

Prep #21: Normal distributions

- The distribution of the diameters of ball bearings made under a given manufacturing process is normally distributed with a mean of 4 cm and a standard deviation of 0.2 cm. What proportion of the ball bearings will have a diameter less than 3.7 cm?

3. There are 440 students at Thomas Paine High School enrolled in U.S. History. On the April report card, the students' grades are approximately normally distributed with a mean of 79 and a standard deviation of 7. Students who earn a grade less than or equal to 64.9 must attend summer school. (sketch and label the distribution)

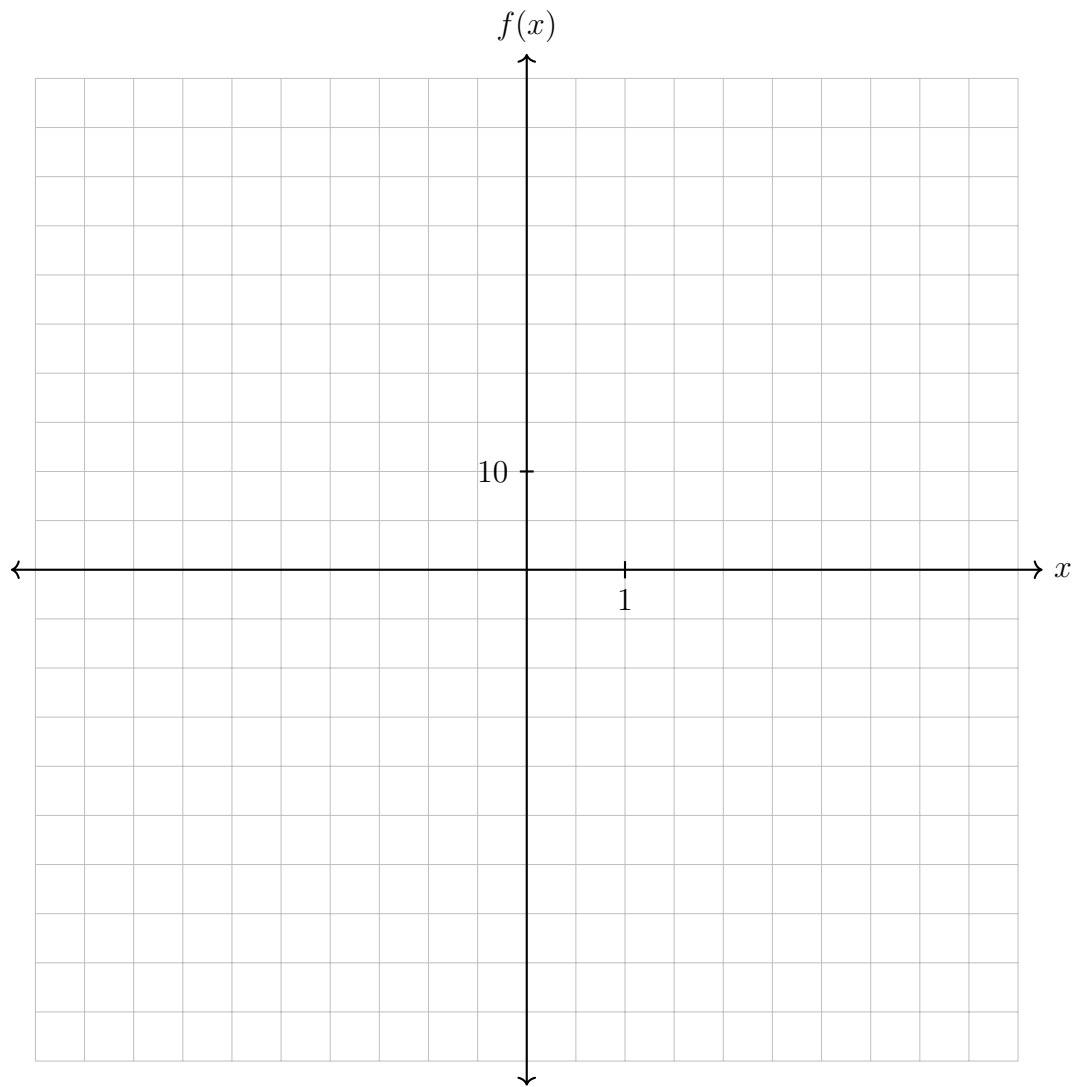
- (a) Calculate the number of standard deviations the 64.9 cutoff score is from the mean.

