Spiral Review: 6-1 P1 (No Calculator) Calculus Tangents - A

1. 10N.1.sl.TZ0.2 (Mild)
   Let $g(x) = 2x \sin x$.
   (a) Find $g'(x)$ [4 marks]
   (b) Find the gradient of the graph of g at $x = \pi$. [3 marks]

2. 10M.1.sl.TZ2.5 [6 marks] (Medium)
   Let $f(x) = kx^4$. The point $P(1, k)$ lies on the curve of f. At P, the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k.

3. 13M.1.sl.TZ2.9 (Spicy)
   Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \leq x \leq \pi$.
   (a) Find $f'(x)$. [3 marks]
   (b) Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g.
   Find $g(4)$. [3 marks]
   (c) The function g can be expressed in the form $g(x) = a(x - h)^2 + 3$.
      i. Write down the value of h.
      ii. Find the value of a.
   (d) Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g. [6 marks]

Spiral Review: 6-1 P1 (No Calculator) Calculus Tangents - B

1. 12M.1.sl.TZ1.3 (Mild)
   Let $f(x) = e^{6x}$.
   (a) Write down $f'(x)$ [1 mark]
   (b) The tangent to the graph of f at the point $P(0, b)$ has gradient m. [4 marks]
      i. Show that $m = 6$.
      ii. Find b.
   (c) Hence, write down the equation of this tangent. [1 mark]

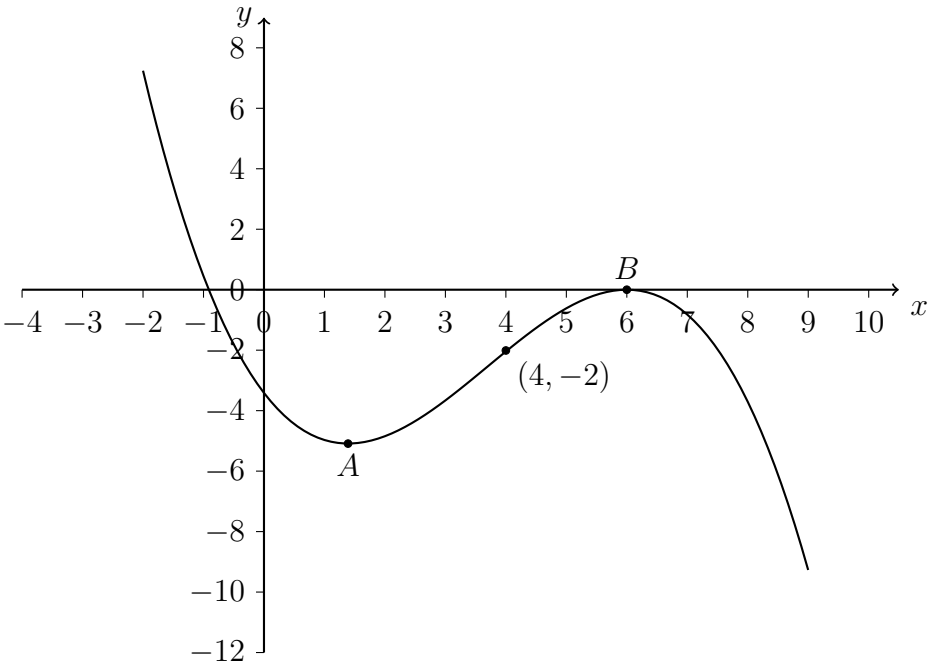
2. 13N.1.sl.TZ0.6 [6 marks] (Medium)
   Let $f(x) = e^{2x}$. The line L is the tangent to the curve of f at $(1, e^2)$.
   Find the equation of L in the form $y = ax + b$.

3. 15M.1.sl.TZ1.9 (Spicy)
   A function f has its derivative given by $f'(x) = 3x^2 - 2kx - 9$, where k is a constant.
   (a) Find $f''(x)$. [2 marks]
   (b) The graph of f has a point of inflexion when $x = 1$.
   Show that $k = 3$. [3 marks]
   (c) Find $f'(-2)$. [2 marks]
   (d) Find the equation of the tangent to the curve of f at $(-2, 1)$, giving your answer in the form $y = ax - b$. [4 marks]
   (e) Given that $f'(-1) = 0$, explain why the graph of f has a local maximum when $x = -1$. [3 marks]

Spiral Review: 6-1 P1 (No Calculator) Calculus Tangents - C

1. 09M.1.sl.TZ1.3 (Mild) [6 marks]
   Let $f(x) = e^x \cos x$. Find the gradient of the normal to the curve of f at $x = \pi$.

