Exam: Probability, Venn diagrams, descriptive statistics, trigonometry

1. Given:
   $U = \{\text{the letters in the alphabet}\}$ $A = \{b, e, c, a\}$ $B = \{r, u, l, e, s\}$

   (a) What is $A \cap B$? $\{e\}$ 1

   (b) What is $A \cup B$? $\{a, b, c, e, l, r, s, u\}$ 1

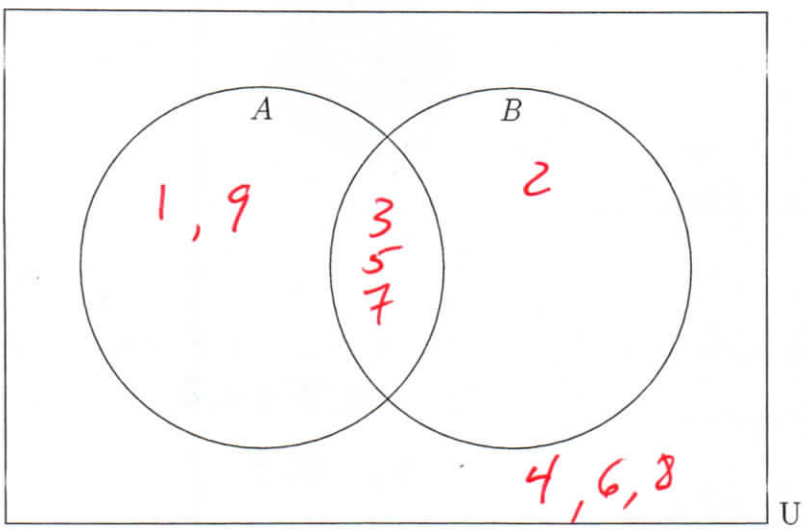
2. The universal set U is defined as the set of positive integers less than 10. The subsets A and B are defined as follows:

   $A = \{\text{the odd numbers}\}$ $B = \{\text{prime numbers}\}$

   (a) List the members of A' $\{2, 4, 6, 8\}$ 1

   (b) List the members of $(A \cup B)'$ $4, 6, 8$ 1

   (c) Place the elements of A and B in the appropriate regions in the Venn diagram below.



(1 2 wrong)
2 all

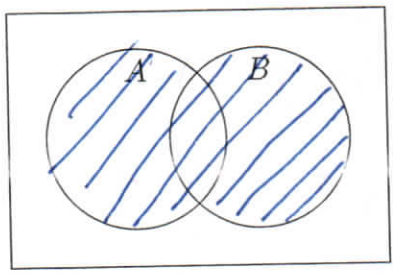
   (d) List the items in $A \cap B$. $3, 5, 7$ 1

   (e) If an element is selected at random, what is the probability that it is a member of both sets, $(A \cap B)$? $\frac{3}{9}$ 1

8

3. For each Venn diagram, shade the area representing the expression.

(a) $A \cup B$



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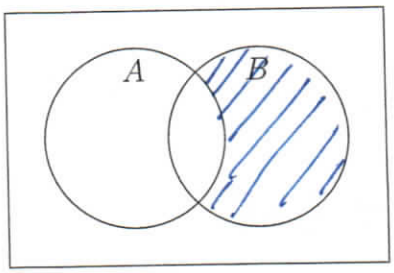
(b) $A' \cap B$



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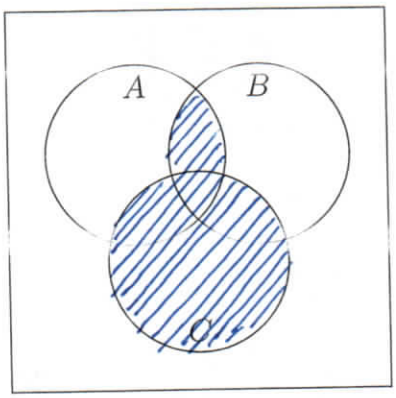
(c) $(A \cap B) \cup C$



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4. The events A and B are mutually exclusive with $\mathrm{P}(A) = 0.7$ and $\mathrm{P}(B) = 0.2$.

