Solutions

Lesson 1 Practice Problems

1. Select **all** sequences that could be geometric.

   $\times 2$ $\times 7/4$

   A. 2, 4, 7, 11, . . .

   $\times 3$

   B. $\frac{1}{3}$, 1, 3, 9, . . .

   $+2$

   C. 1, 3, 5, 7, . . .

   $\times 4$ $\times 4$

   D. $\frac{1}{2}$, 2, 8, 32, . . .

   $\div 5$ $\times \frac{1}{5}$

   E. 1,000, 200, 40, 8, . . .

   -100

   F. 999, 899, 799, 699, . . .

2. A blogger had 400 subscribers to her blog in January. The number of subscribers has grown by a factor of 1.5 every month since then. Write a sequence to represent the number of subscribers in the 3 months that followed.

   400, 600, 900, 1350

3. Tyler says that the sequence 1, 1, 1,... of repeating 1s is not exponential because it does not change. Do you agree with Tyler? Explain your reasoning.

   disagree $r = 1$

   $1 \times 1 = 1$
   $1 \times 1 = 1$ etc

4. In 2000, an invasive plant species covered 0.2% of an island. For the 5 years that followed, the area covered by the plant tripled every year.

   A student said, "That means that about half of the island's area was covered by the plant in 2005!"

   Do you agree with his statement? Explain your reasoning.

year	
0	0.2%
1	0.6%
2	1.8%
3	5.4%
4	16.2%
5	48.6%

5. A square picture with side length 30 cm is scaled by 60% on a photocopier. The copy is then scaled by 60% again.

   a. What is the side length of the second copy of the picture?

   $30 \times (0.60)^2 = 10.8$

   b. What is the side length of the picture after it has been successively scaled by 60% 4 times? Show your reasoning.

   $30 \times 0.6^4 = 3.888$

6. A geometric sequence g starts 5, 15, ... Explain how you would calculate the value of the 50th term.

   $\curvearrowright$ $*3$

   $r = \frac{15}{5} = 3$

   $a_{50} = 5 \cdot 3^{(50-1)} \approx 1.2 \times 10^{24}$

   (very big!)

(From Unit 1, Lesson 8.)

7. Select **all** the expressions equivalent to 9^4. $= (3^2)^4 = 3^8$

   A. 3^6

   B. 3^8

   C. $9^2 \times 9^2$

   D. $\frac{9^4}{9^{-2}}$

   E. $3^4 \times 3^4$

(From Unit 3, Lesson 1.)

Lesson 2 Practice Problems

1. In 1990, the value of a home is \$170,000. Since then, its value has increased 5% per year.

   a. What is the approximate value of the home in the year 1993?

   $v(1993) = 170{,}000 \cdot (1.05)^3 = 196{,}796.25$

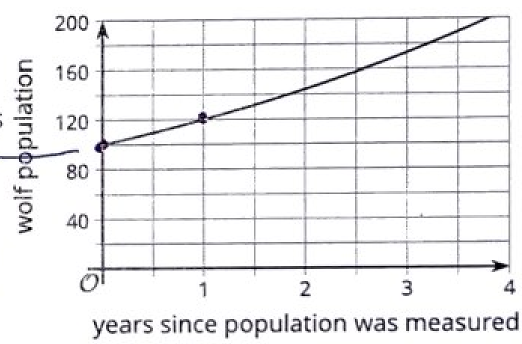
   b. Write an equation, in function notation, to represent the value of the home as a function of time in years since 1990, t.

   $v(t) = 170{,}000\,(1.05^{t})$

   c. Will the value of the home be more than \$500,000 in 2020 (assuming that the trend continues)? Show your reasoning.

   yes. $v(30) = 170{,}000\,(1.05^{30}) = 734{,}730.20\ldots > 500{,}000$

2. The graph shows a wolf population which has been growing exponentially.



   a. What was the population when it was first measured? 100

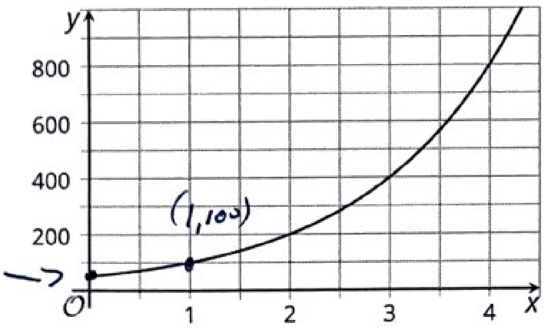
   b. By what factor did the population grow in the first year? $\frac{120}{100} = 1.2$

   c. Write an equation relating the wolf population, w, and the number of years since it was measured, t.

   $w(t) = 100 \times 1.2^{t}$

3. Here is the graph of an exponential function f.

(0, 50) →



Find an equation defining f. Explain your reasoning.

$f(x) = 50 * 2^x$

$\frac{100}{50} = 2$ growth factor

4. The equation $f(t) = 24{,}500 \cdot (0.88)^t$ represents the value of a car, in dollars, t years after it was purchased.

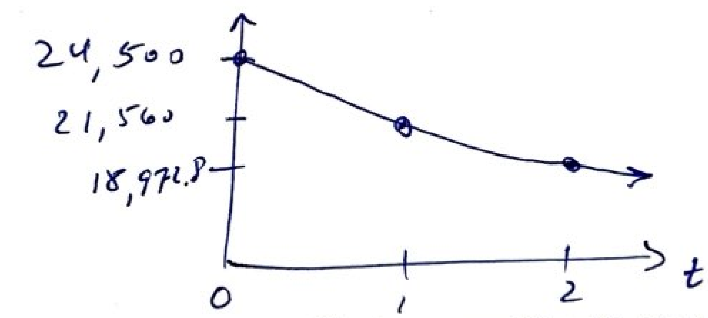
   a. What do the numbers 24,500 and 0.88 mean?

   the initial value of the car is \$24,500
   The car maintains 0.88 of its value each year

   b. What does $f(9)$ represent?

