Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

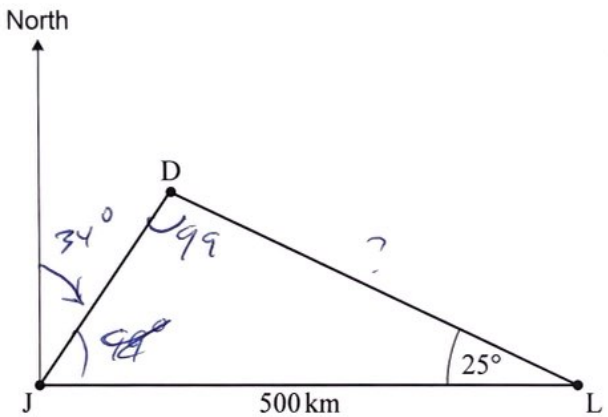
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The cities Lucknow (L), Jaipur (J) and Delhi (D) are represented in the following diagram. Lucknow lies 500 km directly east of Jaipur, and $\hat{JLD} = 25°$.

diagram not to scale



The bearing of D from J is 034°.

(a) Find $\hat{JDL}$. [2]

(b) Find the distance between Lucknow and Delhi. [3]

(a) $\hat{DJL} = 90 - 34 = 56°$ (M1)

$\hat{JDL} = 180 - (25 + 56) = 99$ A1

(b) $\frac{DL}{\sin 56°} = \frac{500}{\sin 99}$ (M1) (A1)

$DL = 419.686...$

≈ 420 km A1

2. [Maximum mark: 6]

The value of a car is given by the function $C = 40\,000(0.91)^t$, where t is in years since 1 January 2023 and C is in USD($).

(a) Write down the annual rate of depreciation of the car. [1]

(b) Find the value of the car on 1 January 2028. [2]

Alvie wants to buy this car. On 1 January 2023, he invested $\$15\,000$ in an account that earns 3% annual interest compounded yearly. He makes no further deposits to, or withdrawals from, the account.

Alvie wishes to buy this car for its value on 1 January 2028. In addition to the money in his account, he will need an extra $\$M$.

(c) Find the value of M. [3]

a) 9% A1

b) $C_5 = 40{,}000(0.91)^5$ (A1)

$= 24{,}961.2858...$

$\approx 24{,}961.29$

$\approx$ ~~25,000~~ 25,000 A1

(A1)

c) $P_5 = 15000(1+0.03)^5$

$= 17{,}389.1111...$

$C_5 - P_5 = 24{,}961.285... - 17{,}389.111...$ (M1)

$= 7572.17468...$

$\approx 7{,}572.17$

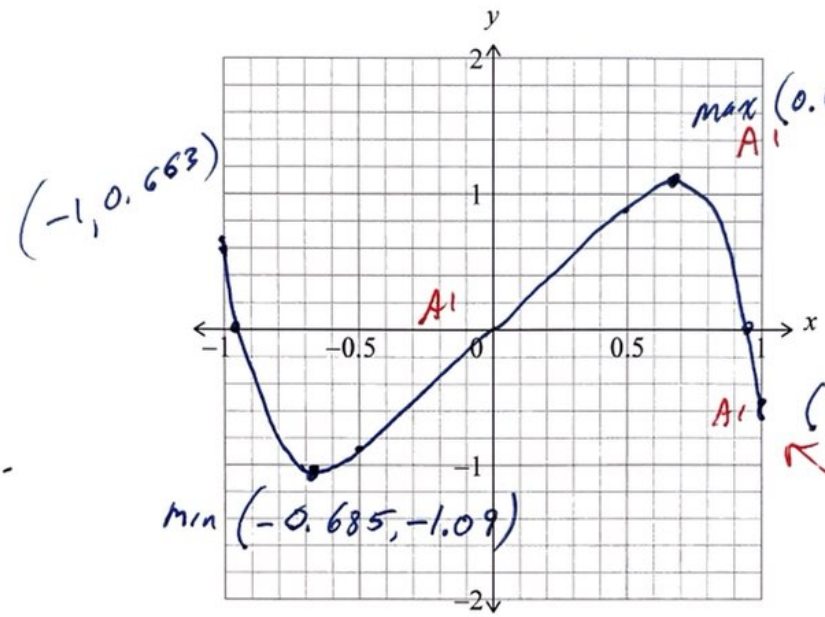
$\approx 7{,}570$ A1

3. [Maximum mark: 5]

The functions f and g are defined by $f(x) = 2x - x^3$ and $g(x) = \tan x$.

