7.2 Tangent lines & derivatives (12.1 IB SL)

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Drui

Definition of the derivative

Tangent and secant

Definition of the derivative

Secant in the limit

Definition of the derivative

Example slope calculation

Definition of the derivative

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Definition of the derivative

Notation

Calculation of the derivative

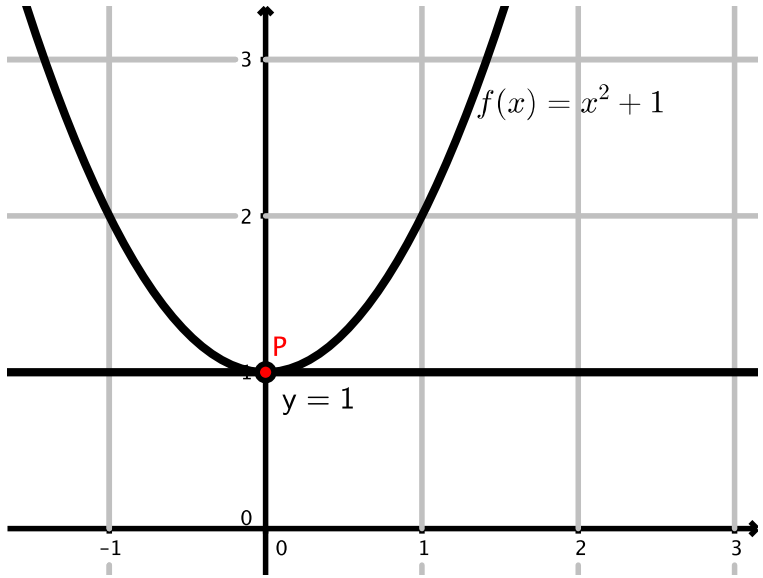
Example slope calculation

GQ: What is the slope of a curve?

- CCSS: Derivatives
- Do Now: Graph $f(x) = x^2 + 1$ and $y = 1$
  (on the same graph)
- Lesson: Tangents to functions, derivatives (p. 200)
- Homework:

