

# 0507CW-applications [36 marks]

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Ten students were surveyed about the number of hours,  $x$ , they spent browsing the Internet during week 1 of the school year. The results of the survey are given below.

$$\sum_{i=1}^{10} x_i = 252, \sigma = 5 \text{ and median} = 27.$$

- 1a. Find the mean number of hours spent browsing the Internet.

[2 marks]

## Markscheme

attempt to substitute into formula for mean (M1)

eg  $\frac{\Sigma x}{10}, \frac{252}{n}, \frac{252}{10}$

mean = 25.2 (hours) A1 N2

[2 marks]

- 1b. During week 2, the students worked on a major project and they each spent an additional five hours browsing the Internet. For week 2, write down

[2 marks]

- (i) the mean;
- (ii) the standard deviation.

## Markscheme

(i) mean = 30.2 (hours) A1 N1

(ii)

$\sigma = 5$  (hours) A1 N1

[2 marks]

- 1c. During week 3 each student spent 5% less time browsing the Internet than during week 1. For week 3, find

[6 marks]

- (i) the median;
- (ii) the variance.

# Markscheme

(i) valid approach **(M1)**

*eg*

95%, 5% of 27

correct working **(A1)**

*eg*  $0.95 \times 27$ ,  $27 - (5\% \text{ of } 27)$

median = 25.65 (exact), 25.7 (hours) **A1 N2**

(ii) **METHOD 1**

variance = (standard deviation)<sup>2</sup> (seen anywhere) **(A1)**

valid attempt to find new standard deviation **(M1)**

*eg*  $\sigma_{\text{new}} = 0.95 \times 5$ , 4.75

variance = 22.5625 (exact), 22.6 **A1 N2**

**METHOD 2**

variance = (standard deviation)<sup>2</sup> (seen anywhere) **(A1)**

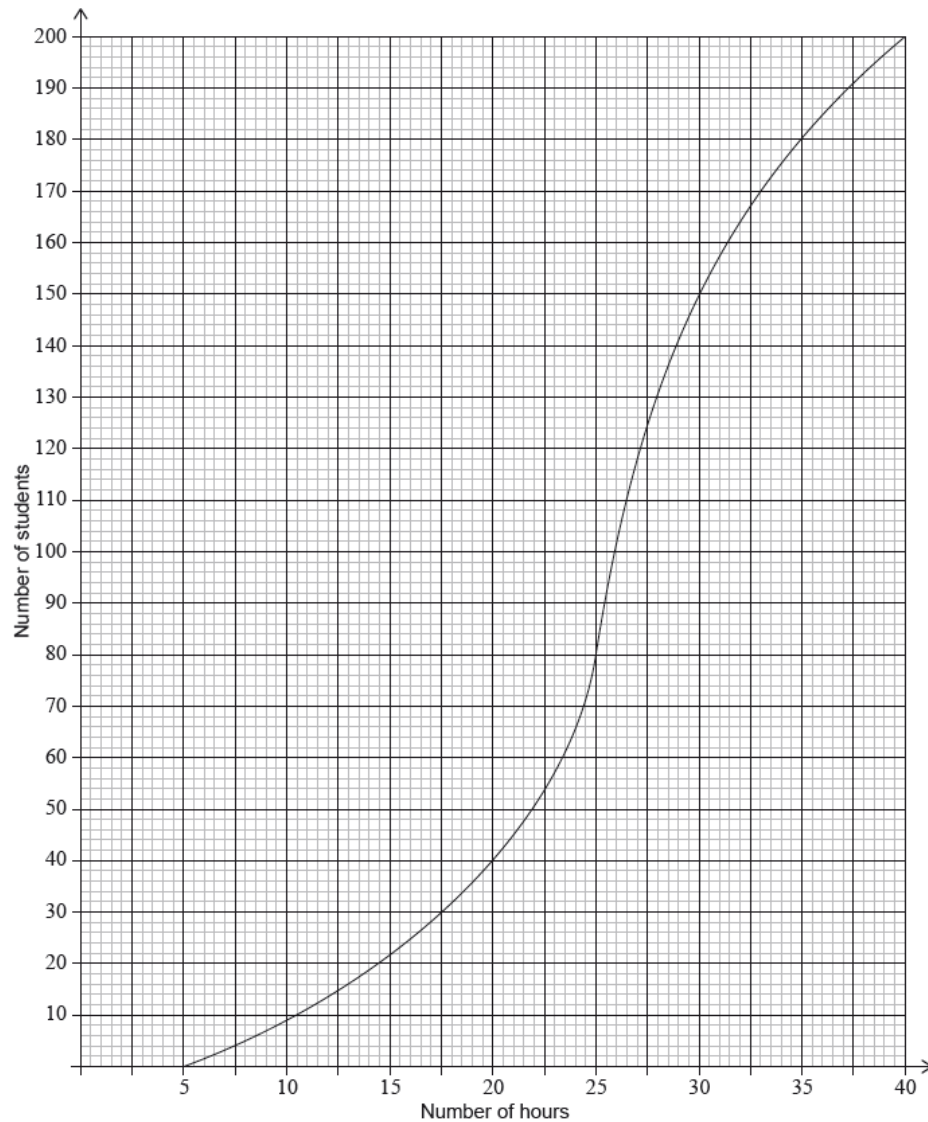
valid attempt to find new variance **(M1)**

*eg*  $0.95^2$ ,  $0.9025 \times \sigma^2$

new variance = 22.5625 (exact), 22.6 **A1 N2**

**[6 marks]**

During week 4, the survey was extended to all 200 students in the school. The results are shown in the cumulative frequency graph:



- 1d. (i) Find the number of students who spent between 25 and 30 hours browsing the Internet. [6 marks]
- (ii) Given that 10% of the students spent more than  $k$  hours browsing the Internet, find the maximum value of  $k$ .

