6-1-P1_Calculus-tangents *[321 marks]*

Let
$g(x) = 2x \sin x$.

1a. Find
$g'(x)$. *[4 marks]*

Markscheme

evidence of choosing the product rule ***(M1)***

e.g.
$uv' + vu'$

correct derivatives
$\cos x$, 2 ***(A1)(A1)***

$g'(x) = 2x \cos x + 2 \sin x$ ***A1 N4***

[4 marks]

1b. Find the gradient of the graph of *g* at
$x = \pi$. *[3 marks]*

Markscheme

attempt to substitute into gradient function ***(M1)***

e.g.
$g'(\pi)$

correct substitution ***(A1)***

e.g.
$2\pi \cos \pi + 2 \sin \pi$

$\text{gradient} = -2\pi$ ***A1 N2***

[3 marks]

Let
$f(x) = \mathrm{e}^{6x}$.

2a. Write down
$f'(x)$. *[1 mark]*

Markscheme

$f'(x) = 6\mathrm{e}^{6x}$ ***A1 N1***

[1 mark]

2b. The tangent to the graph of f at the point $\mathrm{P}(0, b)$ has gradient m. *[4 marks]*

(i) Show that $m = 6$.

(ii) Find b.

Markscheme

(i) evidence of valid approach ***(M1)***

e.g. $f'(0)$, $6\mathrm{e}^{6\times 0}$

correct manipulation ***A1***

e.g. $6\mathrm{e}^{0}$, 6×1

$m = 6$ ***AG N0***

(ii) evidence of finding $f(0)$ ***(M1)***

e.g. $y = \mathrm{e}^{6(0)}$

$b = 1$ ***A1 N2***

[4 marks]

2c. Hence, write down the equation of this tangent. *[1 mark]*

Markscheme

$y = 6x + 1$ ***A1 N1***

[1 mark]

3. Let $f(x) = \mathrm{e}^{x}\cos x$. Find the gradient of the normal to the curve of f at $x = \pi$. *[6 marks]*

Markscheme

evidence of choosing the product rule ***(M1)***

$f'(x) = \mathrm{e}^x \times (-\sin x) + \cos x \times \mathrm{e}^x$
$(= \mathrm{e}^x \cos x - \mathrm{e}^x \sin x)$ ***A1A1***

substituting
π ***(M1)***

e.g.
$f'(\pi) = \mathrm{e}^\pi \cos \pi - \mathrm{e}^\pi \sin \pi$,
$\mathrm{e}^\pi(-1 - 0)$,
$-\mathrm{e}^\pi$

taking negative reciprocal ***(M1)***

e.g.
$-\frac{1}{f'(\pi)}$

gradient is
$\frac{1}{\mathrm{e}^\pi}$ ***A1 N3***

[6 marks]

Consider
$f(x) = x^2 \sin x$.

4a. Find
$f'(x)$. *[4 marks]*

Markscheme

evidence of choosing product rule ***(M1)***

eg
$uv' + vu'$

correct derivatives (must be seen in the product rule)
$\cos x$,
$2x$ ***(A1)(A1)***

$f'(x) = x^2 \cos x + 2x \sin x$ ***A1 N4***

[4 marks]

4b. Find the gradient of the curve of
f at
$x = \frac{\pi}{2}$. *[3 marks]*

Markscheme

substituting $\frac{\pi}{2}$ into **their** $f'(x)$ ***(M1)***

eg

$f'\left(\frac{\pi}{2}\right)$,

$\left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right) \sin\left(\frac{\pi}{2}\right)$

correct values for **both** $\sin\frac{\pi}{2}$ and $\cos\frac{\pi}{2}$ seen in $f'(x)$ ***(A1)***

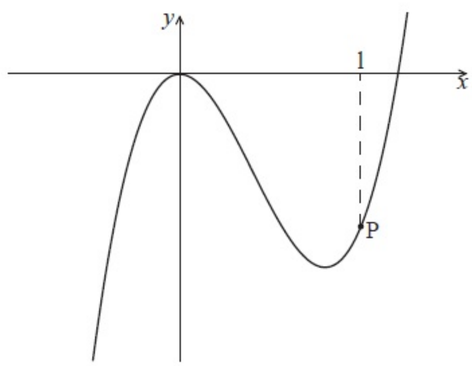
eg

$0 + 2\left(\frac{\pi}{2}\right) \times 1$

$f'\left(\frac{\pi}{2}\right) = \pi$ ***A1 N2***

[3 marks]

Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.



The point P lies on the graph of f . At P, $x = 1$.

5a. Find $f'(x)$. *[2 marks]*

Markscheme

$f'(x) = 3ax^2 - 12x$ ***A1A1 N2***

Note: Award ***A1*** for each correct term.

[2 marks]

5b. The graph of f has a gradient of 3 at the point P. Find the value of a . *[4 marks]*

Markscheme

setting their derivative equal to 3 (seen anywhere) ***A1***

e.g.
$f'(x) = 3$

attempt to substitute
$x = 1$ into
$f'(x)$ ***(M1)***

e.g.
$3a(1)^2 - 12(1)$

correct substitution into
$f'(x)$ ***(A1)***

e.g.
$3a - 12$,
$3a = 15$

$a = 5$ ***A1*** ***N2***

[4 marks]

Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

6a. Find $(g \circ f)(x)$. *[2 marks]*

Markscheme

attempt to form composite ***(M1)***

eg $g(1 + e^{-x})$

correct function ***A1*** ***N2***

eg $(g \circ f)(x) = 2 + b + 2e^{-x},\ 2(1 + e^{-x}) + b$

[2 marks]

6b. Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b. *[4 marks]*

Markscheme

evidence of $\lim_{x \to \infty}(2 + b + 2e^{-x}) = 2 + b + \lim_{x \to \infty}(2e^{-x})$ ***(M1)***

eg $2 + b + 2e^{-\infty}$, graph with horizontal asymptote when $x \to \infty$

Note: Award ***M0*** if candidate clearly has incorrect limit, such as $x \to 0,\ e^{\infty},\ 2e^{0}$.

evidence that $e^{-x} \to 0$ (seen anywhere) ***(A1)***

eg $\lim_{x \to \infty}(e^{-x}) = 0,\ 1 + e^{-x} \to 1,\ 2(1) + b = -3,\ e^{\text{large negative number}} \to 0$, graph of $y = e^{-x}$ or $y = 2e^{-x}$ with asymptote $y = 0$, graph of composite function with asymptote $y = -3$

correct working ***(A1)***

eg $2 + b = -3$

$b = -5$ ***A1*** ***N2***

[4 marks]

7. Let
$f(x) = kx^4$. The point
$\mathrm{P}(1, k)$ lies on the curve of f . At P, the normal to the curve is parallel to
$y = -\frac{1}{8}x$. Find the value of *k*.

[6 marks]

Markscheme

gradient of tangent
$= 8$ (seen anywhere) ***(A1)***

$f'(x) = 4kx^3$ (seen anywhere) ***A1***

recognizing the gradient of the tangent is the derivative ***(M1)***

setting the derivative equal to 8 ***(A1)***

e.g.
$4kx^3 = 8$,
$kx^3 = 2$

substituting
$x = 1$ (seen anywhere) ***(M1)***

$k = 2$ ***A1*** ***N4***

[6 marks]

8. Let
$f(x) = \mathrm{e}^{2x}$. The line
L is the tangent to the curve of
f at
$(1,\ \mathrm{e}^2)$.

[6 marks]

Find the equation of
L in the form
$y = ax + b$.

Markscheme

recognising need to differentiate (seen anywhere) ***R1***

eg
$f',\ 2\mathrm{e}^{2x}$

attempt to find the gradient when
$x = 1$ ***(M1)***

eg
$f'(1)$

$f'(1) = 2\mathrm{e}^2$ ***(A1)***

attempt to substitute coordinates (in any order) into equation of a straight line ***(M1)***

eg
$y - \mathrm{e}^2 = 2\mathrm{e}^2(x - 1),\ \mathrm{e}^2 = 2\mathrm{e}^2(1) + b$

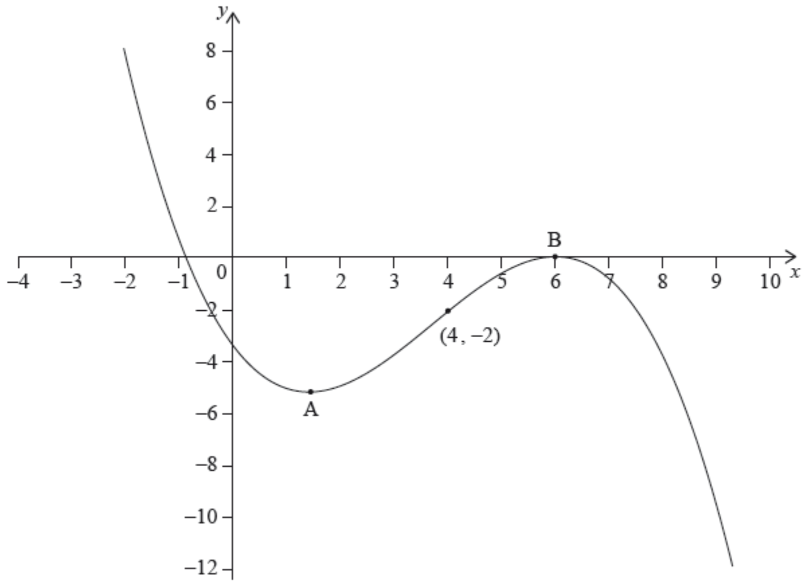
correct working ***(A1)***

eg
$y - \mathrm{e}^2 = 2\mathrm{e}^2 x - 2\mathrm{e}^2,\ b = -\mathrm{e}^2$

$y = 2\mathrm{e}^2 x - \mathrm{e}^2$ ***A1*** ***N3***

[6 marks]

The following diagram shows the graph of f', the derivative of f.



The graph of f' has a local minimum at A, a local maximum at B and passes through $(4,\ -2)$.

The point $\mathrm{P}(4,\ 3)$ lies on the graph of the function, f.

