Geometry Unit 3: Transversals

Bronx Early College Academy

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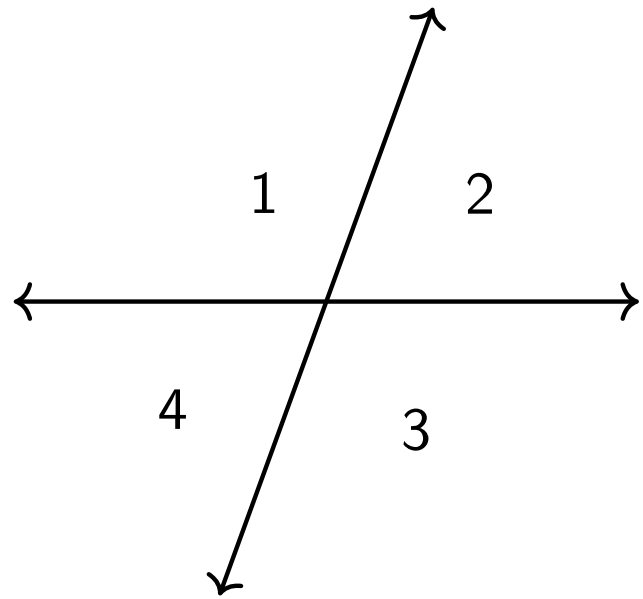
11 October - 21 October 2022

Learning Target: I can name parallel lines transversal angles

HSG.CO.C.9 Prove theorems about lines and angles 3.1 Tuesday 11 October

Do Now: Identify the true statements

1. $\angle 1 \cong \angle 2$
2. $\angle 2 \cong \angle 4$
3. $m\angle 1 + m\angle 4 = 180°$
4. $m\angle 2 + m\angle 3 = 90°$

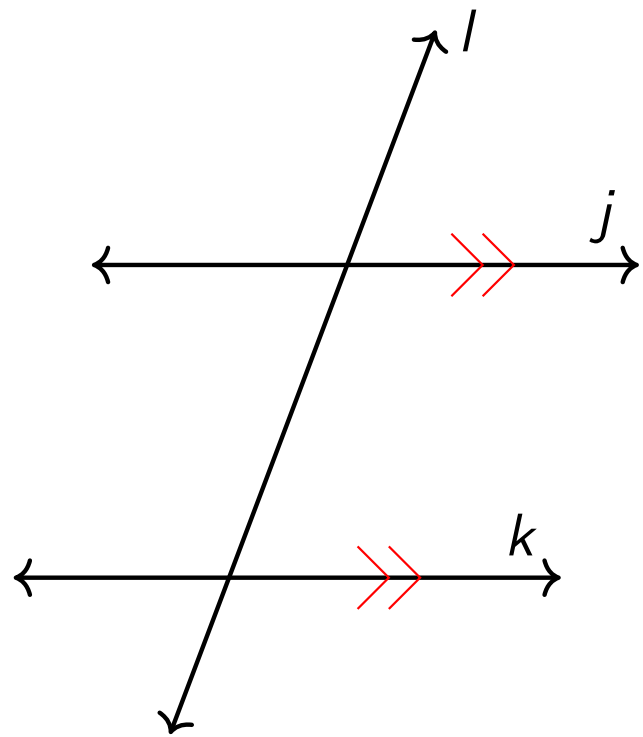


Lesson: Parallel lines crossed by a transverse line, horizontal and vertical directions

New terminology for parallel lines

Parallel lines are in the same plane and never intersect

Parallel lines	$j \parallel k$, mark with arrows
Transversal	Line l, crosses parallel lines
Interior	Inside ($\angle$s)
Exterior	Outside ($\angle$s)
Same side	On the left or right of l
Alternate	Across l from each other
Horizontal	Sideways direction
Vertical	Up and down direction



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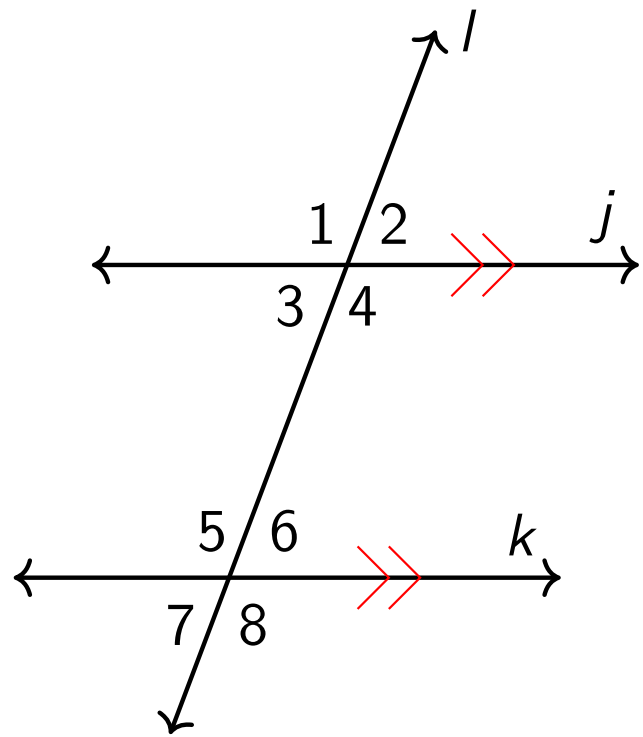


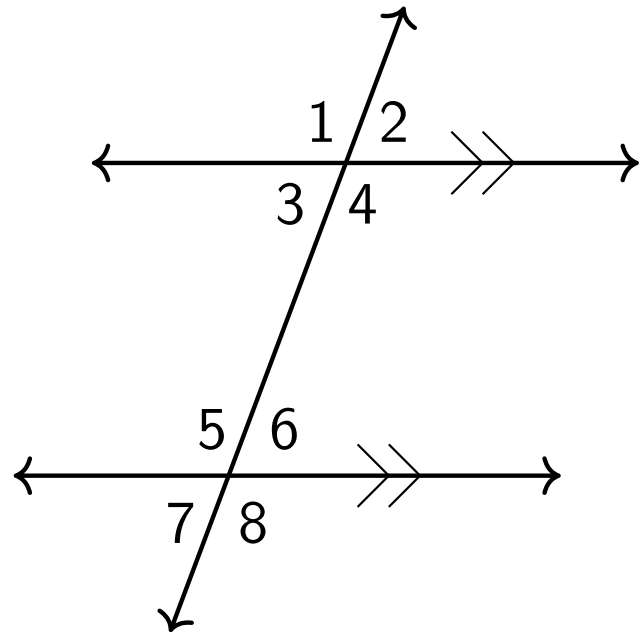
We often number the angles this way.

