0409Mixed-NoCalc-extended-response *[76 marks]*

A line
L_1 passes through the points $\mathrm{A}(0,\ -3,\ 1)$ and $\mathrm{B}(-2,\ 5,\ 3)$.

1a. *[3 marks]*

(i) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$.

(ii) Write down a vector equation for L_1.

Markscheme

(i) correct approach ***A1***

eg $\mathrm{OB} - \mathrm{OA},\ \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix},\ \mathrm{B} - \mathrm{A}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$ ***AG*** ***N0***

(ii) **any** correct equation in the form $r = a + tb$ (accept any parameter for t)

where a is $\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$, and b is a scalar multiple of $\begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$ ***A2*** ***N2***

eg$r$ = \left({

$$\begin{matrix} 0 \\ -3 \\ 1 \end{matrix}$$

} \right) + t\left({

$$\begin{matrix} -2 \\ 8 \\ 2 \end{matrix}$$

} \right),r = \left({

$$\begin{matrix} -2 - 2s \\ 5 + 8s \\ 3 + 2s \end{matrix}$$

} \right),$r = 2i + 5j + 3k + t(-2i + 8j + 2k)$

Note: Award ***A1*** for the form a + tb, ***A1*** for the form $L =$ \(a + tb,

A0 for the form r = b + ta.

[3 marks]

Examiners report

[N/A]

1b. *[5 marks]*

A line L_2 has equation $\mathrm{r} = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$. The lines L_1 and L_2 intersect at a point

C.

Show that the coordinates of C are $(-1,\ 1,\ 2)$.

Markscheme

valid approach ***(M1)***

eg equating lines, $L_1 = L_2$

one correct equation in one variable ***A1***

eg $-2t = -1,\ -2 - 2t = -1$

valid attempt to solve ***(M1)***

eg $2t = 1,\ -2t = 1$

one correct parameter ***A1***

eg $t = \frac{1}{2},\ t = -\frac{1}{2},\ s = -6$

correct substitution of either parameter ***A1***

eg $r = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix},\ r = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix},\ r = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} - 6\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

the coordinates of C are $(-1,\ 1,\ 2)$, or position vector of C is $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ ***AG*** ***N0***

Note: If candidate uses the same parameter in both vector equations and working shown, award ***M1A1M1A0A0***.

[5 marks]

Examiners report

[N/A]

1c. A point D lies on line L_2 so that $\left|\overrightarrow{\mathrm{CD}}\right| = \sqrt{18}$ and $\overrightarrow{\mathrm{CA}} \bullet \overrightarrow{\mathrm{CD}} = -9$. Find $\mathrm{A\hat{C}D}$. *[7 marks]*

Markscheme

valid approach *(M1)*

eg attempt to find $\overrightarrow{\mathrm{CA}}$, $\cos \mathrm{A\hat{C}D} = \frac{\overrightarrow{\mathrm{CA}} \bullet \overrightarrow{\mathrm{CD}}}{\left|\overrightarrow{\mathrm{CA}}\right|\left|\overrightarrow{\mathrm{CD}}\right|}$, $\mathrm{A\hat{C}D}$ formed by $\overrightarrow{\mathrm{CA}}$ and $\overrightarrow{\mathrm{CD}}$

$\overrightarrow{\mathrm{CA}} = \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}$ *(A1)*

Notes: Exceptions to ***FT:***

1 if candidate indicates that they are finding $\overrightarrow{\mathrm{CA}}$, but makes an error, award ***M1A0***;

2 if candidate finds an incorrect vector (including $\overrightarrow{\mathrm{AC}}$), award ***M0A0***.

In both cases, if working shown, full ***FT*** may be awarded for subsequent correct ***FT*** work.

Award the final (***A1***) for simplification of **their** value for $\mathrm{A\hat{C}D}$.

Award the final ***A2*** for finding **their** arc cos. If their value of cos does not allow them to find an angle, they cannot be awarded this ***A2***.

finding $\left|\overrightarrow{\mathrm{CA}}\right|$ (may be seen in cosine formula) ***A1***

eg $\sqrt{1^2 + (-4)^2 + (-1)^2}$, $\sqrt{18}$

correct substitution into cosine formula *(A1)*

eg $\frac{-9}{\sqrt{18}\sqrt{18}}$

finding $\cos \mathrm{A\hat{C}D} - \frac{1}{2}$ *(A1)*

$\mathrm{A\hat{C}D} = \frac{2\pi}{3}$ (120°) ***A2 N2***

Notes: Award ***A1*** if additional answers are given.

