0329Test_statistics *[67 marks]*

Let
A and
B be independent events, where
$\mathrm{P}(A) = 0.3$ and
$\mathrm{P}(B) = 0.6$.

1a. Find $\mathrm{P}(A \cap B)$. *[2 marks]*

Markscheme

correct substitution ***(A1)***

eg

0.3×0.6

$\mathrm{P}(A \cap B) = 0.18$ ***A1 N2***

[2 marks]

1b. Find $\mathrm{P}(A \cup B)$. *[2 marks]*

Markscheme

correct substitution ***(A1)***

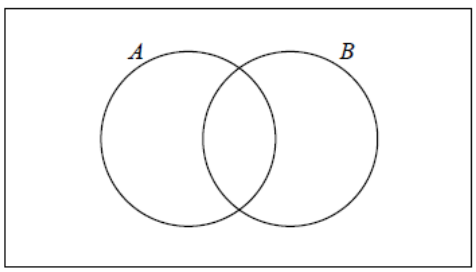
eg

$\mathrm{P}(A \cup B) = 0.3 + 0.6 - 0.18$

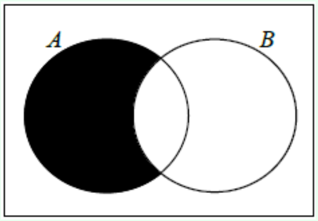
$\mathrm{P}(A \cup B) = 0.72$ ***A1 N2***

[2 marks]

1c. On the following Venn diagram, shade the region that represents $A \cap B'$. *[1 mark]*



Markscheme



A1 ***N1***

1d. Find $\mathrm{P}(A \cap B')$. *[2 marks]*

Markscheme

appropriate approach ***(M1)***

eg

$0.3 - 0.18,\ \mathrm{P}(A) \times \mathrm{P}(B')$

$\mathrm{P}(A \cap B') = 0.12$ (may be seen in Venn diagram) ***A1*** ***N2***

[2 marks]

2a. A factory makes lamps. The probability that a lamp is defective is 0.05. A randomsample of 30 lamps is tested. *[4 marks]*

Find the probability that there is at least one defective lamp in the sample.

Markscheme

evidence of recognizing binomial (seen anywhere) ***(M1)***

e.g.
$\mathrm{B}(n, p)$,
0.95^{30}

finding
$\mathrm{P}(X = 0) = 0.21463876$ ***(A1)***

appropriate approach ***(M1)***

e.g. complement, summing probabilities

0.785361

probability is
0.785 ***A1*** ***N3***

[4 marks]

2b. A factory makes lamps. The probability that a lamp is defective is 0.05. A randomsample of 30 lamps is tested. *[4 marks]*

Given that there is at least one defective lamp in the sample, find the probabilitythat there are at most two defective lamps.

Markscheme

identifying correct outcomes (seen anywhere) ***(A1)***

e.g.
$\mathrm{P}(X=1)+\mathrm{P}(X=2)$, 1 or 2 defective,
$0.3389\ldots+0.2586\ldots$

recognizing conditional probability (seen anywhere) ***R1***

e.g.
$\mathrm{P}(A|B)$,
$\mathrm{P}(X\le 2|X\ge 1)$, P(at most 2|at least 1)

appropriate approach involving conditional probability ***(M1)***

e.g.
$\frac{\mathrm{P}(X=1)+\mathrm{P}(X=2)}{\mathrm{P}(X\ge 1)}$,
$\frac{0.3389\ldots+0.2586\ldots}{0.785\ldots}$,
$\frac{1 \text{ or } 2}{0.785}$

0.760847

probability is
0.761 ***A1*** ***N2***

[4 marks]

The following table shows the amount of fuel (y litres) used by a car to travel certain distances (x km).

Distance (x km)	40	75	120	150	195
Amount of fuel (y litres)	3.6	6.5	9.9	13.1	16.2

This data can be modelled by the regression line with equation $y=ax+b$.