- (b) What percentage of the students scored at or below 64.9?

- (c) Find the number of students who must attend summer school for U.S. History rounded to the *nearest whole* student.

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4. Graph the function $f(x) = x^4 - 2x^3 - 5x^2 + 3x + 4$.



Mark and label the zeros of the function to the *nearest hundredth*.

Describe the behavior of the given function as x approaches positive infinity.

5. Convert between radical and rational exponent forms. (assume $x > 0$)

(a) $\frac{(9x)^{\frac{1}{2}}y}{y^{\frac{1}{2}}} =$

(b) $\frac{\sqrt[3]{8x^8}}{2\sqrt{x^4}} =$

6. Explain what a rational exponent, such as $\frac{3}{2}$ means. Use this explanation to evaluate $4^{\frac{3}{2}}$.

7. Simplify each complex expression to the form $a + bi$.

(a) $i^2 =$

(c) $(8 + 7i) - (5 + 3i) =$

(b) $(2 - 2i)(10 + i) =$

(d) $\frac{1}{3}i(\sqrt{-81} + 6i) + 5i =$

8. Solve for x over the complex numbers.

$$x^2 - 4x + 13 = 0$$

9. Solve algebraically for all values of x .

$$\sqrt{x+12} = x$$

10. Solve algebraically for all values of x .

$$\frac{1}{x+12} = x$$

11. Write a recursive formula for the sequence 16, 8, 0, -8 , \dots

12. A sequence is defined by the recursive formula

$$\begin{aligned} a_1 &= 30 \\ a_n &= a_{n-1} + 5 \end{aligned}$$

Write an explicit formula for the sequence.

13. The sum of the first n terms of the geometric sequence beginning 1, 1.5, 2.25, \dots is 171, rounded to *the nearest integer*. Find n .

14. Complete the table for the geometric sequence a .

n	1	2	3	4	5
a_n	100	80			

Model the sequence with an exponential function.

15. A survey was conducted to compare the dietary habits of American and Japanese families. Families were asked which they had eaten for dinner most recently, meat or fish. The proportions of each answer are shown in the table below.

Nationality	Meat	Fish	Total
Americans	0.78	0.22	1.00
Japanese	0.43	0.57	1.00

- (a) Does the survey data indicate that Americans and Japanese families have similar dietary habits? Justify your answer.
- (b) 200 American families and 100 Japanese families participated in the survey. Calculate the number of each category of response and enter it in the appropriate cell in the table below.

