BECA / Huson / 12.1 IB Math SL Name:
14 December 2017
Test: Vector and calculus

1a. Line L_1 passes through points **A**(3, 0, 7) and **B**(4, -1, 8).

Find $\overrightarrow{\mathrm{AB}}$. *[2 marks]*

1b. Find an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[2 marks]*

1c. Line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

Find the angle between L_1 and L_2. *[7 marks]*

1d. The lines L_1 and L_2 intersect at point C. Find the coordinates of C. *[6 marks]*

2a. The line L_1 is represented by the vector equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$.

A second line L_2 is parallel to L_1 and passes through the point B($-8, -5, 25$).

Write down a vector equation for L_2 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[2 marks]*

2b. A third line L_3 is perpendicular to L_1 and is represented by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q\begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$.

Show that $k = -2$. *[5 marks]*

2c. The lines L_1 and L_3 intersect at the point A. Find the coordinates of A. *[6 marks]*

2d. The lines L_2 and L_3 intersect at point C where $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$.

(i) Find $\overrightarrow{\mathrm{AB}}$.

(ii) Hence, find $|\overrightarrow{\mathrm{AC}}|$. *[5 marks]*

3a. Let $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$. Find $\overrightarrow{\mathrm{BC}}$. *[2 marks]*

3b. Find a unit vector in the direction of $\overrightarrow{AB}$. *[3 marks]*

3c. Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$. *[3 marks]*

4a. Let $f(x) = \frac{6x}{x+1}$, for $x > 0$. Find $f'(x)$. *[5 marks]*

4b. Let $g(x) = \ln\left(\frac{6x}{x+1}\right)$, for $x > 0$.

Show that $g'(x) = \frac{1}{x(x+1)}$. *[4 marks]*

5a. Let $f(x) = \mathrm{e}^{6x}$. Write down $f'(x)$. *[1 mark]*

5b. The tangent to the graph of *f* at the point $\mathrm{P}(0, b)$ has gradient *m* .

(i) Show that $m = 6$.

(ii) Find *b* . *[4 marks]*

5c. Hence, write down the equation of this tangent. *[1 mark]*

6a. The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

Distance, x km	11 500	7500	13 600	10 800	9500	12 200	10 400
Price, y dollars	15 000	21 500	12 000	16 000	19 000	14 500	17 000

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

(i) Find the correlation coefficient.

(ii) Write down the value of a and of b. *[4 marks]*

6b. On 1 January 2010, Lina buys a car which has travelled 11,000 km.

Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest 100 dollars. *[3 marks]*