

# Geometry Unit 3: Transversals

Bronx Early College Academy

Christopher J. Huson PhD

11 October - 21 October 2022

3.1 Identify transversal angles	17 October
3.2 Transversals problems	18 October
3.3 Triangle sum theorem	20 October
3.4 Parallelograms	21 October
3.5 External angles	24 October
3.6 Transversal situations	25 October
3.7 Parallelogram situations	27 October
3.8 Transversals review	31 October
3.9 Transversals test	1 November

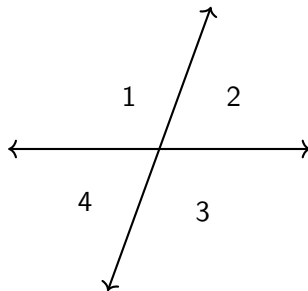
# Learning Target: I can name parallel lines transversal angles

HSG.CO.C.9 Prove theorems about lines and angles

3.1 Monday 17 October

Do Now: Identify the true statements

1.  $\angle 1 \cong \angle 2$
2.  $\angle 2 \cong \angle 4$
3.  $m\angle 1 + m\angle 4 = 180^\circ$
4.  $m\angle 2 + m\angle 3 = 90^\circ$



Lesson: Parallel lines crossed by a transversal line, horizontal and vertical directions

# New terminology for parallel lines

Parallel lines are in the same plane and never intersect

**Parallel lines**  $j \parallel k$ , mark with arrows

**Transversal** Line  $l$ , crosses parallel lines

**Interior** Inside ( $\angle$ s)

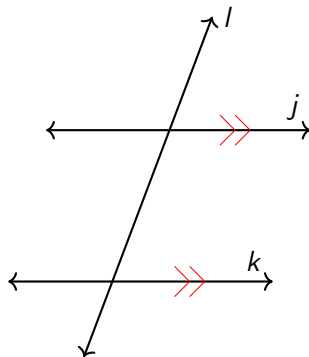
**Exterior** Outside ( $\angle$ s)

**Same side** On the left or right of  $l$

**Alternate** Across  $l$  from each other

**Horizontal** Sideways direction

**Vertical** Up and down direction



## New terminology for parallel lines

Parallel lines are in the same plane and never intersect

**Parallel lines**  $j \parallel k$ , mark with arrows

**Transversal** Line  $l$ , crosses parallel lines

**Interior** Inside ( $\angle$ s)

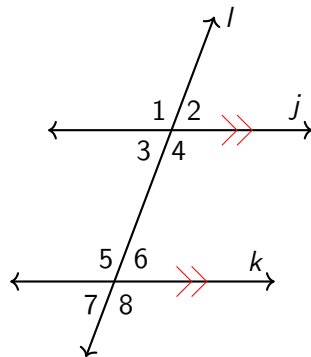
**Exterior** Outside ( $\angle$ s)

**Same side** On the left or right of  $l$

**Alternate** Across  $l$  from each other

**Horizontal** Sideways direction

**Vertical** Up and down direction



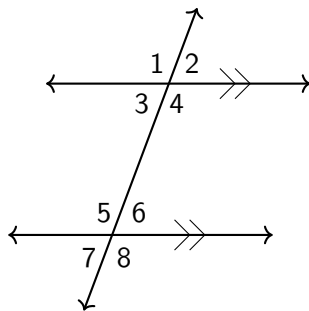
We often number the angles this way.

## New theorems for parallel lines

**Corresponding** Having the same position. e.g.  $\angle 2$  and  $\angle 6$

**Postulate** Corresponding  $\angle$ s of  $\parallel$  lines are congruent,  $\angle 2 \cong \angle 6$

1. Alternate interior  $\angle$ s are  $\cong$   
 $\angle 4 \cong \angle 5$
2. Same-side interior  $\angle$ s are supplementary  
 $m\angle 3 + m\angle 5 = 180$
3. Alternate exterior  $\angle$ s are  $\cong$   
 $\angle 1 \cong \angle 8$



There are only two angle measures, the acute  $\angle$ s and the obtuse  $\angle$ s  
And they add to  $180^\circ$ , i.e. supplementary

## Apply the theorems of parallel lines with a transversal

Given two parallel lines and a transversal, with  $m\angle 6 = 70^\circ$ . Write down the value of each angle measure.

