Mathematics Class Slides

Bronx Early College Academy

Chris Huson

April 2018

GQ: How do we use the inverse normal function?

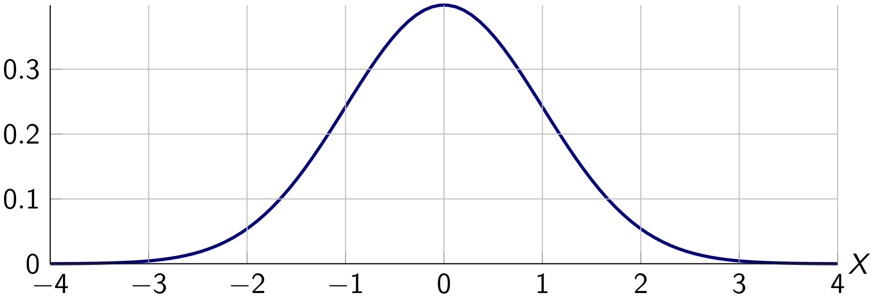
CCSS: HSS.MD.A.3 Develop a probability distribution for a random variable 12.1

Do Now: Review problem set solutions

Pretest packet homework review
Lesson: Chapter 15 summary p. 553
Task: Review exercises p. 551
Assessment: Exam tomorrow
Homework: Break Paper 1, Paper 2, corrections due?

Bell Shaped Gaussian Distribution

$X \sim N(0,1)$



How do we solve for missing parameters given a series?

CCSS: HSA.SSE.B.4 Arithmetic series 11.1

Do Now: Group share of IA criteria scores

1. Make a table of your group's scores
2. Each person: A, B, C, D, E, total
3. Write on paper to turn in, group's names

Lesson: Finance domain knowledge
Task: Review pretest solutions
Assessment: (test tomorrow)
Homework: Break mini-IA paper due Monday April 9th (email me a draft on Monday - optional)
follow proposal on math.huson.com

Standards for writing technical papers

Practice writing mathematics according to IB requirements, as per IA criteria.

Criterion C: Personal engagement (0-4 points)

1. Address a personal interest; "make it your own"
2. Think independently and/or creatively
3. Present mathematical ideas in your own way

Criterion D: Reflection (0-3 points)

1. Review, analyze, and evaluate the mathematics throughout the paper. Go beyond just describing results
2. Link to the aims, comment on what has been learned, consider limitations, and compare different mathematical approaches
3. Consider what's next, discuss the implications of results, strengths and weaknesses of approaches, and consider different perspectives

Standard conventions for mathematical notation

Practice writing mathematics according to IB requirements, as per exam rubrics.

1. Use the formula sheet.
2. Chose the appropriate formula (M1).
   (you do not have to copy the formula)
3. Substitute values correctly (A1).
4. Solve, showing key steps (A1).
   (skip routine algebra if you like)
5. Write down the exact solution or copy the calculator display.
   An ellipsis (. . .) indicates more digits (A1).
6. Round to 3 significant digits (use $\approx$)(A1).

Standard conventions for mathematical notation

Practice writing mathematics according to IB requirements, as per exam rubrics.

Examples of key algebraic techniques

1. Setting a quadratic function $= 0$
2. Converting an exponent to a log
3. Reading a value from a graph
4. When writing lists, you may write only the first two and the last terms. For example,

$$\sum_{k=1}^{5} 3 \cdot 2.25^k = 3 + 6.75 + \ldots + 76.8867\ldots$$

$$= 135.99609\ldots \approx 136$$

Aim and rationale

There are many important and amazing sequences and series in mathematics.

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}\dots \text{ Zeno's paradox (Greek)}$$

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\dots \text{ harmonic series}$$

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\frac{1}{13}\dots \text{Madhava (India)}$$

0.99999999...

The aim of this exploration is to discover the patterns, formulas, and rules of geometric series by experimenting and investigating using a spreadsheet to total various example series.

*Sequence: list of numbers. Series: sum of a sequence of numbers. Geometric sequence: consecutive terms have a constant ratio.

Descriptive statistics terminology

Make a list of these terms, find their definitions in the textbook.

Univariate data, bivariate
Population, sample, random/biased sample, survey, census
Discrete/continuous data, quantitative/qualitative
Central tendency, mean ($\overline{x}, \mu$), median, mode; quartiles, percentiles
5-figure summary, box & whisker plots, range, interquartile range, outlier
Dispersion, standard deviation (σ), variance ($v = \sigma^2$)
Frequency distributions (tables/bar charts/histograms)
Grouped data, class, mid-interval value, boundaries, modal class
Cumulative frequency distributions

Bias and fairness, random variation, & combinations

When rolling two dice, why aren't all the possible totals equally likely?

Definition:
A fair (p. 67) or unbiased (p. 79) process

In mathematics we usually simplify and assume a random process follows exact, idealized probabilities. For example, we assume heads and tails are equally likely results of a coin toss.

Bias and fairness, random variation, & combinations

When rolling two dice, why aren't all the possible totals equally likely?

