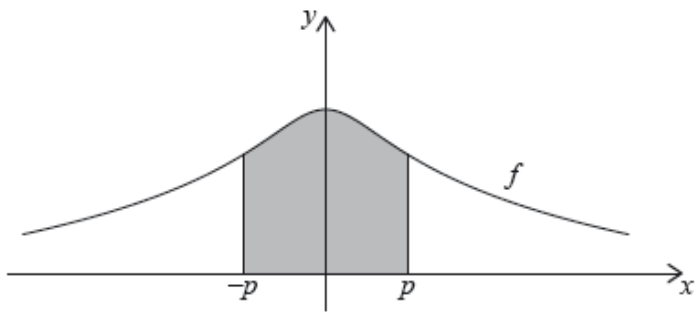
BECA / Huson / 12.1 IB Math SL Name:
13 April 2018

Homework: Integration exam problems

1a. Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \mathbb{R}$. The graph of f passes through the point $(p,\ 4)$, where $p > 0$.

Find the value of p. *[2 marks]*

1b. The following diagram shows part of the graph of f.



The region enclosed by the graph of f, the x-axis and the lines $x = -p$ and $x = p$ is rotated 360° about the x-axis. Find the volume of the solid formed. *[3 marks]*

2a. Find $\int x\mathrm{e}^{x^2-1}\,\mathrm{d}x$. *[4 marks]*

2b. Find $f(x)$, given that $f'(x) = x\mathrm{e}^{x^2-1}$ and $f(-1) = 3$. *[3 marks]*

3. Let $f'(x) = \frac{3x^2}{(x^3+1)^5}$. Given that $f(0) = 1$, find $f(x)$. *[6 marks]*

4. Let $f'(x) = \sin^3(2x)\cos(2x)$. Find $f(x)$, given that $f\left(\frac{\pi}{4}\right) = 1$. *[7 marks]*

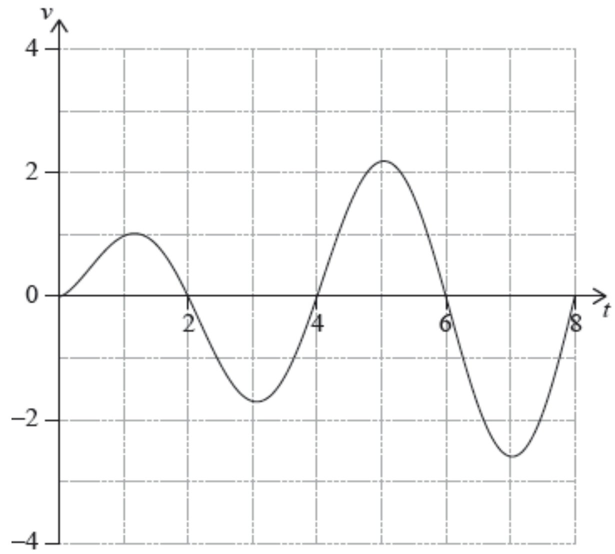
5a. Let $f(x) = x\mathrm{e}^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

Find the value of p and of q. *[3 marks]*

5b. Hence, find the area of the region enclosed by the graphs of f and g. *[3 marks]*

6a. A particle P moves along a straight line. Its velocity $v_{\mathrm{P}}\ \mathrm{m\,s^{-1}}$ after t seconds is given by $v_{\mathrm{P}} = \sqrt{t}\sin\left(\frac{\pi}{2}t\right)$, for $0 \leqslant t \leqslant 8$. The following diagram shows the graph of v_{P}.



