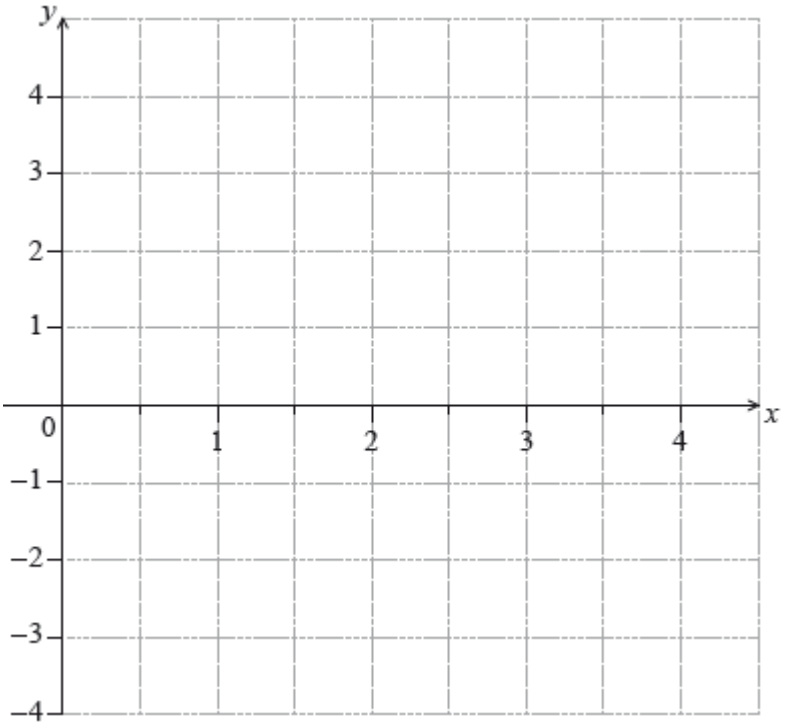
3.4 Periodic-functions, trigonometry (Paper 2, with calculator)

1a. Let $f(x) = 3\sin\left(\frac{\pi}{2}x\right)$, for $0 \leqslant x \leqslant 4$.

(i) Write down the amplitude of f.

(ii) Find the period of f. *[3 marks]*

1b. On the following grid sketch the graph of f.



[4 marks]

2a. Let $f(x) = \frac{3x}{2} + 1$, $g(x) = 4\cos\left(\frac{x}{3}\right) - 1$. Let $h(x) = (g \circ f)(x)$.

Find an expression for $h(x)$. *[3 marks]*

2b. Write down the period of h. *[1 mark]*

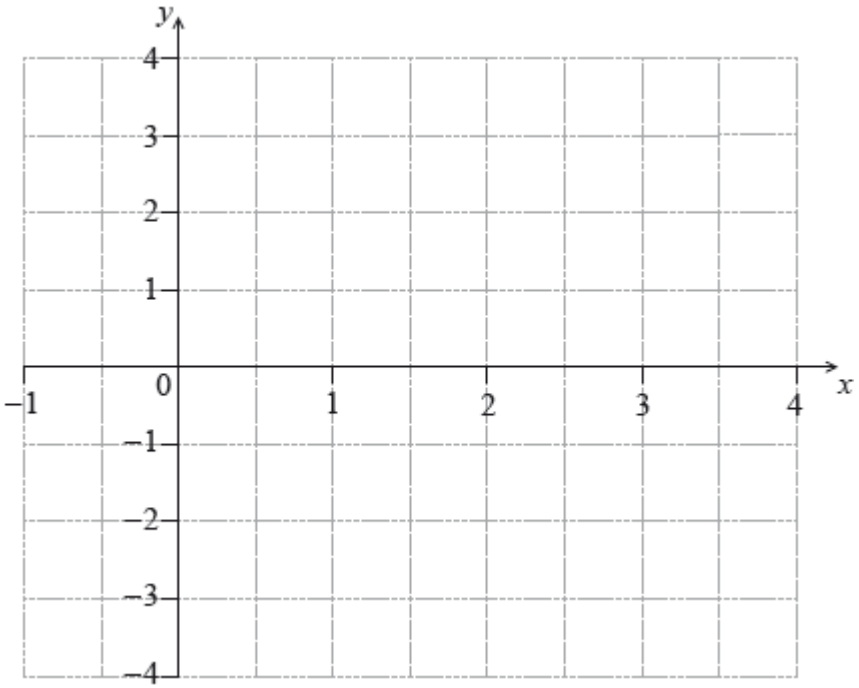
2c. Write down the range of h. *[2 marks]*

3a. Let $f(x) = 3\sin(\pi x)$.

