0305Pre-Test-Trimester-review *[143 marks]*

Let $f(x) = 5x$ and $g(x) = x^2 + 1$, for $x \in \mathbb{R}$.

1a. Find $f^{-1}(x)$. *[2 marks]*

Markscheme

interchanging x and x ***(M1)***

eg $x = 5y$

$f^{-1}(x) = \frac{x}{5}$ ***A1 N2***

[2 marks]

1b. Find $(f \circ g)(7)$. *[3 marks]*

Markscheme

METHOD 1

attempt to substitute 7 into $g(x)$ or $f(x)$ ***(M1)***

eg $7^2 + 1,\ 5 \times 7$

$g(7) = 50$ ***(A1)***

$f(50) = 250$ ***A1 N2***

METHOD 2

attempt to form composite function (in any order) ***(M1)***

eg $5(x^2 + 1),\ (5x)^2 + 1$

correct substitution ***(A1)***

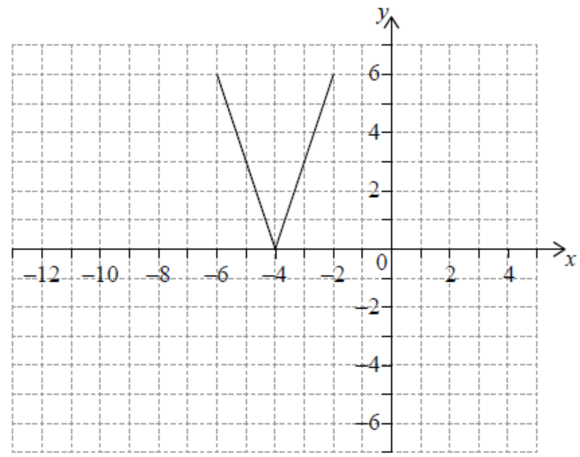
eg $5 \times (7^2 + 1)$

$(f \circ g)(7) = 250$ ***A1 N2***

[3 marks]

The following diagram shows the graph of a function $y = f(x)$, for $-6 \leqslant x \leqslant -2$.

The points $(-6,\ 6)$ and $(-2,\ 6)$ lie on the graph of f. There is a minimum point at $(-4,\ 0)$.



2a. Write down the range of f. *[2 marks]*

Markscheme

correct interval ***A2*** ***N2***

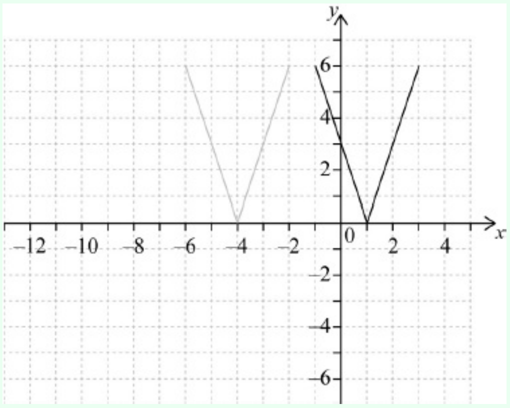
eg $0 \leqslant y \leqslant 6,\ [0,\ 6]$, from 0 to 6

[2 marks]

Let $g(x) = f(x - 5)$.

2b. On the grid above, sketch the graph of g. *[2 marks]*

Markscheme



M1A1 ***N2***

Note: Award ***M1*** for a horizontal shift of the whole shape, 5 units to the left or right and ***A1*** for the correct graph.

[2 marks]

2c. Write down the domain of g. *[2 marks]*

Markscheme

correct interval ***A2 N2***

eg $-1 \leqslant x \leqslant 3$, $[-1, 3]$, from -1 to 3

[2 marks]

Let $f(x) = 8x + 3$ and $g(x) = 4x$, for $x \in \mathbb{R}$.

3a. Write down $g(2)$. *[1 mark]*

Markscheme

$g(2) = 8$ ***A1 N1***

[1 mark]

3b. Find $(f \circ g)(x)$. *[2 marks]*

Markscheme

attempt to form composite (in any order) ***(M1)***

eg

$f(4x),\ 4 \times (8x + 3)$

$(f \circ g)(x) = 32x + 3$ ***A1 N2***

[2 marks]

3c. Find $f^{-1}(x)$. *[2 marks]*

Markscheme

interchanging x and y (may be seen at any time) ***(M1)***

eg $x = 8y + 3$

$f^{-1}(x) = \frac{x-3}{8}$ $\left(\text{accept } \frac{x-3}{8},\ y = \frac{x-3}{8}\right)$ ***A1 N2***

[2 marks]

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x - 5$, for $x \in \mathbb{R}$.

