Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $f(x) = a(x-h)^2 + k$. The vertex of the graph of f is at $(2, 3)$ and the graph passes through $(1, 7)$.

(a) Write down the value of h and of k. *[2]*

(b) Find the value of a. *[3]*

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

The following diagram shows a right-angled triangle, ABC, where $\sin A = \frac{5}{13}$.

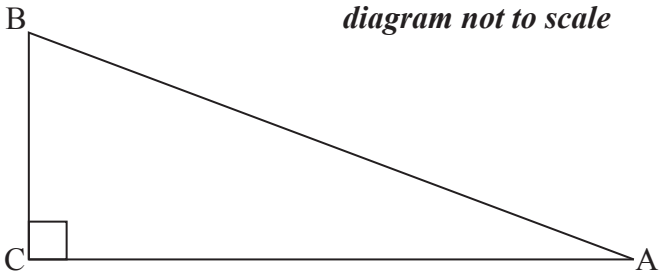

diagram not to scale

(a) Show that $\cos A = \frac{12}{13}$. *[2]*

(b) Find $\cos 2A$. *[3]*

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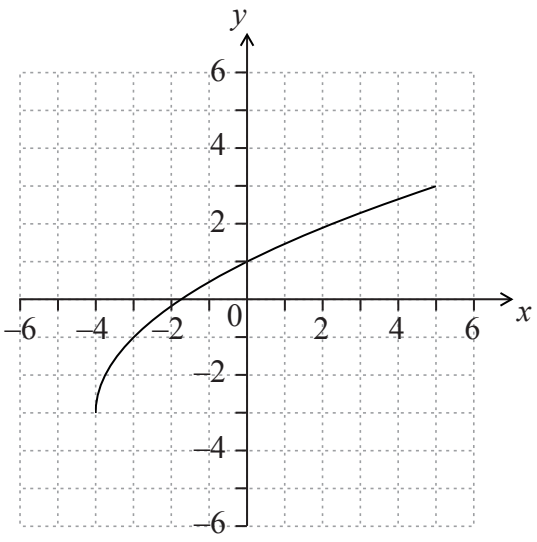
3. *[Maximum mark: 6]*

The following diagram shows the graph of $y = f(x)$, for $-4 \le x \le 5$.



(a) Write down the value of

(i) $f(-3)$;

(ii) $f^{-1}(1)$. *[2]*

(b) Find the domain of f^{-1}. *[2]*

(c) On the grid above, sketch the graph of f^{-1}. *[2]*

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4. *[Maximum mark: 6]*

(a) Write down the value of

(i) $\log_3 27$;

(ii) $\log_8 \frac{1}{8}$;

(iii) $\log_{16} 4$. *[3]*

(b) Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$. *[3]*

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4. *[Maximum mark: 6]*

(a) Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$. *[3]*

(b) Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$. *[3]*

5. *[Maximum mark: 6]*

Celeste wishes to hire a taxicab from a company which has a large number of taxicabs. The taxicabs are randomly assigned by the company.

The probability that a taxicab is yellow is 0.4.
The probability that a taxicab is a Fiat is 0.3.
The probability that a taxicab is yellow or a Fiat is 0.6.

Find the probability that the taxicab hired by Celeste is **not** a yellow Fiat.

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5. *[Maximum mark: 6]*

Let $f(x) = p + \frac{9}{x-q}$, for $x \neq q$. The line $x = 3$ is a vertical asymptote to the graph of f.

(a) Write down the value of q. *[1]*

The graph of f has a y-intercept at $(0, 4)$.

(b) Find the value of p. *[4]*

(c) Write down the equation of the horizontal asymptote of the graph of f. *[1]*

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*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

Let $f(x) = 3x^2 - 6x + p$. The equation $f(x) = 0$ has two equal roots.

(a) (i) Write down the **value** of the discriminant.

(ii) Hence, show that $p = 3$. *[3]*

The graph of f has its vertex on the x-axis.

