0323Pretest_Descriptive-statistics *[80 marks]*

The time taken for a student to complete a task is normally distributed with a mean of
20 minutes and a standard deviation of
1.25 minutes.

1a. A student is selected at random. Find the probability that the student completes the task in less than 21.8 minutes. *[2 marks]*

Markscheme

Note: There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with **their** working and check carefully for ***FT***.

attempt to standardize ***(M1)***
eg
$z = \frac{21.8-20}{1.25},\ 1.44$
$\mathrm{P}(T < 21.8) = 0.925$ ***A1 N2***
[2 marks]

1b. The probability that a student takes between k and 21.8 minutes is 0.3. Find the value of k. *[5 marks]*

Markscheme

Note: There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with **their** working and check carefully for ***FT***.

attempt to subtract probabilities ***(M1)***
eg
$\mathrm{P}(T < 21.8) - \mathrm{P}(T < k) = 0.3,\ 0.925 - 0.3$
$\mathrm{P}(T < k) = 0.625$ ***A1***
EITHER
finding the z-value for 0.625 ***(A1)***
eg
$z = 0.3186$ (from tables),
$z = 0.3188$
attempt to set up equation using **their** z-value ***(M1)***
eg
$0.3186 = \frac{k-20}{1.25},\ -0.524 \times 1.25 = k - 20$
$k = 20.4$ ***A1 N3***
OR
$k = 20.4$ ***A3 N3***
[5 marks]

2. A random variable X is normally distributed with $\mu = 150$ and $\sigma = 10$. *[7 marks]*

Find the interquartile range of X.

Markscheme

recognizing one quartile probability (may be seen in a sketch) ***(M1)***

eg $\mathrm{P}(X < Q_3) = 0.75$, 0.25

finding standardized value for either quartile ***(A1)***

eg $z = 0.67448\ldots$, $z = -0.67448\ldots$

attempt to set up equation (must be with z-values) ***(M1)***

eg $0.67 = \frac{Q_3 - 150}{10}$, $-0.67448 = \frac{x-150}{10}$

one correct quartile

eg $Q_3 = 156.74\ldots$, $Q_1 = 143.25\ldots$

correct working ***(A1)***

eg other correct quartile, $Q_3 - \mu = 6.744\ldots$

valid approach for IQR (seen anywhere) ***(A1)***

eg $Q_3 - Q_1$, $2(Q_3 - \mu)$

IQR $= 13.5$ ***A1 N4***

[7 marks]

A competition consists of two independent events, shooting at 100 targets and running for one hour.

The number of targets a contestant hits is the S score. The S scores are normally distributed with mean 65 and standard deviation 10.

3a. A contestant is chosen at random. Find the probability that their S score is less than 50. *[2 marks]*

Markscheme

0.0668072

$\mathrm{P}(S < 50) = 0.0668$ (accept $\mathrm{P}(S \leqslant 49) = 0.0548$) ***A2 N2***

[2 marks]

A competition consists of two independent events, shooting at 100 targets and running for one hour.

The number of targets a contestant hits is the S score. The S scores are normally distributed with mean 65 and standard deviation 10.

The distance in km that a contestant runs in one hour is the R score. The R scores are normally distributed with mean 12 and standard deviation 2.5. The R score is independent of the S score.

Contestants are disqualified if their S score is less than 50 **and** their R score is less than x km.

3b. Given that 1% of the contestants are disqualified, find the value of x. *[4 marks]*

Markscheme

valid approach ***(M1)***

Eg $\mathrm{P}(S < 50) \times \mathrm{P}(R < x)$

correct equation (accept any variable) ***A1***

eg $\mathrm{P}(S < 50) \times \mathrm{P}(R < x) = 1\%,\ 0.0668072 \times p = 0.01,\ \mathrm{P}(R < x) = \frac{0.01}{0.0668}$

finding the value of $\mathrm{P}(R < x)$ ***(A1)***

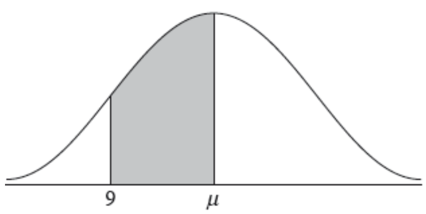
eg $\frac{0.01}{0.0668},\ 0.149684$

9.40553

$x = 9.41\ (\text{accept } x = 9.74 \text{ from } 0.0548)$ ***A1 N3***

[4 marks]

A random variable X is normally distributed with mean, μ. In the following diagram, the shaded region between 9 and μ represents 30% of the distribution.



