Name: ____________________________________ Date: __________________

RATIONAL EXPONENTS
COMMON CORE ALGEBRA II

When you first learned about exponents, they were always **positive integers**, and just represented **repeated multiplication**. And then we had to go and introduce **negative exponents**, which really just represent **repeated division**. Today we will introduce **rational (or fractional) exponents** and extend your exponential knowledge that much further.

***Exercise* #1:** Recall the **Product Property of Exponents** and use it to rewrite each of the following as a simplified exponential expression. There is no need to find a final numerical value.

(a) $\left(2^3\right)^4$ (b) $\left(5^{-2}\right)^5$ (c) $\left(3^7\right)^0$ (d) $\left(\left(4^2\right)^{-2}\right)^2$

We will now use the Product Property to extend our understanding of exponents to include **unit fraction** exponents (those of the form $1/n$ where n is a positive integer).

***Exercise* #2:** Consider the expression $16^{1/2}$.

(a) Apply the Product Property to simplify $\left(16^{1/2}\right)^2$. What other number squared yields 16?

(b) You can now say that $16^{1/2}$ is equivalent to what more familiar quantity?

This is remarkable! An exponent of $1/2$ is equivalent to a square root of a number!!!

***Exercise* #3:** Test the equivalence of the $1/2$ exponent to the square root by using your calculator to evaluate each of the following. Be careful in how you enter each expression.

(a) $25^{1/2} =$ (b) $81^{1/2} =$ (c) $100^{1/2} =$

We can extend this now to all levels of roots, that is square roots, cubic roots, fourth roots, etcetera.

UNIT FRACTION EXPONENTS

For n given as a positive integer: $b^{1/n} = \sqrt[n]{b}$

***Exercise* #4:** Rewrite each of the following using roots instead of fractional exponents. Then, if necessary, evaluate using your calculator to guess and check to find the roots (don't use the generic root function). Check with your calculator.

(a) $125^{1/3}$ (b) $16^{1/4}$ (c) $9^{-1/2}$ (d) $32^{-1/5}$

We can now combine traditional integer powers with unit fractions in order interpret any exponent that is a **rational number**, i.e. the **ratio of two integers**. The next exercise will illustrate the thinking. Remember, we want our exponent properties to be consistent with the structure of the expression.

***Exercise* #5:** Let's think about the expression $4^{3/2}$.

(a) Fill in the missing blank and then evaluate this expression:

$$4^{3/2}=(\quad)^{1/2}$$

(b) Fill in the missing blank and then evaluate this expression:

$$4^{3/2}=(\quad)^{3}$$

(c) Verify both (a) and (b) using your calculator.

(d) Evaluate $27^{2/3}$ without your calculator. Show your thinking. Verify with your calculator.

RATIONAL EXPONENT CONNECTION TO ROOTS

For the rational number m/n we define $b^{m/n}$ to be: $\sqrt[n]{b^m}$ or $\left(\sqrt[n]{b}\right)^m$.

***Exercise* #6:** Evaluate each of the following exponential expressions involving rational exponents without the use of your calculator. Show your work. Then, check your final answers with the calculator.

(a) $16^{3/4}$ (b) $25^{3/2}$ (c) $8^{-2/3}$

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RATIONAL EXPONENTS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Rewrite the following as equivalent roots and then evaluate as many as possible **without your calculator.**

(a) $36^{1/2}$ (b) $27^{1/3}$ (c) $32^{1/5}$ (d) $100^{-1/2}$

(e) $625^{1/4}$ (f) $49^{1/2}$ (g) $81^{-1/4}$ (h) $343^{1/3}$

2. Evaluate each of the following by considering the root and power indicated by the exponent. Do as many as possible **without your calculator**.

(a) $8^{2/3}$ (b) $4^{3/2}$ (c) $16^{-3/4}$ (d) $81^{5/4}$

(e) $4^{-5/2}$ (f) $128^{3/7}$ (g) $625^{3/4}$ (h) $243^{3/5}$

3. Given the function $f(x)=5(x+4)^{3/2}$, which of the following represents its y-intercept?

(1) 40 (3) 4

(2) 20 (4) 30

____

4. Which of the following is equivalent to $x^{-1/2}$?

(1) $-\frac{1}{2}x$ (3) $\frac{1}{\sqrt{x}}$

(2) $-\sqrt{x}$ (4) $-\frac{1}{2x}$ ________

5. Written without fractional or negative exponents, $x^{-3/2}$ is equal to

(1) $-\frac{3x}{2}$ (3) $\frac{1}{\sqrt{x^3}}$

(2) $\frac{1}{\sqrt[3]{x^2}}$ (4) $-\frac{1}{\sqrt{x}}$ ________

6. Which of the following is *not* equivalent to $16^{3/2}$?

(1) $\sqrt{4096}$ (3) 64

(2) 8^3 (4) $\sqrt{16^3}$ ________

Reasoning

7. Marlene claims that the square root of a cube root is a sixth root? Is she correct? To start, try rewriting the expression below in terms of fractional exponents. Then apply the **Product Property of Exponents**.

$\sqrt{\sqrt[3]{a}}$

8. We should know that $\sqrt[3]{8}=2$. To see how this is equivalent to $8^{1/3}=2$ we can solve the equation $8^n=2$. To do this, we can rewrite the equation as:

$$\left(2^3\right)^n=2^1$$

How can we now use this equation to see that $8^{1/3}=2$?