0308HW_Integration-areas *[33 marks]*

Consider a function
$f(x)$ such that
$\int_1^6 f(x)\mathrm{d}x = 8.$

1a. Find *[2 marks]*
$\int_1^6 2f(x)\mathrm{d}x.$

Markscheme

appropriate approach ***(M1)***

eg

$2\int f(x),\ 2(8)$

$\int_1^6 2f(x)\mathrm{d}x = 16$ ***A1*** ***N2***

[2 marks]

1b. Find *[4 marks]*
$\int_1^6 (f(x)+2)\,\mathrm{d}x.$

Markscheme

appropriate approach ***(M1)***

eg

$\int f(x) + \int 2,\ 8 + \int 2$

$\int 2\mathrm{d}x = 2x$ (seen anywhere) ***(A1)***

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg

$2(6) - 2(1),\ 8 + 12 - 2$

$\int_1^6 (f(x)+2)\,\mathrm{d}x = 18$ ***A1*** ***N3***

[4 marks]

2a. Find *[4 marks]*
$\int_4^{10} (x-4)\mathrm{d}x$.

Markscheme

correct integration ***A1A1***

e.g.
$\frac{x^2}{2} - 4x,$
$\left[\frac{x^2}{2} - 4x\right]_4^{10},$
$\frac{(x-4)^2}{2}$

Notes: In the first 2 examples, award ***A1*** for each correct term.

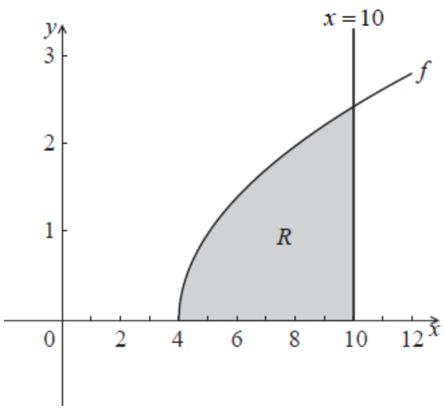
In the third example, award ***A1*** for
$\frac{1}{2}$ and ***A1*** for
$(x-4)^2$.

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

e.g.
$\left(\frac{10^2}{2} - 4(10)\right) - \left(\frac{4^2}{2} - 4(4)\right), 10 - (-8), \frac{1}{2}(6^2 - 0)$

$\int_4^{10} (x-4)\mathrm{d}x = 18$ ***A1*** ***N2***

2b. Part of the graph of
$f(x) = \sqrt{x-4}$, for
$x \geq 4$, is shown below. The shaded region *R* is enclosed by the graph of
f, the line
$x = 10$, and the *x*-axis. *[3 marks]*



The region *R* is rotated
360° about the *x*-axis. Find the volume of the solidformed.

Markscheme

attempt to substitute either limits or the function into volume formula ***(M1)***

e.g.
$\pi \int_4^{10} f^2 \mathrm{d}x,\ \int_a^b \left(\sqrt{x-4}\right)^2,\ \pi \int_4^{10} \sqrt{x-4}$

Note: Do not penalise for missing
π or dx.

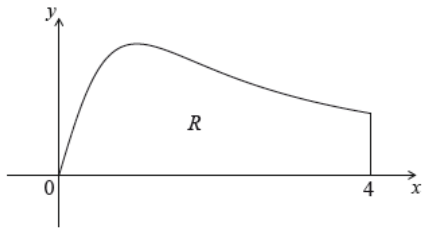
correct substitution (accept absence of dx and
π) ***(A1)***

e.g.
$\pi \int_4^{10} \left(\sqrt{x-4}\right)^2,\ \pi \int_4^{10} (x-4)\mathrm{d}x,\ \int_4^{10} (x-4)\mathrm{d}x$

volume =
18π ***A1*** ***N2***

[3 marks]

3. The following diagram shows the graph of $f(x) = \frac{x}{x^2+1}$, for $0 \le x \le 4$, and the line $x = 4$. *[6 marks]*



Let R be the region enclosed by the graph of f, the x-axis and the line $x = 4$.

Find the area of R.

