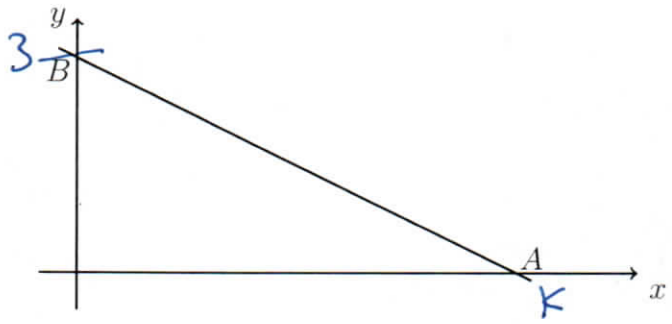
/8
52 +13 Name: Solutions

4.11 Exam: Linear equations, function operations, regression

1. [Maximum mark: 6]

The diagram shows the straight line L_1, which intersects the x-axis at $A(k, 0)$ and the y-axis at $B(0, 3)$.

diagram is not to scale



The gradient of L_1 is $-\frac{3}{4}$.

(a) Write down the equation of the line L_1. [1]

(b) Find the value of k. [2]

(c) The line L_2 is perpendicular to L_1 and passes through $(2, 1)$.

i. Write down the gradient of the line L_2. [1]

ii. Hence, write down the equation of L_2. Leave your answer in the form $y - a = m(x - b)$. [2]

a) $y = -\frac{3}{4}x + 3$

b) $f(k) = -\frac{3}{4}k + 3 = 0$

$k = 4$

c) i $\frac{4}{3}$

ii $y - 1 = \frac{4}{3}(x - 2)$

M1 A
N2

2. [Maximum mark: 7]

Let $f(x) = 2x + 8$ and $g(x) = \sqrt{x} - 1$, for $x \geq 0$.

(a) Write down $g(9)$. [1]

(b) Find $(f - g)(x)$. [1]

(c) Find $(g \circ f)(4)$. [1]

(d) Write down $g^{-1}(4)$. [2]

(e) Find $f^{-1}(x)$. [2]

Working:

a) $g(9) = \sqrt{9} - 1 = 2$

b) $2x + 8 - (\sqrt{x} - 1) = 2x - \sqrt{x} + 9$

c) $g(f(x)) = \sqrt{2x+8} - 1$

$g(f(4)) = \sqrt{2 \cdot 4 + 8} - 1$

$= 3$

d) $\sqrt{x} - 1 = 4$

$x = 25$

e) $f: x = 2y + 8$

$f^{-1}: y = \frac{x-8}{2}$

$y = \frac{1}{2}x - 4$

Answers:

(a) 2

(b) $2x - \sqrt{x} + 9$

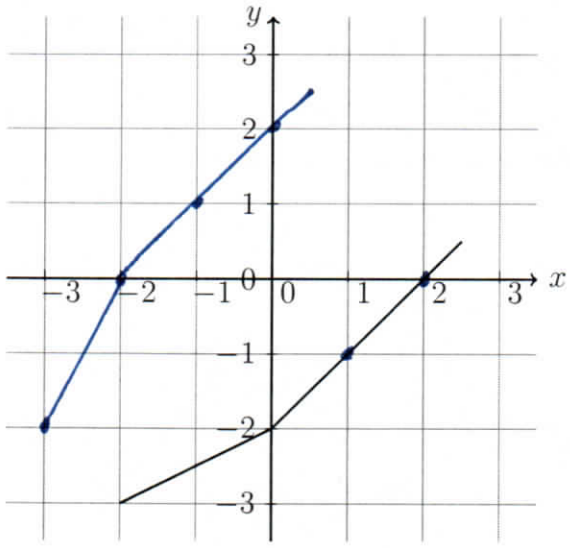
(c) 3

(d) 25

(e) $\frac{1}{2}x - 4$

3. [Maximum mark: 6]

Early finishers: The diagram below shows the graph of a function f for $-2 \leq x \leq 2.5$.



