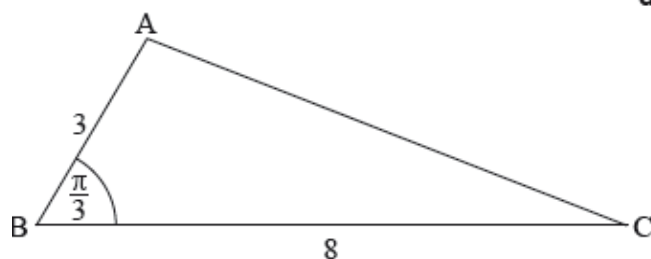


6-3 Problem Set: Trig functions

- 1a.** The following diagram shows triangle ABC, with $AB = 3$ cm, $BC = 8$ cm, and $\hat{ABC} = \frac{\pi}{3}$.

diagram not to scale

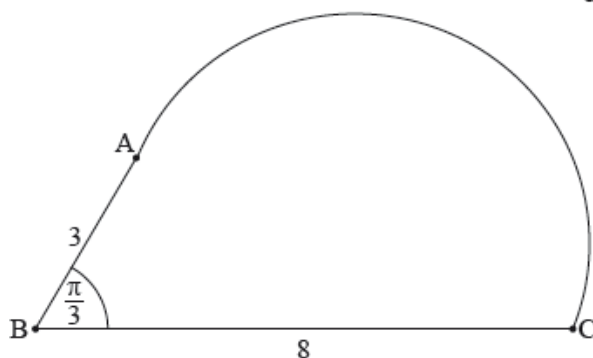


Show that $AC = 7$ cm.

[4 marks]

- 1b.** The shape in the following diagram is formed by adding a semicircle with diameter $[AC]$ to the triangle.

diagram not to scale

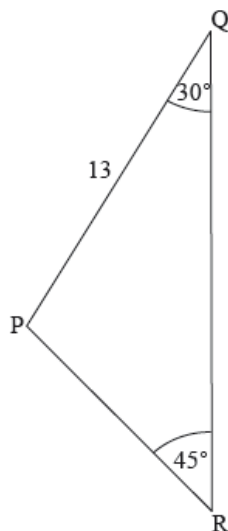


Find the exact perimeter of this shape.

[3 marks]

- 2.** The following diagram shows triangle PQR.

diagram not to scale



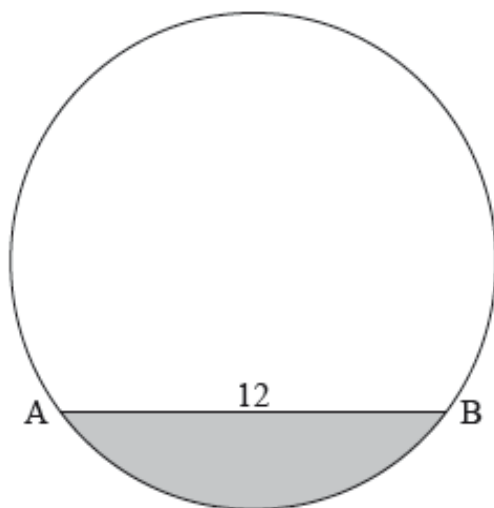
$\hat{PQR} = 30^\circ$, $\hat{QRP} = 45^\circ$ and $PQ = 13$ cm.

Find PR.

[6 marks]

3. The following diagram shows the chord $[AB]$ in a circle of radius 8 cm, where $AB = 12$ cm.

diagram not to scale



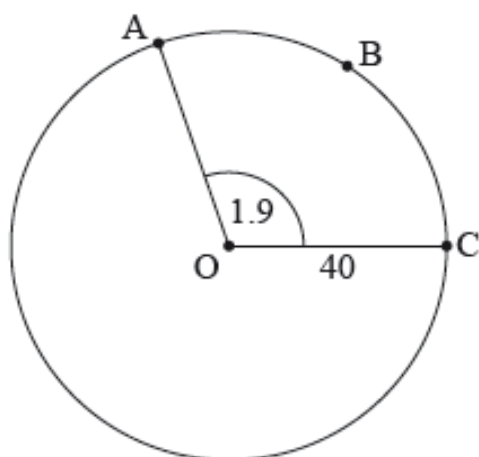
Find the area of the shaded segment.

[7 marks]

4a. [2 marks]

The following diagram shows a circle with centre O and radius 40 cm.

diagram not to scale



The points A, B and C are on the circumference of the circle and $\angle AOC = 1.9$ radians.

Find the length of arc ABC.

4b. Find the perimeter of sector OABC.

[2 marks]

4c. Find the area of sector OABC.

[2 marks]

5a. The depth of water in a port is modelled by the function $d(t) = p \cos qt + 7.5$, for $0 \leq t \leq 12$, where t is the number of hours after high tide.

At high tide, the depth is 9.7 metres.

