

PROPERTIES OF PARALLELOGRAMS COMMON CORE GEOMETRY



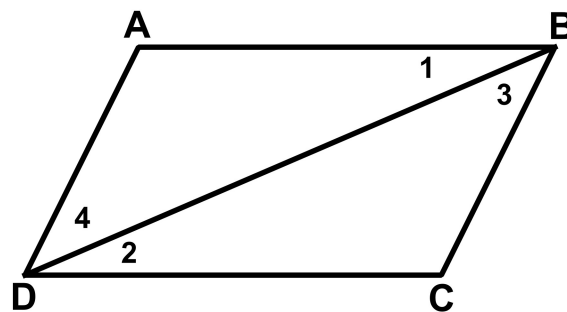
In the last lesson we saw the definition of a **parallelogram**, i.e. a quadrilateral (any four-sided figure) with **both pairs of opposite sides being parallel**. There are a remarkable number of extra properties that come from this one condition. They may seem obvious, but we will prove each of them in the lesson today. Here are just a few:

PROPERTIES OF A PARALLELOGRAM

1. Opposite sides are congruent (have the same length).
2. Opposite angles are congruent (have the same measure).

Exercise #1: In the diagram below, parallelogram $ABCD$ is shown with $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Diagonal \overline{BD} is drawn with certain angles numbered.

- (a) Using tracing paper and a ruler, carefully copy the parallelogram and verify that opposite sides are of equal length and opposite angles are of equal measure.



- (b) We would now like to prove these two properties of parallelograms using Euclidean proof. It is enough to show that $\overline{AB} \cong \overline{DC}$ and $\angle A \cong \angle C$. Fill in the reasons to the proof below.

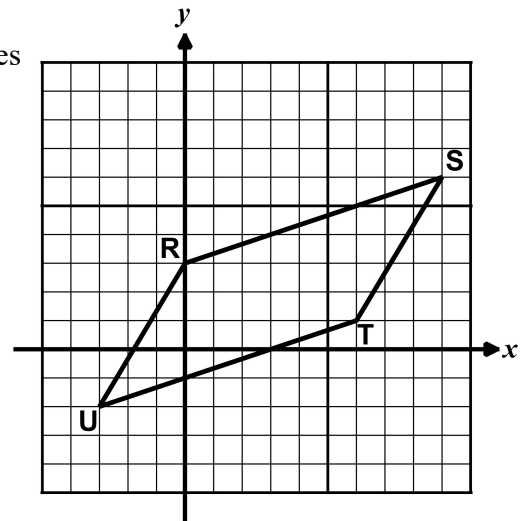
Statements	Reasons
(1) $ABCD$ is a parallelogram	(1) _____
(2) $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$	(2) _____ _____
(3) $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$	(3) _____ _____
(4) $\overline{BD} \cong \overline{BD}$	(4) _____
(5) $\triangle ABD \cong \triangle CDB$	(5) _____
(6) $\overline{AB} \cong \overline{DC}$ and $\angle A \cong \angle C$	(6) _____

- (c) Draw in diagonal \overline{AC} using a straightedge. Are the two diagonals congruent? Test using tracing paper or a ruler. If not, which one is larger?



Parallelogram properties can be explored and then proven using Euclidean proof. They can also be explored using the tools of coordinate geometry.

Exercise #2: On the diagram, quadrilateral $RSTU$ is shown with vertices $R(0, 3)$, $S(9, 6)$, $T(6, 1)$ and $U(-3, -2)$.



(a) Prove that $RSTU$ is a parallelogram using coordinate geometry.

(b) Show that $\overline{RU} \cong \overline{ST}$ using coordinate geometry.

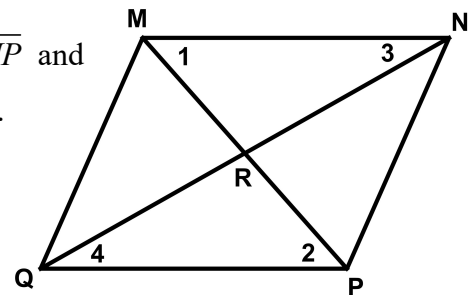
(c) Using the midpoint formula, find the midpoint of the diagonals \overline{RT} and \overline{SU} . What observation can you make about these? What does it tell you about the diagonals? Draw them in to visualize.

