

1.20 PreExam: Probability, Venn diagrams

1. A survey question has three possible responses: A , B , and C . Among 100 surveys, the frequency of the answers collected were as follows: $n(A) = 10$, $n(B) = 35$, and $n(C) = 55$.

- (a) If a survey is selected at random, what is the probability the response was B or C ?

$$\frac{n(B) + n(C)}{n(U)} = \frac{35 + 55}{100} = 0.9$$

- (b) What is the probability a survey selected at random was an answer other than B or C ?

$$\frac{n(A)}{n(U)} = \frac{10}{100} = 0.1$$

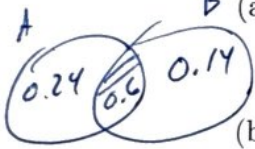
2. The events A and B are independent with $P(A) = 0.3$ and $P(B) = 0.2$.

- (a) What is $P(A \cap B)$?

$$P(A \cap B) = (P(A))(P(B)) = 0.3 \cdot 0.2 = 0.06$$

- (b) What is $P(A \cup B)$?

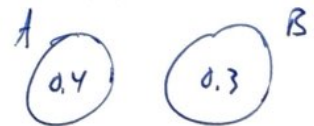
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.2 - 0.06 = 0.44$$



3. The events A and B are mutually exclusive with $P(A) = 0.4$ and $P(B) = 0.3$.

- (a) What is $P(A \cap B)$?

0



- (b) What is $P(A' \cup B)$?

0.3

4. Given:

$U = \{\text{the letters in the alphabet}\}$

$A = \{b, e, c, a\}$

$B = \{r, u, l, e, s\}$

- (a) List the elements of $A \cap B$.

$\{e\}$

[1 mark]

- (b) List the members of $A \cup B$.

$\{b, e, c, a, r, u, l, s\}$

[1 mark]

↑
do not
duplicate
the
"e"

5. The universal set U is defined as the set of positive integers less than 10. The subsets A and B are defined as follows:

$$A = \{\text{the odd numbers}\} \quad B = \{\text{prime numbers}\}$$

$$U = \{1, 2, \dots, 8, 9\}$$

- (a) List the members of A' .

$$\{2, 4, 6, 8\}$$

[1 mark]

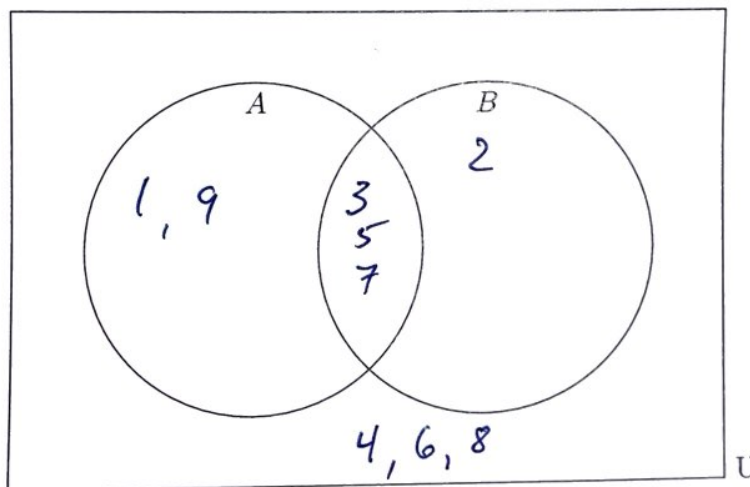
- (b) List the members of $(A \cup B)'$.

$$\{4, 6, 8\}$$

[1 mark]

- (c) Place the elements of A and B in the appropriate regions in the Venn diagram below.

[2 marks]



- (d) List the items in $A \cap B$.

$$\{3, 5, 7\}$$

[1 mark]

- (e) If an element is selected at random, what is the probability that it is a member of both sets, $(A \cap B)$?