2. 17M.1.sl.TZ1.6 (Medium)
   The following diagram shows the graph of f', the derivative of f.



   The graph of f' has a local minimum at A, a local maximum at B and passes through $(4, 2)$. The point $P(4, 3)$ lies on the graph of the function, f.

   (a) Write down the gradient of the curve of f at P. [1 mark]

   (b) Find the equation of the normal to the curve of f at P. [3 marks]

   (c) Determine the concavity of the graph of f when $4 < x < 5$ **and** justify your answer. [2 marks]

3. 17M.1.sl.TZ1.9 (Spicy)
   A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has an axis of symmetry $x = 2.5$ and y-intercept at $(0, -6)$.

   (a) Find the value of p. [3 marks]

   (b) Find the value of a. [3 marks]

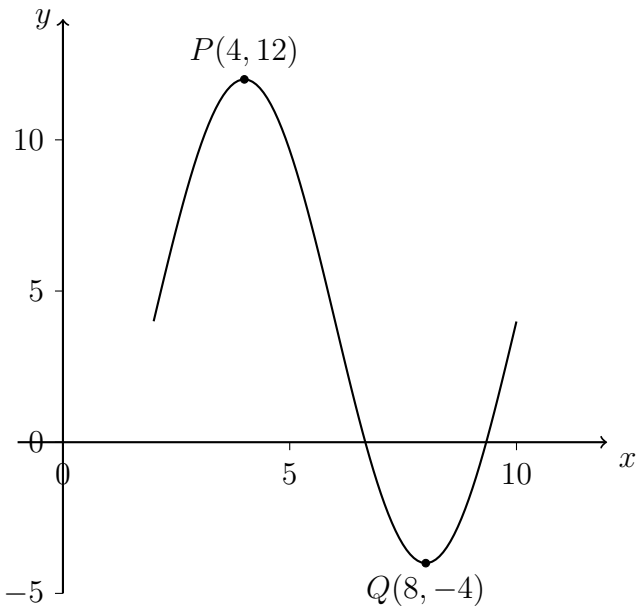
   (c) The line $y = kx - 5$ is a tangent to the curve of f. Find the values of k. [8 marks]

Spiral Review: 6-1 P1 (No Calculator) Calculus Tangents - D

1. 13M.1.sl.TZ1.3 (Mild)
   Consider $f(x) = x^2 \sin x$.
   (a) Find $f'(x)$. [4 marks]
   (b) Find the gradient of the curve of f at $x = \frac{\pi}{2}$. [3 marks]

2. 18M.1.sl.TZ1.7 [7 marks] (Medium)
   Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in \mathbb{R}$ where $h(x) = (f \circ g)(x)$.
   Given that $g(3) = 7$, $g'(3) = 4$, and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$.

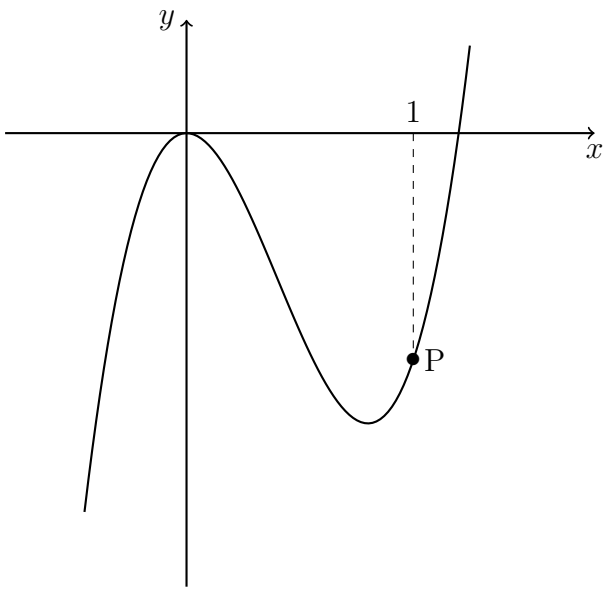
3. 11N.1.sl.TZ0.9 (Spicy)
   The following diagram shows the graph of $f(x) = a\sin(b(x - c)) + d$, for $2 \leq x \leq 10$.



   There is a maximum point at $P(4, 12)$ and a minimum point ast $Q(8, -4)$.
   (a) Use the graph to write down the value of [3 marks]
      i. a;
      ii. c;
      iii. d.
   (b) Show that $b = \frac{\pi}{4}$. [2 marks]
   (c) Find $f'(x)$. [3 marks]
   (d) At a point R, the gradient is -2π. Find the x-coordinate of R. [6 marks]

Spiral Review: 6-1 P1 (No Calculator) Calculus Tangents - E

1. 12N.1.sl.TZ0.4 (Mild)
Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.