   The value of the car after nine years

   c. Sketch a graph that represents the function f and shows $f(0)$, $f(1)$, and $f(2)$.



5. The first two terms of an exponential sequence are 18 and 6. What are the next 3 terms of this sequence?

   $18, 6, 2, \frac{2}{3}, \frac{2}{9}$ ($\frac{1}{3}$)

   (From Unit 4, Lesson 1.)

6. A bacteria population has been doubling each day for the last 5 days. It is currently 100,000. What was the bacterial population 5 days ago? Explain how you know.

$f(5) = A \cdot 2^5 = 100{,}000$

$A \cdot 32 = 100{,}000$

$A = 100{,}000/32 = 3125$

100,000 5
50,000 4
25,000 3
12,500 2
6,250 1
3125

(From Unit 4, Lesson 1.)

7. Select all expressions that are equivalent to $27^{\frac{1}{3}}$. $= \sqrt[3]{27} = 3$

A. 9

B. 3

C. $\sqrt{27}$

D. $\sqrt[3]{27}$

E. $\sqrt[3]{3^3}$

F. $\frac{1}{27}$

G. $\frac{1}{27^3}$

(From Unit 3, Lesson 3.)

Lesson 3 Practice Problems

1. Select **all** solutions to $m \cdot m \cdot m = 729$. $9 \cdot 9 \cdot 9 = 729$

   A. $\sqrt{729}$ x

   B. $\frac{729}{3}$ x

   C. $\frac{\sqrt{729}}{3}$ x

   D. $\frac{1}{3}\sqrt{729}$ x

   E. $729^{\frac{1}{3}}$

   F. $\sqrt[3]{729}$

2. In a pond, the area that is covered by algae doubles each week. When the algae was first spotted, the area it covered was about 12.5 square meters.

   a. Find the area, in square meters, covered by algae 10 days after it was spotted. Show your reasoning.

   $t = \frac{10}{7}$

   $A = 12.5 \times 2^{\left(\frac{10}{7}\right)}$

   $= 33.6475\ldots \approx 33.6$ sq. m

   b. Explain why we can find the area covered by algae 1 day after it was spotted by multiplying 12.5 by $\sqrt[7]{2}$.

   $t = \frac{1}{7}$

   $A = 12.5 \cdot 2^{\frac{1}{7}}$

   $= 12.5 \times \sqrt[7]{2}$

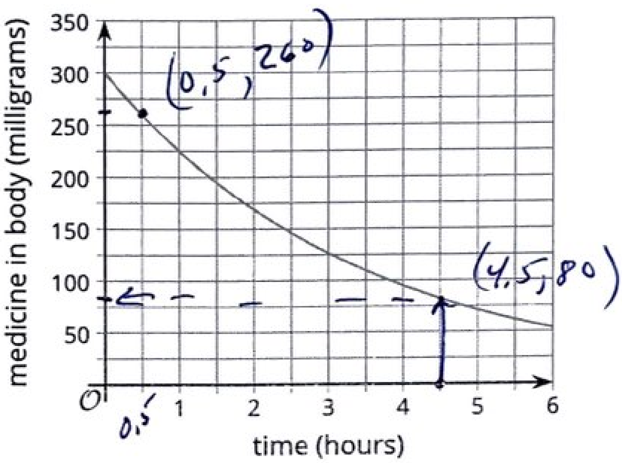
   fractional exponent is a radical

3. The function m, defined by $m(h) = 300 \cdot \left(\frac{3}{4}\right)^h$, represents the amount of a medicine, in milligrams, in a patient's body. h represents the number of hours after the medicine is administered.

   a. What does $m(0.5)$ represent in this situation?

   the amount after 0.5 hours

   b. This graph represents the function m. Use the graph to estimate $m(0.5)$.



   ≈ 260

   c. Suppose the medicine is administered at noon. Use the graph to estimate the amount of medicine in the body at 4:30 p.m. on the same day.

   4.5 hours later about 80 mg

4. The area covered by a lake is 11 square kilometers. It is decreasing exponentially at a rate of 2 percent each year and can be modeled by $A(t) = 11 \cdot (0.98)^t$.

   a. By what factor does the area decrease in 10 years?

   $0.98^{10} = 0.817072\ldots \approx 0.817$

   b. By what factor does the area decrease each month?

   $0.98^{1/12} = 0.998317\ldots \approx 0.9983$

5. The third and fourth numbers in an exponential sequence are 100 and 500. What are the first and second numbers in this sequence?

   (From Unit 4, Lesson 1.)

   $*5$

   4, 20, 100, 500

   $* 1/5$ $* \frac{1}{5}$

6. The population of a city in thousands is modeled by the function $f(t) = 250 \cdot (1.01)^t$ where t is the number of years after 1950. Which of the following are predicted by the model? Select **all** that apply.

   A. The population in 1950 was 250.

   B. The population in 1950 was 250,000.

   C. The population grows by 1 percent each year.

   D. The population in 1951 was 275,000.

   E. The population grows exponentially.

(From Unit 4, Lesson 2.)