Definition:
Experimental or empirical (p. 65) results

In real life, the results of any experiment have a degree of random variation. The observed relative frequencies are estimates of the underlying theoretical probabilities, which grow more accurate with additional trials.

Bias and fairness, random variation, & combinations

When rolling two dice, why aren't all the possible totals equally likely?

Counting events in a sample space (p. 78) or calculating combinations (p. 184)

The six possible results of rolling a single die are equally likely, $\mathrm{P}(x) = \frac{1}{6}$, if we assume the die is fair. Similarly, the probability of any of the 36 (6×6) possible results of rolling two dice are equally likely, $\mathrm{P}(x) = (\frac{1}{6})^2$. However, the probability of a particular total varies according to how many combinations lead to that total. Thus, for example, 7 can be rolled six different ways, so $\mathrm{P}(7) = \frac{6}{36}$, while 2 can only result one way, $\mathrm{P}(2) = \frac{1}{36}$.

Sets, subsets, & proper subsets

Definitions:
A set is an unordered collection of elements.
e.g. $\{$red, white, blue$\}$ (do not repeat elements)

Subset: Set A is a subset of set B if and only if all of the elements of A are elements of B.
Written: $A \subseteq B$

Proper subset: $A \subseteq B$ and A is not equal to B. Written: $A \subset B$

The empty set is a subset of all sets. $\{\}$ or $\emptyset$

GQ: Combinatorics problem

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function

Show the formula and then use your calculator function

1. You have a \$1 bill, a \$5 bill, a \$10 bill, a \$20 bill, a quarter, a dime, a nickel, and a penny. How many different total amounts can you make by choosing six bills and coins?

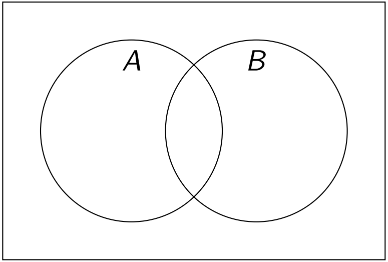
What is the number of the set you are choosing from?
How many are you picking?
Does their order matter?

Do Now #1: Phone preferences by gender

Given the frequency table, make a Venn diagram

	Android	iPhone
Boys	15	5
Girls	5	15

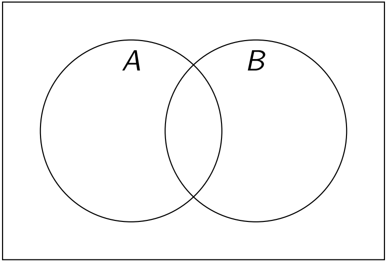
$A = \{\text{prefers Android}\}$ and $B = \{\text{is a boy}\}$



Do Now #2: Independence

Given the situation, make a Venn diagram, frequency table, and tree representing

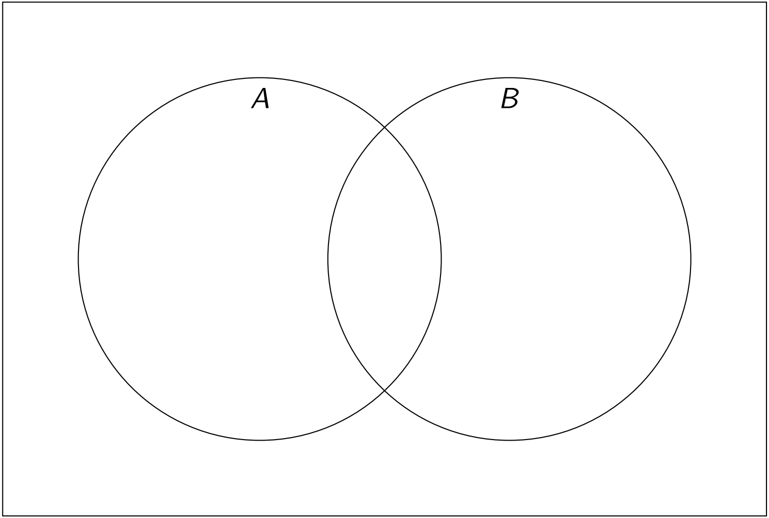
$$\mathrm{P}(A) = 0.6,\ \mathrm{P}(B) = 0.5,\ \mathrm{P}(A \cap B) = 0.3$$



	A	A'
B		
B'		

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

The addition rule



Distributions

Tables and charts used to summarize a problem situation

A frequency distribution displays the number of times each event in the sample space occurs, either in tabular or graphical form.

A probability distribution shows the same data, normalizing the totals to one.

Technical writing

Write a short paper answering the query:
"How many subsets can be picked from a group of four students?"