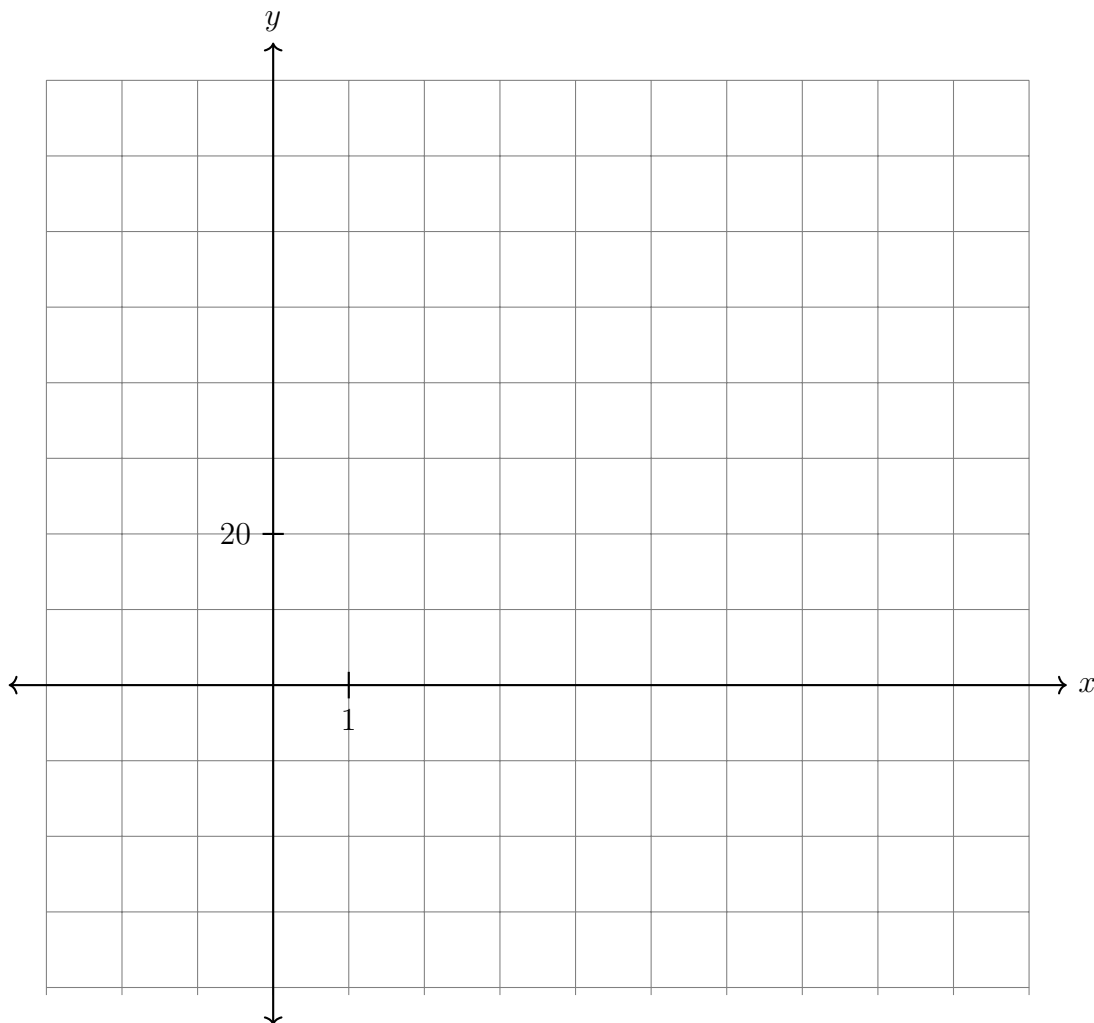
Nationality	Meat	Fish	Total
Americans			200
Japanese			100

- (c) The survey was conducted in Kansas City (an inland city) and Tokyo (a city on the Pacific Ocean). How might that affect the survey's findings?

16. A manufacture of portable speakers finds that its profit varies depending on the number of speakers it makes. The profit, $p(x)$, in thousands of dollars as a function of the number of speakers, x , in thousands is modeled by

$$p(x) = -x^3 + 7x^2 + 6x - 25$$

- (a) Graph $y = p(x)$, over the interval $0 \leq x \leq 8$ on the set of axes below.
- (b) Over the given interval, state the coordinates of the maximum of p rounding all values to the *nearest integer*. Explain what the point means in terms of the number of speakers and profit.