3a. Write down the value of a and of b. *[2 marks]*

Markscheme

$a=0.0823604, b=0.306186$

$a=0.0824, b=0.306$ ***A1A1*** ***N2***

[2 marks]

3b. Explain what the gradient a represents. *[1 mark]*

Markscheme

correct explanation with reference to number of litres

required for

1 km ***A1*** ***N1***

eg

a represents the (average) amount of fuel (litres) required to drive

1 km, (average) litres per kilometre, (average) rate of change in fuel used for each km travelled

[1 marks]

3c. Use the model to estimate the amount of fuel the car would use if it is driven 110 km. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg

$y = 0.0824(110) + 0.306$, sketch

9.36583

9.37 (litres) ***A1 N2***

[2 marks]

The vectors ***a*** = $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and ***b*** = $\begin{pmatrix} k+3 \\ k \end{pmatrix}$ are perpendicular to each other.

4a. Find the value of k. *[4 marks]*

Markscheme

evidence of scalar product ***M1***

eg **a • b**, $4(k+3) + 2k$

recognizing scalar product must be zero ***(M1)***

eg **a • b** $= 0,\ 4k + 12 + 2k = 0$

correct working (must involve combining terms) ***(A1)***

eg $6k + 12,\ 6k = -12$

$k = -2$ ***A1*** ***N2***

[4 marks]

4b. Given that ***c*** = ***a*** + 2***b***, find ***c***. *[3 marks]*

Markscheme

attempt to substitute **their** value of k (seen anywhere) ***(M1)***

eg $\boldsymbol{b} = \begin{pmatrix} -2+3 \\ -2 \end{pmatrix}$, $2\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

correct working ***(A1)***

eg $\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 4+2k+6 \\ 2+2k \end{pmatrix}$

$\boldsymbol{c} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ ***A1 N2***

[3 marks]

A standard die is rolled 36 times. The results are shown in the following table.

Score	1	2	3	4	5	6
Frequency	3	5	4	6	10	8

5a. Write down the standard deviation. *[2 marks]*

Markscheme

$\sigma = 1.61$ ***A2 N2***

[2 marks]

5b. Write down the median score. *[1 mark]*

Markscheme

median
$= 4.5$ ***A1 N1***

[1 mark]

5c. Find the interquartile range. *[3 marks]*

Markscheme

$Q_1 = 3$,
$Q_3 = 5$ (may be seen in a box plot) ***(A1)(A1)***

$\text{IQR} = 2$ (accept any notation that suggests the interval 3 to 5) ***A1 N3***

[3 marks]

Consider a function
$f(x)$ such that
$\int_1^6 f(x)\mathrm{d}x = 8.$

6a. Find
$\int_1^6 2f(x)\mathrm{d}x.$ *[2 marks]*

Markscheme

appropriate approach ***(M1)***

eg

$2\int f(x),\ 2(8)$

$\int_1^6 2f(x)\mathrm{d}x = 16$ ***A1 N2***

[2 marks]

6b. Find

$\int_1^6 \left(f(x) + 2\right) \mathrm{d}x.$

[4 marks]

Markscheme

appropriate approach ***(M1)***

eg

$\int f(x) + \int 2,\ 8 + \int 2$

$\int 2\mathrm{d}x = 2x$ (seen anywhere) ***(A1)***

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg

$2(6) - 2(1),\ 8 + 12 - 2$

$\int_1^6 \left(f(x) + 2\right) \mathrm{d}x = 18$ ***A1 N3***

[4 marks]

A van can take either Route A or Route B for a particular journey.

If Route A is taken, the journey time may be assumed to be normally distributed with mean 46 minutes and a standard deviation 10 minutes.

If Route B is taken, the journey time may be assumed to be normally distributed with mean μ minutes and standard deviation 12 minutes.

7a. For Route A, find the probability that the journey takes **more** than 60 minutes.

