

Geometry Unit 3: Transversals

Bronx Early College Academy

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11 October - 21 October 2022

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3.2 Transversals problems	12 October
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Learning Target: I can name parallel lines transversal angles

HSG.CO.C.9 Prove theorems about lines and angles

3.1 Tuesday 11 October

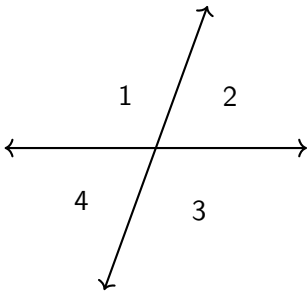
Do Now: Identify the true statements

1. $\angle 1 \cong \angle 2$

2. $\angle 2 \cong \angle 4$

3. $m\angle 1 + m\angle 4 = 180^\circ$

4. $m\angle 2 + m\angle 3 = 90^\circ$



Lesson: Parallel lines crossed by a transversal line, horizontal and vertical directions

New terminology for parallel lines

Parallel lines are in the same plane and never intersect

Parallel lines $j \parallel k$, mark with arrows

Transversal Line l , crosses parallel lines

Interior Inside (\angle s)

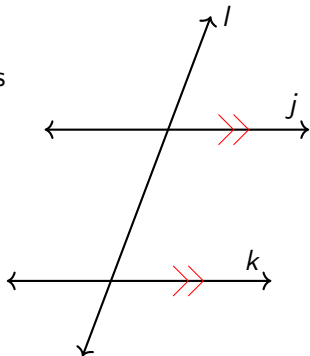
Exterior Outside (\angle s)

Same side On the left or right of l

Alternate Across l from each other

Horizontal Sideways direction

Vertical Up and down direction



New terminology for parallel lines

Parallel lines are in the same plane and never intersect

Parallel lines $j \parallel k$, mark with arrows

Transversal Line l , crosses parallel lines

Interior Inside (\angle s)

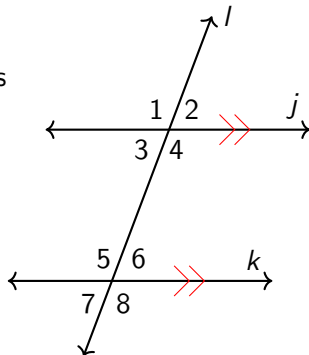
Exterior Outside (\angle s)

Same side On the left or right of l

Alternate Across l from each other

Horizontal Sideways direction

Vertical Up and down direction



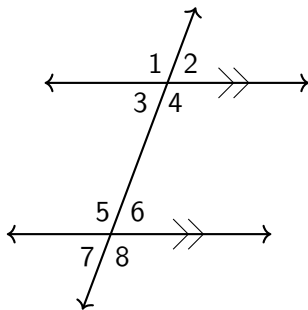
We often number the angles this way.

New theorems for parallel lines

Corresponding Having the same position. e.g. $\angle 2$ and $\angle 6$

Postulate Corresponding \angle s of \parallel lines are congruent, $\angle 2 \cong \angle 6$

1. Alternate interior \angle s are \cong
 $\angle 4 \cong \angle 5$
2. Same-side interior \angle s are supplementary
 $m\angle 3 + m\angle 5 = 180$
3. Alternate exterior \angle s are \cong
 $\angle 1 \cong \angle 8$

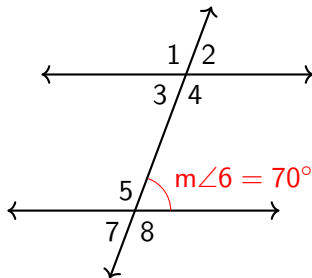


There are only two angle measures, the acute \angle s and the obtuse \angle s
And they add to 180° , i.e. supplementary

Apply the theorems of parallel lines with a transversal

Given two parallel lines and a transversal, with $m\angle 6 = 70^\circ$. Write down the value of each angle measure.

