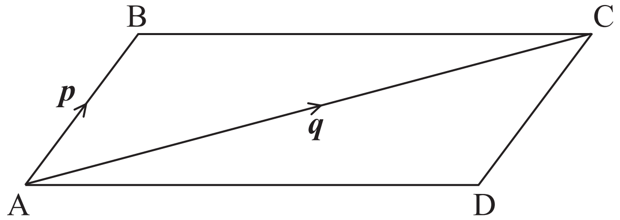
2. [Maximum mark: 7]

The following diagram shows the parallelogram ABCD.



Let $\vec{AB} = \boldsymbol{p}$ and $\vec{AC} = \boldsymbol{q}$. Find each of the following vectors in terms of $\boldsymbol{p}$ and/or $\boldsymbol{q}$.

(a) $\vec{CB}$ [2]

(b) $\vec{CD}$ [2]

(c) $\vec{DB}$ [3]

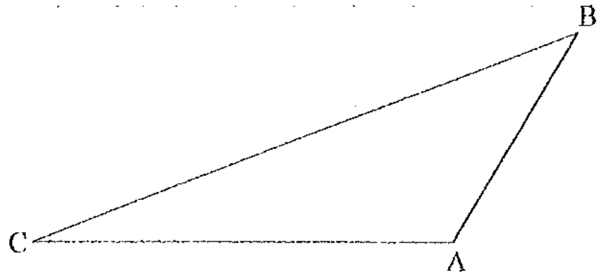
7. [Maximum mark: 7]

Let $\boldsymbol{u} = -3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{v} = m\boldsymbol{j} + n\boldsymbol{k}$, where $m, n \in \mathbb{R}$. Given that $\boldsymbol{v}$ is a unit vector perpendicular to $\boldsymbol{u}$, find the possible values of m and of n.

7. *[Maximum mark: 6]*

The following diagram shows triangle ABC.

diagram not to scale



Let $\vec{AB}\cdot\vec{AC}=-5\sqrt{3}$ and $\left|\vec{AB}\right|\left|\vec{AC}\right|=10$. Find the area of triangle ABC.

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

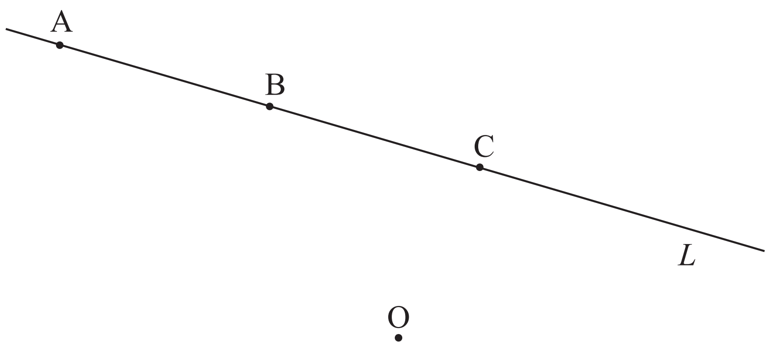
A line L passes through points $\mathrm{A}(-2, 4, 3)$ and $\mathrm{B}(-1, 3, 1)$.

(a) (i) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$.

(ii) Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [3]

(b) Find a vector equation for L. [2]

The following diagram shows the line L and the origin O. The point C also lies on L.



Point C has position vector $\begin{pmatrix} 0 \\ y \\ -1 \end{pmatrix}$.

(c) Show that $y = 2$. [4]

(d) (i) Find $\overrightarrow{\mathrm{OC}} \bullet \overrightarrow{\mathrm{AB}}$.

(ii) Hence, write down the size of the angle between OC and L. [3]

(e) Hence or otherwise, find the area of triangle OAB. [4]

Turn over

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A line L_1 passes through the points $\mathrm{A}(0, -3, 1)$ and $\mathrm{B}(-2, 5, 3)$.

(a) (i) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$.

(ii) Write down a vector equation for L_1. [3]

A line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$. The lines L_1 and L_2 intersect at a point C.

(b) Show that the coordinates of C are $(-1, 1, 2)$. [5]

(c) A point D lies on line L_2 so that $\left|\overrightarrow{\mathrm{CD}}\right| = \sqrt{18}$ and $\overrightarrow{\mathrm{CA}} \bullet \overrightarrow{\mathrm{CD}} = -9$. Find $\mathrm{A\hat{C}D}$. [7]

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9. [Maximum mark: 15]

Let P and Q have coordinates $(1, 0, 2)$ and $(-11, 8, m)$ respectively.

(a) Express $\overrightarrow{\mathrm{PQ}}$ in terms of m. [2]

Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be perpendicular vectors, where $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ n \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$.

(b) Find n. [4]

(c) Given that $\overrightarrow{\mathrm{PQ}}$ is parallel to $\boldsymbol{b}$,

(i) express $\overrightarrow{\mathrm{PQ}}$ in terms of $\boldsymbol{b}$;

(ii) hence find m. [5]

In part (d), distance is in metres, time is in seconds.

(d) A particle moves along a straight line through Q so that its position is given by $\boldsymbol{r} = \boldsymbol{c} + t\boldsymbol{a}$.

(i) Write down a possible vector $\boldsymbol{c}$.

(ii) Find the speed of the particle. [4]

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 17]*

Let L_x be a family of lines with equation given by $r = \begin{pmatrix} x \\ \frac{2}{x} \end{pmatrix} + t\begin{pmatrix} x^2 \\ -2 \end{pmatrix}$, where $x > 0$.

(a) Write down the equation of L_1. *[2]*

A line L_a crosses the y-axis at a point P.

(b) Show that P has coordinates $\left(0, \frac{4}{a}\right)$. *[6]*

The line L_a crosses the x-axis at $\mathrm{Q}(2a, 0)$. Let $d = \mathrm{PQ}^2$.

(c) Show that $d = 4a^2 + \frac{16}{a^2}$. *[2]*

(d) There is a minimum value for d. Find the value of a that gives this minimum value. *[7]*