Vector Test Name: REFERENCE
17 January 2020

1. Find a value for n that will give vector $\vec{a}$ a magnitude of 7:

$$\vec{a} = \begin{pmatrix} -3 \\ 2 \\ n \end{pmatrix}$$

$|\vec{a}| = 7$

$|\vec{a}| = \sqrt{(-3)^2 + 2^2 + n^2}$ $\quad 49 = 9 + 4 + n^2$

$n^2 = 36 \quad n = 6$ or -6

2. Find values for b_x, b_y, and b_z that will make $\vec{b}$ perpendicular to $\vec{a}$ regardless of the value of m in $\vec{a}$:

$$\vec{a} = \begin{pmatrix} -2 \\ 1 \\ m \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$$

$\vec{a} \perp \vec{b} \rightarrow \vec{a} \cdot \vec{b} = 0$

$-2b_x + b_y + mb_z = 0$

Since we can't account for the m value in the z dimension in $\vec{a}$ without knowing the value of m ahead of time, we must set b_z to zero.

$b_z = 0 \quad -2b_x + b_y = 0 \rightarrow b_x = 1,\ b_y = 2$ works

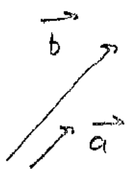
3. Let $\vec{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$:

(a) Find the unit vector for $\vec{a}$: $= \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$|\vec{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$= \begin{pmatrix} \frac{1}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} \end{pmatrix}$

(b) Consider some vector $\vec{b} = k\vec{a}$ for some positive value of k (that is, $\vec{b}$ is parallel to $\vec{a}$). What is the unit vector of $\vec{b}$:

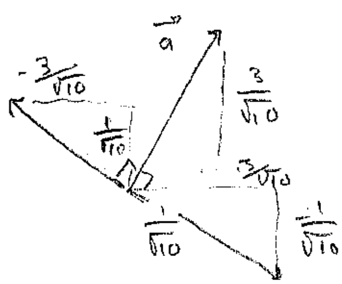


Since $\vec{a}$ and $\vec{b}$ are parallel, (with positive k) they must have the same unit vector:

$\begin{pmatrix} \frac{1}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} \end{pmatrix}$

(c) Consider some vector $\vec{b}$ which is perpendicular to $\vec{a}$. Find a possible value for the unit vector of $\vec{b}$:

Perpendicular vector has negative reciprocal slope. $\begin{pmatrix} \frac{-3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{pmatrix}$ or $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$



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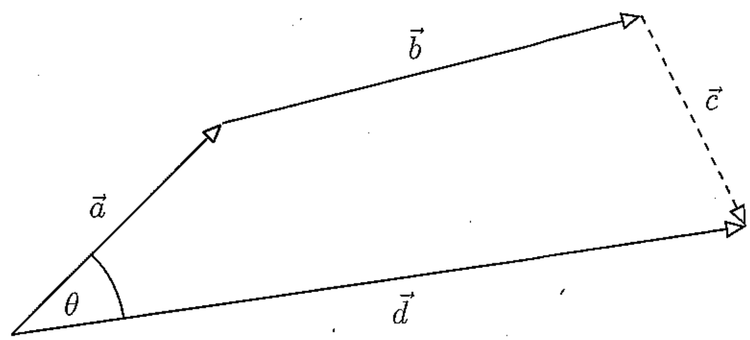
4. Consider the path formed by the 4 vectors in the diagram below:



(a) Write an equation for $\vec{c}$ in terms of $\vec{a}$, $\vec{b}$, and $\vec{d}$:

$$\vec{a} + \vec{b} + \vec{c} = \vec{d}$$
$$\vec{c} = \vec{d} - \vec{a} - \vec{b}$$

(b) Given the following values for $\vec{a}$, $\vec{b}$, and $\vec{d}$, calculate the value of $\vec{c}$:

$$\vec{a} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \qquad \vec{d} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

$$\vec{c} = \begin{pmatrix} 7 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

(c) For the above values vector values, calculate the angle θ between the $\vec{a}$ and $\vec{d}$:

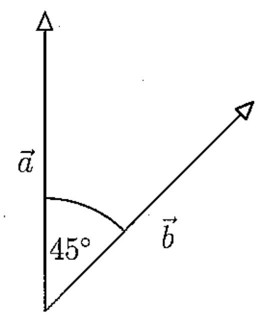
$$|\vec{a}| = \sqrt{2^2+2^2} = \sqrt{8}$$
$$|\vec{b}| = \sqrt{7^2+1^2} = \sqrt{50}$$

$$\vec{a} \cdot \vec{d} = |\vec{a}||\vec{d}| \cos\theta$$
$$16 = \sqrt{8}\sqrt{50} \cos\theta$$

$$\vec{a} \cdot \vec{d} = 2 \cdot 7 + 2 \cdot 1 = 16$$

$$\cos\theta = \frac{16}{\sqrt{8}\sqrt{50}} = \frac{16}{\sqrt{400}} = \frac{16}{20} = \frac{4}{5} = \cos\theta$$
$$\theta = \cos^{-1}\left(\frac{4}{5}\right) = 36.87^\circ$$

5. In the diagram below, $\vec{a}$ and $\vec{b}$ are both unit vectors with a 45° angle between them. Find the value of the dot product $\vec{a} \cdot \vec{b}$:



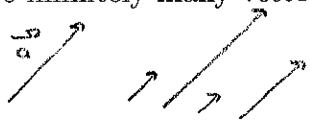
$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$

$= (1)(1) \cos 45°$

$= \cos 45° = \frac{1}{\sqrt{2}} = .7071$

6. Mark each of the following statements as either True or False:

   (a) For a given vector $\vec{a}$, there are infinitely many vectors $\vec{b}$ such that $\vec{b}$ is parallel to $\vec{a}$. True or False?

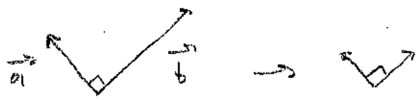
   

   can scale $\vec{b}$ by any length

   (b) For a given vector $\vec{a}$, there are infinitely many *unit* vectors $\vec{b}$ such that $\vec{b}$ is parallel to $\vec{a}$. True or False?

   $\vec{b}$ can only be the unit vector of $\vec{a}$ (or of $-\vec{a}$). So only 2 choices

   (c) If $\vec{a}$ and $\vec{b}$ are perpendicular, then the unit vectors for $\vec{a}$ and $\vec{b}$ are also perpendicular. True or False?

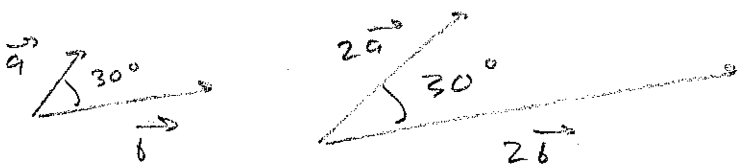
   (d) If $\vec{a}$ and $\vec{b}$ are parallel and $\vec{a} \cdot \vec{b} = |\vec{a}|^2$ (the magnitude of $\vec{a}$ squared), then $\vec{a}$ must $\vec{b}$ must have the same magnitude. True or False?

   $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| = |\vec{a}||\vec{a}|$

   cosine of angle between $\vec{a}$ and $\vec{b}$ is 1, so $|\vec{a}| = |\vec{b}|$

   (e) If the angle between $\vec{a}$ and $\vec{b}$ is 30°, then the angle between $2\vec{a}$ and $2\vec{b}$ must be 60°. True or False?

   angle is still 30°



7. Challenge Problem (extra credit):

Consider the following three vectors:

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix} \qquad \vec{c} = \begin{pmatrix} 1 \\ c_y \\ c_z \end{pmatrix}$$

Find the values for c_y and c_z that make $\vec{c}$ perpendicular to both $\vec{a}$ and $\vec{b}$ at the same time: (*hint: start by setting up the dot-product equations*)

$\vec{a} \cdot \vec{c} = 0 = 1 + 2C_y - 3C_z$

$\vec{b} \cdot \vec{c} = 0 = 5 - C_y + C_z$

$C_y = C_z + 5$

$0 = 1 + 2(C_z + 5) - 3C_z$

$0 = 1 + 10 - C_z$ $\quad \underline{C_z = 11}$ $\quad \underline{C_y = 11 + 5 = 16}$

$$\boxed{\vec{c} = \begin{pmatrix} 1 \\ 16 \\ 11 \end{pmatrix}}$$

$\vec{a} \cdot \vec{c} = 1 \cdot 1 + 2 \cdot 16 - 3 \cdot 11 = 33 - 33 = 0$ ✓

$\vec{b} \cdot \vec{c} = 5 \cdot 1 + -1 \cdot 16 + 1 \cdot 11 = 5 - 16 + 11 = 0$ ✓