At low tide, which is 7 hours later, the depth is 5.3 metres.

Find the value of p .

[2 marks]

5b. Find the value of q .

[2 marks]

5c. Use the model to find the depth of the water 10 hours after high tide.

[2 marks]

6a. [3 marks]

Let $\sin \theta = \frac{\sqrt{5}}{3}$, where θ is acute.

Find $\cos \theta$.

6b. [2 marks]

Find $\cos 2\theta$.

7a. Let $f(x) = 3 \sin\left(\frac{\pi}{2}x\right)$, for $0 \leq x \leq 4$.

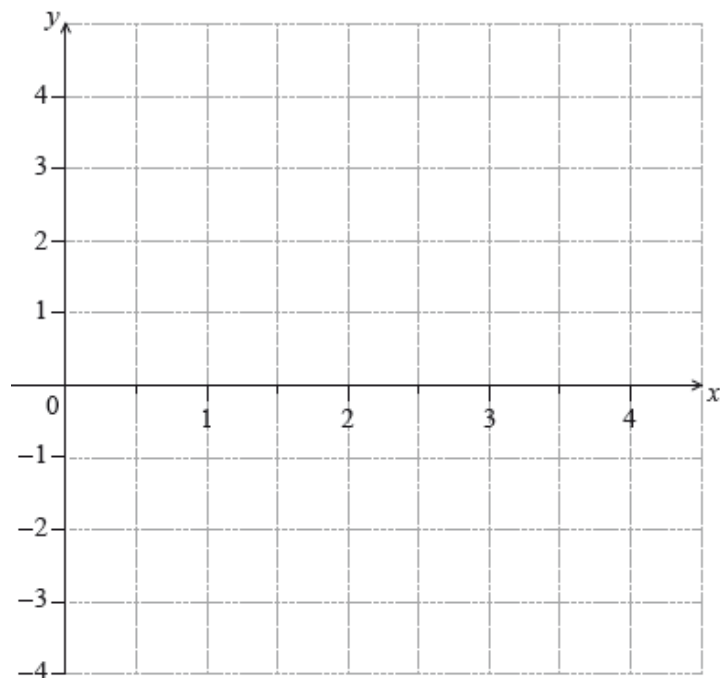
(i) Write down the amplitude of f .

(ii) Find the period of f .

[3 marks]

7b. [4 marks]

On the following grid sketch the graph of f .



8a. [5 marks]

Let $f(x) = 6x\sqrt{1-x^2}$, for $-1 \leq x \leq 1$, and $g(x) = \cos(x)$, for $0 \leq x \leq \pi$.

Let $h(x) = (f \circ g)(x)$.

Write $h(x)$ in the form $a \sin(bx)$, where $a, b \in \mathbb{Z}$.

8b. [2 marks]

Hence find the range of h .

9a. The height, h metres, of a seat on a Ferris wheel after t minutes is given by

$$h(t) = -15 \cos 1.2t + 17, \text{ for } t \geq 0.$$

Find the height of the seat when $t = 0$.

[2 marks]

9b. The seat first reaches a height of 20 m after k minutes. Find k .

[3 marks]

9c. Calculate the time needed for the seat to complete a full rotation, giving your answer correct to one decimal place.

[3 marks]

10a. Let $f(x) = 3 \sin(\pi x)$.

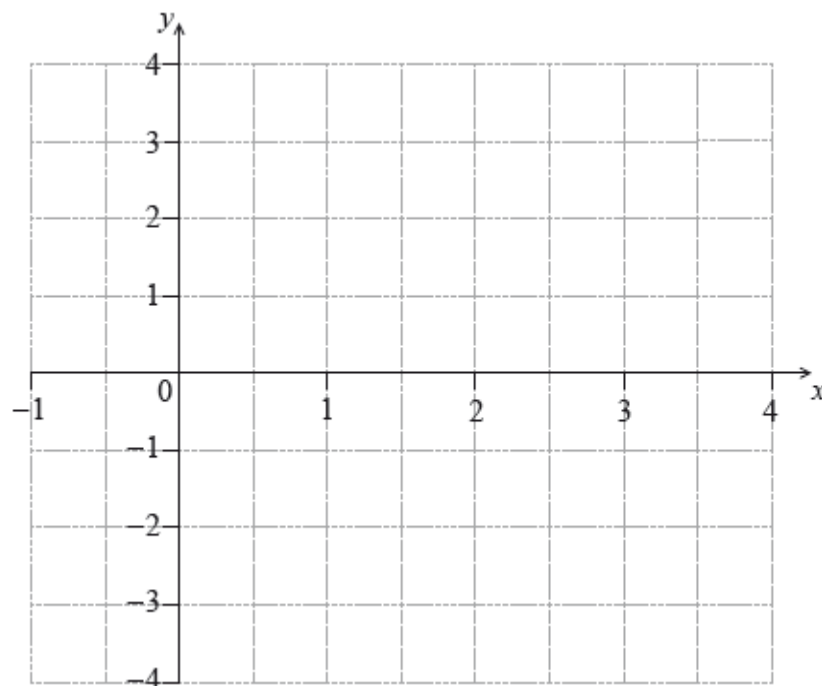
Write down the amplitude of f .