(a) Write down the value of $f(2)$. [1]

(b) Write down the value of $f^{-1}(-1)$. [2]

(c) Sketch the graph of f^{-1} on the grid. [3]

a) 0

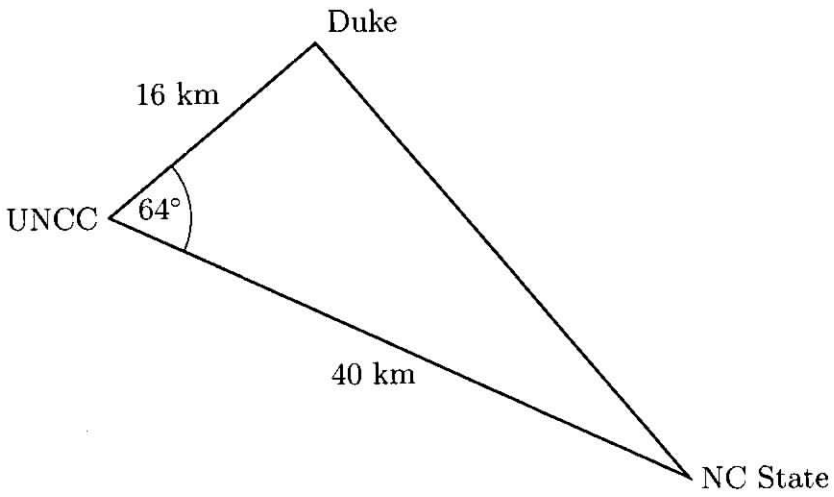
b) 1

c)

4. [Maximum mark: 6]

The North Carolina Research Triangle is one of the world's leading regions for high tech businesses and research. The three universities that anchor the area, Duke University, University of North Carolina at Chapel Hill, and North Carolina State University, form a triangle as shown below.

Assume that the distance from UNCC to NC State is 40 km, from UNCC to Duke is 16 km, and that the angle made by Duke, UNCC, and NC State is 64°.



(a) Calculate the distance from Duke to NC State. [3]

(b) Find the area of the triangle formed by the three universities. [3]

Working:

a) $c^2 = 16^2 + 40^2 - 2(16)(40)\cos 64°$

$= 1294.81...$

$c = 35.984509...$

≈ 36.0

b) $A = \frac{1}{2}(16)(40)\sin 64°$

$= 287.614...$

≈ 288

Answers:

(a) 36.0 km

(b) 288 km^2

5. [Maximum mark: 6]

The following table shows the Diploma score x and university entrance mark y for seven IB Diploma students.

Diploma score (x)	28	30	27	31	32	25	27
University entrance mark (y)	73.9	78.1	70.2	82.2	85.5	62.7	69.4

(a) Find the correlation coefficient. [2]

The relationship can be modelled by the regression line with equation $y = ax + b$.

(b) Write down the value of a and of b [2]

Rita scored a total of 26 in her IB Diploma.

(c) Use your regression line to estimate Rita's university entrance mark. [2]

Working:

a) $a = 3.15037877...$
≈ 3.15

$r = 0.9960109...$
≈ 0.996

b) $b = -15.4399...$
≈ -15.4

c) $f(26) = 3.15(26) + -15.4$
$= 66.4704...$
≈ 66.5

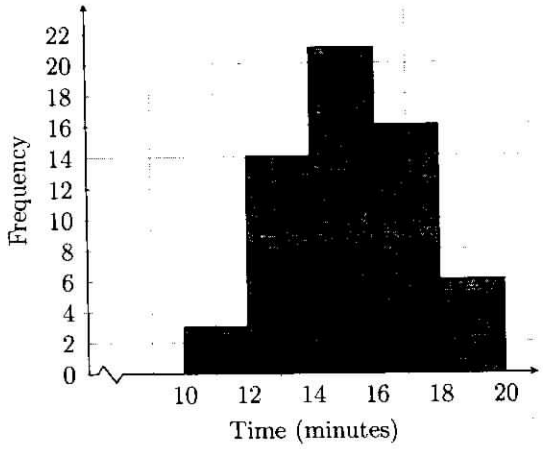
Answers:

(a) ~~3.15~~ 0.996

(b) 3.15, -15.4

(c) + 66.5

6. The chart shows the time t in minutes for 60 marines to complete an obstacle course.



The following is the frequency table for the distribution of t.

Time (t)	$10 \leq t < 12$	$12 \leq t < 14$	$14 \leq t < 16$	$16 \leq t < 18$	$18 \leq t < 20$
Freq	3	14	21	p	6

(a) Write down the value of p. [1]

(b) Write down the modal class. [1]

(c) A marine is selected at random. Find the probability that the marine completed the course in less than 14 minutes. [2]

(d) Write down the mid-interval value for the class $18 \leq t < 20$. [1]

