

# Geometry Unit 1: Segments, Length, and Area

## Bronx Early College Academy

Christopher J. Huson PhD

8-23 September 2022

1.1 Segment addition	8 September
1.2 Solve for length	9 September
1.3 Terminology and notation	12 September
1.4 Midpoint and bisector	13 September
1.5 Equilateral and isosceles triangles, perimeter	14 September
1.6 Roundtable review	15 September
1.7 Unit conversion, Exit note quiz	16 September

# Learning Target: I can measure my world

CCSS: HSG.CO.A.1 Know precise geometric definitions

1.1 Thursday 8 Sept

Do Now: Make simple measurements on paper

1. Diagram the desks *adjacent* to yours and their distances
2. Early finishers: Calculate diagonal distances

ToDo: add classroom desk image, diagram

Lesson: Points, line segments, length; Segment addition postulate

Homework (on looseleaf, due tomorrow):

1. Write for me your "math autobiography."
2. Set one Math goal for the year.
3. Optional: spicy absolute value worksheet

A *diagram* is a simplified image representing a situation

This is an example diagram of a desk arrangement

When making diagrams

Include common elements: labels, titles, distances

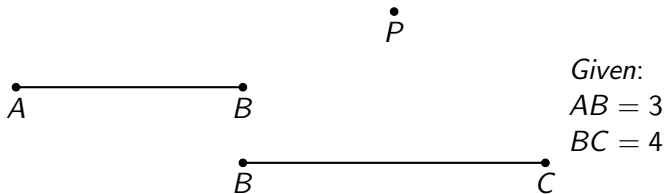
**Conventions** Standard ways of doing things to make it easier to work with other people

**Adjacent** Positioned next to each other

Write down vocabulary and terminology in your notebook with definitions and examples. (I write new terms in *italics*)

## Line segments and their endpoints

Points  $P$ ,  $A$ ,  $B$ ,  $C$ , and line segments  $\overline{AB}$ ,  $\overline{BC}$  are shown.

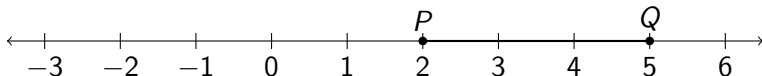


The *length* of a line segment is the distance between the two endpoints. The length of segment  $\overline{AB}$  is written  $AB$  (no bar over).

A *number line* is useful for calculating length or distance

Take the difference in the points' values

Given  $\overline{PQ}$  as shown on the number line.

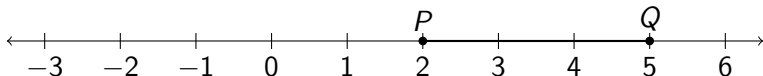


Find the distance on the number line between the points  $P$  and  $Q$ .

A *number line* is useful for calculating length or distance

Take the difference in the points' values

Given  $\overline{PQ}$  as shown on the number line.



Find the distance on the number line between the points  $P$  and  $Q$ .

$$PQ = 5 - 2 = 3$$

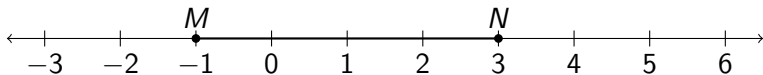
Can a length be a negative number?

Most of the lengths on our problem sets are in centimeters.

## Negative number practice on a number line

Take the difference in the points' values. Check by counting the marks.

Given  $\overline{MN}$  with  $M(-1)$  and  $N(3)$ , as shown on the number line.



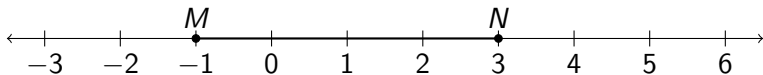
What is the length of the segment  $\overline{MN}$ ? Show your work as an equation.



## Negative number practice on a number line

Take the difference in the points' values. Check by counting the marks.

Given  $\overline{MN}$  with  $M(-1)$  and  $N(3)$ , as shown on the number line.



What is the length of the segment  $\overline{MN}$ ? Show your work as an equation.

$$MN = 3 - (-1) = 4$$

Why is “minus a negative” like adding a positive?

## Decimal practice on a number line

Mark the points then take the difference in the points' values.

Given  $\overline{GH}$  with  $G(1)$  and  $H(4.5)$ .