New theorems for parallel lines

Corresponding Having the same position. e.g. $\angle 2$ and $\angle 6$

Postulate Corresponding $\angle$s of $\parallel$ lines are congruent, $\angle 2 \cong \angle 6$

1. Alternate interior $\angle$s are $\cong$
   $\angle 4 \cong \angle 5$
2. Same-side interior $\angle$s are supplementary
   $m\angle 3 + m\angle 5 = 180$
3. Alternate exterior $\angle$s are $\cong$
   $\angle 1 \cong \angle 8$

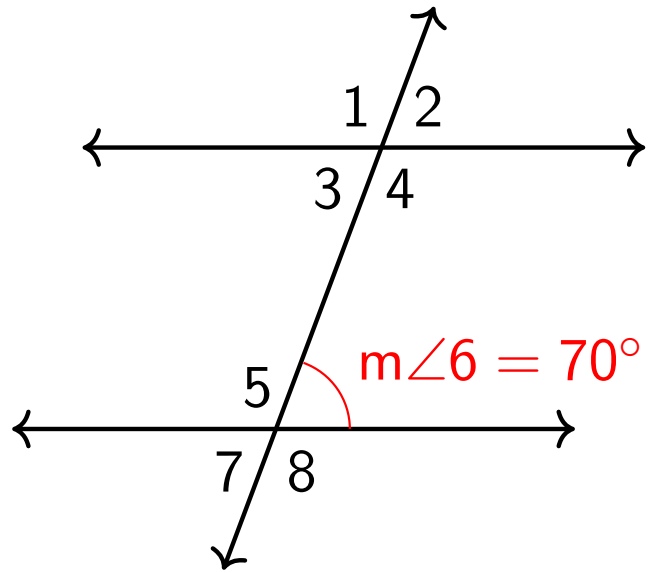


There are only two angle measures, the acute $\angle$s and the obtuse $\angle$s
And they add to $180°$, i.e. supplementary

Apply the theorems of parallel lines with a transversal

Given two parallel lines and a transversal, with $m\angle 6 = 70°$. Write down the value of each angle measure.

1. $m\angle 1 =$
2. $m\angle 2 =$
3. $m\angle 3 =$
4. $m\angle 4 =$
5. $m\angle 5 =$
6. $m\angle 6 = 70°$
7. $m\angle 7 =$
8. $m\angle 8 =$



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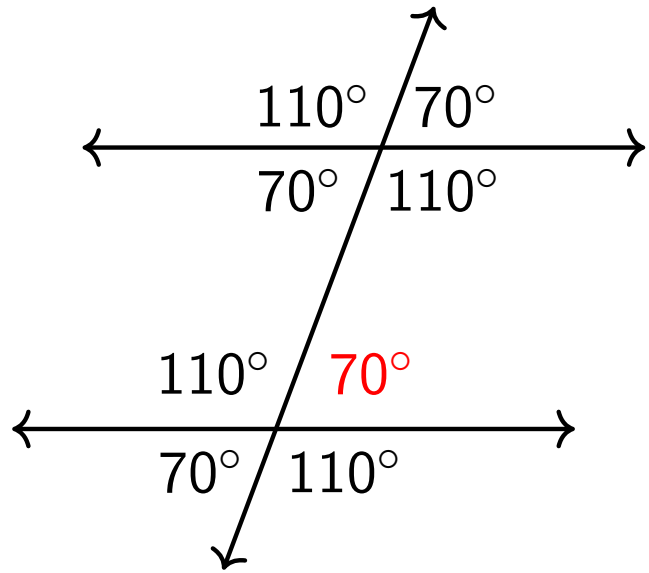


Solution

Extension: Ratios are fractions

We often state proportions as ratios

Example: Divide a distance into equal parts, i.e.

$$1 : 1$$

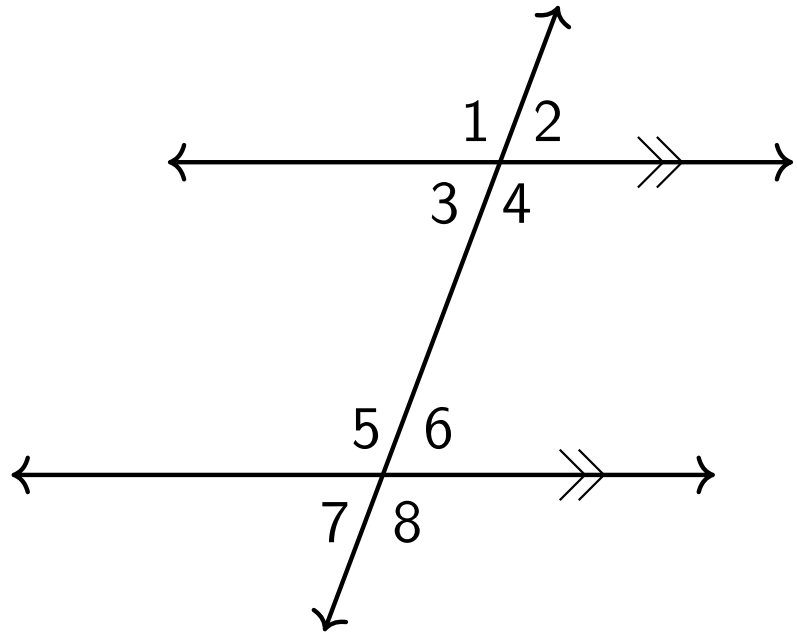
We say "one to one", or "in a one to one ratio."
A rectangle's length to width ratio is two to one. $2 : 1$