9a. Write down the gradient of the curve of f at P. *[1 mark]*

Markscheme

-2 ***A1*** ***N1***

[1 mark]

9b. Find the equation of the normal to the curve of f at P. *[3 marks]*

Markscheme

gradient of normal $= \frac{1}{2}$ ***(A1)***

attempt to substitute their normal gradient and coordinates of P (in any order) ***(M1)***

eg $y - 4 = \frac{1}{2}(x - 3),\ 3 = \frac{1}{2}(4) + b,\ b = 1$

$y - 3 = \frac{1}{2}(x - 4),\ y = \frac{1}{2}x + 1,\ x - 2y + 2 = 0$ ***A1*** ***N3***

[3 marks]

9c. Determine the concavity of the graph of f when $4 < x < 5$ **and** justify your answer. *[2 marks]*

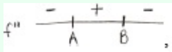
Markscheme

correct answer **and** valid reasoning *A2* *N2*

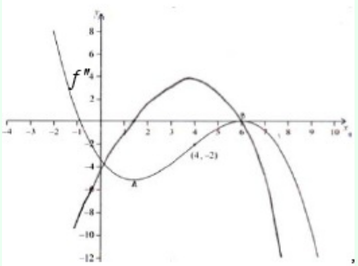
answer: *eg* graph of f is concave up, concavity is positive (between $4 < x < 5$)

reason: *eg* slope of f' is positive, f' is increasing, $f'' > 0$,

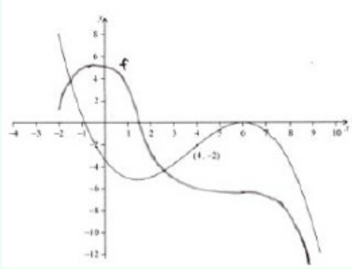
sign chart (must clearly be for f'' and show A and B)



,



,



Note: The reason given must refer to a specific function/graph. Referring to “the graph” or “it” is not sufficient.

[2 marks]

10. Consider *f*(*x*), *g*(*x*) and *h*(*x*), for x∈$\mathbb{R}$ where *h*(*x*) = $(f \circ g)(x)$. *[7 marks]*

Given that *g*(3) = 7 , *g*´(3) = 4 and *f* ´(7) = −5 , find the gradient of the normal to the curve of *h* at *x* = 3.

Markscheme

recognizing the need to find *h*´ ***(M1)***

recognizing the need to find *h*´(3) (seen anywhere) ***(M1)***

evidence of choosing chain rule ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$, $f'(g(3)) \times g'(3)$, $f'(g) \times g'$

correct working ***(A1)***

eg $f'(7) \times 4$, -5×4

$h'(3) = -20$ ***(A1)***

evidence of taking **their** negative reciprocal for normal ***(M1)***

eg $-\frac{1}{h'(3)}$, $m_1 m_2 = -1$

gradient of normal is $\frac{1}{20}$ ***A1 N4***

[7 marks]

The following diagram shows part of the graph of the function
$f(x) = 2x^2$.

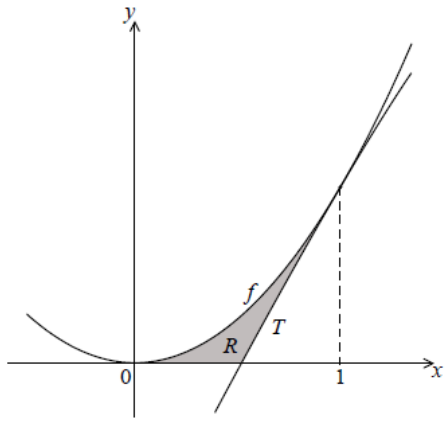


diagram not to scale

The line T is the tangent to the graph of f at
$x = 1$.

11a. Show that the equation of T is
$y = 4x - 2$. *[5 marks]*

Markscheme

$f(1) = 2$ ***(A1)***

$f'(x) = 4x$ ***A1***

evidence of finding the gradient of f at
$x = 1$ ***M1***

e.g. substituting
$x = 1$ into
$f'(x)$

finding gradient of f at
$x = 1$ ***A1***

e.g.
$f'(1) = 4$

evidence of finding equation of the line ***M1***

e.g.
$y - 2 = 4(x - 1)$,
$2 = 4(1) + b$

$y = 4x - 2$ ***AG*** ***N0***

[5 marks]

11b. Find the x-intercept of T . *[2 marks]*

Markscheme

appropriate approach ***(M1)***

e.g.
$4x - 2 = 0$

$x = \frac{1}{2}$ ***A1*** ***N2***

[2 marks]

11c. The shaded region R is enclosed by the graph of f, the line T, and the x-axis. *[9 marks]*

(i) Write down an expression for the area of R.

(ii) Find the area of R.

Markscheme

(i) bottom limit
$x = 0$ (seen anywhere) ***(A1)***

approach involving subtraction of integrals/areas ***(M1)***

e.g.
$\int f(x) - \text{area of triangle}$,
$\int f - \int l$

correct expression ***A2 N4***

e.g.
$\int_0^1 2x^2 \mathrm{d}x - \int_{0.5}^1 (4x-2)\mathrm{d}x$,
$\int_0^1 f(x)\mathrm{d}x - \frac{1}{2}$,
$\int_0^{0.5} 2x^2 \mathrm{d}x + \int_{0.5}^1 (f(x) - (4x-2))\mathrm{d}x$

(ii) **METHOD 1 (using only integrals)**

correct integration ***(A1)(A1)(A1)***

$\int 2x^2 \mathrm{d}x = \frac{2x^3}{3}$,
$\int (4x-2)\mathrm{d}x = 2x^2 - 2x$

substitution of limits ***(M1)***

e.g.
$\frac{1}{12} + \frac{2}{3} - 2 + 2 - \left(\frac{1}{12} - \frac{1}{2} + 1\right)$

area =
$\frac{1}{6}$ ***A1 N4***

METHOD 2 (using integral and triangle)

area of triangle =
$\frac{1}{2}$ ***(A1)***

correct integration ***(A1)***

$\int 2x^2 \mathrm{d}x = \frac{2x^3}{3}$

substitution of limits ***(M1)***

e.g.
$\frac{2}{3}(1)^3 - \frac{2}{3}(0)^3$,
$\frac{2}{3} - 0$

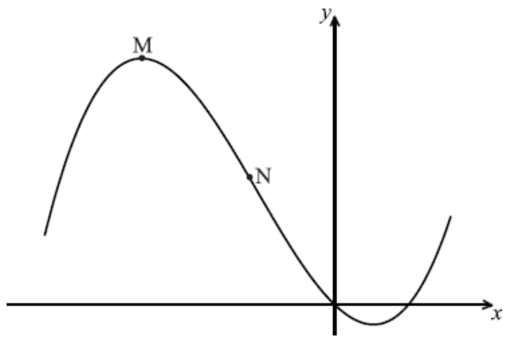
correct simplification ***(A1)***

e.g.
$\frac{2}{3} - \frac{1}{2}$

area =
$\frac{1}{6}$ ***A1 N4***

[9 marks]

Consider
$f(x) = \frac{1}{3}x^3 + 2x^2 - 5x$. Part of the graph of *f* is shown below. There is a maximum point at M, and a point of inflexion at N.



12a. Find
$f'(x)$.

[3 marks]

Markscheme

$f'(x) = x^2 + 4x - 5$ ***A1A1A1*** ***N3***

[3 marks]

12b. Find the *x*-coordinate of M.

[4 marks]

Markscheme

evidence of attempting to solve
$f'(x) = 0$ ***(M1)***

evidence of correct working ***A1***

e.g.
$(x+5)(x-1)$,
$\frac{-4 \pm \sqrt{16+20}}{2}$, sketch

$x = -5,$
$x = 1$ ***(A1)***

so
$x = -5$ ***A1*** ***N2***

[4 marks]

12c. Find the *x*-coordinate of N.

[3 marks]

Markscheme

METHOD 1

$f''(x) = 2x + 4$ (may be seen later) ***A1***

evidence of setting second derivative = 0 ***(M1)***

e.g.
$2x + 4 = 0$

$x = -2$ ***A1*** ***N2***

METHOD 2

evidence of use of symmetry ***(M1)***

e.g. midpoint of max/min, reference to shape of cubic

correct calculation ***A1***

e.g.
$\frac{-5+1}{2}$

$x = -2$ ***A1*** ***N2***

[3 marks]

12d. The line L is the tangent to the curve of f at $(3, 12)$. Find the equation of L in the form $y = ax + b$. *[4 marks]*

Markscheme

attempting to find the value of the derivative when
$x = 3$ ***(M1)***

$f'(3) = 16$ ***A1***

valid approach to finding the equation of a line ***M1***

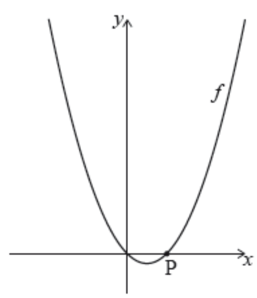
e.g.
$y - 12 = 16(x - 3)$,
$12 = 16 \times 3 + b$

$y = 16x - 36$ ***A1*** ***N2***

[4 marks]

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.

diagram not to scale



The graph of f crosses the x-axis at the origin and at the point $\mathrm{P}(1, 0)$.

13a. Show that $f'(1) = 1$. *[3 marks]*

Markscheme

$f'(x) = 2x - 1$ ***A1A1***

correct substitution ***A1***

eg $2(1) - 1,\ 2 - 1$

$f'(1) = 1$ ***AG*** ***N0***

[3 marks]

The line *L* is the normal to the graph of *f* at P.

13b. Find the equation of L in the form $y = ax + b$. *[3 marks]*

Markscheme

correct approach to find the gradient of the normal ***(A1)***

eg $\frac{-1}{f'(1)},\ m_1 m_2 = -1,\ \text{slope} = -1$

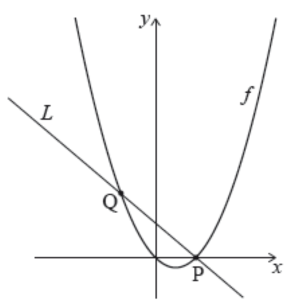
attempt to substitute correct normal gradient and coordinates into equation of a line ***(M1)***

eg $y - 0 = -1(x - 1),\ 0 = -1 + b,\ b = 1,\ L = -x + 1$

$y = -x + 1$ ***A1*** ***N2***

[3 marks]

The line L intersects the graph of f at another point Q, as shown in the following diagram.



