BECA / Huson / 12.1 IB Math SL Name:
8 December 2017
Homework: Challenging vector and calculus problems

1a. Consider the points $\mathrm{A}\ (1,\ 5,\ -7)$ and $\mathrm{B}\ (-9,\ 9,\ -6)$.

Find $\overrightarrow{\mathrm{AB}}$. *[2 marks]*

1b. Let C be a point such that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$.

Find the coordinates of C. *[2 marks]*

1c. The line L passes through B and is parallel to (AC).

Write down a vector equation for L. *[2 marks]*

1d. Given that $\left|\overrightarrow{\mathrm{AB}}\right| = k\left|\overrightarrow{\mathrm{AC}}\right|$, find k. *[3 marks]*

1e. The point D lies on L such that $\left|\overrightarrow{\mathrm{AB}}\right| = \left|\overrightarrow{\mathrm{BD}}\right|$. Find the possible coordinates of D. *[6 marks]*

2a. *[3 marks]* A line L_1 passes through the points $\mathrm{A}(0,\ -3,\ 1)$ and $\mathrm{B}(-2,\ 5,\ 3)$.

(i) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$.

(ii) Write down a vector equation for L_1.

2b. A line L_2 has equation $\mathbf{r} = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$. The lines L_1 and L_2 intersect at a point C.

Show that the coordinates of C are $(-1,\ 1,\ 2)$. *[5 marks]*

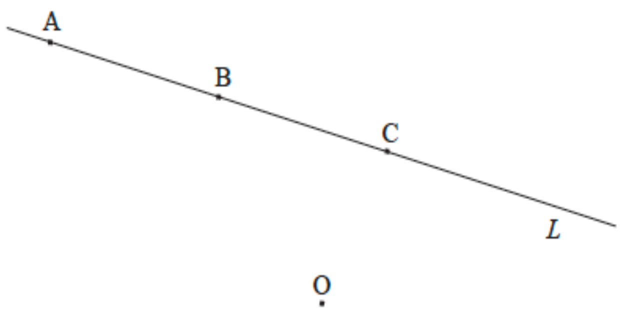
2c. A point D lies on line L_2 so that $\left|\overrightarrow{\mathrm{CD}}\right| = \sqrt{18}$ and $\overrightarrow{\mathrm{CA}} \bullet \overrightarrow{\mathrm{CD}} = -9$. Find $\mathrm{A\hat{C}D}$. *[7 marks]*

3a. A line L passes through points $\mathrm{A}(-2,\ 4,\ 3)$ and $\mathrm{B}(-1,\ 3,\ 1)$. *[3 marks]*

(i) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$.

(ii) Find $\left|\overrightarrow{\mathrm{AB}}\right|$.

3b. The following diagram shows the line L and the origin O. The point C also lies on L.



Point C has position vector $\begin{pmatrix} 0 \\ y \\ -1 \end{pmatrix}$.

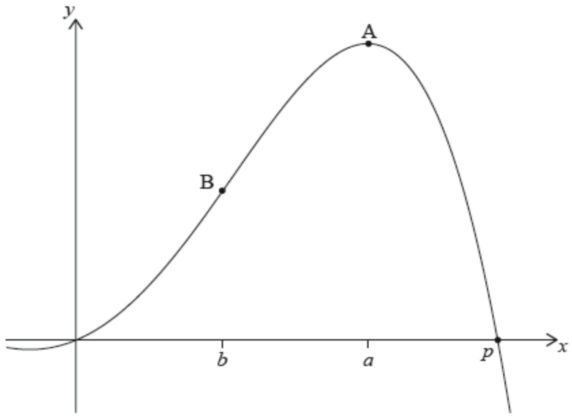
Show that $y = 2$. *[4 marks]*

3c. (i) Find $\overrightarrow{\mathrm{OC}} \bullet \overrightarrow{\mathrm{AB}}$.

(ii) Hence, write down the size of the angle between C and L. *[3 marks]*

3d. Hence or otherwise, find the area of triangle OAB. *[4 marks]*

4a. Let $f(x) = -0.5x^4 + 3x^2 + 2x$. The following diagram shows part of the graph of f.



There are x-intercepts at $x = 0$ and at $x = p$. There is a maximum at A where $x = a$, and a point of inflexion at B where $x = b$.

Find the value of p. *[2 marks]*

4b. Write down the coordinates of A. *[2 marks]*

4c. Write down the rate of change of f at A. *[1 mark]*

4d. Find the coordinates of B. *[4 marks]*

4e. Find the the rate of change of f at B. *[3 marks]*

4f. Let R be the region enclosed by the graph of f , the x-axis, the line $x = b$ and the line $x = a$. The region R is rotated 360° about the x-axis. Find the volume of the solid formed. *[3 marks]*

5a. Let $f(x) = \cos x$. *[4 marks]*

(i) Find the first four derivatives of $f(x)$.

(ii) Find $f^{(19)}(x)$.

5b. Let $g(x) = x^k$, where $k \in \mathbb{Z}^+$. *[5 marks]*

(i) Find the first three derivatives of $g(x)$.

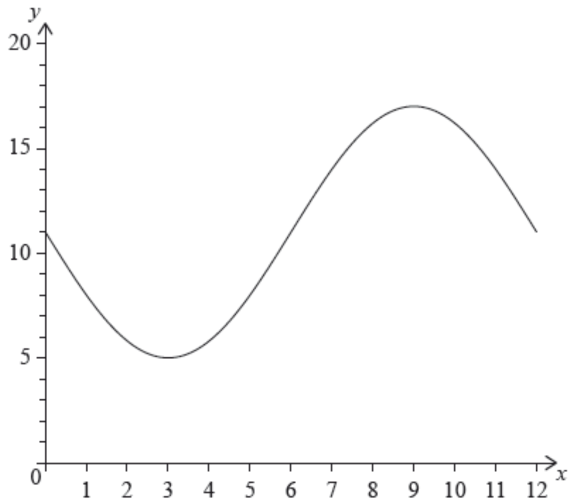
(ii) Given that $g^{(19)}(x) = \frac{k!}{(k-p)!}\left(x^{k-19}\right)$, find p.

5c. Let $k = 21$ and $h(x) = \left(f^{(19)}(x) \times g^{(19)}(x)\right)$. *[7 marks]*

(i) Find $h'(x)$.

(ii) Hence, show that $h'(\pi) = \frac{-21!}{2}\pi^2$.

6a. *[6 marks]* The following diagram shows the graph of $f(x) = a\sin bx + c$, for $0 \leqslant x \leqslant 12$.



The graph of f has a minimum point at $(3,\ 5)$ and a maximum point at $(9,\ 17)$.

(i) Find the value of c.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a.

6b. *[3 marks]* The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5,\ 17)$.

(i) Write down the value of k.

(ii) Find $g(x)$.

6c. *[6 marks]* The graph of g changes from concave-up to concave-down when $x = w$.

(i) Find w.

(ii) Hence or otherwise, find the maximum positive rate of change of g.