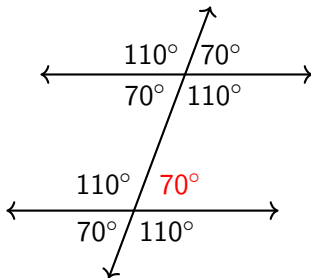
1. $m\angle 1 =$
2. $m\angle 2 =$
3. $m\angle 3 =$
4. $m\angle 4 =$
5. $m\angle 5 =$
6. $m\angle 6 = 70^\circ$
7. $m\angle 7 =$
8. $m\angle 8 =$



Apply the theorems of parallel lines with a transversal

Given two parallel lines and a transversal, with $m\angle 6 = 70^\circ$. Write down the value of each angle measure.

1. $m\angle 1 =$
2. $m\angle 2 =$
3. $m\angle 3 =$
4. $m\angle 4 =$
5. $m\angle 5 =$
6. $m\angle 6 = 70^\circ$
7. $m\angle 7 =$
8. $m\angle 8 =$



Solution

Extension: Ratios are fractions

We often state proportions as ratios

Example: Divide a distance into equal parts, i.e.

$$1 : 1$$

We say “one to one”, or “in a one to one ratio.”

A rectangle's length to width ratio is two to one. $2 : 1$

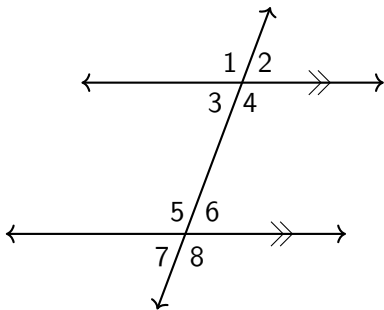
Learning Target: I can calculate transversal angles

HSG.CO.C.9 Prove theorems about lines and angles

3.2 Wednesday 12 October

Do Now: Identify each angle

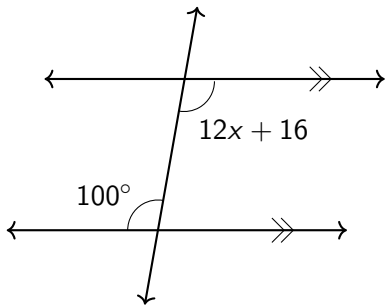
1. Opposite $\angle 4$
2. Corresponding to $\angle 3$
3. Alternate exterior to $\angle 8$
4. Same side interior to $\angle 5$
5. Alternate interior to $\angle 4$



Lesson: Solve for angle measures

Parallel lines intersected by a transversal. Find x .

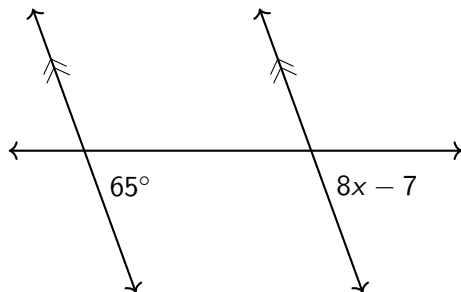
Alternate interior angles measure 100° and $12x + 16$, as shown.



Are the angles congruent or supplementary?

Parallel lines intersected by a transversal. Find x .

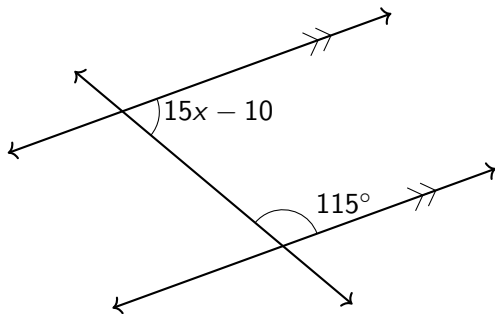
Parallel lines do not have to be horizontal.



State the postulate or theorem you are employing.

Parallel lines intersected by a transversal. Find x .

Given: Same side interior angles measure 115° and $15x - 10$.