Learning Target: I can calculate transversal angles

HSG.CO.C.9 Prove theorems about lines and angles 3.2 Wednesday 12 October

Do Now: Identify each angle

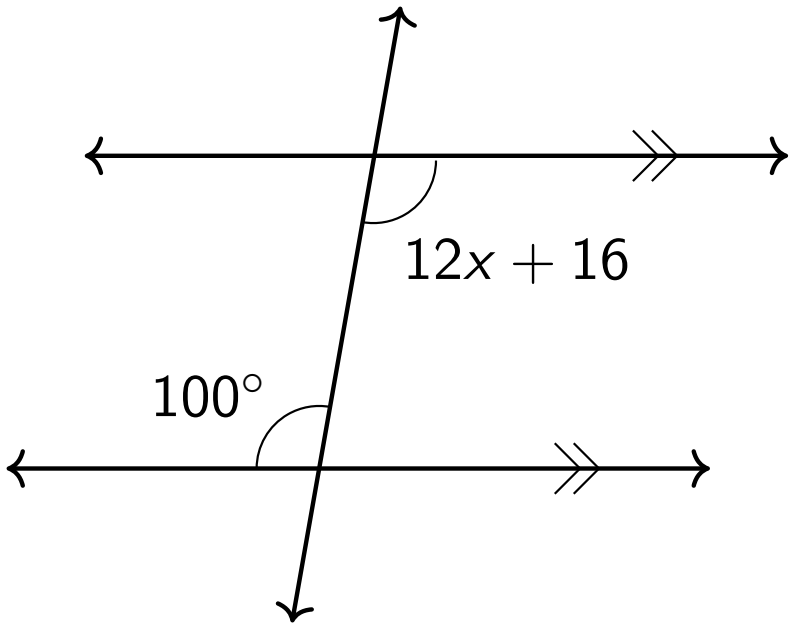
1. Opposite $\angle 4$
2. Corresponding to $\angle 3$
3. Alternate exterior to $\angle 8$
4. Same side interior to $\angle 5$
5. Alternate interior to $\angle 4$



Lesson: Solve for angle measures

Parallel lines intersected by a transversal. Find x.

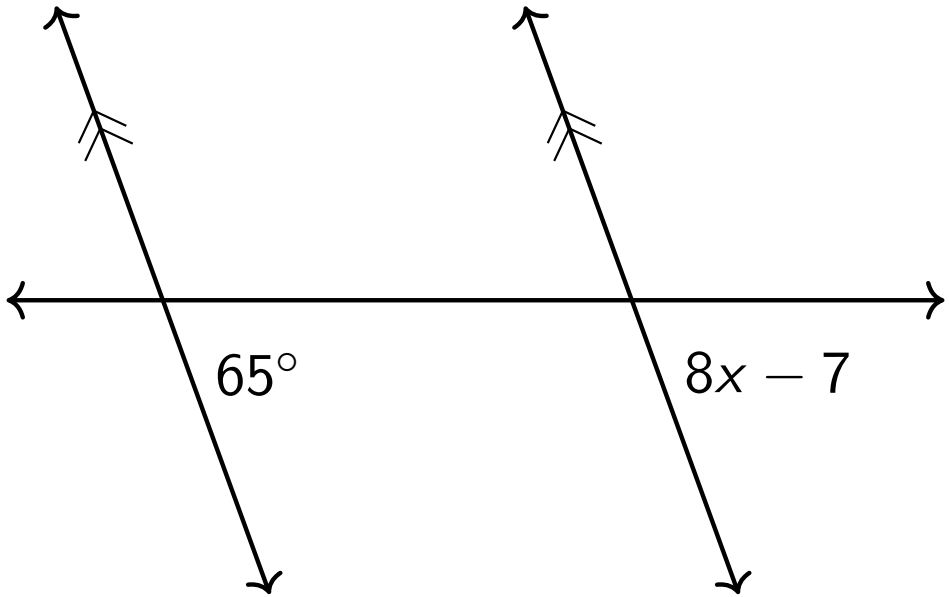
Alternate interior angles measure 100° and $12x + 16$, as shown.



Are the angles congruent or supplementary?

Parallel lines intersected by a transversal. Find x.

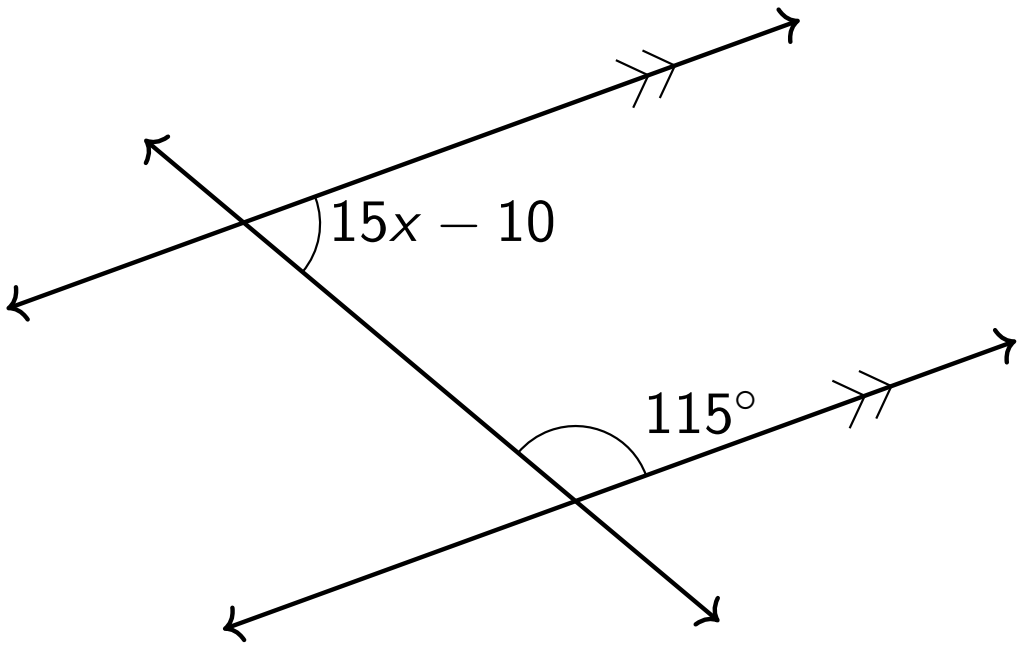
Parallel lines do not have to be horizontal.