4a. Find $\mathrm{P}(X < 9)$. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}(X < \mu) = 0.5,\ 0.5 - 0.3$

$\mathrm{P}(X < 9) = 0.2$ (exact) ***A1 N2***

[2 marks]

The standard deviation of X is 2.1.

4b. Find the value of μ. *[3 marks]*

Markscheme

$z = -0.841621$ (may be seen in equation) ***(A1)***

valid attempt to set up an equation with **their** z ***(M1)***

eg $-0.842 = \frac{\mu - X}{\sigma}, \ -0.842 = \frac{X-\mu}{\sigma}, \ z = \frac{9-\mu}{2.1}$

10.7674

$\mu = 10.8$ ***A1 N3***

[3 marks]

The random variable Y is normally distributed with mean λ and standard deviation 3.5. The events $X > 9$ and $Y > 9$ are independent, and $P((X > 9) \cap (Y > 9)) = 0.4$.

4c. Find λ. *[5 marks]*

Markscheme

$\mathrm{P}(X > 9) = 0.8$ (seen anywhere) ***(A1)***

valid approach ***(M1)***

eg $\mathrm{P}(A) \times \mathrm{P}(B)$

correct equation ***(A1)***

eg $0.8 \times \mathrm{P}(Y > 9) = 0.4$

$\mathrm{P}(Y > 9) = 0.5$ ***A1***

$\lambda = 9$ ***A1 N3***

[5 marks]

4d. Given that $Y > 9$, find $\mathrm{P}(Y < 13)$. *[5 marks]*

Markscheme

finding $\mathrm{P}(9 < Y < 13) = 0.373450$ (seen anywhere) ***(A2)***

recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A|B), \ \mathrm{P}(Y < 13|Y > 9)$

correct working ***(A1)***

eg $\frac{0.373}{0.5}$

0.746901

0.747 ***A1 N3***

[5 marks]

A company makes containers of yogurt. The volume of yogurt in the containers is normally distributed with a mean of 260 ml and standard deviation of 6 ml.

A container which contains less than 250 ml of yogurt is **underfilled**.

5a. A container is chosen at random. Find the probability that it is underfilled. *[2 marks]*

Markscheme

0.0477903

probability $= 0.0478$ ***A2*** ***N2***

[2 marks]

5b. The company decides that the probability of a container being underfilled should be reduced to 0.02. It decreases the standard deviation to σ and leaves the mean unchanged. *[4 marks]*

Find σ.

Markscheme

$\mathrm{P}(\text{volume} < 250) = 0.02$ ***(M1)***

$z = -2.05374$ (may be seen in equation) ***A1***

attempt to set up equation with z ***(M1)***

eg $\frac{\mu - 260}{\sigma} = z,\ 260 - 2.05(\sigma) = 250$

4.86914

$\sigma = 4.87\ (\text{ml})$ ***A1*** ***N3***

[4 marks]

5c. The company changes to the new standard deviation, σ, and leaves the mean unchanged. *[6 marks]*

A container is chosen at random for inspection. It passes inspection if its volume of yogurt is between 250 and 271 ml.

(i) Find the probability that it passes inspection.

(ii) Given that the container is **not** underfilled, find the probability that it passes inspection.

Markscheme

(i) 0.968062

$\mathrm{P}(250 < \mathrm{Vol} < 271) = 0.968$ ***A2*** ***N2***

(ii) recognizing conditional probability (seen anywhere, including in correct working) ***R1***

eg $\mathrm{P}(A|B),\ \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)},\ \mathrm{P}(A \cap B) = \mathrm{P}(A|B)\mathrm{P}(B)$

correct value or expression for P (not underfilled) ***(A1)***

eg $0.98, 1 - 0.02,\ 1 - \mathrm{P}(X < 250)$

probability $= \frac{0.968}{0.98}$ ***A1***

0.987818

probability $= 0.988$ ***A1*** ***N2***

[6 marks]

5d. A sample of 50 containers is chosen at random. Find the probability that 48 or more of the containers pass inspection. *[4 marks]*