Remember the check.

Extension: *Partitioning* a segment or angle in a ratio

Point B divides \overline{AC} in a $2 : 1$ ratio, i.e. $AB = 2BC$

Ray \overrightarrow{BD} divides $\angle ABC$ in a $2 : 1$ ratio. Find x .

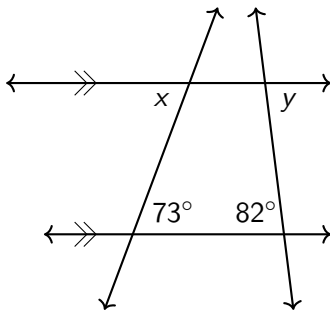
Learning Target: I can calculate triangle angles

HSG.CO.C.9 Prove theorems about lines and angles

3.3 Thursday 13 October

Do Now:

1. Given two parallel lines, two transversals
2. Find x , y
3. What relationship are you using? (e.g. vertical angles, same-side exterior angles, alternate interior angles)



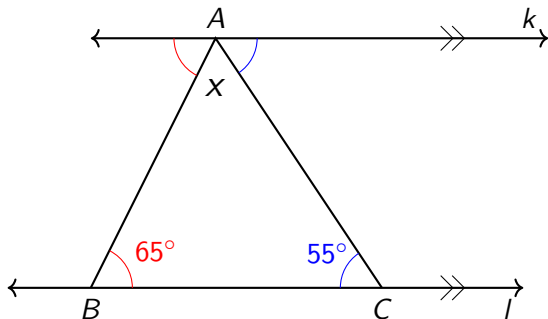
Lesson: The sum of a triangle's *interior* angles is 180°

Triangle sum theorem

Triangle sum theorem

Given parallel lines $k \parallel l$, $\triangle ABC$, $m\angle B = 65^\circ$, $m\angle C = 55^\circ$.

Find $m\angle BAC = x$.

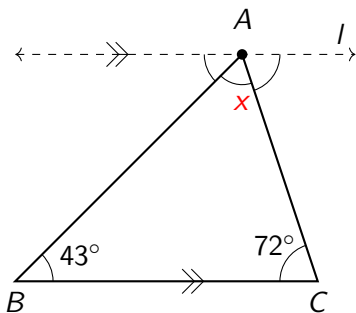


Interior The three angles that are *inside* the triangle

Theorem The sum of the measures of the three internal angles of a triangle is 180°

Mark 3 missing angle measures to make a straight angle

An *auxiliary* line l is drawn through A , parallel to triangle base \overline{BC} .
Find $m\angle BAC$.

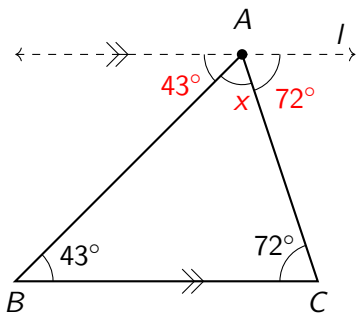


Auxiliary An extra line added to a diagram

Linear triple Three adjacent angles that make a straight line

Mark 3 missing angle measures to make a straight angle

An *auxiliary* line l is drawn through A , parallel to triangle base \overline{BC} .
Find $m\angle BAC$.



$$43 + x + 72 = 180$$

$$x = 60^\circ$$

Theorem:

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

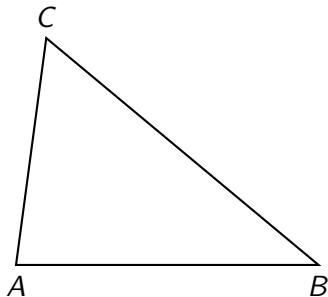
for any triangle

Auxiliary An extra line added to a diagram

Linear triple Three adjacent angles that make a straight line

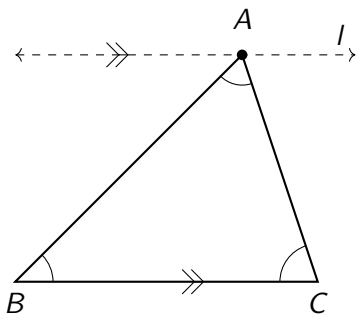
Find the missing angle measure

Given $\triangle ABC$, $m\angle A = 82^\circ$, $m\angle C = 59^\circ$. Find $m\angle B$.



