0420Mixed-NoCalc- *[57 marks]*

In an arithmetic sequence, the first term is 8 and the second term is 5.

1a. Find the common difference. *[2 marks]*

Markscheme

subtracting terms ***(M1)***

eg $5-8,\ u_2-u_1$

$d=-3$ ***A1*** ***N2***

[2 marks]

1b. Find the tenth term. *[2 marks]*

Markscheme

correct substitution into formula ***(A1)***

eg $u_{10}=8+(10-1)(-3),\ 8-27,\ -3(10)+11$

$u_{10}=-19$ ***A1*** ***N2***

[2 marks]

1c. Find the sum of the first ten terms. *[2 marks]*

Markscheme

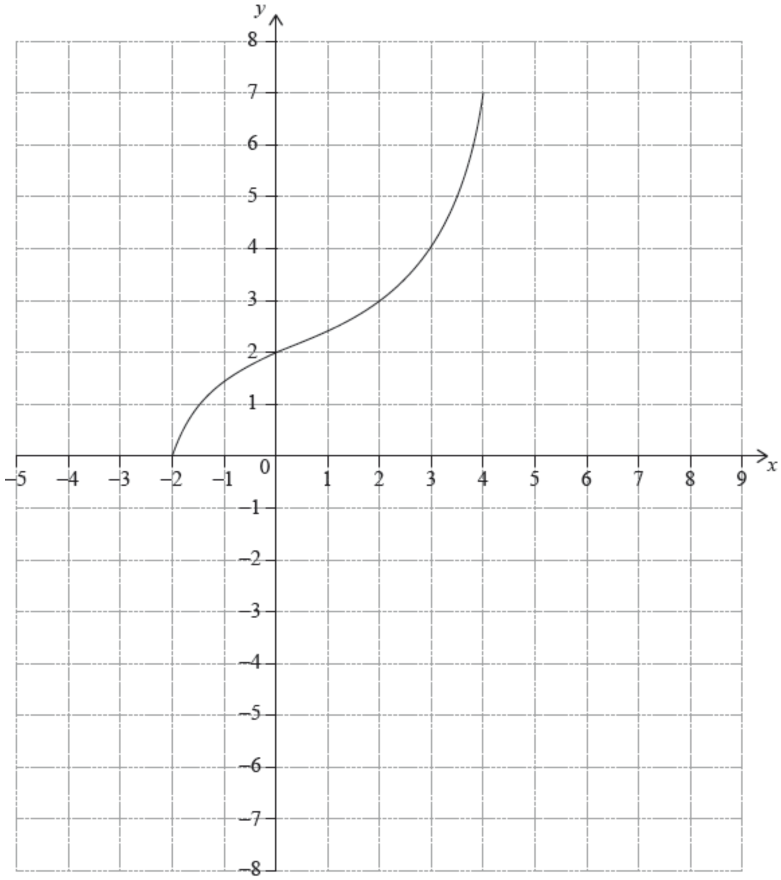
correct substitution into formula for sum ***(A1)***

eg $S_{10}=\frac{10}{2}(8-19),\ 5\left(2(8)+(10-1)(-3)\right)$

$S_{10}=-55$ ***A1*** ***N2***

[2 marks]

The following diagram shows the graph of a function f, with domain $-2 \leqslant x \leqslant 4$.



The points $(-2,\ 0)$ and $(4,\ 7)$ lie on the graph of f.

2a. Write down the range of f. *[1 mark]*

Markscheme

correct range (do not accept $0 \leqslant x \leqslant 7$) ***A1 N1***

eg $[0,\ 7]$, $0 \leqslant y \leqslant 7$

[1 mark]

2b. Write down $f(2)$; *[1 mark]*

Markscheme

$f(2) = 3$ ***A1 N1***

[1 mark]