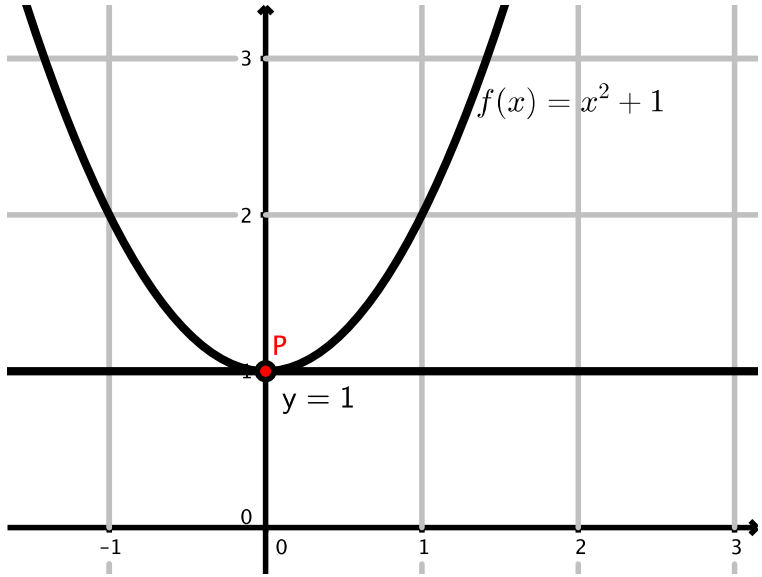
Definition of tangent and secant

What is the "gradient" of a curve? $f(x) = x^2 + 1$ and $y = 1$



Definition of tangent and secant

$f(x) = x^2 + 1$ and $y = 1$



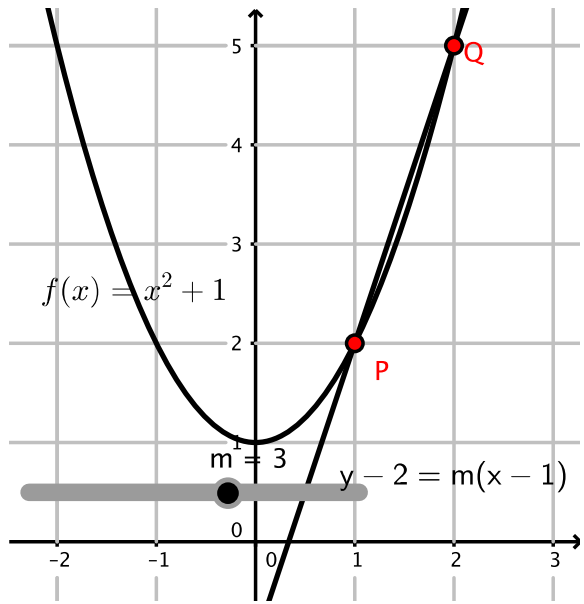
Definition

A secant intersects a curve

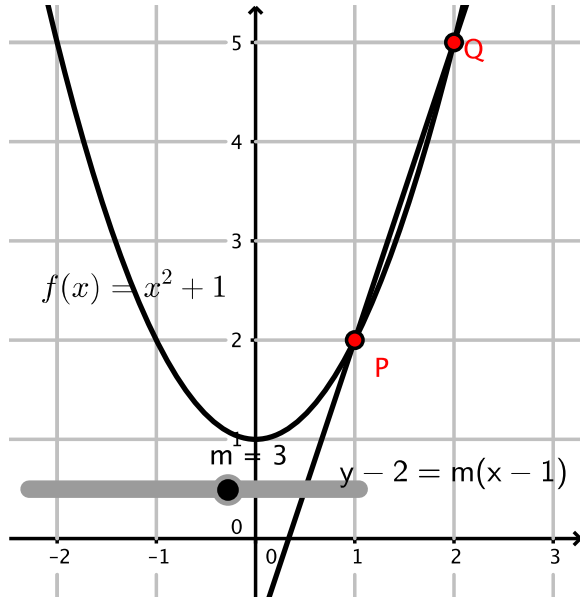
A tangent "touches" a curve in one point (locally)

Definition of the difference quotient

What is the "gradient" of the secant through points P and Q?
$f(x) = x^2 + 1$ and $y - 2 = m(x - 1)$ link



Definition of the difference quotient



Given a small Δx from P to Q there will be a small change in y values. The gradient will be:

$$m = \frac{f(x+h) - f(x)}{(x+h) - h} = \frac{f(x+h) - f(x)}{h}$$

This is called the difference quotient

Definition of the derivative

What is the "gradient" of the secant through any point $(x, f(x))$?

$$f(x) = x^2 + 1$$

Definition of the derivative

What is the "gradient" of the secant through any point $(x, f(x))$?

$$f(x) = x^2 + 1$$

Hint: use two points on the curve, $(x, f(x))$ and $(x + h, f(x + h))$, and the slope definition:

$$m = \frac{\Delta y}{\Delta x}$$

Definition of the derivative

What is the "gradient" of the secant through any point $(x, f(x))$?

$$f(x) = x^2 + 1$$

Substitute for $f(x)$, expand and simplify:

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{((x+h)^2 + 1) - (x^2 + 1)}{h}$$

Definition of the derivative

What is the "gradient" of the secant through any point $(x, f(x))$?

$$f(x) = x^2 + 1$$

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{((x+h)^2 + 1) - (x^2 + 1)}{h}$$

$$= \frac{(x^2 + 2xh + h^2 + 1) - (x^2 + 1)}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= \frac{h(2x + h)}{h}$$

$$= 2x + h$$

Definition of the derivative

As h gets small, the secant approaches the tangent line, and the slope approaches the local steepness of the curve at that point.

Definition

The derivative of a function is the slope of a tangent line, expressed as a limit as h gets very small.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Definition of the derivative

The derivative is also called the differential or the instantaneous rate of change, or change at the margin.

Notation:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Also written as $\frac{dy}{dx}$ ("dee y, dee x") and y' ("y prime").

Calculation of the derivative. Example 4 p202

What is the derivative of $f(x) = x^2 + 1$ and *hence* the slope of the tangent when $x = 3$?

Calculation of the derivative. Example 4 p202

$$f(x) = x^2 + 1$$

Definition of derivative:

$$f'(x) = lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Calculation of the derivative. Example 4 p202

$$f(x) = x^2 + 1$$

Definition of derivative:

$$f'(x) = lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= lim_{h \to 0} \frac{((x+h)^2 + 1) - (x^2 + 1)}{h}$$

$$= lim_{h \to 0} (2x + h)$$

Calculation of the derivative. Example 4 p202

$$f(x) = x^2 + 1$$

Definition of derivative:

$$f'(x) = lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= lim_{h \to 0} \frac{((x+h)^2 + 1) - (x^2 + 1)}{h}$$

$$= lim_{h \to 0}(2x + h) = 2x + 0$$

$$f'(x) = 2x$$

Calculation of the derivative. Example 4 p202

What is the derivative of $f(x) = x^2 + 1$ and *hence* the slope of the tangent when $x = 3$?

$$f'(x) = 2x$$

$$f'(3) = 2(3) = 6$$