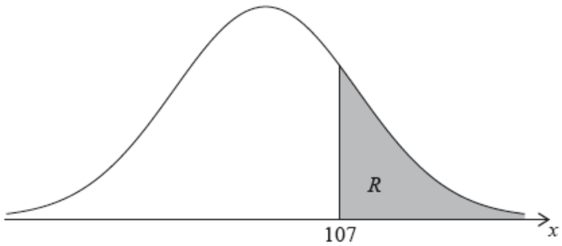
0319HW_Normal-distribution *[56 marks]*

The random variable X is normally distributed with a mean of 100. The following diagram shows the normal curve for X.



Let R be the shaded region under the curve, to the right of 107. The area of R is 0.24.

1a. Write down $\mathrm{P}(X > 107)$. *[1 mark]*

Markscheme

$\mathrm{P}(X > 107) = 0.24 \ \left(= \frac{6}{25}, 24\%\right)$ ***A1 N1***

[1 mark]

1b. Find $\mathrm{P}(100 < X < 107)$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}(X > 100) = 0.5, \mathrm{P}(X > 100) - \mathrm{P}(X > 107)$

correct working ***(A1)***

eg $0.5 - 0.24, 0.76 - 0.5$

$\mathrm{P}(100 < X < 107) = 0.26 \ \left(= \frac{13}{50}, 26\%\right)$ ***A1 N2***

[3 marks]

1c. Find $\mathrm{P}(93 < X < 107)$. *[2 marks]*

Markscheme

valid approach ***(M1)***

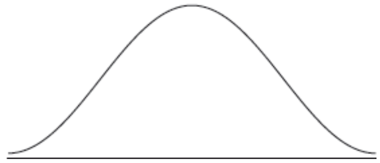
eg $2 \times 0.26, 1 - 2(0.24), \mathrm{P}(93 < X < 100) = \mathrm{P}(100 < X < 107)$

$\mathrm{P}(93 < X < 107) = 0.52 \ \left(= \frac{13}{25}, 52\%\right)$ ***A1 N2***

[2 marks]

A random variable X is distributed normally with a mean of 20 and standard deviation of 4.

2a. On the following diagram, shade the region representing $\mathrm{P}(X \leqslant 25)$. *[2 marks]*



Markscheme

A1A1 ***N2***

Note: Award ***A1*** for vertical line clearly to right of mean,

A1 for shading to left of their vertical line.

2b. Write down $\mathrm{P}(X \leqslant 25)$, correct to two decimal places. *[2 marks]*

Markscheme

$\mathrm{P}(X \leqslant 25) = 0.894350$ ***(A1)***

$\mathrm{P}(X \leqslant 25) = 0.89$ (must be 2 d.p.) ***A1*** ***N2***

[2 marks]

2c. Let $\mathrm{P}(X \leqslant c) = 0.7$. Write down the value of c. *[2 marks]*

Markscheme

$c = 22.0976$

$c = 22.1$ ***A2*** ***N2***

[2 marks]

The time taken for a student to complete a task is normally distributed with a mean of 20 minutes and a standard deviation of 1.25 minutes.

3a. A student is selected at random. Find the probability that the student completes the task in less than 21.8 minutes. *[2 marks]*

Markscheme

Note: There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with **their** working and check carefully for ***FT***.

attempt to standardize ***(M1)***

eg

$z = \frac{21.8-20}{1.25},\ 1.44$

$\mathrm{P}(T < 21.8) = 0.925$ ***A1*** ***N2***

[2 marks]

3b. The probability that a student takes between k and 21.8 minutes is 0.3. Find the value of k. *[5 marks]*

Markscheme

Note: There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with **their** working and check carefully for ***FT***.

attempt to subtract probabilities ***(M1)***

eg

$\mathrm{P}(T < 21.8) - \mathrm{P}(T < k) = 0.3,\ 0.925 - 0.3$

$\mathrm{P}(T < k) = 0.625$ ***A1***

EITHER

finding the z-value for 0.625 ***(A1)***

eg

$z = 0.3186$ (from tables),

$z = 0.3188$

attempt to set up equation using**their** z-value ***(M1)***

eg

$0.3186 = \frac{k-20}{1.25},\ -0.524 \times 1.25 = k - 20$

$k = 20.4$ ***A1*** ***N3***

OR

$k = 20.4$ ***A3*** ***N3***

[5 marks]

The weights, W, of newborn babies in Australia are normally distributed with a mean 3.41 kg and standard deviation 0.57 kg. A newborn baby has a low birth weight if it weighs less than w kg.

4a. Given that 5.3% of newborn babies have a low birth weight, find w. *[3 marks]*

Markscheme

valid approach *(M1)*

eg $z = -1.61643$, 

2.48863

$w = 2.49\ (\text{kg})$ ***A2*** ***N3***

[3 marks]

4b. A newborn baby has a low birth weight. *[3 marks]*

Find the probability that the baby weighs at least 2.15 kg.

Markscheme

correct value or expression (seen anywhere)

eg $0.053 - \text{P}(X \leqslant 2.15),\ 0.039465$ ***(A1)***

evidence of conditional probability ***(M1)***

eg $\frac{\text{P}(2.15 \leqslant X \leqslant w}{\text{P}(X \leqslant w)},\ \frac{0.039465}{0.053}$

0.744631

0.745 ***A1*** ***N2***

[3 marks]

The masses of watermelons grown on a farm are normally distributed with a mean of 10 kg.

The watermelons are classified as small, medium or large.

A watermelon is small if its mass is less than
4 kg. Five percent of the watermelons are classified as small.

