0306CW_Binomial-distributions *[93 marks]*

A discrete random variable X has the following probability distribution.

x	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	p

1a. Find p. *[3 marks]*

Markscheme

summing probabilities to 1 ***(M1)***

eg, $\sum = 1,\ 3+4+2+x=10$

correct working ***(A1)***

$\frac{3}{10}+\frac{4}{10}+\frac{2}{10}+p=1,\ p=1-\frac{9}{10}$

$p=\frac{1}{10}$ ***A1 N3***

[3 marks]

1b. Find $\mathrm{E}(X)$. *[3 marks]*

Markscheme

correct substitution into formula for $\mathrm{E}(X)$ ***(A1)***

eg $0\left(\frac{3}{10}\right)+\ldots+3(p)$

correct working ***(A1)***

eg $\frac{4}{10}+\frac{4}{10}+\frac{3}{10}$

$\mathrm{E}(X)=\frac{11}{10}\quad(1.1)$ ***A1 N2***

[3 marks]

Total [6 marks]

The following table shows the probability distribution of a discrete random variable X.

x	0	1	2	3
$\mathrm{P}(X=x)$	0.15	k	0.1	$2k$

2a. Find the value of k. *[3 marks]*

Markscheme

evidence of using $\sum p_i = 1$ ***(M1)***

correct substitution ***A1***

eg $0.15 + k + 0.1 + 2k = 1,\ 3k + 0.25 = 1$

$k = 0.25$ ***A1*** ***N2***

[3 marks]

2b. Find $\mathrm{E}(X)$. *[2 marks]*

Markscheme

correct substitution ***(A1)***

eg $0 \times 0.15 + 1 \times 0.25 + 2 \times 0.1 + 3 \times 0.5$

$\mathrm{E}(X) = 1.95$ ***A1*** ***N2***

[2 marks]

Total [5 marks]

3. The random variable X has the following probability distribution. *[6 marks]*

x	1	2	3
$\mathrm{P}(X = x)$	s	0.3	q

Given that
$\mathrm{E}(X) = 1.7$, find q .

Markscheme

correct substitution into
$\mathrm{E}(X) = \sum px$ (seen anywhere) ***A1***

e.g.
$1s + 2 \times 0.3 + 3q = 1.7$,
$s + 3q = 1.1$

recognizing
$\sum p = 1$ (seen anywhere) ***(M1)***

correct substitution into
$\sum p = 1$ ***A1***

e.g.
$s + 0.3 + q = 1$

attempt to solve simultaneous equations ***(M1)***

correct working ***(A1)***

e.g.
$0.3 + 2q = 0.7$,
$2s = 1$

$q = 0.2$ ***A1*** ***N4***

[6 marks]

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X = x) = \frac{x^2}{14}, x \in \{1, 2, k\}, \text{ where} k > 0$$

.

4a. Write down $\mathrm{P}(X = 2)$. *[1 mark]*

Markscheme

$\mathrm{P}(X = 2) = \frac{4}{14}$
$\left(= \frac{2}{7}\right)$ ***A1 N1***

[1 mark]

4b. Show that $k = 3$. *[4 marks]*

Markscheme

$\mathrm{P}(X = 1) = \frac{1}{14}$ ***(A1)***

$\mathrm{P}(X = k) = \frac{k^2}{14}$ ***(A1)***

setting the sum of probabilities $= 1$ ***M1***

e.g. $\frac{1}{14} + \frac{4}{14} + \frac{k^2}{14} = 1$, $5 + k^2 = 14$

$k^2 = 9$ (accept $\frac{k^2}{14} = \frac{9}{14}$) ***A1***

$k = 3$ ***AG N0***

[4 marks]

4c. Find $\mathrm{E}(X)$. *[2 marks]*

Markscheme

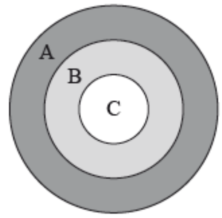
correct substitution into $\mathrm{E}(X) = \sum x\mathrm{P}(X = x)$ ***A1***

e.g. $1\left(\frac{1}{14}\right) + 2\left(\frac{4}{14}\right) + 3\left(\frac{9}{14}\right)$

$\mathrm{E}(X) = \frac{36}{14}$ $\left(= \frac{18}{7}\right)$ ***A1 N1***

[2 marks]

The following diagram shows a board which is divided into three regions A, B and C.



A game consists of a contestant throwing one dart at the board. The probability of hitting each region is given in the following table.

Region	A	B	C
Probability	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{1}{20}$

5a. Find the probability that the dart does **not** hit the board. *[3 marks]*

Markscheme

evidence of summing probabilities to 1 ***(M1)***

eg $\frac{5}{20}+\frac{4}{20}+\frac{1}{20}+p=1,\quad \sum=1$

correct working ***(A1)***

eg $p=1-\frac{10}{20}$

$p=\frac{10}{20}\quad\left(=\frac{1}{2}\right)$ ***A1 N2***

[3 marks]

5b. The contestant scores points as shown in the following table. *[4 marks]*

Region	A	B	C	Does not hit the board
Points	0	q	10	−3

Given that the game is fair, find the value of q.

