Geometry

9.2 Sectors

SOLUTIONS

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

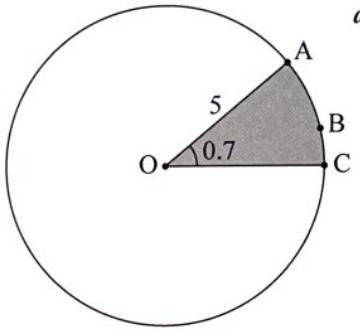
SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The following diagram shows a circle with centre O and radius 5 cm.

diagram not to scale



The points A, B and C lie on the circumference of the circle, and $A\hat{O}C = 0.7$ radians.

(a) (i) Find the length of the arc ABC.

(ii) Find the perimeter of the shaded sector. *[4]*

(b) Find the area of the shaded sector. *[2]*

(a) i) $\ell = 5 \cdot 0.7 = 3.5$ cm

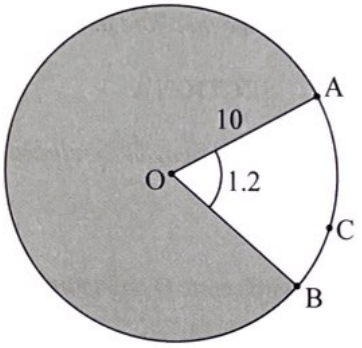
ii) $P = 3.5 + 2(5) = 13.5$ cm

(This question continues on the following page)

(b) $A = 5^2\pi \cdot \frac{0.7}{2\pi} = 8.75 \text{ cm}^2$

2. [Maximum mark: 5]

The following diagram shows a circle with centre O and a radius of 10 cm. Points A, B and C lie on the circle.



Angle AOB is 1.2 radians.

(a) Find the length of arc ACB. [2]

(b) Find the perimeter of the shaded region. [3]

(a) $l = 10 \cdot 1.2 = 12$ cm

(b) $P = 2(10) + (2(10)\pi - 12)$

$= 70.8319\ldots$

≈ 70.8 cm

3. *[Maximum mark: 7]*

The following diagram shows a circle with centre O and radius 8 cm.

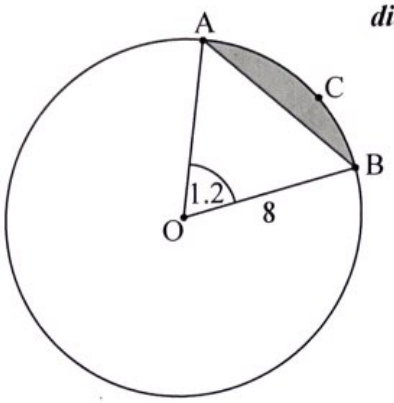


diagram not to scale

The points A, B and C are on the circumference of the circle, and $A\hat{O}B = 1.2$ radians.

(a) Find the length of arc ACB. *[2]*

(b) Find AB. *[3]*

(c) Hence, find the perimeter of the shaded segment ABC. *[2]*

(a) $l = 8(1.2) = 9.6$ cm

(b) $AB^2 = 8^2 + 8^2 - 2(8)(8)\cos 1.2$

$= 81.6182\ldots$

$AB = 9.03428\ldots$

≈ 9.03 cm

(c) $P = 9.6 + 9.03$

$= 18.63$

≈ 18.6 cm

5. *[Maximum mark: 7]*

In triangle ABC, AB = 6 cm and AC = 8 cm. The area of the triangle is $16\,\text{cm}^2$.

(a) Find the two possible values for $\hat{A}$. *[4]*

(b) Given that $\hat{A}$ is obtuse, find BC. *[3]*

(a) $A = \frac{1}{2}(6)(8)\sin A = 16$

$\sin A = \frac{2}{3}$

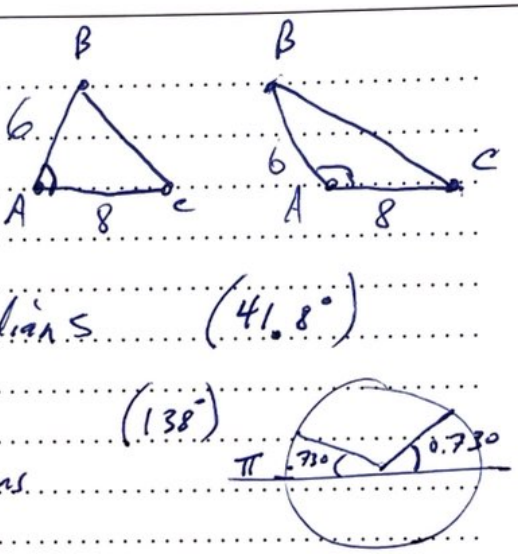
$A = \sin^{-1}\left(\frac{2}{3}\right)$

$= 0.729728\ldots$ radians $(41.8°)$

≈ 0.730

$\pi - 0.7297\ldots = 2.41186\ldots$ $(138°)$

≈ 2.41 radians



(b) $BC^2 = 6^2 + 8^2 - 2(6)(8)\cos 2.41$

$= 171.554\ldots$

$BC = 13.0579\ldots$

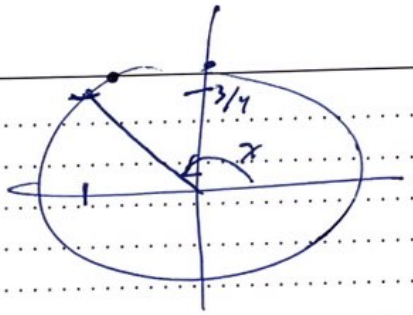
≈ 13.1 cm

5. [Maximum mark: 7]

Given that $\sin x = \frac{3}{4}$, where x is an obtuse angle, find the value of No Calculator

(a) $\cos x$; [4]

(b) $\cos 2x$. [3]



(a) $1-\left(\frac{3}{4}\right)^2 = \frac{7}{16}$

$\cos x = -\frac{\sqrt{7}}{4}$

(b) $\cos 2x = 1-2\left(\frac{3}{4}\right)^2$

$= -\frac{1}{8}$

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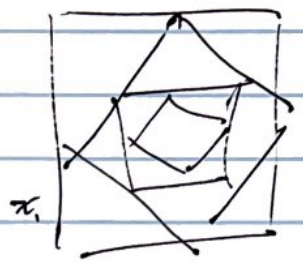
SOLUTIONS

10 a)



n	1	2	3
x_n	8	$4\sqrt{2}$	4
A_n	32	16	8

b) $A_n = 32\left(\frac{1}{2}\right)^{n-1}$

$A_6 = 32\left(\frac{1}{2}\right)^{6-1} = 1$

(c) New triangle areas B_n

$B_1 = \frac{1}{2}\left(\frac{k}{2}\right)^2$ $\quad r = \frac{1}{2}$

$S_\infty = \frac{1}{2}\left(\frac{k}{2}\right)^2\left(\frac{1}{1-\frac{1}{2}}\right) = k$

$\frac{k^2}{8}(2) = k$

$\frac{1}{4}k^2 - k = 0$

$k(k-4) = 0 \qquad k \neq 0$

$k = 4$