SECTION A

1. (a) $h = 2, k = 3$ ***A1A1*** ***N2***

[2 marks]

(b) attempt to substitute $(1, 7)$ in any order into **their** $f(x)$ ***(M1)***

eg $7 = a(1-2)^2 + 3$, $7 = a(1-3)^2 + 2$, $1 = a(7-2)^2 + 3$

correct equation ***(A1)***

eg $7 = a + 3$

$a = 4$ ***A1*** ***N2***

[3 marks]

Total [5 marks]

2. (a) attempt to find d ***(M1)***

eg $\frac{16-10}{2}$, $10 - 2d = 16 - 4d$, $2d = 6$, $d = 6$

$d = 3$ ***A1*** ***N2***

[2 marks]

(b) correct approach ***(A1)***

eg $10 = u_1 + 2 \times 3, 10 - 3 - 3$

$u_1 = 4$ ***A1*** ***N2***

[2 marks]

(c) correct substitution into sum or term formula ***(A1)***

eg $\frac{20}{2}\left(2 \times 4 + 19 \times 3\right)$, $u_{20} = 4 + 19 \times 3$

correct simplification ***(A1)***

eg $8 + 57$, $4 + 61$

$S_{20} = 650$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

SECTION A

1. (a) **METHOD 1**

approach involving Pythagoras' theorem *(M1)*

eg $5^2 + x^2 = 13^2$, labelling correct sides on triangle

finding third side is 12 (may be seen on diagram) *A1*

$\cos A = \dfrac{12}{13}$ *AG* *N0*

METHOD 2

approach involving $\sin^2\theta + \cos^2\theta = 1$ *(M1)*

eg $\left(\dfrac{5}{13}\right)^2 + \cos^2\theta = 1$, $x^2 + \dfrac{25}{169} = 1$

correct working *A1*

eg $\cos^2\theta = \dfrac{144}{169}$

$\cos A = \dfrac{12}{13}$ *AG* *N0*

[2 marks]

(b) correct substitution into $\cos 2\theta$ *(A1)*

eg $1 - 2\left(\dfrac{5}{13}\right)^2$, $2\left(\dfrac{12}{13}\right)^2 - 1$, $\left(\dfrac{12}{13}\right)^2 - \left(\dfrac{5}{13}\right)^2$

correct working *(A1)*

eg $1 - \dfrac{50}{169}$, $\dfrac{288}{169} - 1$, $\dfrac{144}{169} - \dfrac{25}{169}$

$\cos 2A = \dfrac{119}{169}$ *A1* *N2*

[3 marks]

Total [5 marks]

3. (a) (i) $f(-3) = -1$ ***A1*** ***N1***

(ii) $f^{-1}(1) = 0$ (accept $y = 0$) ***A1*** ***N1***

[2 marks]

(b) domain of f^{-1} is range of f ***(R1)***

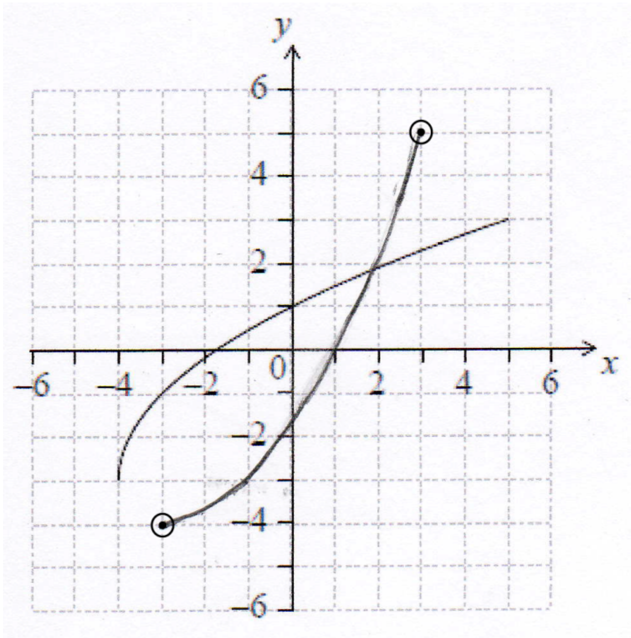
eg $\mathrm{R}f = \mathrm{D}f^{-1}$

correct answer ***A1*** ***N2***

eg $-3 \le x \le 3$, $x \in [-3, 3]$ (accept $-3 < x < 3$, $-3 \le y \le 3$)

[2 marks]

(c)



A1A1 ***N2***

Note:	Graph must be approximately correct reflection in $y = x$. **Only** if the shape is approximately correct, award the following: ***A1*** for x-intercept at 1, and ***A1*** for endpoints within circles.