4a. Find $f(8)$. *[2 marks]*

Markscheme

attempt to substitute $x = 8$ ***(M1)***

eg $8^2 + 2 \times 8 + 1$

$f(8) = 81$ ***A1 N2***

[2 marks]

4b. Find $(g \circ f)(x)$. *[2 marks]*

Markscheme

attempt to form composition (in any order) ***(M1)***

eg $f(x-5),\ g(f(x)),\ \left(x^2+2x+1\right)-5$

$(g \circ f)(x) = x^2+2x-4$ ***A1 N2***

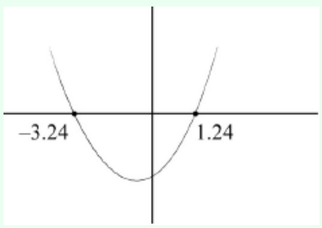
[2 marks]

4c. Solve $(g \circ f)(x) = 0$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg

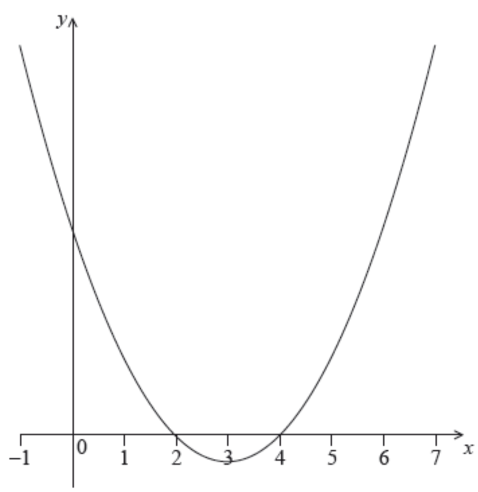
$x = \frac{-2 \pm \sqrt{20}}{2}$, 

$1.23606,\ -3.23606$

$x = 1.24,\ x = -3.24$ ***A1A1 N3***

[3 marks]

The following diagram shows part of the graph of a quadratic function f.



The vertex is at $(3,\ -1)$ and the x-intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x-h)^2 + k$.

5a. Write down the value of h and of k. *[2 marks]*

Markscheme

$h = 3,\ k = -1$ ***A1A1 N2***

[2 marks]

The function can also be written in the form $f(x) = (x - a)(x - b)$.

5b. Write down the value of a and of b. *[2 marks]*

Markscheme

$a = 2,\ b = 4$ (or $a = 4,\ b = 2$) ***A1A1*** ***N2***

[2 marks]

5c. Find the y-intercept. *[2 marks]*

Markscheme

attempt to substitute $x = 0$ into their f ***(M1)***

eg $(0 - 3)^2 - 1,\ (0 - 2)(0 - 4)$

$y = 8$ ***A1*** ***N2***

[2 marks]

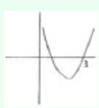
A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y-intercept at $(0,\ -6)$

6a. Find the value of p. *[3 marks]*

Markscheme

METHOD 1 (using *x*-intercept)

determining that 3 is an x-intercept ***(M1)***

eg $x - 3 = 0$, 

valid approach ***(M1)***

eg $3 - 2.5,\ \frac{p+3}{2} = 2.5$

$p = 2$ ***A1*** ***N2***

METHOD 2 (expanding *f* (*x*))

correct expansion (accept absence of a) ***(A1)***

eg $ax^2 - a(3 + p)x + 3ap,\ x^2 - (3 + p)x + 3p$

valid approach involving equation of axis of symmetry ***(M1)***

eg $\frac{-b}{2a} = 2.5,\ \frac{a(3+p)}{2a} = \frac{5}{2},\ \frac{3+p}{2} = \frac{5}{2}$

$p = 2$ ***A1*** ***N2***

METHOD 3 (using derivative)

correct derivative (accept absence of a) ***(A1)***

eg $a(2x - 3 - p),\ 2x - 3 - p$

valid approach ***(M1)***

eg $f'(2.5) = 0$

$p = 2$ ***A1*** ***N2***

[3 marks]

A quadratic function f can be written in the form $f(x)=a(x-p)(x-3)$. The graph of f has axis of symmetry $x=2.5$ and y-intercept at $(0,\ -6)$

6b. Find the value of a. *[3 marks]*

Markscheme

attempt to substitute $(0,\ -6)$ ***(M1)***

eg $-6=a(0-2)(0-3),\ 0=a(-8)(-9),\ a(0)^2-5a(0)+6a=-6$

correct working ***(A1)***

eg $-6=6a$

$a=-1$ ***A1 N2***

[3 marks]

A quadratic function f can be written in the form $f(x)=a(x-p)(x-3)$. The graph of f has axis of symmetry $x=2.5$ and y-intercept at $(0,\ -6)$