[1 mark]

10b. Find the period of f .

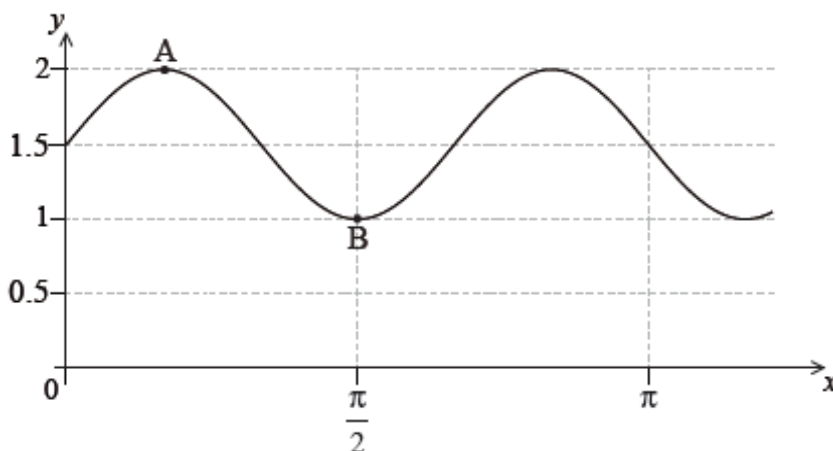
[2 marks]

10c. On the following grid, sketch the graph of $y = f(x)$, for $0 \leq x \leq 3$.



[4 marks]

11a. The following diagram shows part of the graph of $y = p \sin(qx) + r$.



The point $A \left(\frac{\pi}{6}, 2 \right)$ is a maximum point and the point $B \left(\frac{\pi}{6}, 1 \right)$ is a minimum point.

Find the value of p ;

[2 marks]

11b. r ;

[2 marks]

11c. q .

[2 marks]

12a. [3 marks]

Let $f(x) = \cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right)$, for $-4 \leq x \leq 4$.

Sketch the graph of f .

12b. [5 marks] Find the values of x where the function is decreasing.

12c. [3 marks]

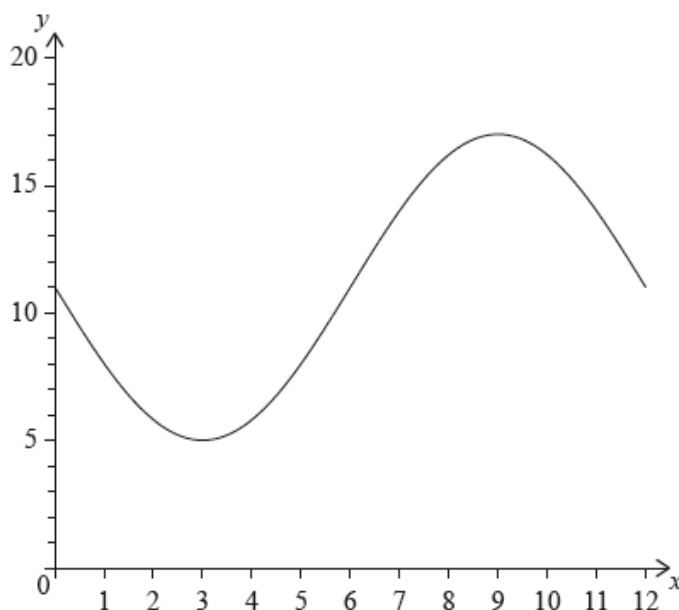
The function f can also be written in the form $f(x) = a \sin\left(\frac{\pi}{4}(x + c)\right)$, where $a \in \mathbb{R}$, and $0 \leq c \leq 2$. Find the value of a ;

12d. [4 marks]

The function f can also be written in the form $f(x) = a \sin\left(\frac{\pi}{4}(x + c)\right)$, where $a \in \mathbb{R}$, and $0 \leq c \leq 2$. Find the value of c .

13a. [6 marks]

The following diagram shows the graph of $f(x) = a \sin bx + c$, for $0 \leq x \leq 12$.



The graph of f has a minimum point at $(3, 5)$ and a maximum point at $(9, 17)$.

(i) Find the value of c .

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a .

13b. [3 marks]

The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5, 17)$.

(i) Write down the value of k .

(ii) Find $g(x)$.

13c. [6 marks]

The graph of g changes from concave-up to concave-down when $x = w$.

(i) Find w .

(ii) Hence or otherwise, find the maximum positive rate of change of g .