Triangle sum theorem (180°)

Check your notes



Auxiliary line An extra line added to a diagram

Linear triple Three adjacent angles that make a straight line

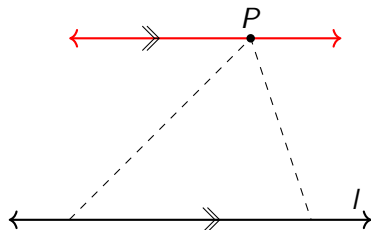
Interior angles The three angles that are inside the triangle

Theorem The sum of a triangle's angles is 180°

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

Extension: Euclid's fifth postulate (the Parallel Postulate)

Given a line and a point, there exists one line through the point parallel to the line.



Euclid Greek author of the most successful math book of all time, *The Elements*

Postulate A statement we assume is true as the basis of all further mathematical theorems and proofs

Non-Euclidean geometries Alternative mathematics not using the Parallel Postulate. Lobachevsky (1826 Russian), Bolyai (1832 Hungarian), Einstein (1916 German)

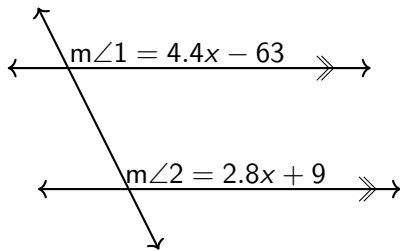
Learning Target: I can find the angles of a parallelogram

HSG.CO.C.9 Prove theorems about lines and angles

3.4 Friday 14 October

Do Now: Two parallel lines intersect a transversal. Given corresponding angles $m\angle 1 = 4.4x - 63$ and $m\angle 2 = 2.8x + 9$.

Find the measure of $\angle 1$.



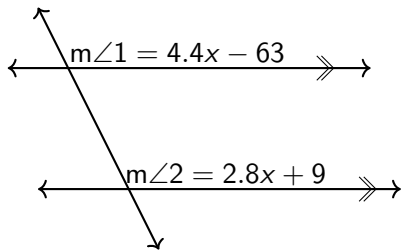
Learning Target: I can find the angles of a parallelogram

HSG.CO.C.9 Prove theorems about lines and angles

3.4 Friday 14 October

Do Now: Two parallel lines intersect a transversal. Given corresponding angles $m\angle 1 = 4.4x - 63$ and $m\angle 2 = 2.8x + 9$.

Find the measure of $\angle 1$.



Corresponding angles are \cong

$$4.4x - 63 = 2.8x + 9$$

$$1.6x = 72$$

$$x = 45$$

$$m\angle 1 = 4.4(45) - 63 = 135^\circ$$

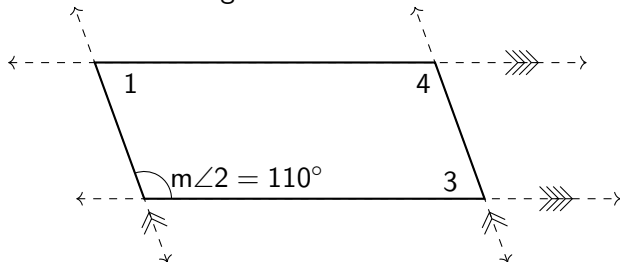
Check:

$$m\angle 2 = 2.8(45) + 9 = 135$$

A parallelogram's opposite sides are parallel and congruent

Consecutive angles are supplementary. Opposite angles are congruent.

Find the other angle measures.