Write down the first value of t at which P changes direction. *[1 mark]*

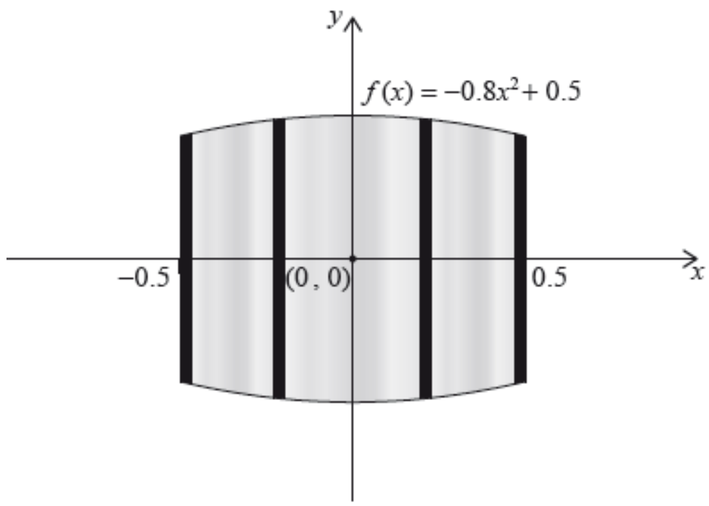
6b. Find the **total** distance travelled by P, for $0 \leqslant t \leqslant 8$. *[2 marks]*

6c. A second particle Q also moves along a straight line. Its velocity, $v_{\mathrm{Q}}\ \mathrm{m\,s^{-1}}$ after t seconds is given by $v_{\mathrm{Q}} = \sqrt{t}$ for $0 \leqslant t \leqslant 8$. After k seconds Q has travelled the same total distance as P.

Find k. *[4 marks]*

7a. All lengths in this question are in metres.

Let $f(x) = -0.8x^2 + 0.5$, for $-0.5 \leqslant x \leqslant 0.5$. Mark uses $f(x)$ as a model to create a barrel. The region enclosed by the graph of f, the x-axis, the line $x = -0.5$ and the line $x = 0.5$ is rotated 360° about the x-axis. This is shown in the following diagram.



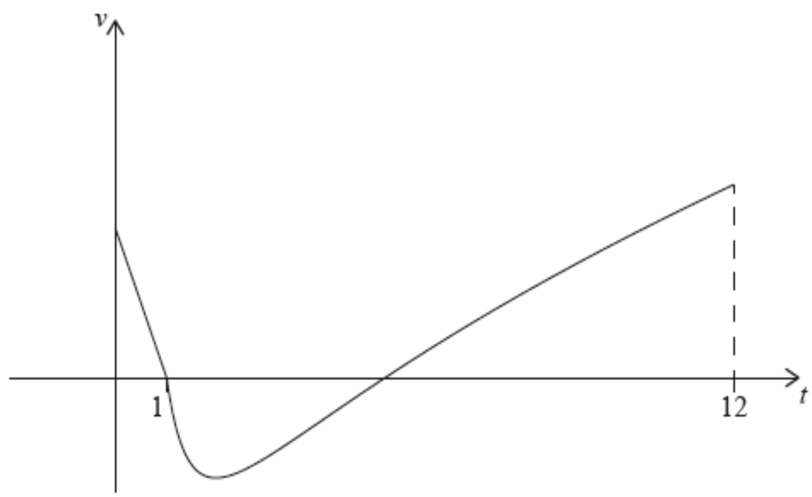
Use the model to find the volume of the barrel. *[3 marks]*

7b. The empty barrel is being filled with water. The volume $V\ \mathrm{m}^3$ of water in the barrel after t minutes is given by $V = 0.8(1 - \mathrm{e}^{-0.1t})$. How long will it take for the barrel to be half-full? *[3 marks]*

8a. A particle P starts from a point A and moves along a horizontal straight line. Its velocity $v\ \mathrm{cm\,s^{-1}}$ after t seconds is given by

$$v(t) = \begin{cases} -2t + 2, & \text{for } 0 \leqslant t \leqslant 1 \\ 3\sqrt{t} + \frac{4}{t^2} - 7, & \text{for } 1 \leqslant t \leqslant 12 \end{cases}$$

The following diagram shows the graph of v.



Find the initial velocity of P. *[2 marks]*

8b. P is at rest when $t = 1$ and $t = p$.

Find the value of p. *[2 marks]*

8c. When $t = q$, the acceleration of P is zero.