2c. Write down $f^{-1}(2)$. *[1 mark]*

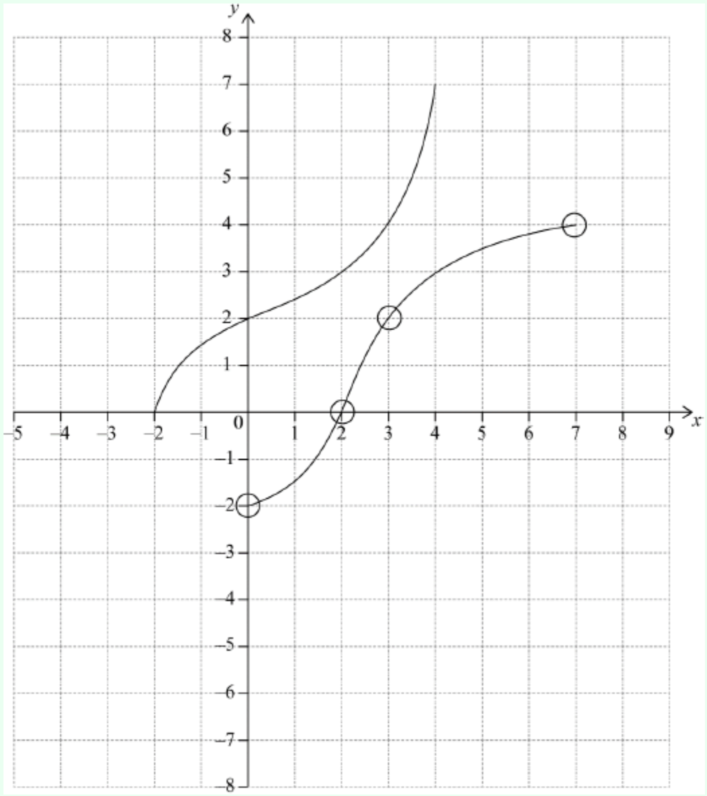
Markscheme

$f^{-1}(2) = 0$ ***A1 N1***

[1 mark]

2d. On the grid, sketch the graph of f^{-1}. *[3 marks]*

Markscheme



A1A1A1 ***N3***

Notes: Award ***A1*** for both end points within circles,

A1 for images of $(2,\ 3)$ and $(0,\ 2)$ within circles,

A1 for approximately correct reflection in $y = x$, concave up then concave down shape (do not accept line segments).

[3 marks]

Let $f(x) = 1 + \mathrm{e}^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

3a. Find $(g \circ f)(x)$. *[2 marks]*

Markscheme

attempt to form composite ***(M1)***

eg $g(1 + \mathrm{e}^{-x})$

correct function ***A1*** ***N2***

eg $(g \circ f)(x) = 2 + b + 2\mathrm{e}^{-x},\ 2(1 + \mathrm{e}^{-x}) + b$

[2 marks]

3b. Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b. *[4 marks]*

Markscheme

evidence of $\lim_{x\to\infty}(2+b+2e^{-x}) = 2+b+\lim_{x\to\infty}(2e^{-x})$ ***(M1)***

eg $2+b+2e^{-\infty}$, graph with horizontal asymptote when $x\to\infty$

Note: Award ***M0*** if candidate clearly has incorrect limit, such as $x\to 0,\ e^{\infty},\ 2e^{0}$.

evidence that $e^{-x}\to 0$ (seen anywhere) ***(A1)***

eg $\lim_{x\to\infty}(e^{-x})=0,\ 1+e^{-x}\to 1,\ 2(1)+b=-3,\ e^{\text{large negative number}}\to 0$, graph of $y=e^{-x}$ or $y=2e^{-x}$ with asymptote $y=0$, graph of composite function with asymptote $y=-3$

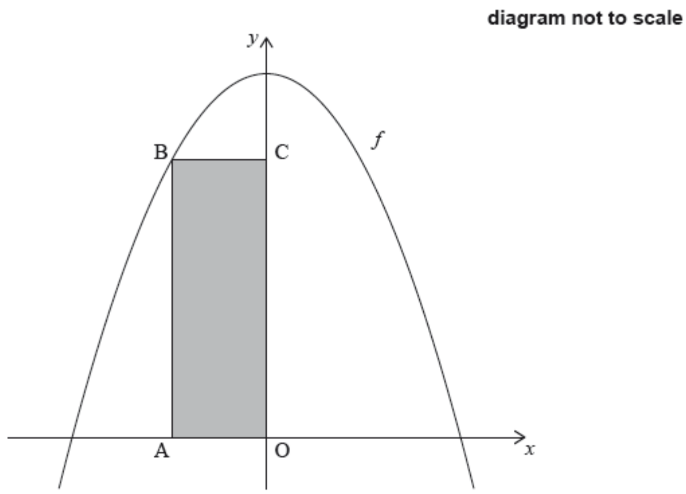
correct working ***(A1)***

eg $2+b=-3$

$b=-5$ ***A1 N2***

[4 marks]

4. Let $f(x)=15-x^2$, for $x\in\mathbb{R}$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x-axis, B is on the graph of f, and C is on the y-axis. *[7 marks]*



Find the x-coordinate of A that gives the maximum area of OABC.

