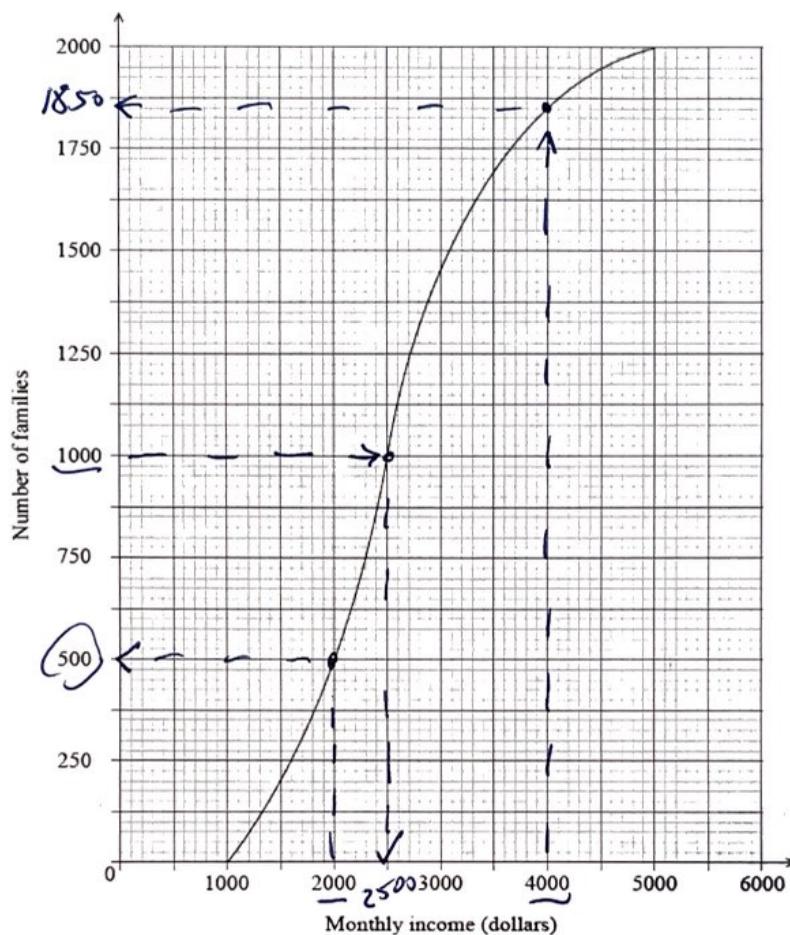


#### 4.6 Classwork: Statistics and probability

**1a.** The following cumulative frequency graph shows the monthly income,  $I$  dollars, of 2000 families.



Find the median monthly income.

*2500 dollars*

[2 marks]

**1b. [4 marks]**

(i) Write down the number of families who have a monthly income of 2000 dollars or less.

*500 families*

(ii) Find the number of families who have a monthly income of more than 4000 dollars.

$$2000 - 1850 = \frac{150}{1}$$

**1c.** The 2000 families live in two different types of housing. The following table gives information about the number of families living in each type of housing and their monthly income  $I$ .

	$1000 < I \leq 2000$	$2000 < I \leq 4000$	$4000 < I \leq 5000$
Apartment	436	765	28
Villa	64	$p$	122

Find the value of  $p$ .

$$n = 436 + 765 + 28 + 64 + p + 122 = 2000 \quad [2 \text{ marks}]$$

$$p = 585$$

**14d.** [2 marks]

A family is chosen at random.

