BECA / Huson / 12.1 IB Math SL Name:
20 September 2017

Test: Introduction to differential calculus

Show working for all problems. State answers exactly or to three significant figures.

1. Write down the derivative of the function $f(x) = 3x^2 - 3x + 1$.

2. A function is given as $y = ax^2 + bx + 5$.

   (a) Find $\dfrac{dy}{dx}$.

   (b) If the gradient of this function is 2 when x is 3, write an equation in terms of a and b.

   (c) If the point $(1, -5)$ lies on the graph of the function find a second equation in terms of a and b.

3. Find $f'(x)$ for the following function. Express your final result without negative exponents:
$$f(x) = \frac{3x^2 - x + 3}{x}$$

4. Sketch the function $f(x) = x^2 - 3x - 4$.

   (a) Find $f(3)$.

   (b) Find $f'(x)$.

   (c) What is the slope of a tangent to f when $x = 3$?

   (d) What is the equation of the line tangent to f when $x = 3$?

   (e) What is the equation of the line normal to f when $x = 3$?

   (e) Add the tangent and normal lines to the sketch, labeling the point of tangency.

5. Find the equation of the tangent to $f(x) = \dfrac{1}{2x^2}$ when $x = 2$.

6. Show that the derivative of $f(x) = 3x^2 - x$ is $f'(x) = 6x - 1$ from first principles using the definition of the derivative as a limit.

Review of function inverses and composition

7. For the function $f(x) = 2x - 7$
   (a) What is the value of $f(3)$?
   (b) Solve for x if $f(x) = 0$.
   (c) Find $f(1 - x)$.
   (d) Find the inverse of $f(x)$, $f^{-1}(x)$.
8. For the function $g(x) = x^2 - 4$ with $x > 0$
   (a) What is the value of $g(x - 3)$?
   (b) Find $g^{-1}(x)$.
9. For the functions $f(x) = 2 - x^2$ and $g(x) = 2x - 5$
   (a) What is the value of $g(3)$?
   (b) Find $(f \circ g)(3) = 0$.
   (c) Find $(f \circ g)(x) = 0$.
10. Find the inverse of $f(x) = \dfrac{4x - 2}{5}$
11. Given that $g(x) = \frac{1}{3}x + 2$
    (a) Find the inverse of $g(x)$.
    (b) Graph the function $g(x)$ and its inverse on the same axes, using the scale 1 unit equals 1 cm and labeling the graph following IB conventions.
12. For the functions defined by $f(x) = 2x$ and $g(x) = x + 4$
    (a) Find an expression for $(f \circ g)(x)$.
    (b) Find an expression for $(g \circ f)(x)$.
    (c) Solve $(f \circ g)(x) = (g \circ f)(x)$.
13. Write down the domain and range of $f(x) = x^2 - 6$
14. Using a GDC to analyze the function $f(x) = (3x + 2)/(x + 1)$
    (a) Write down the equations for the asymptotes.
    (b) Write down the domain and range of $f(x)$.

15. Write down the domain and range of the function graphed in Figure-1.

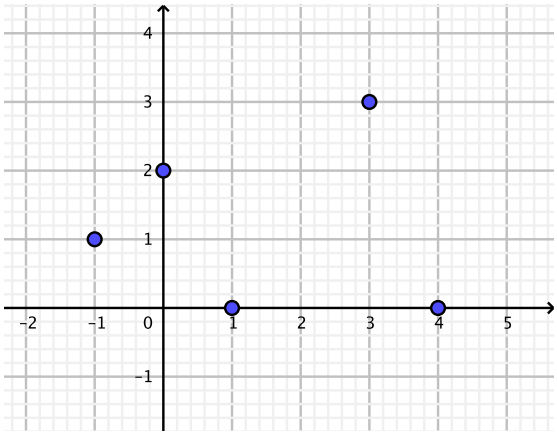

Figure 1: Write down domain and range.