1.  $m\angle 1 =$

2.  $m\angle 2 =$

3.  $m\angle 3 =$

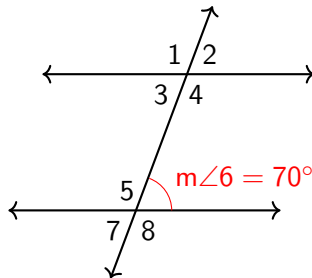
4.  $m\angle 4 =$

5.  $m\angle 5 =$

6.  $m\angle 6 = 70^\circ$

7.  $m\angle 7 =$

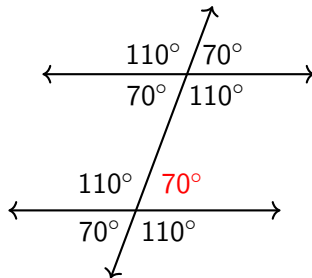
8.  $m\angle 8 =$



## Apply the theorems of parallel lines with a transversal

Given two parallel lines and a transversal, with  $m\angle 6 = 70^\circ$ . Write down the value of each angle measure.

1.  $m\angle 1 =$
2.  $m\angle 2 =$
3.  $m\angle 3 =$
4.  $m\angle 4 =$
5.  $m\angle 5 =$
6.  $m\angle 6 = 70^\circ$
7.  $m\angle 7 =$
8.  $m\angle 8 =$



Solution



## Extension: Ratios are fractions

We often state proportions as ratios

Example: Divide a distance into equal parts, i.e.

$$1 : 1$$

We say “one to one”, or “in a one to one ratio.”

A rectangle's length to width ratio is two to one.  $2 : 1$

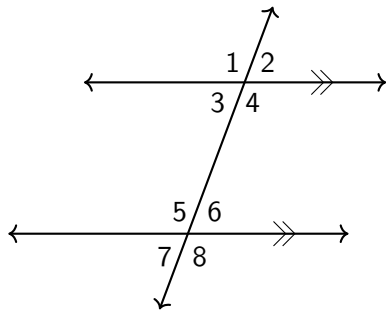
# Learning Target: I can calculate transversal angles

HSG.CO.C.9 Prove theorems about lines and angles

3.2 Tuesday 18 October

Do Now: Identify each angle

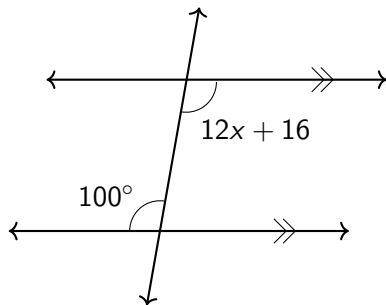
1. Opposite  $\angle 4$
2. Corresponding to  $\angle 3$
3. Alternate exterior to  $\angle 8$
4. Same side interior to  $\angle 5$
5. Alternate interior to  $\angle 4$



Lesson: Solve for angle measures

## Parallel lines intersected by a transversal. Find $x$ .

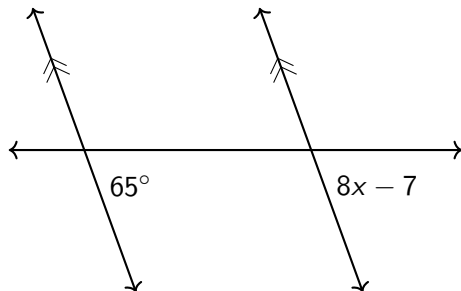
Alternate interior angles measure  $100^\circ$  and  $12x + 16$ , as shown.



Are the angles congruent or supplementary?

## Parallel lines intersected by a transversal. Find $x$ .

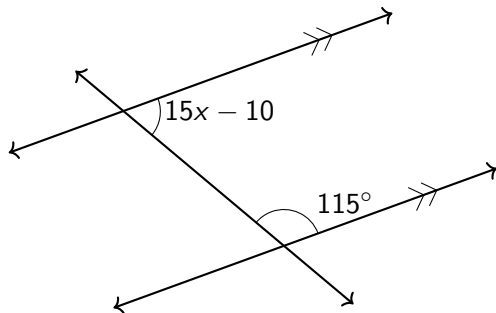
Parallel lines do not have to be horizontal.