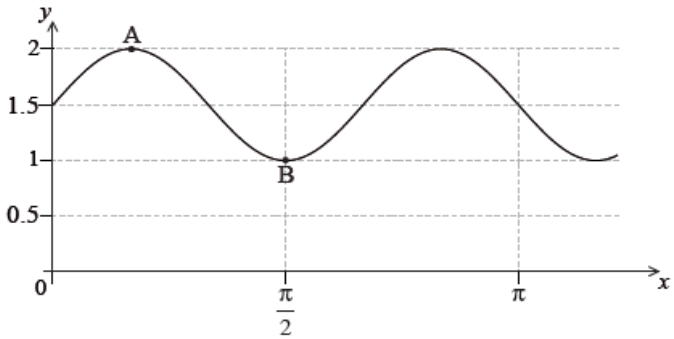
Write down the amplitude of f. *[1 mark]*

3b. Find the period of f. *[2 marks]*

3c. On the following grid, sketch the graph of $y = f(x)$, for $0 \leq x \leq 3$. *[4 marks]*



4a. The following diagram shows part of the graph of $y = p\sin(qx) + r$.



The point $\mathrm{A}\left(\frac{\pi}{6},\ 2\right)$ is a maximum point and the point $\mathrm{B}\left(\frac{\pi}{6},\ 1\right)$ is a minimum point.

Find the value of p; *[2 marks]*

4b. r; *[2 marks]*

4c. q. *[2 marks]*

5a. The depth of water in a port is modelled by the function $d(t) = p\cos qt + 7.5$, for $0 \leqslant t \leqslant 12$, where t is the number of hours after high tide.

At high tide, the depth is 9.7 metres.

At low tide, which is 7 hours later, the depth is 5.3 metres.

Find the value of p. *[2 marks]*

5b. Find the value of q. *[2 marks]*

5c. Use the model to find the depth of the water 10 hours after high tide. *[2 marks]*

6a. *[2 marks]*

The height, h metros, of a seat on a Ferris wheel after t minutes is given by

$$h(t) = -15\cos 1.2t + 17, \text{ for } t \geqslant 0.$$

Find the height of the seat when $t = 0$.

6b. *[3 marks]*

The seat first reaches a height of 20 m after k minutes. Find k.

6c. *[3 marks]*

Calculate the time needed for the seat to complete a full rotation, giving your answer correct to one decimal place.

7a. *[3 marks]*

The population of deer in an enclosed game reserve is modelled by the function $P(t) = 210\sin(0.5t - 2.6) + 990$, where t is in months, and $t = 1$ corresponds to 1 January 2014.

Find the number of deer in the reserve on 1 May 2014.

7b. *[2 marks]*

Find the rate of change of the deer population on 1 May 2014.

7c. *[1 mark]*

Interpret the answer to part (i) with reference to the deer population size on 1 May 2014.

8a. Let $f(x) = \sin\left(x + \frac{\pi}{4}\right) + k$. The graph of f passes through the point $\left(\frac{\pi}{4},\ 6\right)$.