State the postulate or theorem you are employing.

Parallel lines intersected by a transversal. Find x.

Given: Same side interior angles measure 115° and $15x - 10$.



Remember the check.

Extension: *Partitioning* a segment or angle in a ratio

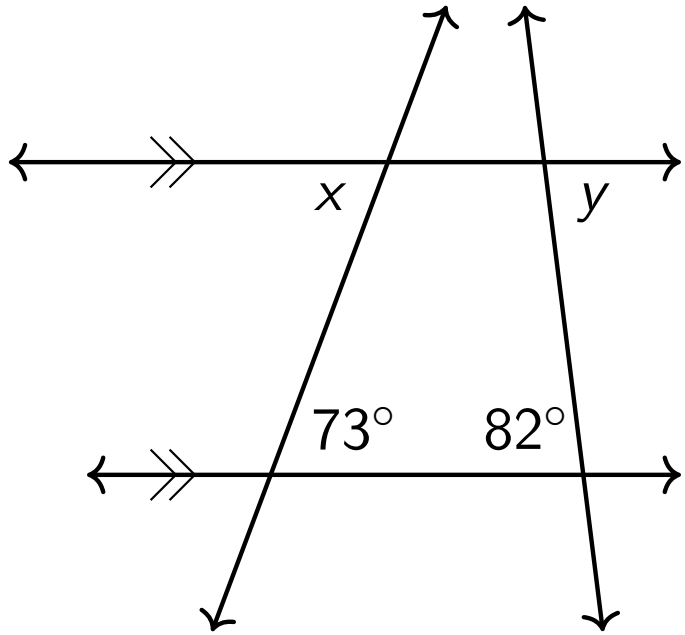
Point B divides $\overline{AC}$ in a $2:1$ ratio, i.e. $AB = 2BC$
Ray $\overrightarrow{BD}$ divides $\angle ABC$ in a $2:1$ ratio. Find x.

Learning Target: I can calculate triangle angles

HSG.CO.C.9 Prove theorems about lines and angles 3.3 Thursday 13 October

Do Now:

1. Given two parallel lines, two transversals
2. Find x, y
3. What relationship are you using? (e.g. vertical angles, same-side exterior angles, alternate interior angles)



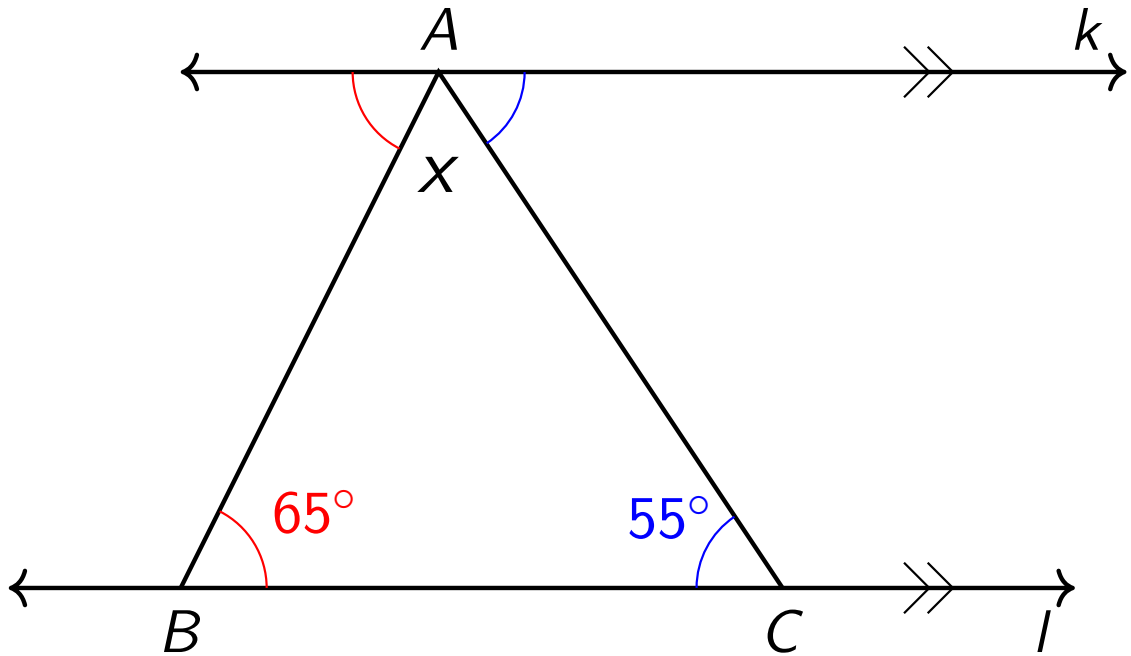
Lesson: The sum of a triangle's *interior* angles is 180°

Triangle sum theorem

Triangle sum theorem

Given parallel lines $k \parallel l$, $\triangle ABC$, $\text{m}\angle B = 65°$, $\text{m}\angle C = 55°$.

Find $\text{m}\angle BAC = x$.



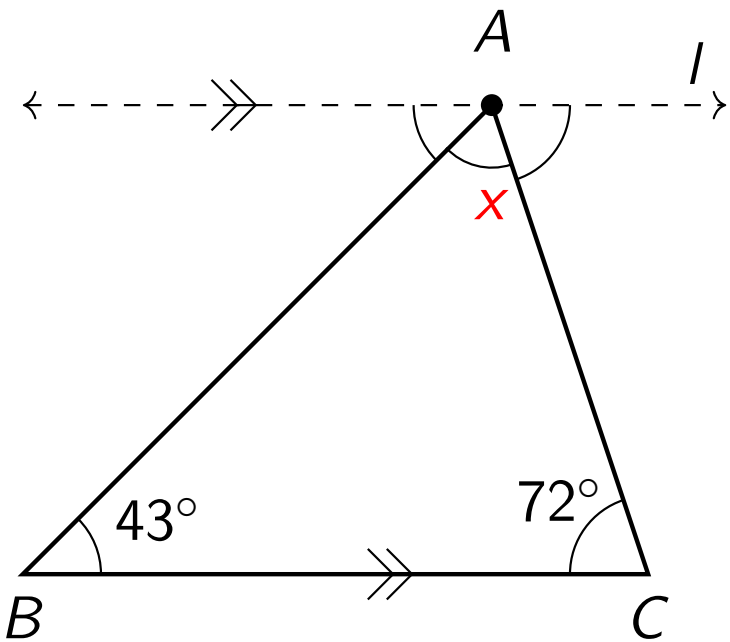
Interior The three angles that are *inside* the triangle

Theorem The sum of the measures of the three internal angles of a triangle is 180°

Mark 3 missing angle measures to make a straight angle

An *auxilary* line l is drawn through A, parallel to triangle base $\overline{BC}$.

