

7 May 2018

Name: .

**Homework: Exponential functions, imaginary numbers, sequences, logs**

1. A bank account earns interest at a continuous interest rate of 5% per year. The initial deposit is \$225.

(a) Express the balance in the account as a function in the form  $P(t) = P_0 \cdot e^{rt}$

(b) Convert the function to one without a coefficient in the exponent.

(c) What is the interest rate expressed as a simple, annual rate?

2. Judith puts \$5000 into an investment account with interest compounded continuously. If the annual interest rate is 3.25% what is the balance after 30 years?

3. Lisa puts \$1000 into an investment account with interest compounded continuously. What is the approximate annual rate needed for the account to grow to \$1529.59 after 10 years?

4. The function below models the average price of gas in a small town since January 1st.

$$G(t) = -0.0049t^4 + 0.0923t^3 - 0.56t^2 + 1.166t + 3.23, \text{ where } 0 \leq t \leq 10.$$

If  $G(t)$  is the average price of gas in dollars and  $t$  represents the number of months since January 1st, the absolute maximum  $G(t)$  reaches over the given domain is about what value, to the nearest cent? (graph the function in your calculator and use the Max function)

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5. Write  $\sqrt[3]{x^8}$  as a single term with a rational exponent.
6. Write  $\sqrt{a^3} \div a^{\frac{1}{2}}$  as an expression with positive, integer exponents.
7. If  $n = \sqrt{z^5}$  and  $m = z^{\frac{7}{2}}$ , where  $a > 0$ , express  $\frac{n}{m}$  as a radical with positive, integer exponents.
8. What is the expression  $5i^3(-2i + 5)$  equivalent to? Express your answer in the form  $a + bi$ , where  $a, b \in \mathbb{R}$ .
9. Simplify the expression  $(2x - i)^2$ , where  $i$  is the imaginary unit. Express your answer in the form  $a + bi$ , where  $a, b \in \mathbb{R}$ .
10. Algebraically determine the values of  $h$  and  $k$  to correctly complete the identity stated below.

$$3x^3 - 7x^2 + 5x - 7 = (x - 2)(3x^2 + hx + 3) + k$$

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11. The expression  $(x + a)(x + b)$  can not be written as

(a)  $a(x + b) + x(x + b)$

(b)  $x^2 + (a + b)x + ab$

(c)  $x^2 + abx + ab$

(d)  $x(x + a) + b(x + a)$

12. In an arithmetic sequence, the first term is 3 and the second term is 7.

(a) Find the common difference.

[2]

(b) Find the tenth term.

[2]

(c) Find the sum of the first ten terms of the sequence.

[2]

13. Consider a geometric sequence where the first term is 768 and the second term is 576.  
Find the least value of  $n$  such that the  $n$ th term of the sequence is less than 7.

[6]

14. Let  $x = \ln 7$  and  $y = \ln 3$ . Write the following expressions in terms of  $x$  and  $y$ .

(a)  $\ln\left(\frac{3}{7}\right)$ .

[2]

(b)  $\ln 63$ .

[4]

15. Let  $f(x) = k \log_2 x$ .

(a) Given that  $f^{-1}(1) = 8$ , find the value of  $k$ .

[3]

(b) Find  $f^{-1}\left(\frac{2}{3}\right)$

[4]