Markscheme

METHOD 1

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***

eg $X \quad \mathrm{B}(50,\ 0.968),\ \text{binomial cdf},\ p = 0.968,\ r = 47$

$\mathrm{P}(X \le 47)$ = 0.214106\) ***(A1)***

evidence of using complement ***(M1)***

eg $1 - \mathrm{P}(X \le 47)$

0.785894

probability $= 0.786$ ***A1*** ***N3***

METHOD 2

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***

eg $X \quad \mathrm{B}(50,\ 0.968),\ \text{binomial cdf},\ p = 0.968,\ r = 47$

$\mathrm{P}(\text{not pass}) = 1 - \mathrm{P}(\text{pass}) = 0.0319378$ ***(A1)***

evidence of attempt to find P (2 or fewer fail) ***(M1)***

eg $0,\ 1,$ or 2 not pass, $\mathrm{B}(50,\ 2)$

0.785894

probability $= 0.786$ ***A1*** ***N3***

METHOD 3

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***

eg $X \quad \mathrm{B}(50,\ 0.968),\ \text{binomial cdf},\ p = 0.968,\ r = 47$

evidence of summing probabilities ***(M1)***

eg $\mathrm{P}(X = 48) + \mathrm{P}(X = 49) + \mathrm{P}(X = 50)$

correct working

eg $0.263088 + 0.325488 + 0.197317$ ***(A1)***

0.785894

probability $= 0.786$ ***A1*** ***N3***

[4 marks]

Total [16 marks]

Adam is a beekeeper who collected data about monthly honey production in his bee hives. The data for six of his hives is shown in the following table.

Number of bees (N)	190	220	250	285	305	320
Monthly honey production in grams (P)	900	1100	1200	1500	1700	1800

The relationship between the variables is modelled by the regression line with equation $P = aN + b$.

6a. Write down the value of a and of b. *[3 marks]*

Markscheme

evidence of setup ***(M1)***

eg correct value for a or b

$a = 6.96103,\ b = -454.805$

$a = 6.96,\ b = -455$ $(\text{accept } 6.96x - 455)$ ***A1A1*** ***N3***

[3 marks]

6b. Use this regression line to estimate the monthly honey production from a hive that has 270 bees. *[2 marks]*

Markscheme

substituting $N = 270$ into **their** equation ***(M1)***

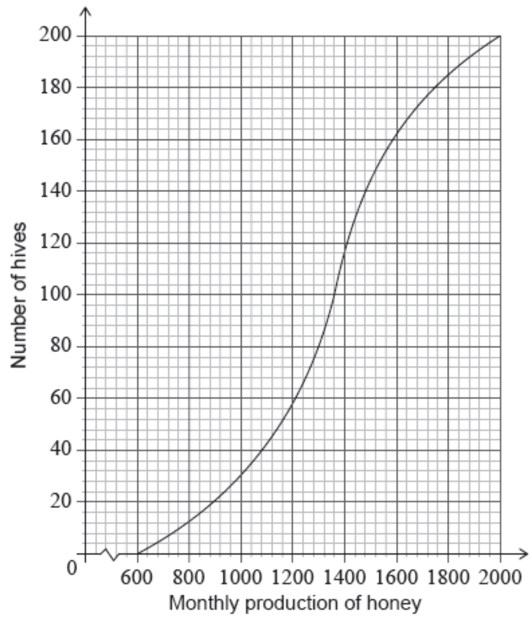
eg $6.96(270) - 455$

1424.67

$P = 1420$ (g) ***A1*** ***N2***

[2 marks]

Adam has 200 hives in total. He collects data on the monthly honey production of all the hives. This data is shown in the following cumulative frequency graph.



Adam's hives are labelled as low, regular or high production, as defined in the following table.

Type of hive	low	regular	high
Monthly honey production in grams (P)	$P \le 1080$	$1080 < P \le k$	$P > k$

6c. Write down the number of low production hives. *[1 mark]*

Markscheme

40 (hives) ***A1*** ***N1***

[1 mark]

Adam knows that 128 of his hives have a regular production.

