

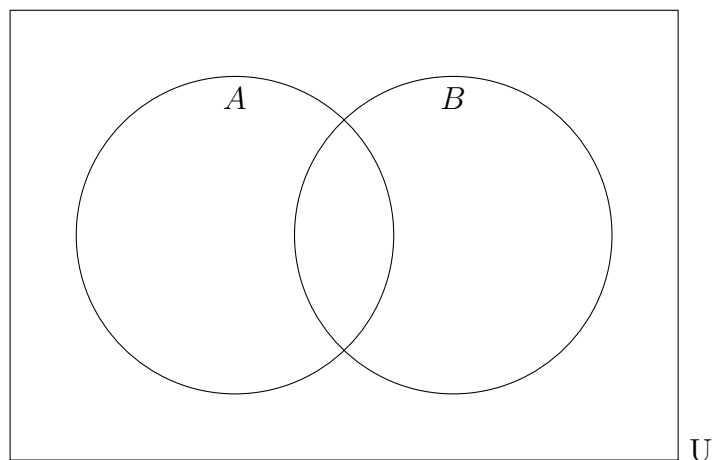
1.23 Exam: Probability, Venn diagrams

1. Given:

$$U = \{\text{the letters in the alphabet}\} \quad A = \{s, o, u, t, h\} \quad B = \{b, r, o, n, x\}$$

(a) List the members of $A \cap B$. [1 mark](b) List the elements of $A \cup B$. [1 mark](c) A letter is selected at random. What is the probability that it is a member of either or both sets, $(A \cup B)$? [1 mark]2. Events A and B are independent with $P(A) = 0.8$, $P(B) = 0.25$. Find each probability.(a) $P(A \cap B)$ [2 mark](b) $P(A \cup B)$ [2 mark](c) $P(A \cap B')$ [2 mark](d) $P(B|A)$ [2 mark]

(e) Mark the Venn diagram with the probabilities for each area. [2 marks]

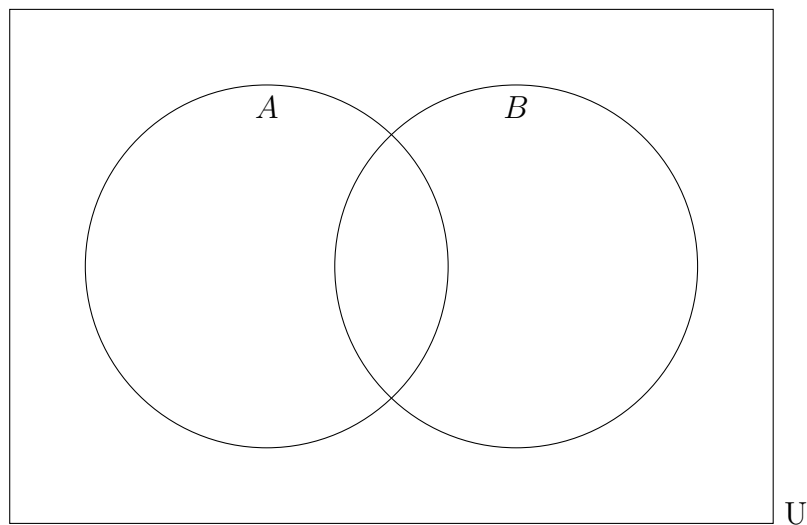


3. The universal set U is defined as the set of positive integers less than 9.

(a) Subset is defined as $A = \{\text{multiples of two}\}$. List its elements. [1 mark]

(b) Subset $B = \{\text{prime numbers}\}$. List the members of set B . [1 mark]

(c) Place the elements of U in the appropriate regions in the Venn diagram. [2 marks]



(d) List the members of $(A' \cap B)$. [1 mark]

(e) If an element is selected at random, what is the probability that it is a member of both sets, $(A \cap B)$? [1 mark]

(f) If a member of set A is selected at random, what is the probability that it is also a member of set B , i.e. the conditional probability $(B|A)$? [2 marks]

4. A jar contains 20 marbles, 12 of which are red, 5 are blue, and 3 are green.
- (a) A marble is selected at random. Find the probability it is *not* red. [1 mark]
 - (b) The marble is replaced and a second marble is selected. Given that the second marble is not red, find the probability it is green. [1 mark]
 - (c) The marbles are returned to the jar and two marbles are selected at random. Find the probability that both are blue. [2 mark]
5. Draw a tree diagram to represent the taxi cab problem in the textbook. First, there are two cab companies, 85% are black and the rest are yellow. Then, the witness identifies the color of the cab correctly 80% of the time. [3 marks]
- (a) Label the branches with the probabilities. [1 marks]
 - (b) Calculate the probabilities of each four outcomes. [2 marks]
 - (c) Given that the witness identified the cab as yellow, find the probability that it was black, i.e. that she was wrong. [3 marks]

6. Jake and Elisa are given a mathematics problem.

The probability that Jake can solve it is 0.35.

