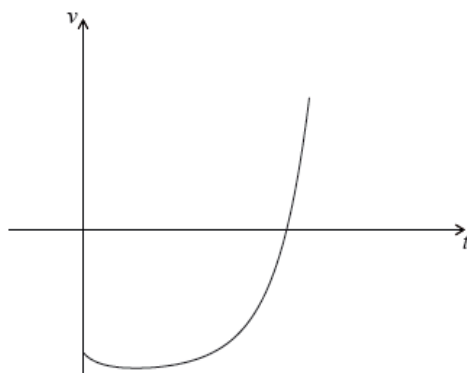**Homework: Calculus review**

1. Let $f'(x) = 6x^2 - 5$. Given that $f(2) = -3$, find $f(x)$. *[6 marks]*

2a. The velocity $v \text{ ms}^{-1}$ of a particle after t seconds is given by

$v(t) = (0.3t + 0.1)^t - 4$, for $0 \le t \le 5$

The following diagram shows the graph of v.

Find the value of t when the particle is at rest.

[3 marks]

2b. Find the value of t when the acceleration of the particle is 0. *[3 marks]*

3. Let $f(x) = \frac{\ln(4x)}{x}$ for $0 < x \le 5$.

Points $P(0.25, 0)$ and Q are on the curve of f. The tangent to the curve of f at P is perpendicular to the tangent at Q. Find the coordinates of Q. *[7 marks]*

4a. Let $f(x) = -x^4 + 2x^3 - 1$, for $0 \le x \le 2$.

Sketch the graph of f on the following grid. *[3 marks]*

4b. Solve $f(x) = 0$. *[2 marks]*

4c. The region enclosed by the graph of f and the x-axis is rotated $360°$ about the x-axis.

Find the volume of the solid formed. *[3 marks]*

5a. Let $f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.

Find $f'(x)$.   [2 marks]

5b. Find $\int f(x)\mathrm{d}x$.   [4 marks]

6a. *[4 marks]*

Consider $f(x) = x^2 \sin x$.

Find $f'(x)$.

6b. *[3 marks]*

Find the gradient of the curve of f at $x = \frac{\pi}{2}$.
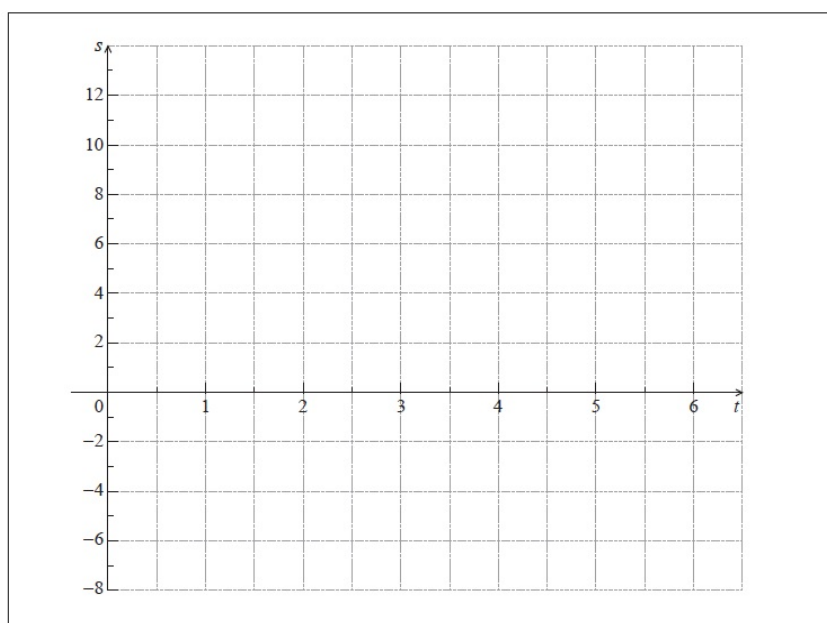
7. *[7 marks]*

A rocket moving in a straight line has velocity v km s⁻¹ and displacement s km at time t seconds. The velocity v is given by $v(t) = 6e^{2t} + t$. When $t = 0$, $s = 10$.