State the postulate or theorem you are employing.

## Parallel lines intersected by a transversal. Find $x$ .

Given: Same side interior angles measure  $115^\circ$  and  $15x - 10$ .



Remember the check.

## Extension: *Partitioning* a segment or angle in a ratio

Point  $B$  divides  $\overline{AC}$  in a  $2 : 1$  ratio, i.e.  $AB = 2BC$   
Ray  $\overrightarrow{BD}$  divides  $\angle ABC$  in a  $2 : 1$  ratio. Find  $x$ .

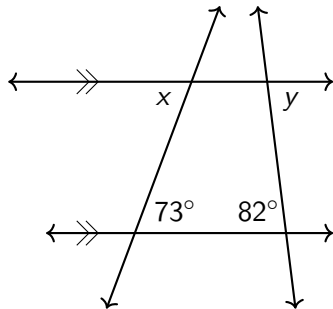
# Learning Target: I can calculate triangle angles

HSG.CO.C.9 Prove theorems about lines and angles

3.3 Thursday 20 October

Do Now:

1. Given two parallel lines, two transversals
2. Find  $x$ ,  $y$
3. What relationship are you using? (e.g. vertical angles, same-side exterior angles, alternate interior angles)



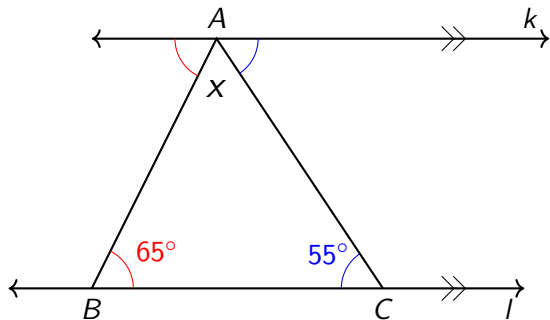
Lesson: The sum of a triangle's *interior* angles is  $180^\circ$

*Triangle sum theorem*

## Triangle sum theorem

Given parallel lines  $k \parallel l$ ,  $\triangle ABC$ ,  $m\angle B = 65^\circ$ ,  $m\angle C = 55^\circ$ .

Find  $m\angle BAC = x$ .



**Interior** The three angles that are *inside* the triangle

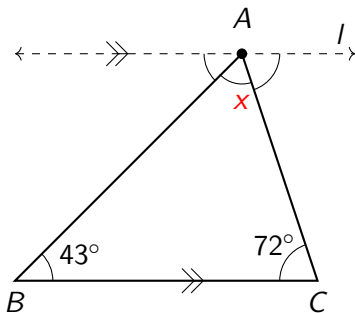
**Theorem** The sum of the measures of the three internal angles of a triangle is  $180^\circ$



## Mark 3 missing angle measures to make a straight angle

An *auxiliary* line  $l$  is drawn through  $A$ , parallel to triangle base  $\overline{BC}$ .

Find  $m\angle BAC$ .



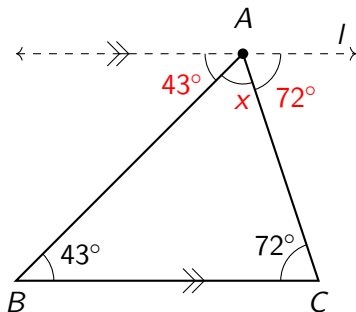
**Auxiliary** An extra line added to a diagram

**Linear triple** Three adjacent angles that make a straight line

## Mark 3 missing angle measures to make a straight angle

An *auxiliary* line  $l$  is drawn through  $A$ , parallel to triangle base  $\overline{BC}$ .