16. For the function shown in Figure-2

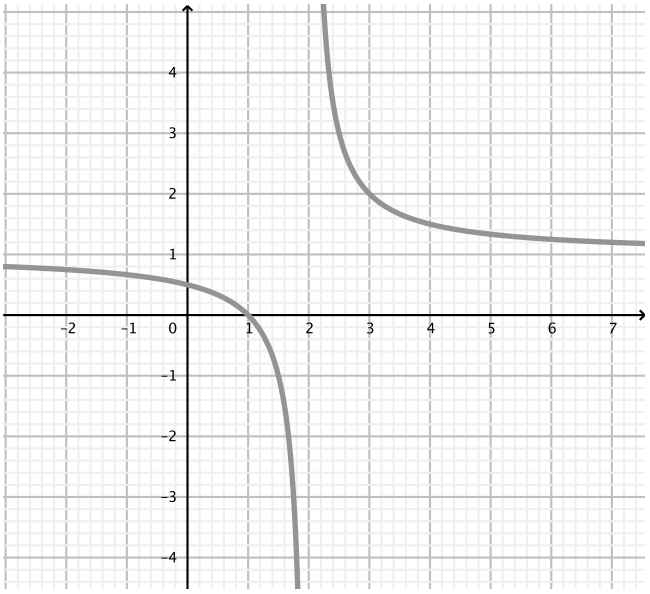

Figure 2: Determine asymptotes.

(a) Write down the equations for the asymptotes.

(b) Write down the domain and range of $f(x)$.

BECA / Huson / 11.1 IB Math SL Name:
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Graph accurately in pencil using a straight edge or smooth curve.

17. Given the graph of the function $f(x)$ shown in Figure-3

    (a) Label the points on the function representing $f(-1) = -2$ and $f(4) = -1$

    (b) Graph the inverse of $f(x)$ on the same axes. Label the inverses of the points named in part (a)

    (c) Write down the domain and range of $f(x)$.

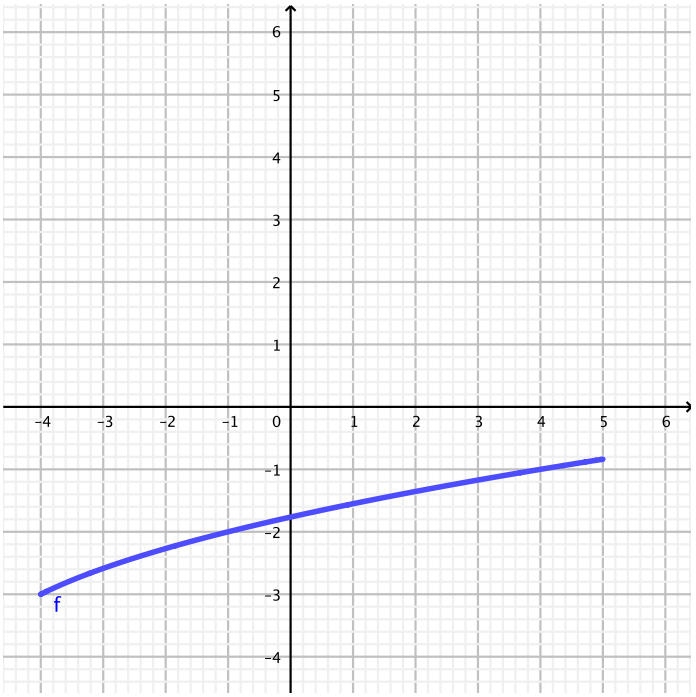

Figure 3: Label given points and plot inverse.

18. Consider the function $f(x) = x^3 - 4x^2 - 3x + 18$.

    (a) Find the values of $f(x)$ for a and b in the table below:

x	-3	-2	-1	0	1	2	3	4	5
$f(x)$	-36	a	16	b	12	4	0	6	28

    (b) Using a scale of 1 cm for each unit on the x-axis and 1 cm for each 5 units on the y-axis, draw the graph of $f(x)$ for $-3 \leq x \leq 5$. Label it clearly using IB conventions on the graph paper provided (other side).