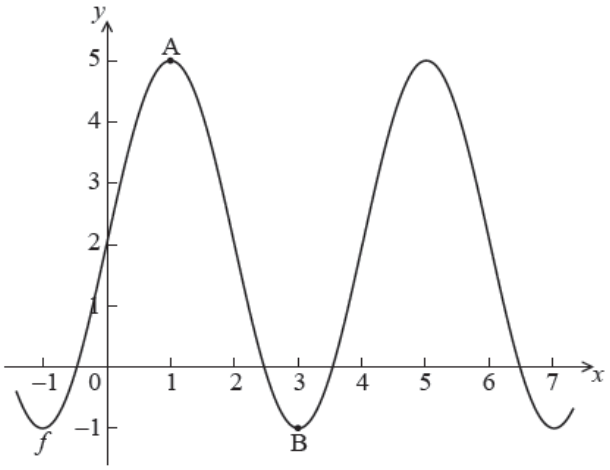
Find the value of k. *[3 marks]*

8b. Find the minimum value of $f(x)$. *[2 marks]*

8c. Let $g(x) = \sin x$. The graph of g is translated to the graph of f by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of p and of q. *[2 marks]*

9a. The diagram below shows part of the graph of a function f.



The graph has a maximum at A$(1, 5)$ and a minimum at B$(3, -1)$.

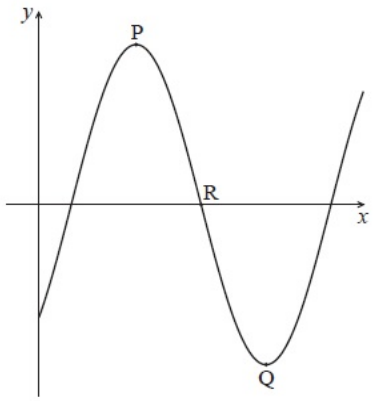
The function f can be written in the form $f(x) = p\sin(qx) + r$. Find the value of

(a) p

(b) q

(c) r. *[6 marks]*

10a. Let $f(x) = a\cos(b(x-c))$. The diagram below shows part of the graph of f, for $0 \leq x \leq 10$.



The graph has a local maximum at P$(3, 5)$, a local minimum at Q$(7, -5)$, and crosses the x-axis at R.

Write down the value of

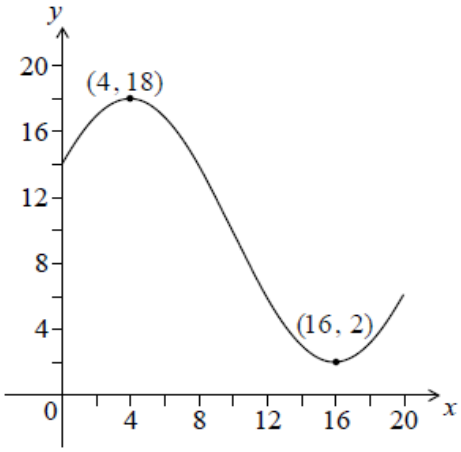
(i) a;

(ii) c. *[2 marks]*

10b. Find the value of b. *[2 marks]*

10c. Find the x-coordinate of R. *[2 marks]*

11a. Let $f(x) = p\cos(q(x+r)) + 10$, for $0 \leqslant x \leqslant 20$. The following diagram shows the graph of f.



The graph has a maximum at $(4, 18)$ and a minimum at $(16, 2)$.

Write down the value of r. *[2 marks]*

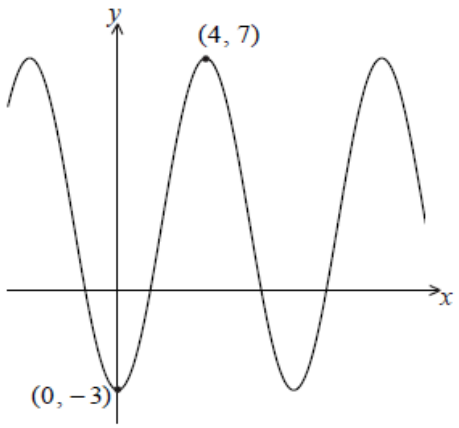
11b. Find p. *[2 marks]*

11c. Find q. *[2 marks]*

11d. Solve $f(x) = 7$. *[2 marks]*

12a. *[6 marks]*

The graph of $y = p\cos qx + r$, for $-5 \leq x \leq 14$, is shown below.



There is a minimum point at (0, −3) and a maximum point at (4, 7) .

Find the value of

(i) p ;

(ii) q ;

(iii) r.

12b. *[1 mark]*

The equation $y = k$ has exactly **two** solutions. Write down the value of k.