13c. Find the x-coordinate of Q. *[4 marks]*

Markscheme

equating expressions ***(M1)***

eg $f(x) = L,\ -x + 1 = x^2 - x$

correct working (must involve combining terms) ***(A1)***

eg $x^2 - 1 = 0,\ x^2 = 1,\ x = 1$

$x = -1 \quad (\text{accept } Q(-1,\ 2))$ ***A2*** ***N3***

[4 marks]

13d. Find the area of the region enclosed by the graph of f and the line L. *[6 marks]*

Markscheme

valid approach ***(M1)***

eg $\int L - f,\ \int_{-1}^{1}(1-x^2)\mathrm{d}x$, splitting area into triangles and integrals

correct integration ***(A1)(A1)***

eg $\left[x - \frac{x^3}{3}\right]_{-1}^{1},\ -\frac{x^3}{3} - \frac{x^2}{2} + \frac{x^2}{2} + x$

substituting **their** limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $1 - \frac{1}{3} - \left(-1 - \frac{-1}{3}\right)$

Note: Award ***M0*** for substituting into original or differentiated function.

area $= \frac{4}{3}$ ***A2 N3***

[6 marks]

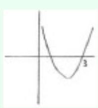
A quadratic function f can be written in the form $f(x) = a(x-p)(x-3)$. The graph of f has axis of symmetry $x = 2.5$ and y-intercept at $(0,\ -6)$

14a. Find the value of p. *[3 marks]*

Markscheme

METHOD 1 (using *x*-intercept)

determining that 3 is an x-intercept ***(M1)***

eg $x - 3 = 0,$ 

valid approach ***(M1)***

eg $3 - 2.5,\ \frac{p+3}{2} = 2.5$

$p = 2$ ***A1 N2***

METHOD 2 (expanding *f* (*x*))

correct expansion (accept absence of a) ***(A1)***

eg $ax^2 - a(3+p)x + 3ap,\ x^2 - (3+p)x + 3p$

valid approach involving equation of axis of symmetry ***(M1)***

eg $\frac{-b}{2a} = 2.5,\ \frac{a(3+p)}{2a} = \frac{5}{2},\ \frac{3+p}{2} = \frac{5}{2}$

$p = 2$ ***A1 N2***

METHOD 3 (using derivative)

correct derivative (accept absence of a) ***(A1)***

eg $a(2x-3-p),\ 2x-3-p$

valid approach ***(M1)***

eg $f'(2.5) = 0$

$p = 2$ ***A1 N2***

[3 marks]

14b. Find the value of a. *[3 marks]*

Markscheme

attempt to substitute $(0,\ -6)$ ***(M1)***

eg $-6=a(0-2)(0-3),\ 0=a(-8)(-9),\ a(0)^2-5a(0)+6a=-6$

correct working ***(A1)***

eg $-6=6a$

$a=-1$ ***A1 N2***

[3 marks]

14c. The line $y=kx-5$ is a tangent to the curve of f. Find the values of k. *[8 marks]*

Markscheme

METHOD 1 (using discriminant)

recognizing tangent intersects curve once ***(M1)***

recognizing one solution when discriminant = 0 ***M1***

attempt to set up equation ***(M1)***

eg $g=f,\ kx-5=-x^2+5x-6$

rearranging their equation to equal zero ***(M1)***

eg $x^2-5x+kx+1=0$

correct discriminant (if seen explicitly, not just in quadratic formula) ***A1***

eg $(k-5)^2-4,\ 25-10k+k^2-4$

correct working ***(A1)***

eg $k-5=\pm 2,\ (k-3)(k-7)=0,\ \frac{10\pm\sqrt{100-4\times 21}}{2}$

$k=3,\ 7$ ***A1A1 N0***

METHOD 2 (using derivatives)

attempt to set up equation ***(M1)***

eg $g=f,\ kx-5=-x^2+5x-6$

recognizing derivative/slope are equal ***(M1)***

eg $f'=m_T,\ f'=k$

correct derivative of f ***(A1)***

eg $-2x+5$

attempt to set up equation in terms of either x or k ***M1***

eg $(-2x+5)x-5=-x^2+5x-6,\ k\left(\frac{5-k}{2}\right)-5=-\left(\frac{5-k}{2}\right)^2+5\left(\frac{5-k}{2}\right)-6$

rearranging their equation to equal zero ***(M1)***

eg $x^2-1=0,\ k^2-10k+21=0$

correct working ***(A1)***

eg $x=\pm 1,\ (k-3)(k-7)=0,\ \frac{10\pm\sqrt{100-4\times 21}}{2}$

$k=3,\ 7$ ***A1A1 N0***

[8 marks]

A function f has its derivative given by $f'(x)=3x^2-2kx-9$, where k is a constant.

15a. Find $f''(x)$. *[2 marks]*

Markscheme

$f''(x) = 6x - 2k$ ***A1A1*** ***N2***

[2 marks]

15b. The graph of f has a point of inflexion when $x = 1$. *[3 marks]*

Show that $k = 3$.

Markscheme

substituting $x = 1$ into f'' ***(M1)***

eg $f''(1),\ 6(1) - 2k$

recognizing $f''(x) = 0$ (seen anywhere) ***M1***

correct equation ***A1***

eg $6 - 2k = 0$

$k = 3$ ***AG*** ***N0***

[3 marks]

15c. Find $f'(-2)$. *[2 marks]*

Markscheme

correct substitution into $f'(x)$ ***(A1)***

eg $3(-2)^2 - 6(-2) - 9$

$f'(-2) = 15$ ***A1*** ***N2***

[2 marks]

15d. Find the equation of the tangent to the curve of f at $(-2,\ 1)$, giving your answer in the form $y = ax + b$. *[4 marks]*

Markscheme

recognizing gradient value (may be seen in equation) ***M1***

eg $a = 15,\ y = 15x + b$

attempt to substitute $(-2,\ 1)$ into equation of a straight line ***M1***

eg $1 = 15(-2) + b,\ (y - 1) = m(x + 2),\ (y + 2) = 15(x - 1)$

correct working ***(A1)***

eg $31 = b,\ y = 15x + 30 + 1$

$y = 15x + 31$ ***A1*** ***N2***

[4 marks]

15e. Given that $f'(-1) = 0$, explain why the graph of f has a local maximum when $x = -1$. *[3 marks]*

Markscheme

METHOD 1 (2^{nd} derivative)

recognizing $f'' < 0$ (seen anywhere) ***R1***

substituting $x = -1$ into f'' ***(M1)***

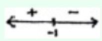
eg $f''(-1),\ 6(-1) - 6$

$f''(-1) = -12$ ***A1***

therefore the graph of f has a local maximum when $x = -1$ ***AG*** ***N0***

METHOD 2 (1^{st} derivative)

recognizing change of sign of $f'(x)$ (seen anywhere) ***R1***

eg *sign chart* 

correct value of f' for $-1 < x < 3$ ***A1***

eg $f'(0) = -9$

correct value of f' for x value to the left of -1 ***A1***

eg $f'(-2) = 15$

therefore the graph of f has a local maximum when $x = -1$ ***AG*** ***N0***

[3 marks]

Total [14 marks]

Let
$f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for
$0 \leq x \leq \pi$.

16a. Find
$f'(x)$. *[3 marks]*

Markscheme

$f'(x) = \cos x + x - 2$ ***A1A1A1*** ***N3***

Note: Award ***A1*** for each term.

[3 marks]

Let
g be a quadratic function such that
$g(0) = 5$. The line
$x = 2$ is the axis of symmetry of the graph of
g .

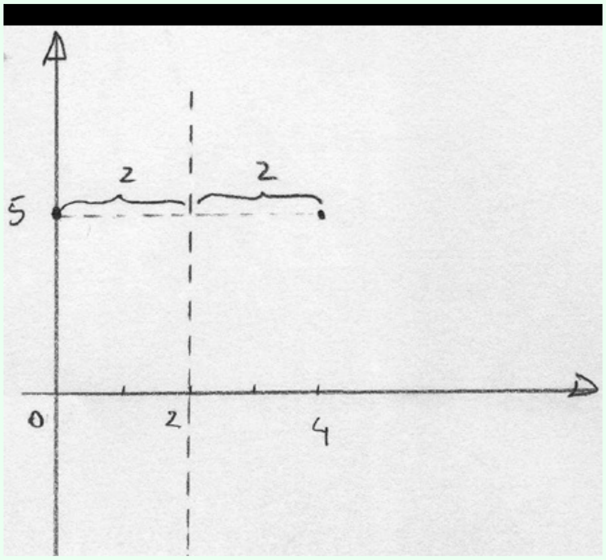
16b. Find
$g(4)$. *[3 marks]*

Markscheme

recognizing $g(0)=5$ gives the point $(0, 5)$ ***(R1)***

recognize symmetry ***(M1)***

eg vertex, sketch



$g(4)=5$ ***A1*** ***N3***

[3 marks]

The function g can be expressed in the form $g(x)=a(x-h)^2+3$.

16c. (i) Write down the value of h . *[4 marks]*

(ii) Find the value of a .

