BECA / Huson / 12.1 IB Math SL Name:
13 November 2017

Homework: Vectors (Paper 1 problems)

1a. Let $u = 6i + 3j + 6k$ and $v = 2i + 2j + k$. *[5 marks]*

Find

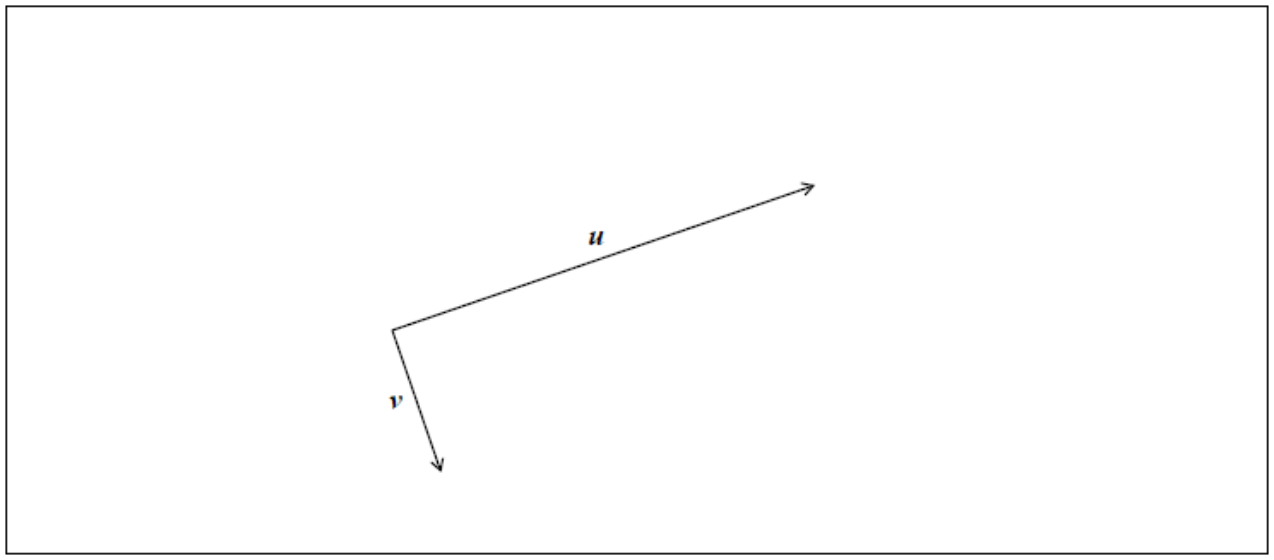
(i) $u \bullet v$;

(ii) $|u|$;

(iii) $|v|$.

1b. Find the angle between u and v. *[2 marks]*

2a. The following diagram shows two perpendicular vectors ***u*** and ***v***.



Let $w = u - v$. Represent w on the diagram above. *[2 marks]*

2b. Given that $u = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} 5 \\ n \\ 3 \end{pmatrix}$, where $n \in \mathbb{Z}$, find *n*. *[4 marks]*

3a. The vectors $\boldsymbol{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} k+3 \\ k \end{pmatrix}$ are perpendicular to each other. *[4 marks]*

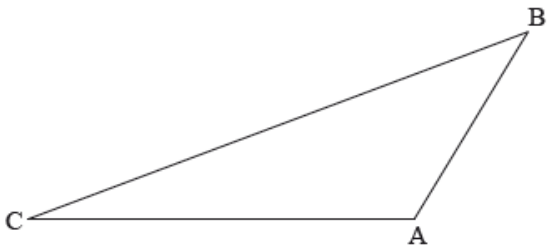
Find the value of k.

3b. Given that $\boldsymbol{c} = \boldsymbol{a} + 2\boldsymbol{b}$, find $\boldsymbol{c}$. *[3 marks]*

4. Let $\boldsymbol{u} = -3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{v} = m\boldsymbol{j} + n\boldsymbol{k}$, where $m,\ n \in \mathbb{R}$. Given that $\boldsymbol{v}$ is a unit vector perpendicular to $\boldsymbol{u}$, find the possible values of m and of n. *[7 marks]*

5. The following diagram shows triangle ABC.

diagram not to scale



Let $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}} = -5\sqrt{3}$ and $\left|\overrightarrow{\mathrm{AB}}\right| \left|\overrightarrow{\mathrm{AC}}\right| = 10$. Find the area of triangle ABC. *[6 marks]*

6a. Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$. *[6 marks]*

(a) Find

(i) $2\boldsymbol{a} + \boldsymbol{b}$;

(ii) $|2\boldsymbol{a} + \boldsymbol{b}|$. *[4 marks]*

6b. Let $2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = 0$, where 0 is the zero vector.

(b) Find $\boldsymbol{c}$. *[2 marks]*

7a. *[2 marks]*

Note: In this question, distance is in metres and time is in seconds.

Two particles P_1 and P_2 start moving from a point A at the same time, along different straight lines.

After t seconds, the position of P_1 is given by $\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

Find the coordinates of A.

7b. *[3 marks]*

Two seconds after leaving A, P_1 is at point B.

Find $\overrightarrow{\mathrm{AB}}$;

7c. *[2 marks]*

Find $\left|\overrightarrow{\mathrm{AB}}\right|$.

7d. *[5 marks]*

Two seconds after leaving A, P_2 is at point C, where $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$.

Find $\cos \mathrm{B\hat{A}C}$.

7e. *[4 marks]*

Hence or otherwise, find the distance between P_1 and P_2 two seconds after they leave A.