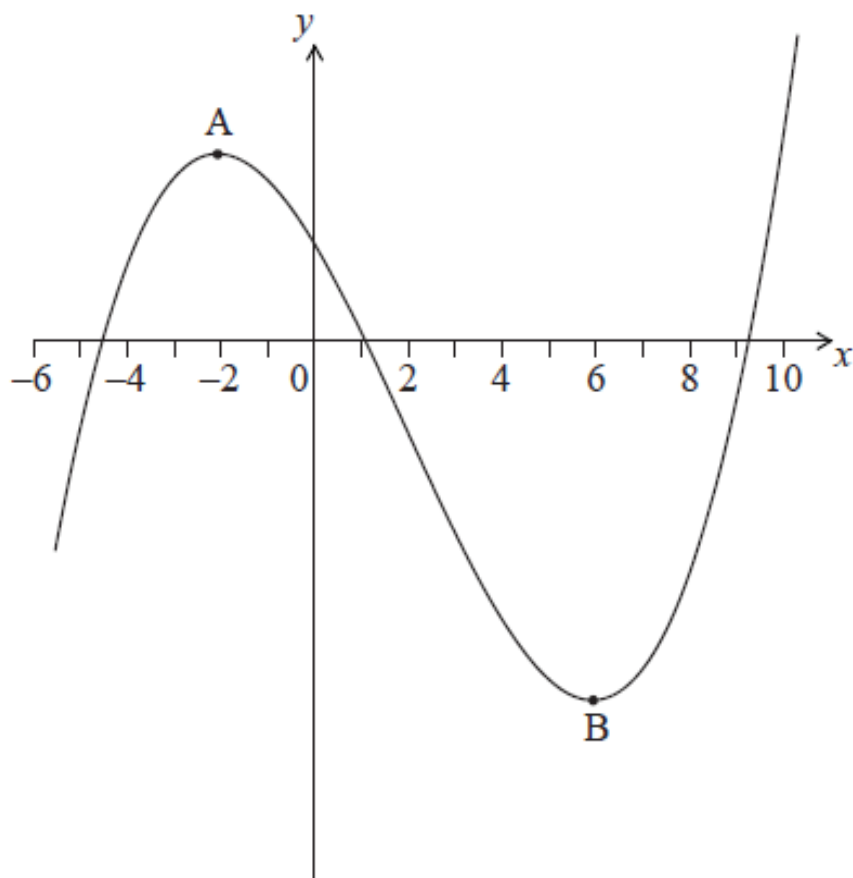


Calculus review: Function graphs

Answer the first four problems in the space provided.

1. The following diagram shows part of the graph of $y = f(x)$.

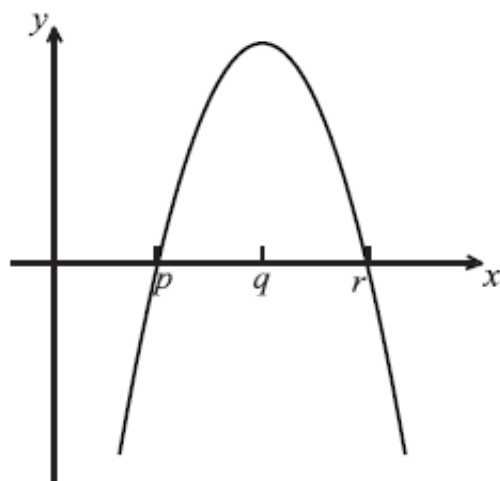


The graph has a local maximum at A , where $x = -2$, and a local minimum at B , where $x = 6$.

On the graph above, sketch the graph of $y = f'(x)$.

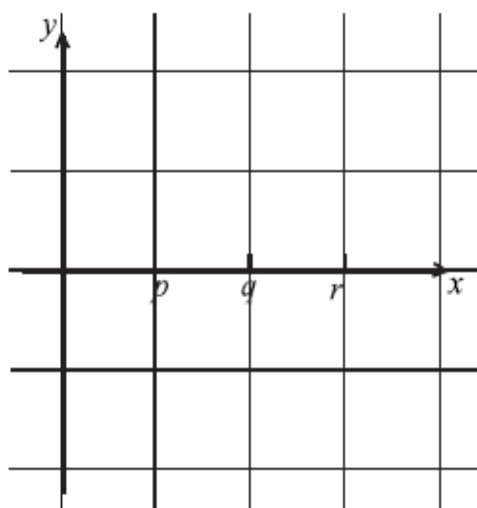
[4 marks]

2a. The diagram below shows part of the graph of the gradient function, $y = f'(x)$.