Markscheme

substitution of limits or function ***(A1)***

eg $A = \int_0^4 f(x),\ \int \frac{x}{x^2+1}\mathrm{d}x$

correct integration by substitution/inspection ***A2***

$\frac{1}{2}\ln(x^2+1)$

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $\frac{1}{2}\left(\ln(4^2+1) - \ln(0^2+1)\right)$

correct working ***A1***

eg $\frac{1}{2}\left(\ln(4^2+1) - \ln(0^2+1)\right),\ \frac{1}{2}(\ln(17) - \ln(1)),\ \frac{1}{2}\ln 17 - 0$

$A = \frac{1}{2}\ln(17)$ ***A1*** ***N3***

Note: Exception to ***FT*** rule. Allow full ***FT*** on incorrect integration involving a $\ln$ function.

[6 marks]

Let $f(x) = x^2$ and $g(x) = 3\ln(x+1)$, for $x > -1$.

4a. Solve $f(x) = g(x)$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg sketch

0, 1.73843

$x = 0,\ x = 1.74\ (\text{accept } (0,\ 0) \text{ and } (1.74,\ 3.02))$ ***A1A1*** ***N3***

[3 marks]

4b. Find the area of the region enclosed by the graphs of f and g. *[3 marks]*

Markscheme

integrating and subtracting functions (in any order) ***(M1)***

eg $\int g - f$

correct substitution of their limits **or** function (accept missing $\mathrm{d}x$)

(A1)

eg $\int_0^{1.74} g - f,\ \int 3\ln(x+1) - x^2$

Note: Do not award ***A1*** if there is an error in the substitution.

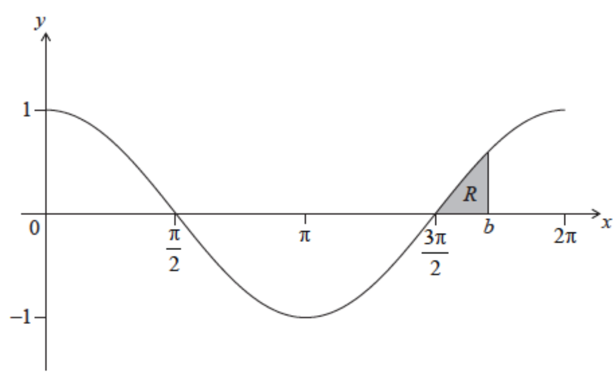
1.30940

1.31 ***A1*** ***N3***

[3 marks]

5. Let $f(x) = \cos x$, for $0 \le x \le 2\pi$. The following diagram shows the graph of f. *[8 marks]*

There are x-intercepts at $x = \frac{\pi}{2},\ \frac{3\pi}{2}$.



The shaded region R is enclosed by the graph of f, the line $x = b$, where $b > \frac{3\pi}{2}$, and the x-axis. The area of R is $\left(1 - \frac{\sqrt{3}}{2}\right)$. Find the value of b.

Markscheme

attempt to set up integral (accept missing or incorrect limits and missing $\mathrm{d}x$) ***M1***

eg $\int_{\frac{3\pi}{2}}^{b} \cos x \mathrm{d}x,\ \int_{a}^{b} \cos x \mathrm{d}x,\ \int_{\frac{3\pi}{2}}^{b} f \mathrm{d}x,\ \int \cos x$

correct integration (accept missing or incorrect limits) ***(A1)***

eg $[\sin x]_{\frac{3\pi}{2}}^{b},\ \sin x$

substituting correct limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $\sin b - \sin\left(\frac{3\pi}{2}\right),\ \sin\left(\frac{3\pi}{2}\right) - \sin b$

$\sin\left(\frac{3\pi}{2}\right) = -1$ (seen anywhere) ***(A1)***

setting **their** result from an integrated function equal to $\left(1 - \frac{\sqrt{3}}{2}\right)$ ***M1***

eg $\sin b = -\frac{\sqrt{3}}{2}$

evaluating $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ ***(A1)***

eg $b = \frac{\pi}{3},\ -60^{\circ}$

identifying correct value ***(A1)***

eg $2\pi - \frac{\pi}{3},\ 360 - 60$

$b = \frac{5\pi}{3}$ ***A1 N3***

[8 marks]