(a) Find $(f \circ g)(x)$. [2]

(b) On the following grid, sketch the graph of $y = (f \circ g)(x)$ for $-1 \leq x \leq 1$. Write down and clearly label the coordinates of any local maximum or minimum points. [3]



max (0.685, 1.09) A1

(−1, 0.663) A1

A1

(1, −0.663) A1

end points between $-0.6 > y \geq 0.8$

min (−0.685, −1.09)

a) $f(g(x)) = 2\tan x - \tan^3 x$ (M1) A1

b)

4. [Maximum mark: 7]

The total number of children, y, visiting a park depends on the highest temperature, T, in degrees Celsius (°C). A park official predicts the total number of children visiting his park on any given day using the model $y = -0.6T^2 + 23T + 110$, where $10 \leq T \leq 35$.

(a) Use this model to estimate the number of children in the park on a day when the highest temperature is $25\,°C$. [2]

An ice cream vendor investigates the relationship between the total number of children visiting the park and the number of ice creams sold, x. The following table shows the data collected on five different days.

Total number of children (y)	81	175	202	346	360
Ice creams sold (x)	15	27	23	35	46

(b) Find an appropriate regression equation that will allow the vendor to predict the number of ice creams sold on a day when there are y children in the park. [3]

(c) Hence, use your regression equation to predict the number of ice creams that the vendor sells on a day when the highest temperature is $25\,°C$. [2]

a) $y_{25} = -0.6(25)^2 + 23(25) + 110$ (M1)

$= 310$ A1

b) $m = 0.0935\ldots$ (M1)

≈ 0.0935

$b = 7.43054\ldots$ (A1)

≈ 7.43

$x = 0.0935y + 7.43$ A1

c) $x = 0.0935\left(\frac{25}{310}\right) + 7.43$ (M1)

~~9.76833~~ $36.4191\ldots \approx 36.4$ A1

$\approx$ ~~977~~ ice creams (also 36)

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

A botanist is conducting an experiment which studies the growth of plants.

The heights of the plants are measured on seven different days.

The following table shows the number of days, d, that the experiment has been running and the average height, h cm, of the plants on each of those days.

Number of days (d)	2	5	13	24	33	37	42
Average height (h)	10	16	30	59	76	79	82

(a) The regression line of h on d for this data can be written in the form $h = ad + b$.

Find the value of a and the value of b. [2]

(b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

(c) Use your regression line to estimate the average height of the plants when the experiment has been running for 20 days. [2]

a) $a = 1.93259...$
≈ 1.93 A1
$b = 7.2166...$
≈ 7.23 A1

b) $r = 0.991087...$
≈ 0.991 A1

c) $h = 1.93(20) + 7.23$ M1 (45.8 with rounded values)
$= 45.868...$
≈ 45.9 cm Note use exact parameter values – round only at the end

3. [Maximum mark: 5]

The amount of a drug, in milligrams (mg), in a patient's body can be modelled by the function $A(t) = 500e^{-kt}$, where k is a positive constant and t is the time in hours after the initial dose is given.

(a) Write down the amount of the drug in the patient's body when $t = 0$. [1]

After three hours, the amount of the drug in the patient's body has decreased to $280\,\text{mg}$.

(b) Find the value of k. [2]

The second dose is given T hours after the initial dose, when the amount of the drug in the patient's body is $140\,\text{mg}$.

(c) Find the value of T. [2]

a) 500 A1

b) $A(3) = 500e^{-k\times 3} = 280$ (A1)

$-3k = \ln\frac{280}{500}$

$K = 0.19327\ldots$

≈ 0.193 A1

c) $A(T) = 500e^{-0.193T} = 140$ (A1)

$-0.193T = \ln\frac{140}{500}$

$T = 6.5863\ldots$ A1

≈ 6.59 hours

Mock Paper 2 — Solutions

7. Cup of tea

a) i) $H(0) = 21 + 75 = 96$ °C A1

ii) $H(3) = 21 + 75e^{-0.08(3)}$

$= 79.887...$

≈ 80.0 °C A1

b) $H'(t) = -6e^{-0.08t}$

$H'(3) = -6e^{-0.08(3)}$

$= -4.7177...$

≈ -4.72 degrees/minute A2

(c) After three minutes the tea is cooling at a rate of 4.72 degrees/minute A2

(d)