6d. Find the value of k; *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $128 + 40$

168 hives have a production less than k ***(A1)***

$k = 1640$ ***A1*** ***N3***

[3 marks]

6e. Find the number of hives that have a high production. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $200 - 168$

32 (hives) ***A1*** ***N2***

[2 marks]

6f. Adam decides to increase the number of bees in each low production hive. Research suggests that there is a probability of 0.75 that a low production hive becomes a regular production hive. Calculate the probability that 30 low production hives become regular production hives. *[3 marks]*

Markscheme

recognize binomial distribution (seen anywhere) ***(M1)***

eg $X \sim \mathrm{B}(n,\ p),\ \binom{n}{r} p^r (1-p)^{n-r}$

correct values ***(A1)***

eg $n = 40$ (check ***FT***) and $p = 0.75$ and $r = 30,\ \binom{40}{30} 0.75^{30}(1-0.75)^{10}$

0.144364

0.144 ***A1*** ***N2***

[3 marks]

A jar contains 5 red discs, 10 blue discs and m green discs. A disc is selected at random and replaced. This process is performed four times.

7a. Write down the probability that the first disc selected is red. *[1 mark]*

Markscheme

$\mathrm{P(red)} = \frac{5}{15+m}$ ***A1*** ***N1***

[1 mark]

7b. Let X be the number of red discs selected. Find the smallest value of m for which $\mathrm{Var}(X) < 0.6$. *[5 marks]*

Markscheme

recognizing binomial distribution *(M1)*

eg $X \sim B(n,\, p)$

correct value for the complement of **their** p (seen anywhere) ***A1***

eg $1 - \frac{5}{15+m},\ \frac{10+m}{15+m}$

correct substitution into $\mathrm{Var}(X) = np(1-p)$ ***(A1)***

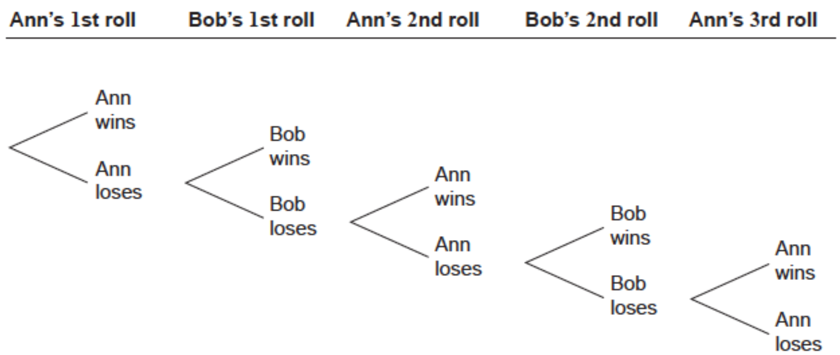
eg $4\left(\frac{5}{15+m}\right)\left(\frac{10+m}{15+m}\right),\ \frac{20(10+m)}{(15+m)^2} < 0.6$

$m > 12.2075$ ***(A1)***

$m = 13$ ***A1*** ***N3***

[5 marks]

Ann and Bob play a game where they each have an eight-sided die. Ann's die has three green faces and five red faces; Bob's die has four green faces and four red faces. They take turns rolling their own die and note what colour faces up. The first player to roll green wins. Ann rolls first. Part of a tree diagram of the game is shown below.



8a. Find the probability that Ann wins on her first roll. *[2 marks]*

Markscheme

recognizing Ann rolls green ***(M1)***

eg $\mathrm{P(G)}$

$\frac{3}{8}$ ***A1*** ***N2***

[2 marks]

8b. Find the probability that Ann wins the game. *[7 marks]*

Markscheme

recognize the probability is an infinite sum ***(M1)***

eg Ann wins on her 1^{st} roll or 2^{nd} roll or 3^{rd} roll…, S_∞

recognizing GP ***(M1)***

$u_1 = \frac{3}{8}$ (seen anywhere) ***A1***

$r = \frac{20}{64}$ (seen anywhere) ***A1***

correct substitution into infinite sum of GP ***A1***

eg $\frac{\frac{3}{8}}{1-\frac{5}{16}}, \frac{3}{8}\left(\frac{1}{1-\left(\frac{5}{8}\times\frac{4}{8}\right)}\right), \frac{1}{1-\frac{5}{16}}$

correct working ***(A1)***

eg $\frac{\frac{3}{8}}{\frac{11}{16}}, \frac{3}{8}\times\frac{16}{11}$

$\mathrm{P}\left(\text{Ann wins}\right) = \frac{48}{88}$ $\left(=\frac{6}{11}\right)$ ***A1 N1***

[7 marks]

Total [15 marks]