6c. The line $y=kx-5$ is a tangent to the curve of f. Find the values of k. *[8 marks]*

Markscheme

METHOD 1 (using discriminant)

recognizing tangent intersects curve once ***(M1)***

recognizing one solution when discriminant = 0 ***M1***

attempt to set up equation ***(M1)***

eg $g=f,\ kx-5=-x^2+5x-6$

rearranging their equation to equal zero ***(M1)***

eg $x^2-5x+kx+1=0$

correct discriminant (if seen explicitly, not just in quadratic formula) ***A1***

eg $(k-5)^2-4,\ 25-10k+k^2-4$

correct working ***(A1)***

eg $k-5=\pm 2,\ (k-3)(k-7)=0,\ \frac{10\pm\sqrt{100-4\times 21}}{2}$

$k=3,\ 7$ ***A1A1 N0***

METHOD 2 (using derivatives)

attempt to set up equation ***(M1)***

eg $g=f,\ kx-5=-x^2+5x-6$

recognizing derivative/slope are equal ***(M1)***

eg $f'=m_T,\ f'=k$

correct derivative of f ***(A1)***

eg $-2x+5$

attempt to set up equation in terms of either x or k ***M1***

eg $(-2x+5)x-5=-x^2+5x-6,\ k\left(\frac{5-k}{2}\right)-5=-\left(\frac{5-k}{2}\right)^2+5\left(\frac{5-k}{2}\right)-6$

rearranging their equation to equal zero ***(M1)***

eg $x^2-1=0,\ k^2-10k+21=0$

correct working ***(A1)***

eg $x=\pm 1,\ (k-3)(k-7)=0,\ \frac{10\pm\sqrt{100-4\times 21}}{2}$

$k=3,\ 7$ ***A1A1 N0***

[8 marks]

Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$.

The distance between the two zeros of f is 9.

7a. Show that the two zeros are 3 and -6. *[2 marks]*

Markscheme

recognition that the x-coordinate of the vertex is -1.5 (seen anywhere) ***(M1)***

eg axis of symmetry is -1.5, sketch, $f'(-1.5) = 0$

correct working to find the zeroes ***A1***

eg -1.5 ± 4.5

$x = -6$ and $x = 3$ ***AG*** ***N0***

[2 marks]

7b. Find the value of q and of r. *[4 marks]*

Markscheme

METHOD 1 (using factors)

attempt to write factors ***(M1)***

eg $(x-6)(x+3)$

correct factors ***A1***

eg $(x-3)(x+6)$

$q = 3,\ r = -18$ ***A1A1*** ***N3***

METHOD 2 (using derivative or vertex)

valid approach to find q ***(M1)***

eg $f'(-1.5) = 0,\ -\frac{q}{2a} = -1.5$

$q = 3$ ***A1***

correct substitution ***A1***

eg $3^2 + 3(3) + r = 0,\ (-6)^2 + 3(-6) + r = 0$

$r = -18$ ***A1***

$q = 3,\ r = -18$ ***N3***

METHOD 3 (solving simultaneously)

valid approach setting up system of two equations ***(M1)***

eg $9 + 3q + r = 0,\ 36 - 6q + r = 0$

one correct value

eg $q = 3,\ r = -18$ ***A1***

correct substitution ***A1***

eg $3^2 + 3(3) + r = 0,\ (-6)^2 + 3(-6) + r = 0,\ 3^2 + 3q - 18 = 0,\ 36 - 6q - 18 = 0$

second correct value ***A1***

eg $q = 3,\ r = -18$

$q = 3,\ r = -18$ ***N3***

[4 marks]

Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y.

8a. $\ln\left(\frac{5}{3}\right)$. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $\ln 5 - \ln 3$

$\ln\left(\frac{5}{3}\right) = y - x$ ***A1 N2***

[2 marks]

8b. $\ln 45$. *[4 marks]*

Markscheme

recognizing factors of 45 (may be seen in log expansion) ***(M1)***

eg $\ln(9 \times 5),\ 3 \times 3 \times 5,\ \log 3^2 \times \log 5$

correct application of $\log(ab) = \log a + \log b$ ***(A1)***

eg $\ln 9 + \ln 5,\ \ln 3 + \ln 3 + \ln 5,\ \ln 3^2 + \ln 5$

correct working ***(A1)***

eg $2\ln 3 + \ln 5,\ x + x + y$

$\ln 45 = 2x + y$ ***A1 N3***

[4 marks]

9a. Find the value of $\log_2 40 - \log_2 5$. *[3 marks]*

Markscheme

evidence of correct formula ***(M1)***

eg
$\log a - \log b = \log \frac{a}{b}$,
$\log\left(\frac{40}{5}\right)$,
$\log 8 + \log 5 - \log 5$

