

**1a.** Line  $L_1$  passes through points  $A(3, 0, 7)$  and  $B(4, -1, 8)$ .

Find  $\overrightarrow{AB}$ .

[2 marks]

**1b.** Find an equation for  $L_1$  in the form  $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ .

[2 marks]

$$\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

**1c.** Line  $L_2$  has equation

Find the angle between  $L_1$  and  $L_2$ .

[7 marks]

**1d.** The lines  $L_1$  and  $L_2$  intersect at point C. Find the coordinates of C.

[6 marks]

$$\mathbf{r} = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$$

**2a.** The line  $L_1$  is represented by the vector equation

A second line  $L_2$  is parallel to  $L_1$  and passes through the point  $B(-8, -5, 25)$ .

Write down a vector equation for  $L_2$  in the form  $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ .

[2 marks]

$$\mathbf{r} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$$

**2b.** A third line  $L_3$  is perpendicular to  $L_1$  and is represented by

Show that  $k = -2$ .

[5 marks]

**2c.** The lines  $L_1$  and  $L_3$  intersect at the point A. Find the coordinates of A.

[6 marks]

$$\overrightarrow{BC} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$$

**2d.** The lines  $L_2$  and  $L_3$  intersect at point C where

(i) Find  $\overrightarrow{AB}$ .

(ii) Hence, find  $|\overrightarrow{AC}|$ .

[5 marks]

**3a.** Let  $\vec{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$  and  $\vec{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$ . Find  $\vec{BC}$ . [2 marks]

**3b.** Find a unit vector in the direction of  $\vec{AB}$ . [3 marks]

**3c.** Show that  $\vec{AB}$  is perpendicular to  $\vec{AC}$ . [3 marks]

**4a.** Let  $f(x) = \frac{6x}{x+1}$ , for  $x > 0$ . Find  $f'(x)$ . [5 marks]

**4b.** Let  $g(x) = \ln\left(\frac{6x}{x+1}\right)$ , for  $x > 0$ .

Show that  $g'(x) = \frac{1}{x(x+1)}$ . [4 marks]

**5a.** Let  $f(x) = e^{6x}$ . Write down  $f'(x)$ . [1 mark]

**5b.** The tangent to the graph of  $f$  at the point  $P(0, b)$  has gradient  $m$ .

(i) Show that  $m = 6$ .

(ii) Find  $b$ . [4 marks]

**5c.** Hence, write down the equation of this tangent. [1 mark]

**6a.** The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

<b>Distance, <math>x</math> km</b>	11 500	7500	13 600	10 800	9500	12 200	10 400
<b>Price, <math>y</math> dollars</b>	15 000	21 500	12 000	16 000	19 000	14 500	17 000

The relationship between  $x$  and  $y$  can be modelled by the regression equation  $y = ax + b$ .

(i) Find the correlation coefficient.

(ii) Write down the value of  $a$  and of  $b$ . [4 marks]

**6b.** On 1 January 2010, Lina buys a car which has travelled 11,000 km.

Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest 100 dollars. [3 marks]

**17a.** Let  $f(x) = \frac{1}{x-1} + 2$ , for  $x > 1$ .

Write down the equation of the horizontal asymptote of the graph of  $f$ . [2 marks]

**17b.** Find  $f'(x)$ . [2 marks]

**17c.** Let  $g(x) = ae^{-x} + b$ , for  $x \geq 1$ . The graphs of  $f$  and  $g$  have the same horizontal asymptote.

Write down the value of  $b$ . [2 marks]

**17d.** Given that  $g'(1) = -e$ , find the value of  $a$ . [4 marks]

**17e.** There is a value of  $x$ , for  $1 < x < 4$ , for which the graphs of  $f$  and  $g$  have the same gradient. Find this gradient. [4 marks]

**18a.** Let  $f(x) = (x - 5)^3$ , for  $x \in \mathbb{R}$ .

Find  $f^{-1}(x)$ . [3 marks]

**18b.** Let  $g$  be a function so that  $(f \circ g)(x) = 8x^6$ . Find  $g(x)$ . [3 marks]

**19a.** The following diagram shows part of the graph of a quadratic function  $f$ .

The vertex is at  $(1, -9)$ , and the graph crosses the  $y$ -axis at the point  $(0, c)$ .

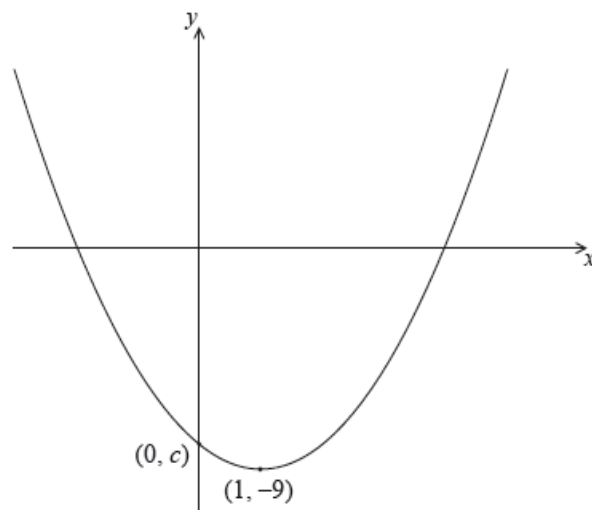
The function can be written in the form

$$f(x) = (x - h)^2 + k.$$

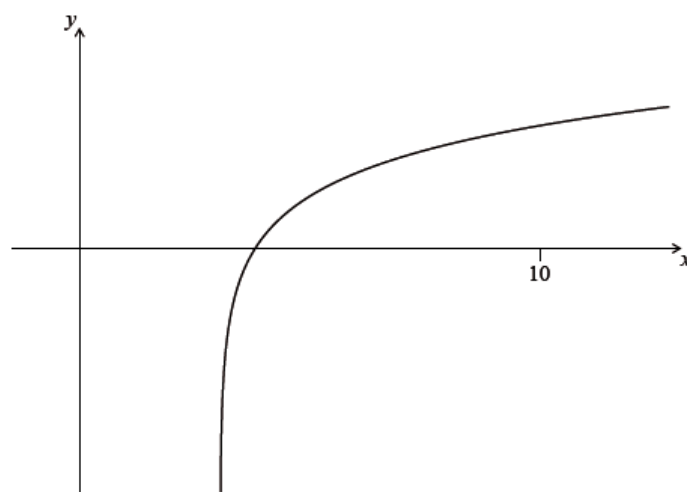
Write down the value of  $h$  and of  $k$ .

**19b.** Let  $g(x) = -(x - 3)^2 + 1$ . The graph of  $g$  is obtained by a reflection of the graph of  $f$  in the  $x$ -axis, followed by a translation of  $\begin{pmatrix} p \\ q \end{pmatrix}$ .

Find the value of  $p$  and of  $q$ . [5 marks]



**20a.** Let  $f(x) = 2 \ln(x - 3)$ , for  $x > 3$ . The diagram shows part of the graph of  $f$ . Find the equation of the vertical asymptote to the graph of  $f$ .



**20b.** Find the  $x$ -intercept of the graph of  $f$ .

**21a.** The first three terms of a geometric sequence are  $u_1 = 0.64$ ,  $u_2 = 1.6$ , and  $u_3 = 4$ .

Find the value of  $r$ .

**21b.** Find the value of  $S_6$ .

[2 marks]

**21c.** Find the least value of  $n$  such that  $S_n > 75\,000$ .

[3 marks]