[2 marks]

Total [6 marks]

3. (a) substituting for $\left(f(x)\right)^2$ (may be seen in integral) ***A1***

eg $\left(x^2\right)^2$, x^4

correct integration, $\int x^4\,\mathrm{d}x = \frac{1}{5}x^5$ ***(A1)***

substituting limits into **their integrated** function and subtracting (in any order) ***(M1)***

eg $\frac{2^5}{5} - \frac{1}{5}$, $\frac{1}{5}(1-4)$

$\int_1^2 \left(f(x)\right)^2 \mathrm{d}x = \frac{31}{5}$ $(=6.2)$ ***A1*** ***N2***

[4 marks]

(b) attempt to substitute limits or function into formula involving f^2 ***(M1)***

eg $\int_1^2 \left(f(x)\right)^2 \mathrm{d}x$, $\pi\int x^4 \mathrm{d}x$

$\frac{31}{5}\pi$ $(=6.2\pi)$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

4. (a) (i) $\log_3 27 = 3$ ***A1*** ***N1***

(ii) $\log_8 \frac{1}{8} = -1$ ***A1*** ***N1***

(iii) $\log_{16} 4 = \frac{1}{2}$ ***A1*** ***N1***

[3 marks]

(b) correct equation with **their** three values ***(A1)***

eg $\frac{3}{2} = \log_4 x$, $3 + (-1) - \frac{1}{2} = \log_4 x$

correct working involving powers ***(A1)***

eg $x = 4^{\frac{3}{2}}$, $4^{\frac{3}{2}} = 4^{\log_4 x}$

$x = 8$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

5. recognize need for intersection of Y and F ***(R1)***

eg $\mathrm{P}(Y \cap F)$, 0.3×0.4

valid approach to find $\mathrm{P}(Y \cap F)$ ***(M1)***

eg $\mathrm{P}(Y) + \mathrm{P}(F) - \mathrm{P}(Y \cup F)$, Venn diagram

correct working (may be seen in Venn diagram) ***(A1)***

eg $0.4 + 0.3 - 0.6$

$\mathrm{P}(Y \cap F) = 0.1$ ***A1***

recognize need for complement of $Y \cap F$ ***(M1)***

eg $1 - \mathrm{P}(Y \cap F)$, $1 - 0.1$

$\mathrm{P}\left((Y \cap F)'\right) = 0.9$ ***A1*** ***N3***

[6 marks]

6. correct integration (ignore absence of limits and "$+C$") ***(A1)***

eg $\dfrac{\sin(2x)}{2}$, $\int_{\pi}^{a} \cos 2x = \left[\dfrac{1}{2}\sin(2x)\right]_{\pi}^{a}$

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $\dfrac{1}{2}\sin(2a) - \dfrac{1}{2}\sin(2\pi)$, $\sin(2\pi) - \sin(2a)$

$\sin(2\pi) = 0$ ***(A1)***

setting **their** result from an integrated function equal to $\dfrac{1}{2}$ ***M1***

eg $\dfrac{1}{2}\sin 2a = \dfrac{1}{2}$, $\sin(2a) = 1$

recognizing $\sin^{-1} 1 = \dfrac{\pi}{2}$ ***(A1)***

eg $2a = \dfrac{\pi}{2}$, $a = \dfrac{\pi}{4}$

correct value ***(A1)***

eg $\dfrac{\pi}{2} + 2\pi, 2a = \dfrac{5\pi}{2}$, $a = \dfrac{\pi}{4} + \pi$

$a = \dfrac{5\pi}{4}$ ***A1*** ***N3***

[7 marks]

7. (a) $f'(x) = 3px^2 + 2px + q$ ***A2*** ***N2***

Note: Award ***A1*** if only 1 error.

[2 marks]

(b) evidence of discriminant (must be seen explicitly, not in quadratic formula) ***(M1)***

eg $b^2 - 4ac$

correct substitution into discriminant (may be seen in inequality) ***A1***

eg $(2p)^2 - 4 \times 3p \times q$, $4p^2 - 12pq$

$f'(x) \geq 0$ then f' has two equal roots or no roots ***(R1)***

recognizing discriminant less or equal than zero ***R1***

eg $\Delta \leq 0, 4p^2 - 12pq \leq 0$

correct working that clearly leads to the required answer ***A1***

eg $p^2 - 3pq \leq 0$, $4p^2 \leq 12pq$

$p^2 \leq 3pq$ ***AG*** ***N0***

[5 marks]

Total [7 marks]

