Illustrative Mathematics

Solutions

Lesson 14 Practice Problems

1. Select **all** expressions that are equivalent to $8 + 16i$.

   A. $2(4 + 8i)$

   B. $2i(8 - 4i)$

   C. $4(2i - 4)$ ×

   D. $4i(4 - 2i)$

   E. $-2i(-8 - 4i)$ ×

2. Which expression is equivalent to $(-4 + 3i)(2 - 7i)$?

   A. $-29 - 22i$

   B. $-29 + 34i$

   C. $13 - 22i$

   D. $13 + 34i$

   $-8 + 28i + 6i - 21i^2$

   $13 + 34i$

3. Match the equivalent expressions.

   A. $i^2(3 + i)$ $= -3 - i$

   B. $-4i \cdot 5i = +20$

   C. $5i(4 - 3i) = 15 + 20i$

   D. $(1 + 2i)(-1 + 3i)$

   $= -1 + 3i - 2i + 6i^2$

   $= -7 + i$

   1. $(3 + 5i) - (10 + 4i) = -7 - i$

   2. $(2 + 4i)(2 - 4i) = 4 + 16 = 20$

   3. $(1 - 4i) + (-4 + 3i) = -3 - i$

   4. $(-6 + 12i) - (-21 - 8i) = 15 + 20i$

4. Write each expression in $a + bi$ form.

a. $(-8 + 3i) - (2 + 5i)$ $= -10 - 2i$

b. $7i(4 - i)$ $= 7 + 28i$

c. $(3i)^3$ $= -27i$

d. $(3 + 5i)(4 + 3i)$ $= 12 + 9i + 20i + 15i^2$
$= -3 + 29i$

e. $(3i)(-2i)(4i)$ $= 24i$

5. Here is a method for solving the equation $\sqrt{5 + x} + 10 = 6$. Does the method produce the correct solution to the equation? Explain how you know.

$$\sqrt{5 + x} + 10 = 6$$

$\sqrt{5 + x} = -4$ (after subtracting 10 from each side)

$5 + x = 16$ $\leftarrow$ squaring (after squaring both sides)

$x = 11$

$\sqrt{5 + (11)} + 10 \stackrel{?}{=} 6$

$\sqrt{16} + 10 \stackrel{?}{=} 6$

$4 + 10 \stackrel{?}{=} 6$

$14 \neq 6$

No. The squaring step introduces an extraneous solution.

Checking through substitution shows $x \neq 11$

(From Unit 3, Lesson 7.)

6. Write each expression in the form $a + bi$, where a and b are real numbers.

a. $4(3 - i)$ $= 12 - 4i$

b. $(4 + 2i) + (8 - 2i)$ $= 12 + 0i$

c. $(1 + 3i)(4 + i)$ $= 4 + i + 12i + 3i^2 = 1 + 13i$

d. $i(3 + 5i)$ $= -5 + 3i$

e. $2i \cdot 7i$ $= -14$

(From Unit 3, Lesson 13.)

Lesson 15 Practice Problems

1. Select **all** the expressions that are equivalent to $(3 - 5i)(\text{-}8 + 2i)$.

   $= -24 + 6i + 40i - 10i^2$
   $= -14 + 46i$

   A. $\text{-}24 + 6i - 40i + 10i^2$ $= -34 - 34i$

   B. $\text{-}24 + 46i - 10$ $= -34 + 46i$

   C. $\text{-}24 + 6i + 40i - 10i^2$ $= -14 + 46i$

   D. $\text{-}14 - 34i$

   E. $\text{-}34 - 34i$

   F. $\text{-}24 + 46i + 10$ $= -14 + 46i$

   G. $46i - 14$

   H. $\text{-}34 + 46i$

2. Explain or show how to write $(20 - i)(8 + 4i)$ in the form $a + bi$, where a and b are real numbers.

   $= 160 + 80i - 8i - 4i^2$ $\quad i^2 = -1$

   $= 164 + 72i$ $\quad$ Collect like terms

3. Without going through all the trouble of writing the left side in the form $a + bi$, how could you tell that this equation is false?

