BECA / Huson / 12.1 IB Math SL Name:
8 February 2019

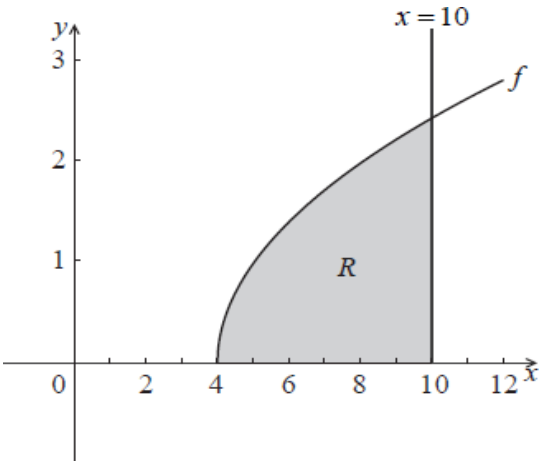
Unit test: Integration as the area under a curve, volumes of rotation (46 marks)

1a. Consider a function $f(x)$ such that $\int_1^6 f(x)\mathrm{d}x = 8$. Find $\int_1^6 2f(x)\mathrm{d}x$. *[2 marks]*

1b. Find $\int_1^6 (f(x)+2)\,\mathrm{d}x$. *[4 marks]*

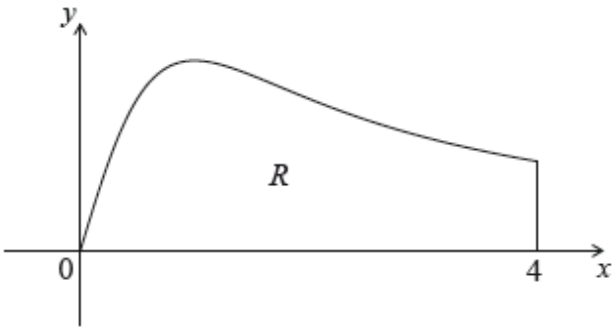
2a. Find $\int_4^{10} (x-4)\mathrm{d}x$. *[4 marks]*

2b. Part of the graph of $f(x) = \sqrt{x-4}$, for $x \geq 4$, is shown below. The shaded region *R* is enclosed by the graph of f, the line $x = 10$, and the *x*-axis.



Find the area of the shaded region. *[3 marks]*

3. The following diagram shows the graph of $f(x) = \frac{x}{x^2+1}$, for $0 \leq x \leq 4$, and the line $x = 4$.



Let R be the region enclosed by the graph of f, the x-axis and the line $x = 4$.

Find the area of R. *[6 marks]*

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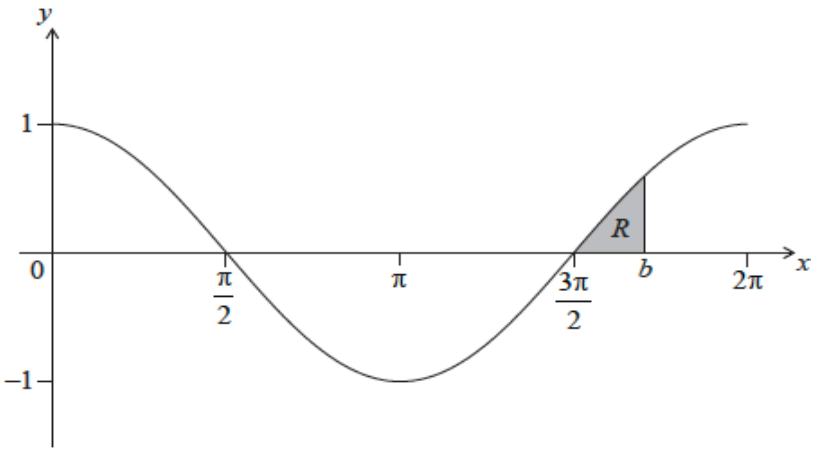
4a. Let $f(x) = x^2$ and $g(x) = 3\ln(x+1)$, for $x > -1$.