Note: Ignore missing or incorrect base.

correct working ***(A1)***

eg
$\log_2 8$,
$2^3 = 8$

$\log_2 40 - \log_2 5 = 3$ ***A1 N2***

[3 marks]

9b. Find the value of $8^{\log_2 5}$. *[4 marks]*

Markscheme

attempt to write
8 as a power of
2 (seen anywhere)   ***(M1)***

eg
$(2^3)^{\log_2 5}$,
$2^3 = 8$,
2^a

multiplying powers   ***(M1)***

eg
$2^{3\log_2 5}$,
$a\log_2 5$

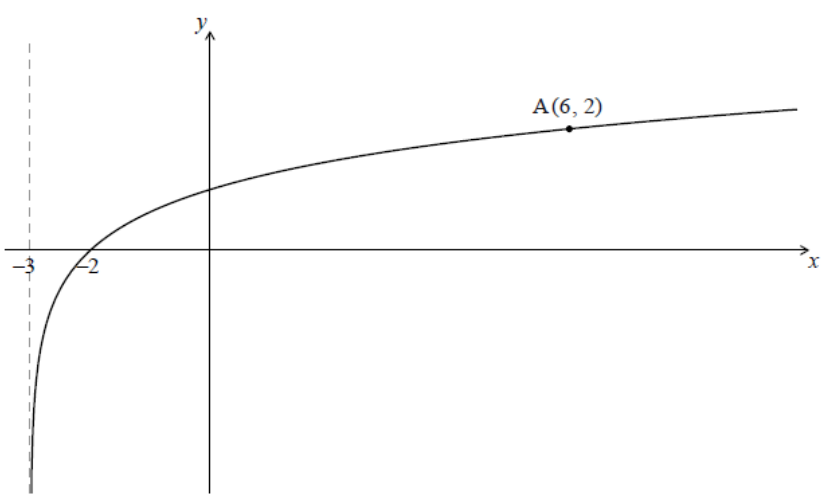
correct working   ***(A1)***

eg
$2^{\log_2 125}$,
$\log_2 5^3$,
$\left(2^{\log_2 5}\right)^3$

$8^{\log_2 5} = 125$ ***A1***   ***N3***

[4 marks]

Let
$f(x) = \log_p(x+3)$ for
$x > -3$. Part of the graph of *f* is shown below.



The graph passes through A(6, 2) , has an *x*-intercept at (–2, 0) and has an asymptote at
$x = -3$.

10a. Find *p* .   *[4 marks]*

Markscheme

evidence of substituting the point A ***(M1)***

e.g.
$2 = \log_p(6+3)$

manipulating logs ***A1***

e.g.
$p^2 = 9$

$p = 3$ ***A2*** ***N2***

[4 marks]

10b. The graph of *f* is reflected in the line $y = x$ to give the graph of g . *[5 marks]*

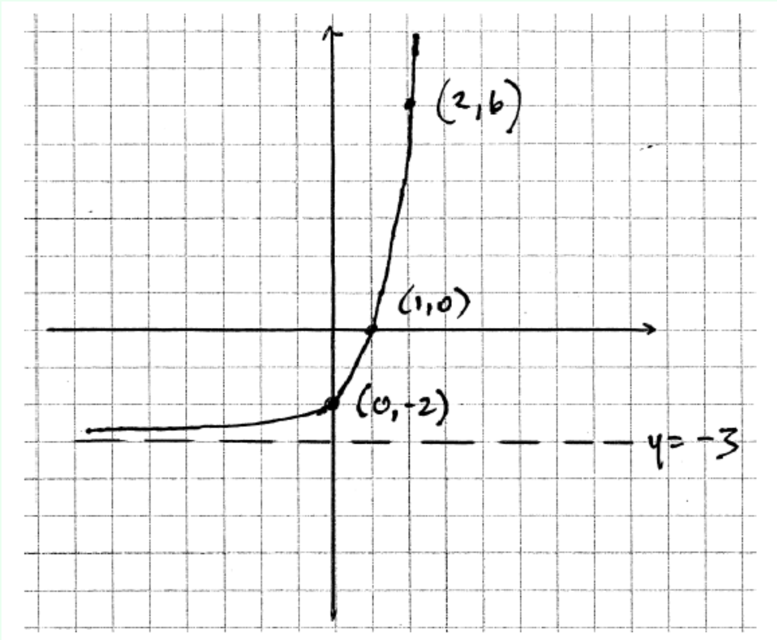
(i) Write down the *y*-intercept of the graph of g .

(ii) Sketch the graph of g , noting clearly any asymptotes and the image of A.

