0419HW_Trig-FR *[51 marks]*

1. Solve the equation
$2\cos x = \sin 2x$, for
$0 \le x \le 3\pi$. *[7 marks]*

Markscheme

METHOD 1

using double-angle identity (seen anywhere) ***A1***

e.g.
$\sin 2x = 2\sin x\cos x$,
$2\cos x = 2\sin x\cos x$

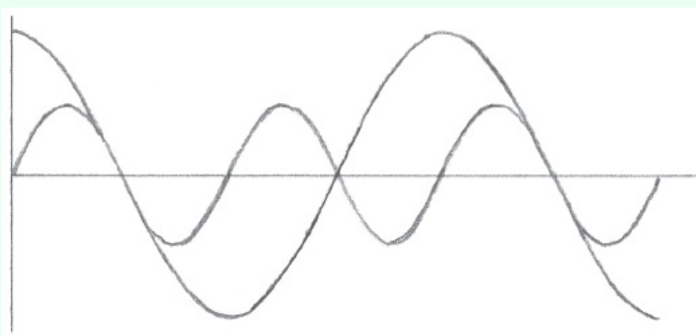
evidence of valid attempt to solve equation ***(M1)***

e.g.
$0 = 2\sin x\cos x - 2\cos x$,
$2\cos x(1 - \sin x) = 0$

$\cos x = 0$,
$\sin x = 1$ ***A1A1***

$x = \frac{\pi}{2}$,
$x = \frac{3\pi}{2}$,
$x = \frac{5\pi}{2}$ ***A1A1A1*** ***N4***

METHOD 2



A1A1M1A1

Notes: Award ***A1*** for sketch of
$\sin 2x$, ***A1*** for a sketch of
$2\cos x$, ***M1*** for at least one intersection point seen, and ***A1*** for 3 approximately correct intersection points. Accept sketches drawn outside
$[0, 3\pi]$, even those with more than 3 intersections.

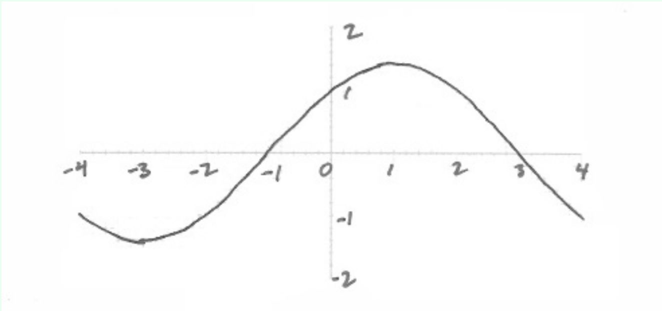
$x = \frac{\pi}{2}$,
$x = \frac{3\pi}{2}$,
$x = \frac{5\pi}{2}$ ***A1A1A1*** ***N4***

[7 marks]

Let
$f(x) = \cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right), \text{ for } -4 \leqslant x \leqslant 4.$

2a. Sketch the graph of
f. *[3 marks]*

Markscheme



A1A1A1 N3

Note: Award ***A1*** for approximately correct sinusoidal shape.

Only if this ***A1*** is awarded, award the following:

A1 for correct domain,

A1 for approximately correct range.

[3 marks]

2b. Find the values of x where the function is decreasing. *[5 marks]*

Markscheme

recognizes decreasing to the left of minimum or right of maximum,

eg

$f'(x) < 0$ ***(R1)***

x-values of minimum and maximum (may be seen on sketch in part (a)) ***(A1)(A1)***

eg

$x = -3,\ (1, 1.4)$

two correct intervals ***A1A1 N5***

eg

$-4 < x < -3,\ 1 \leqslant x \leqslant 4;\ x < -3,\ x \geqslant 1$

[5 marks]

2c. The function f can also be written in the form $f(x) = a\sin\left(\frac{\pi}{4}(x + c)\right)$, where $a \in \mathbb{R}$, and $0 \leqslant c \leqslant 2$. Find the value of a; *[3 marks]*

Markscheme

recognizes that

a is found from amplitude of wave ***(R1)***

y-value of minimum or maximum ***(A1)***

eg (–3, –1.41) , (1, 1.41)

$a = 1.41421$

$a = \sqrt{2}, \ \ (\text{exact}), 1.41,$ ***A1 N3***

[3 marks]

2d. The function *[4 marks]*

f can also be written in the form

$f(x) = a\sin\left(\frac{\pi}{4}(x+c)\right)$, where

$a \in \mathbb{R}$, and

$0 \leqslant c \leqslant 2$. Find the value of

c.