Find m$\angle BAC$.



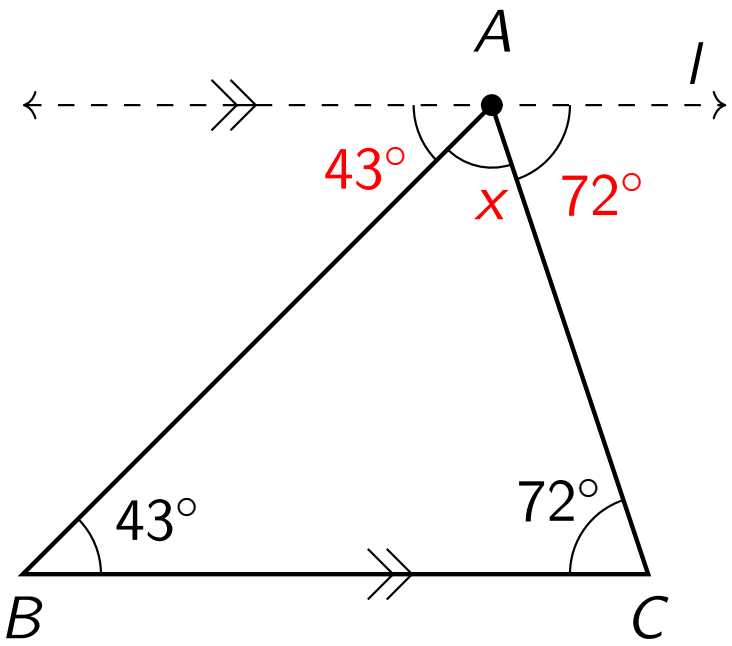
Auxilary An extra line added to a diagram

Linear triple Three adjacent angles that make a straight line

Mark 3 missing angle measures to make a straight angle

An *auxilary* line l is drawn through A, parallel to triangle base $\overline{BC}$.

Find $\text{m}\angle BAC$.



$$43 + x + 72 = 180$$

$$x = 60°$$

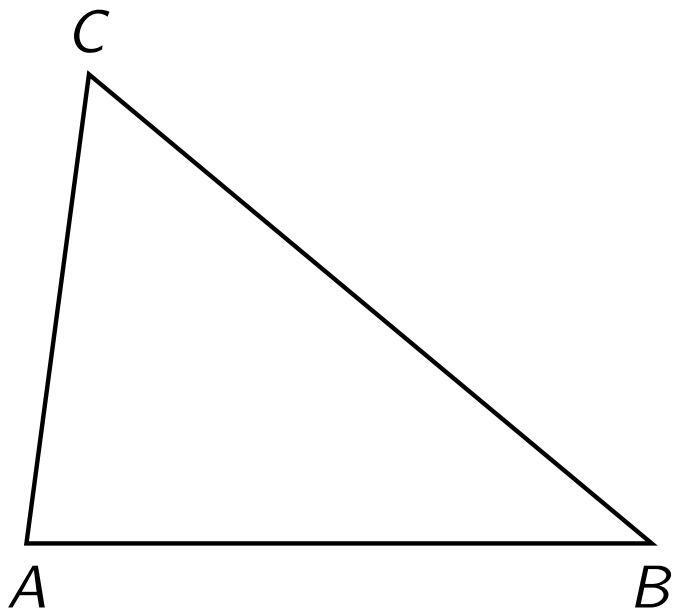
Theorem:
$\text{m}\angle A + \text{m}\angle B + \text{m}\angle C = 180°$
for any triangle

Auxilary An extra line added to a diagram

Linear triple Three adjacent angles that make a straight line

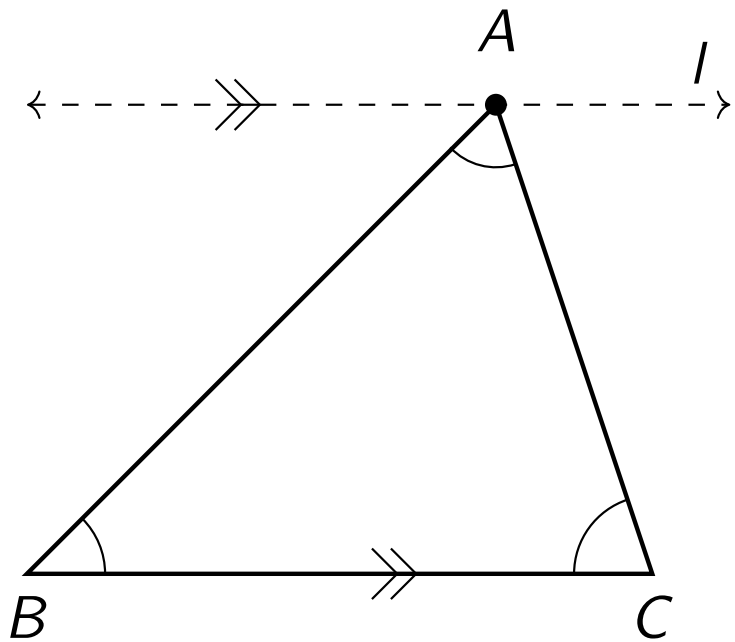
Find the missing angle measure

Given $\triangle ABC$, $\text{m}\angle A = 82°$, $\text{m}\angle C = 59°$. Find $\text{m}\angle B$.