If Jake has solved it, the probability that Elisa can solve it is 0.6. Otherwise, the probability that Elisa can solve it is 0.45.

- a** Draw a tree diagram to illustrate the above question, showing clearly the probabilities on each branch.
(3 marks)
- b** Find the probability that at least one of the students can solve the problem.
(2 marks)
- c** Find the probability that Jake solves the problem, given that Elisa has.
(4 marks)

7. The events A and B are mutually exclusive with $P(A) = 0.70$ and $P(B) = 0.2$.

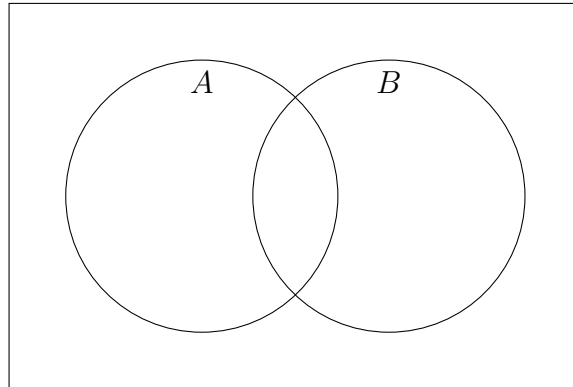
(a) Write down $P(A \cup B)$. [1 mark]

(b) Write down $P(A \cap B)$. [1 mark]

4 March 2024

8. Given events A and B with $P(A) = 0.7$, $P(B) = 0.5$, $P(A \cap B) = 0.35$.

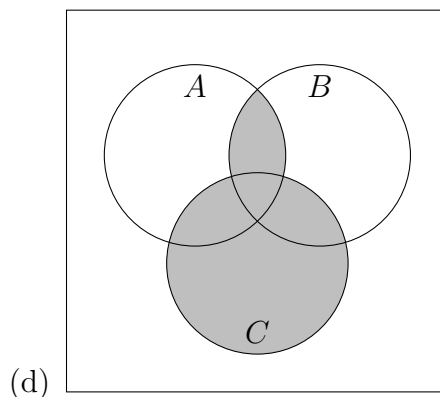
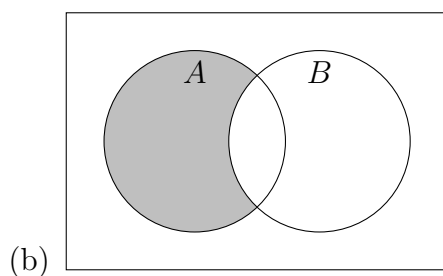
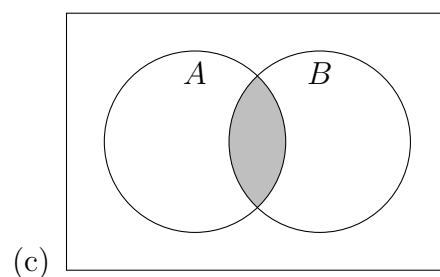
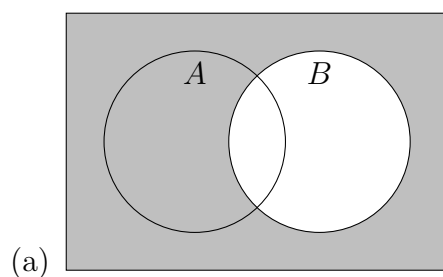
(a) Completely mark the Venn diagram with probabilities for each area. [2 marks]



(b) Find $P(A \cup B)$. [2 marks]

(c) State whether events A and B are independent. Justify your answer. [3 marks]

9. For each Venn diagram, write an expression representing the shaded area. [5 marks]



10. A survey of fruit lovers is taken, all of whom like at least one of the three fruits: apples, bananas, and cherries. The following information is gathered:

- 50 people like apples
- 40 like bananas
- 30 like cherries
- 25 like apples and bananas
- 20 like apples and cherries
- 15 like bananas and cherries
- 10 like all three fruits

Complete the Venn diagram below with the number of individuals in each region to represent the situation. How many people in total were surveyed? [4 marks]