Markscheme

METHOD 1

recognize that shift for sine is found at *x*-intercept ***(R1)***

attempt to find *x*-intercept ***(M1)***

eg

$\cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right) = 0, \ x = 3 + 4k, \ k \in \mathbb{Z}$

$x = -1$ ***(A1)***

$c = 1$ ***A1 N4***

METHOD 2

attempt to use a coordinate to make an equation ***(R1)***

eg

$\sqrt{2}\sin\left(\frac{\pi}{4}c\right) = 1, \ \sqrt{2}\sin\left(\frac{\pi}{4}(3-c)\right) = 0$

attempt to solve resulting equation ***(M1)***

eg sketch,

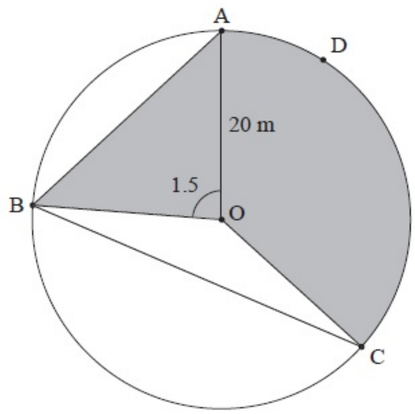
$x = 3 + 4k, \ k \in \mathbb{Z}$

$x = -1$ ***(A1)***

$c = 1$ ***A1 N4***

[4 marks]

The following diagram shows a circular play area for children.



The circle has centre O and a radius of 20 m, and the points A, B, C and D lie onthe circle. Angle AOB is 1.5 radians.

3a. Find the length of the chord [AB]. *[3 marks]*

Markscheme

Note: In this question, do not penalise for missing or incorrect units. They are not included in the markscheme, to avoid complex answer lines.

METHOD 1

choosing cosine rule (must have cos in it) ***(M1)***

e.g.
$c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution (into rhs) ***A1***

e.g.
$20^2 + 20^2 - 2(20)(20)\cos 1.5$,
$\mathrm{AB} = \sqrt{800 - 800\cos 1.5}$

$\mathrm{AB} = 27.26555\ldots$

$\mathrm{AB} = 27.3$
$[27.2, 27.3]$ ***A1 N2***

[3 marks]

METHOD 2

choosing sine rule ***(M1)***

e.g.
$\frac{\sin A}{a} = \frac{\sin B}{b}$,
$\frac{\mathrm{AB}}{\sin O} = \frac{\mathrm{AO}}{\sin B}$

correct substitution ***A1***

e.g.
$\frac{\mathrm{AB}}{\sin 1.5} = \frac{20}{\sin(0.5(\pi - 1.5))}$

$\mathrm{AB} = 27.26555\ldots$

$\mathrm{AB} = 27.3$
$[27.2, 27.3]$ ***A1 N2***

[3 marks]

3b. Find the area of triangle AOB. *[2 marks]*

Markscheme

correct substitution into area formula ***A1***

e.g.
$\frac{1}{2}(20)(20)\sin 1.5$,
$\frac{1}{2}(20)(27.2655504\ldots)\sin(0.5(\pi - 1.5))$

$\text{area} = 199.498997\ldots$ (accept
$199.75106 = 200$, from using 27.3)

$\text{area} = 199$
$[199, 200]$ ***A1*** ***N1***

[2 marks]

3c. Angle BOC is 2.4 radians. *[3 marks]*

Find the length of arc ADC.

Markscheme

appropriate method to find angle AOC ***(M1)***

e.g.
$2\pi - 1.5 - 2.4$

correct substitution into arc length formula ***(A1)***

e.g.
$(2\pi - 3.9) \times 20$,
$2.3831853\ldots \times 20$

$\text{arc length} = 47.6637\ldots$

$\text{arc length} = 47.7$
$(47.6, 47.7]$ (i.e. do **not** accept
$47.6)$ ***A1*** ***N2***

Notes: Candidates may misread the question and use
$\mathrm{A\hat{O}C} = 2.4$. If working shown, award ***M0*** then ***A0MRA1*** for the answer 48. Do not then penalize
$\mathrm{A\hat{O}C}$ in part (d) which, if used, leads to the answer
$679.498\ldots$

However, if they use the prematurely rounded value of 2.4 for
$\mathrm{A\hat{O}C}$, penalise 1 mark for premature rounding for the answer 48 in (c). Do not then penalize for this in (d).