(a) Write down $\mathrm{P}(A \cup B)$. $= 0.7 + 0.2$

$= 0.9$

2

(b) Find $\mathrm{P}(A' \cup B)$. $= 1 - 0.7$

$= 0.3$

2

10

5. The events A and B are independent with $P(A) = 0.5$ and $P(B) = 0.8$.

(a) Find $P(A \cap B)$.

$= 0.5 \cdot 0.8$

$= 0.4$ 2

(b) Find $P(A \cup B)$.

$= 0.5 + 0.8 - 0.4$

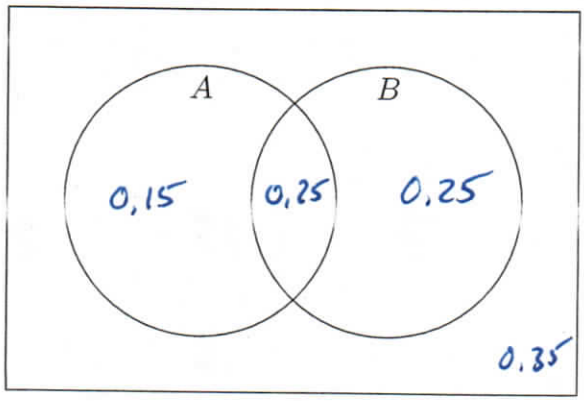
$= 0.9$ 2

(c) Find $P(B|A)$.

$= \frac{0.4}{0.5} = 0.8$ 2 (6)

6. Given events A and B with $P(A) = 0.4$, $P(B) = 0.5$, $P(A \cap B) = 0.25$.

(a) Completely mark the Venn diagram with probabilities for each area.



(1 w 1 error)
2

(b) Find $P(A \cup B)$.

$= 0.4 + 0.5 - 0.25$ M1

$= 0.65$ A1

(c) State whether events A and B are independent. Justify your answer.

No. $P(A) \times P(B) \neq P(A \cap B)$
$0.4 \times 0.5 \neq 0.25$ 3

(d) Find $P(A|B)$.

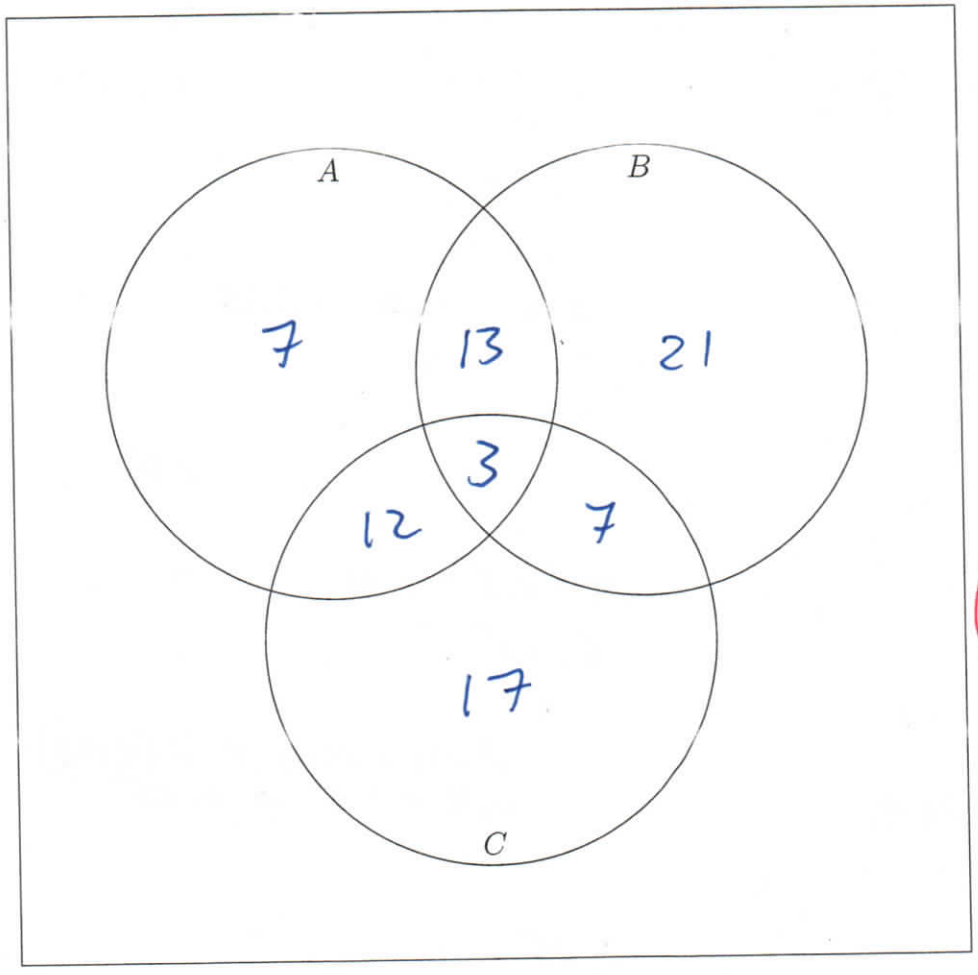
$= \frac{0.25}{0.50} = 0.5$ 2 (4)