Markscheme

(i)

$h = 2$ ***A1 N1***

(ii) substituting into

$g(x) = a(x - 2)^2 + 3$ (not the vertex) ***(M1)***

eg

$5 = a(0 - 2)^2 + 3$,

$5 = a(4 - 2)^2 + 3$

working towards solution ***(A1)***

eg

$5 = 4a + 3$,

$4a = 2$

$a = \frac{1}{2}$ ***A1 N2***

[4 marks]

16d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g . *[6 marks]*

Markscheme

$g(x) = \frac{1}{2}(x - 2)^2 + 3 = \frac{1}{2}x^2 - 2x + 5$

correct derivative of g ***A1A1***

eg

$2 \times \frac{1}{2}(x - 2)$,

$x - 2$

evidence of equating both derivatives ***(M1)***

eg

$f' = g'$

correct equation ***(A1)***

eg

$\cos x + x - 2 = x - 2$

working towards a solution ***(A1)***

eg

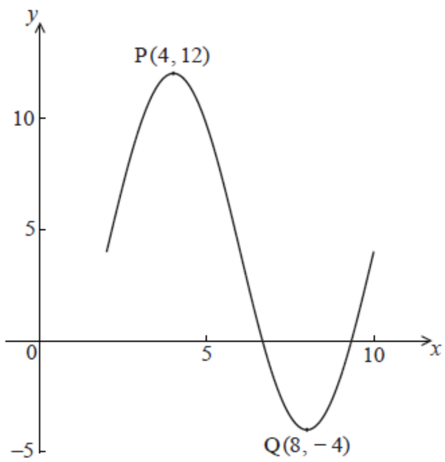
$\cos x = 0$, combining like terms

$x = \frac{\pi}{2}$ ***A1 N0***

Note: Do not award final ***A1*** if additional values are given.

[6 marks]

The following diagram shows the graph of $f(x) = a\sin(b(x - c)) + d$, for $2 \leq x \leq 10$.



There is a maximum point at P(4, 12) and a minimum point at Q(8, −4) .

17a. Use the graph to write down the value of *[3 marks]*

(i) *a* ;

(ii) *c* ;

(iii) *d* .

Markscheme

(i)

$a = 8$ ***A1 N1***

(ii)

$c = 2$ ***A1 N1***

(iii)

$d = 4$ ***A1 N1***

[3 marks]

17b. Show that $b = \frac{\pi}{4}$. *[2 marks]*

Markscheme

METHOD 1

recognizing that period $= 8$ ***(A1)***

correct working ***A1***

e.g. $8 = \frac{2\pi}{b}$, $b = \frac{2\pi}{8}$

$b = \frac{\pi}{4}$ ***AG N0***

METHOD 2

attempt to substitute ***M1***

e.g. $12 = 8\sin(b(4-2)) + 4$

correct working ***A1***

e.g. $\sin 2b = 1$

$b = \frac{\pi}{4}$ ***AG N0***

[2 marks]

17c. Find $f'(x)$. *[3 marks]*

Markscheme

evidence of attempt to differentiate or choosing chain rule ***(M1)***

e.g. $\cos\frac{\pi}{4}(x-2)$, $\frac{\pi}{4}\times 8$

$f'(x) = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$ (accept $2\pi\cos\frac{\pi}{4}(x-2)$) ***A2 N3***

[3 marks]

17d. At a point R, the gradient is -2π. Find the *x*-coordinate of R. *[6 marks]*

Markscheme

recognizing that gradient is
$f'(x)$ ***(M1)***

e.g.
$f'(x) = m$

correct equation ***A1***

e.g.
$-2\pi = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$,
$-1 = \cos\left(\frac{\pi}{4}(x-2)\right)$

correct working ***(A1)***

e.g.
$\cos^{-1}(-1) = \frac{\pi}{4}(x-2)$

using
$\cos^{-1}(-1) = \pi$ (seen anywhere) ***(A1)***

e.g.
$\pi = \frac{\pi}{4}(x-2)$

simplifying ***(A1)***

e.g.
$4 = (x-2)$

$x = 6$ ***A1 N4***

[6 marks]

Let $f(x) = \sqrt{4x+5}$, for $x \geqslant -1.25$.

18a. Find $f'(1)$. *[4 marks]*

Markscheme

choosing chain rule ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ u = 4x+5,\ u' = 4$

correct derivative of f ***A2***

eg $\frac{1}{2}(4x+5)^{-\frac{1}{2}} \times 4,\ f'(x) = \frac{2}{\sqrt{4x+5}}$

$f'(1) = \frac{2}{3}$ ***A1 N2***

[4 marks]

Consider another function g. Let R be a point on the graph of g. The x-coordinate of R is 1. The equation of the tangent to the graph at R is $y = 3x + 6$.

18b. Write down $g'(1)$. *[2 marks]*

Markscheme

recognize that $g'(x)$ is the gradient of the tangent ***(M1)***

eg $g'(x) = m$

$g'(1) = 3$ ***A1 N2***

[2 marks]

18c. Find $g(1)$. *[2 marks]*

Markscheme

recognize that R is on the tangent ***(M1)***

eg $g(1) = 3 \times 1 + 6$, sketch

$g(1) = 9$ ***A1 N2***

[2 marks]

18d. Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent to the graph of h at the point where $x = 1$. *[7 marks]*

Markscheme

$f(1) = \sqrt{4+5}\ (=3)$ (seen anywhere) ***A1***

$h(1) = 3 \times 9\ (=27)$ (seen anywhere) ***A1***

choosing product rule to find $h'(x)$ ***(M1)***

eg $uv' + u'v$

correct substitution to find $h'(1)$ ***(A1)***

eg $f(1) \times g'(1) + f'(1) \times g(1)$

$h'(1) = 3 \times 3 + \frac{2}{3} \times 9\ (=15)$ ***A1***

EITHER

attempt to substitute coordinates (in any order) into the equation of a straight line ***(M1)***

eg $y - 27 = h'(1)(x-1),\ y - 1 = 15(x - 27)$

$y - 27 = 15(x-1)$ ***A1 N2***

OR

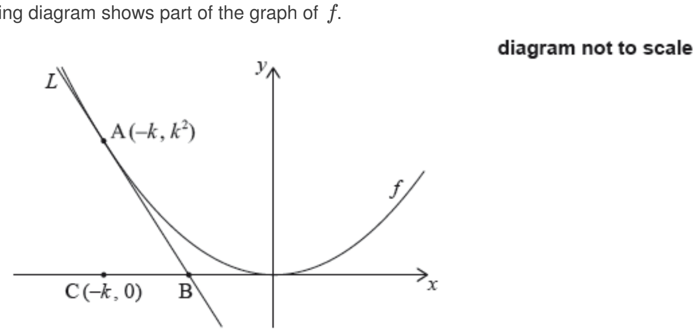
attempt to substitute coordinates (in any order) to find the y-intercept ***(M1)***

eg $27 = 15 \times 1 + b,\ 1 = 15 \times 27 + b$

$y = 15x + 12$ ***A1 N2***

[7 marks]

Let $f(x) = x^2$. The following diagram shows part of the graph of f.



The line L is the tangent to the graph of f at the point $A(-k,\ k^2)$, and intersects the x-axis at point B. The point C is $(-k,\ 0)$.

19a. Write down $f'(x)$. *[1 mark]*

Markscheme

$f'(x) = 2x$ ***A1 N1***

[1 mark]

19b. Find the gradient of L. *[2 marks]*

Markscheme

attempt to substitute $x = -k$ into their derivative ***(M1)***

gradient of L is $-2k$ ***A1 N2***

[2 marks]

19c. Show that the x-coordinate of B is $-\frac{k}{2}$. *[5 marks]*

Markscheme

METHOD 1

attempt to substitute coordinates of A and their gradient into equation of a line ***(M1)***

eg $k^2 = -2k(-k) + b$

correct equation of L in any form ***(A1)***

eg $y - k^2 = -2k(x + k)$, $y = -2kx - k^2$

valid approach ***(M1)***

eg $y = 0$

correct substitution into L equation ***A1***

eg $-k^2 = -2kx - 2k^2$, $0 = -2kx - k^2$

correct working ***A1***

eg $2kx = -k^2$

$x = -\frac{k}{2}$ ***AG N0***

METHOD 2

valid approach ***(M1)***

eg $\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$, $-2k = \frac{\text{rise}}{\text{run}}$

recognizing $y = 0$ at B ***(A1)***

attempt to substitute coordinates of A and B into slope formula ***(M1)***

eg $\frac{k^2 - 0}{-k - x}$, $\frac{-k^2}{x + k}$

correct equation ***A1***

eg $\frac{k^2 - 0}{-k - x} = -2k$, $\frac{-k^2}{x + k} = -2k$, $-k^2 = -2k(x + k)$

correct working ***A1***

eg $2kx = -k^2$

$x = -\frac{k}{2}$ ***AG N0***

[5 marks]

19d. Find the area of triangle ABC, giving your answer in terms of k. *[2 marks]*

Markscheme

valid approach to find area of triangle ***(M1)***

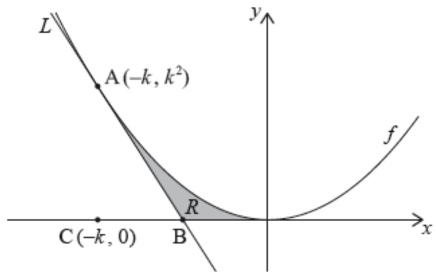
eg $\frac{1}{2}(k^2)\left(\frac{k}{2}\right)$

area of $\text{ABC} = \frac{k^3}{4}$ ***A1 N2***

[2 marks]

The region R is enclosed by L, the graph of f, and the x-axis. This is shown in the following diagram.

diagram not to scale



19e. Given that the area of triangle ABC is p times the area of R, find the value of p. *[7 marks]*

Markscheme

METHOD 1 ($\int f -$ triangle)

valid approach to find area from $-k$ to 0 ***(M1)***

eg $\int_{-k}^{0} x^2 \mathrm{d}x,\ \int_{0}^{-k} f$

correct integration (seen anywhere, even if ***M0*** awarded) ***A1***

eg $\frac{x^3}{3},\ \left[\frac{1}{3}x^3\right]_{-k}^{0}$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $0 - \frac{(-k)^3}{3}$, area from $-k$ to 0 is $\frac{k^3}{3}$

Note: Award ***M0*** for substituting into original or differentiated function.

attempt to find area of R ***(M1)***

eg $\int_{-k}^{0} f(x)\mathrm{d}x -$ triangle

correct working for R ***(A1)***

eg $\frac{k^3}{3} - \frac{k^3}{4},\ R = \frac{k^3}{12}$

correct substitution into $\text{triangle} = pR$ ***(A1)***

eg $\frac{k^3}{4} = p\left(\frac{k^3}{3} - \frac{k^3}{4}\right),\ \frac{k^3}{4} = p\left(\frac{k^3}{12}\right)$