Award ***A1*** for answer $\frac{\pi}{3}(60^\circ)$.

[7 marks]

Total [15 marks]

Examiners report

[N/A]

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

2a. Find $f^{-1}(x)$. *[2 marks]*

Markscheme

interchanging
x and
y ***(M1)***

eg
$x = 3y - 2$
$f^{-1}(x) = \frac{x+2}{3}$ $\left(\text{accept } y = \frac{x+2}{3}, \ \frac{x+2}{3}\right)$ ***A1*** ***N2***

[2 marks]

Examiners report

[N/A]

2b. Show that *[2 marks]*
$\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$.

Markscheme

attempt to form composite (in any order) ***(M1)***

eg
$g\left(\frac{x+2}{3}\right), \ \frac{\frac{5}{3x}+2}{3}$

correct substitution ***A1***

eg
$\frac{5}{3\left(\frac{x+2}{3}\right)}$
$\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$ ***AG*** ***N0***

[2 marks]

Examiners report

[N/A]

Let
$h(x) = \frac{5}{x+2}$, for
$x \geqslant 0$. The graph of *h* has a horizontal asymptote at
$y = 0$.

2c. Find the *[2 marks]*
y-intercept of the graph of
h.

Markscheme

valid approach ***(M1)***

eg
$h(0), \ \frac{5}{0+2}$
$y = \frac{5}{2}$ $(\text{accept } (0, 2.5))$ ***A1*** ***N2***

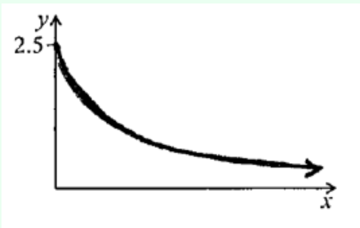
[2 marks]

Examiners report

[N/A]

2d. Hence, sketch the graph of h. *[3 marks]*

Markscheme



A1A2 N3

Notes: Award ***A1*** for approximately correct shape (reciprocal, decreasing, concave up).

Only if this ***A1*** is awarded, award ***A2*** for all the following approximately correct features: *y*-intercept at $(0, 2.5)$, asymptotic to *x*-axis, correct domain $x \geqslant 0$.

If only two of these features are correct, award***A1***.

[3 marks]

Examiners report

[N/A]

2e. For the graph of h^{-1}, write down the x-intercept; *[1 mark]*

Markscheme

$x = \frac{5}{2}$ $(\text{accept } (2.5, 0))$ ***A1 N1***

[1 mark]

Examiners report

[N/A]

2f. For the graph of h^{-1}, write down the equation of the vertical asymptote. *[1 mark]*

Markscheme

$x = 0$ (must be an equation) ***A1 N1***

[1 mark]

Examiners report

[N/A]

2g. Given that $h^{-1}(a) = 3$, find the value of a.

[3 marks]

Markscheme

METHOD 1

attempt to substitute 3 into h (seen anywhere) ***(M1)***

eg $h(3),\ \frac{5}{3+2}$

correct equation ***(A1)***

eg $a = \frac{5}{3+2},\ h(3) = a$

$a = 1$ ***A1 N2***

[3 marks]

METHOD 2

attempt to find inverse (may be seen in (d)) ***(M1)***

eg $x = \frac{5}{y+2},\ h^{-1} = \frac{5}{x} - 2,\ \frac{5}{x} + 2$

correct equation, $\frac{5}{x} - 2 = 3$ ***(A1)***

$a = 1$ ***A1 N2***

[3 marks]

Examiners report

[N/A]

Consider $f(x) = \ln(x^4 + 1)$.

3a. Find the value of $f(0)$.

[2 marks]

Markscheme

substitute 0 into f ***(M1)***

eg $\ln(0 + 1)$, $\ln 1$

$f(0) = 0$ ***A1 N2***

[2 marks]

Examiners report

Many candidates left their answer to part (a) as
$\ln 1$. While this shows an understanding for substituting a value into a function, it leaves an unfinished answer that should be expressed as an integer.

