Name:

Sequences & series

1. In an arithmetic sequence, the first term is 3 and the second term is 7.

   (a) Find the common difference.

   [2]

   (b) Find the tenth term.

   [2]

   (c) Find the sum of the first ten terms of the sequence.

   [2]

2. The first three terms of a geometric sequence are $u_1 = 0.64$, $u_2 = 1.6$, and $u_3 = 4$.

   (a) Find the value of r.

   [2]

   (b) Find the value of S_6.

   [2]

   (c) Find the least value of n such that $S_n > 75000$.

   [3]

3. Consider a geometric sequence where the first term is 768 and the second term is 576. Find the least value of n such that the nth term of the sequence is less than 7.

   [6]

4. In a geometric sequence, the fourth term is 8 times the first term. The sum of the first 10 terms is 2557.5. Find the 10th term of this sequence.

   [6]

5. Three consecutive terms of a geometric sequence are $x - 3$, 6, and $x + 2$.
   Find the possible values of x.

   [6]

6. An arithmetic sequence has the first term $\ln a$ and a common difference $\ln 3$. The 13th term in the sequence is $8 \ln 9$. Find the value of a.

   [6]

Name:

7. The first three terms of a geometric sequence are $\ln x^1 6$, $\ln x^8$, $\ln x^4$, for $x > 0$.

   (a) Find the common ratio.

   [3]

   (b) Solve $\sum_{k=1}^{\infty} 2^{5-k} \ln x = 64$.

   [5]

8. The first two terms of an infinite geometric sequence, in order, are $2\log_2 x$, $\log_2 x$, where $x > 0$.
   The first three terms of an arithmetic sequence, in order, are $\log_2 x$, $\log_2\left(\frac{x}{2}\right)$, $\log_2\left(\frac{x}{4}\right)$, where $x > 0$.
   Let S_{12} be the sum of the first 12 terms of the arithmetic sequence.

   (a) Find r.

   [2]

   (b) Show that the sum of the infinite sequence is $4\log_2 x$

   [2]

   (c) Find d, giving your answer as an integer.

   [4]

   (d) Show that $S_{12} = 12\log_2 x - 66$.

   [2]

   (e) Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x, giving your answer in the form 2^p, where $p \in \mathbb{Q}$.

   [5]

Logarithms (no calculator)

9. Find the value of each of the following, giving your answer as an integer.

   (a) $\log_6 36$.

   [2]

   (b) $\log_6 4 + \log_6 9$.

   [2]

   (c) $\log_6 2 - \log_6 12$.

   [3]

Name:

10. (a) Write down the value of

i. $\log_3 27$.

[1]

ii. $\log_8 \frac{1}{8}$.

[1]

iii. $\log_{16} 4$.

[1]

(b) Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$

[3]

11. Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y.

(a) $\ln\left(\frac{5}{3}\right)$.

[2]

(b) $\ln 45$.

[4]

12. Let $x = \ln 7$ and $y = \ln 3$. Write the following expressions in terms of x and y.

(a) $\ln\left(\frac{3}{7}\right)$.

[2]

(b) $\ln 63$.

[4]

13. (a) Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n.

[2]

(b) Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$.

[4]

14. Let $\log_3 p = 6$ and $\log_3 q = 7$

(a) Find $\log_3 p^2$.

[2]

(b) Find $\log_3\left(\frac{p}{q}\right)$.

[2]

(c) Find $\log_3(9p)$

[3]

Name:

15. (a) Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$.

[3]

(b) Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$.

[3]

16. (a) Find the value of $\log_2 40 - \log_2 5$.

[3]

(b) Find the value of $8^{\log_2 5}$.

[4]

17. (a) Find $\log_2 32$.

[1]

(b) Given that $\log_2\left(\frac{32^x}{8^y}\right)$ can be written as $px + qy$, find the value of p and of q.

[4]

18. Solve $\log_2 x + \log_2(x-2) = 3$, for $x > 2$.

[7]

19. Let $f(x) = 3\ln x$ and $g(x) = \ln 5x^3$.

(a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in Z^+$.

[4]

(b) The graph of g is a transformation of the graph of f. Give a full geometric description of this transformation.

[3]

20. Let $f(x) = k\log_2 x$.

(a) Given that $f^{-1}(1) = 8$, find the value of k.

[3]

(b) Find $f^{-1}(\frac{2}{3})$

[4]

21. Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a) Show that $f^{-1}(x) = 3^{2x}$.

[2]

Name:

(b) Write down the range of f^{-1}.

[1]

(c) Let $g(x) = \log_3 x$, for $x > 0$.
Find $(f^{-1} \circ g)(2)$, giving your answer as an integer.

[4]

22. Let $f(x) = e^{x+3}$.

(a) i. Show that $f^{-1}(x) = \ln x - 3$.

[3]

ii. Write down the domain of f^{-1}.

(b) Solve the equation $f^{-1}(x) = \ln \frac{1}{x}$.

[4]

23. Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$. [calculator allowed]

(a) Show that $f(x) = \log_3 2x$.

[2]

(b) Find the value of $f(0.5)$ and $f(4.5)$.

[3]

(c) The function f can also be written in the form $f(x) = \log_3 \frac{\ln ax}{\ln b}$

i. Write down the value of a and b.

ii. Hence on graph paper, sketch the graph of f, for $-5 \leq x \leq 5, -5 \leq y \leq 5$, using a scale of 1 cm to 1 unit on each axis.

iii. Write down the equation of the asymptote.

[6]

(d) Write down the value of $f^{-1}(0)$.

[1]

(e) The point A lies on the graph of f. At A, $x = 4.5$.
On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A.

[4]

Graphing calculator equation solving

24. Solve the equation $e^x = 4 \sin x$, for $0 \leq x \leq 2\pi$.

[5]

Name:

25. Let $f(x) = 4x - e^{x-2} - 3$, for $0 \leq x \leq 5$.
Find the x-intercepts of the graph of f.

[3]