1. Mark and label the points and segment on the number line.
2. What is the length of the segment  $\overline{GH}$ ? Show your work as an equation.

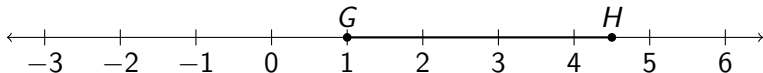


## Decimal practice on a number line

Mark the points then take the difference in the points' values.

Given  $\overline{GH}$  with  $G(1)$  and  $H(4.5)$ .

1. Mark and label the points and segment on the number line.
2. What is the length of the segment  $\overline{GH}$ ? Show your work as an equation.



$$GH = 4.5 - 1 = 3.5$$

# Take class notes in a composition book

Copy definitions using your own words. Write down example diagrams and problems

## Terminology:

**Point** A location, has no size; label with capital letter,  $P$

**Endpoint** A point at the end of a line segment

**Line segment** Two points and all the points between them; label with *endpoints* and a bar, e.g.  $\overline{AB}$

**Distance** The positive difference between two points on a number line (length is the same thing).  $AB = 3$  inches

**Number line** A line with lengths marked on it

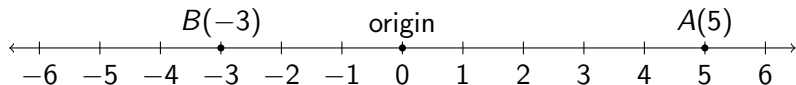
**Conventions** Standard ways of doing things to make it easier to work with other people

**Diagram** Simplified image of a situation

**Adjacent** Positioned next to each other

**Spicy:** *Absolute value* is the distance from a point to zero

“Spicy”, or extension topics, must be written in your notebook, but homework and tests are optional.



The absolute value of 5 is 5.  $|5| = 5$

The absolute value of  $-3$  is 3.  $|-3| = 3$

The absolute value of a number is always a positive number, or zero

Write the absolute value of a number  $x$  using vertical bars  $|x|$  or  $abs(x)$

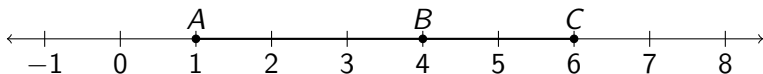
## Learning Target: I can solve for segment lengths

CCSS: HSG.CO.A.1 Know precise geometric definitions

1.2 Friday 9 September

Do Now: Given  $A(1)$ ,  $B(4)$ ,  $C(6)$ .

Write down  $AB$ ,  $BC$ , and  $AC$ .



Lesson: Segment addition, solving algebraic models

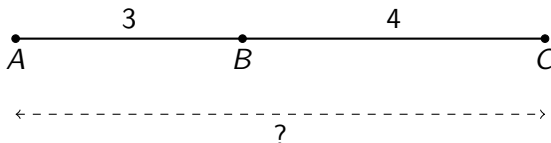
Homework: Problem set 1.2 (plus optional spicy worksheet)

# Lengths add up on a straight line

## Segment Addition Postulate

Shown *collinear* points  $A$ ,  $B$ ,  $C$ . Given  $AB = 3$ ,  $BC = 4$ .

Find  $AC$ .



Definitions:

**Collinear** Points that lie on the same straight line

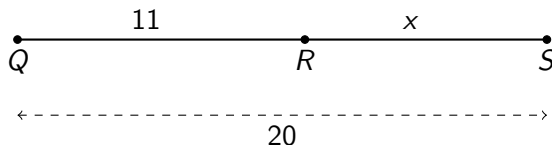
**Postulate** A rule that we assume is true

# Use a variable ( $x$ ) to represent an unknown value

An equation is a *model* of a situation

Given collinear points  $Q$ ,  $R$ ,  $S$ , with  $QR = 11$ ,  $QS = 20$ .

Find  $RS$ .



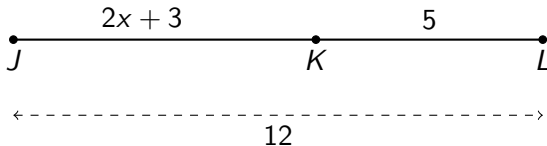
1. How would you check your answer?
2. Which equation represents the situation?

$$11 + x = 20 \qquad x = 20 - 11$$



## Step-by-step modeling

Given  $\overline{JKL}$ ,  $JK = 2x + 3$ ,  $KL = 5$ ,  $JL = 12$ . Find  $x$ .