Markscheme

correct substitution into $\mathrm{E}(X)$ ***(A1)***

eg $\frac{4}{20}(q)+\frac{1}{20}(10)+\frac{10}{20}(-3)$

valid reasoning for fair game (seen anywhere, including equation) ***(M1)***

eg $\mathrm{E}(X)=0$, points lost=points gained

correct working ***(A1)***

eg $4q+10-30=0,\quad \frac{4}{20}q+\frac{10}{20}=\frac{30}{20}$

$q=5$ ***A1 N2***

[4 marks]

Total [7 marks]

In a large university the probability that a student is left handed is 0.08. A sample of 150 students is randomly selected from the university. Let k be the expected number of left-handed students in this sample.

6a. Find k. *[2 marks]*

Markscheme

evidence of binomial distribution (may be seen in part (b)) ***(M1)***

eg $np,\ 150 \times 0.08$

$k = 12$ ***A1*** ***N2***

[2 marks]

6b. Hence, find the probability that exactly k students are left handed; *[2 marks]*

Markscheme

$\mathrm{P}\left(X = 12\right) = \binom{150}{12}(0.08)^{12}(0.92)^{138}$ ***(A1)***

0.119231

probability $= 0.119$ ***A1*** ***N2***

[2 marks]

6c. Hence, find the probability that fewer than k students are left handed. *[2 marks]*

Markscheme

recognition that $X \leqslant 11$ ***(M1)***

0.456800

$\mathrm{P}(X < 12) = 0.457$ ***A1*** ***N2***

[2 marks]

The following table shows a probability distribution for the random variable X, where $\mathrm{E}(X) = 1.2$.

x	0	1	2	3
$\mathrm{P}(X=x)$	p	$\frac{1}{2}$	$\frac{3}{10}$	q

7a. Find q. *[2 marks]*

Markscheme

correct substitution into $\mathrm{E}(X)$ formula ***(A1)***

eg $0(p) + 1(0.5) + 2(0.3) + 3(q) = 1.2$

$q = \frac{1}{30}$, 0.0333 ***A1*** ***N2***

[2 marks]

7b. Find p. *[2 marks]*

Markscheme

evidence of summing probabilities to 1 ***(M1)***

eg $p + 0.5 + 0.3 + q = 1$

$p = \frac{1}{6}$, 0.167 ***A1 N2***

[2 marks]

A bag contains white and blue marbles, with at least three of each colour. Three marbles are drawn from the bag, without replacement. The number of blue marbles drawn is given by the random variable X.

7c. Write down the probability of drawing three blue marbles. *[1 mark]*

Markscheme

$\mathrm{P}\ (3\ \text{blue}) = \frac{1}{30}$, 0.0333 ***A1 N1***

[1 mark]

7d. Explain why the probability of drawing three white marbles is $\frac{1}{6}$. *[1 mark]*

Markscheme

valid reasoning ***R1***

eg $\mathrm{P}\ (3\ \text{white}) = \mathrm{P}(0\ \text{blue})$

$\mathrm{P}(3\ \text{white}) = \frac{1}{6}$ ***AG N0***

[1 mark]

7e. The bag contains a total of ten marbles of which w are white. Find w. *[3 marks]*

Markscheme

valid method ***(M1)***

eg $\mathrm{P}(3\ \text{white}) = \frac{w}{10} \times \frac{w-1}{9} \times \frac{w-2}{8}$, $\frac{{}_wC_3}{{}_{10}C_3}$

correct equation ***A1***

eg $\frac{w}{10} \times \frac{w-1}{9} \times \frac{w-2}{8} = \frac{1}{6}$, $\frac{{}_wC_3}{{}_{10}C_3} = 0.167$

$w = 6$ ***A1 N2***

[3 marks]

A game is played in which three marbles are drawn from the bag of ten marbles, without replacement. A player wins a prize if three white marbles are drawn.