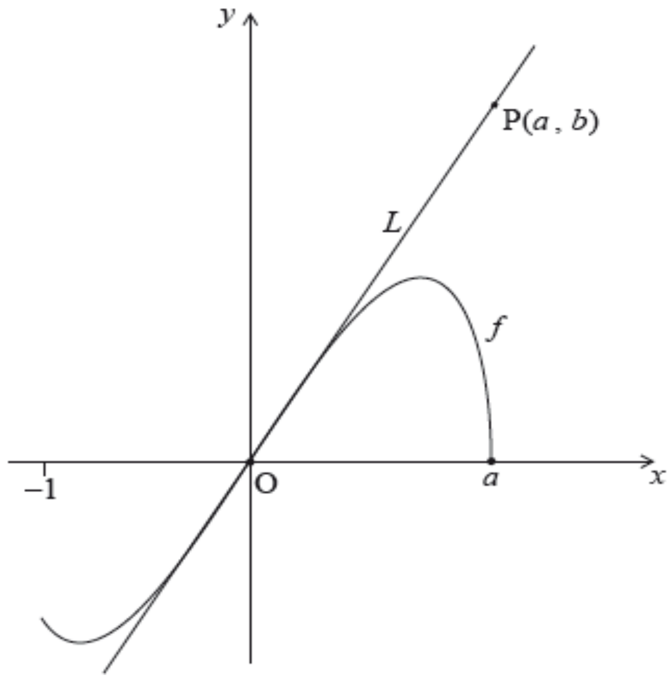
(i) Find the value of q.

(ii) Hence, find the **speed** of P when $t = q$. *[4 marks]*

8d. (i) Find the total distance travelled by P between $t = 1$ and $t = p$.

(ii) Hence or otherwise, find the displacement of P from A when $t = p$. *[6 marks]*

9a. The following diagram shows the graph of $f(x) = 2x\sqrt{a^2 - x^2}$, for $-1 \leqslant x \leqslant a$, where $a > 1$.

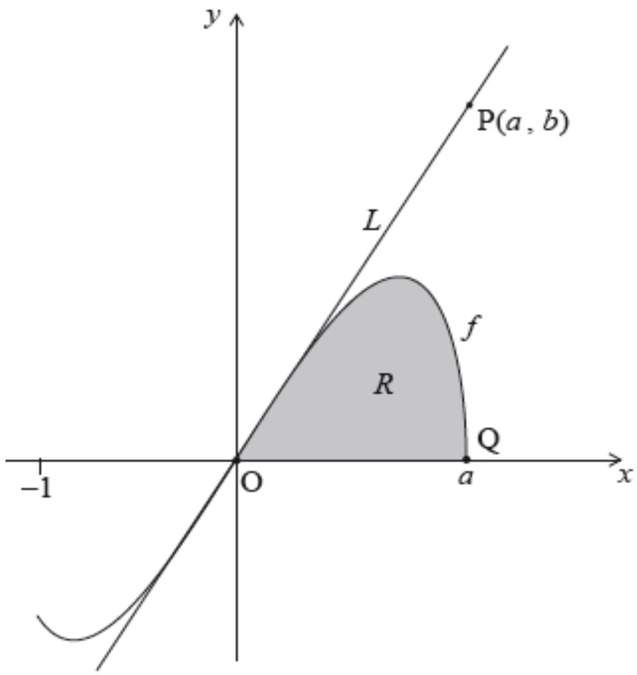


The line L is the tangent to the graph of f at the origin, O. The point $\mathrm{P}(a,\ b)$ lies on L.

(i) Given that $f'(x) = \frac{2a^2-4x^2}{\sqrt{a^2-x^2}}$, for $-1 \leqslant x < a$, find the equation of L.

(ii) Hence or otherwise, find an expression for b in terms of a. *[6 marks]*

9b. The point $Q(a, 0)$ lies on the graph of f. Let R be the region enclosed by the graph of f and the x-axis. This information is shown in the following diagram. *[6 marks]*



Let A_R be the area of the region R.

Show that $A_R = \frac{2}{3}a^3$.

9c. Let A_T be the area of the triangle OPQ. Given that $A_T = kA_R$, find the value of k. *[4 marks]*