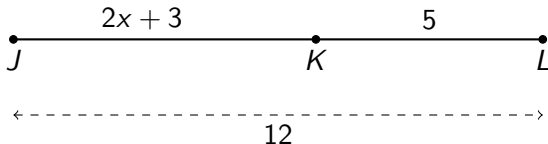


1. Write down an equation to represent the situation.
2. Solve for  $x$ .
3. Check your answer.

The diagram may be given, or you may have to sketch it

Write the steps in your notebook

Given  $\overline{JKL}$ ,  $JK = 2x + 3$ ,  $KL = 5$ ,  $JL = 12$ . Find  $x$ .



$$JK + KL = JL$$

$$(2x + 3) + 5 = 12$$

$$2x + 8 = 12$$

$$2x = 4$$

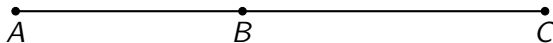
$$x = 2$$

$$2(2) + 3 + 5 = 12?$$

1. Sketch and label the situation
2. Write a geometric equation
3. Substitute algebraic values
4. Solve for  $x$
5. Answer the question
6. **Check** your answer

Mark the diagram, find  $x$ , answer  $AB = ?$

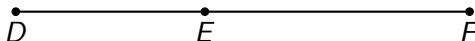
Given  $\overline{ABC}$ ,  $AB = 3x - 7$ ,  $BC = x + 5$ ,  $AC = 14$ .



Find  $AB$ .

## More practice: Solve an equation with $x$ on both sides

Given  $\overrightarrow{DEF}$ ,  $DE = x + 1$ ,  $EF = 9$ ,  $DF = 3x$ . Find  $DE$ .



# Lengths in a straight line add up

Check your notebook for completeness

## Segment Addition Postulate

Mathematics is constructed of fundamental rules or postulates, and basic objects like points, lines, and numbers.

Vocabulary:

**Collinear** Points that lie on the same straight line

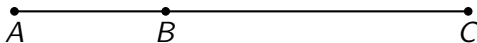
**Postulate** A rule that we assume is true (also called *axioms*)

**Modeling** Using an equation (algebra) to represent a situation in a simplified way

**Check** Substitute the value of  $x$  into the equation to test whether it is correct

## Spicy: Fractional *coefficients*

Given  $\overline{ABC}$ ,  $AB = \frac{1}{2}x$ ,  $BC = x$ ,  $AC = 21$ . Find  $x$ .



**Term** An expression representing a number, for example  $\frac{1}{2}x$

**Variable** The unknown value represented by a letter ( $x$ )

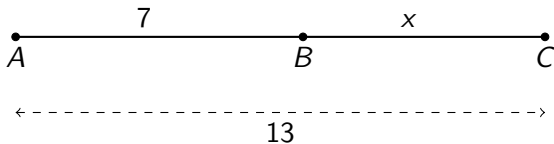
**Coefficient** The fixed number in front of the variable. (e.g.  $\frac{1}{2}$ )

## Learning Target: I can use geometric conventions

CCSS: HSG.CO.A.1 Know precise geometric definitions

1.3 Monday 12 Sept

Do Now: Given collinear points  $A$ ,  $B$ ,  $C$ , with  $AB = 7$ ,  $AC = 13$ .



1. Circle the equation that most simply represents the situation.

$$7 + x = 13$$

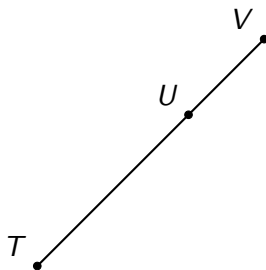
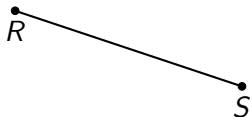
$$x = 13 - 7$$

2. Find  $BC$ .

Write down an example of each geometric object.

Use proper notation.