Find  $m\angle BAC$ .



$$43 + x + 72 = 180$$

$$x = 65^\circ$$

Theorem:

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

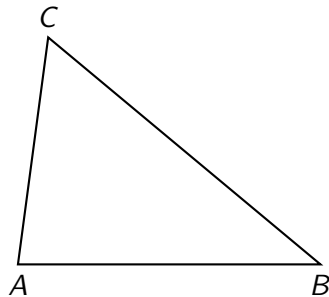
for any triangle

**Auxiliary** An extra line added to a diagram

**Linear triple** Three adjacent angles that make a straight line

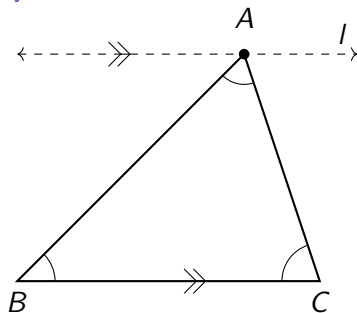
## Find the missing angle measure

Given  $\triangle ABC$ ,  $m\angle A = 82^\circ$ ,  $m\angle C = 59^\circ$ . Find  $m\angle B$ .



# Triangle sum theorem ( $180^\circ$ )

Check your notes



**Auxiliary line** An extra line added to a diagram

**Linear triple** Three adjacent angles that make a straight line

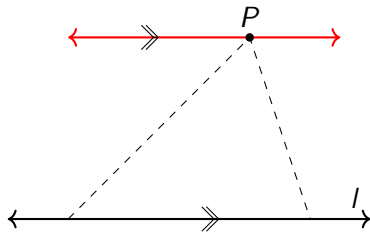
**Interior angles** The three angles that are inside the triangle

**Theorem** The sum of a triangle's angles is  $180^\circ$

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

## Extension: Euclid's fifth postulate (the Parallel Postulate)

Given a line and a point, there exists one line through the point parallel to the line.



**Euclid** Greek author of the most successful math book of all time, *The Elements*

**Postulate** A statement we assume is true as the basis of all further mathematical theorems and proofs

**Non-Euclidean geometries** Alternative mathematics not using the Parallel Postulate. Lobachevsky (1826 Russian), Bolyai (1832 Hungarian), Einstein (1916 German)

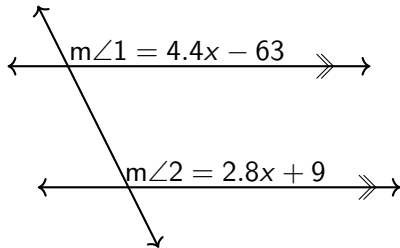
# Learning Target: I can find the angles of a parallelogram

HSG.CO.C.9 Prove theorems about lines and angles

3.4 Friday 21 October

Do Now: Two parallel lines intersect a transversal. Given corresponding angles  $m\angle 1 = 4.4x - 63$  and  $m\angle 2 = 2.8x + 9$ .

Find the measure of  $\angle 1$ .



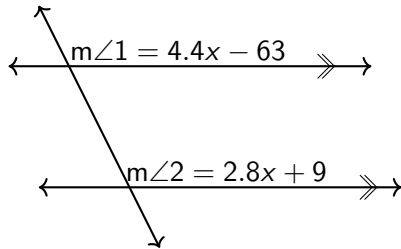
# Learning Target: I can find the angles of a parallelogram

HSG.CO.C.9 Prove theorems about lines and angles

3.4 Friday 21 October

Do Now: Two parallel lines intersect a transversal. Given corresponding angles  $m\angle 1 = 4.4x - 63$  and  $m\angle 2 = 2.8x + 9$ .

Find the measure of  $\angle 1$ .



Corresponding angles are  $\cong$

$$4.4x - 63 = 2.8x + 9$$

$$1.6x = 72$$

$$x = 45$$

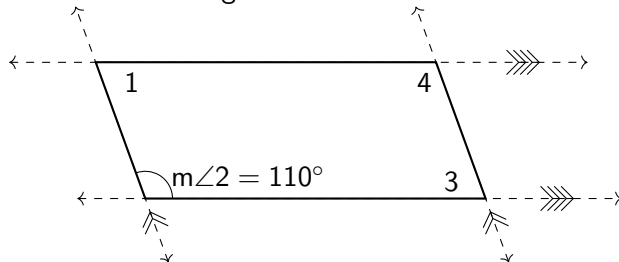
$$m\angle 1 = 4.4(45) - 63 = 135^\circ$$

$$\text{Check: } m\angle 2 = 2.8(45) + 9 = 135$$

# A parallelogram's opposite sides are parallel and congruent

Consecutive angles are supplementary. Opposite angles are congruent.