5a. Find the standard deviation of the masses of the watermelons. *[4 marks]*

Markscheme

finding standardized value for 4 kg (seen anywhere) ***(A1)***

eg $z = -1.64485$

attempt to standardize ***(M1)***

eg $\sigma = \frac{x-\mu}{z},\ \frac{4-10}{\sigma}$

correct substitution ***(A1)***

eg $-1.64 = \frac{4-10}{\sigma},\ \frac{4-10}{-1.64}$

$\sigma = 3.64774$

$\sigma = 3.65$ ***A1*** ***N2***

[4 marks]

5b. The following table shows the percentages of small, medium and large watermelons grown on the farm. *[2 marks]*

small	medium	large
5%	57%	38%

A watermelon is large if its mass is greater than w kg.

Find the value of w.

Markscheme

valid approach ***(M1)***

eg $1-p,\ 0.62,\ \frac{w-10}{3.65}=0.305$

$w=11.1143$

$w=11.1$ ***A1 N2***

[2 marks]

5c. All the medium and large watermelons are delivered to a grocer. *[3 marks]*

The grocer selects a watermelon at random from **this** delivery. Find the probability that it is medium.

Markscheme

attempt to restrict melon population ***(M1)***

eg 95% are delivered, $\mathrm{P}(\text{medium}|\text{delivered}),\ 57+38$

correct probability for medium watermelons ***(A1)***

eg $\frac{0.57}{0.95}$

$\frac{57}{95},\ 0.6,\ 60\%$ ***A1 N3***

[3 marks]

The maximum temperature T, in degrees Celsius, in a park on six randomly selected days is shown in the following table. The table also shows the number of visitors, N, to the park on each of those six days.

Maximum temperature (T)	4	5	17	31	29	11
Number of visitors (N)	24	26	36	38	46	28

The relationship between the variables can be modelled by the regression equation $N=aT+b$.

6a. Find the value of a and of b. *[3 marks]*

Markscheme

evidence of set up ***(M1)***

eg correct value for a or b

0.667315, 22.2117

$a=0.667,\ b=22.2$ ***A1A1 N3***

[3 marks]

6b. Write down the value of r. *[1 mark]*

Markscheme

0.922958

$r = 0.923$ ***A1*** ***N1***

[1 marks]

6c. Use the regression equation to estimate the number of visitors on a day when the maximum temperature is 15 °C. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $0.667(15) + 22.2,\ N(15)$

32.2214 ***(A1)***

32 (visitors) (must be an integer) ***A1*** ***N2***

[3 marks]

The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

Distance, x km	11 500	7500	13 600	10 800	9500	12 200	10 400
Price, y dollars	15 000	21 500	12 000	16 000	19 000	14 500	17 000

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

7a. (i) Find the correlation coefficient. *[4 marks]*

(ii) Write down the value of a and of b.

Markscheme

Note: There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

(i) valid approach ***(M1)***

eg correct value for r (or for a or b seen in (ii))

-0.994347

$r = -0.994$ ***A1*** ***N2***

(ii)
$-1.58095,\ 33480.3$

$a = -1.58,\ b = 33500$ ***A1A1*** ***N2***

[4 marks]

On 1 January 2010, Lina buys a car which has travelled $11\,000$ km.

7b. Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest 100 dollars. *[3 marks]*

Markscheme

Note: There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

correct substitution into **their** regression equation

eg $-1.58095(11000)\ +33480.3$ ***(A1)***

$16\,089.85\ (16\,120 \text{ from } 3\text{sf})$ ***(A1)***

$\text{price} = 16\,100\ (\text{dollars})$ (must be rounded to the nearest 100 dollars) ***A1 N3***

[3 marks]

The price of a car decreases by 5% each year.

7c. Calculate the price of Lina's car after 6 years. *[4 marks]*

Markscheme

Note: There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

METHOD 1

valid approach ***(M1)***

eg $P \times (\text{rate})^t$

$\text{rate} = 0.95$ (may be seen in their expression) ***(A1)***

correct expression ***(A1)***

eg 16100×0.95^6

$11\,834.97$

$11\,800\ (\text{dollars})$ ***A1 N2***

METHOD 2

attempt to find all six terms ***(M1)***

eg

$(((16\,100 \times 0.95) \times 0.95) \ldots) \times 0.95$, table of values

5 correct values (accept values that round correctly to the nearest dollar)

$15\,295,\ 14\,530,\ 13\,804,\ 13\,114,\ 12\,458$ ***A2***

$11\,835$

$11\,800\ (\text{dollars})$ ***A1 N2***

[4 marks]

Lina will sell her car when its price reaches $10\,000$ dollars.

7d. Find the year when Lina sells her car. *[4 marks]*

Markscheme

Note: There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

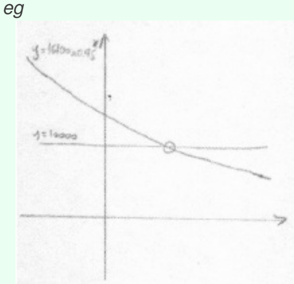
METHOD 1

correct equation ***(A1)***

eg $16\,100 \times 0.95^x = 10\,000$

valid attempt to solve ***(M1)***

eg



, using logs

9.28453 ***(A1)***

year 2019 ***A1*** ***N2***

METHOD 2

valid approach using table of values ***(M1)***

both crossover values (accept values that round correctly to the nearest dollar) ***A2***

eg $\mathrm{P} = 10\,147 \text{ (1 Jan 2019)}, \mathrm{P} = 9\,639.7 \text{ (1 Jan 2020)}$

year 2019 ***A1*** ***N2***

[4 marks]