1. point
2. line segment
3. endpoint
4. three collinear points

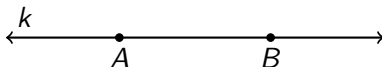


5. Given  $TU = 1.4$ ,  $UV = 0.6$ . Find  $TV$ . (label the diagram first)

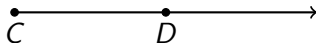


## More definitions: lines, rays, planes

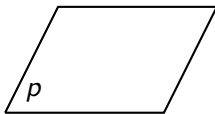
A *line* extends infinitely in both directions,  $\overleftrightarrow{AB}$ .  
(sometimes labeled with a small letter, for example, line  $k$ )



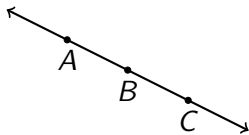
A *ray* has one endpoint and extends infinitely in one direction,  $\overrightarrow{CD}$ .



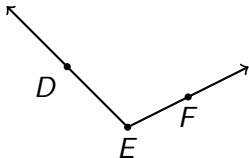
A *plane* is flat and extends infinitely in two directions,  $p$ .



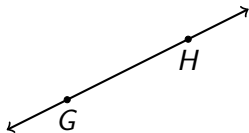
*Opposite rays* are collinear rays with a common endpoint.



$\overrightarrow{BA}$  and  $\overrightarrow{BC}$  are opposite rays.



These rays do not make a straight line.

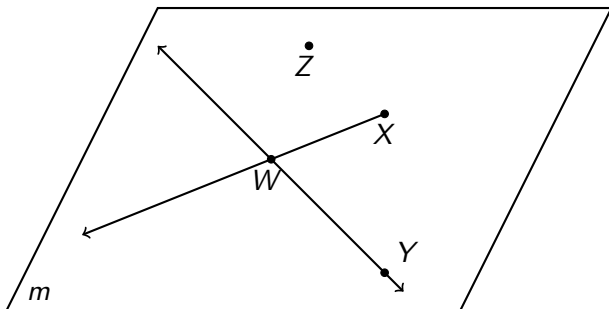


The rays  $\overrightarrow{GH}$  and  $\overrightarrow{HG}$  do not share a common endpoint.

## Several objects are shown in a plane

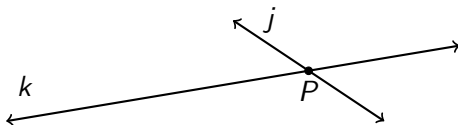
Circle true or false

1. T F The name of the plane is  $m$ .
2. T F The line  $\overleftrightarrow{WY}$  is in the plane.
3. T F The ray  $\overrightarrow{WX}$  is shown in the plane.
4. T F Points  $W$ ,  $X$ , and  $Z$  are collinear.
5. T F  $\overleftrightarrow{WY}$  and  $\overleftrightarrow{YW}$  are the same line.

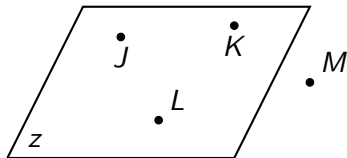


## More definitions: intersections, coplanar

Two lines *intersect* if they cross. Their common point is the *intersection*. (shown here, lines  $j$  and  $k$  intersect at point  $P$ )



*Coplanar* means to lie in the same plane. Three points are always coplanar, but four points may not be.



# Learn and practice using formal language and notation

**Line** An infinite collection of points extending straight in both directions indefinitely,  $\overleftrightarrow{AB}$  or  $l$

**Ray** An endpoint and half of a straight line extending away from the endpoint,  $\overrightarrow{JK}$

**Plane** A flat surface extending infinitely in two dimensions,  $p$

**Opposite rays** Collinear rays with a common endpoint.

**Coplanar** Points or objects all in the same plane

**Intersection** Where two lines cross, the common point

Spicy: Which is the more efficient method,  
*distribute* or multiply both sides by 3?

$$\frac{2}{3}(x + 5) = 4$$

$$\frac{2}{3}(x + 5) = 4$$

**Distribute** Multiply both terms in parentheses by the coefficient

**Numerator** The top of a fraction (i.e.  $p$  in  $\frac{p}{q}$ )

**Denominator** The bottom of a fraction (i.e.  $q$  in  $\frac{p}{q}$ )

**LCD** Converting to the *Lowest Common Denominator* is the most efficient way to add fractions

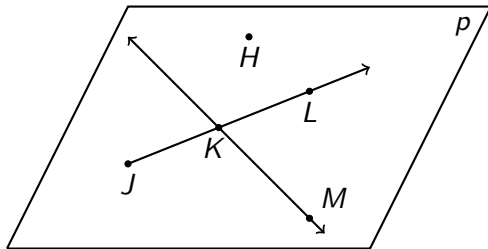
# Learning Target: I can *bisect* a length