3b. Find the set of values of x for which f is increasing. *[5 marks]*

Markscheme

$f'(x) = \frac{1}{x^4+1} \times 4x^3$ (seen anywhere) ***A1A1***

Note: Award ***A1*** for $\frac{1}{x^4+1}$ and ***A1*** for $4x^3$.

recognizing f increasing where $f'(x) > 0$ (seen anywhere) ***R1***

eg
$f'(x) > 0$, diagram of signs

attempt to solve $f'(x) > 0$ ***(M1)***

eg
$4x^3 = 0$,
$x^3 > 0$

f increasing for $x > 0$ (accept $x \geq 0$) ***A1*** ***N1***

[5 marks]

Examiners report

Candidates who attempted to consider where f is increasing generally understood the derivative is needed. However, a number of candidates did not apply the chain rule, which commonly led to answers such as "increasing for all x". Many set their derivative equal to zero, while neglecting to indicate in their working that $f'(x) > 0$ for an increasing function. Some created a diagram of signs, which provides appropriate evidence as long as it is clear that the signs represent f'.

The second derivative is given by $f''(x) = \frac{4x^2(3-x^4)}{(x^4+1)^2}$.

The equation $f''(x) = 0$ has only three solutions, when $x = 0$, $\pm\sqrt[4]{3}$ $(\pm 1.316\ldots)$.

3c. (i) Find $f''(1)$. *[5 marks]*

(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x = 0$.

Markscheme

(i) substituting $x = 1$ into f'' ***(A1)***

eg $\frac{4(3-1)}{(1+1)^2}$, $\frac{4 \times 2}{4}$

$f''(1) = 2$ ***A1*** ***N2***

(ii) valid interpretation of point of inflexion (seen anywhere) ***R1***

eg no change of sign in $f''(x)$, no change in concavity,

f' increasing both sides of zero

attempt to find $f''(x)$ for $x < 0$ ***(M1)***

eg $f''(-1)$, $\frac{4(-1)^2(3-(-1)^4)}{((-1)^4+1)^2}$, diagram of signs

correct working leading to positive value ***A1***

eg $f''(-1) = 2$, discussing signs of numerator **and** denominator

there is no point of inflexion at $x = 0$ ***AG*** ***N0***

[5 marks]

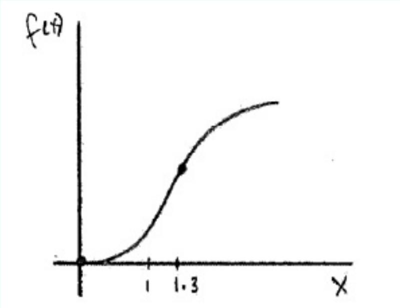
Examiners report

Finding $f''(1)$ proved no challenge, however, using this value to **show that** no point of inflexion exists proved elusive for many. Some candidates recognized the signs must not change in the second derivative. Few candidates presented evidence in the form of a calculation, which follows from the "hence" command of the question. In this case, a sign diagram without numerical evidence was not sufficient.

3d. There is a point of inflexion on the graph of f at $x = \sqrt[4]{3}$ $(x = 1.316\ldots)$. *[3 marks]*

Sketch the graph of f , for $x \geq 0$.

Markscheme



A1A1A1 ***N3***

Notes: Award ***A1*** for shape concave up left of POI and concave down right of POI.

Only if this ***A1*** is awarded, then award the following:

A1 for curve through $(0, 0)$, ***A1*** for increasing throughout.

Sketch need not be drawn to scale. Only essential features need to be clear.

[3 marks]

Examiners report

Few candidates created a correct graph from the information given or found in the question. This included the point $(0, 0)$, the fact that the function is always increasing for $x > 0$, the concavity at $x = 1$ and the change in concavity at the given point of inflexion. Many incorrect attempts showed a graph concave down to the right of $x = 0$, changing to concave up.

Jar A contains three red marbles and five green marbles. Two marbles are drawnfrom the jar, one after the other, without replacement.

4a. Find the probability that *[5 marks]*

(i) none of the marbles are green;

(ii) exactly one marble is green.

