BECA / Huson / Unit 12: Integral Calculus
18 May 2023

Name: Solutions

12.9 Pre-Quiz: Integral calculus

Find the anti-derivative of each polynomial function (include the constant of integration)

1. $f(x) = 4x^3 + 2x$

   $F(x) =$ $x^4 + x^2 + C$

2. $f(x) = 12x^3 + 9x^2 - 1$

   $F(x) =$ $3x^4 + 3x^3 - x + C$

3. A portion of the function $f(x) = 2x + 1$ is plotted below.

   (a) Write down a definite integral that represents the area of the shaded region **R**.

   $$\int_1^3 2x+1 \, dx$$

   (b) Calculate the area using geometric formulas.

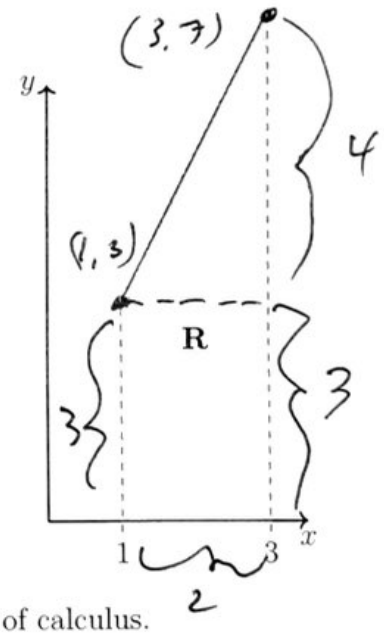
$f(1) = 2(1)+1 = 3$

$f(3) = 2(3)+1 = 7$

$A = 2\times 3 + \frac{1}{2}(2\times 4)$

$= 6 + 4$

$= 10$



   (c) Find the area using a definite integral and the methods of calculus.

$F(x) = x^2 + x$

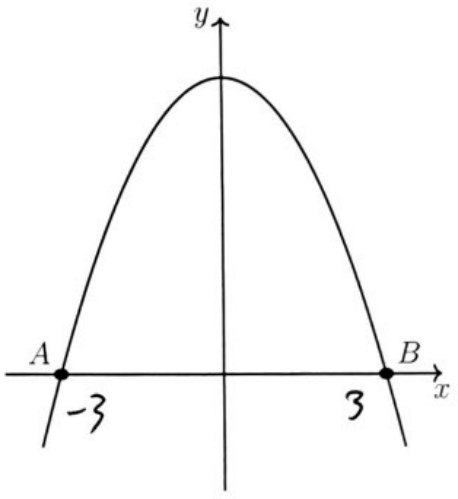
$$\int_1^3 2x+1 \, dx = F(3) - F(1)$$

$= (3^2+3) - (1^2+1)$

$= 12 - 2$

$= 10$

4. Let $f(x) = 9 - x^2$. Part of the graph of f is shown in the following diagram.



(a) The graph crosses the x-axis at the points A and B.
Find the x-coordinates of A and of B.

$f(x) = 9 - x^2 = 0$

$x = 3, -3$

(b) The region enclosed by the graph of f and the x-axis has the area A. Write down a definite integral that represents A.

$$\int_{-3}^{3} 9 - x^2 \, dx$$

(c) Find A by using the antiderivative and applying the fundamental theorem of calculus.

$F(x) = 9x - \frac{1}{3}x^3 + C$

$$\int_{-3}^{3} 9 - x^2 \, dx = F(3) - F(-3)$$

$$= \left[9(3) - \tfrac{1}{3}(3^3)\right] - \left[9(-3) - \tfrac{1}{3}(-3)^3\right]$$

$$= (27 - 9) - (-27 + 9)$$

$$= 18 - (-18)$$

$$= 36$$

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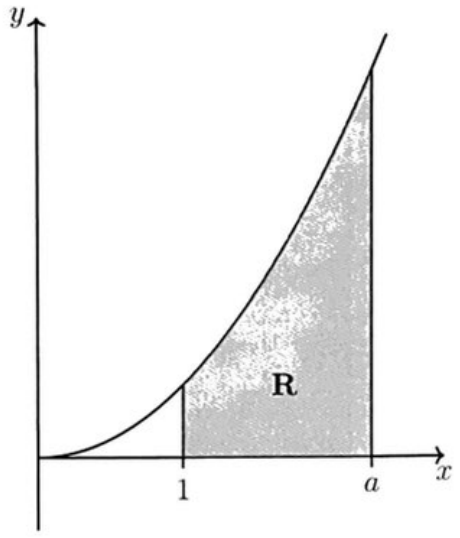
Calculator section

5. The following diagram shows part of the graph of $f(x) = x^2$.



(a) Find $\int_0^1 f(x)\,\mathrm{d}x$

$= \frac{1}{3}$ (use calculator)

(b) The shaded region R is enclosed by the graph of f, the x-axis, and the lines $x = 1$ and $x = a$. Find the value of a so that $R \approx 4$.

$a \approx 2.35\ldots$

(use calculator repeatedly)