CCSS: HSG.CO.A.1 Know precise geometric definitions

1.4 Tuesday 13 Sept

Do Now: Circle or mark each object in the plane

1. The point  $H$
2. The ray  $\overrightarrow{JL}$
3. The name of the plane shown

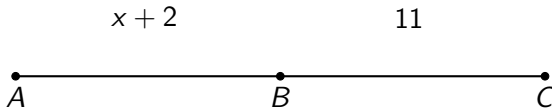


Lesson: Midpoint, congruence, bisection

The point  $B$  *bisects* the segment  $\overline{AC}$

Point  $B$  is in the exact middle between  $A$  and  $C$

Given  $AB = x + 2$ ,  $BC = 11$ . Find  $x$ .



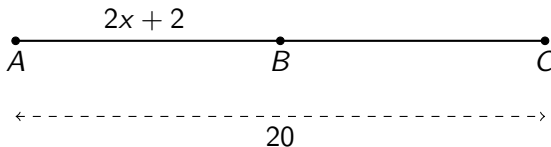
Hint: The line segment is split into two equal lengths.



## The *midpoint* of a line segment

Given  $\overline{ABC}$ , with  $AB = 2x + 2$ ,  $AC = 20$ .  $AB = BC$

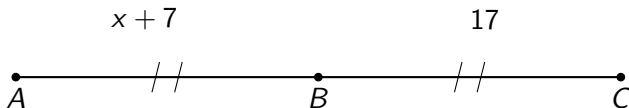
Find  $x$ .



A *bisector* creates two line segments with the same length

*Congruent* line segments are the same length

Given point  $B$  is the midpoint of  $\overline{AC}$ , with  $AB = x + 7$ ,  $BC = 17$ .  
Find  $x$ .



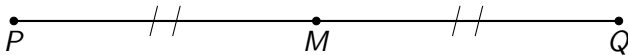
The *midpoint* or *bisector* of a line segment divides it exactly in half.

*Congruent* means equal in length,  $\overline{AB} \cong \overline{BC}$  (also  $AB = BC$ )

Mark congruent segments in diagrams with cross “*hash*” marks.

## Check your notes

$M$  bisects  $\overline{PQ}$



**Bisect** Divide exactly in half

**Midpoint** The point in the exact middle of a line segment

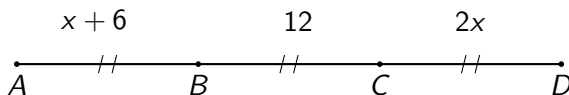
**Congruent** Equal in length or measure.  $\overline{AB} \cong \overline{BC}$

**Hash marks** Mark congruent segments with small crossways lines (also called “tick” marks)

## Spicy: *Trisect* a segment into three congruent parts

Points  $B$  and  $C$  trisect segment  $\overline{AD}$  with segment lengths as shown.

Find  $x$ .



**Trisect** Divide exactly in three equal parts

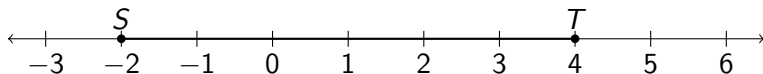
**Partition** Cut into parts (not necessarily evenly)

# Learning Target: I can work with objects having congruent parts

CCSS: HSG.CO.A.1 Know precise geometric definitions

1.5 Wednesday 14 Sept

Do Now: Given  $\overline{ST}$  with  $S(-2)$  and  $T(4)$



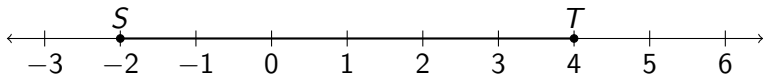
What is the length of the segment  $\overline{ST}$ ? Show your work as an equation.

Lesson: Perimeter, congruent line segments in rectangles & isosceles triangles

## Negative number practice on a number line

Take the difference in the points' values. Check by counting the marks.

Given  $\overline{ST}$  with  $S(-2)$  and  $T(4)$ , as shown on the number line.



What is the length of the segment  $\overline{ST}$ ? Show your work as an equation.