7f. Grant plays the game until he wins two prizes. Find the probability that he wins his second prize on his eighth attempt. *[4 marks]*

Markscheme

recognizing one prize in first seven attempts *(M1)*

eg $\binom{7}{1}$, $\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6$

correct working *(A1)*

eg $\binom{7}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6$, 0.390714

correct approach *(A1)*

eg $\binom{7}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6 \times \frac{1}{6}$

0.065119

0.0651 ***A1*** ***N2***

[4 marks]

8a. The probability of obtaining "tails" when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining **at least** four tails. *[4 marks]*

Markscheme

evidence of recognizing binomial distribution *(M1)*

e.g.
$X \sim \mathrm{B}(10, 0.57)$,
$p = 0.57$,
$q = 0.43$

EITHER

$\mathrm{P}(X \le 3) = 2.16 \times 10^{-4} + 0.00286 + 0.01709 + 0.06041$
$(= 0.08057)$ *(A1)*

evidence of using complement *(M1)*

e.g.
$1-$ any probability,
$\mathrm{P}(X \ge 4) = 1 - \mathrm{P}(X \le 3)$

$0.919423\ldots$

$\mathrm{P}(X \ge 4) = 0.919$ ***A1*** ***N3***

OR

summing the probabilities from
$X = 4$ to
$X = 10$ *(M1)*

correct expression or values *(A1)*

e.g.
$\sum_{r=4}^{10}\binom{10}{r}(0.57)^r(0.43)^{10-r}$,
$0.14013 + 0.2229 + \ldots + 0.02731 + 0.00362$

0.919424

$\mathrm{P}(X \ge 4) = 0.919$ ***A1*** ***N3***

[4 marks]

8b. The probability of obtaining "tails" when a biased coin is tossed is 0.57. The coin istossed ten times. Find the probability of obtaining the fourth tail on the tenth toss. *[3 marks]*

Markscheme

evidence of valid approach *(M1)*

e.g. three tails in nine tosses,

$\binom{9}{3}(0.57)^3(0.43)^6$

correct calculation

e.g.

$\binom{9}{3}(0.57)^3(0.43)^6 \times 0.57$,

0.09834×0.57 *(A1)*

$0.05605178\ldots$

$\mathrm{P}(\text{4th tail on 10th toss}) = 0.0561$ ***A1*** ***N2***

[3 marks]

A jar contains 5 red discs, 10 blue discs and m green discs. A disc is selected at random and replaced. This process is performed four times.

9a. Write down the probability that the first disc selected is red. *[1 mark]*

Markscheme

$\mathrm{P}(\text{red}) = \frac{5}{15+m}$ ***A1*** ***N1***

[1 mark]

9b. Let X be the number of red discs selected. Find the smallest value of m for which $\mathrm{Var}(X) < 0.6$. *[5 marks]*

Markscheme

recognizing binomial distribution ***(M1)***

eg $X \sim B(n,\, p)$

correct value for the complement of **their** p (seen anywhere) ***A1***

eg $1 - \frac{5}{15+m},\ \frac{10+m}{15+m}$

correct substitution into $\mathrm{Var}(X) = np(1-p)$ ***(A1)***

eg $4\left(\frac{5}{15+m}\right)\left(\frac{10+m}{15+m}\right),\ \frac{20(10+m)}{(15+m)^2} < 0.6$

$m > 12.2075$ ***(A1)***

$m = 13$ ***A1*** ***N3***

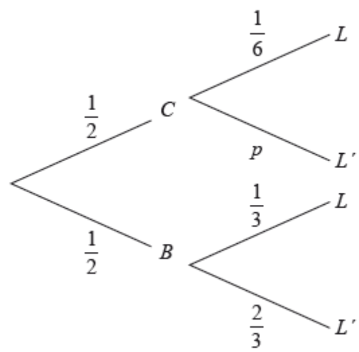
[5 marks]

Adam travels to school by car (C) or by bicycle (B). On any particular day he is equally likely to travel by car or by bicycle.

The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car.

The probability of being late for school is $\frac{1}{3}$ if he travels by bicycle.

This information is represented by the following tree diagram.



10a. Find the value of p. *[2 marks]*

Markscheme

correct working ***(A1)***

eg $1-\frac{1}{6}$

$p=\frac{5}{6}$ ***A1 N2***

[2 marks]

10b. Find the probability that Adam will travel by car and be late for school. *[2 marks]*

Markscheme

multiplying along correct branches ***(A1)***

eg $\frac{1}{2}\times\frac{1}{6}$

$\mathrm{P}(C\cap L)=\frac{1}{12}$ ***A1 N2***

[2 marks]

10c. Find the probability that Adam will be late for school. *[4 marks]*

Markscheme

multiplying along the other branch ***(M1)***

eg $\frac{1}{2}\times\frac{1}{3}$

adding probabilities of their 2 mutually exclusive paths ***(M1)***

eg $\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{3}$

correct working ***(A1)***

eg $\frac{1}{12}+\frac{1}{6}$

$\mathrm{P}(L)=\frac{3}{12}\ \left(=\frac{1}{4}\right)$ ***A1 N3***

[4 marks]

10d. Given that Adam is late for school, find the probability that he travelled by car. *[3 marks]*

Markscheme

recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P}(C|L)$

correct substitution of **their** values into formula ***(A1)***

eg $\frac{\frac{1}{12}}{\frac{3}{12}}$

$\mathrm{P}(C|L) = \frac{1}{3}$ ***A1 N2***

[3 marks]

10e. Adam will go to school three times next week. *[4 marks]*

Find the probability that Adam will be late exactly once.