On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the x -intercept.

[2 marks]



2b. Complete the table, for the graph of $y = f(x)$.

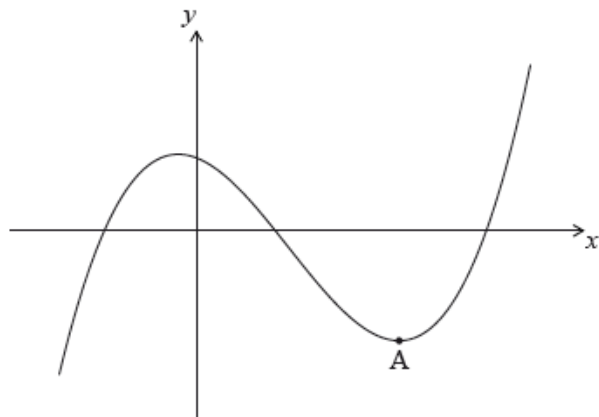
[2 marks]

	x -coordinate
(i) Maximum point on f	
(ii) Inflexion point on f	

2c. Justify your answer to part (b) (ii).

[2 marks]

3a. The following diagram shows the graph of a function f . There is a local minimum point at A , where $x > 0$.



The derivative of f is given by $f'(x) = 3x^2 - 8x - 3$.

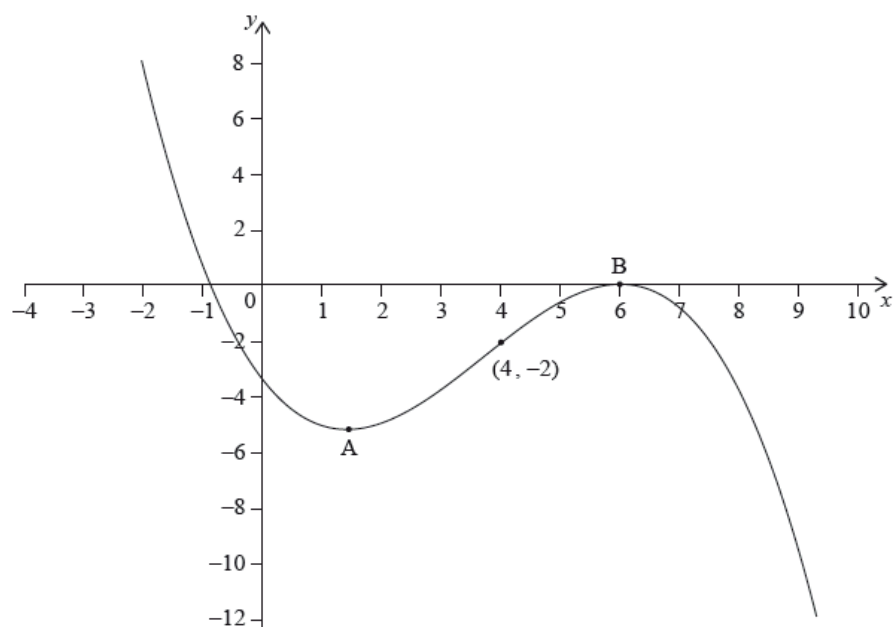
Find the x -coordinate of A .

[5 marks]

3b. The y -intercept of the graph is at $(0, 6)$. Find an expression for $f(x)$.

[6 marks]

4a. The following diagram shows the graph of f' , the derivative of f .



The graph of f' has a local minimum at A, a local maximum at B and passes through $(4, -2)$.

The point $P(4, 3)$ lies on the graph of the function, f .

Write down the gradient of the curve of f at P.

[1 mark]

4b. Find the equation of the normal to the curve of f at P.