Triangle sum theorem (180°)

Check your notes



Auxilary line An extra line added to a diagram

Linear triple Three adjacent angles that make a straight line

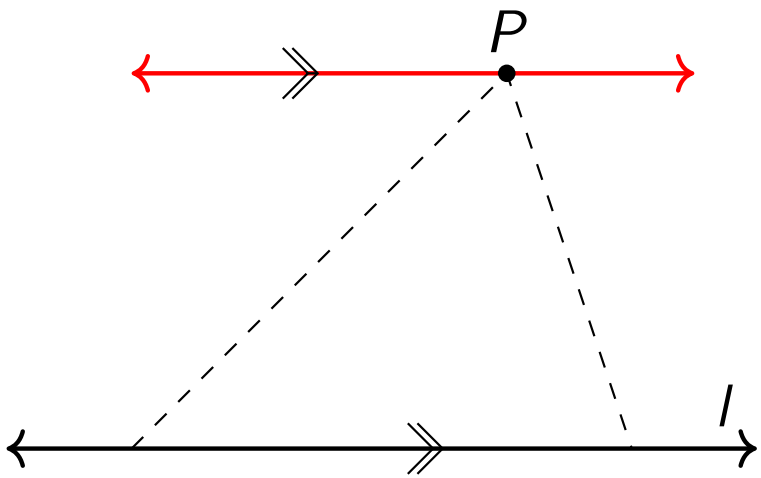
Interior angles The three angles that are inside the triangle

Theorem The sum of a triangle's angles is 180°

$$\mathrm{m}\angle A + \mathrm{m}\angle B + \mathrm{m}\angle C = 180°$$

Extension: Euclid's fifth postulate (the Parallel Postulate)

Given a line and a point, there exists one line through the point parallel to the line.



Euclid Greek author of the most successful math book of all time, *The Elements*

Postulate A statement we assume is true as the basis of all further mathematical theorems and proofs

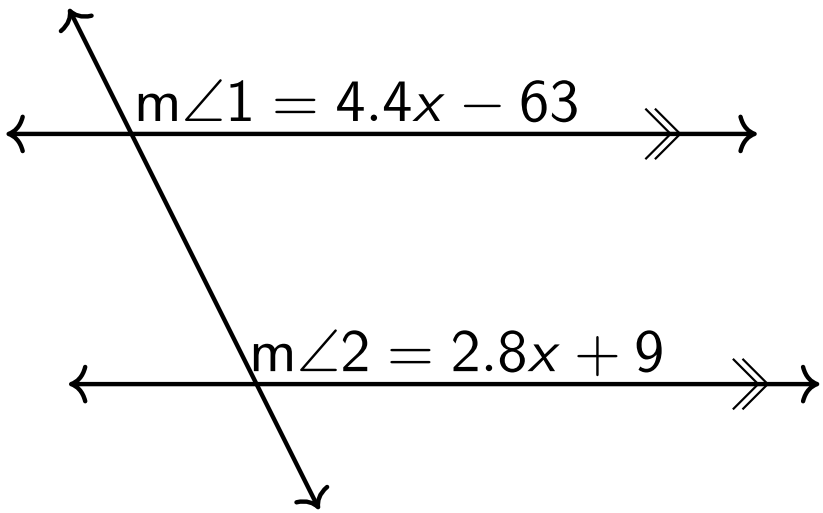
Non-Euclidean geometries Alternative mathematics not using the Parallel Postulate. Lobachevsky (1826 Russian), Bolyai (1832 Hungarian), Einstein (1916 German)

Learning Target: I can find the angles of a parallelogram

HSG.CO.C.9 Prove theorems about lines and angles 3.4 Friday 14 October

Do Now: Two parallel lines intersect a transversal. Given corresponding angles $m\angle 1 = 4.4x - 63$ and $m\angle 2 = 2.8x + 9$.

Find the measure of $\angle 1$.



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HSG.CO.C.9 Prove theorems about lines and angles 3.4 Friday 14 October

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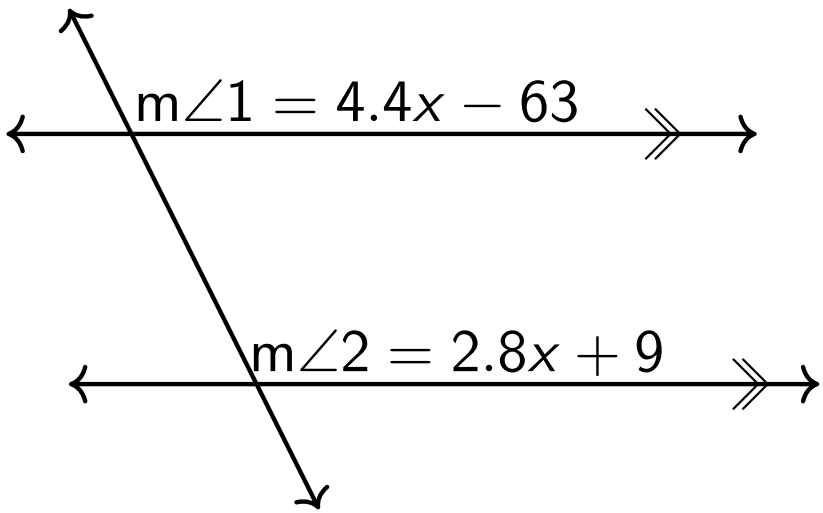