Solution

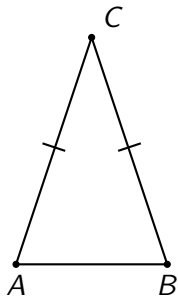
$$ST = 4 - (-2) = 6$$

Why is “minus a negative” the same as add a positive?

## An *isosceles* triangle has two congruent sides

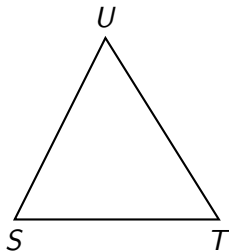
Given isosceles  $\triangle ABC$ . Which two sides are congruent?

Write your answer using symbols (i.e. two segments and  $\cong$ )



On the diagram mark the congruent line segments with tick marks.

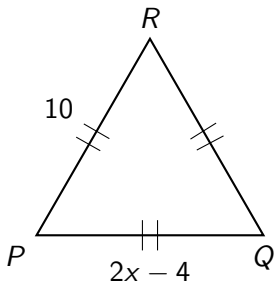
Given isosceles  $\triangle STU$  with  $\overline{ST} \cong \overline{TU}$ .





An *equilateral* triangle has all three sides congruent

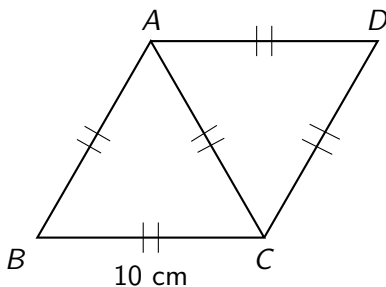
Given equilateral  $\triangle PQR$  with  $PQ = 2x - 4$ ,  $PR = 10$ . Find  $x$ .



The *perimeter* is the distance around the triangle. Find the perimeter of  $\triangle PQR$ .

## A *quadrilateral* has four sides

Given two *adjacent* equilateral  $\triangle$ s,  $\triangle ABC$  and  $\triangle ACD$ . All sides measure 10 cm.



Find the perimeter of the quadrilateral  $ABCD$ .

# Check your notes

**Equilateral** Triangle with all three sides congruent

**Isosceles** Triangle having two sides of the same length

**Scalene** Triangle without any sides of matching lengths

**Quadrilateral** A four-sided figure (examples: square, rectangle, parallelogram, rhombus, kite)

**Polygon** Objects with multiple sides (e.g. triangle, quadrilateral, pentagon, hexagon)

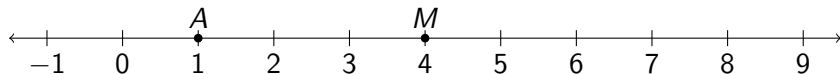
**Perimeter** The total length around a figure (all sides added)

**Adjacent** “next to”, two things that are side by side

## Spicy: Given the midpoint, find an end point

Points  $A(1)$ ,  $M(4)$ , and  $B$  lie on a numberline.  $M$  bisects  $\overline{AB}$ .

Find  $B$ .



# Learning Target: I can collaborate in review

CCSS: HSG.CO.A.1 Know precise geometric definitions 1.6 Thursday 15 September

Do Now: Given the points  $X$  and  $Y$ , draw  $\overrightarrow{YX}$ .

(careful! which direction does it go?)

$\dot{X}$

$\dot{Y}$

Lesson: Roundtable quiz review

# Groupwork review for quiz tomorrow

“Roundtable” of four students, with four topics assigned

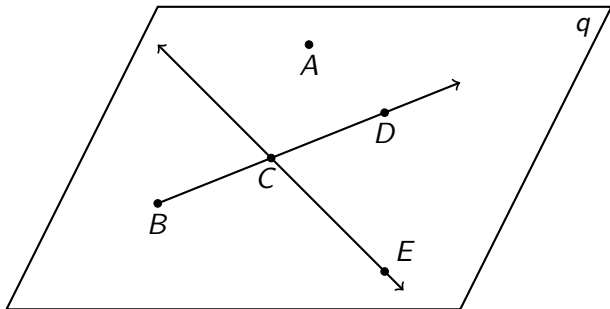
## Geometry skills to study / teach

1. Conventions: terminology, notation, diagramming
2. Modeling situations with algebra
3. Perimeter and special shapes:
  - ▶ Scalene, isosceles, and equilateral  $\triangle$ s
  - ▶ Squares, rectangles, parallelograms, trapezoids, rhombuses, kites (quadrilateral side  $\cong$ s will be marked)
4. Solving algebraic equations for one variable

# 1. Identify each item.

Example of Topic 1: Conventions: terminology, notation, diagramming

1. The point  $A$
2. The ray  $\overrightarrow{BD}$
3. The name of the plane



## 2. Write down an equation to represent the situation

Example of Topic 3: Modeling situations with algebra

Given  $M$  is the midpoint of  $\overline{AB}$ ,  $AM = 4x + 2$ ,  $AB = 20$ .

