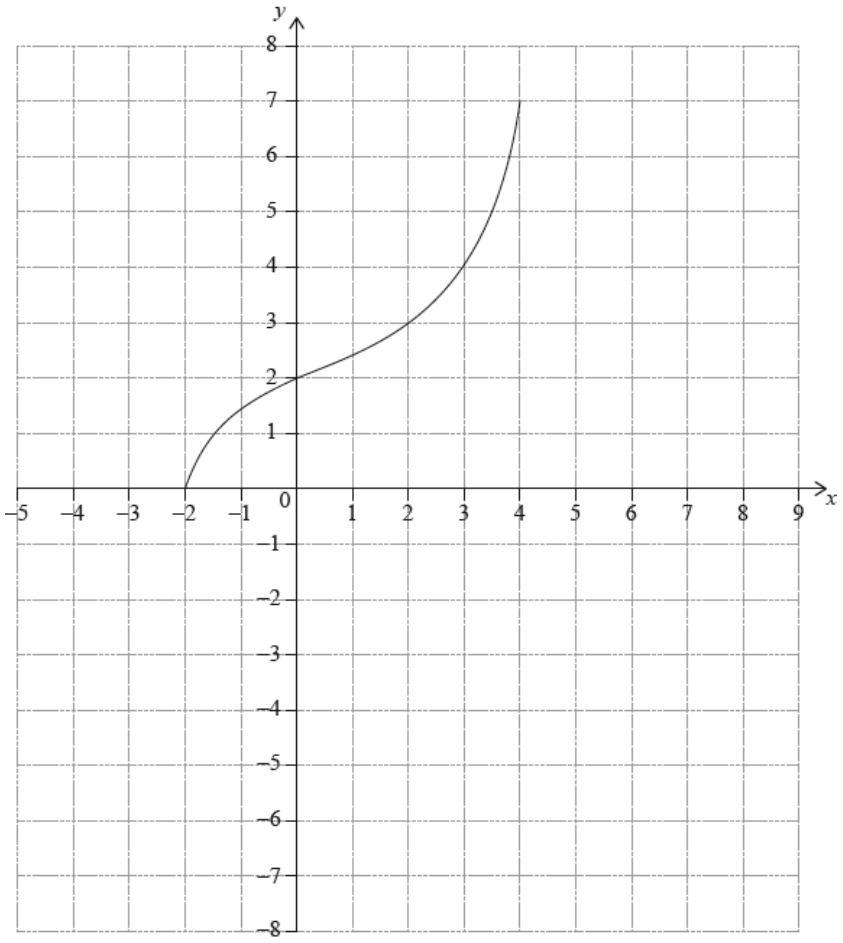
BECA / Huson / 11.1 IB Math SL Name:
10 May 2018

Homework: Function operations (these are calculator-intensive problems)

1a. The following diagram shows the graph of a function f, with domain $-2 \leqslant x \leqslant 4$.



The points $(-2,\ 0)$ and $(4,\ 7)$ lie on the graph of f.

Write down the range of f. *[1 mark]*

1b. Write down $f(2)$;

[1 mark]

1c. Write down $f^{-1}(2)$.

[1 mark]