(e) Hence find an estimate for the

i. mean; [2]

ii. standard deviation. [2]

a) $p = 16$

b) $14 \leq t < 16$

c) $P(t < 14) = \frac{3+14}{60} = \frac{17}{60}$

d) 19

e) i $\bar{x} = 15.2666\ldots \approx 15.3$

$\sigma = 2.08059\ldots \approx 2.08$

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

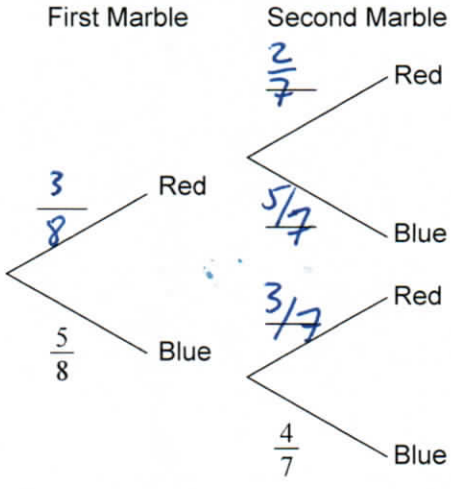
1. [Maximum mark: 6]

A bag contains eight marbles. Three marbles are red and five are blue. Two marbles are drawn from the bag without replacement.

(a) Write down the probability that the first marble drawn is red. [1]

$\frac{3}{8}$

(b) Complete the following tree diagram. [3]

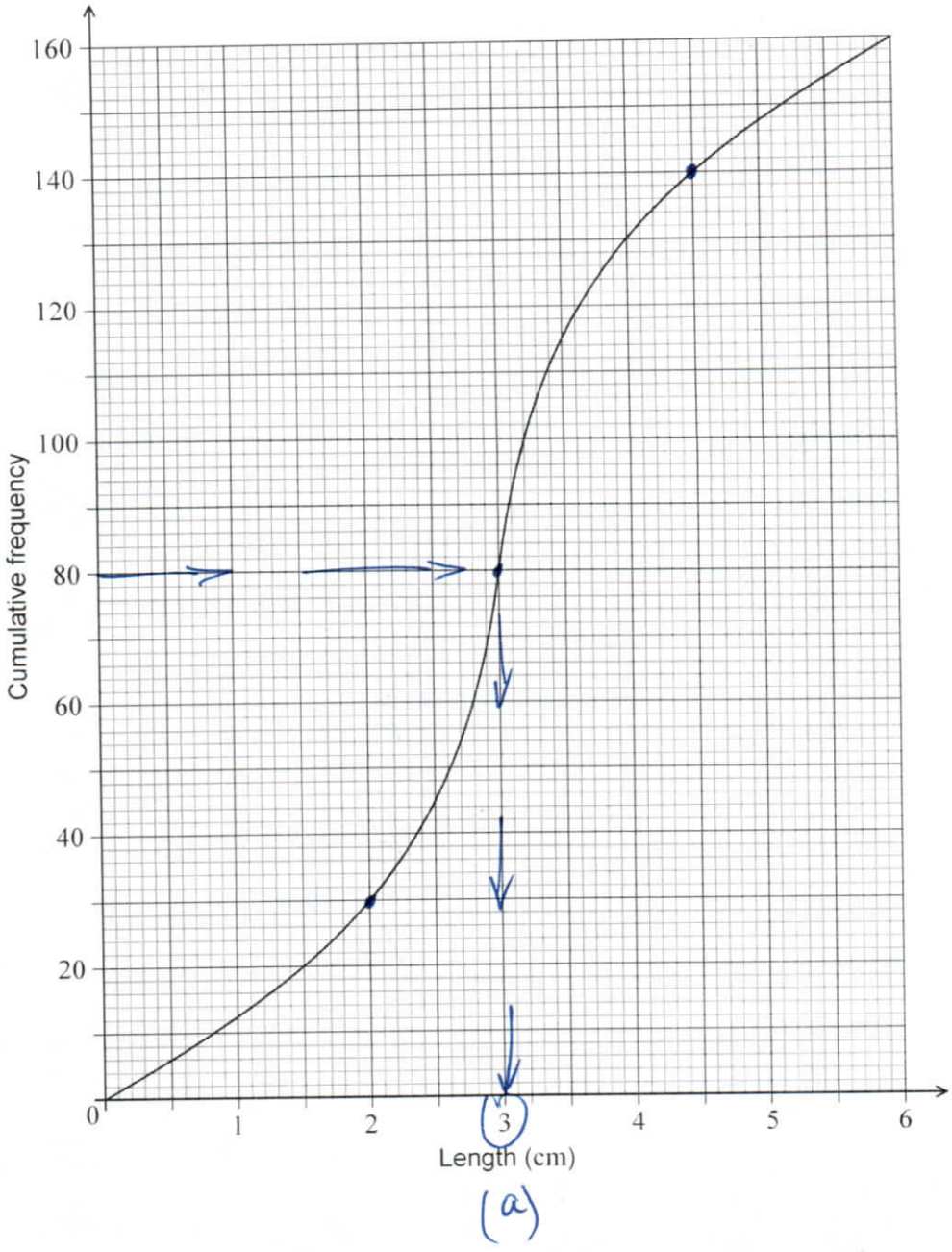


(c) Find the probability that both marbles are blue. [2]

$\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$ $\left(= \frac{5}{14}\right)$

3. [Maximum mark: 6]

The following cumulative frequency diagram shows the lengths of 160 fish, in cm.