Markscheme

(i)
$y = -2$ (accept
$(0,\ -2)$) ***A1*** ***N1***

(ii)



A1A1A1A1 ***N4***

Note: Award ***A1*** for asymptote at
$y = -3$, ***A1*** for an increasing function that is concave up, ***A1*** for a positive *x*-intercept and a negative *y*-intercept, ***A1*** for passing through the point
$(2, 6)$.

[5 marks]

10c. The graph of f is reflected in the line $y = x$ to give the graph of g .

[4 marks]

Find $g(x)$.

Markscheme

METHOD 1

recognizing that $g = f^{-1}$ ***(R1)***

evidence of valid approach ***(M1)***

e.g. switching *x* and *y* (seen anywhere), solving for *x*

correct manipulation ***(A1)***

e.g. $3^x = y + 3$

$g(x) = 3^x - 3$ ***A1 N3***

METHOD 2

recognizing that $g(x) = a^x + b$ ***(R1)***

identifying vertical translation ***(A1)***

e.g. graph shifted down 3 units, $f(x) - 3$

evidence of valid approach ***(M1)***

e.g. substituting point to identify the base

$g(x) = 3^x - 3$ ***A1 N3***

[4 marks]

A population of rare birds, P_t, can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$.

11a. (i) Find the value of k.

[3 marks]

(ii) Interpret the meaning of the value of k.

Markscheme

(i) valid approach ***(M1)***

eg $0.9 = e^{k(1)}$

$k = -0.105360$

$k = \ln 0.9 \text{ (exact)}, \ -0.105$ ***A1 N2***

(ii) correct interpretation ***R1 N1***

eg population is decreasing, growth rate is negative

[3 marks]

A population of rare birds, P_t, can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$.

11b. Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.75$.

[5 marks]

Markscheme

METHOD 1

valid approach (accept an equality, but do not accept 0.74) ***(M1)***

eg $P < 0.75P_0,\ P_0\mathrm{e}^{kt} < 0.75P_0,\ 0.75 = \mathrm{e}^{t\ln 0.9}$

valid approach to solve **their** inequality ***(M1)***

eg logs, graph

$t > 2.73045$ (accept $t = 2.73045$) (2.73982 from -0.105) ***A1***

28 years ***A2*** ***N2***

METHOD 2

valid approach which gives both crossover values accurate to at least 2 sf ***A2***

eg $\frac{P_{2.7}}{P_0} = 0.75241\ldots,\ \frac{P_{2.8}}{P_0} = 0.74452\ldots$

$t = 2.8$ ***(A1)***

28 years ***A2*** ***N2***

[5 marks]

Events A and B are independent with $\mathrm{P}(A \cap B) = 0.2$ and $\mathrm{P}(A' \cap B) = 0.6$.

12a. Find $\mathrm{P}(B)$. *[2 marks]*

Markscheme

valid interpretation (may be seen on a Venn diagram) ***(M1)***

eg $\mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B),\ 0.2 + 0.6$

$\mathrm{P}(B) = 0.8$ ***A1*** ***N2***

[2 marks]

12b. Find $\mathrm{P}(A \cup B)$. *[4 marks]*

Markscheme

valid attempt to find $\mathrm{P}(A)$ ***(M1)***

eg $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B),\ 0.8 \times A = 0.2$

correct working for $\mathrm{P}(A)$ ***(A1)***

eg $0.25,\ \frac{0.2}{0.8}$

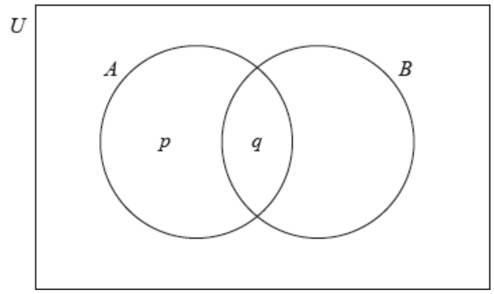
correct working for $\mathrm{P}(A \cup B)$ ***(A1)***

eg $0.25 + 0.8 - 0.2,\ 0.6 + 0.2 + 0.05$

$\mathrm{P}(A \cup B) = 0.85$ ***A1*** ***N3***

[4 marks]

The following Venn diagram shows the events A and B, where
$\mathrm{P}(A) = 0.4,\ \mathrm{P}(A \cup B) = 0.8$ and
$\mathrm{P}(A \cap B) = 0.1$. The values p and q are probabilities.



13a. (i) Write down the value of q. *[3 marks]*

(ii) Find the value of p.