Solve $f(x) = g(x)$. *[3 marks]*

4b. Find the area of the region enclosed by the graphs of f and g. *[3 marks]*

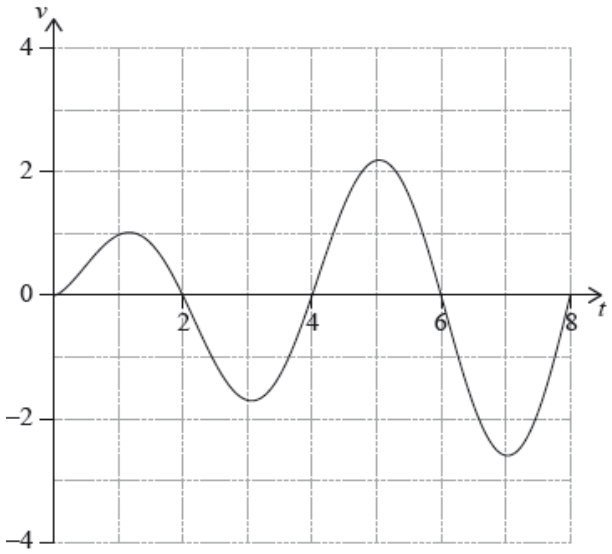
5. Let $f(x) = \cos x$, for $0 \leq x \leq 2\pi$. The following diagram shows the graph of f.

There are x-intercepts at $x = \frac{\pi}{2}, \frac{3\pi}{2}$.



The shaded region R is enclosed by the graph of f, the line $x = b$, where $b > \frac{3\pi}{2}$, and the x-axis. The area of R is $\left(1 - \frac{\sqrt{3}}{2}\right)$. Find the value of b. *[8 marks]*

6a. A particle P moves along a straight line. Its velocity v_{P} m s^{-1} after t seconds is given by $v_{\text{P}} = \sqrt{t}\sin\left(\frac{\pi}{2}t\right)$, for $0 \leqslant t \leqslant 8$. The following diagram shows the graph of v_{P}.



Write down the first value of t at which P changes direction. *[1 mark]*

6b. Find the **total** distance travelled by P, for $0 \leqslant t \leqslant 8$. *[2 marks]*

6c. A second particle Q also moves along a straight line. Its velocity, v_{Q} m s^{-1} after t seconds is given by $v_{\text{Q}} = \sqrt{t}$ for $0 \leqslant t \leqslant 8$. After k seconds Q has travelled the same total distance as P.

Find k. *[4 marks]*

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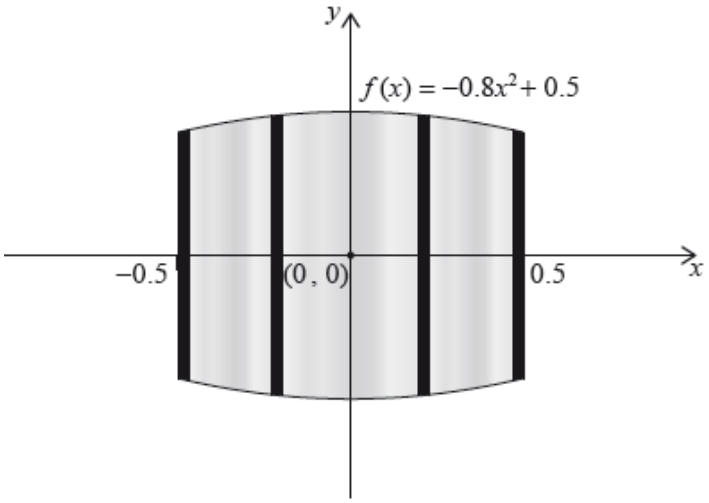
7a. All lengths in this question are in metres.

Let $f(x) = -0.8x^2 + 0.5$, for $-0.5 \leqslant x \leqslant 0.5$. Mark uses $f(x)$ as a model to create a barrel. The region enclosed by the graph of f, the x-axis, the line $x = -0.5$ and the line $x = 0.5$ is rotated 360° about the x-axis. This is shown in the following diagram.



Use the model to find the volume of the barrel. *[3 marks]*

7b. The empty barrel is being filled with water. The volume $V\ \text{m}^3$ of water in the barrel after t minutes is given by $V = 0.8(1 - \text{e}^{-0.1t})$. How long will it take for the barrel to be half-full? *[3 marks]*