24 February 2020 Name:

Spiral Review: 1-3 P1 (No Calculator) Algebra Logarithms

1. 14M.1.sl.TZ2.2
   Find the value of each of the following, giving your answer as an integer.
   (a) $\log_6 36$ [2 marks]
   (b) $\log_6 4 + \log_6 9$ [2 marks]
   (c) $\log_6 2 - \log_6 12$ [2 marks]

2. 16M.1.sl.TZ2.3
   Let $x = \ln 3$ and $x = \ln 5$. Write the following expressions in terms of x and y.
   (a) $\ln \frac{5}{3}$ [2 marks]
   (b) $\ln 45$ [4 marks]

3. 13M.1.sl.TZ2.3
   Let $\log_3 p = 6$ and $\log_3 q = 7$.
   (a) Find $\log_3 p^2$ [2 marks]
   (b) Find $\log_3 \frac{p}{q}$ [2 marks]
   (c) Find $\log_3(9p)$ [3 marks]

4. 15M.1.sl.TZ1.3
   (a) Given that $2^m = 8$ and $2^n = 16$, write down the value of m and n. [2 marks]
   (b) Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. [4 marks]

5. 14N.1.sl.TZ0.4
   (a) Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$. [3 marks]
   (b) Hence or otherwise solve $3\ln 2 - \ln 4 = -\ln x$. [3 marks]

6. 14M.1.sl.TZ1.4
   (a) Write down the value of
      i. $\log_3 27$ [1 mark]
      ii. $\log_8 \frac{1}{8}$ [1 mark]
      iii. $\log_{16} 4$ [1 mark]
   (b) Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$. [3 marks]

7. 09M.1.sl.TZ2.4
   (a) Find $\log_2 32$. [1 mark]
   (b) Given that $\log_2(\frac{32^x}{8^y})$ can be written as $px + qy$, find the value of p and q. [4 marks]

8. 11M.1.sl.TZ2.5

(a) Let $f(x) = \ln x$ and $g(x) = \ln 5x^3$.
Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$. [4 marks]

(b) The graph of g is a transformation of the graph of f. Give a full geometric description of this transformation. [3 marks]

9. 17M.1.sl.TZ2.7
Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$. [7 marks]

10. 10M.1.sl.TZ2.6
Solve $\log_2 x + \log_2(x-2) = -3$, for $x > 2$. [7 marks]

11. 09M.1.sl.TZ1.6

(a) Let $f(x) = e^{x+3}$. [3 marks]

i. Show that $f^{-1}(x) = \ln x - 3$.

ii. Write down the domain of f^{-1}.

(b) Solve the equation $f^{-1}(x) = \ln \frac{1}{x}$. [4 marks]

12. 13M.1.sl.TZ1.7

(a) Find the value of $\log_2 40 - \log_2 5$. [3 marks]

(b) Find the value of $8^{\log_2 5}$. [4 marks]

13. 10M.1.sl.TZ1.7

(a) Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.
Show that $f^{-1}(x) = 3^{2x}$. [2 marks]

(b) Write down the range of f^{-1}. [1 mark]

(c) Let $g(x) = \log_3 x$, for $x > 0$.
Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer. [4 marks]

14. 09N.1.sl.TZ0.7

(a) Let $f(x) = k \log_2 x$.
Given that $f^{-1}(1) = 8$, find the value of k. [3 marks]

(b) Find $f^{-1}(\frac{2}{3})$. [4 marks]

15. 16M.1.sl.TZ1.9

(a) Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.
The graph of f has a maximum point at P.
Find the x-coordinate of P. [3 marks]

(b) The y-coordinate of P is $\ln 27$.
Find $f(x)$, expressing your answer as a single logarithm. [8 marks]

(c) The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates (a, b). Find the value of a and of b, where $a, b \in \mathbb{N}$. [4 marks]