[2 marks]

Markscheme

$A \sim N(46, 10^2)$
$B \sim N(\mu, 12^2)$

$\mathrm{P}(A > 60) = 0.0808$ ***A2 N2***

[2 marks]

7b. For Route B, the probability that the journey takes **less** than 60 minutes is 0.85.

Find the value of μ .

[3 marks]

Markscheme

correct approach ***(A1)***

e.g.

$\mathrm{P}\left(Z < \frac{60-\mu}{12}\right) = 0.85$, sketch

$\frac{60-\mu}{12} = 1.036\ldots$ ***(A1)***

$\mu = 47.6$ ***A1*** ***N2***

[3 marks]

7c. The van sets out at 06:00 and needs to arrive before 07:00. *[3 marks]*

(i) Which route should it take?

(ii) Justify your answer.

Markscheme

(i) route A ***A1*** ***N1***

(ii) **METHOD 1**

$\mathrm{P}(A < 60) = 1 - 0.0808 = 0.9192$ ***A1***

valid reason ***R1***

e.g. probability of *A* getting there on time is greater than probability of *B*

$0.9192 > 0.85$ ***N2***

METHOD 2

$\mathrm{P}(B > 60) = 1 - 0.85 = 0.15$ ***A1***

valid reason ***R1***

e.g. probability of *A* getting there late is less than probability of *B*

$0.0808 < 0.15$ ***N2***

[3 marks]

7d. On five consecutive days the van sets out at 06:00 and takes Route B. Find the probability that *[5 marks]*

(i) it arrives before 07:00 on all five days;

(ii) it arrives before 07:00 on at least three days.

Markscheme

(i) let X be the number of days when the van arrives before 07:00

$\mathrm{P}(X = 5) = (0.85)^5$ ***(A1)***

$= 0.444$ ***A1 N2***

(ii) **METHOD 1**

evidence of adding correct probabilities ***(M1)***

e.g.
$\mathrm{P}(X \geq 3) = \mathrm{P}(X = 3) + \mathrm{P}(X = 4) + \mathrm{P}(X = 5)$

correct values
$0.1382 + 0.3915 + 0.4437$ ***(A1)***

$\mathrm{P}(X \geq 3) = 0.973$ ***A1 N3***

METHOD 2

evidence of using the complement ***(M1)***

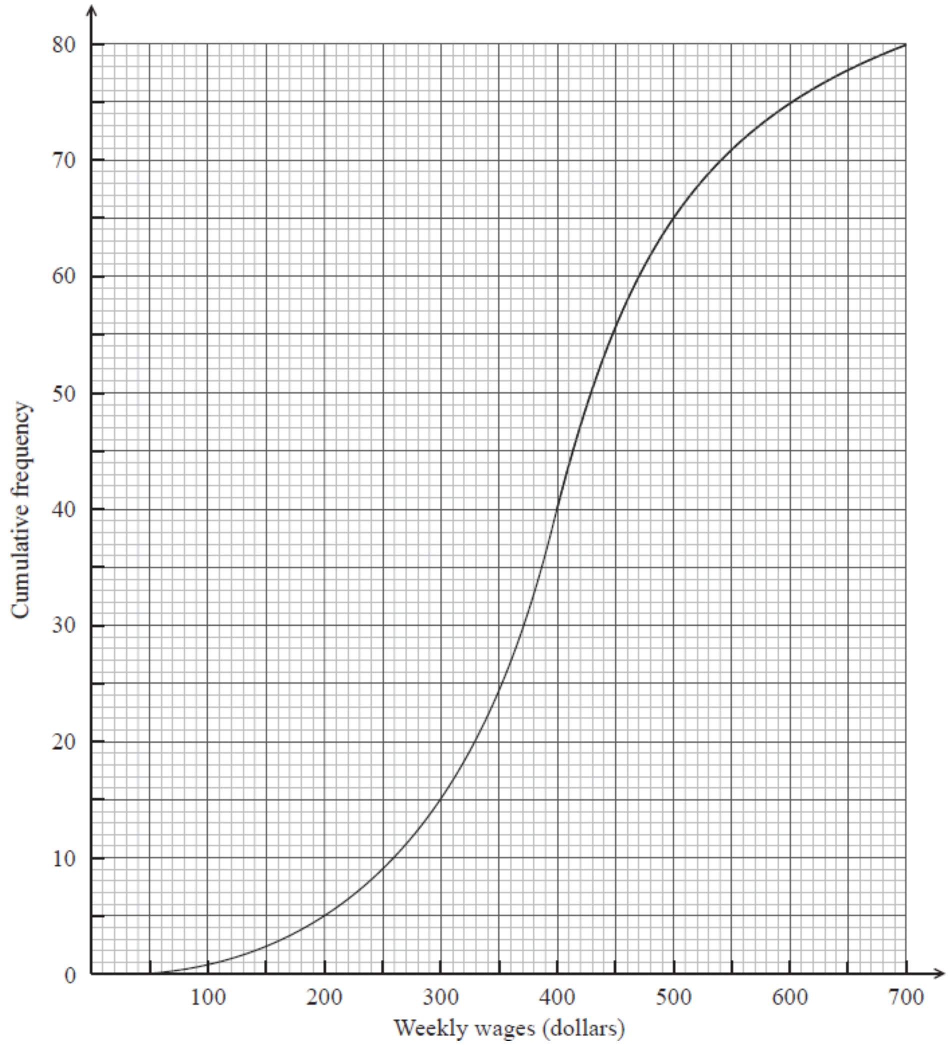
e.g.
$\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2)$,
$1 - p$