   $(\text{-}9 + 2i)(10 - 13i) = \text{-}68 - \underline{97i}$

   both coefficients of the i term are positive,
   $[2 \cdot 10 + (-9)(-13)]\, i$ $\quad$ but $(-97i)$ on the right

4. Andre spilled something on his math notebook and some parts of the problems he was working on were erased. Here is one of the problems:

$(\ 7\ -2i)(\ 5\ +2i) = \qquad -10i$

a. What could go in the blanks?

7 & 5

b. Could other numbers work, or is this the only possibility? Explain your reasoning.

yes. a lot of values could be the first blank

5. Find the exact solution(s) to each of these equations, or explain why there is no solution.

   a. $x^2 = 49$

   ± 7

   b. $x^3 = 49$

   $\sqrt[3]{49}$

   c. $x^2 = -49$

   No real solution.
   square less than zero

   d. $x^3 = -49$

   $-\sqrt[3]{49}$

(From Unit 3, Lesson 8.)

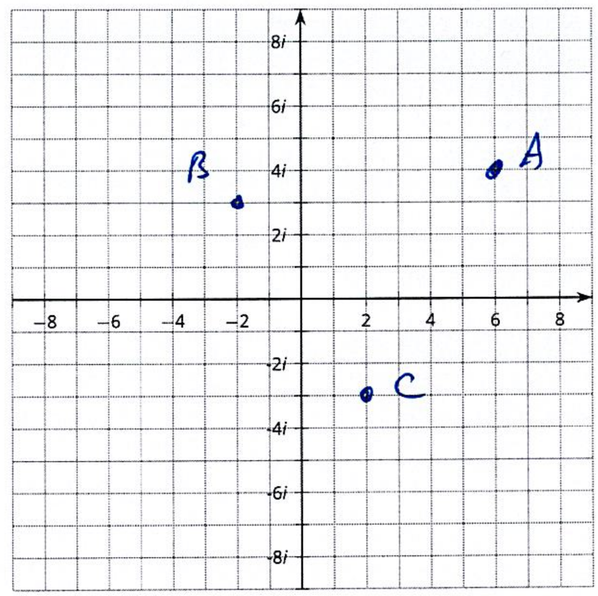
6. Write each expression in the form $a + bi$, where a and b are real numbers. Optionally, plot $3 + 2i$ in the complex plane. Then plot and label each of your answers.

a. $2(3 + 2i)$ $= 6 + 4i$

b. $i(3 + 2i)$ $= -2 + 3i$

c. $-i(3 + 2i)$ $= 2 - 3i$

d. $(3 - 2i)(3 + 2i)$
$9 - 4i^2 = 13$



$d = 13$ →

(From Unit 3, Lesson 13.)

7. The table shows two investment account balances growing over time.

time (years since 2000)	account A (thousands of dollars)	account B (thousands of dollars)
0	5	10
1	5.1	10.15
2	5.2	10.3
3	5.3	10.45
4	5.4	10.6

+0.1

0.15

a. Describe a pattern in how each account balance changed from one year to the next.

A : arithmetic $d = 0.1$

B : arithmetic $d = 0.15$

b. Define the amount of money, in thousands of dollars, in accounts A and B as functions of time t, where t is years since 2000, using function notation.

$A(t) = 5 + 0.1t$ $B(t) = 10 + 0.15t$

c. Will account A ever have the same balance as account B? If so, when? Explain how you know.

No, B > A ~~at~~ at the beginning and B continues to grow faster.

(From Unit 1, Lesson 10.)