Midpoint of \overline{RT} :

Midpoint of \overline{SU} :

Observation and conclusion:

Exercise #3: Given parallelogram $MNPQ$ shown below with diagonals \overline{MP} and \overline{NQ} intersecting at point R , prove that \overline{MP} and \overline{NQ} must bisect each other.



Name: _____

Date: _____



PROPERTIES OF PARALLELOGRAMS COMMON CORE GEOMETRY HOMEWORK

PROBLEM SOLVING

1. Which of the following is *not* a property of *all* parallelograms?
 - (1) opposite sides are congruent
 - (2) diagonals are congruent
 - (3) opposite angles are congruent
 - (4) opposite sides are parallel

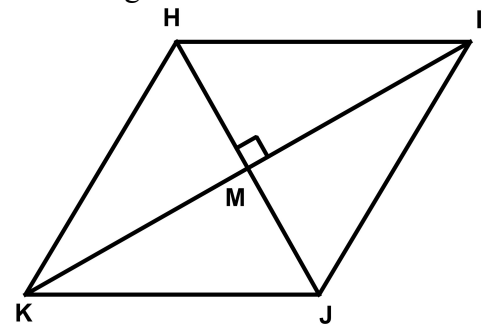
2. In parallelogram $ABCD$ diagonals \overline{AC} and \overline{BD} intersect at point L . If $AL = x - 2$ and $AC = 3x - 16$, then which of the following represents the length of \overline{CL} ?

- (1) 5
 - (2) 7

- (3) 10
 - (4) 12

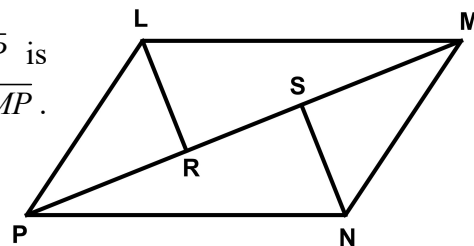
3. In parallelogram $MNPQ$, side \overline{MN} is seven inches longer than twice the length of side \overline{NP} . If the perimeter of $MNPQ$ is 68 inches, then determine the length of \overline{NP} . Show how you solved for your answer.

4. In the diagram of parallelogram $HIJK$ below, the diagonals \overline{HJ} and \overline{IK} are perpendicular, as shown. If \overline{HJ} is 10 inches long and \overline{IK} is 24 inches long, then why must \overline{HK} be 13 inches long?

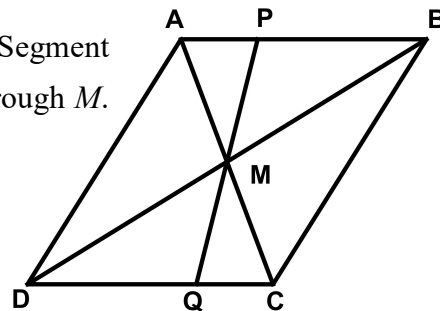


REASONING

5. In the diagram shown, parallelogram $LMNP$ is shown. Diagonal \overline{MP} is drawn and contains points R and S such that $\overline{LR} \perp \overline{MP}$ and $\overline{NS} \perp \overline{MP}$. Prove that $\overline{RM} \cong \overline{SP}$.



6. Given that $ABCD$ is a parallelogram with diagonals intersecting at M . Segment \overline{PQ} is drawn such that P lies on \overline{AB} and Q lies on \overline{DC} and it passes through M . Prove that $\overline{PM} \cong \overline{QM}$.



7. Parallelogram $ABCD$ has coordinates of $A(7,1)$, $B(-2,-3)$, and $C(0,3)$. What must be the coordinates of point D ? Explain how you found your answer.

