Test: Vector algebra and differential calculus

1a. Let $f(x) = e^{6x}$.

Write down $f'(x)$.   *[1 mark]*

1b. The tangent to the graph of f at the point $P(0, b)$ has gradient m .

(i) Show that $m = 6$.

(ii) Find b.   *[4 marks]*

1c. Hence, write down the equation of this tangent.   *[1 mark]*

2a. Let $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

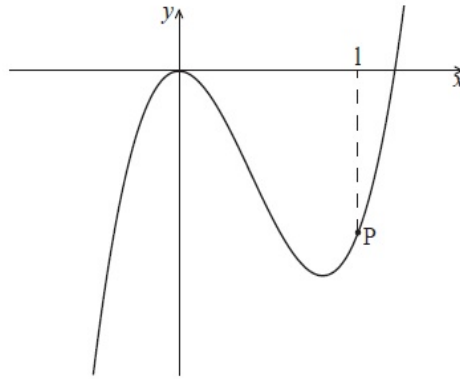Find $\overrightarrow{BC}$.   *[2 marks]*

2b. Find a unit vector in the direction of $\overrightarrow{AB}$.   *[3 marks]*

2c. Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$.   *[3 marks]*

3a. Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.

The point P lies on the graph of f . At P, x = 1.
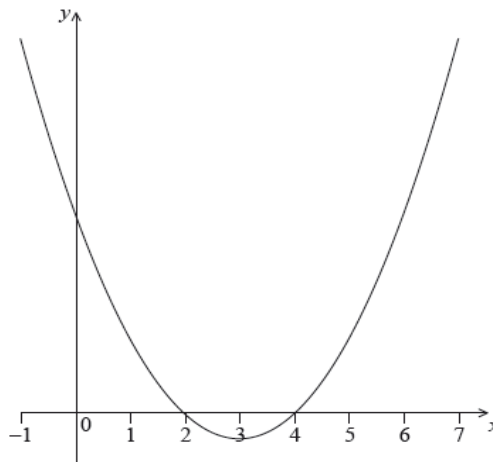
Find $f'(x)$.   [2 marks]

3b. The graph of f has a gradient of 3 at the point P. Find the value of a .   [4 marks]

4a. The following diagram shows part of the graph of a quadratic function f.

The vertex is at $(3, \ -1)$ and the x-intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x - h)^2 + k$.

Write down the value of h and of k.   [2 marks]

4b. The function can also be written in the form $f(x) = (x - a)(x - b)$.

Write down the value of a and of b.   [2 marks]

4c. Find the y-intercept.   [2 marks]

5a. The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

Distance, x km	11 500	7500	13 600	10 800	9500	12 200	10 400
Price, y dollars	15 000	21 500	12 000	16 000	19 000	14 500	17 000

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

(i) Find the correlation coefficient.

(ii) Write down the value of a and of b.   *[4 marks]*

5b. On 1 January 2010, Lina buys a car which has travelled $11\,000$ km.

Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest 100 dollars.   *[3 marks]*

5c. The price of a car decreases by 5% each year.

Calculate the price of Lina's car after 6 years.   *[4 marks]*

5d. Lina will sell her car when its price reaches $10\,000$ dollars.

Find the year when Lina sells her car.   *[4 marks]*

6a. Line L_1 passes through points $A(3, 0, 7)$ and $B(4, -1, 8)$.

Find $\overrightarrow{AB}$.   *[2 marks]*

6b. Find an equation for L_1 in the form $r = a + tb$.   *[2 marks]*

$$r = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

6c. Line L_2 has equation   .

Find the angle between L_1 and L_2 .   *[7 marks]*

6d. The lines L_1 and L_2 intersect at point C. Find the coordinates of C.   *[6 marks]*

Spicy

7a. *In this question, distance is in metres.*

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, p seconds after it has passed through A, is given by
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}.$$

(i) Write down the coordinates of A.

(ii) Find the speed of the airplane in ms^{-1}.   *[4 marks]*

7b. After seven seconds the airplane passes through a point B.

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds.   *[5 marks]*

7c. Airplane 2 passes through a point C. Its position q seconds after it passes through C is given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}, a \in \mathbb{R}.$$

The angle between the flight paths of Airplane 1 and Airplane 2 is $40°$. Find the two values of a.

[7 marks]