SECTION B

8. (a) correct approach *A1*

eg $\begin{pmatrix}1\\1\\5\end{pmatrix}-\begin{pmatrix}2\\1\\4\end{pmatrix}$, $\text{AO}+\text{OB}$, $\boldsymbol{b}-\boldsymbol{a}$

$\overrightarrow{\text{AB}}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$ *AG* *N0*

[1 mark]

(b) (i) correct vector (or any multiple) *A1* *N1*

eg $\boldsymbol{d}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$

(ii) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix}2\\1\\4\end{pmatrix}$ or $\begin{pmatrix}1\\1\\5\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}-1\\0\\1\end{pmatrix}$ *A2* *N2*

eg $\boldsymbol{r}=\begin{pmatrix}1\\1\\5\end{pmatrix}+t\begin{pmatrix}-1\\0\\1\end{pmatrix}$, $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2-s\\1\\4+s\end{pmatrix}$

Note: Award ***A1*** for $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for $L_1=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[3 marks]

continued ...

Question 8 continued

(c) valid approach **(M1)**

eg $r_1 = r_2$, $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

one correct equation in one parameter **A1**
eg $2 - t = 4, 1 = 7 - s, 1 - t = 4$

attempt to solve **(M1)**
eg $2 - 4 = t, s = 7 - 1, t = 1 - 4$

one correct parameter **A1**
eg $t = -2, s = 6, t = -3$,

attempt to substitute **their** parameter into vector equation **(M1)**

eg $\begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + 6\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$\text{P}(4, 1, 2)$ (accept position vector) **A1** **N2**

[6 marks]

(d) (i) correct direction vector for L_2 **A1** **N1**

eg $\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$

(ii) correct scalar product and magnitudes for **their** direction vectors **(A1)(A1)(A1)**

scalar product $= 0 \times -1 + -1 \times 0 + 1 \times 1 \ (= 1)$

magnitudes $= \sqrt{0^2 + (-1)^2 + 1^2}, \sqrt{-1^2 + 0^2 + 1^2} \ \left(\sqrt{2}, \sqrt{2}\right)$

attempt to substitute **their** values into formula **M1**

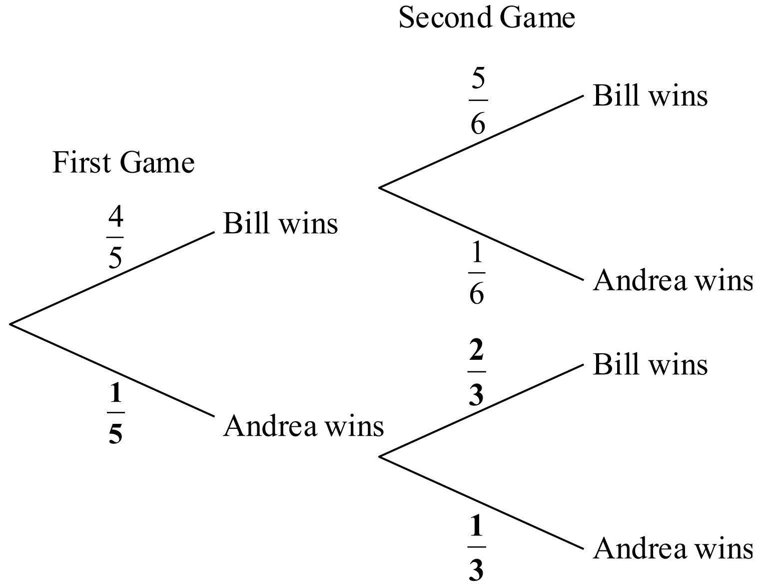
eg $\dfrac{0 + 0 + 1}{\left(\sqrt{0^2 + (-1)^2 + 1^2}\right) \times \left(\sqrt{-1^2 + 0^2 + 1^2}\right)}, \dfrac{1}{\sqrt{2} \times \sqrt{2}}$

correct value for cosine, $\dfrac{1}{2}$ **A1**

angle is $\dfrac{\pi}{3}\ (= 60^\circ)$ **A1** **N1**

[7 marks]
Total [17 marks]

9. (a)



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** probability.