1. Logical, step-by-step explanation, using an example
2. Precise terminology, succinct: combination, permutation, order (matters), event, sample space, set, subset, with /without replacement, factorial
3. Notation: algebra symbols, tables, trees, grids
4. Summary, big-picture, conceptual idea
5. Audience: student peers

Combinatorics formulas

Combinations, when order doesn't matter

$$_nC_r = \frac{n!}{(n-r)!r!} \qquad \text{"n pick r"}$$

Permutations, when order does matter

$$_nP_r = \frac{n!}{(n-r)!}$$

Definition of theoretical probability

The theoretical probability of an event A is $\mathrm{P}(A) = \dfrac{n(A)}{n(U)}$

where $n(A)$ is the number of ways an event can occur

and $n(U)$ is the total number of possible outcomes (p. 65)

Theoretically, in n trials, one would expect the event to occur $n \times \mathrm{P}(A)$ times

Probabilities are between 0 and 1, inclusive. $0 \leq \mathrm{P}(X) \leq 1$

Empirical (experimental) probability

The relative frequency of an event can be used as an estimate of its probability.

$$\mathrm{P}(A) = \frac{\text{number of occurrences of event } A}{\text{total number of trials}}$$

The larger the number of trials the more reliable the estimate of probability.

Independence and mutual exclusivity

Two events are independent if the occurrence of one does not affect the probability of the other.

$$\mathrm{P}(\text{both } A \text{ and } B \text{ occur}) = \mathrm{P}(A) \times \mathrm{P}(B)$$

Two events are mutually exclusive if they never occur together.

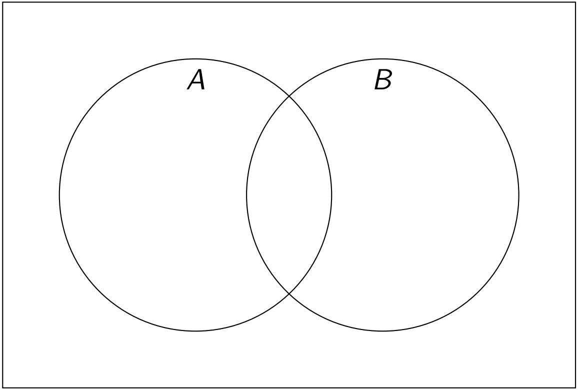
$$\mathrm{P}(\text{both } A \text{ and } B \text{ occur}) = 0 \qquad \text{and}$$

$$\mathrm{P}(\text{either } A \text{ or } B \text{ occur}) = \mathrm{P}(A) + \mathrm{P}(B)$$

Venn diagrams

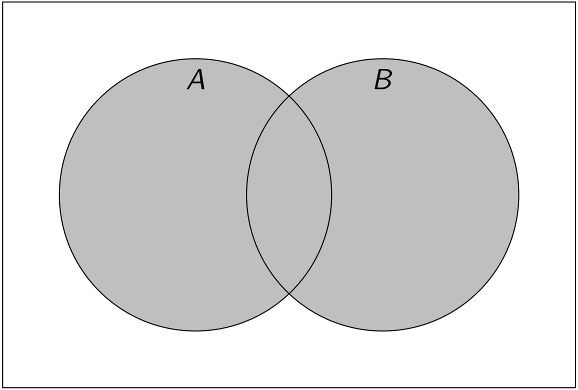
For organizing compound events

When two events can occur, and perhaps both, or neither.



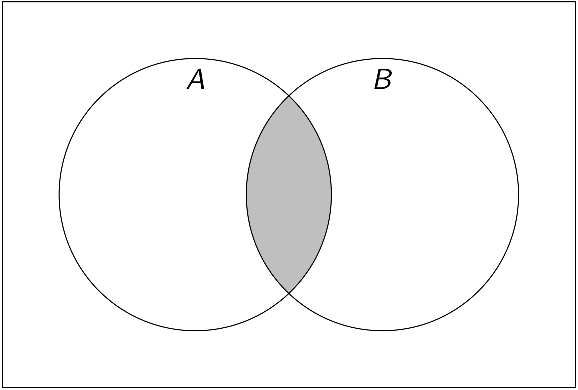
The union of sets: $A \cup B$

That A happens, or B happens, or both



The intersection of sets: $A \cap B$

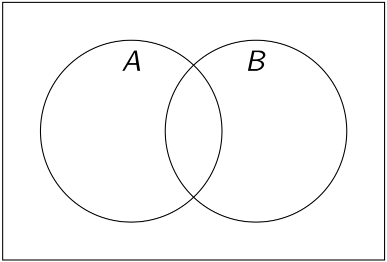
That both A and B happen



The addition rule

That A or B or both occur

When two events can occur, and perhaps both



$$P(\text{either } A \text{ or } B \text{ occur}) = P(A) + P(B) - P(\text{both } A \text{ and } B \text{ occur})$$

Vocabulary for probability & statistics

event, experiment, random

probability, P(A), values [0,1]

theoretical, empirical, subjective

sample space, U; frequency, trials

n(U) = number of possibilities

P(A) = n(A)/n(U); expected = n * P

Interpreting a displacement vs time graph

CCSS: F.IF.B.6 Calculate & interpret the rate of change of a function

Consider the function $f(x) = -x^2 + 2x + 3$

1. Factor f and state its zeros.
2. Restate f in vertex form. Write down the vertex as an ordered pair.
3. Over what intervals is the function increasing, decreasing, and neither?
4. If $f(x)$ represents the height of a diver over the domain $0 \leq x \leq 3$, interpret $f(0)$, the vertex, and $f(3)$
5. What does the "slope" of the curve represent?