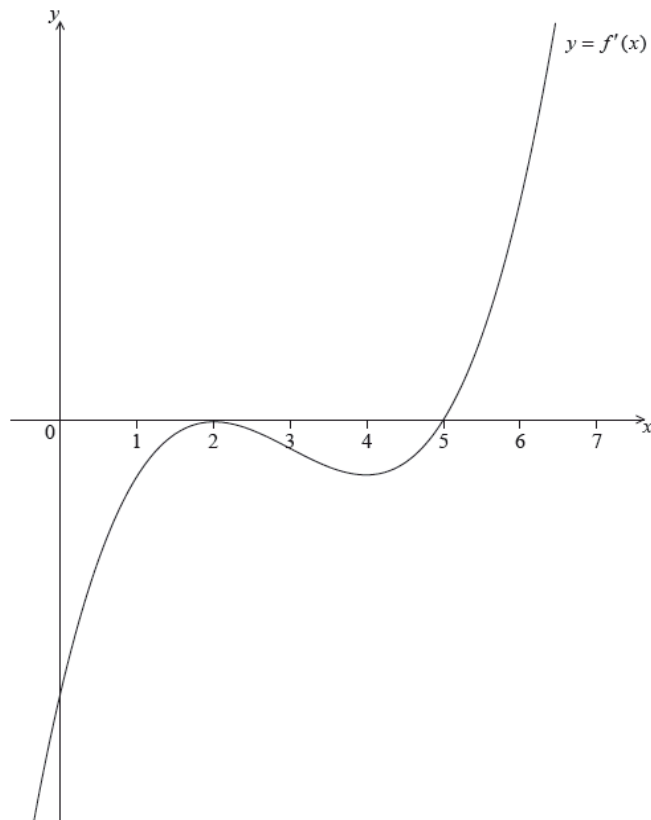
[3 marks]

4c. Determine the concavity of the graph of f when $4 < x < 5$ and justify your answer.

[2 marks]

For the remaining problems, answer on lined paper.

5a. Let $y = f(x)$, for $-0.5 \leq x \leq 6.5$. The following diagram shows the graph of f' , the derivative of f .



The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the x -axis at the point $(5, 0)$.

Explain why the graph of f has a local minimum when $x = 5$. [2 marks]

5b. Find the set of values of x for which the graph of f is concave down. [2 marks]

6a. Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

Use the quotient rule to show that $g'(x) = \frac{1-2 \ln x}{x^3}$. [4 marks]

6b. The graph of g has a maximum point at A. Find the x -coordinate of A. [3 marks]

7a. Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

Show that $f'(x) = \frac{\ln x}{x}$.

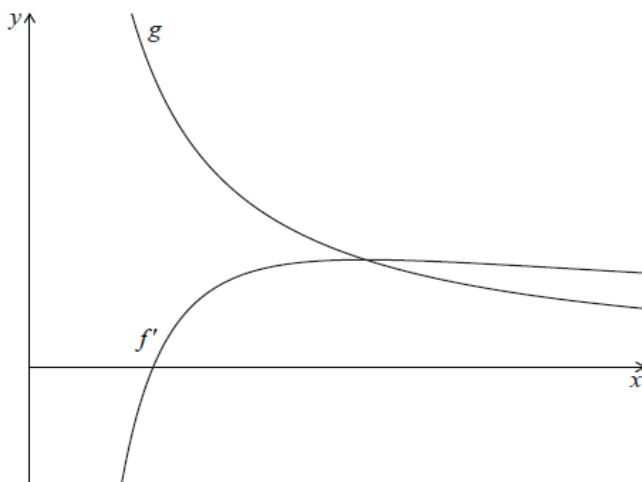
[2 marks]

7b. There is a minimum on the graph of f . Find the x -coordinate of this minimum.

[3 marks]

7c. Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g .

[2 marks]



The graph of f' has an x -intercept at $x = p$.

Write down the value of p .

7d. The graph of g intersects the graph of f' when $x = q$.

Find the value of q .

[3 marks]

7e. Let R be the region enclosed by the graph of f' , the graph of g and the line $x = p$.

Show that the area of R is $\frac{1}{2}$.

[5 marks]