(b) Find the coordinates of the vertex of the graph of f. *[4]*

(c) Write down the solution of $f(x) = 0$. *[1]*

(d) The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of

(i) a;

(ii) h;

(iii) k. *[3]*

(e) The graph of a function g is obtained from the graph of f by a reflection of f in the x-axis, followed by a translation by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. Find g, giving your answer in the form $g(x) = Ax^2 + Bx + C$. *[4]*

*Do **NOT** write solutions on this page.*

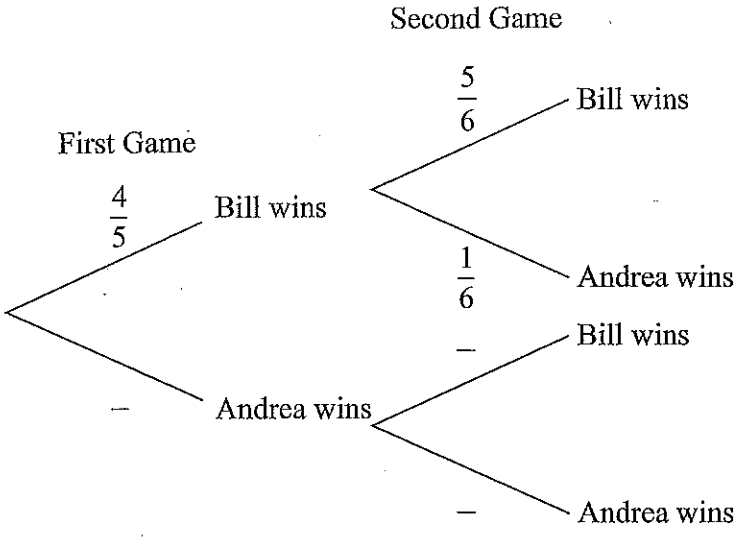
9. *[Maximum mark: 14]*

Bill and Andrea play two games of tennis. The probability that Bill wins the first game is $\frac{4}{5}$.

If Bill wins the first game, the probability that he wins the second game is $\frac{5}{6}$.

If Bill loses the first game, the probability that he wins the second game is $\frac{2}{3}$.

(a) **Copy** and complete the following tree diagram. *(Do **not** write on this page.)* *[3]*



(b) Find the probability that Bill wins the first game and Andrea wins the second game. *[2]*

(c) Find the probability that Bill wins at least one game. *[4]*

(d) Given that Bill wins at least one game, find the probability that he wins both games. *[5]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

The sides of a square are 16 cm in length. The midpoints of the sides of this square are joined to form a new square and four triangles (diagram 1). The process is repeated twice, as shown in diagrams 2 and 3.

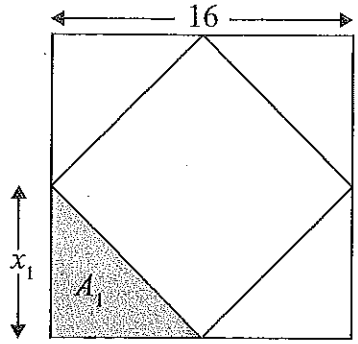


diagram 1

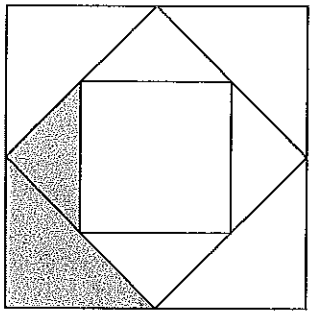


diagram 2

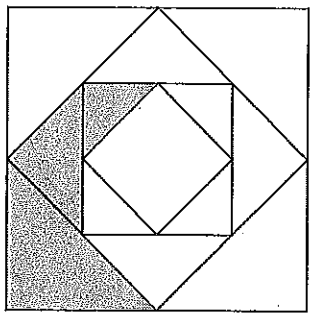


diagram 3

Let x_n denote the length of one of the equal sides of each new triangle.
Let A_n denote the area of each new triangle.

(a) The following table gives the values of x_n and A_n, for $1 \leq n \leq 3$. **Copy** and complete the table. *(Do **not** write on this page.)* *[4]*

n	1	2	3
x_n	8		4
A_n	32	16	

(b) The process described above is repeated. Find A_6. *[4]*

(c) Consider an initial square of side length k cm. The process described above is repeated indefinitely. The total area of the shaded regions is $k\text{cm}^2$. Find the value of k. *[7]*