

Unit 5 Pretest: Integral Calculus

Using the calculator for definite integrals

1. 16M.2.sl.TZ1.2 [6 marks]

Let $f(x) = x^2$ and $g(x) = 3 \ln(x + 1)$, for $x > -1$.

(a) Solve $f(x) = g(x)$ [3]

(b) Find the area of the region enclosed by the graphs of f and g . [3]

2. 16N.2.sl.TZ0.4 [6 marks]

Let $f(x) = xe^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

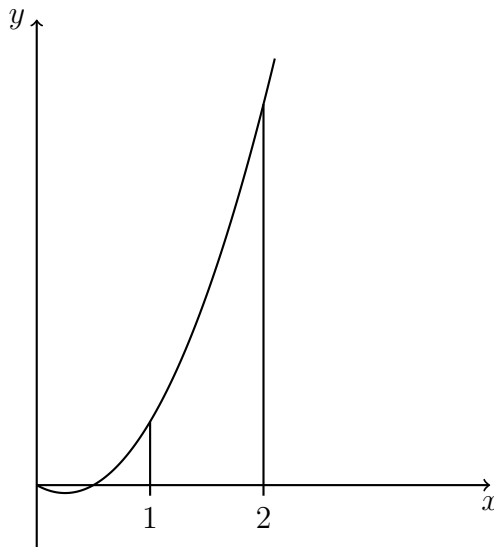
(a) Find the values of p and q . [3]

(b) Hence, find the area of the region enclosed by the graphs of f and g . [3]

No Calculator section

3. 18M.1.sl.TZ2.2 [6 marks]

Let $f(x) = 6x^2 - 3x$. The graph of f is shown in the following diagram.



(a) Find $f'(x)$. [2]

(b) Find the area of the region enclosed by the graph of f , the x -axis and the lines $x = 1$ and $x = 2$. [4]

4. 15N.1.sl.TZ0.3 [6 marks]

Let $f'(x) = 6x^2 - 5$. and $f(2) = -3$, find $f(x)$. [6]

5. 13N.1.sl.TZ0.4

[6 marks]

Consider a function $f(x)$ such that $\int_1^6 f(x) dx = 8$.

(a) Find $\int_1^6 2f(x) dx$.

[2]

(b) Find $\int_1^6 (f(x) + 2) dx$.

[4]

6. 18M.2.sl.TZ1.4

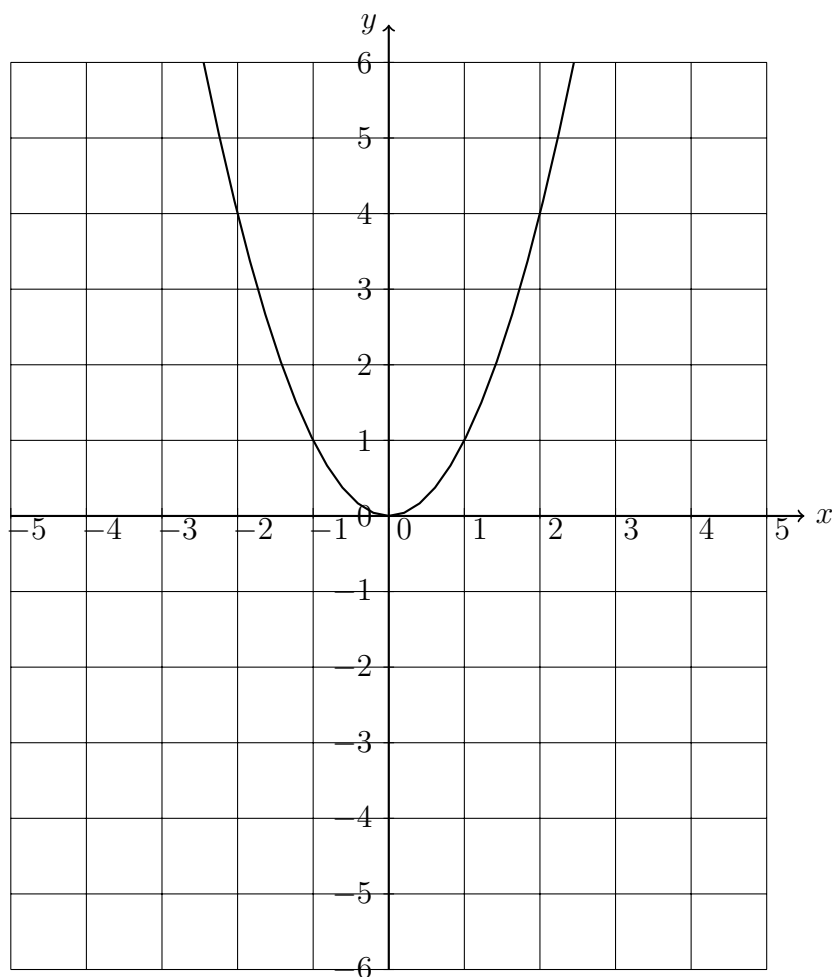
[7 marks]

Let $g(x) = -(x - 1)^2 + 5$.