Lesson 16 Practice Problems

1. What number should be added to the expression $x^2 - 15x$ to result in an expression equivalent to a perfect square?

$\frac{b}{2} = -\frac{15}{2} = -7.5$

$\left(\frac{b}{2}\right)^2 = \cancel{6.}\ (-7.5) = 56.25$

   A. -7.5

   B. 7.5

   C. -56.25

   D. 56.25

2. Noah uses the quadratic formula to solve the equation $2x^2 + 3x - 5 = 4$. He finds $x =$ -2.5 or 1. But, when he checks his answer, he finds that neither -2.5 nor 1 are solutions to the equation. Here are his steps:

$c = -9$

$a = 2$, $b = 3$, $c =$ -5

$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot -5}}{2 \cdot 2}$

$x = \frac{-3 \pm \sqrt{49}}{4}$

$x =$ -2.5 or 1

   a. Explain what Noah's mistake was.

$c = -9$

   b. Solve the equation correctly.

$2x^2 + 3x - 9 = 0$

$x = \frac{-(3) \pm \sqrt{3^2 - 4(2)(-9)}}{2(2)}$

$= \frac{-3 \pm \sqrt{9 + 72}}{4} = \frac{-3 \pm \sqrt{81}}{4} = \frac{-3 \pm 9}{4}$

$= \frac{6}{4}, -\frac{12}{4}$ or $1\frac{1}{2}, -3$

3. Solve each quadratic equation with the method of your choice.

a. $x^2 - 2x = -1$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1$

b. $x^2 + 8x + 14 = 23$

$x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$x = -9, 1$

c. $x^2 - 15 = 0$

$(x - \sqrt{15})(x + \sqrt{15}) = 0$

$x = \pm\sqrt{15}$

d. $7x^2 - 2x - 5 = 0$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(-5)}}{2(7)} = \dfrac{2 \pm \sqrt{4+140}}{14}$

$\sqrt{144} = 12$

$= \dfrac{2 \pm 12}{14} = -\dfrac{10}{14}, 1$ or $-\dfrac{5}{7}, 1$

e. $2x^2 + 12x = 8$

$\frac{6}{2} = 3$

$(\frac{6}{2})^2 = 9$

$x^2 + 6x - 4 = 0$

$x^2 + 6x + 9 = 13$

$(x-3)^2 = 13$

$x = 3 \pm \sqrt{13}$

4. What are the solutions to the equation $x^2 - 4x = -3$?

$x^2 - 4x + 3 = 0$

A. $\dfrac{4 \pm \sqrt{16 - 4 \cdot 0 \cdot 3}}{2 \cdot 0}$ x

B. $\dfrac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot -3}}{2 \cdot 1}$ x

C. $\dfrac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$ ✓

D. $\dfrac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$ x

5. Which expression is equivalent to $\sqrt{-23}$?

A. $-23i$

B. $23i$

C. $-i\sqrt{23}$

D. $i\sqrt{23}$

(From Unit 3, Lesson 11.)

6. Write each expression in the form $a + bi$, where a and b are real numbers.

a. $5i^2$ $= -5 + 0i$

b. $i^2 \cdot i^2$ $= 1 + 0i$

c. $(-3i)^2$ $= 3 + 0i$

d. $7 \cdot 4i$ $= 0 + 28i$

e. $(5 + 4i) - (-3 + 2i)$ $= 2 + 2i$

(From Unit 3, Lesson 12.)

7. Let $m = (7 - 2i)$ and $k = 3i$. Write each expression in the form $a + bi$, where a and b are real numbers.

a. $k - m$ $= 3i - (7 - 2i) = -7 + 1i$

b. k^2 $= (3i)^2 = -9 + 0i$

c. m^2 $= (7 - 2i)^2 = 49 - 28i + 4i^2 = 45 - 28i$

d. $k \cdot m$ $= (3i)(7 - 2i) = 6 + 21i$

(From Unit 3, Lesson 13.)

Lesson 17 Practice Problems

1. Find the solution or solutions to each equation.

   a. $x^2 + 0.5x - 14 = 0$ $\quad x = 3.5, -4$

   b. $x^2 + 12x + 36 = 0$ $\quad x = -6$

   c. $x^2 - 3x + 8 = 0$ $\quad \dfrac{3 \pm i\sqrt{23}}{2}$

   d. $x^2 + 4 = 0$ $\quad x = \pm 2i$

2. Which describes the solutions to the equation $x^2 + 7 = 0$?

   A. One real solution

   B. Two real solutions

   C. One non-real solution

   D. Two non-real solutions

3. Explain how you know $\sqrt{3x+2} = \text{-}16$ has no solutions.

   radicals are alway ~~non~~ non-negative

(From Unit 3, Lesson 7.)