$p = 3$ ***A1 N2***

METHOD 2 ($\int (f - L)$)

valid approach to find area of R ***(M1)***

eg $\int_{-k}^{-\frac{k}{2}} x^2 - (-2kx - k^2)\mathrm{d}x + \int_{-\frac{k}{2}}^{0} x^2 \mathrm{d}x,\ \int_{-k}^{-\frac{k}{2}} (f - L) + \int_{-\frac{k}{2}}^{0} f$

correct integration (seen anywhere, even if ***M0*** awarded) ***A2***

eg $\frac{x^3}{3} + kx^2 + k^2x,\ \left[\frac{x^3}{3} + kx^2 + k^2x\right]_{-k}^{-\frac{k}{2}} + \left[\frac{x^3}{3}\right]_{-\frac{k}{2}}^{0}$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $\left(\frac{\left(-\frac{k}{2}\right)^3}{3} + k\left(-\frac{k}{2}\right)^2 + k^2\left(-\frac{k}{2}\right)\right) - \left(\frac{(-k)^3}{3} + k(-k)^2 + k^2(-k)\right) + (0) - \left(\frac{\left(-\frac{k}{2}\right)^3}{3}\right)$

Note: Award ***M0*** for substituting into original or differentiated function.

correct working for R ***(A1)***

eg $\frac{k^3}{24} + \frac{k^3}{24},\ -\frac{k^3}{24} + \frac{k^3}{4} - \frac{k^3}{2} + \frac{k^3}{3} - k^3 + k^3 + \frac{k^3}{24},\ R = \frac{k^3}{12}$

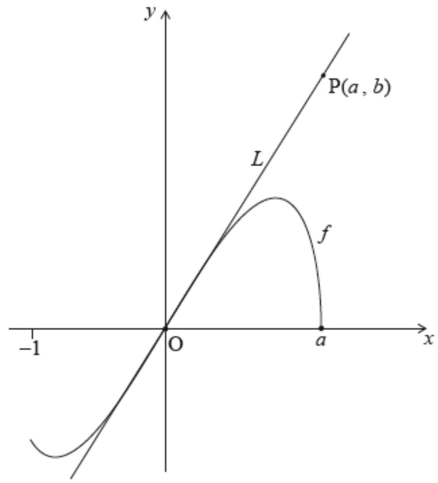
correct substitution into $\text{triangle} = pR$ ***(A1)***

eg $\frac{k^3}{4} = p\left(\frac{k^3}{24} + \frac{k^3}{24}\right),\ \frac{k^3}{4} = p\left(\frac{k^3}{12}\right)$

$p = 3$ ***A1 N2***

[7 marks]

The following diagram shows the graph of $f(x) = 2x\sqrt{a^2 - x^2}$, for $-1 \leqslant x \leqslant a$, where $a > 1$.



The line L is the tangent to the graph of f at the origin, O. The point $\mathrm{P}(a,\, b)$ lies on L.

20a. (i) Given that $f'(x) = \frac{2a^2 - 4x^2}{\sqrt{a^2 - x^2}}$, for $-1 \leqslant x < a$, find the equation of L. *[6 marks]*

(ii) Hence or otherwise, find an expression for b in terms of a.

Markscheme

(i) recognizing the need to find the gradient when $x = 0$ (seen anywhere) ***R1***

eg $f'(0)$

correct substitution ***(A1)***

$f'(0) = \frac{2a^2 - 4(0)}{\sqrt{a^2 - 0}}$

$f'(0) = 2a$ ***(A1)***

correct equation with gradient 2
a (do not accept equations of the form $L = 2ax$) ***A1 N3***

eg $y = 2ax,\ y - b = 2a(x - a),\ y = 2ax - 2a^2 + b$

(ii) **METHOD 1**

attempt to substitute $x = a$ into **their** equation of L ***(M1)***

eg $y = 2a \times a$

$b = 2a^2$ ***A1 N2***

METHOD 2

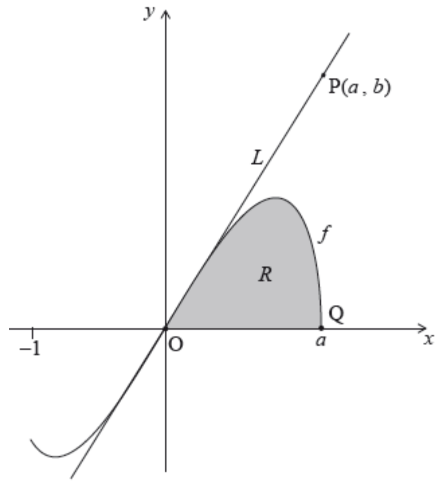
equating gradients ***(M1)***

eg $\frac{b}{a} = 2a$

$b = 2a^2$ ***A1 N2***

[6 marks]

The point $\mathrm{Q}(a,\ 0)$ lies on the graph of f. Let R be the region enclosed by the graph of f and the x-axis. This information is shown in the following diagram.



Let A_R be the area of the region R.

20b. Show that $A_R = \frac{2}{3}a^3$. *[6 marks]*

Markscheme

METHOD 1

recognizing that area $= \int_0^a f(x)\mathrm{d}x$ (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

eg $\int 2x\sqrt{u}\mathrm{d}x,\ u = a^2 - x^2,\ \mathrm{d}u = -2x\mathrm{d}x,\ \frac{2}{3}(a^2 - x^2)^{\frac{3}{2}}$

correct working ***(A1)***

eg $\int 2x\sqrt{a^2 - x^2}\mathrm{d}x = \int -\sqrt{u}\mathrm{d}u$

$\int -\sqrt{u}\mathrm{d}u = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ ***(A1)***

$\int f(x)\mathrm{d}x = -\frac{2}{3}(a^2 - x^2)^{\frac{3}{2}} + c$ ***(A1)***

substituting limits and subtracting ***A1***

eg $A_R = -\frac{2}{3}(a^2 - a^2)^{\frac{3}{2}} + \frac{2}{3}(a^2 - 0)^{\frac{3}{2}},\ \frac{2}{3}(a^2)^{\frac{3}{2}}$

$A_R = \frac{2}{3}a^3$ ***AG N0***

METHOD 2

recognizing that area $= \int_0^a f(x)\mathrm{d}x$ (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

eg $\int 2x\sqrt{u}\mathrm{d}x,\ u = a^2 - x^2,\ \mathrm{d}u = -2x\mathrm{d}x,\ \frac{2}{3}(a^2 - x^2)^{\frac{3}{2}}$

correct working ***(A1)***

eg $\int 2x\sqrt{a^2 - x^2}\mathrm{d}x = \int -\sqrt{u}\mathrm{d}u$

$\int -\sqrt{u}\mathrm{d}u = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ ***(A1)***

new limits for *u* (even if integration is incorrect) ***(A1)***

eg $u = 0 \text{ and } u = a^2,\ \int_0^{a^2} u^{\frac{1}{2}}\mathrm{d}u,\ \left[-\frac{2}{3}u^{\frac{3}{2}}\right]_{a^2}^{0}$

substituting limits and subtracting ***A1***

eg $A_R = -\left(0 - \frac{2}{3}a^3\right),\ \frac{2}{3}(a^2)^{\frac{3}{2}}$

$A_R = \frac{2}{3}a^3$ ***AG N0***

[6 marks]

20c. Let A_T be the area of the triangle OPQ. Given that $A_T = kA_R$, find the value of k. *[4 marks]*

Markscheme

METHOD 1

valid approach to find area of triangle ***(M1)***

eg $\frac{1}{2}(\mathrm{OQ})(\mathrm{PQ}),\ \frac{1}{2}ab$

correct substitution into formula for A_T (seen anywhere) ***(A1)***

eg $A_T = \frac{1}{2} \times a \times 2a^2,\ a^3$

valid attempt to find k (must be in terms of a) ***(M1)***

eg $a^3 = k\frac{2}{3}a^3,\ k = \frac{a^3}{\frac{2}{3}a^3}$

$k = \frac{3}{2}$ ***A1 N2***

METHOD 2

valid approach to find area of triangle ***(M1)***

eg $\int_0^a (2ax)\mathrm{d}x$

correct working ***(A1)***

eg $[ax^2]_0^a,\ a^3$

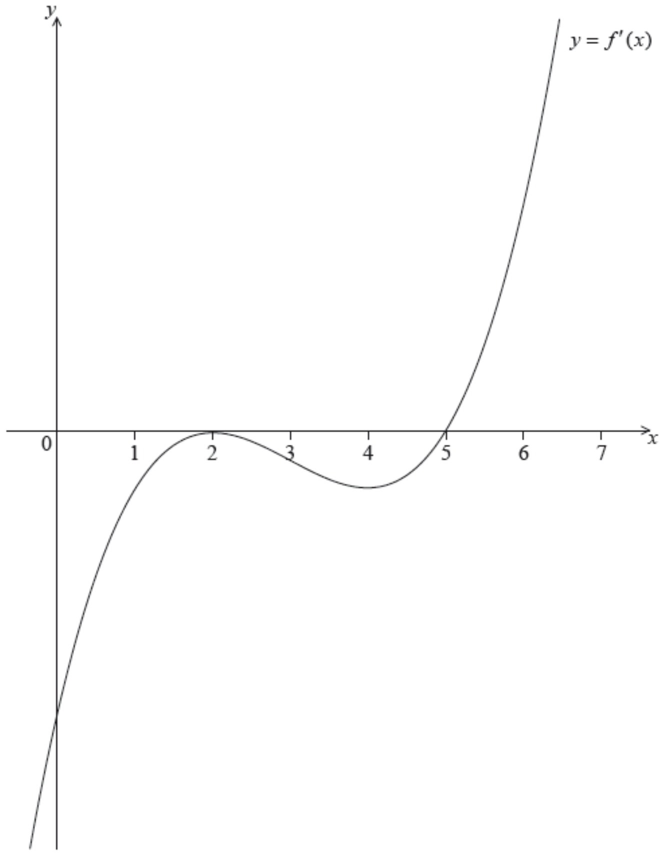
valid attempt to find k (must be in terms of a) ***(M1)***

eg $a^3 = k\frac{2}{3}a^3,\ k = \frac{a^3}{\frac{2}{3}a^3}$

$k = \frac{3}{2}$ ***A1 N2***

[4 marks]

Let $y = f(x)$, for
$-0.5 \leq$ x
$\leq$
6.5. The following diagram shows the graph of f', the derivative of f.