Learning Target: I can calculate external triangle angles

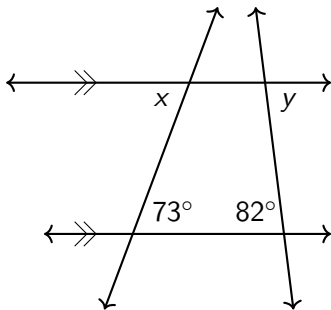
HSG.CO.C.9 Prove theorems about lines and angles

3.5 Monday 17 October

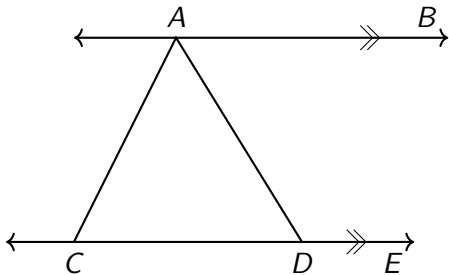
Do Now:

1. Given two parallel lines, two transversals
2. Find x , y
3. What relationship are you using? (e.g. vertical angles, same-side exterior angles, alternate interior angles, etc.)

Lesson: Triangle external angle theorem



Given parallel lines $\overleftrightarrow{AB} \parallel \overleftrightarrow{CDE}$ with $\overline{AC} \cong \overline{CD}$. If $m\angle BAD = 80$ find $m\angle ACD$.



Learning Target: I can calculate transversal angles

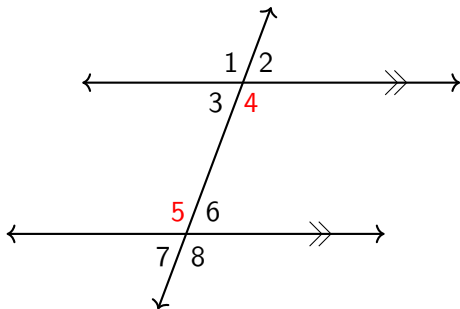
HSG.CO.C.9 Prove theorems about lines and angles

3.6 Tuesday 18 October

Given two parallel lines and a transversal,

$$m\angle 4 = 3x \text{ and } m\angle 5 = x + 70.$$

Write an equation, then solve for x .



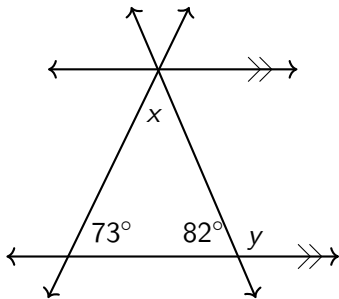
Learning Target: I can calculate angles in parallelograms

HSG.CO.C.9 Prove theorems about lines and angles

3.7 Wednesday 19 October

Do Now:

1. Given a triangle, shown
2. Find angle measures x , y
3. What relationships are you using? (e.g. vertical angles, same-side exterior angles, alternate interior angles)



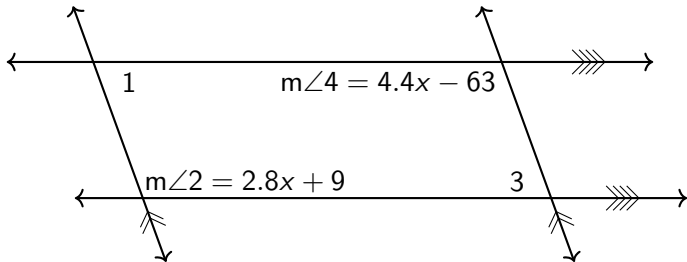
Lesson: Triangle's exterior angles

Learning Target: I can review with my classmates

HSG.CO.C.9 Prove theorems about lines and angles

3.8 Thursday 20 October

Two parallel lines intersect a second set of parallel lines. Given $m\angle 2 = 2.8x + 9$ and $m\angle 4 = 4.4x - 63$, find the measure of $\angle 1$.



Learning Target: I can review with my classmates

HSG.CO.C.9 Prove theorems about lines and angles

3.9 Friday 21 October