SOLUDINS

2.4 Classwork: Review Exam Problems

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

In Lucy's music academy, eight students took their piano diploma examination and achieved scores out of 150. For her records, Lucy decided to record the average number of hours per week each student reported practising in the weeks prior to their examination. These results are summarized in the table below.

Average weekly practice time (h)	28	13	45	33	17	29	39	36
Diploma score (D)	115	82	120	116	79	101	110	121

(a) Find Pearson's product-moment correlation coefficient, r, for these data. [2]

(b) The relationship between the variables can be modelled by the regression equation $D = ah + b$. Write down the value of a and the value of b. [1]

(c) One of these eight students was disappointed with her result and wished she had practised more. Based on the given data, determine how her score could have been expected to alter had she practised an extra five hours per week. [2]

(a) $r = 0.88353\ldots$
≈ 0.884

(b) $a = 1.36609\ldots$
≈ 1.37
$b = 64.5172\ldots$
≈ 64.5

(c) $5 \times 1.37 = 6.83047\ldots$
≈ 6.83

improved almost 7 points

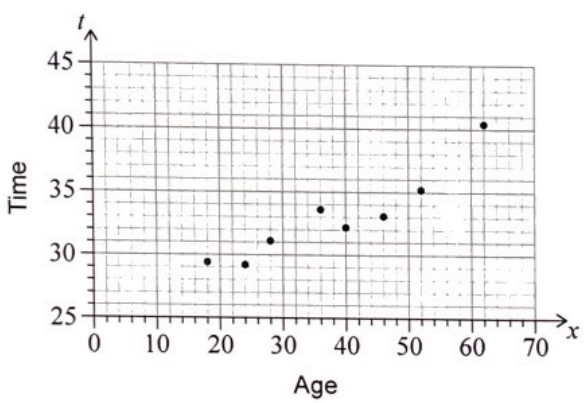
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Eduardo believes that there is a linear relationship between the age of a male runner and the time it takes them to run 5000 metres.

To test this, he recorded the age, x years, and the time, t minutes, for eight males in a single 5000 m race. His results are presented in the following table and scatter diagram.

x, years	18	24	28	36	40	46	52	62
t, minutes	29.4	29.2	31.1	33.6	32.2	33.1	35.2	40.4



(a) For this data, find the value of the Pearson's product-moment correlation coefficient, r. [2]

Eduardo looked in a sports science text book. He found that the following information about r was appropriate for athletic performance.

Value of $\lvert r \rvert$	**Description of the correlation**
$0 \leq \lvert r \rvert < 0.4$	weak
$0.4 \leq \lvert r \rvert < 0.8$	moderate
$0.8 \leq \lvert r \rvert \leq 1$	strong

(b) Comment on your answer to part (a), using the information that Eduardo found. [1]

(c) Write down the equation of the regression line of t on x, in the form $t = ax + b$. [1]

(This question continues on the following page)

(Question 1 continued)

A 57-year-old male also ran in the 5000 m race.

(d) Use the equation of the regression line to estimate the time he took to complete the 5000 m race. [2]

(a) $r = 0.933419...$
≈ 0.933

(b) This is a strong positive correlation

(c) $t = 0.227714...x + 24.3153...$
$t = 0.228x + 24.3$

(d) $t = 0.228(57) + 24.3$
$= 37.296$
≈ 37.3 minutes

3. [Maximum mark: 4]

Natasha carries out an experiment on the growth of mould. She believes that the growth can be modelled by an exponential function

$$P(t) = Ae^{kt},$$

where P is the area covered by mould in mm^2, t is the time in days since the start of the experiment and A and k are constants.

The area covered by mould is $112\,\text{mm}^2$ at the start of the experiment and $360\,\text{mm}^2$ after 5 days.

(a) Write down the value of A. [1]

(b) Find the value of k. [3]

(a) $A = 112 \text{ mm}^2$

(b) $P(5) = 112 \cdot e^{k(5)} = 360$

$5k = \ln\left(\frac{360}{112}\right) = 1.16761\ldots$

$k = 0.233521\ldots$

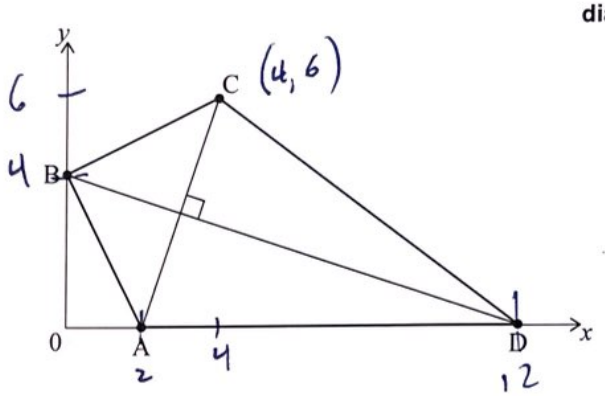
≈ 0.234

4. [Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents 10 cm.

The coordinates of A, B and C are (2, 0), (0, 4) and (4, 6) respectively. Point D lies on the x-axis. [AC] is perpendicular to [BD]. This information is shown in the following diagram.

diagram not to scale



(a) Find the gradient of the line through A and C. [2]

(b) Write down the gradient of the line through B and D. [1]

(c) Find the equation of the line through B and D. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. [2]

(d) Write down the x-coordinate of point D. [1]

Answer below

(a) $m_{AC} = \frac{6-0}{4-2} = 3$

(b) $-\frac{1}{3}$

(c) $y - 4 = -\frac{1}{3}(x-0)$

$-3y + 12 = x$

$x + 3y - 12 = 0$

(d) 12

2. [Maximum mark: 16]

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12300
2	12669

(a) Calculate the percentage increase in applications from the first year to the second year. [2]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

(b) (i) Write down the common ratio of the sequence.

(ii) Find an expression for u_n.

(iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [4]

In the first year there were 10380 places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

(c) Write down an expression for v_n. [2]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an \$80 acceptance fee.

(d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

(e) Find k. [3]

(f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]

Answer on lined paper.

2.4 Review Exam Problems Solutions

p3 #2

(a) % increase $= \frac{12,669 - 12,300}{12,300} = 3\%$

(b) i) $r = 1.03$

ii) $u_n = 12,300 \cdot 1.03^{(n-1)}$

iii) $u_{11} = 12,300 \cdot 1.03^{(11-1)} = 16530.2$

≈ 16530

(c) $V_n = 10,380 + 600(n-1)$

(d) $S_{10} \cdot 80 = 80\left(\frac{10}{2}(2 \cdot 10,380 + 600 \cdot 9)\right)$

$= \$10,464,000$

(e) $u_k \overset{?}{<} V_k$

$12,300 \cdot 1.03^{k-1} \overset{?}{<} 10,380 + 600(k-1)$

13^{th} year $V_{13} = 17,580 > 17,536.9 = u_{13}$

(note $k = 12$ $16,980 < 17,026.1$

(f) No. Exponential growth will eventually win out

$u_{24} > V_{24}$ for example