Markscheme

(i)

$q = 0.1$ ***A1 N1***

(ii) appropriate approach ***(M1)***

eg $\mathrm{P}(A) - q,\ 0.4 - 0.1$

$p = 0.3$ ***A1 N2***

[3 marks]

13b. Find $\mathrm{P}(B)$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B),\ \mathrm{P}(A \cap B) + \mathrm{P}(B \cap A')$

correct values ***(A1)***

eg $0.8 = 0.4 + \mathrm{P}(B) - 0.1,\ 0.1 + 0.4$

$\mathrm{P}(B) = 0.5$ ***A1 N2***

[3 marks]

Let
C and
D be independent events, with $\mathrm{P}(C) = 2k$ and $\mathrm{P}(D) = 3k^2$, where $0 < k < 0.5$.

14a. Write down an expression for $\mathrm{P}(C \cap D)$ in terms of k. *[2 marks]*

Markscheme

$\mathrm{P}(C \cap D) = 2k \times 3k^2$ ***(A1)***

$\mathrm{P}(C \cap D) = 6k^3$ ***A1 N2***

[2 marks]

14b. Find $\mathrm{P}(C'|D)$. *[3 marks]*

Markscheme

METHOD 1

finding **their** $\mathrm{P}(C' \cap D)$ (seen anywhere) ***(A1)***

eg $0.4 \times 0.27, 0.27 - 0.162, 0.108$

correct substitution into conditional probability formula ***(A1)***

eg $\mathrm{P}(C'|D) = \frac{\mathrm{P}(C' \cap D)}{0.27}, \frac{(1-2k)(3k^2)}{3k^2}$

$\mathrm{P}(C'|D) = 0.4$ ***A1 N2***

METHOD 2

recognizing $\mathrm{P}(C'|D) = \mathrm{P}(C')$ ***A1***

finding **their** $\mathrm{P}(C') = 1 - \mathrm{P}(C)$ (only if first line seen) ***(A1)***

eg $1 - 2k,\ 1 - 0.6$

$\mathrm{P}(C'|D) = 0.4$ ***A1 N2***

[3 marks]

Total [7 marks]

Let
A and
B be independent events, where
$\mathrm{P}(A) = 0.3$ and
$\mathrm{P}(B) = 0.6$.

15a. Find *[2 marks]*
$\mathrm{P}(A \cap B)$.

Markscheme

correct substitution ***(A1)***

eg

0.3×0.6

$\mathrm{P}(A \cap B) = 0.18$ ***A1 N2***

[2 marks]

15b. Find *[2 marks]*
$\mathrm{P}(A \cup B)$.

Markscheme

correct substitution ***(A1)***

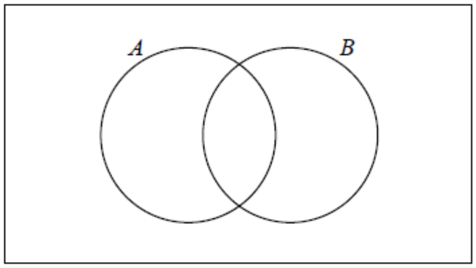
eg

$\mathrm{P}(A \cup B) = 0.3 + 0.6 - 0.18$

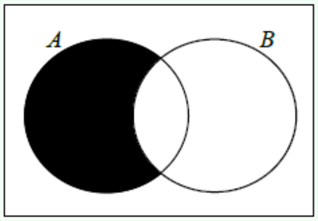
$\mathrm{P}(A \cup B) = 0.72$ ***A1 N2***

[2 marks]

15c. On the following Venn diagram, shade the region that represents $A \cap B'$. *[1 mark]*



Markscheme



A1 N1

15d. Find $\mathrm{P}(A \cap B')$. *[2 marks]*

Markscheme

appropriate approach ***(M1)***

eg

$0.3 - 0.18,\ \mathrm{P}(A) \times \mathrm{P}(B')$

$\mathrm{P}(A \cap B') = 0.12$ (may be seen in Venn diagram) ***A1 N2***

[2 marks]

Two events A and B are such that $\mathrm{P}(A) = 0.2$ and $\mathrm{P}(A \cup B) = 0.5$.

16a. Given that A and B are mutually exclusive, find $\mathrm{P}(B)$. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg

$0.5 = 0.2 + \mathrm{P}(B),\ \mathrm{P}(A \cap B) = 0$

$\mathrm{P}(B) = 0.3$ ***A1 N2***

[2 marks]

16b. Given that A and B are independent, find $\mathrm{P}(B)$. *[4 marks]*

Markscheme

Correct expression for $\mathrm{P}(A \cap B)$ (seen anywhere) ***A1***

eg $\mathrm{P}(A \cap B) = 0.2\mathrm{P}(B),\ 0.2x$

attempt to substitute into correct formula for $\mathrm{P}(A \cup B)$ ***(M1)***

eg $\mathrm{P}(A \cup B) = 0.2 + \mathrm{P}(B) - \mathrm{P}(A \cap B),\ \mathrm{P}(A \cup B) = 0.2 + x - 0.2x$

correct working ***(A1)***

eg $0.5 = 0.2 + \mathrm{P}(B) - 0.2\mathrm{P}(B),\ 0.8x = 0.3$

$\mathrm{P}(B) = \frac{3}{8}\ (= 0.375,\ \text{exact})$ ***A1*** ***N3***

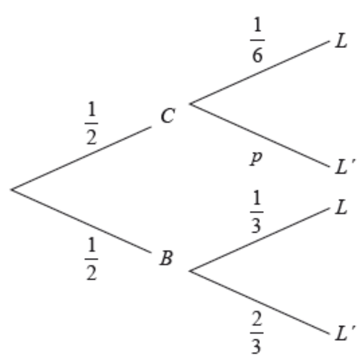
[4 marks]

Adam travels to school by car (C) or by bicycle (B). On any particular day he is equally likely to travel by car or by bicycle.

The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car.

The probability of being late for school is $\frac{1}{3}$ if he travels by bicycle.

This information is represented by the following tree diagram.



17a. Find the value of p. *[2 marks]*

Markscheme

correct working ***(A1)***

eg $1 - \frac{1}{6}$

$p = \frac{5}{6}$ ***A1*** ***N2***

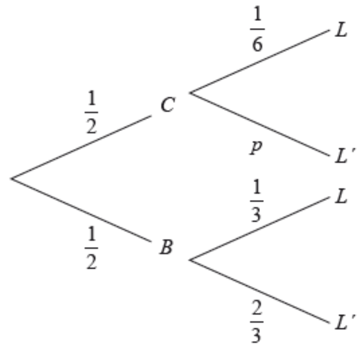
[2 marks]

Adam travels to school by car (C) or by bicycle (B). On any particular day he is equally likely to travel by car or by bicycle.

The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car.

The probability of being late for school is $\frac{1}{3}$ if he travels by bicycle.

This information is represented by the following tree diagram.



17b. Find the probability that Adam will travel by car and be late for school. *[2 marks]*

Markscheme

multiplying along correct branches ***(A1)***

eg $\frac{1}{2} \times \frac{1}{6}$

$\mathrm{P}(C \cap L) = \frac{1}{12}$ ***A1 N2***

[2 marks]

Adam travels to school by car (C) or by bicycle (B). On any particular day he is equally likely to travel by car or by bicycle.

The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car.

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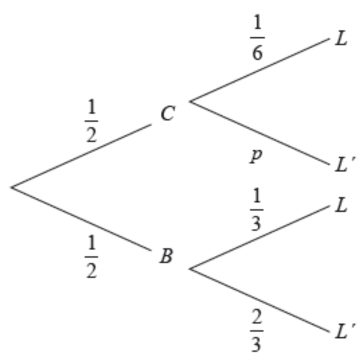


17c. Find the probability that Adam will be late for school. *[4 marks]*

Markscheme

multiplying along the other branch ***(M1)***

eg $\frac{1}{2} \times \frac{1}{3}$

adding probabilities of their 2 mutually exclusive paths ***(M1)***

eg $\frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{3}$

correct working ***(A1)***

eg $\frac{1}{12} + \frac{1}{6}$

$\mathrm{P}(L) = \frac{3}{12}$ $\left(= \frac{1}{4}\right)$ ***A1 N3***

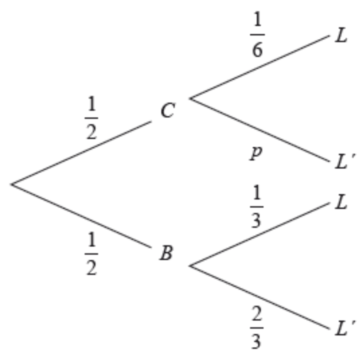
[4 marks]

Adam travels to school by car (C) or by bicycle (B). On any particular day he is equally likely to travel by car or by bicycle.

The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car.

The probability of being late for school is $\frac{1}{3}$ if he travels by bicycle.

This information is represented by the following tree diagram.



17d. Given that Adam is late for school, find the probability that he travelled by car. *[3 marks]*

Markscheme

recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P}(C|L)$

correct substitution of **their** values into formula ***(A1)***

eg $\frac{\frac{1}{12}}{\frac{3}{12}}$

$\mathrm{P}(C|L) = \frac{1}{3}$ ***A1 N2***

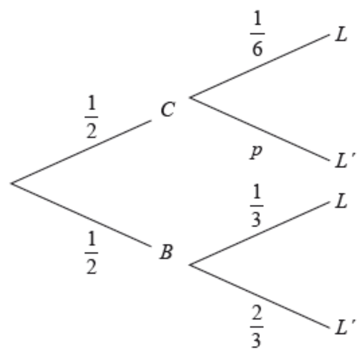
[3 marks]

Adam travels to school by car (C) or by bicycle (B). On any particular day he is equally likely to travel by car or by bicycle.