(i) Find the probability that this family lives in an apartment.

$$P(A) = \frac{436 + 765 + 28}{2000} = \frac{1229}{2000}$$

$$= 0.6145 \approx 0.615$$

(ii) Find the probability that this family lives in an apartment, given that its monthly income is greater than 4000 dollars.

$$P(A | I > 4000) = \frac{28}{28 + 122} = \frac{28}{150}$$

$$= 0.1866 \approx 0.187$$

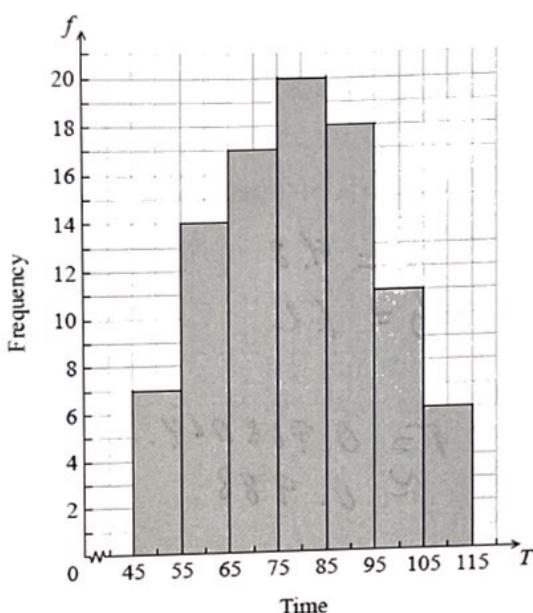
**14e.** Estimate the mean monthly income for families living in a villa.

[2 marks]

$$\bar{I} | V = \frac{64 \cdot 1500 + 585 \cdot 3000 + 122 \cdot 4500}{64 + 585 + 122}$$

$$= 3112.84... \approx 3110 \text{ dollars}$$

2a. The histogram below shows the time  $T$  seconds taken by 93 children to solve a puzzle. [3 marks]



The following is the frequency distribution for  $T$ .

Time	$45 \leq T < 55$	$55 \leq T < 65$	$65 \leq T < 75$	$75 \leq T < 85$	$85 \leq T < 95$	$95 \leq T < 105$	$105 \leq T < 115$
Frequency	7	14	$p$	20	18	$q$	6
			17			11	

(i) Write down the value of  $p$  and of  $q$ .

$$p = 17, \quad q = 11$$

(ii) Write down the median class.

$$\frac{93+1}{2} = 47 \quad 75 \leq T < 85$$

$$7 + 14 + 17 + 20 + 18 = 76$$

2b. A child is selected at random. Find the probability that the child takes less than 95 seconds to solve the puzzle.

$$P(T < 95) = \frac{7 + 14 + 17 + 20 + 18}{93} = \frac{76}{93} \quad [2 \text{ marks}] \quad (\approx 0.817)$$

2c. Consider the class interval  $45 \leq T < 55$ .

(i) Write down the interval width.

$$55 - 45 = 10$$

(ii) Write down the mid-interval value.

$$50$$

[2 marks]

2d. Hence find an estimate for the

$$(i) \text{ mean: } \bar{x} = 79.1398 \dots \approx 79.1$$

(ii) standard deviation.

$$\sigma = 16.4386 \dots \approx 16.4$$

[4 marks]

3. The following table shows the sales,  $y$  millions of dollars, of a company,  $x$  years after it opened.

Time after opening ( $x$ years)	2	4	6	8	10
Sales ( $y$ millions of dollars)	12	20	30	36	52

The relationship between the variables is modelled by the regression line with equation  $y = ax + b$ .

(i) Find the value of  $a$  and of  $b$ .

$$a = 4.8$$

$$b = 1.2$$

(ii) Write down the value of  $r$ .

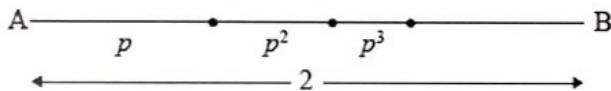
[4 marks]

$$r = 0.988064 \dots$$

$$\approx 0.988$$

13. The following diagram shows [AB], with length 2 cm. The line is divided into an infinite number of line segments. The diagram shows the first three segments.

diagram not to scale



The lengths of the line segments are  $p$  cm,  $p^2$  cm,  $p^3$  cm, ..., where  $0 < p < 1$ .

Show that  $p = \frac{2}{3}$ .

geometric series

$$u_1 = p$$

$$r = p$$

$$S_{\infty} = \frac{P}{1-P} = 2$$

$$p = 2 - 2p$$

$$3p = 2$$

$$p = \frac{2}{3}$$

[5 marks]