Corresponding angles are $\cong$

$$4.4x - 63 = 2.8x + 9$$
$$1.6x = 72$$
$$x = 45$$

$$m\angle 1 = 4.4(45) - 63 = 135^\circ$$

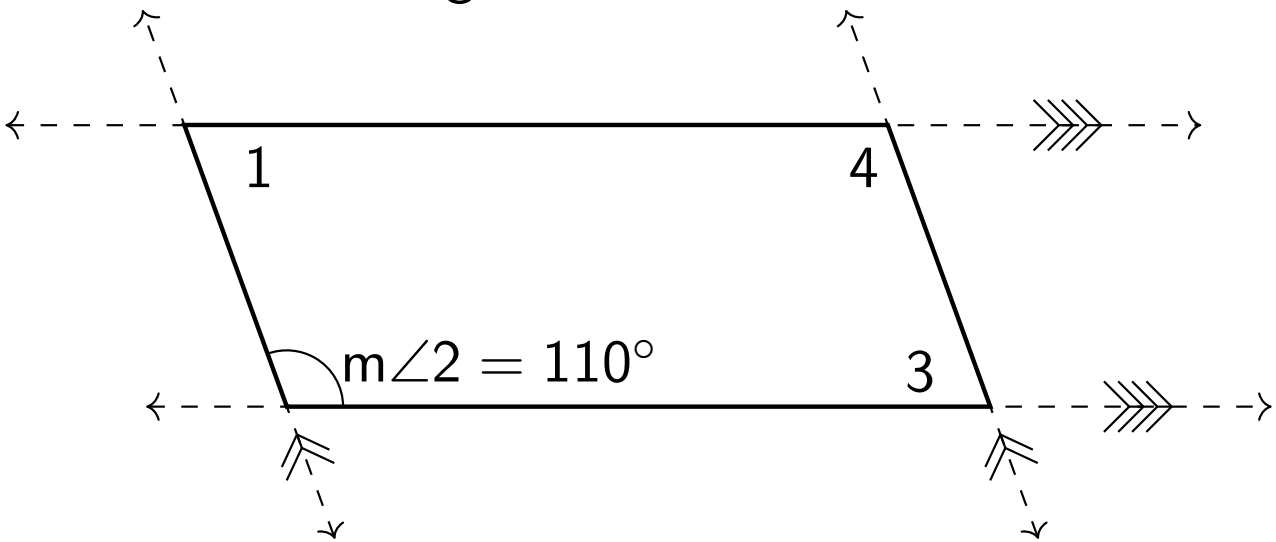
Check:

$$m\angle 2 = 2.8(45) + 9 = 135$$

A parallelogram's opposite sides are parallel and congruent

Consecutive angles are supplementary. Opposite angles are congruent.

Find the other angle measures.

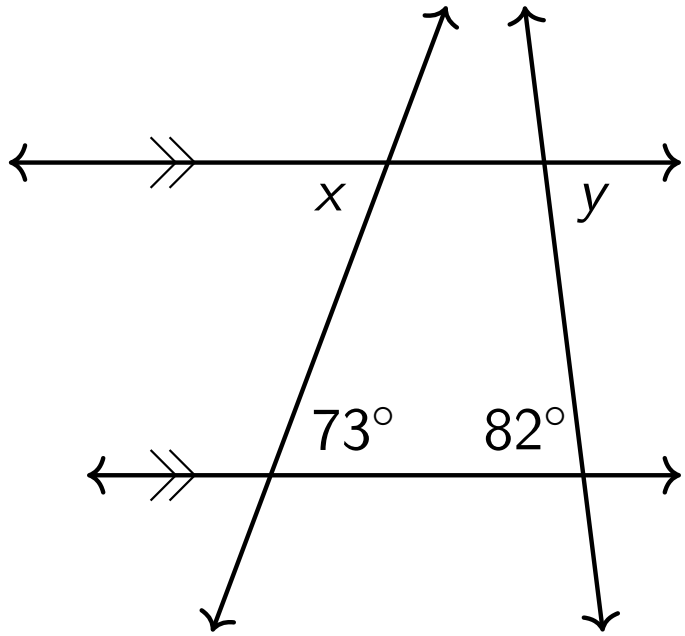


Learning Target: I can calculate external triangle angles

HSG.CO.C.9 Prove theorems about lines and angles 3.5 Monday 17 October

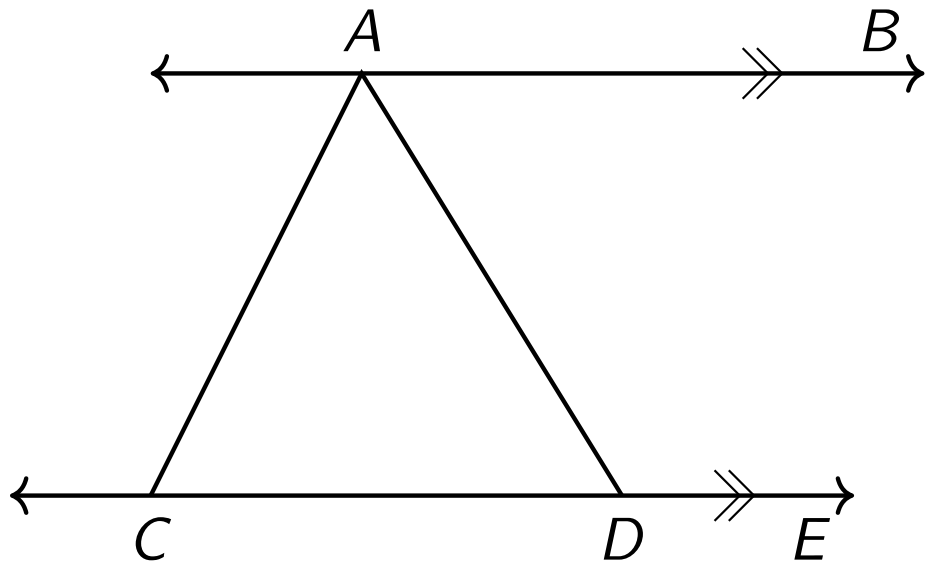
Do Now:

1. Given two parallel lines, two transversals
2. Find x, y
3. What relationship are you using? (e.g. vertical angles, same-side exterior angles, alternate interior angles, etc.)



Lesson: Triangle external angle theorem

Given parallel lines $\overleftrightarrow{AB} \parallel \overleftrightarrow{CDE}$ with $\overline{AC} \cong \overline{CD}$. If $m\angle BAD = 80$ find $m\angle ACD$.