4. Determine the number of real solutions and non-real solutions to each equation. Use the graphs; don't do any calculations to find the solutions.

a. $x^2 - 6x + 7 = 0$ two

b. $3x^2 + 2x + 1 = 0$ none

c. $-x^2 - 3x + 2 = 0$ two

d. $x^2 - 6x + 7 = -2$

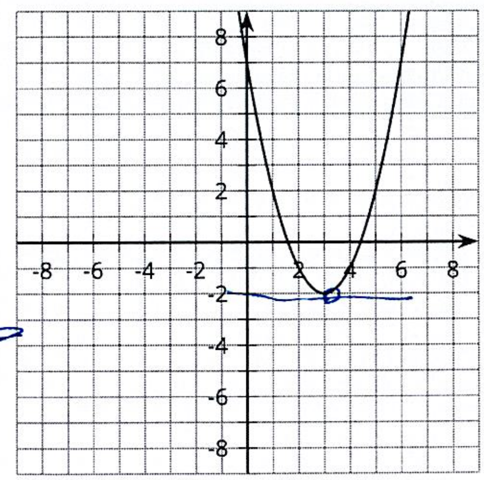
one

e. $-x^2 - 3x + 2 = 6$ none

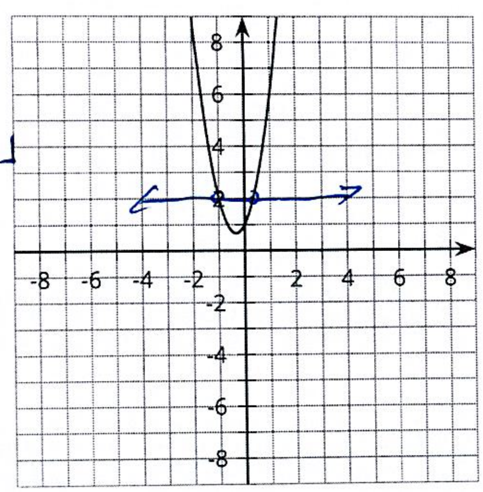
f. $3x^2 + 2x + 1 = 2$

2

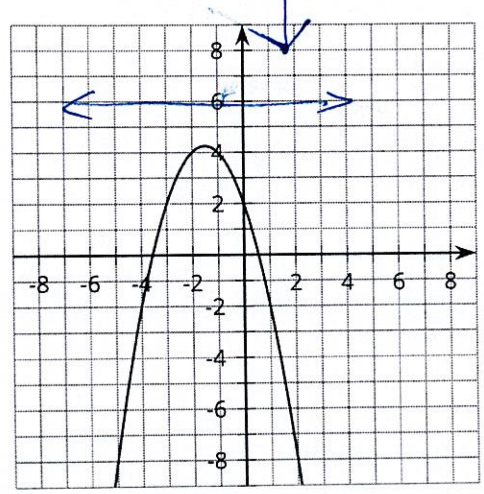
$y = x^2 - 6x + 7$



$y = 3x^2 + 2x + 1$ None



$y = -x^2 - 3x + 2$



5. a. Write $(5 - 5i)^2$ in the form $a + bi$, where a and b are real numbers.

$= 25 - 50i + 25i^2$

$= 0 - 50i$

b. Write $(5 - 5i)^4$ in the form $a + bi$, where a and b are real numbers.

$((5-5i)^2)^2 = (-50i)^2 = +-2500 + 0i$

(From Unit 3, Lesson 14.)