The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the x-axis at the point
$(5,\ 0)$.

21a. Explain why the graph of f has a local minimum when $x = 5$. *[2 marks]*

Markscheme

METHOD 1

$f'(5) = 0$ ***(A1)***

valid reasoning including reference to the graph of f' ***R1***

eg f' changes sign from negative to positive at $x = 5$, labelled sign chart for f'

so f has a local minimum at $x = 5$ ***AG*** ***N0***

Note: It must be clear that any description is referring to the graph of f', simply giving the conditions for a minimum without relating them to f' does not gain the ***R1***.

METHOD 2

$f'(5) = 0$ ***A1***

valid reasoning referring to second derivative ***R1***

eg $f''(5) > 0$

so f has a local minimum at $x = 5$ ***AG*** ***N0***

[2 marks]

21b. Find the set of values of x for which the graph of f is concave down. *[2 marks]*

Markscheme

attempt to find relevant interval *(M1)*

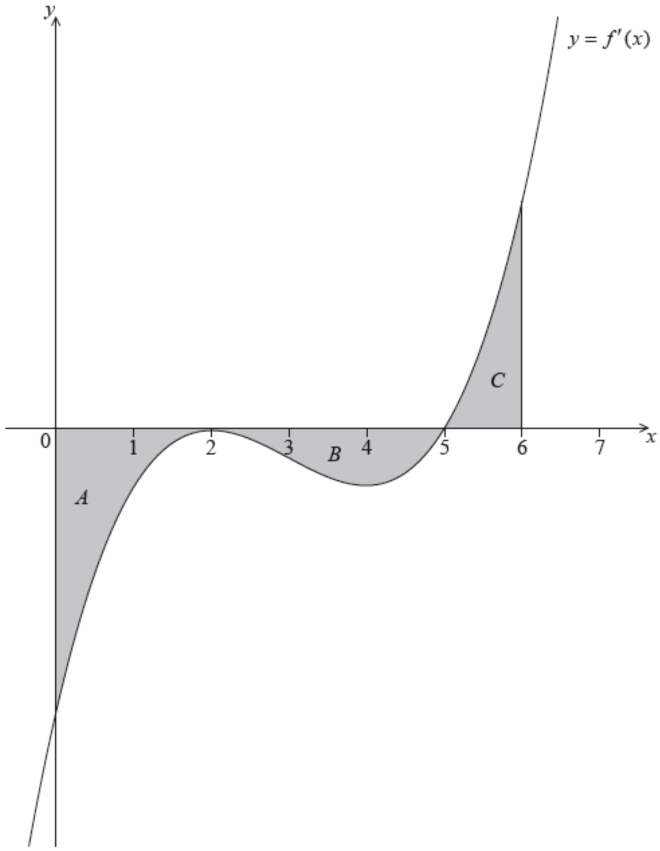
eg f' is decreasing, gradient of f' is negative, $f'' < 0$

$2 < x < 4$ (accept "between 2 and 4") ***A1 N2***

Notes: If no other working shown, award ***M1A0*** for incorrect inequalities such as $2 \le x \le 4$, or "from 2 to 4"

[2 marks]

21c. The following diagram shows the shaded regions A, B and C. *[5 marks]*



The regions are enclosed by the graph of f', the x-axis, the y-axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Given that $f(0) = 14$, find $f(6)$.

Markscheme

METHOD 1 (one integral)

correct application of Fundamental Theorem of Calculus ***(A1)***

eg $\int_0^6 f'(x)\mathrm{d}x = f(6) - f(0),\ f(6) = 14 + \int_0^6 f'(x)\mathrm{d}x$

attempt to link definite integral with areas ***(M1)***

eg $\int_0^6 f'(x)\mathrm{d}x = -12 - 6.75 + 6.75,\ \int_0^6 f'(x)\mathrm{d}x = \text{Area } A + \text{Area } B + \text{ Area } C$

correct value for $\int_0^6 f'(x)\mathrm{d}x$ ***(A1)***

eg $\int_0^6 f'(x)\mathrm{d}x = -12$

correct working ***A1***

eg $f(6) - 14 = -12,\ f(6) = -12 + f(0)$

$f(6) = 2$ ***A1 N3***

METHOD 2 (more than one integral)

correct application of Fundamental Theorem of Calculus ***(A1)***

eg $\int_0^2 f'(x)\mathrm{d}x = f(2) - f(0),\ f(2) = 14 + \int_0^2 f'(x)$

attempt to link definite integrals with areas ***(M1)***

eg $\int_0^2 f'(x)\mathrm{d}x = 12,\ \int_2^5 f'(x)\mathrm{d}x = -6.75,\ \int_0^6 f'(x) = 0$

correct values for integrals ***(A1)***

eg $\int_0^2 f'(x)\mathrm{d}x = -12,\ \int_5^2 f'(x)\mathrm{d}x = 6.75,\ f(6) - f(2) = 0$

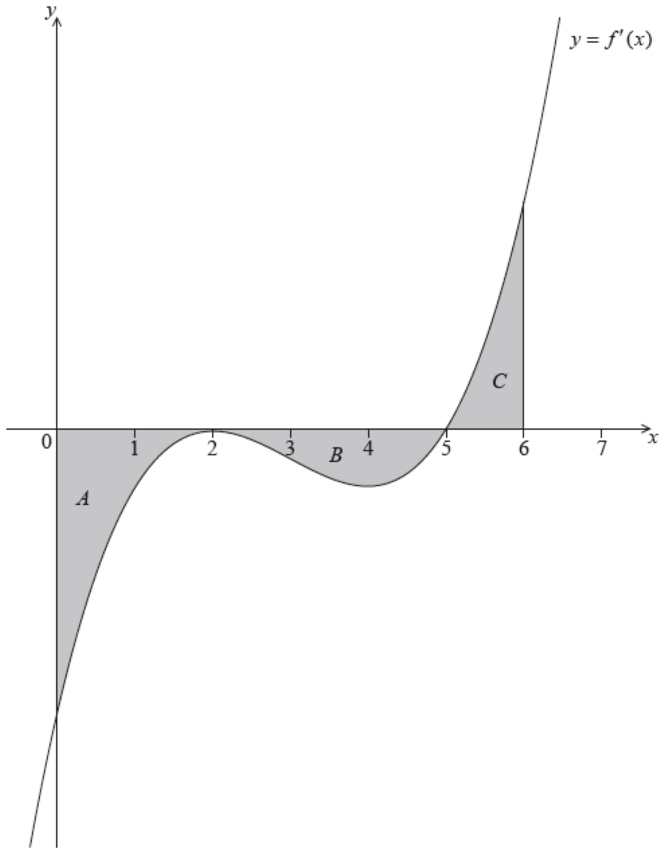
one correct intermediate value ***A1***

eg $f(2) = 2,\ f(5) = -4.75$

$f(6) = 2$ ***A1 N3***

[5 marks]

21d. The following diagram shows the shaded regions A, B and C. *[6 marks]*



The regions are enclosed by the graph of f', the *x*-axis, the *y*-axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Let $g(x) = (f(x))^2$. Given that $f'(6) = 16$, find the equation of the tangent to the graph of g at the point where $x = 6$.

Markscheme

correct calculation of $g(6)$ (seen anywhere) ***A1***

eg $2^2,\ g(6) = 4$

choosing chain rule or product rule ***(M1)***

eg $g'(f(x))\,f'(x),\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ f(x)f'(x) + f'(x)f(x)$

correct derivative ***(A1)***

eg $g'(x) = 2f(x)f'(x),\ f(x)f'(x) + f'(x)f(x)$

correct calculation of $g'(6)$ (seen anywhere) ***A1***

eg $2(2)(16),\ g'(6) = 64$

attempt to substitute **their** values of $g'(6)$ and $g(6)$ (in any order) into equation of a line ***(M1)***

eg $2^2 = (2 \times 2 \times 16)6 + b,\ y - 6 = 64(x - 4)$

correct equation in any form ***A1*** ***N2***

eg $y - 4 = 64(x - 6),\ y = 64x - 380$

[6 marks]

[Total 15 marks]

Consider
$f(x) = \ln(x^4 + 1)$.

22a. Find the value of
$f(0)$. *[2 marks]*

Markscheme

substitute 0 into f ***(M1)***

eg $\ln(0+1)$, $\ln 1$

$f(0) = 0$ ***A1 N2***

[2 marks]

22b. Find the set of values of x for which f is increasing. *[5 marks]*

Markscheme

$f'(x) = \frac{1}{x^4+1} \times 4x^3$ (seen anywhere) ***A1A1***

Note: Award ***A1*** for $\frac{1}{x^4+1}$ and ***A1*** for $4x^3$.

recognizing f increasing where $f'(x) > 0$ (seen anywhere) ***R1***

eg $f'(x) > 0$, diagram of signs

attempt to solve $f'(x) > 0$ ***(M1)***

eg $4x^3 = 0$, $x^3 > 0$

f increasing for $x > 0$ (accept $x \geq 0$) ***A1 N1***

[5 marks]

The second derivative is given by $f''(x) = \frac{4x^2(3-x^4)}{(x^4+1)^2}$.

The equation $f''(x) = 0$ has only three solutions, when $x = 0$, $\pm\sqrt[4]{3}$ $(\pm 1.316\ldots)$.

22c. (i) Find $f''(1)$. *[5 marks]*

(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x = 0$.