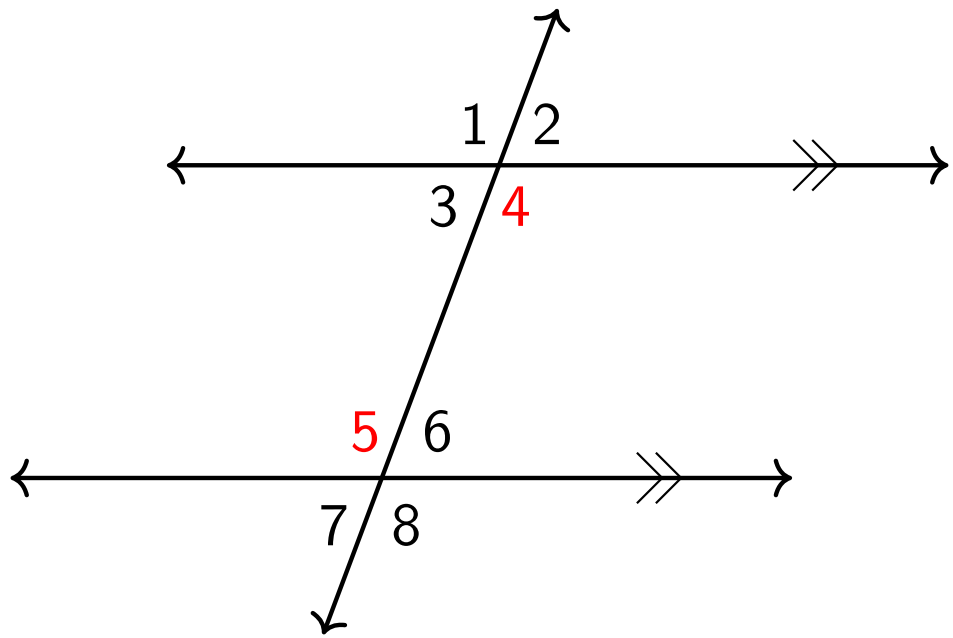
Learning Target: I can calculate transversal angles

HSG.CO.C.9 Prove theorems about lines and angles 3.6 Tuesday 18 October

Given two parallel lines and a transversal,

$m\angle 4 = 3x$ and $m\angle 5 = x + 70$.

Write an equation, then solve for x.

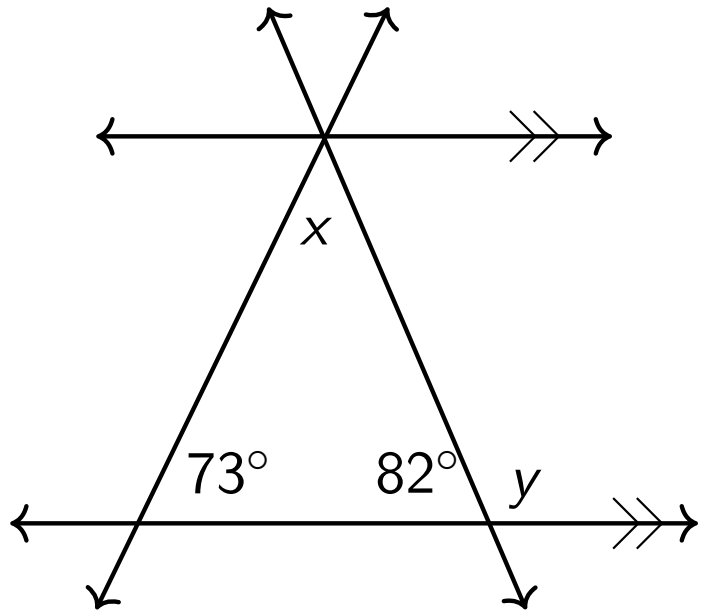


Learning Target: I can calculate angles in parallelograms

HSG.CO.C.9 Prove theorems about lines and angles 3.7 Wednesday 19 October

Do Now:

1. Given a triangle, shown
2. Find angle measures x, y
3. What relationships are you using? (e.g. vertical angles, same-side exterior angles, alternate interior angles)

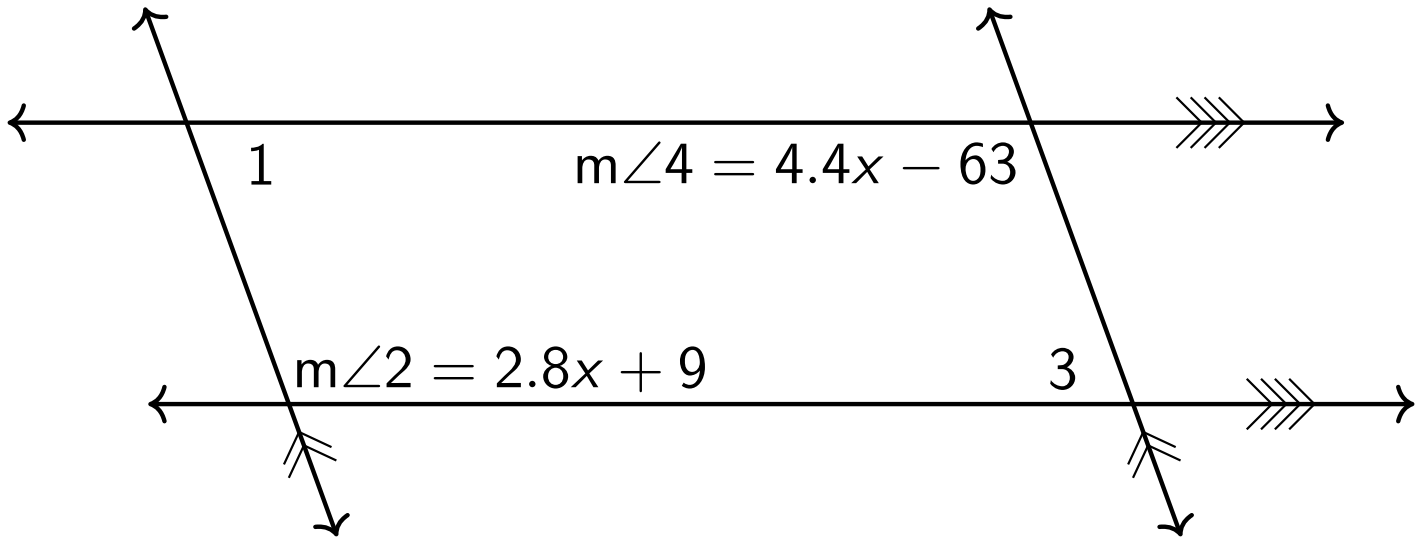


Lesson: Triangle's exterior angles

Learning Target: I can review with my classmates

HSG.CO.C.9 Prove theorems about lines and angles 3.8 Thursday 20 October

Two parallel lines intersect a second set of parallel lines. Given $m\angle 2 = 2.8x + 9$ and $m\angle 4 = 4.4x - 63$, find the measure of $\angle 1$.



Learning Target: I can review with my classmates

HSG.CO.C.9 Prove theorems about lines and angles 3.9 Friday 21 October