6. What number n makes this equation true?

$x^2 + 11x + \frac{121}{4} = (x + n)^2$

$\frac{b}{2} = \frac{11}{2}$

A. $\frac{11}{4}$

B. $\frac{11}{2}$

C. 11

D. $\frac{121}{4}$

(From Unit 3, Lesson 16.)

Lesson 18 Practice Problems

1. Clare solves the quadratic equation $4x^2 + 12x + 58 = 0$, but when she checks her answer, she realizes she made a mistake. Explain what Clare's mistake was.

$$x = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 4 \cdot 58}}{2 \cdot 4}$$
$$x = \frac{-12 \pm \sqrt{144 - 928}}{8}$$
$$x = \frac{-12 \pm \sqrt{-784}}{8}$$
$$x = \frac{-12 \pm 28i}{8}$$
$$x = -1.5 \pm 28i$$

$-1.5 \pm \frac{28}{8}i$

2. Write in the form $a + bi$, where a and b are real numbers:

a. $\frac{5 \pm \sqrt{-4}}{3}$ $= \frac{5}{3} \pm \frac{2}{3}i$

b. $\frac{10 \pm \sqrt{-16}}{2}$ $= 5 \pm 2i$

c. $\frac{-3 \pm \sqrt{-144}}{6}$ $= -\frac{1}{2} \pm 2i$

3. Priya is using the quadratic formula to solve two different quadratic equations.

For the first equation, she writes $x = \frac{4 \pm \sqrt{16-72}}{12}$ complex – square root of a negative number is imaginary

For the second equation, she writes $x = \frac{8 \pm \sqrt{64-24}}{6}$ real

Which equation(s) will have real solutions? Which equation(s) will have non-real solutions? Explain how you know.

4. Find the exact solution(s) to each of these equations, or explain why there is no solution.

a. $x^2 = 25$ $x = \pm 5$

b. $x^3 = 27$ $x = 3$

c. $x^2 = 12$ $x = \pm\sqrt{12} = \pm 2\sqrt{3}$

d. $x^3 = 12$ $x = \sqrt[3]{12}$

(From Unit 3, Lesson 8.)

5. Kiran is solving the equation $\sqrt{x+2} - 5 = 11$ and decides to start by squaring both sides. Which equation results if Kiran squares both sides as his first step?

A. $x + 2 - 25 = 121$

B. $x + 2 + 25 = 121$

C. $x + 2 - 10\sqrt{x+2} + 25 = 121$

D. $x + 2 + 10\sqrt{x+2} + 25 = 121$

(From Unit 3, Lesson 9.)

6. Plot each number on the real or imaginary number line.

a. $-\sqrt{4}$ $= -2$

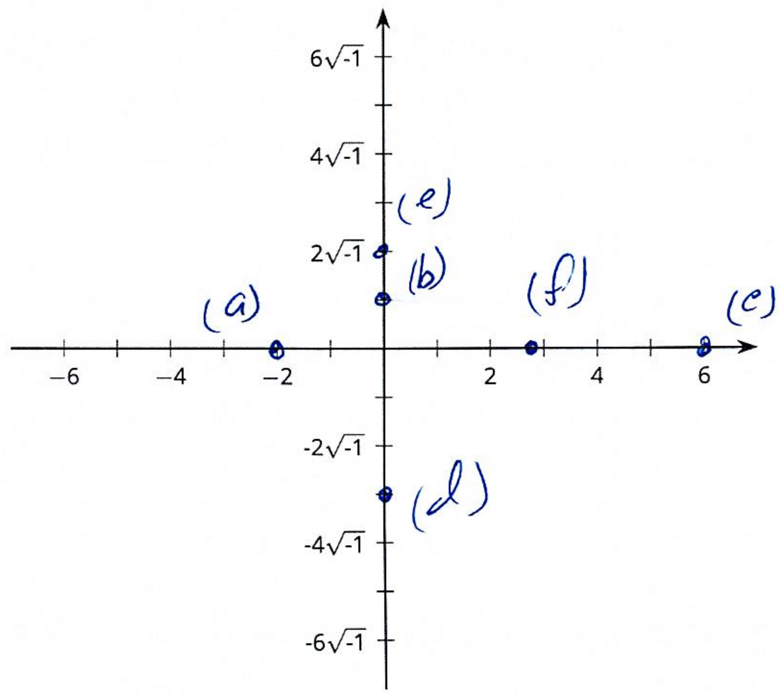
b. $\sqrt{-1}$ $= i$

c. $3\sqrt{4}$ $= 6$

d. $-3\sqrt{-1}$ $= -3i$

e. $4\sqrt{-1}$ $= 2i$

f. $2\sqrt{2}$



(From Unit 3, Lesson 10.)

Solutions

Lesson 19 Practice Problems

1. Without calculating the solutions, determine whether each equation has real solutions or not.

   a. $-0.5x^2 + 3x = 0$ yes. 2

   b. $x^2 - 4x + 7 = 0$ No

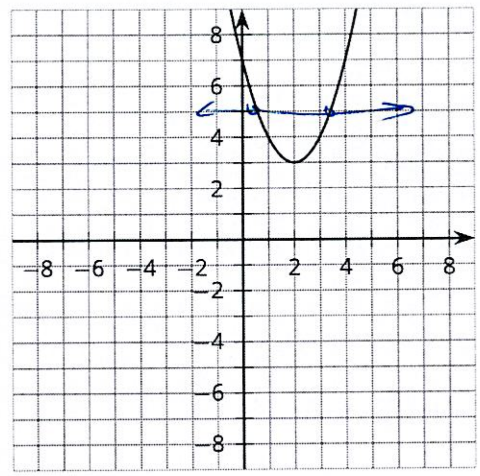
   c. $2x^2 - 2x - 1 = 0$ yes 2

   d. $-0.5x^2 + 3x = 3$ yes, two

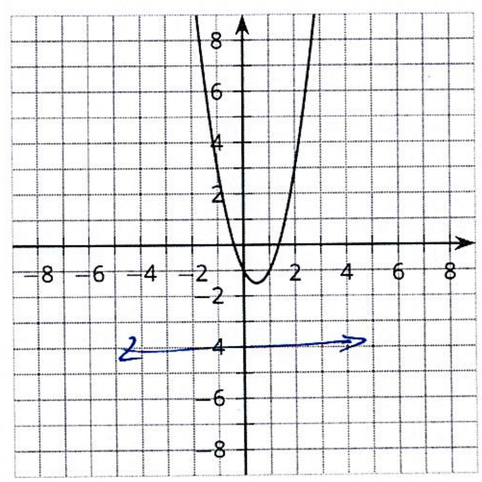
   e. $x^2 - 4x + 7 = 5$ yes two

   f. $2x^2 - 2x - 1 = -4$ No

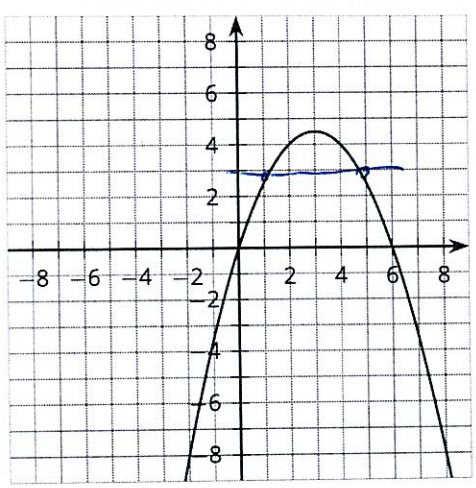
$y = x^2 - 4x + 7$



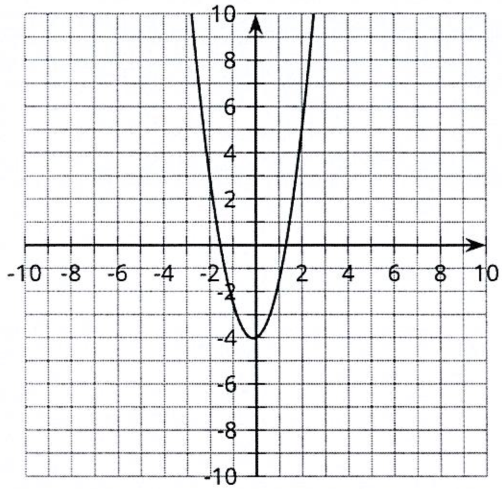
$y = 2x^2 - 2x - 1$



$y = -0.5x^2 + 3x$



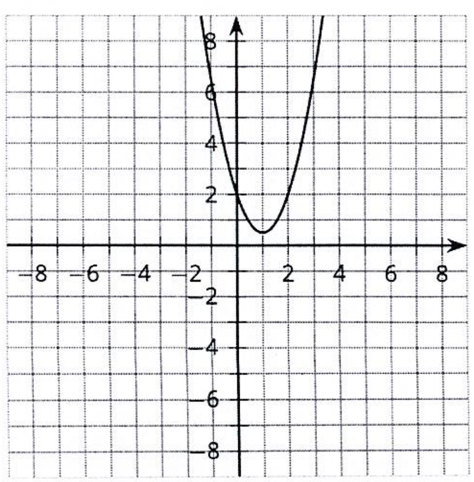
2. The graph shows the equation $y = 2x^2 + 0.5x - 4$.



Based on the graph, what number could you put in the box to create an equation that has no real solutions?

$$2x^2 + 0.5x - 4 = \boxed{-5}$$

3. The graph shows the equation $y = 1.5x^2 - 3x + 2$.



a. Without calculating the solutions, determine whether $1.5x^2 - 3x + 2 = 0$ has real solutions.

NO

b. Show how to solve $1.5x^2 - 3x + 2 = 0$.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1.5)(2)}}{2(1.5)}$$

$$= \frac{3 \pm \sqrt{9 - 12}}{3}$$

$$= 1 \pm \frac{i\sqrt{3}}{3}$$

4. Write a quadratic equation that has two non-real solutions. How did you decide what equation to write?

$x^2 = -1$

quadratic with a negative result

5. Find the solution or solutions to each equation.

a. $-2x^2 + 2x = 2.5$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(-2)(-2.5)}}{2(-2)}$$

$$= \frac{-2 \pm \sqrt{-16}}{-4} = \frac{1}{2} \pm i$$

b. $4.5x^2 + 3x + \frac{1}{2} = 0$

$$x = \frac{-(3) \pm \sqrt{3^2 - 4(4.5)(\frac{1}{2})}}{2(4.5)} = \frac{-3 \pm \sqrt{0}}{9} = -\frac{1}{3}$$

c. $\frac{1}{2}x^2 + 5x = -14$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(\frac{1}{2})(+14)}}{2(\frac{1}{2})} = -5 \pm \sqrt{-3} = -5 \pm i\sqrt{3}$$

d. $-x^2 - 1.5x + 5 = 7$

$x^2 + 1.5x + 2 = 0$

$$x = \frac{-1.5 \pm \sqrt{(1.5)^2 - 4(1)(2)}}{2}$$

$$= \frac{-1.5 \pm i\sqrt{5.75}}{2} = -\frac{3}{4} \pm \frac{i}{4}\sqrt{23}$$

6. Elena and Kiran were solving the equation $2x^2 - 4x + 3 = 0$ and they got different answers. Elena wrote $1 \pm i\sqrt{0.5}$, and Kiran wrote $1 \pm \frac{i\sqrt{8}}{4}$. Are their answers equivalent? Say how you know.

$$1 \pm i\sqrt{0.5} = 1 \pm i\sqrt{\frac{1}{2}}$$

$$= 1 \pm \frac{i}{\sqrt{2}} \qquad \left(* \frac{\sqrt{8}}{\sqrt{8}}\right)$$

$$= 1 \pm i\,\frac{\sqrt{8}}{\sqrt{2}\sqrt{8}}$$

$$= 1 \pm i\,\frac{\sqrt{8}}{4} \quad \checkmark$$