Markscheme

attempt to find the area of OABC ***(M1)***

eg $\text{OA}\times\text{OC},\ x\times f(x),\ f(x)\times(-x)$

correct expression for area in one variable ***(A1)***

eg $\text{area}=x(15-x^2),\ 15x-x^3,\ x^3-15x$

valid approach to find maximum **area** (seen anywhere) ***(M1)***

eg $A'(x)=0$

correct derivative ***A1***

eg $15-3x^2,\ (15-x^2)+x(-2x)=0,\ -15+3x^2$

correct working ***(A1)***

eg $15=3x^2,\ x^2=5,\ x=\sqrt{5}$

$x=-\sqrt{5}\ \left(\text{accept A}\left(-\sqrt{5},\ 0\right)\right)$ ***A2 N3***

[7 marks]

5. Consider $f(x) = \log k(6x - 3x^2)$, for $0 < x < 2$, where $k > 0$. *[7 marks]*

The equation $f(x) = 2$ has exactly one solution. Find the value of k.

Markscheme

METHOD 1 – using discriminant

correct equation without logs ***(A1)***

eg $6x - 3x^2 = k^2$

valid approach ***(M1)***

eg $-3x^2 + 6x - k^2 = 0,\ 3x^2 - 6x + k^2 = 0$

recognizing discriminant must be zero (seen anywhere) ***M1***

eg $\Delta = 0$

correct discriminant ***(A1)***

eg $6^2 - 4(-3)(-k^2),\ 36 - 12k^2 = 0$

correct working ***(A1)***

eg $12k^2 = 36,\ k^2 = 3$

$k = \sqrt{3}$ ***A2 N2***

METHOD 2 – completing the square

correct equation without logs ***(A1)***

eg $6x - 3x^2 = k^2$

valid approach to complete the square ***(M1)***

eg $3(x^2 - 2x + 1) = -k^2 + 3,\ x^2 - 2x + 1 - 1 + \frac{k^2}{3} = 0$

correct working ***(A1)***

eg $3(x-1)^2 = -k^2 + 3,\ (x-1)^2 - 1 + \frac{k^2}{3} = 0$

recognizing conditions for one solution ***M1***

eg $(x-1)^2 = 0,\ -1 + \frac{k^2}{3} = 0$

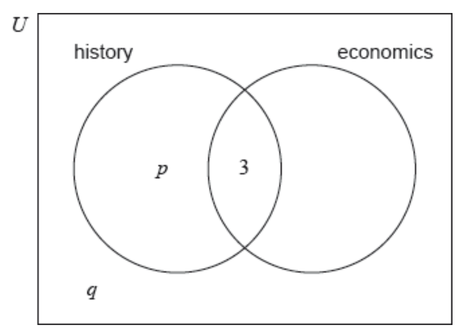
correct working ***(A1)***

eg $\frac{k^2}{3} = 1,\ k^2 = 3$

$k = \sqrt{3}$ ***A2 N2***

[7 marks]

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.



6a. Find the value of p; *[2 marks]*

Markscheme

valid approach *(M1)*

eg $p + 3 = 13,\ 13 - 3$

$p = 10$ ***A1 N2***

[2 marks]

6b. Find the value of q. *[2 marks]*

Markscheme

valid approach *(M1)*

eg $p + 3 + 5 + q = 20,\ 10 - 10 - 8$

$q = 2$ ***A1 N2***

[2 marks]

6c. A girl is selected at random. Find the probability that she takes economics but not history. *[2 marks]*

Markscheme

valid approach *(M1)*

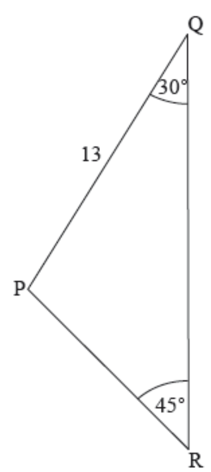
eg $20 - p - q - 3,\ 1 - \frac{15}{20},\ n(E \cap H') = 5$

$\frac{5}{20}\ \left(\frac{1}{4}\right)$ ***A1 N2***

[2 marks]

7. The following diagram shows triangle PQR. *[6 marks]*

diagram not to scale



$P\hat{Q}R = 30°$, $Q\hat{R}P = 45°$ and $PQ = 13\ \text{cm}$.

Find PR.

Markscheme

METHOD 1

evidence of choosing the sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution ***A1***

eg $\frac{x}{\sin 30} = \frac{13}{\sin 45}$, $\frac{13 \sin 30}{\sin 45}$

$\sin 30 = \frac{1}{2}$, $\sin 45 = \frac{1}{\sqrt{2}}$ ***(A1)(A1)***

correct working ***A1***

eg $\frac{1}{2} \times \frac{13}{\frac{1}{\sqrt{2}}}$, $\frac{1}{2} \times 13 \times \frac{2}{\sqrt{2}}$, $13 \times \frac{1}{2} \times \sqrt{2}$

correct answer ***A1*** ***N3***

eg $\text{PR} = \frac{13\sqrt{2}}{2}$, $\frac{13}{\sqrt{2}}$ (cm)

METHOD 2 (using height of ΔPQR)

valid approach to find height of ΔPQR ***(M1)***

eg $\sin 30 = \frac{x}{13}$, $\cos 60 = \frac{x}{13}$

$\sin 30 = \frac{1}{2}$ or $\cos 60 = \frac{1}{2}$ ***(A1)***

$\text{height} = 6.5$ ***A1***

correct working ***A1***

eg $\sin 45 = \frac{6.5}{\text{PR}}$, $\sqrt{6.5^2 + 6.5^2}$

correct working ***(A1)***

eg $\sin 45 = \frac{1}{\sqrt{2}}$, $\cos 45 = \frac{1}{\sqrt{2}}$, $\sqrt{\frac{169 \times 2}{4}}$

correct answer ***A1*** ***N3***

eg $\text{PR} = \frac{13\sqrt{2}}{2}$, $\frac{13}{\sqrt{2}}$ (cm)

[6 marks]

Jim heated a liquid until it boiled. He measured the temperature of the liquid as it cooled. The following table shows its temperature, d degrees Celsius, t minutes after it boiled.

t (min)	0	4	8	12	16	20
d (°C)	105	98.4	85.4	74.8	68.7	62.1

8a. Write down the independent variable. *[1 mark]*

Markscheme

t ***A1*** ***N1***

[1 mark]

8b. Write down the boiling temperature of the liquid. *[1 mark]*

Markscheme

105 ***A1*** ***N1***

[1 mark]

Jim believes that the relationship between d and t can be modelled by a linear regression equation.

8c. Jim describes the correlation as **very strong**. Circle the value below which best represents the correlation coefficient. *[2 marks]*

$$0.992 \qquad 0.251 \qquad 0 \qquad -0.251 \qquad -0.992$$

Markscheme

-0.992 ***A2*** ***N2***

[2 marks]

8d. Jim's model is $d = -2.24t + 105$, for $0 \leqslant t \leqslant 20$. Use his model to predict the decrease in temperature for any 2 minute interval. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $\frac{\mathrm{d}d}{\mathrm{d}t} = -2.24;\ 2 \times 2.24,\ 2 \times -2.24,\ d(2) = -2 \times 2.24 \times 105,$

finding $d(t_2) - d(t_1)$ where $t_2 = t_1 + 2$

4.48 (degrees) ***A1*** ***N2***

Notes: Award no marks for answers that **directly** use the table to find the decrease in temperature for 2 minutes *eg* $\frac{105-98.4}{2} = 3.3$.

[2 marks]

9a. Find $\int x\mathrm{e}^{x^2-1}\mathrm{d}x$. *[4 marks]*

Markscheme

valid approach to set up integration by substitution/inspection ***(M1)***

eg $u = x^2 - 1,\ \mathrm{d}u = 2x,\ \int 2x\mathrm{e}^{x^2-1}\mathrm{d}x$

correct expression ***(A1)***

eg $\frac{1}{2}\int 2x\mathrm{e}^{x^2-1}\mathrm{d}x,\ \frac{1}{2}\int \mathrm{e}^{u}\mathrm{d}u$

$\frac{1}{2}\mathrm{e}^{x^2-1} + c$ ***A2*** ***N4***

Notes: Award ***A1*** if missing "$+c$".

[4 marks]

9b. Find $f(x)$, given that $f'(x) = x\mathrm{e}^{x^2-1}$ and $f(-1) = 3$. *[3 marks]*

Markscheme

substituting $x = -1$ into **their** answer from (a) ***(M1)***

eg $\frac{1}{2}\mathrm{e}^{0},\ \frac{1}{2}\mathrm{e}^{1-1} = 3$

correct working ***(A1)***

eg $\frac{1}{2} + c = 3,\ c = 2.5$

$f(x) = \frac{1}{2}\mathrm{e}^{x^{2}-1} + 2.5$ ***A1*** ***N2***

[3 marks]