Geometry Unit 8: Congruence transformations

Bronx Early College Academy

Christopher J. Huson PhD

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Learning Target: I can translate objects

CCSS: HSG.CO.C.9 Prove geometric theorems 4.1

Four pages of △ duplication constructions for binder

1. Side-side-side (SSS)
2. Side-angle-side (SAS)
3. Angle-side-angle (ASA)
4. Side-side-angle (SSA), false, "ambiguous case"

SAS triangle congruence

SAS $\triangle$ congruence

1. SAS $\triangle$ congruence Angle must be the *included* angle, between the two sides
2. Duplicate a side, duplicate an angle, duplicate a side.
3. $\triangle ABC \cong \triangle A'B'C'$ iff
   $\overline{AB} \cong \overline{A'B'}, \angle A \cong \angle A'$, and $\overline{AC} \cong \overline{A'C'}$
4. Angle-side-angle (ASA) $\triangle ABC \cong \triangle A'B'C'$ iff
   $\angle A \cong \angle A', \overline{AB} \cong \overline{A'B'}$, and $\angle B \cong \angle B'$
5. Duplicate an angle, duplicate a side, duplicate an angle
6. SSA $\triangle$ congruence (or ASS, "jack ass theorem")
7. Duplicate an angle, duplicate a side, duplicate an side
8. Given $\triangle ABC$ if $\angle A \cong \angle A', \overline{AB} \cong \overline{A'B'}$, and $\overline{BC} \cong \overline{B'C'}$ then two possible $\triangle$s may result.
9. ff

When does a transformations maintain length and angle measures?

Triangle $A'B'C'$ is the image of triangle ABC after a translation of 2 units to the right and 3 units up. Is triangle ABC congruent to triangle $A'B'C'$? Explain why.

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Yes, the $\triangle$'s are $\cong$ because a translation is a rigid motion so it preserves side lengths. ~~and angle measures~~ Because corr. sides have the same lengths, the $\triangle$'s are $\cong$ by SSS.

Symmetry

When is an object unchanged by a transformation?

If when an object $A \rightarrow A'$ and $A = A'$ then we say it is symmetric.
Reflection: *axis of symmetry*
Rotation: *center and angle of rotation*

Example: Regular polygons are symmetrical

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Which transformation would *not* carry a square onto itself?

(1) a reflection over one of its diagonals

(2) a 90° rotation clockwise about its center

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(4) a reflection over the perpendicular bisector of one side

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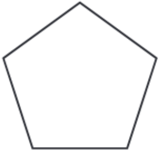
The regular polygon below is rotated about its center.



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Four pages of $\triangle$ duplication for your binder

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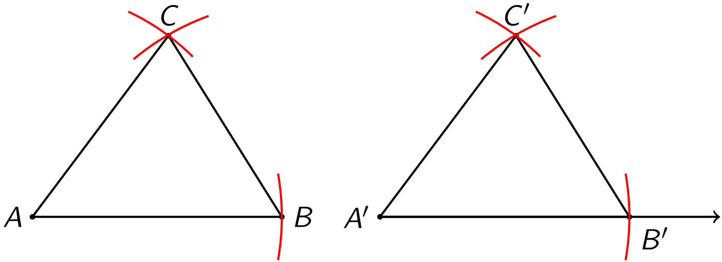
   $\triangle ABC \cong \triangle A'B'C'$ iff
   $\overline{AB} \cong \overline{A'B'}, \overline{BC} \cong \overline{B'C'}$, and $\overline{AC} \cong \overline{A'C'}$
2. Side-angle-side (SAS)
3. Angle-side-angle (ASA)
4. Side-side-angle (SSA), false, "ambiguous case"

Function notation: $A \rightarrow A'$ is pronounced "A gets mapped to A prime," or "A corresponds to A prime."