[3 marks]

(b) multiplying along the branches (may be seen on diagram) ***(M1)***

eg $\frac{4}{5}\times\frac{1}{6}$

$\frac{4}{30}\left(\frac{2}{15}\right)$ ***A1*** ***N2***

[2 marks]

(c) **METHOD 1**

multiplying along the branches (may be seen on diagram) ***(M1)***

eg $\frac{4}{5}\times\frac{5}{6},\ \frac{4}{5}\times\frac{1}{6},\ \frac{1}{5}\times\frac{2}{3}$

adding their probabilities of three mutually exclusive paths ***(M1)***

eg $\frac{4}{5}\times\frac{5}{6}+\frac{4}{5}\times\frac{1}{6}+\frac{1}{5}\times\frac{2}{3},\ \frac{4}{5}+\frac{1}{5}\times\frac{2}{3}$

correct simplification ***(A1)***

eg $\frac{20}{30}+\frac{4}{30}+\frac{2}{15},\ \frac{2}{3}+\frac{2}{15}+\frac{2}{15}$

$\frac{28}{30}\left(=\frac{14}{15}\right)$ ***A1*** ***N3***

continued ...

Question 9 continued

METHOD 2

recognizing "Bill wins at least one" is complement of "Andrea wins 2" ***(R1)***
eg finding P (Andrea wins 2)

P (Andrea wins both) $= \frac{1}{5} \times \frac{1}{3}$ ***(A1)***

evidence of complement ***(M1)***
eg $1 - p,\ 1 - \frac{1}{15}$

$\frac{14}{15}$ ***A1*** ***N3***

[4 marks]

(d) P (B wins both) $= \frac{4}{5} \times \frac{5}{6} \left(= \frac{2}{3} \right)$ ***A1***

evidence of recognizing conditional probability ***(R1)***
eg $\mathrm{P}(A \mid B)$, P (Bill wins both |Bill wins at least one), tree diagram

correct substitution ***(A2)***

eg $\dfrac{\frac{4}{5} \times \frac{5}{6}}{\frac{14}{15}}$

$\frac{20}{28} \left(= \frac{5}{7} \right)$ ***A1*** ***N3***

[5 marks]

Total [14 marks]

10. (a) valid method for finding side length ***(M1)***

eg $8^2+8^2=c^2$, $45-45-90$ side ratios, $8\sqrt{2}$, $\frac{1}{2}s^2=16$, $x^2+x^2=8^2$

correct working for area ***(A1)***

eg $\frac{1}{2}\times 4\times 4$

n	1	2	3
x_n	8	$\mathbf{\sqrt{32}}$	4
A_n	32	16	**8**

A1A1 ***N2N2***

[4 marks]

(b) **METHOD 1**

recognize geometric progression for A_n ***(R1)***

eg $u_n=u_1r^{n-1}$

$r=\frac{1}{2}$ ***(A1)***

correct working ***(A1)***

eg $32\left(\frac{1}{2}\right)^5$; 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, …

$A_6=1$ ***A1*** ***N3***

METHOD 2

attempt to find x_6 ***(M1)***

eg $8\left(\frac{1}{\sqrt{2}}\right)^5$, $2\sqrt{2}$, 2, $\sqrt{2}$, 1, …

$x_6=\sqrt{2}$ ***(A1)***

correct working ***(A1)***

eg $\frac{1}{2}\left(\sqrt{2}\right)^2$

$A_6=1$ ***A1*** ***N3***

[4 marks]

continued …

Question 10 continued

(c) **METHOD 1**

recognize infinite geometric series ***(R1)***

eg $S_n = \dfrac{a}{1-r}$, $|r| < 1$

area of first triangle in terms of k ***(A1)***

eg $\dfrac{1}{2}\left(\dfrac{k}{2}\right)^2$

attempt to substitute into sum of infinite geometric series (must have k) ***(M1)***

eg $\dfrac{\frac{1}{2}\left(\frac{k}{2}\right)^2}{1-\frac{1}{2}}$, $\dfrac{k}{1-\frac{1}{2}}$

correct equation ***A1***

eg $\dfrac{\frac{1}{2}\left(\frac{k}{2}\right)^2}{1-\frac{1}{2}} = k$, $k = \dfrac{\frac{k^2}{8}}{\frac{1}{2}}$

correct working ***(A1)***

eg $k^2 = 4k$

valid attempt to solve **their** quadratic ***(M1)***

eg $k(k-4)$, $k = 4$ or $k = 0$

$k = 4$ ***A1*** ***N2***

METHOD 2

recognizing that there are four sets of infinitely shaded regions with equal area ***R1***

area of original square is k^2 ***(A1)***

so total shaded area is $\dfrac{k^2}{4}$ ***(A1)***

correct equation $\dfrac{k^2}{4} = k$ ***A1***

$k^2 = 4k$ ***(A1)***

valid attempt to solve **their** quadratic ***(M1)***

eg $k(k-4)$, $k = 4$ or $k = 0$

$k = 4$ ***A1*** ***N2***

[7 marks]

Total [15 marks]