Find an expression for the displacement of the rocket in terms of t.

8a. A particle's displacement, in metres, is given by $s(t) = 2t \cos t$, for $0 \le t \le 6$, where t is the time in seconds.

On the grid below, sketch the graph of s.   [4 marks]

8b. *[3 marks]*

Find the maximum velocity of the particle.

2

9a. *[3 marks]*

Let $f(x) = \frac{20x}{e^{0.3x}}$, for $0 \le x \le 20$.

Sketch the graph of f.

9b. *[3 marks]*

(i) Write down the x-coordinate of the maximum point on the graph of f.

(ii) Write down the interval where f is increasing.

9c. *[5 marks]*

Show that $f'(x) = \frac{20-6x}{e^{0.3x}}$.

9d. *[4 marks]*

Find the interval where the rate of change of f is increasing.

10a. *[1 mark]*

The velocity v ms^{-1} of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \le t \le 2$.

Write down the velocity of the particle when $t = 0$.

10b. *[8 marks]*

When $t = k$, the acceleration is zero.

(i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$.

10c. *[4 marks]*

When $t < \frac{\pi}{4}$, $\frac{dv}{dt} > 0$ and when $t > \frac{\pi}{4}$, $\frac{dv}{dt} > 0$.

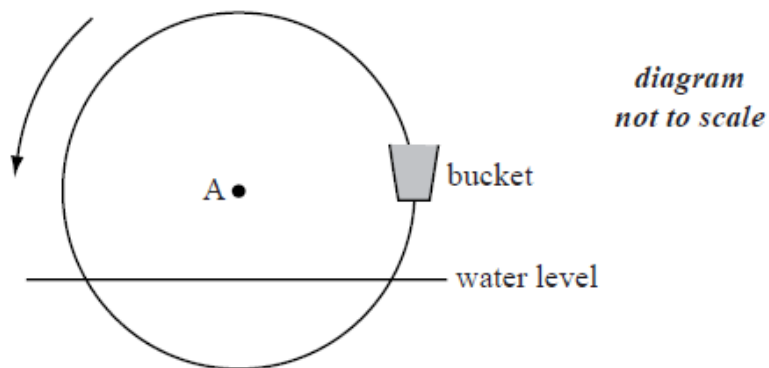Sketch a graph of v against t .

10d. *[3 marks]*

Let d be the distance travelled by the particle for $0 \le t \le 1$.

(i) Write down an expression for d .

(ii) Represent d on your sketch.

11a. *[2 marks]*

The following diagram shows a waterwheel with a bucket. The wheel rotates at a constant rate in an anticlockwise (counter-clockwise) direction.

diagram
not to scale

A •   bucket

water level

The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metres above the water level. After t seconds, the height of the bucket above the water level is given by $h = a \sin bt + 2$.

Show that $a = 4$.

11b. *[2 marks]*

The wheel turns at a rate of one rotation every 30 seconds.

Show that $b = \frac{\pi}{15}$.

11c. *[6 marks]*

In the first rotation, there are two values of t when the bucket is **descending** at a rate of 0.5 ms^{-1}.

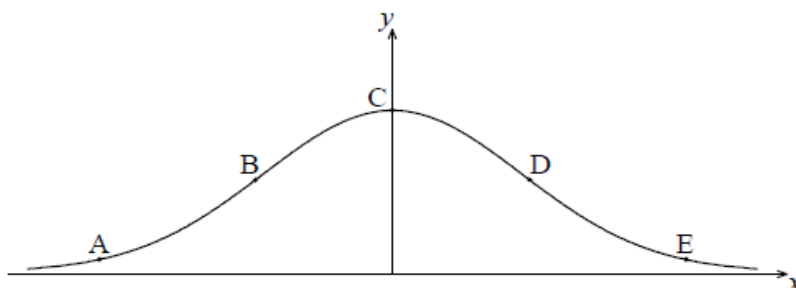Find these values of t.

11d. *[4 marks]*

In the first rotation, there are two values of t when the bucket is **descending** at a rate of 0.5 ms^{-1}.

Determine whether the bucket is underwater at the second value of t.

12a. *[2 marks]*

The following diagram shows the graph of $f(x) = e^{-x^2}$.

The points A, B, C, D and E lie on the graph of f. Two of these are points of inflexion.

Identify the **two** points of inflexion.

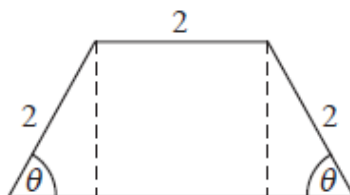**12b.** (i) Find $f'(x)$.

(ii) Show that $f''(x) = (4x^2 - 2)e^{-x^2}$.   *[5 marks]*

12c. Find the x-coordinate of each point of inflexion.   *[4 marks]*

12d. *[4 marks]*

Use the second derivative to show that one of these points is a point of inflexion.

13a. The diagram below shows a plan for a window in the shape of a trapezium.

Three sides of the window are 2 m long. The angle between the sloping sides of the window and the base is θ, where $0 < \theta < \frac{\pi}{2}$.

Show that the area of the window is given by $y = 4\sin\theta + 2\sin 2\theta$.   *[5 marks]*

13b. *[4 marks]*

Zoe wants a window to have an area of 5 m^2. Find the two possible values of θ.

13c. *[7 marks]*

John wants two windows which have the same area A but different values of θ.

Find all possible values for A.

14a. *[2 marks]*

A particle moves in a straight line. Its velocity, $v \text{ ms}^{-1}$, at time t seconds, is given by

$$v = \left(t^2 - 4\right)^3, \text{ for } 0 \leqslant t \leqslant 3.$$

Find the velocity of the particle when $t = 1$.

14b. *[3 marks]*

Find the value of t for which the particle is at rest.

14c. *[3 marks]*

Find the total distance the particle travels during the first three seconds.

14d. *[3 marks]*

Show that the acceleration of the particle is given by $a = 6t(t^2 - 4)^2$.

14e. *[4 marks]*

Find all possible values of t for which the velocity and acceleration are both positive or both negative.

15a. *[2 marks]*

The first three terms of a infinite geometric sequence are $m - 1, \ 6, \ m + 4$, where $m \in \mathbb{Z}$.

Write down an expression for the common ratio, r.

15b. *[2 marks]*

Hence, show that m satisfies the equation $m^2 + 3m - 40 = 0$.

15c. *[3 marks]*

Find the two possible values of m.

15d. *[3 marks]*

Find the possible values of r.

15e. *[3 marks]*

The sequence has a finite sum.

State which value of r leads to this sum **and** justify your answer.

15f. *[3 marks]*

The sequence has a finite sum.

Calculate the sum of the sequence.

16a. Consider the lines L_1 and L_2 with equations L_1 : $r = \begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ and L_2 :

$r = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$.

The lines intersect at point P. Find the coordinates of P.   *[6 marks]*

16b. Show that the lines are perpendicular.   *[5 marks]*

16c. The point $Q(7, 5, 3)$ lies on L_1. The point R is the reflection of Q in the line L_2.

Find the coordinates of R.   *[6 marks]*

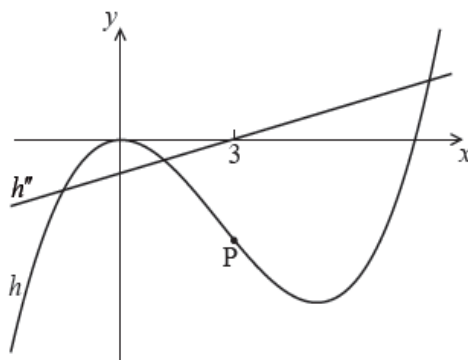**17a.** Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	−14	−18
$f'(x)$	1	1
$g'(x)$	−5	−3
$h''(x)$	−6	0

Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$.   *[3 marks]*

17b. The following diagram shows parts of the graphs of h and h''.

There is a point of inflexion on the graph of h at P, when $x = 3$.

Explain why P is a point of inflexion.   *[2 marks]*

17c. Given that $h(x) = f(x) \times g(x)$,

find the y-coordinate of P.   *[2 marks]*

17d. find the equation of the normal to the graph of h at P.   *[7 marks]*