The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car.

The probability of being late for school is $\frac{1}{3}$ if he travels by bicycle.

This information is represented by the following tree diagram.



17e. Adam will go to school three times next week. *[4 marks]*

Find the probability that Adam will be late exactly once.

Markscheme

valid approach ***(M1)***

eg $X \sim B\left(3, \frac{1}{4}\right)$, $\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2$, $\binom{3}{1}$, three ways it could happen

correct substitution ***(A1)***

eg $\binom{3}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2$, $\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4}+\frac{3}{4}\times\frac{1}{4}\times\frac{3}{4}+\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}$

correct working ***(A1)***

eg $3\left(\frac{1}{4}\right)\left(\frac{9}{16}\right)$, $\frac{9}{64}+\frac{9}{64}+\frac{9}{64}$

$\frac{27}{64}$ ***A1 N2***

[4 marks]

Total [15 marks]

A running club organizes a race to select girls to represent the club in a competition.

The times taken by the group of girls to complete the race are shown in the table below.

Time t minutes	$10 \le t < 12$	$12 \le t < 14$	$14 \le t < 20$	$20 \le t < 26$	$26 \le t < 28$	$28 \le t < 30$
Frequency	50	20	p	40	20	20
Cumulative Frequency	50	70	120	q	180	200

18a. Find the value of p and of q. *[4 marks]*

Markscheme

attempt to find
p ***(M1)***

eg
$120 - 70$,
$50 + 20 + x = 120$

$p = 50$ ***A1*** ***N2***

attempt to find
q ***(M1)***

eg
$180 - 20$,
$200 - 20 - 20$

$q = 160$ ***A1*** ***N2***

[4 marks]

18b. A girl is chosen at random. *[3 marks]*

(i) Find the probability that the time she takes is less than
14 minutes.

(ii) Find the probability that the time she takes is at least
26 minutes.

Markscheme

(i)
$\frac{70}{200}$
$\left(= \frac{7}{20}\right)$ ***A1*** ***N1***

(ii) valid approach ***(M1)***

eg
$20 + 20$,
$200 - 160$

$\frac{40}{200}$
$\left(= \frac{1}{5}\right)$ ***A1*** ***N2***

[3 marks]

18c. A girl is selected for the competition if she takes less than
x minutes to complete the race. *[4 marks]*

Given that
40% of the girls are not selected,

(i) find the number of girls who are not selected;

(ii) find
x .

Markscheme

(i) attempt to find number of girls ***(M1)***

eg
$0.4,$
$\frac{40}{100} \times 200$

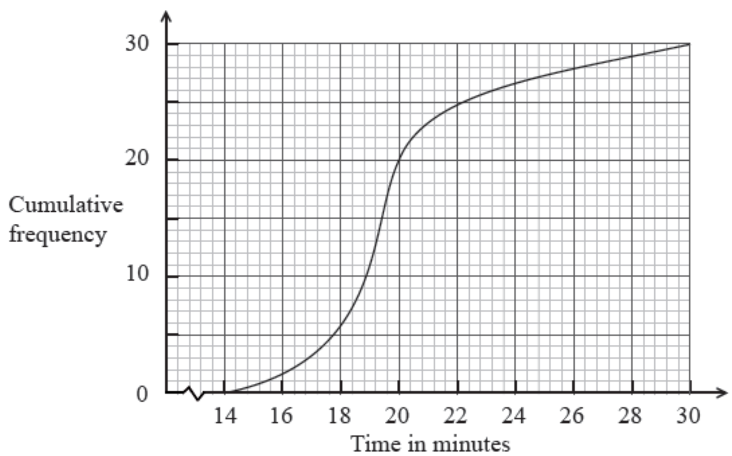
80 are not selected ***A1 N2***

(ii)
120 are selected ***(A1)***

$x = 20$ ***A1 N2***

[4 marks]

18d. Girls who are not selected, but took less than 25 minutes to complete the race, are allowed another chance to be selected. The new times taken by these girls are shown in the cumulative frequency diagram below. *[4 marks]*



(i) Write down the number of girls who were allowed another chance.

(ii) Find the percentage of the whole group who were selected.

Markscheme

(i)
30 given second chance *A1* *N1*

(ii)
20 took less than
20 minutes *(A1)*

attempt to find **their** selected total (may be seen in
$\%$ calculation) *(M1)*

eg
$120 + 20$
$(= 140)$,
$120+$ **their** answer from (i)

70 (
$\%$) *A1* *N3*

[4 marks]