***

[4 marks]