*First mark the diagram with hash marks and values.*



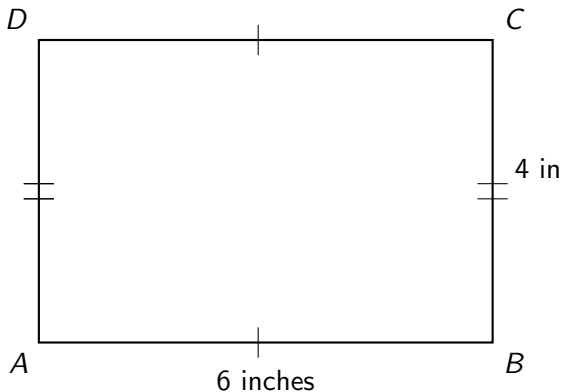
*Sometimes you will not be asked to solve the equation.*



### 3. Find the perimeter of the rectangle $ABCD$

Example of Topic 2: Perimeter and special triangles and quadrilaterals

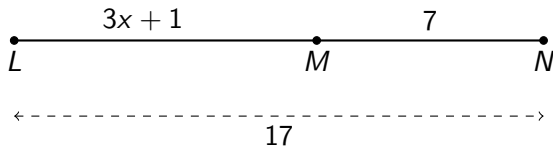
Given  $AB = 6$  inches,  $BC = 4$  inches.



## 4. Solve for $x$

Example of Topic 4: Solving algebraic equations for one variable

Given  $\overline{LMN}$ ,  $LM = 3x + 1$ ,  $MN = 7$ ,  $LN = 17$ .



$$(3x + 1) + 7 = 17$$

*You must check the solution.*

# Learning Target: I can change units of length

CCSS: HSG.CO.A.1 Know precise geometric definitions

1.7 Friday 16 September

Do Now: Mike is six feet tall. How many inches is that?

Conversion: 1 foot = 12 inches

Exit note quiz today

# Multiply by *conversion factors* to change units

reference: [Wikipedia Dimension analysis](#)

Mike is six feet tall. How many inches is that?

$$H = 6 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} = 72 \text{ inches}$$

**Conversion factor** is a ratio of units equal to one, for example,

$$\frac{12 \text{ inches}}{1 \text{ foot}} = 1$$

## Numerator vs denominator of conversion factors

An American football field is 100 yards long. How many feet is that?

$$1 \text{ yard} = 3 \text{ feet}$$

## Numerator vs denominator of conversion factors

An American football field is 100 yards long. How many feet is that?

$$1 \text{ yard} = 3 \text{ feet}$$

$$L = 100 \text{ yards} \times \frac{3 \text{ feet}}{1 \text{ yard}} = 300 \text{ feet}$$

Each conversion factor ratio has two forms:

$$\frac{1 \text{ yards}}{3 \text{ feet}} = \frac{3 \text{ feet}}{1 \text{ yards}} = 1$$

## Cancel units when choosing correct conversion factor

reference: [NY State Regents Exam formula sheet](#)

Stephen's height is  $H = 69$  inches. Find his height in meters.

$$1 \text{ meter} = 39.37 \text{ inches}$$

## Cancel units when choosing correct conversion factor

reference: NY State Regents Exam formula sheet

Stephen's height is  $H = 69$  inches. Find his height in meters.

$$1 \text{ meter} = 39.37 \text{ inches}$$

$$H = 69 \text{ inches} \times \frac{1 \text{ meter}}{39.37 \text{ inches}} = 1.7526 \dots \text{ meter}$$

Select the ratio with inches in the denominator:

$$\frac{39.37 \text{ inches}}{1 \text{ meter}} = \frac{1 \text{ meter}}{39.37 \text{ inches}} = 1$$