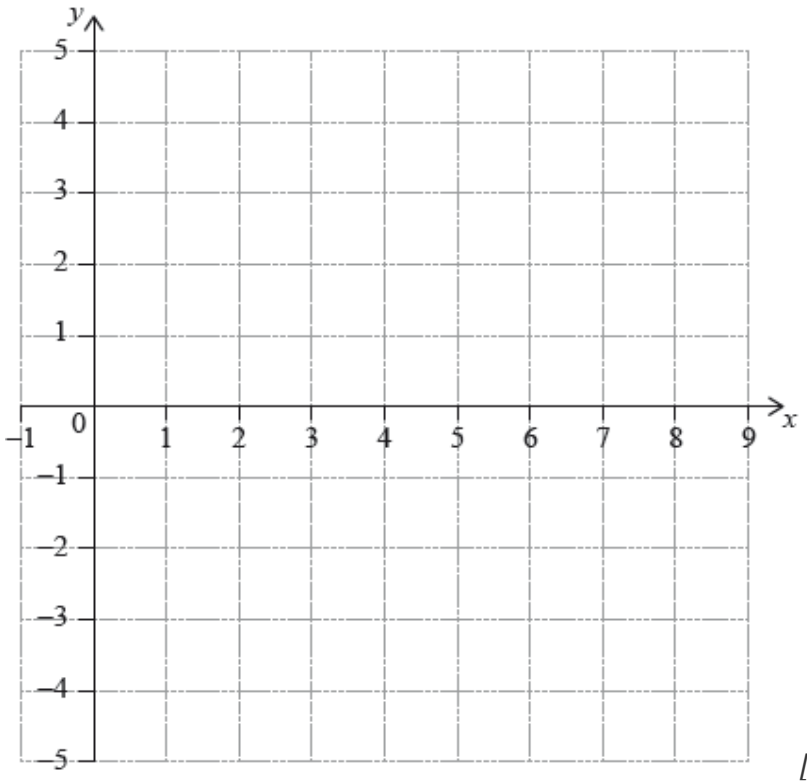
1d. On the grid, sketch the graph of f^{-1}. *[3 marks]*

2a. Let $f(x) = \frac{6x^2 - 4}{e^x}$, for $0 \leqslant x \leqslant 7$.

Find the x-intercept of the graph of f. *[2 marks]*

2b. The graph of f has a maximum at the point A. Write down the coordinates of A. *[2 marks]*

2c. On the following grid, sketch the graph of f.



[3 marks]

3. Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \mathbb{R}$. The graph of f passes through the point $(p,\ 4)$, where $p > 0$.

Find the value of p. *[2 marks]*

4a. Let $f(x) = 5x$ and $g(x) = x^2 + 1$, for $x \in \mathbb{R}$.

Find $f^{-1}(x)$. *[2 marks]*

4b. Find $(f \circ g)(7)$. *[3 marks]*

5a. Consider the graph of $f(x) = \frac{e^x}{5x-10} + 3$, for $x \neq 2$.

