2. [Maximum mark: 4]

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos\left(x - \frac{\pi}{4}\right)$ and $y = 2$ when $x = \frac{3\pi}{4}$, find y in terms of x.

3. [Maximum mark: 5]

The function f is defined by $f(x) = \dfrac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f;

(ii) the horizontal asymptote of the graph of f. [2]

(b) Find the coordinates where the graph of f crosses

(i) the x-axis;

(ii) the y-axis. [2]

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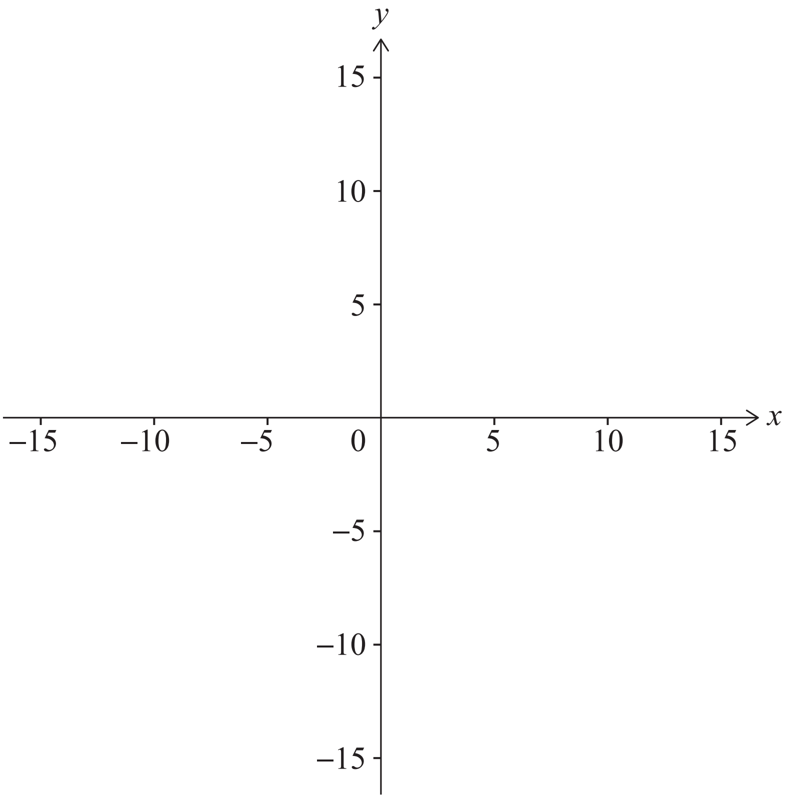
(This question continues on the following page)

(Question 3 continued)

(c) Sketch the graph of f on the axes below. [1]



6. [Maximum mark: 7]

(a) Show that $2x-3-\dfrac{6}{x-1}=\dfrac{2x^2-5x-3}{x-1}$, $x\in\mathbb{R}$, $x\neq 1$. [2]

(b) Hence or otherwise, solve the equation $2\sin 2\theta-3-\dfrac{6}{\sin 2\theta-1}=0$ for $0\leq\theta\leq\pi$, $\theta\neq\dfrac{\pi}{4}$. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle P moves along the x-axis. The velocity of P is $v\,\mathrm{m\,s^{-1}}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \le t \le 3$. When $t = 0$, P is at the origin O.

(a) (i) Find the value of t when P reaches its maximum velocity.

(ii) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [7]

(b) Sketch a graph of v against t, clearly showing any points of intersection with the axes. [4]

(c) Find the total distance travelled by P. [5]

8. [Maximum mark: 15]

Consider the function $f(x) = a^x$ where $x, a \in \mathbb{R}$ and $x > 0$, $a > 1$.

The graph of f contains the point $\left(\frac{2}{3}, 4\right)$.