Markscheme

(i) substituting $x = 1$ into f'' ***(A1)***

eg $\frac{4(3-1)}{(1+1)^2}$, $\frac{4\times 2}{4}$

$f''(1) = 2$ ***A1 N2***

(ii) valid interpretation of point of inflexion (seen anywhere) ***R1***

eg no change of sign in $f''(x)$, no change in concavity,

f' increasing both sides of zero

attempt to find $f''(x)$ for $x < 0$ ***(M1)***

eg $f''(-1)$, $\frac{4(-1)^2(3-(-1)^4)}{((-1)^4+1)^2}$, diagram of signs

correct working leading to positive value ***A1***

eg $f''(-1) = 2$, discussing signs of numerator **and** denominator

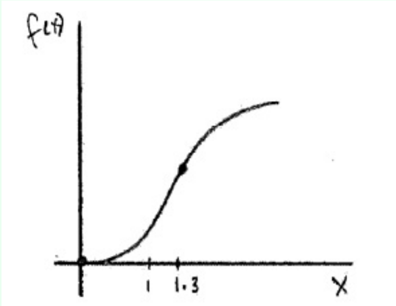
there is no point of inflexion at $x = 0$ ***AG N0***

[5 marks]

22d. There is a point of inflexion on the graph of f at $x = \sqrt[4]{3}$ $(x = 1.316\ldots)$. *[3 marks]*

Sketch the graph of f , for $x \geq 0$.

Markscheme



A1A1A1 N3

Notes: Award ***A1*** for shape concave up left of POI and concave down right of POI.

Only if this ***A1*** is awarded, then award the following:

A1 for curve through $(0, 0)$, ***A1*** for increasing throughout.

Sketch need not be drawn to scale. Only essential features need to be clear.

[3 marks]

Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some valuesassociated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	–14	–18
$f'(x)$	1	1
$g'(x)$	–5	–3
$h''(x)$	–6	0

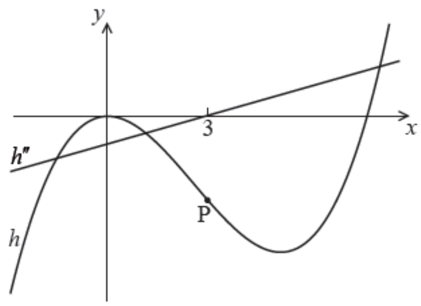
23a. Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$. *[3 marks]*

Markscheme

$g(3) = -18$, $f'(3) = 1$, $h''(2) = -6$ ***A1A1A1 N3***

[3 marks]

The following diagram shows parts of the graphs of h and h''.



There is a point of inflexion on the graph of h at P, when $x = 3$.

23b. Explain why P is a point of inflexion. *[2 marks]*

Markscheme

$h''(3) = 0$ ***(A1)***

valid reasoning ***R1***

eg h'' changes sign at $x = 3$, change in concavity of h at $x = 3$

so P is a point of inflexion ***AG N0***

[2 marks]

Given that $h(x) = f(x) \times g(x)$,

23c. find the y-coordinate of P. *[2 marks]*

Markscheme

writing $h(3)$ as a product of $f(3)$ and $g(3)$ ***A1***

eg $f(3) \times g(3)$, $3 \times (-18)$

$h(3) = -54$ ***A1 N1***

[2 marks]

23d. find the equation of the normal to the graph of h at P. *[7 marks]*

Markscheme

recognizing need to find derivative of h ***(R1)***

eg h' , $h'(3)$

attempt to use the product rule (do **not** accept $h' = f' \times g'$) ***(M1)***

eg $h' = fg' + gf'$, $h'(3) = f(3) \times g'(3) + g(3) \times f'(3)$

correct substitution ***(A1)***

eg $h'(3) = 3(-3) + (-18) \times 1$

$h'(3) = -27$ ***A1***

attempt to find the gradient of the normal ***(M1)***

eg $-\frac{1}{m}$, $-\frac{1}{27}x$

attempt to substitute **their** coordinates and **their** normal gradient into the equation of a line ***(M1)***

eg $-54 = \frac{1}{27}(3) + b$, $0 = \frac{1}{27}(3) + b$, $y + 54 = 27(x - 3)$, $y - 54 = \frac{1}{27}(x + 3)$

correct equation in any form ***A1 N4***

eg $y + 54 = \frac{1}{27}(x - 3)$, $y = \frac{1}{27}x - 54\frac{1}{9}$

[7 marks]

Let $f(x) = \frac{1}{4}x^2 + 2$. The line *L* is the tangent to the curve of *f* at (4, 6) .

24a. Find the equation of *L* . *[4 marks]*

Markscheme

finding $f'(x) = \frac{1}{2}x$ ***A1***

attempt to find $f'(4)$ ***(M1)***

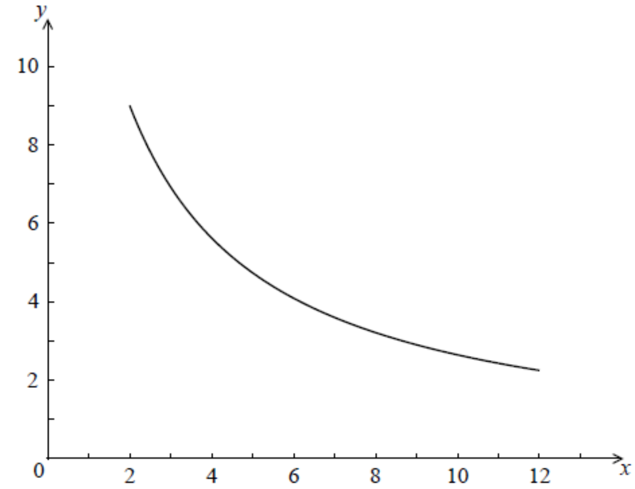
correct value $f'(4) = 2$ ***A1***

correct equation in any form ***A1 N2***

e.g. $y - 6 = 2(x - 4)$, $y = 2x - 2$

[4 marks]

Let
$g(x) = \frac{90}{3x+4}$, for
$2 \le x \le 12$. The following diagram shows the graph of g .



24b. Find the area of the region enclosed by the curve of g , the x-axis, and the lines $x = 2$ and $x = 12$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$. *[6 marks]*

Markscheme

$\text{area} = \int_2^{12} \frac{90}{3x+4} \mathrm{d}x$

correct integral ***A1A1***

e.g.
$30 \ln(3x+4)$

substituting limits and subtracting ***(M1)***

e.g.
$30 \ln(3 \times 12 + 4) - 30 \ln(3 \times 2 + 4)$,
$30 \ln 40 - 30 \ln 10$

correct working ***(A1)***

e.g.
$30(\ln 40 - \ln 10)$

correct application of
$\ln b - \ln a$ ***(A1)***

e.g.
$30 \ln \frac{40}{10}$

$\text{area} = 30 \ln 4$ ***A1 N4***

[6 marks]

24c. The graph of g is reflected in the x-axis to give the graph of h . The area of the region enclosed by the lines L , $x = 2$, $x = 12$ and the x-axis is 120 120 cm^2 . *[3 marks]*

Find the area enclosed by the lines L , $x = 2$, $x = 12$ and the graph of h .

Markscheme

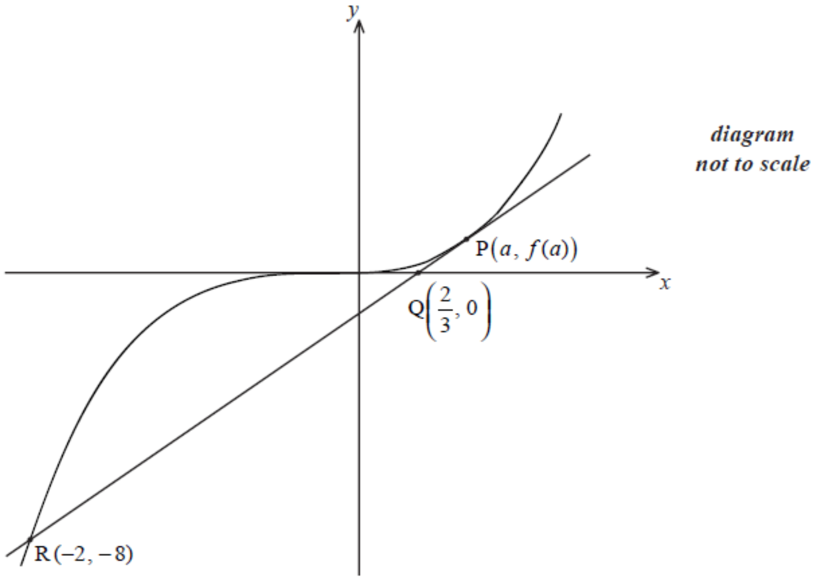
valid approach *(M1)*

e.g. sketch, area *h* = area *g* , 120 + **their** answer from (b)

$\text{area} = 120 + 30\ln 4$ ***A2*** ***N3***

[3 marks]

Let
$f(x) = x^3$. The following diagram shows part of the graph of *f* .



The point
$\mathrm{P}(a, f(a))$, where
$a > 0$, lies on the graph of *f* . The tangent at P crosses the *x*-axis at the point
$\mathrm{Q}\left(\frac{2}{3}, 0\right)$. This tangent intersects the graph of *f* at the point R(−2, −8) .

25a. (i) Show that the gradient of [PQ] is
$\frac{a^3}{a-\frac{2}{3}}$.

[7 marks]

(ii) Find
$f'(a)$.

(iii) Hence show that
$a = 1$.