15 (12)

7. There are 80 athletes playing the following sports:

- 35 play Archery
- 44 play Badminton
- 39 play Cricket
- 16 play Archery and Badminton
- 15 play Archery and Cricket
- 10 play Badminton and Cricket
- 3 play all three of these sports

Complete the Venn diagram below with the number of students in each region to represent the situation.



4

(3 - 1 error
2 3 errors
1 4 errors)

(4)

8. Forty IB high school students range in age from 15 to 18 years old. The following table shows the frequencies of each age.

Age (years)	15	16	17	18
Frequency	5	k	15	7

(a) Calculate the value of k. [1 mark]

$40-(5+15+7) = 13$

(b) Write down the mode. [1 mark]

17

(c) Find the value of the range. [1 marks]

$18-15 = 3$

(d) Find the median. [1 marks]

$\frac{40}{2}=20 \quad x_{20} = 17$

(e) Find the mean. [2 marks]

16.6

(f) Find the standard deviation. [2 marks]

$0,916515153\ldots$
$\approx 0,917$

8

9. A runner records her pace in terms of distance run (d) in miles over time (t) in minutes during a 4.5 mile run. She models her pace with a linear regression equation $d = at+b$.

minutes (t)	0	8	15	22	30
miles (d)	0	1.8	2.7	3.7	4.5

(a) Find the values of a, b, and the correlation r. [3 marks]

$a = 0.147445\ldots \approx 0,147$
$b = 0,328321\ldots \approx 0.328$
$r = 0,987705\ldots \approx 0,988$

(b) What does the value of a represent? [2 marks]

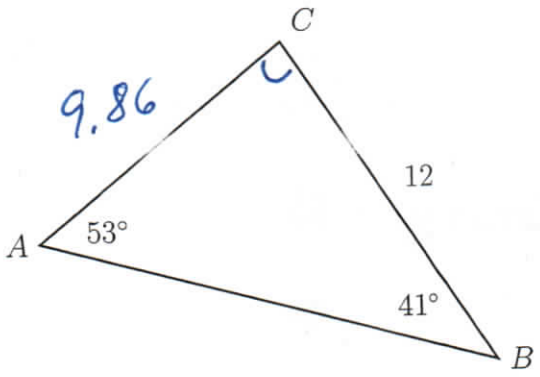
her speed in miles per minute (modeled as a linear regression)

* in "context"

5

(13)

10. The following diagram shows triangle ABC (not drawn to scale).



$BC = 12$, $C\hat{A}B = 53°$, and $A\hat{B}C = 41°$

(a) Find the measure of $A\hat{C}B$. [1 mark]

$\hat{C} = 180 - (53 + 41)$

$= 86°$

(b) Find AC. [3 marks]

$\frac{AC}{\sin 41} = \frac{12}{\sin 53}$

$AC = 9.857698\ldots$

≈ 9.86

(c) Find the area of triangle ABC. [3 marks]