(a) Show that $a = 8$. [2]

(b) Write down an expression for $f^{-1}(x)$. [1]

(c) Find the value of $f^{-1}\left(\sqrt{32}\right)$. [3]

(d) Consider the arithmetic sequence $\log_8 27$, $\log_8 p$, $\log_8 q$, $\log_8 125$, where $p > 1$ and $q > 1$.

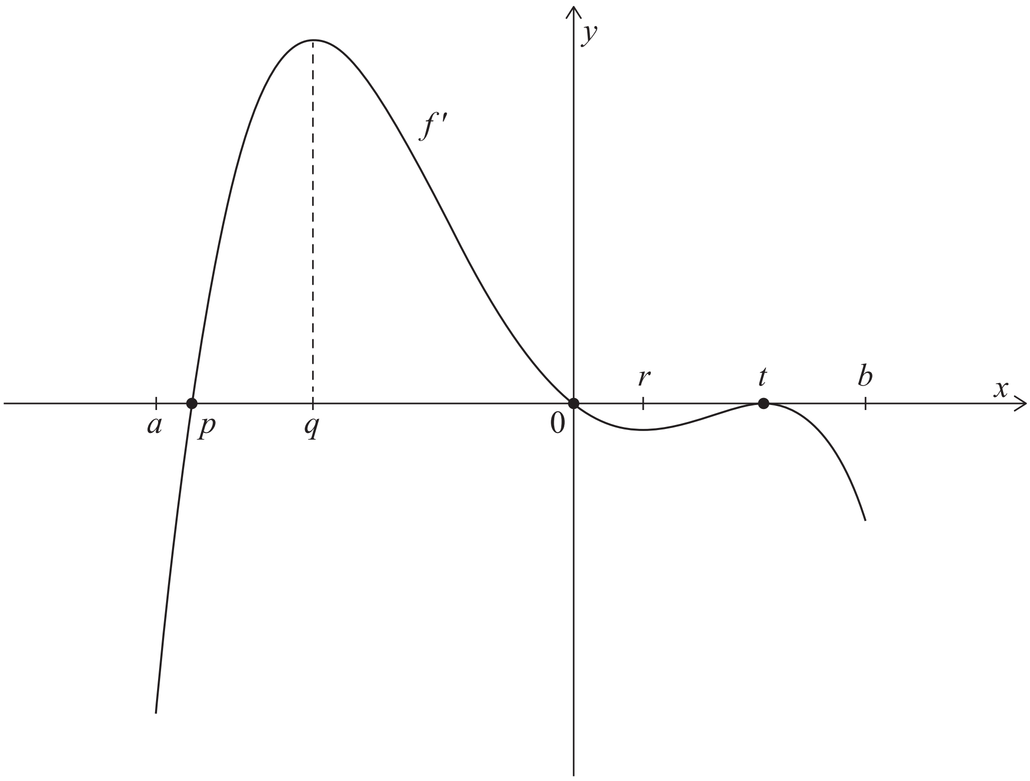
(i) Show that 27, p, q and 125 are four consecutive terms in a geometric sequence.

(ii) Find the value of p and the value of q. [9]

Do **not** write solutions on this page.

9. [Maximum mark: 14]

Consider a function f with domain $a < x < b$. The following diagram shows the graph of f', the derivative of f.



The graph of f', the derivative of f, has x-intercepts at $x = p$, $x = 0$ and $x = t$. There are local maximum points at $x = q$ and $x = t$ and a local minimum point at $x = r$.

(a) Find all the values of x where the graph of f is increasing. Justify your answer. [2]

(b) Find the value of x where the graph of f has a local maximum. [1]

(c) (i) Find the value of x where the graph of f has a local minimum. Justify your answer.

(ii) Find the values of x where the graph of f has points of inflexion. Justify your answer. [5]

(d) The total area of the region enclosed by the graph of f', the derivative of f, and the x-axis is 20.

Given that $f(p) + f(t) = 4$, find the value of $f(0)$. [6]

References: