Solutions

Lesson 4 Practice Problems

1. A bacteria population is tripling every hour. By what factor does the population change in $\frac{1}{2}$ hour? Select **all** that apply.

   A. $\sqrt{3}$

   B. $\frac{3}{2}$

   C. $\sqrt[3]{2}$

   D. $3^{\frac{1}{2}}$

   E. 3^2

2. A medication has a half-life of 4 hours after it enters the bloodstream. A nurse administers a dose of 225 milligrams to a patient at noon.

   a. Write an expression to represent the amount of medication, in milligrams, in the patient's body at:

      i. 1 p.m. on the same day $225 \cdot \left(\frac{1}{2}\right)^{1/4}$

      ii. 7 p.m. on the same day $225 \cdot \left(\frac{1}{2}\right)^{\frac{7}{4}}$

   b. The expression $225 \cdot \left(\frac{1}{2}\right)^{\frac{5}{2}}$ represents the amount of medicine in the body some time after it is administered. What is that time?

      $\frac{5}{2} = \frac{10}{4} =$ 10 pm

3. The number of employees in a company has been growing exponentially by 10% each year. By what factor does the number of employees change:

   a. Each month? $1.10^{1/12}$

   b. Every 3 months? $1.10^{3/12}$

   c. Every 20 months? $1.10^{20/12}$

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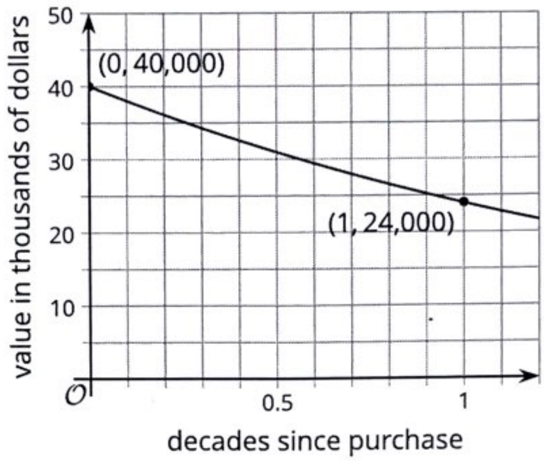
4. The value of a truck decreases exponentially since its purchase. The two points on the graph shows the truck's initial value and its value a decade afterward.



a. Express the car's value, in dollars, as a function of time d, in decades, since purchase.

$\frac{24{,}000}{40{,}000} = \frac{3}{5}$ $\quad V(d) = 40{,}000 \cdot \left(\frac{3}{5}\right)^{d}$

b. Write an expression to represent the car's value 4 years after purchase.

$V\left(\frac{4}{10}\right) = 40{,}000 \cdot \left(\frac{3}{5}\right)^{4/10}$

c. By what factor is the value of the car changing each year? Show your reasoning.

$\left(\frac{3}{5}\right)^{1/10} = \sqrt[10]{3/5}$ $\quad$ the $1/10^{th}$ power of the factor $3/5$

5. The value of a stock increases by 8% each year.

a. Explain why the stock value does not increase by 80% each decade.

It is an exponential increase, not linear

$1.08^{10} \neq 0.08 \times 10$

b. Does the value increase by more or less than 80% each decade?

more

$1.08^{10} = 2.1589\ldots > 1.80$

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6. Decide if each statement is true or false.

   a. $50^{\frac{1}{2}} = 25$ FALSE

   b. $\sqrt{30}$ is a solution to $y^2 = 30$. TRUE

   c. $243^{\frac{1}{3}}$ is equivalent to $\sqrt[3]{243}$. TRUE

   d. $\sqrt{20}$ is a solution to $m^4 = 20$. False

   (From Unit 4, Lesson 3.)

7. Lin is saving \$300 per year in an account that pays 4.5% interest per year, compounded annually. About how much money will she have 20 years after she started?

   A. \$545.45

   B. \$3,748.78

   C. \$9,411.43

   D. \$1,124,634.54

   $$S = 300 \cdot \frac{(1-1.045^{20})}{(1-1.045)}$$

   $$= \$9411.43$$

   (From Unit 2, Lesson 26.)

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Illustrative Mathematics

Solutions

Lesson 5 Practice Problems

1. The table shows the monthly revenue of a business rising exponentially since it opened an online store.

months since online store opened	monthly revenue in dollars
0	72,000
1	80,498.45
3	90,000
4	
6	112,500

$\times 3$ $\times 3$

$\frac{90,000}{72,000}$ $\frac{112,500}{90,000}$

a. Describe how the monthly revenue is growing.

exponentially by a factor of

$\sqrt[3]{\frac{90,000}{72,000}} \approx \cancel{1.118}\ldots\ 1.0772\ldots$

b. Write an equation to represent the revenue, R, as a function of months, m, since the online store opened.

$R(m) = 72,000 \cdot \cancel{1.118}\ldots\ 1.0772^m$

c. Find the monthly revenue 1 month after the online store opened. Record the value in the table. Explain your reasoning.

$72,000 \cdot \cancel{1.118\ldots}^1 = \cancel{80,498.45}$

$1.0772\ldots \approx 77,559.65$

d. Explain how we can use the value of $R(1)$ to find $R(4)$.

Since the factor for an input change of 3 is 1.25

$1.25 \times R(1) = R(4)$

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2. At 7 a.m., a colony of 100 bacteria is placed on a petri dish where the population will triple every 6 hours.

   Select **all** statements that are true about the bacteria population.

   A. When the bacteria population reaches 900, 12 hours have passed since the colony was placed on the petri dish.

   B. Three hours after the colony is placed on the petri dish, there are 200 bacteria.

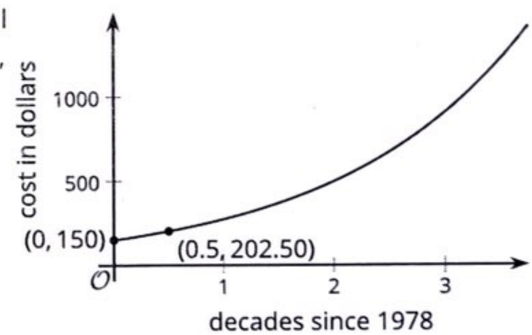
   C. Three hours after the colony is placed on the petri dish, there are about 173 bacteria in the colony.

   D. In the first hour the colony is placed on the petri dish, the population grows by a factor of $3^{\frac{1}{6}}$.

   E. Between 8 a.m. and 9 a.m., the population grows by a factor of $3^{\frac{2}{3}}$.

3. The graph represents the cost of a medical treatment, in dollars, as a function of time, d, in decades since 1978.

   Find the cost of the treatment, in dollars, when $d = 1$. Show your reasoning.



$$\left(\frac{202.50}{150}\right)^2 = 1.8225$$

$$C(1) = 150 \cdot 1.8225 = 273.375$$

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4. The exponential function f is given by $f(x) = 3^x$.

   a. By what factor does f increase when the exponent x increases by 1? Explain how you know.

   3, the base of the exponential is 3

   b. By what factor does f increase when the exponent x increases by 2? Explain how you know.

   $3^2 = 9$

   c. By what factor does f increase when the exponent x increases by $\frac{1}{2}$? Explain how you know.

   $3^{\frac{1}{2}} = \sqrt[2]{3}$

5. A piece of paper has area 93.5 square inches. How many times does it need to be folded in half before the area is less than 1 square inch? Explain how you know.

0	93.5	$\cdot\frac{1}{2}$
1	46.75	$\cdot\frac{1}{2}$
⋮		
6	1.4609...	
7	0.7304	

Seven times. I calculated a table.

(From Unit 4, Lesson 1.)

6. The area covered by an invasive tropical plant triples every year. By what factor does the area covered by the plant increase every month?

(From Unit 4, Lesson 4.)

$3^{\frac{1}{12}} = \sqrt[12]{3}$

$\approx 1.09587...$

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Solutions

Lesson 6 Practice Problems

1. A population of 1,500 insects grows exponentially by a factor of 3 every week. Select **all** equations that represent or approximate the population, p, as a function of time in days, t, since the population was 1,500.

   A. $p(t) = 1{,}500 \cdot 3^t$

   B. $p(t) = 1{,}500 \cdot 3^{\frac{t}{7}}$

   C. $p(t) = 1{,}500 \cdot 3^7 t$

   D. $p(t) = 1{,}500 \cdot \left(3^{\frac{1}{7}}\right)^t$

2 years

2. The tuition at a public university was $21,000 in 2008. Between 2008 and 2010, the tuition had increased by 15%. Since then, it has continued to grow exponentially.

   Select **all** statements that describe the growth in tuition cost.

   A. The tuition cost can be defined by the function $f(y) = 21{,}000 \cdot (1.15)^{\frac{y}{2}}$, where y represents years since 2008.

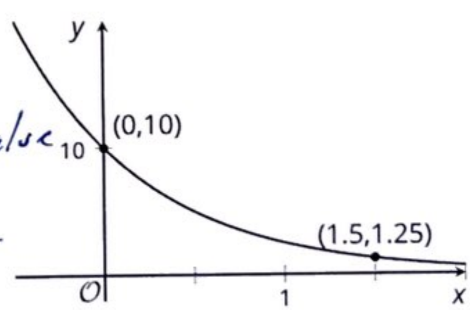
   B. The tuition cost increased 7.5% each year. ×

   C. The tuition cost increased about 7.2% each year. $\sqrt{1.15} \approx 1.07238$

   D. The tuition cost roughly doubles in 10 years. $1.15^5 \approx 2.011\ldots$

   E. The tuition cost can be approximated by the function $f(d) = 21{,}000 \cdot 2^d$, where d represents decades since 2008.

3. Here is a graph that represents $g(x) = a \cdot b^x$. Find the values of a and b. Show your reasoning.



$a = 10$ initial Value

$b = \left(\frac{1.25}{10}\right)^{1/1.5} = 0.25$

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4. The number of fish in a lake is growing exponentially. The table shows the values, in thousands, after different numbers of years since the population was first measured.

years	population
0	10
1	20
2	40
3	80
4	160
5	320
6	640

x4

a. By what factor does the population grow every two years? Use this information to fill out the table for 4 years and 6 years.

b. By what factor does the population grow every year? Explain how you know, and use this information to complete the table.

$\sqrt{4} = 2$

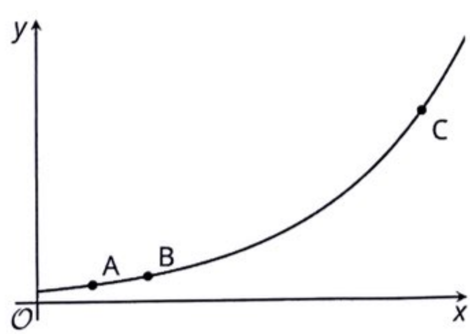
(From Unit 4, Lesson 3.)

5. The value of a home increases by 7% each year. Explain why the value of the home doubles approximately once each decade.

$1.07^{10} = 1.967... \approx 2$

(From Unit 4, Lesson 4.)

6. Here is the graph of an exponential function f.



The coordinates of A are $\left(\frac{1}{4}, 3\right)$. The coordinates of B are $\left(\frac{1}{2}, 4.5\right)$. If the x-coordinate of C is $\frac{7}{4}$, what is its y-coordinate? Explain how you know.

x increases $\frac{1}{4}$

y increases $\frac{4.5}{3} = 1\frac{1}{2}$

x: $\frac{1}{2} \to \frac{7}{4}$ is $3 * \frac{1}{4}$

y : $4.5 \times \left(\frac{3}{2}\right)^3$

$C \left(\frac{7}{4}, 15.1875\right)$

(From Unit 4, Lesson 5.)

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Solutions

Lesson 7 Practice Problems

1. The half-life of carbon-14 is about 5,730 years. A fossil had 6 picograms of carbon-14 at one point in time. (A picogram is a trillionth of a gram or 1×10^{-12} gram.) Which expression describes the amount of carbon-14, in picograms, t years after it was measured to be 6 picograms.

   A. $6 \cdot \left(\frac{1}{2}\right)^{\frac{t}{5,730}}$

   B. $6 \cdot \left(\frac{1}{2}\right)^{5,730t}$

   C. $6 \cdot (5,730)^{\frac{1}{2}t}$

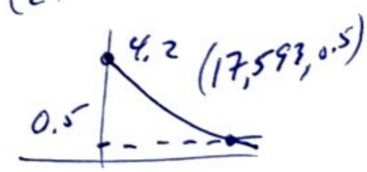
   D. $\frac{1}{2} \cdot (6)^{\frac{t}{5,730}}$

2. The half-life of carbon-14 is about 5,730 years. A tree fossil was estimated to have about 4.2 picograms of carbon-14 when it died. (A picogram is a trillionth of a gram.) The fossil now has about 0.5 picogram of carbon-14. About how many years ago did the tree die? Show your reasoning.

   $c(t) = 4.2 \cdot \left(\frac{1}{2}\right)^{t/5730}$

   $\frac{0.5}{4.2} = 0.119$

   $t = 17,593$ years

3. Nickel-63 is a radioactive substance with a half-life of about 100 years. An artifact had 9.8 milligrams of nickel-63 when it was first measured. Write an equation to represent the mass of nickel-63, in milligrams, as a function of:

   a. t, time in years

   $m(t) = 9.8 \cdot \left(\frac{1}{2}\right)^{t/100}$

   b. d, time in days

   $m(d = 9.5 \cdot \left(\frac{1}{2}\right)^{d/(100 \cdot 365)}$

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4. Tyler says that the function $f(x) = 5^x$ is exponential and so it grows by equal factors over equal intervals. He says that factor must be $\sqrt[10]{5}$ for an interval of $\frac{1}{10}$ because ten of those intervals makes an interval of length 1. Do you agree with Tyler? Explain your reasoning.

yes. $5^{\frac{1}{10}} = \sqrt[10]{5}$

(From Unit 4, Lesson 5.)

5. The population in a city is modeled by the equation $p(d) = 100{,}000 \cdot (1 + 0.3)^d$, where d is the number of decades since 1970.

   a. What do the 0.3 and 100,000 mean in this situation?

100,000 Initial population

0.3 increase per Decade (30%)

   b. Write an equation for the function f to represent the population y years after 1970. Show your reasoning.

$f(y) = 100{,}000 \cdot (1.3)^{y/10}$

   c. Write an equation for the function g to represent the population c centuries after 1970. Show your reasoning.

$g(c) = 100{,}000\ (1.3)^{c \cdot 10}$

(From Unit 4, Lesson 6.)

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$\times 2$ $\times 2$

6. The function f is exponential. Its graph contains the points $(0, 5)$ and $(1.5, 10)$. $3.0, 20$

$+1.5$ $+1.5$

a. Find $f(3)$. Explain your reasoning.

$f(3) = 20$

b. Use the value of $f(3)$ to find $f(1)$. Explain your reasoning.

$\frac{f(3)}{f(0)} = \frac{20}{5} = 4$

$\sqrt[3]{4}$ is the annual factor

$f(1) = 5\sqrt[3]{4} = 7.937...$

c. What is an equation that defines f?

$f(x) = 5 \cdot (\sqrt[3]{4})^x$

(From Unit 4, Lesson 6.)

7. Select **all** expressions that are equal to $8^{\frac{2}{3}}$.

A. $\sqrt[3]{8^2}$

B. $\sqrt[3]{8}^2$

C. $\sqrt{8^3}$

D. 2^2

E. 2^3

F. 4

(From Unit 3, Lesson 4.)

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