[1 mark]

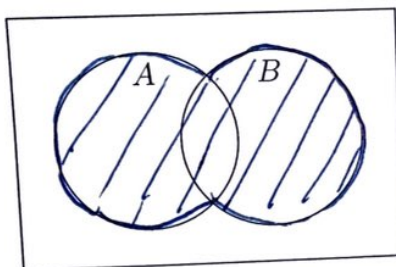
$$\frac{n(\{3, 5, 7\})}{n(U)}$$

$$\frac{3}{9} = \frac{1}{3}$$

Name:

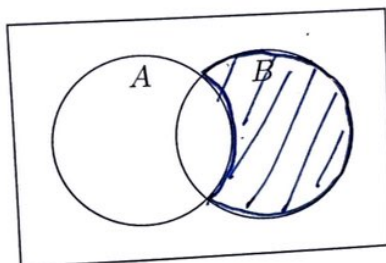
6. For each Venn diagram, shade the area representing the expression. Use pencil.

(a) $A \cup B$



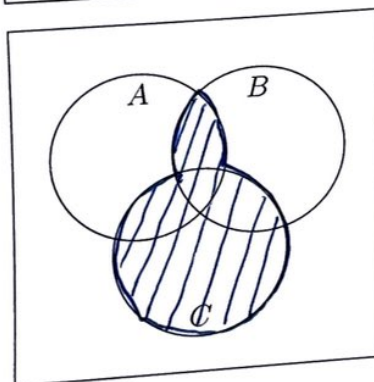
[2 marks]

(b) $A' \cap B$



[2 marks]

(c) $(A \cap B) \cup C$



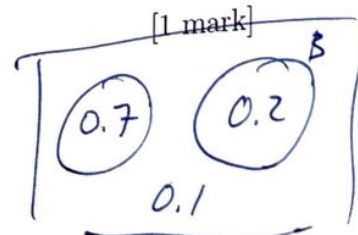
[2 marks]

7. The events A and B are mutually exclusive with $P(A) = 0.7$ and $P(B) = 0.2$.

(a) Write down $P(A \cup B)$.

$$0.7 + 0.2 = 0.9$$

[1 mark]



[1 mark]

(b) Find $P(A' \cup B)$.

$$0.2$$

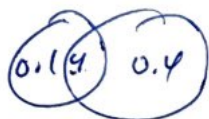
8. The events A and B are independent with $P(A) = 0.5$ and $P(B) = 0.8$.

(a) Find $P(A \cap B)$.

$$P(A) \cdot P(B) = 0.5 \cdot 0.8 = 0.40 \quad [2 \text{ marks}]$$

(b) Find $P(A \cup B)$.

$$= P(A) + P(B) - P(A \cap B) = 0.5 + 0.8 - 0.4 = 0.9 \quad [2 \text{ marks}]$$

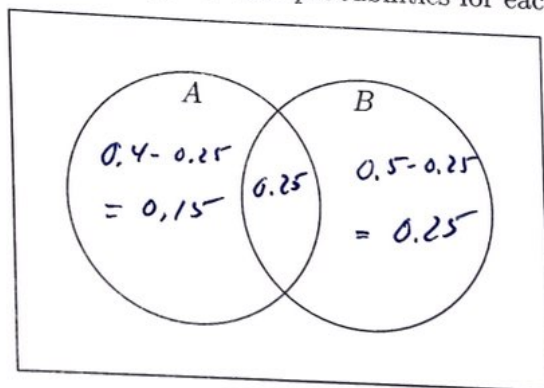


(c) Find $P(B|A)$.

$$= \frac{P(A \cap B)}{P(A)} = \frac{0.4}{0.5} = 0.8 \quad [2 \text{ marks}]$$

9. Given events A and B with $P(A) = 0.4$, $P(B) = 0.5$, $P(A \cap B) = 0.25$.

(a) Completely mark the Venn diagram with probabilities for each area. [2 marks]



(b) Find $P(A \cup B)$.

$$= 0.4 + 0.5 - 0.25 = 0.65 \quad [2 \text{ marks}]$$

(c) State whether events A and B are independent. Justify your answer. [3 marks]

$$P(A) \cdot P(B) = 0.4 \cdot 0.5 = 0.2 \neq 0.25 = P(A \cap B)$$

Not independent

(d) Find $P(A|B)$.

$$= \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.5} = 0.5 \quad [2 \text{ marks}]$$

10. There are 80 athletes playing the following sports:

- 35 play Archery
- 44 play Badminton
- 39 play Cricket
- 16 play Archery and Badminton
- 15 play Archery and Cricket
- 10 play Badminton and Cricket
- 3 play all three of these sports

Complete the Venn diagram below with the number of students in each region to represent the situation. [4 marks]