(a) Write down the coordinates of the vertex of the graph of g .

[1]

(b) Let $f(x) = x^2$. The following diagram shows part of the graph of f .



The graph of g intersects the graph of f at $x = -1$ and $x = 2$.

On the grid above, sketch the graph of g for $-2 \leq x \leq 4$.

[3]

(c) Find the area of the region enclosed by the graphs of f and g .

[3]

7. 14M.1.sl.TZ2.5

[6 marks]

The graph of a function h passes through the point $(\frac{\pi}{12}, 5)$.

Given that $h'(x) = 4 \cos 2x$, find $h(x)$.

[6]

8. 17M.1.sl.TZ2.5 [6 marks]

Let $f'(x) = \frac{3x^2}{(x^3 + 1)^5}$. Given that $f(0) = 1$, find $f(x)$. [6]

9. 17M.1.sl.TZ1.5a [7 marks]

(a) Find $\int xe^{x^2-1} dx$. [4]

(b) Find $f(x)$, given that $f'(x) = xe^{x^2-1}$ and $f(-1) = 3$ [3]

Spicy: Extended response

10. 18M.1.sl.TZ1.8 [13 marks]

A function $f(x)$ has derivative $f'(x) = 3x^2 + 18x$. The graph of f has an x -intercept at $x = -1$.

(a) Find $f(x)$. [6]

(b) The graph of f has a point of inflexion at $x = p$. Find p . [4]

(c) Find the values of x for which the graph of f is concave-down. [3]

11. 16M.1.sl.TZ1.10 [15 marks]

Let $f(x) = \sqrt{4x + 5}$, for $x \geq -1.25$.

(a) Find $f'(1)$. [4]

(b) Consider another function g . Let R be a point on the graph of g . The x -coordinate of R is 1. The equation of the tangent to the graph at R is $y = 3x + 6$.

Write down $g'(1)$. [2]

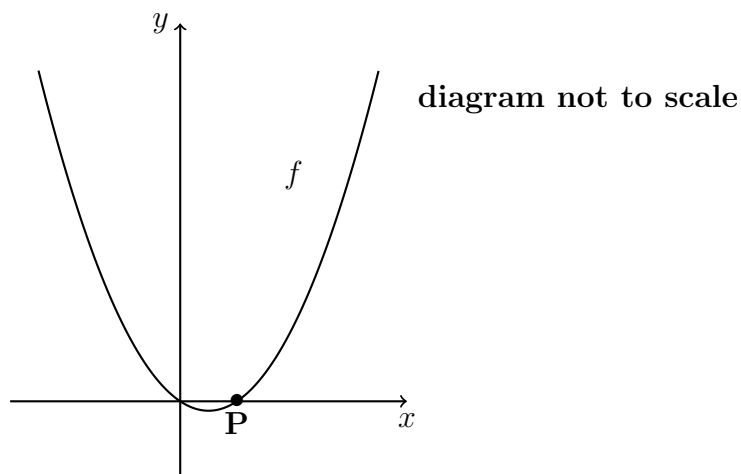
(c) Find $g(1)$. [2]

(d) Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent to the graph of h at the point where $x = 1$. [7]

12. (#19) 17N.1.sl.TZ0.8

[16 marks]

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f .

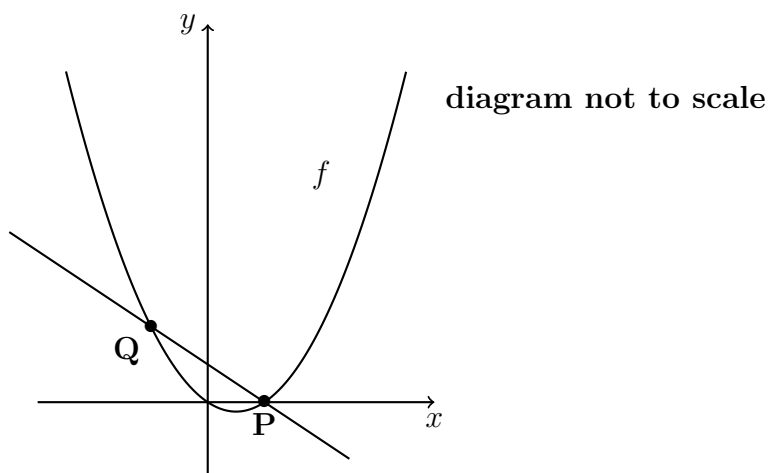


The graph of f crosses the x -axis at the origin and at the point $P(1, 0)$.