Markscheme

(i) attempt to find $\mathrm{P(red)} \times \mathrm{P(red)}$ ***(M1)***

eg $\frac{3}{8} \times \frac{2}{7}$, $\frac{3}{8} \times \frac{3}{8}$, $\frac{3}{8} \times \frac{2}{8}$

$\mathrm{P(none\ green)} = \frac{6}{56}$ $\left(= \frac{3}{28}\right)$ ***A1 N2***

(ii) attempt to find $\mathrm{P(red)} \times \mathrm{P(green)}$ ***(M1)***

eg $\frac{5}{8} \times \frac{3}{7}$, $\frac{3}{8} \times \frac{5}{8}$, $\frac{15}{56}$

recognizing two ways to get one red, one green ***(M1)***

eg $2\mathrm{P}(R) \times \mathrm{P}(G)$, $\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$, $\frac{3}{8} \times \frac{5}{8} \times 2$

$\mathrm{P(exactly\ one\ green)} = \frac{30}{56}$ $\left(= \frac{15}{28}\right)$ ***A1 N2***

[5 marks]

Examiners report

Many candidates correctly found the probability of selecting no green marbles in two draws, although some candidates treated the second draw as if replacing the first. When finding the probability for exactly one green marble, candidates often failed to recognize two pathways for selecting one of each color.

4b. Find the expected number of green marbles drawn from the jar. *[3 marks]*

Markscheme

$\mathrm{P(both\ green)} = \frac{20}{56}$ (seen anywhere) ***(A1)***

correct substitution into formula for $\mathrm{E}(X)$ ***A1***

eg $0 \times \frac{6}{56} + 1 \times \frac{30}{56} + 2 \times \frac{20}{56}$, $\frac{30}{64} + \frac{50}{64}$

expected number of green marbles is $\frac{70}{56}$ $\left(= \frac{5}{4}\right)$ ***A1 N2***

[3 marks]

Examiners report

Few candidates understood the concept of expected value in this context, often leaving this blank or treating as if a binomial experiment. Successful candidates often made a distribution table before making the final calculation.

Jar B contains six red marbles and two green marbles. A fair six-sided die is tossed.If the score is
1 or
2, a marble is drawn from jar A. Otherwise, a marble is drawn fromjar B.

4c. (i) Write down the probability that the marble is drawn from jar B. *[2 marks]*

(ii) Given that the marble was drawn from jar B, write down the probabilitythat it is red.

Markscheme

(i)
$\mathrm{P}(\text{jar B}) = \frac{4}{6}$
$\left(= \frac{2}{3}\right)$ ***A1 N1***

(ii)
$\mathrm{P}(\text{red}|\text{ jar B}) = \frac{6}{8}$
$\left(= \frac{3}{4}\right)$ ***A1 N1***

[2 marks]

Examiners report

Most candidates answered part (c) correctly. However, many overcomplicated (c)(ii) by using the conditional probability formula. Those with a clear understanding of the concept easily followed the “write down” instruction.

4d. Given that the marble is red, find the probability that it was drawn from jar A. *[6 marks]*

Markscheme

recognizing conditional probability *(M1)*

eg
$\mathrm{P}(A|R)$,
$\frac{\mathrm{P(jar\ A\ and\ red)}}{\mathrm{P(red)}}$, tree diagram

attempt to multiply along either branch (may be seen on diagram) *(M1)*

eg
$\mathrm{P(jar\ A\ and\ red)} = \frac{1}{3} \times \frac{3}{8}$
$\left(= \frac{1}{8}\right)$

attempt to multiply along **other** branch *(M1)*

eg
$\mathrm{P(jar\ B\ and\ red)} = \frac{2}{3} \times \frac{6}{8}$
$\left(= \frac{1}{2}\right)$

adding the probabilities of two mutually exclusive paths *(A1)*

eg
$\mathrm{P(red)} = \frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{6}{8}$

correct substitution

eg
$\mathrm{P(jar\ A|red)} = \frac{\frac{1}{3}\times\frac{3}{8}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{6}{8}}$,
$\frac{\frac{1}{8}}{\frac{5}{8}}$ ***A1***

$\mathrm{P(jar\ A|red)} = \frac{1}{5}$ ***A1*** ***N3***

[6 marks]

Examiners report

Only a handful of candidates correctly applied conditional probability to find $P(A|R)$ in part (d). While some wrote down the formula, or drew a tree diagram, few correctly calculated $P(red) = \frac{5}{8}$. A common error was to combine the marbles in the two jars to give $P(red) = \frac{9}{16}$.