$21 + 75e^{-0.08t} \leq 67$ (M1)

$-0.08t \leq \ln\left(\frac{67-21}{75}\right)$

$t \geq 6.1105$ $t \in \mathbb{Z}^+$ (A1)

7 minutes A1

e) as $t \to \infty$ (M1)

$H(t) \to 21 + 0 = 21$ °C A1

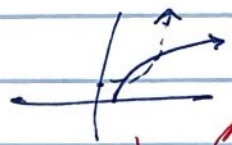
f) $H'(t) = 6e^{-0.08t}$ (M1)

as $t \to \infty$

$H'(t) \to 6/\infty = 0$ A1

Mock Paper 2 — Solutions

7. $h(x) = \sqrt{4x-2}$ $\quad x \geq \frac{1}{2}$



a) i) h^{-1}: $\quad x = \sqrt{4y-2}$ (M1)

$y = \frac{x^2+2}{4}$ $\quad x \geq 0$ A1 A1

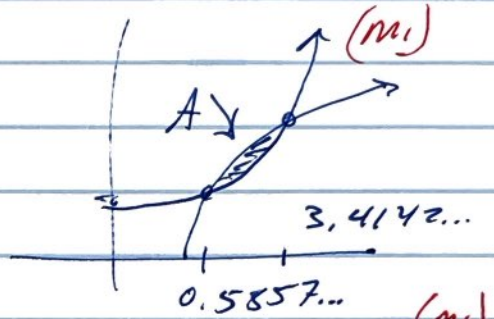
ii) Range: $y \geq \frac{1}{2}$ A1

b) 0.585786...

x 0.586 A1

3.4142...

x 3.41 A1



c) $A = 1.88561...$

≈ 1.89 A1

d) $h'(x) = \frac{1}{2}\left(\frac{1}{\sqrt{4x-2}}\right)(4)$ (M1)

$= \frac{2}{\sqrt{4x-2}}$ A1

e) ~~$h''(x) = \frac{1}{2}\left(2(4x-2)^{-\frac{3}{2}}\right)(4)$~~

$=$

$dh^{-1}/dx = \frac{1}{4}(2)(x) = \frac{x}{2}$ (A1)

$\frac{2}{\sqrt{4x-2}} = \frac{x}{2}$ (M1)

$16 = x^2(4x-2) = 4x^3 - 2x^2$

$4x^3 - 2x^2 - 16 = 0$

$x = 1.77278... \approx 1.77$ A1

Mock Paper 2 — Solutions

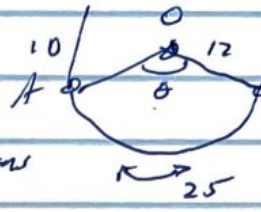
9. gutter

a) $\overset{\frown}{AB} = 45 - 10 - 10$ (M1)

$= 25$ cm (A1)

$\theta = \frac{25}{12} = 2.08333...$ radians A1

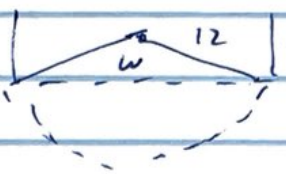
≈ 2.08



b) $w^2 = 12^2 + 12^2 - 2(12)(12)\cos\theta$ (M1) (A1)

$= 429.33...$

$w = 20.7179...$ cm (A1)



$A_{Gutter} = A_{Sector} - A_{\Delta} + A_{Rect}$

$A_{sect} = \frac{1}{2}(2.08...)12^2$ (M1)

$= 150$ (A1)

$A_{\Delta} = \frac{1}{2}(12)(12)\sin 2.08...$

$= 62.7482...$

$A_{Gutter} = 150 - 62.7482... + 10 \cdot 20.7179...$

$= 294.431...$

≈ 294 cm^2 A1 (7 total)

c) Volume of gutter $= 294.43... \times 600$

$= 176,659...$ A1

Average rate over cycle is

3000 cm^3/s

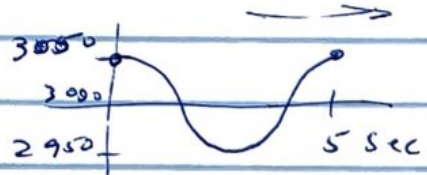
$V = 3000$ cm^3/s $\cdot$ 60 s

$= 180,000$ cm^3

$> 176,659.$

yes. It overflows A1

60 sec. $\rightarrow$ 12 cycles



(M1)(A1)A1