The point P lies on the graph of f. At P, $x = 1$.

(a) Find $f'(x)$. [2 marks]

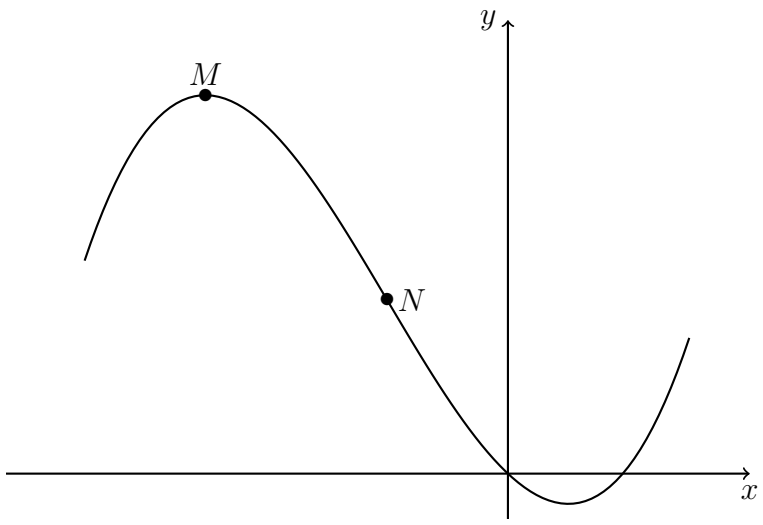
(b) The graph of f has a gradient of 3 at the point P. Find the value of a. [4 marks]

2. 17N.1.sl.TZ0.5 (Medium)
Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

(a) Find $(g \circ f)(x)$. [2 marks]

(b) Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b. [4 marks]

3. 08M.1.sl.TZ1.8 (Spicy)
Consider $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x$. Part of the graph of f is shown below. There is a maximum point at M, and a point of inflexion at N.



(a) Find $f'(x)$. [3 marks]

(b) Find the x-coordinate of M. [4 marks]

(c) Find the x-coordinate of N. [3 marks]

(d) The line L is the tangent to the curve of f at $(3, 12)$. Find the equation of L in the form $y = ax + b$. [4 marks]

4. 16M.1.sl.TZ1.10 (Spicy)
Let $f(x) = \sqrt{4x + 5}$, for $x \geq -1.25$.

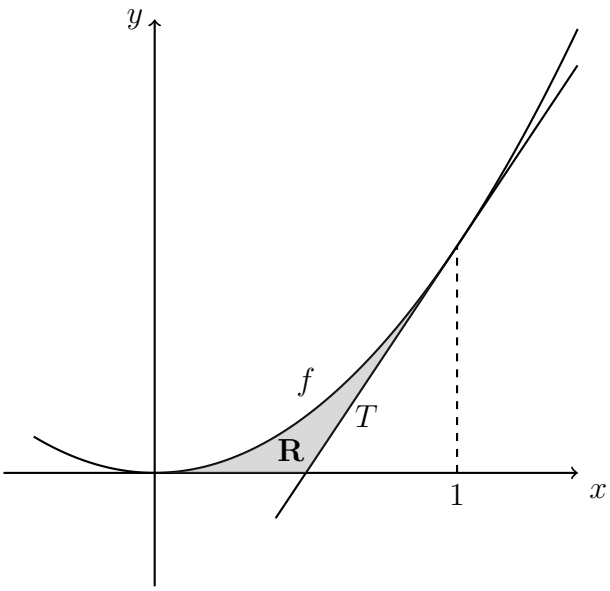
(a) Find $f'(1)$. [4 marks]

(b) Consider another function g. Let R be a point on the graph of g. The x-coordinate of R is 1. The equation of the tangent to the graph at R is $y = 3x + 6$.
Write down $g'(1)$. [2 marks]

(c) Find $g(1)$. [2 marks]

(d) Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent to the graph of h at the point where $x = 1$. [7 marks]

5. 11M.1.sl.TZ2.8 (Spicy)
The following diagram shows part of the graph of the function $f(x) = 2x^2$.



The line T is the tangent to the graph of f at $x = 1$.

(a) Show that the equation of T is $y = 4x - 2$. [5 marks]

(b) Find the x-intercept of T. [2 marks]

(c) The shaded region R is enclosed by the graph of f, the line T, and the x-axis. [9 marks]

i. Write down an expression for the area of R.

ii. Find the area of R.