17. Determine for which polynomial(s) $(x + 2)$ is a factor. Explain your answer.

$$P(x) = x^4 - 3x^3 - 16x - 12$$

$$Q(x) = x^3 - 3x^2 - 16x - 12$$

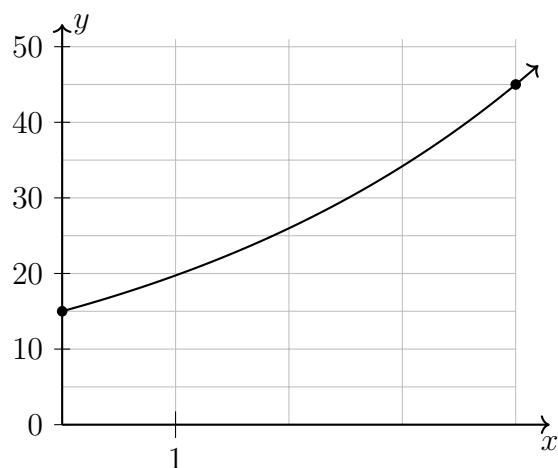
18. Given the function $P(x) = x^3 - 3x^2 - 2x + 4$, find the value of $P(-1)$ and $P(1)$.

Now identify the correct statement.

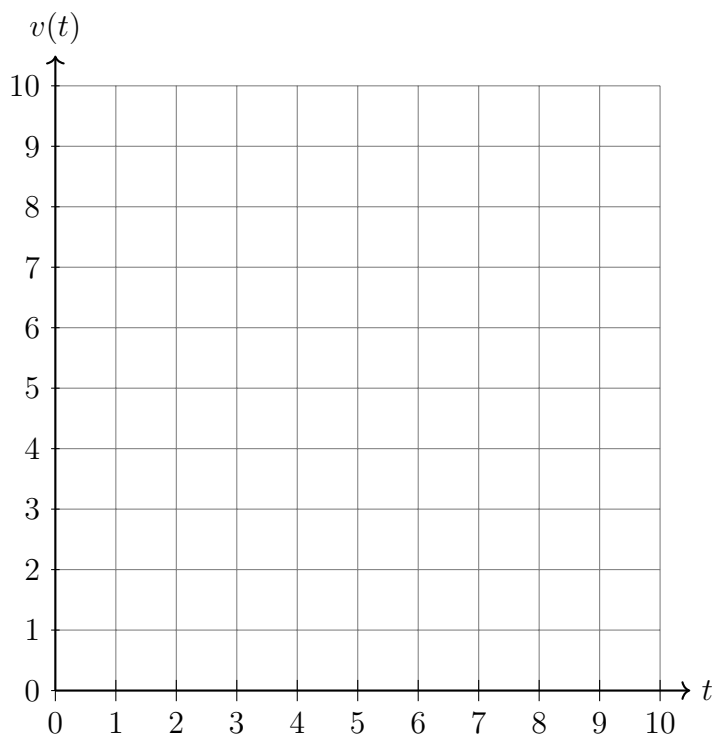
- (a) $(x - 1)$ is a factor because $P(-1) = 2$.
- (b) $(x + 1)$ is a factor because $P(-1) = 2$.
- (c) $(x + 1)$ is a factor because $P(1) = 0$.
- (d) $(x - 1)$ is a factor because $P(1) = 0$.

19. The data in the table was graphed and then a smooth curve was fit through the points. Write down an equation for the exponential function $f(x)$ shown.

n	1	2	3	4	5
a_n	100	80			



20. An investment of \$4000 earns a continuous interest rate of 8%. On the axes below, graph the value of the investment $v(t)$ in thousands of dollars versus time t in years.



Find the value of the investment after 5 years rounding to the *nearest dollar*.

21. Biologists are studying a new bacterium. They create a culture with 100 of the bacteria and anticipate that the number of bacteria will double every 30 hours. Write an equation for the number of bacteria, B , in terms of the number of hours, t , since the experiment began.
22. During the summer, Adam saved \$4000 and Betty saved \$3500. Adam deposited his money in Bank A at an annual rate of 2.4% compounded monthly. Betty deposited her money in Bank B at an annual rate of 4% compounded quarterly. Write two functions that represent the value of each account after t years if no other deposits or withdrawals are made, where Adam's account value is represented by $A(t)$, and Betty's by $B(t)$.
23. Solve algebraically for x that makes the function value 125, to the *nearest hundredth*, $f(x) = 100 \times 10^x$.