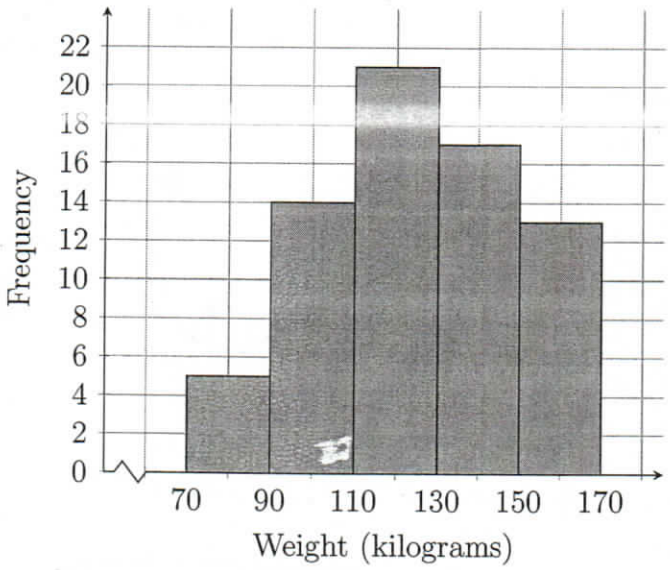
$A_\Delta = \frac{1}{2}(9.86)(12)\sin 86°$

$= 59.002115\ldots$

≈ 59.0

7

11. The histogram below shows the weight w in kilograms for 70 professional football players.



The following is the frequency table for the distribution of w.

HR (x)	$70 \leq x < 90$	$90 \leq x < 110$	$110 \leq x < 130$	$130 \leq x < 150$	$150 \leq x < 170$
Freq	5	14	21	p	13

(a) Write down the value of p. [1 mark]

$5 + 14 + 21 + p + 13 = 70$ $\quad p = 17$

(b) Write down the modal class. [2 marks]

$110 \leq x < 130$

(c) A player is selected at random. Find the probability that the athlete weighs 110 kilograms or less. [2 marks]

$P(x \leq 110) = \frac{5+14}{70} = \frac{19}{70}$

(d) Write down the mid-interval value for the class $110 \leq x < 130$. [1 mark]

120

(e) Hence find an estimate for the

i. mean; [2 marks]

$\bar{x} = \frac{\Sigma(x_i \cdot f_i)}{70} = 125.4285\ldots \approx 125$

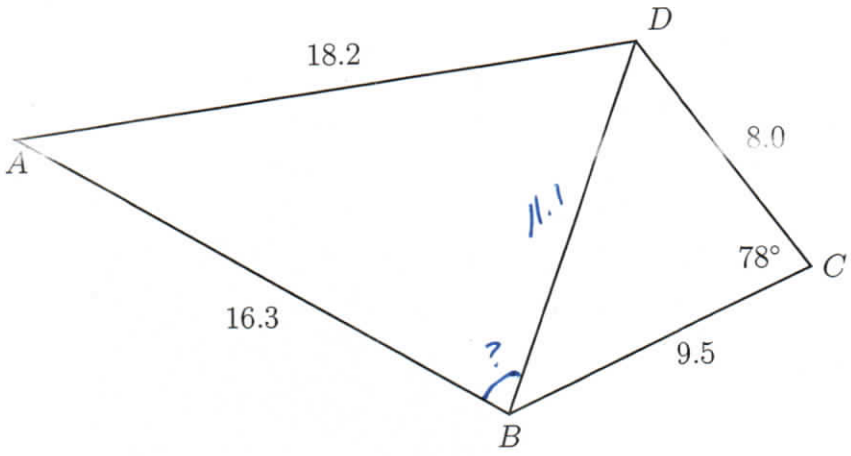
ii. standard deviation. [2 marks]

$\sigma = 23.6453\ldots$

≈ 23.6

10

12. The following diagram shows quadrilateral $ABCD$ (not drawn to scale).



$AB = 16.3$, $BC = 9.5$, $CD = 8.0$, $AD = 18.2$, and $B\hat{C}D = 78°$

(a) Find BD. [3 marks]

$BD = \sqrt{8^2 + 9.5^2 - 2(8)(9.5)\cos 78}$

$= 11.0746297...$

≈ 11.1

(b) Find $A\hat{B}D$. [3 marks]

$A\hat{B}D = \cos^{-1}\left(\dfrac{18.2^2 - 11.1^2 - 16.3^2}{-2(16.3)(11.1)}\right)$

$= 80.83121...$

$\approx 80.8°$

(accept 80.9)