(a) Show that $f'(1) = 1$. [3]

(b) The line L is the normal to the graph of f at P .
Find the equation of L in the form $y = ax + b$. [3]

(c) The line L intersects the graph of f at another point Q , as shown in the following diagram.



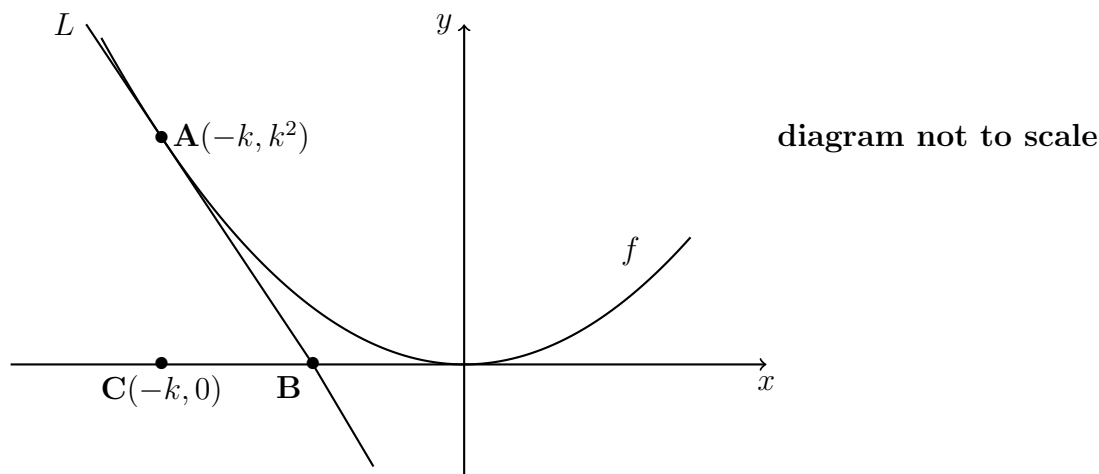
Find the x -coordinate of Q . [4]

(d) Find the area of the region enclosed by the graph of f and the line L . [6]

13. (#23) 17M.1.sl.TZ2.10

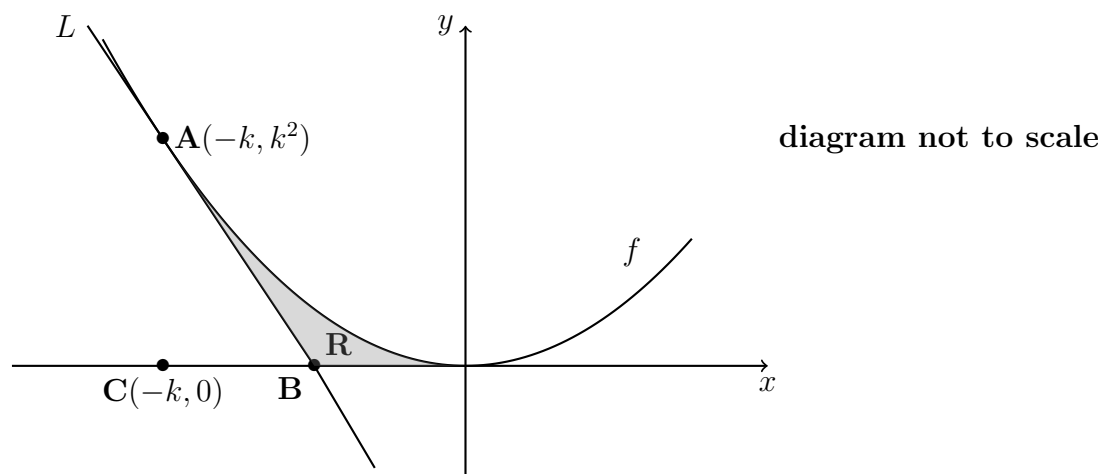
[17 marks]

Let $f(x) = x^2$. The following diagram shows part of the graph of f .



The line L is the tangent to the graph of f at the point $A(-k, k^2)$, and intersects the x -axis at point B . The point C is $(-k, 0)$.

- Write down $f'(x)$. [1]
- Find the gradient of L . [2]
- Show that the x -coordinate of B is $-\frac{k}{2}$. [5]
- Find the area of triangle ABC , giving your answer in terms of k . [2]
- The region R is enclosed by L , the graph of f , and the x -axis. This is shown in the following diagram.



Given that the area of triangle ABC is p times the area of R , find the value of p . [7]