Find the other angle measures.





# Learning Target: I can calculate external triangle angles

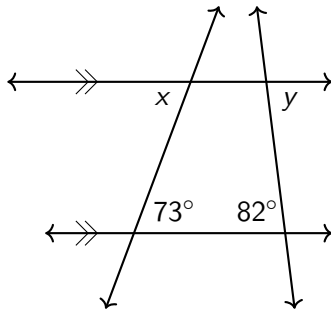
HSG.CO.C.9 Prove theorems about lines and angles

3.5 Monday 24 October

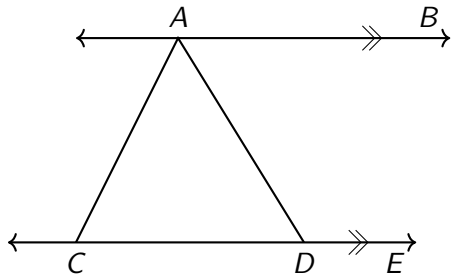
Do Now:

1. Given two parallel lines, two transversals
2. Find  $x$ ,  $y$
3. What relationship are you using? (e.g. vertical angles, same-side exterior angles, alternate interior angles, etc.)

Lesson: Triangle external angle theorem



Given parallel lines  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CDE}$  with  $\overline{AC} \cong \overline{AD}$ . If  $m\angle BAD = 80$  find  $m\angle ACD$ .



# Learning Target: I can calculate transversal angles (algebra review)

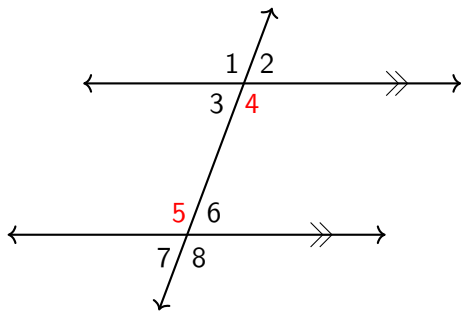
HSG.CO.C.9 Prove theorems about lines and angles

3.6 Tuesday 25 October

Given two parallel lines and a transversal,

$$m\angle 4 = 3x \text{ and } m\angle 5 = x + 70.$$

Write an equation, then solve for  $x$ .



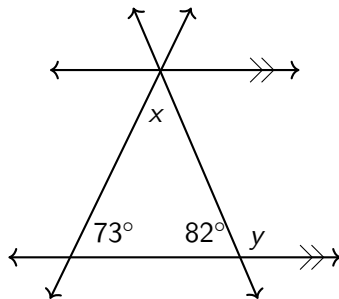
# Learning Target: I can calculate angles in parallelograms

HSG.CO.C.9 Prove theorems about lines and angles

3.7 Wednesday 27 October

Do Now:

1. Given a triangle, shown
2. Find angle measures  $x$ ,  $y$
3. What relationships are you using?  
(e.g. vertical angles, same-side exterior angles, alternate interior angles)



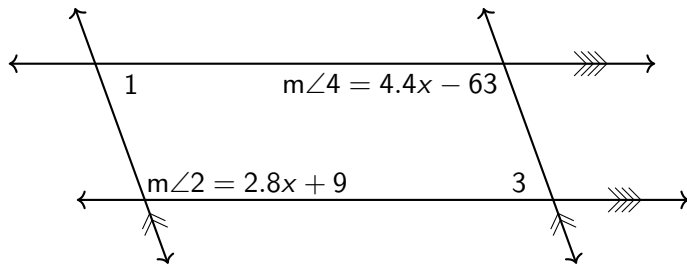
Lesson: Triangle's exterior angles

# Learning Target: I can review with my classmates

HSG.CO.C.9 Prove theorems about lines and angles

3.8 Monday 31 October

Two parallel lines intersect a second set of parallel lines. Given  $m\angle 2 = 2.8x + 9$  and  $m\angle 4 = 4.4x - 63$ , find the measure of  $\angle 1$ .



# Learning Target: I can review with my classmates

HSG.CO.C.9 Prove theorems about lines and angles

3.9 Tuesday 1 November

Unit 3 Test: Parallel lines and transversals