BECA / Huson / Unit 12: Integral Calculus Name:
25 May 2023

12.12 Final Exam: Differential and Integral calculus

1. Find the derivative of each polynomial function.

   (a) $f(x) = x^2 + 7x$

   $\frac{dy}{dx} =$

   (b) $f(x) = x^3 + \frac{1}{2}x^2 - 9$

   $\frac{dy}{dx} =$

2. Evaluate the function and its derivative for a given value of x.

   Given $f(x) = 5x^2 + x$

   (a) Find $f(2)$

   (b) Find $f'(x)$

   (c) Find $f'(2)$

3. Find the anti-derivative of each polynomial function (include the constant of integration)

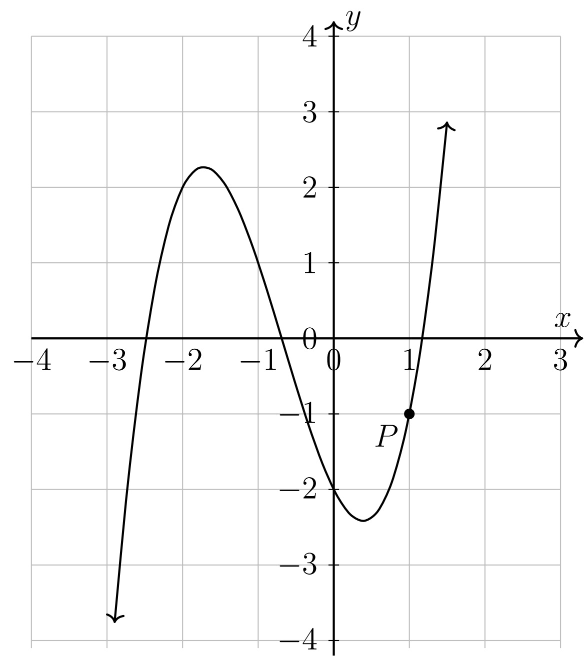
   (a) $f(x) = 3x^2 + 5$

   $F(x) =$

   (b) $f(x) = 16x^3 + 6x^2 - 2x$

   $F(x) =$

4. The graph shows the polynomial function $y = x^3 + 2x^2 - 2x - 2$. Its derivative is $\dfrac{dy}{dx} = 3x^2 + 4x - 2$.



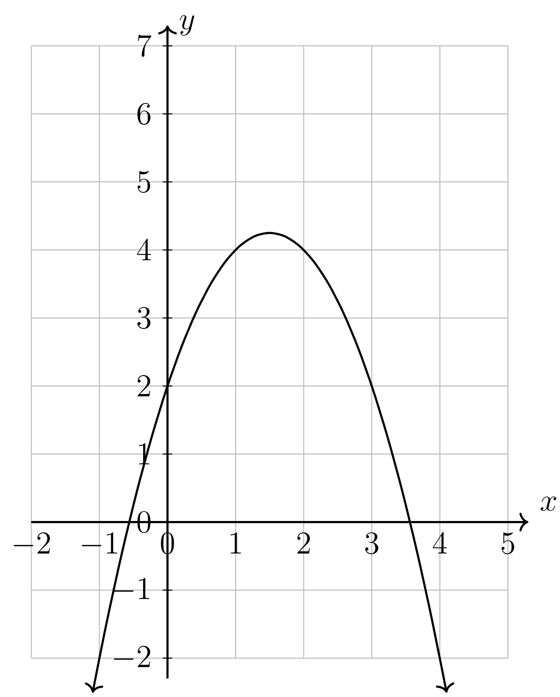
(a) Write down the coordinates of P.

(b) Find the slope of the tangent at P.

(c) Write down the equation of the tangent line through P.

(d) Draw the tangent line on the graph accurately with a straight edge.

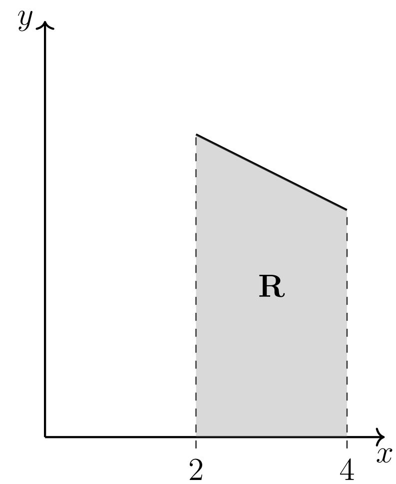
5. The function $y = -x^2 + 3x + 2$ is graphed on the grid below. Find its derivative and the equations of the tangent and normal lines through point $(1, 4)$. Draw the lines.



6. A portion of the function $f(x) = 5 - \frac{1}{2}x$ is plotted below.

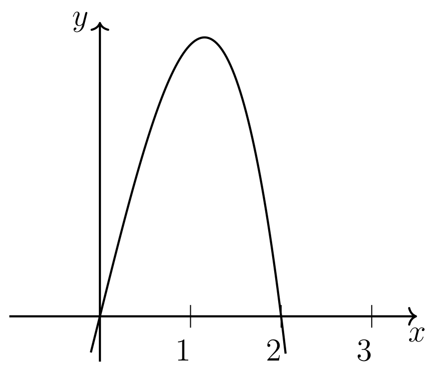
   (a) Write down a definite integral that represents the area of the shaded region **R**.

   (b) Calculate the area using geometric formulas.



   (c) Find the area using a definite integral and the methods of calculus.

7. Part of the graph of $f(x) = -4x^3 + 16x$ is shown in the following diagram.



(a) Write down the antiderivative of $f(x)$. Include the constant of integration.

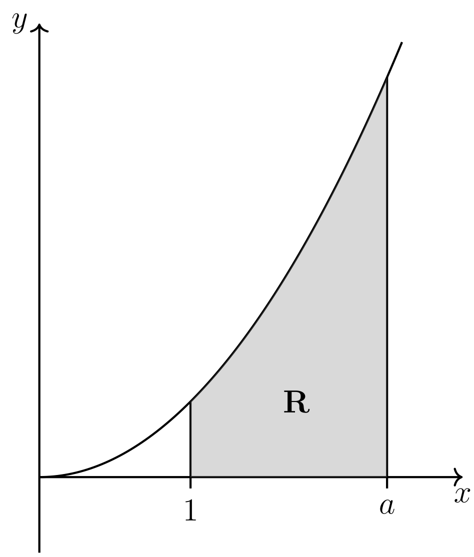
(b) Write down a definite integral that represents the area of the region enclosed by the graph of f and the x-axis from $x = 0$ to $x = 2$.

(c) Find the area of that region using the antiderivative and applying the fundamental theorem of calculus.

BECA / Huson / Unit 12: Integral Calculus Name:
25 May 2023

Calculator section

8. The following diagram shows part of the graph of $f(x) = x^2$.



(a) Find $\displaystyle\int_0^1 f(x)\,\mathrm{d}x$

(b) The shaded region R is enclosed by the graph of f, the x-axis, and the lines $x = 1$ and $x = a$. Find the value of a so that $R \approx 4$.