(a)

(This question continues on the following page)

(Question 3 continued)

(a) Find the median length. [2]

The following frequency table also gives the lengths of the 160 fish.

Length x cm	$0 \leq x \leq 2$	$2 < x \leq 3$	$3 < x \leq 4.5$	$4.5 < x \leq 6$
Frequency	p	50	q	20

(b) (i) Write down the value of p.

(ii) Find the value of q. [4]

a) $\frac{160}{2} = 80$

median = 3 cm

b) i) 30

ii) $140 - 80 = 60$

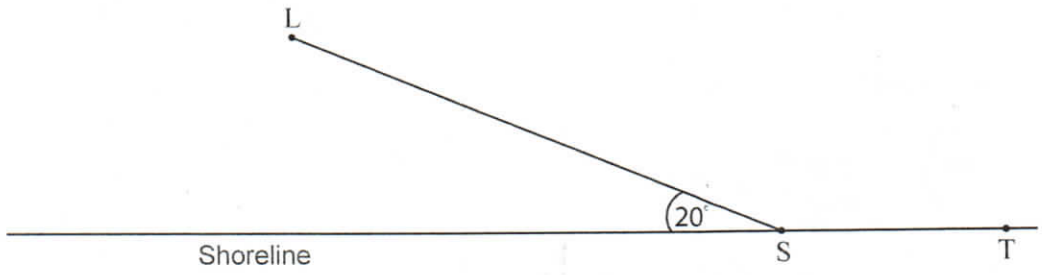
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

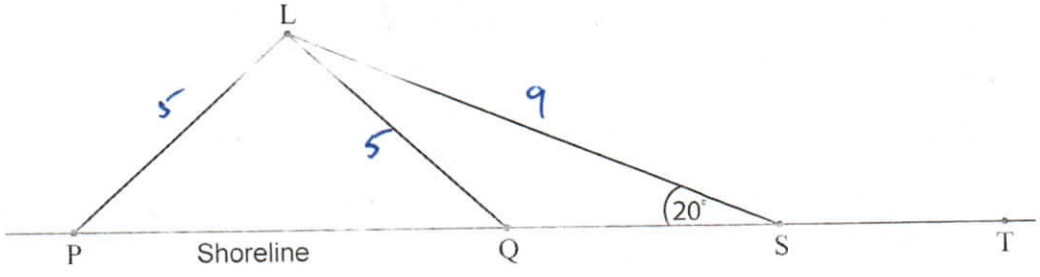
The following diagram shows a straight shoreline, with a supply store at S, a town at T, and an island L.



A boat delivers supplies to the island. The boat leaves S, and sails to the island. Its path makes an angle of 20° with the shoreline.

(a) The boat sails at 6 km per hour, and arrives at L after 1.5 hours. Find the distance from S to L. [2]

It is decided to change the position of the supply store, so that its distance from L is 5 km. The following diagram shows the two possible locations P and Q for the supply store.



(b) Find the size of $S\hat{P}L$ and of $S\hat{Q}L$. [5]

(c) The town wants the new supply store to be as near as possible to the town.

(i) State which of the points P or Q is chosen for the new supply store.

(ii) Hence find the distance between the old supply store and the new one. [6]

17 January 2020
Exam

Solutions

8a) $d = 6 \times 1.5 = 9$ km

b) $\frac{\sin x}{9} = \frac{\sin 20}{5}$

$x = \sin^{-1}\left(\frac{9 \sin 20°}{5}\right)$

$= 28.6088\ldots$ $37.9981\ldots$

$\approx 28.6°$ ≈ 38.0

$S\hat{P}L = 28.6$

$S\hat{Q}L = 180 - 28.6 = 151.4$ 142
38

c) i) Q

ii) $Q\hat{L}S = 180 - (20 + 142) = 18°$

$\Rightarrow \frac{QS}{\sin 18} = \frac{5}{\sin 20}$

or $5^2 = 9^2 + (QS)^2 - 2(9)(QS)\cos 20$

$QS = x$

$x^2 - 16.914\ldots x + 56 = 0$

$x = \frac{-(-16.914) \pm \sqrt{(-16.91\ldots)^2 - 4(1)(56)}}{2}$

$= 4.51798 \approx 4.52$ km