Markscheme

valid approach ***(M1)***

eg $X \sim B\left(3, \frac{1}{4}\right)$, $\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2$, $\binom{3}{1}$, three ways it could happen

correct substitution ***(A1)***

eg $\binom{3}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2$, $\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{1}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$

correct working ***(A1)***

eg $3\left(\frac{1}{4}\right)\left(\frac{9}{16}\right)$, $\frac{9}{64} + \frac{9}{64} + \frac{9}{64}$

$\frac{27}{64}$ ***A1 N2***

[4 marks]

Total [15 marks]

A bag contains four gold balls and six silver balls.

Two balls are drawn at random from the bag, with replacement. Let X be the number of gold balls drawn from the bag.

11a. (i) Find $\mathrm{P}(X = 0)$. *[8 marks]*

(ii) Find $\mathrm{P}(X = 1)$.

(iii) Hence, find $\mathrm{E}(X)$.

Markscheme

METHOD 1

(i) appropriate approach ***(M1)***

eg $\frac{6}{10} \times \frac{6}{10}$, $\frac{6}{10} \times \frac{5}{9}$, $\frac{6}{10} \times \frac{5}{10}$

$\mathrm{P}(X = 0) = \frac{9}{25} = 0.36$ ***A1 N2***

(ii) multiplying one pair of gold and silver probabilities ***(M1)***

eg
$\frac{6}{10} \times \frac{4}{10}$,
$\frac{6}{10} \times \frac{4}{9}$, 0.24

adding the product of both pairs of gold and silver probabilities ***(M1)***

eg
$\frac{6}{10} \times \frac{4}{10} \times 2$,
$\frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{6}{9}$
$\mathrm{P}(X = 1) = \frac{12}{25} = 0.48$ ***A1*** ***N3***

(iii)

$\mathrm{P}(X = 2) = 0.16$ (seen anywhere) (A1)

correct substitution into formula for
$\mathrm{E}(X)$ (A1)

eg
$0 \times 0.36 + 1 \times 0.48 + 2 \times 0.16$,
$0.48 + 0.32$

$\mathrm{E}(X) = \frac{4}{5} = 0.8$ A1 N3

METHOD 2

(i) evidence of recognizing binomial (may be seen in part (ii)) ***(M1)***

eg
$X \sim \mathrm{B}(2, 0.6)$,
$\binom{2}{0}(0.4)^2(0.6)^0$

correct probability for use in binomial ***(A1)***

eg
$p = 0.4$,
$X \sim \mathrm{B}(2, 0.4)$,
${}^2C_0(0.4)^0(0.6)^2$

$\mathrm{P}(X = 0) = \frac{9}{25} = 0.36$ ***A1 N3***

(ii) correct set up ***(A1)***

eg
${}_2C_1(0.4)^1(0.6)^1$

$\mathrm{P}(X = 1) = \frac{12}{25} = 0.48$ ***A1*** ***N2***

(iii)

attempt to substitute into
np (M1)

eg
2×0.6

correct substitution into
np (A1)

eg
2×0.4

$\mathrm{E}(X) = \frac{4}{5} = 0.8$ A1 N3

[8 marks]

Fourteen balls are drawn from the bag, with replacement.

11b. Find the probability that exactly five of the balls are gold. *[2 marks]*

Markscheme

Let
Y be the number of gold balls drawn from the bag.

evidence of recognizing binomial (seen anywhere) ***(M1)***

eg
${}_{14}C_5(0.4)^5(0.6)^9$,
$\mathrm{B}(14, 0.4)$

$\mathrm{P}(Y = 5) = 0.207$ ***A1 N2***

[2 marks]

11c. Find the probability that at most five of the balls are gold. *[2 marks]*

Markscheme

recognize need to find $\mathrm{P}(Y \le 5)$ ***(M1)***

$\mathrm{P}(Y \le 5) = 0.486$ ***A1 N2***

[2 marks]

11d. Given that at most five of the balls are gold, find the probability that exactly fiveof the balls are gold. Give the answer correct to two decimal places. *[3 marks]*

Markscheme

Let
Y be the number of gold balls drawn from the bag.

recognizing conditional probability ***(M1)***

eg
$\mathrm{P}(A|B)$,
$\mathrm{P}(Y = 5|Y \le 5)$,
$\frac{\mathrm{P}(Y=5)}{\mathrm{P}(Y\le5)}$,
$\frac{0.207}{0.486}$

$\mathrm{P}(Y = 5|Y \le 5) = 0.42522518$ ***(A1)***

$\mathrm{P}(Y = 5|Y \le 5) = 0.43$ (to
2 dp) ***A1 N2***

[3 marks]