correct values
$1 - 0.02661$ ***(A1)***

$\mathrm{P}(X \geq 3) = 0.973$ ***A1 N3***

[5 marks]

The weekly wages (in dollars) of 80 employees are displayed in the cumulativefrequency curve below.



8a. (i) Write down the median weekly wage. *[4 marks]*

(ii) Find the interquartile range of the weekly wages.

Markscheme

(i) median weekly wage
$= 400$ (dollars) ***A1*** ***N1***

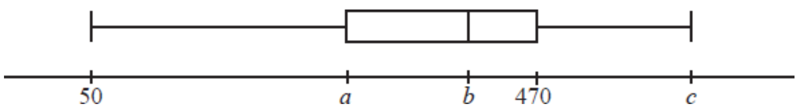
(ii) lower quartile
$= 330$, upper quartile
$= 470$ ***(A1)(A1)***

$\mathrm{IQR} = 140$ (dollars) (accept any notation suggesting interval
330 to
470) ***A1*** ***N3***

Note: Exception to the ***FT*** rule. Award ***A1(FT)*** for an incorrect IQR **only** if both quartiles are explicitly noted.

[4 marks]

8b. The box-and-whisker plot below displays the weekly wages of the employees. *[3 marks]*



Write down the value of

(i)
a ;

(ii)
b ;

(iii)
c .

Markscheme

(i)
330 (dollars) ***A1*** ***N1***

(ii)
400 (dollars) ***A1*** ***N1***

(iii)
700 (dollars) ***A1*** ***N1***

[3 marks]

8c. Employees are paid
$\$\ 20$ per hour. *[3 marks]*

Find the median number of **hours** worked per week.

Markscheme

valid approach ***(M1)***

e.g.
$\text{hours} = \frac{\text{wages}}{\text{rate}}$

correct substitution ***(A1)***

e.g.
$\frac{400}{20}$

median hours per week
$= 20$ ***A1*** ***N2***

[3 marks]

8d. Employees are paid
$\$20$ per hour. *[5 marks]*

Find the number of employees who work more than
25 hours per week.

Markscheme

attempt to find wages for 25 hours per week ***(M1)***

e.g.
$\text{wages} = \text{hours} \times \text{rate}$

correct substitution ***(A1)***

e.g.
25×20

finding wages
$= 500$ ***(A1)***

65 people (earn 500
$\leqslant$) ***(A1)***

15 people (work more than 25 hours) ***A1*** ***N3***

[5 marks]