Markscheme

(i) substitute into gradient
$= \frac{y_1-y_2}{x_1-x_2}$ ***(M1)***

e.g.
$\frac{f(a)-0}{a-\frac{2}{3}}$

substituting
$f(a) = a^3$

e.g.
$\frac{a^3-0}{a-\frac{2}{3}}$ ***A1***

gradient
$\frac{a^3}{a-\frac{2}{3}}$ ***AG N0***

(ii) correct answer ***A1 N1***

e.g.
$3a^2$,
$f'(a) = 3$,
$f'(a) = \frac{a^3}{a-\frac{2}{3}}$

(iii) **METHOD 1**

evidence of approach ***(M1)***

e.g.
$f'(a) = \text{gradient}$,
$3a^2 = \frac{a^3}{a-\frac{2}{3}}$

simplify ***A1***

e.g.
$3a^2\left(a-\frac{2}{3}\right) = a^3$

rearrange ***A1***

e.g.
$3a^3 - 2a^2 = a^3$

evidence of solving ***A1***

e.g.
$2a^3 - 2a^2 = 2a^2(a-1) = 0$

$a = 1$ ***AG N0***

METHOD 2

gradient RQ
$= \frac{-8}{-2-\frac{2}{3}}$ ***A1***

simplify ***A1***

e.g.
$\frac{-8}{-\frac{8}{3}}, 3$

evidence of approach ***(M1)***

e.g.
$f'(a) = \text{gradient}$,
$3a^2 = \frac{-8}{-2-\frac{2}{3}}$,
$\frac{a^3}{a-\frac{2}{3}} = 3$

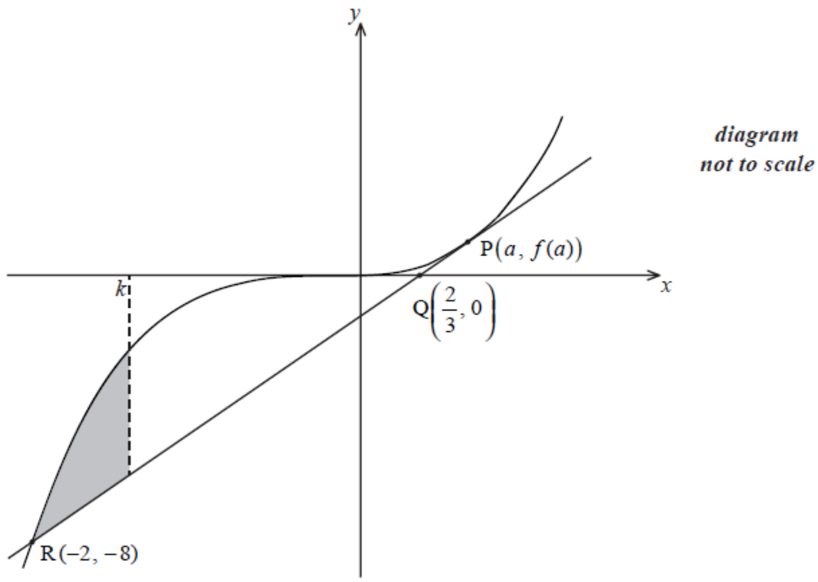
simplify ***A1***

e.g.
$3a^2 = 3$,
$a^2 = 1$

$a = 1$ ***AG N0***

[7 marks]

The equation of the tangent at P is
$y = 3x - 2$. Let *T* be the region enclosed by the graph of *f* , the tangent [PR] and the line
$x = k$, between
$x = -2$ and
$x = k$ where
$-2 < k < 1$. This is shown in the diagram below.



25b. Given that the area of *T* is
$2k + 4$, show that *k* satisfies the equation
$k^4 - 6k^2 + 8 = 0$. *[9 marks]*

Markscheme

approach to find area of T involving subtraction and integrals ***(M1)***

e.g.
$\int f-(3x-2)\mathrm{d}x$,
$\int_{-2}^{k}(3x-2)-\int_{-2}^{k}x^3$,
$\int(x^3-3x+2)$

correct integration with correct signs ***A1A1A1***

e.g.
$\frac{1}{4}x^4-\frac{3}{2}x^2+2x$,
$\frac{3}{2}x^2-2x-\frac{1}{4}x^4$

correct limits
-2 and k (seen anywhere) ***A1***

e.g.
$\int_{-2}^{k}(x^3-3x+2)\mathrm{d}x$,
$\left[\frac{1}{4}x^4-\frac{3}{2}x^2+2x\right]_{-2}^{k}$

attempt to substitute k and
-2 ***(M1)***

correct substitution into **their** integral if 2 or more terms ***A1***

e.g.
$\left(\frac{1}{4}k^4-\frac{3}{2}k^2+2k\right)-(4-6-4)$

setting **their** integral expression equal to
$2k+4$ (seen anywhere) ***(M1)***

simplifying ***A1***

e.g.
$\frac{1}{4}k^4-\frac{3}{2}k^2+2=0$

$k^4-6k^2+8=0$ ***AG*** ***N0***

[9 marks]

Consider a function f. The line L_1 with equation $y=3x+1$ is a tangent to the graph of f when $x=2$

26a. Write down $f'(2)$. *[2 marks]*

Markscheme

recognize that $f'(x)$ is the gradient of the tangent at x ***(M1)***

eg $f'(x)=m$

$f'(2)=3$ (accept m = 3) ***A1 N2***

[2 marks]

26b. Find $f(2)$. *[2 marks]*

Markscheme

recognize that $f(2)=y(2)$ ***(M1)***

eg $f(2)=3\times2+1$

$f(2)=7$ ***A1 N2***

[2 marks]

Let $g(x) = f(x^2 + 1)$ and P be the point on the graph of g where $x = 1$.

26c. Show that the graph of g has a gradient of 6 at P. *[5 marks]*

Markscheme

recognize that the gradient of the graph of g is $g'(x)$ ***(M1)***

choosing chain rule to find $g'(x)$ ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ u = x^2 + 1,\ u' = 2x$

$g'(x) = f'(x^2 + 1) \times 2x$ ***A2***

$g'(1) = 3 \times 2$ ***A1***

$g'(1) = 6$ ***AG N0***

[5 marks]

26d. Let L_2 be the tangent to the graph of g at P. L_1 intersects L_2 at the point Q. *[7 marks]*

Find the y-coordinate of Q.

Markscheme

at Q, $L_1 = L_2$ (seen anywhere) ***(M1)***

recognize that the gradient of L_2 is $g'(1)$ (seen anywhere) ***(M1)***
eg $m = 6$

finding $g(1)$ (seen anywhere) ***(A1)***
eg $g(1) = f(2),\ g(1) = 7$

attempt to substitute gradient and/or coordinates into equation of a straight line ***M1***
eg $y - g(1) = 6(x - 1),\ y - 1 = g'(1)(x - 7),\ 7 = 6(1) + \mathrm{b}$

correct equation for L_2

eg $y - 7 = 6(x - 1),\ y = 6x + 1$ ***A1***

correct working to find Q ***(A1)***
eg same *y*-intercept, $3x = 0$

$y = 1$ ***A1 N2***

[7 marks]

Let $f(x) = \sqrt{x}$. Line L is the normal to the graph of f at the point (4, 2).

27a. Show that the equation of L is $y = -4x + 18$. *[4 marks]*

Markscheme

finding derivative *(A1)*

e.g.

$f'(x) = \frac{1}{2}x^{\frac{1}{2}}, \frac{1}{2\sqrt{x}}$

correct value of derivative or its negative reciprocal (seen anywhere) ***A1***

e.g.

$\frac{1}{2\sqrt{4}}$,

$\frac{1}{4}$

gradient of normal =

$\frac{1}{\text{gradient of tangent}}$ (seen anywhere) ***A1***

e.g.

$-\frac{1}{f'(4)} = -4$,

$-2\sqrt{x}$

substituting into equation of line (for normal) ***M1***

e.g.

$y - 2 = -4(x - 4)$

$y = -4x + 18$ ***AG N0***

[4 marks]

27b. Point A is the *x*-intercept of *L* . Find the *x*-coordinate of A. *[2 marks]*

Markscheme

recognition that

$y = 0$ at A ***(M1)***

e.g.

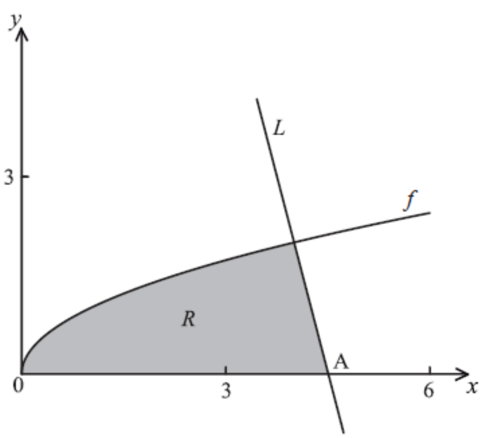
$-4x + 18 = 0$

$x = \frac{18}{4}$

$\left(= \frac{9}{2}\right)$ ***A1 N2***

[2 marks]

In the diagram below, the shaded region*R* is bounded by the *x*-axis, the graph of *f* and the line *L* .



27c. Find an expression for the area of *R* . *[3 marks]*

Markscheme

splitting into two appropriate parts (areas and/or integrals) ***(M1)***

correct expression for area of R ***A2*** ***N3***

e.g. area of R =
$\int_0^4 \sqrt{x}\mathrm{d}x + \int_4^{4.5}(-4x+18)\mathrm{d}x$,
$\int_0^4 \sqrt{x}\mathrm{d}x + \frac{1}{2} \times 0.5 \times 2$ (triangle)

Note: Award ***A1*** if d*x* is missing.

[3 marks]

27d. The region R is rotated
360° about the *x*-axis. Find the volume of the solid formed, giving your answer in terms of
π . *[8 marks]*

Markscheme

correct expression for the volume from
$x=0$ to
$x=4$ ***(A1)***

e.g.
$V=\int_0^4 \pi\left[f(x)^2\right]\mathrm{d}x$,
$\int_0^4 \pi\sqrt{x}^2\mathrm{d}x$,
$\int_0^4 \pi x\mathrm{d}x$

$V=\left[\frac{1}{2}\pi x^2\right]_0^4$ ***A1***

$V=\pi\left(\frac{1}{2}\times 16-\frac{1}{2}\times 0\right)$ ***(A1)***

$V=8\pi$ ***A1***

finding the volume from
$x=4$ to
$x=4.5$

EITHER

recognizing a cone ***(M1)***

e.g.
$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(2)^2\times\frac{1}{2}$ ***(A1)***

$=\frac{2\pi}{3}$ ***A1***

total volume is
$8\pi+\frac{2}{3}\pi$
$\left(=\frac{26}{3}\pi\right)$ ***A1*** ***N4***

OR

$V=\pi\int_4^{4.5}(-4x+18)^2\mathrm{d}x$ ***(M1)***

$=\int_4^{4.5}\pi(16x^2-144x+324)\mathrm{d}x$

$=\pi\left[\frac{16}{3}x^3-72x^2+324x\right]_4^{4.5}$ ***A1***

$=\frac{2\pi}{3}$ ***A1***

total volume is
$8\pi+\frac{2}{3}\pi$
$\left(=\frac{26}{3}\pi\right)$ ***A1*** ***N4***

[8 marks]