Let
$f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for
$0 \leq x \leq \pi$.

5a. Find $f'(x)$. *[3 marks]*

Markscheme

$f'(x) = \cos x + x - 2$ ***A1A1A1*** ***N3***

Note: Award ***A1*** for each term.

[3 marks]

Examiners report

In part (a), most candidates were able to correctly find the derivative of the function.

Let
g be a quadratic function such that
$g(0) = 5$. The line
$x = 2$ is the axis of symmetry of the graph of
g .

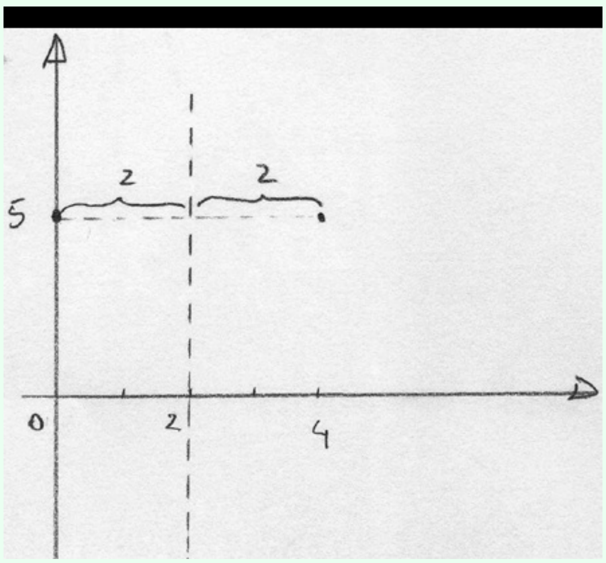
5b. Find $g(4)$. *[3 marks]*

Markscheme

recognizing
$g(0) = 5$ gives the point ($0,$ $5)$ ***(R1)***

recognize symmetry ***(M1)***

eg vertex, sketch



$g(4) = 5$ ***A1 N3***

[3 marks]

Examiners report

In part (b), many candidates did not understand the significance of the axis of symmetry and the known point ($0,$ $5)$, and so were unable to find
$g(4)$ using symmetry. A few used more complicated manipulations of the function, but many algebraic errors were seen.

The function
g can be expressed in the form
$g(x) = a(x - h)^2 + 3$.

5c. (i) Write down the value of h . *[4 marks]*

(ii) Find the value of a .

Markscheme

(i)
$h = 2$ ***A1 N1***

(ii) substituting into
$g(x) = a(x-2)^2 + 3$ (not the vertex) ***(M1)***

eg
$5 = a(0-2)^2 + 3$,
$5 = a(4-2)^2 + 3$

working towards solution ***(A1)***

eg
$5 = 4a + 3$,
$4a = 2$

$a = \frac{1}{2}$ ***A1 N2***

[4 marks]

Examiners report

In part (c), a large number of candidates were able to simply write down the correct value of h, as intended by the command term in this question. A few candidates wrote down the incorrect negative value. Most candidates attempted to substitute the x and y values of the known point correctly into the function, but again many arithmetic and algebraic errors kept them from finding the correct value for a.

5d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g . *[6 marks]*

Markscheme

$g(x) = \frac{1}{2}(x-2)^2 + 3 = \frac{1}{2}x^2 - 2x + 5$

correct derivative of
g ***A1A1***

eg
$2 \times \frac{1}{2}(x-2)$,
$x - 2$

evidence of equating both derivatives ***(M1)***

eg
$f' = g'$

correct equation ***(A1)***

eg
$\cos x + x - 2 = x - 2$

working towards a solution ***(A1)***

eg
$\cos x = 0$, combining like terms

$x = \frac{\pi}{2}$ ***A1*** ***N0***

Note: Do not award final ***A1*** if additional values are given.

[6 marks]

Examiners report

Part (d) required the candidates to find the derivative of
g, and to equate that to their answer from part (a). Although many candidates were able to simplify their equation to
$\cos x = 0$, many did not know how to solve for
x at this point. Candidates who had made errors in parts (a) and/or (c) were still able to earn follow-through marks in part (d).