Homework: Mixed practice, extended-response (76 marks). ***Paper 1, No Calculator***

1a. A line L_1 passes through the points $A(0, \ -3, \ 1)$ and $B(-2, \ 5, \ 3)$.

(i) Show that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}.$$

(ii) Write down a vector equation for L_1.   *[3 marks]*

1b. A line L_2 has equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$$

The lines L_1 and L_2 intersect at a point C.

Show that the coordinates of C are $(-1, \ 1, \ 2)$.   *[5 marks]*

1c. A point D lies on line L_2 so that $\left|\overrightarrow{CD}\right| = \sqrt{18}$ and $\overrightarrow{CA} \bullet \overrightarrow{CD} = -9$. Find $A\hat{C}D$.   *[7 marks]*

2a. Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

Find $f^{-1}(x)$.   *[2 marks]*

2b. Show that $\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$.   *[2 marks]*

2c. Let $h(x) = \frac{5}{x+2}$, for $x \geqslant 0$. The graph of h has a horizontal asymptote at $y = 0$.

Find the y-intercept of the graph of h.   *[2 marks]*

2d. Hence, sketch the graph of h.   *[3 marks]*

2e. For the graph of h^{-1}, write down the x-intercept;   *[1 mark]*

2f. For the graph of h^{-1}, write down the equation of the vertical asymptote.   *[1 mark]*

2g. Given that $h^{-1}(a) = 3$, find the value of a.   *[3 marks]*

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3a. Consider $f(x) = \ln(x^4 + 1)$.

Find the value of $f(0)$.   *[2 marks]*

3b. Find the set of values of x for which f is increasing.   *[5 marks]*

3c. The second derivative is given by $f''(x) = \dfrac{4x^2(3-x^4)}{(x^4+1)^2}$.

The equation $f''(x) = 0$ has only three solutions, when $x = 0$, $\pm\sqrt[4]{3}\,(\pm 1.316\ldots)$.

(i) Find $f''(1)$.

(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x = 0$.   *[5 marks]*

3d. There is a point of inflexion on the graph of f at $x = \sqrt[4]{3}\,(x = 1.316\ldots)$.

Sketch the graph of f, for $x \geq 0$.   *[3 marks]*

4a. Jar A contains three red marbles and five green marbles. Two marbles are drawn from the jar, one after the other, without replacement.

Find the probability that

(i) none of the marbles are green;

(ii) exactly one marble is green.   *[5 marks]*

4b. Find the expected number of green marbles drawn from the jar.   *[3 marks]*

4c. Jar B contains six red marbles and two green marbles. A fair six-sided die is tossed. If the score is 1 or 2, a marble is drawn from jar A. Otherwise, a marble is drawn from jar B.

(i) Write down the probability that the marble is drawn from jar B.

(ii) Given that the marble was drawn from jar B, write down the probability that it is red.   *[2 marks]*

4d. Given that the marble is red, find the probability that it was drawn from jar A.   *[6 marks]*

5a. Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \leq x \leq \pi$.

Find $f'(x)$.   *[3 marks]*

5b. Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g.

Find $g(4)$.   *[3 marks]*

5c. The function g can be expressed in the form $g(x) = a(x - h)^2 + 3$.

(i) Write down the value of h.

(ii) Find the value of a.   *[4 marks]*

5d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g.

[6 marks]

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