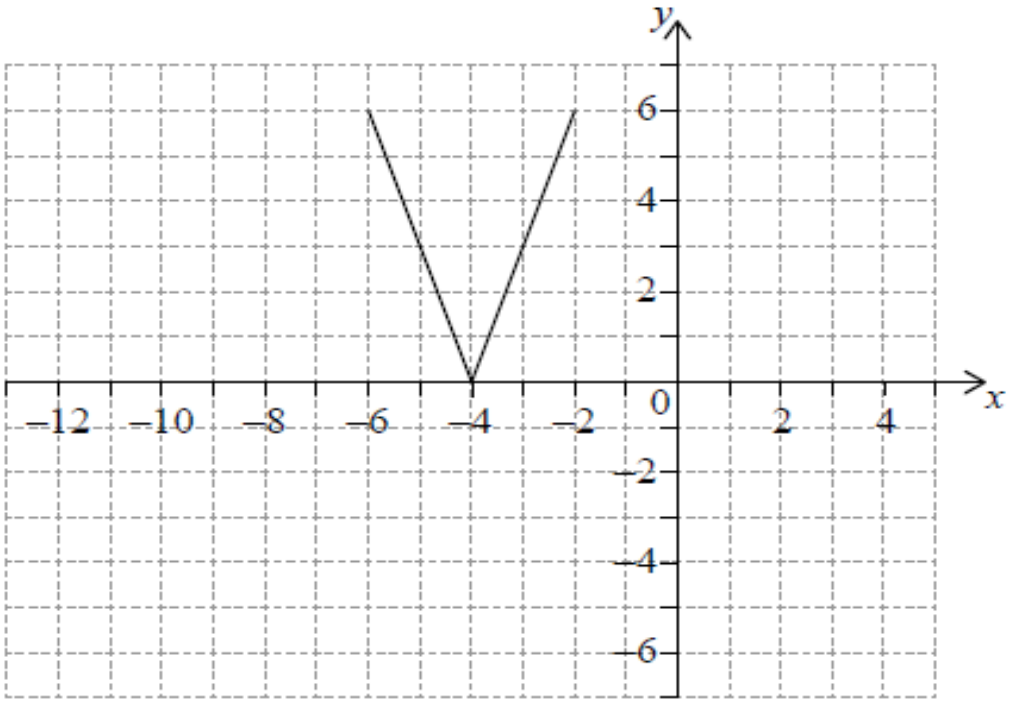
Find the y-intercept. *[2 marks]*

5b. Find the equation of the vertical asymptote. *[2 marks]*

5c. Find the minimum value of $f(x)$ for $x > 2$. *[2 marks]*

6a. The following diagram shows the graph of a function $y = f(x)$, for $-6 \leqslant x \leqslant -2$.

The points $(-6,\ 6)$ and $(-2,\ 6)$ lie on the graph of f. There is a minimum point at $(-4,\ 0)$.



Write down the range of f. *[2 marks]*

6b. Let $g(x) = f(x - 5)$.

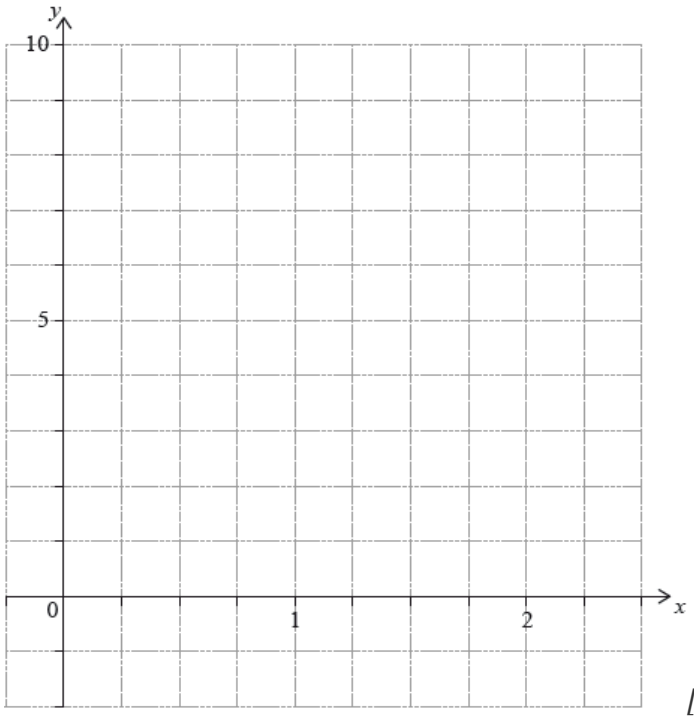
On the grid above, sketch the graph of g. *[2 marks]*

6c. Write down the domain of g. *[2 marks]*

7a. Let $f(x) = x^2 - 1$ and $g(x) = x^2 - 2$, for $x \in \mathbb{R}$.

Show that $(f \circ g)(x) = x^4 - 4x^2 + 3$. *[2 marks]*

7b. On the following grid, sketch the graph of $(f \circ g)(x)$, for $0 \leqslant x \leqslant 2.25$.



[3 marks]

7c. The equation $(f \circ g)(x) = k$ has exactly two solutions, for $0 \leqslant x \leqslant 2.25$. Find the possible values of *k*. *[3 marks]*

8a. Let $f(x) = x^2 - 4x + 5$.

Find the equation of the axis of symmetry of the graph of f. *[2 marks]*

8b. The function can also be expressed in the form $f(x) = (x - h)^2 + k$.

(i) Write down the value of h.

(ii) Find the value of k. *[4 marks]*

9a. Let $f(x) = x^2 + 2x + 1$ and $g(x) = x - 5$, for $x \in \mathbb{R}$.

Find $f(8)$. *[2 marks]*

9b. Find $(g \circ f)(x)$. *[2 marks]*

9c. Solve $(g \circ f)(x) = 0$. *[3 marks]*