SSS Triangle congruence ("side-side-side")

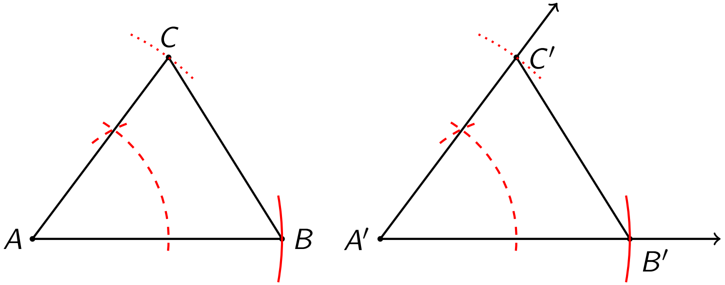
Given $\triangle ABC$, duplicate $\triangle ABC$ by duplicating each side.

1. Construct $\overrightarrow{A'}$.
2. Circle A' with radius AB. Intersection B'.
3. Circle A' with radius AC.
4. Circle B' with radius BC. Intersection C'.
5. $\triangle ABC \cong \triangle A'B'C'$ by the SSS $\triangle \cong$ Postulate.



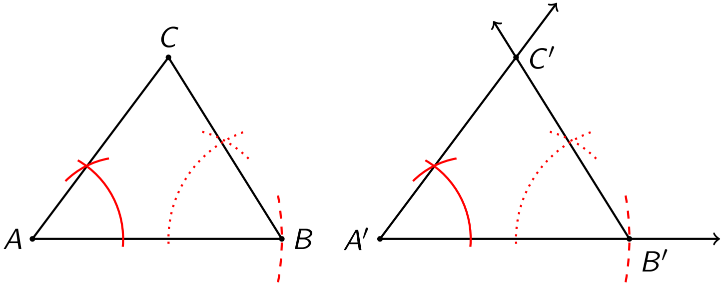
SAS Triangle congruence ("side-angle-side")

1. Given $\triangle ABC$, construct a duplicate $\triangle A'B'C'$
2. Duplicate side $\overline{AB}$, duplicate $\angle A$, duplicate side $\overline{AC}$
3. Angle must be the *included* angle, between the two sides
4. $\triangle ABC \cong \triangle A'B'C'$ iff $\overline{AB} \cong \overline{A'B'}, \angle A \cong \angle A',$ & $\overline{AC} \cong \overline{A'C'}$



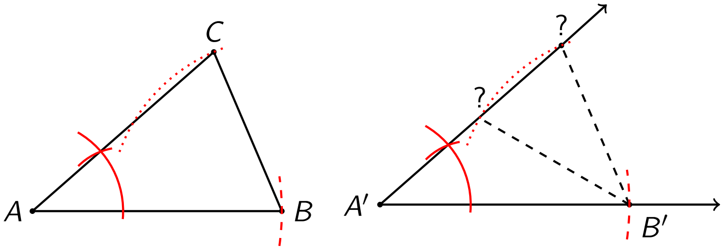
ASA Triangle congruence ("angle-side-angle")

1. Given $\triangle ABC$, construct a duplicate $\triangle A'B'C'$
2. Duplicate $\angle A$, duplicate side $\overline{AB}$, duplicate $\angle B$
3. One side and *any* two angles ("AAS" is ok)
4. $\triangle ABC \cong \triangle A'B'C'$ iff $\angle A \cong \angle A', \overline{AB} \cong \overline{A'B'}$, & $\angle B \cong \angle B'$



SSA *false* congruence (ASS or "jack ass theorem")

1. Given $\triangle ABC$, two $\triangle$s may have two pairs of congruent sides and a *non-included* congruent angle.
2. This is called the "ambiguous case"



HL Triangle congruence ("hypotenuse-leg")

Given right $\triangle ABC$, duplicate $\triangle ABC$ by duplicating a leg, the right angle, and the hypotenuse.

1. Construct $\overrightarrow{A'}$.
2. Circle A' with radius AB. Intersection B'.
3. Construct a perpendicular to $\overline{A'B'}$ through B'.
4. Circle A' with radius AC. Intersection C'.
5. $\triangle ABC \cong \triangle A'B'C'$ by the HL $\triangle \cong$ theorem.