[3 marks]

3d. Angle BOC is 2.4 radians. *[3 marks]*

Find the area of the shaded region.

Markscheme

calculating sector area using **their** angle AOC ***(A1)***

e.g.
$\frac{1}{2}(2.38\ldots)(20^2)$,
$200(2.38\ldots)$,
$476.6370614\ldots$

shaded area = **their** area of triangle AOB + **their** area of sector ***(M1)***

e.g.
$199.4989973\ldots + 476.6370614\ldots$,
$199 + 476.637$

$\text{shaded area} = 676.136\ldots$ (accept
$675.637\ldots = 676$ from using 199)

$\text{shaded area} = 676$
$[676, 677]$ ***A1 N2***

[3 marks]

3e. Angle BOC is 2.4 radians. *[4 marks]*

The shaded region is to be painted red. Red paint is sold in cans which cost
$\$32$ each. One can covers
$140\ \text{m}^2$. How much does it cost to buy the paint?

Markscheme

dividing to find number of cans ***(M1)***

e.g.
$\frac{676}{140}$,
$4.82857\ldots$

5 cans must be purchased ***(A1)***

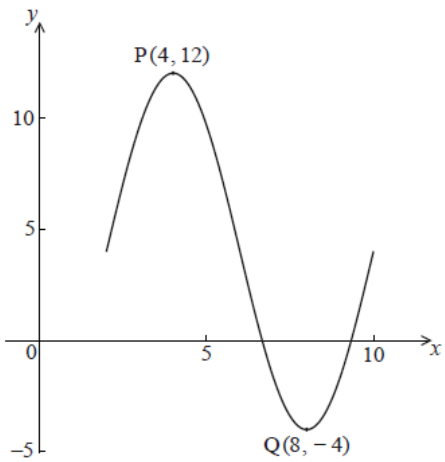
multiplying to find cost of cans ***(M1)***

e.g.
$5(32)$,
$\frac{676}{140} \times 32$

cost is 160 (dollars) ***A1 N3***

[4 marks]

The following diagram shows the graph of
$f(x) = a\sin(b(x-c)) + d$, for
$2 \le x \le 10$.



There is a maximum point at P(4, 12) and a minimum point at Q(8, −4) .

4a. Use the graph to write down the value of *[3 marks]*

(i) *a* ;

(ii) *c* ;

(iii) *d* .

Markscheme

(i)
$a = 8$ ***A1 N1***

(ii)
$c = 2$ ***A1 N1***

(iii)
$d = 4$ ***A1 N1***

[3 marks]

4b. Show that *[2 marks]*
$b = \frac{\pi}{4}$.

Markscheme

METHOD 1

recognizing that period
$= 8$ ***(A1)***

correct working ***A1***

e.g.
$8 = \frac{2\pi}{b}$,
$b = \frac{2\pi}{8}$

$b = \frac{\pi}{4}$ ***AG*** ***N0***

METHOD 2

attempt to substitute ***M1***

e.g.
$12 = 8\sin(b(4-2)) + 4$

correct working ***A1***

e.g.
$\sin 2b = 1$

$b = \frac{\pi}{4}$ ***AG*** ***N0***

[2 marks]

4c. Find
$f'(x)$. *[3 marks]*

Markscheme

evidence of attempt to differentiate or choosing chain rule ***(M1)***

e.g.
$\cos \frac{\pi}{4}(x-2)$,
$\frac{\pi}{4} \times 8$

$f'(x) = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$ (accept
$2\pi\cos\frac{\pi}{4}(x-2)$) ***A2*** ***N3***

[3 marks]

4d. At a point R, the gradient is
-2π . Find the *x*-coordinate of R. *[6 marks]*

Markscheme

recognizing that gradient is
$f'(x)$ ***(M1)***

e.g.
$f'(x) = m$

correct equation ***A1***

e.g.
$-2\pi = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$,
$-1 = \cos\left(\frac{\pi}{4}(x-2)\right)$

correct working ***(A1)***

e.g.
$\cos^{-1}(-1) = \frac{\pi}{4}(x-2)$

using
$\cos^{-1}(-1) = \pi$ (seen anywhere) ***(A1)***

e.g.
$\pi = \frac{\pi}{4}(x-2)$

simplifying ***(A1)***

e.g.
$4 = (x-2)$

$x = 6$ ***A1*** ***N4***

[6 marks]