## Markscheme

(i) both correct frequencies **(A1)**

eg

80, 150

subtracting **their** frequencies in either order **(M1)**

eg  $150 - 80$ ,  $80 - 150$

70 (students) **A1 N2**

(ii) evidence of a valid approach **(M1)**

eg 10% of 200, 90%

correct working **(A1)**

eg

$0.90 \times 200$ ,  $200 - 20$ , 180 students

$k = 35$  **A1 N3**

[6 marks]

Consider the following sequence of figures.

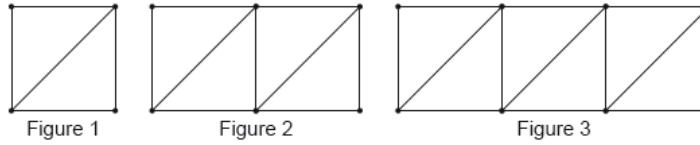


Figure 1 contains 5 line segments.

- 2a. Given that Figure  $n$  contains 801 line segments, show that  $n = 200$ .

[3 marks]

## Markscheme

recognizing that it is an arithmetic sequence **(M1)**

eg  $5, 5 + 4, 5 + 4 + 4, \dots, d = 4, u_n = u_1 + (n - 1)d, 4n + 1$

correct equation **A1**

eg  $5 + 4(n - 1) = 801$

correct working (do not accept substituting  $n = 200$ ) **A1**

eg  $4n - 4 = 796, n - 1 = \frac{796}{4}$

$n = 200$  **AG N0**

[3 marks]

- 2b. Find the total number of line segments in the first 200 figures.

[3 marks]

## Markscheme

recognition of sum **(M1)**

eg  $S_{200}, u_1 + u_2 + \dots + u_{200}, 5 + 9 + 13 + \dots + 801$

correct working for AP **(A1)**

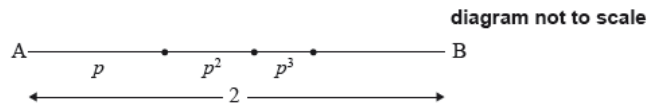
eg  $\frac{200}{2}(5 + 801), \frac{200}{2}(2(5) + 199(4))$

80 600 **A1 N2**

[3 marks]

- 3a. The following diagram shows [AB], with length 2 cm. The line is divided into an infinite number of line segments. The diagram shows the first three segments.

[5 marks]



The length of the line segments are  $p$  cm,  $p^2$  cm,  $p^3$  cm,  $\dots$ , where  $0 < p < 1$ .

Show that  $p = \frac{2}{3}$ .

## Markscheme

infinite sum of segments is 2 (seen anywhere) **(A1)**

eg  $p + p^2 + p^3 + \dots = 2, \frac{u_1}{1-r} = 2$

recognizing GP **(M1)**

eg ratio is  $p, \frac{u_1}{1-r}, u_n = u_1 \times r^{n-1}, \frac{u_1(r^n-1)}{r-1}$

correct substitution into  $S_\infty$  formula (may be seen in equation) **A1**

eg  $\frac{p}{1-p}$

correct equation **(A1)**

eg  $\frac{p}{1-p} = 2, p = 2 - 2p$

correct working leading to answer **A1**

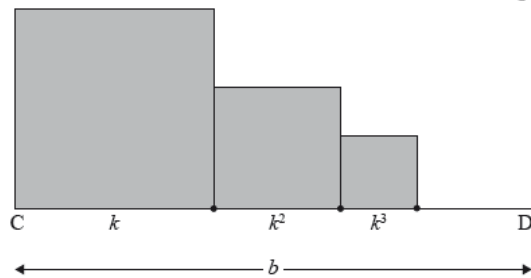
eg  $3p = 2, 2 - 3p = 0$

$p = \frac{2}{3}$  (cm) **AG NO**

**[5 marks]**

- 3b. The following diagram shows [CD], with length  $b$  cm, where  $b > 1$ . Squares with side lengths  $k$  cm,  $k^2$  cm,  $k^3$  cm,  $\dots$ , where  $0 < k < 1$ , are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares. **[9 marks]**

**diagram not to scale**



The **total** sum of the areas of all the squares is  $\frac{9}{16}$ . Find the value of  $b$ .

# Markscheme

recognizing infinite geometric series with squares **(M1)**

eg  $k^2 + k^4 + k^6 + \dots, \frac{k^2}{1-k^2}$

correct substitution into  $S_{\infty} = \frac{9}{16}$  (must substitute into formula) **(A2)**

eg  $\frac{k^2}{1-k^2} = \frac{9}{16}$

correct working **(A1)**

eg  $16k^2 = 9 - 9k^2, 25k^2 = 9, k^2 = \frac{9}{25}$

$k = \frac{3}{5}$  (seen anywhere) **A1**

valid approach with segments and CD (may be seen earlier) **(M1)**

eg  $r = k, S_{\infty} = b$

correct expression for  $b$  in terms of  $k$  (may be seen earlier) **(A1)**

eg  $b = \frac{k}{1-k}, b = \sum_{n=1}^{\infty} k^n, b = k + k^2 + k^3 + \dots$

substituting **their** value of  $k$  into **their** formula for  $b$  **(M1)**

eg  $\frac{\frac{3}{5}}{1-\frac{3}{5}}, \frac{\left(\frac{3}{5}\right)}{\left(\frac{2}{5}\right)}$

$b = \frac{3}{2}$  **A1 N3**

**[9 marks]**