Solutions

Lesson 18 Practice Problems

1. Rewrite the rational function $g(x) = \frac{x-4}{x}$ in the form $g(x) = c + \frac{r}{x}$, where c and r are constants.

$g(x) = 1 - \frac{4}{x}$

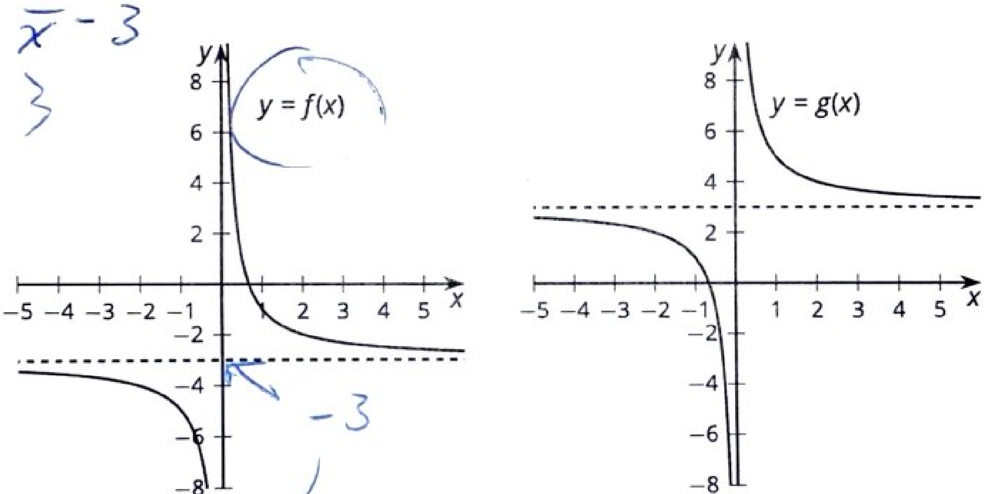
2. The average cost (in dollars) per mile for riding x miles in a cab is $c(x) = \frac{2.5+2x}{x}$. As x gets larger and larger, what does the end behavior of the function tell you about the situation?

$c(x) = \frac{2.5}{x} + 2$ $\quad c(x) \to \$2$ so for longer rides the cost is about \$2/mile

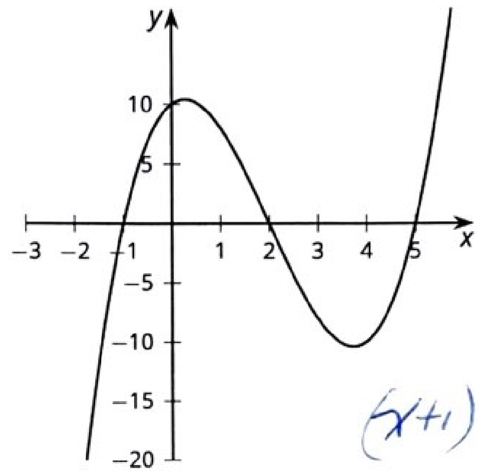
3. The graphs of two rational functions f and g are shown. One of them is given by the expression $\frac{2-3x}{x}$. Which graph is it? Explain how you know.

$= \frac{2}{x} - 3$

minus 3



4. Which polynomial function's graph is shown here?



$(x+1)(x-2)(x-5)$

A. $f(x) = (x+1)(x+2)(x+5)$

B. $f(x) = (x+1)(x-2)(x-5)$

C. $f(x) = (x-1)(x+2)(x+5)$

D. $f(x) = (x-1)(x-2)(x-5)$

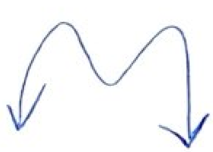
(From Unit 2, Lesson 7.)

5. State the degree and end behavior of $f(x) = 5x^3 - 2x^4 - 6x^2 - 3x + 7$. Explain or show your reasoning.

degree 4

-2 : negative leading coefficient

(From Unit 2, Lesson 9.)

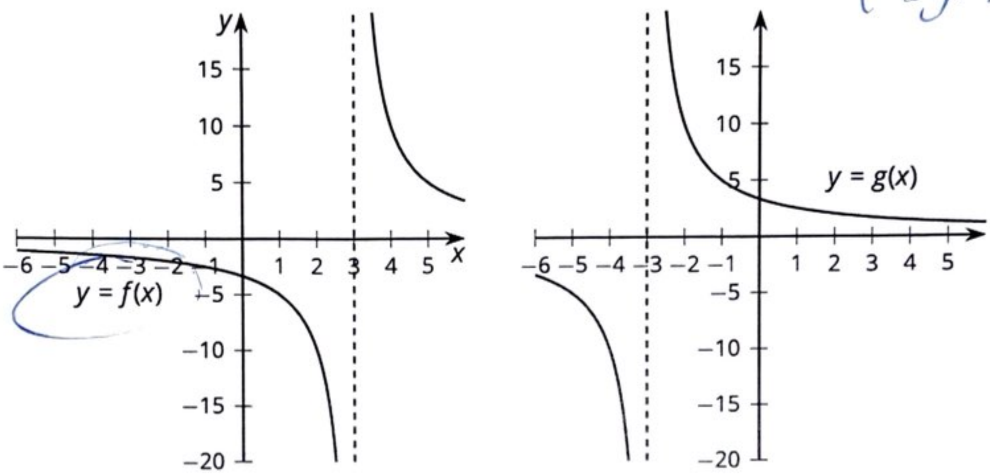


$x \to +\infty \quad y \to -\infty$

$x \to -\infty \quad y \to +\infty$

6. The graphs of two rational functions f and g are shown. Which function must be given by the expression of $\frac{10}{x-3}$? Explain how you know.

denominator $x-3$ $x=3$ undefined (asymptote)



(From Unit 2, Lesson 17.)

Lesson 19 Practice Problems

1. The function $f(x) = \frac{5x+2}{x-3}$ can be rewritten in the form $f(x) = 5 + \frac{17}{x-3}$. What is the end behavior of $y = f(x)$?

horizontal asymptote $y = 5$

$x \to +\infty \quad y \to 5$

$x \to -\infty \quad y \to 5$

Vertical asymptote $x = 3$

2. Rewrite the rational function $g(x) = \frac{x^2+7x-12}{x+2}$ in the form $g(x) = p(x) + \frac{r}{x+2}$, where $p(x)$ is a polynomial and r is an integer.

$g(x) = x + 5 + \frac{-22}{x+2}$

$$\begin{array}{r} x+5 \\ x+2 \,\big)\, x^2+7x-12 \\ x^2+2x \\ \hline 5x-12 \\ 5x+10 \\ \hline -22 \end{array}$$

3. Match each polynomial with its end behavior as x gets larger and larger in the positive and negative directions. (Note: Some of the answer choices are not used and some answer choices are used more than once.)

A. $p(x) = \frac{3}{x-1}$ (5)

B. $q(x) = \frac{2x}{x-1} = 2 + \frac{2}{x-1}$ (1)

C. $r(x) = \frac{2x+3}{x-1} = 2 + \frac{5}{x-1}$ (1)

D. $s(x) = \frac{2x^2+x+3}{x-1} = 2x+3+\frac{6}{x-1}$ (3)

E. $t(x) = \frac{x^3}{x-1} = x^2+x+1+\frac{1}{x-1}$ (4)

1. The graph approaches $y = 2$.
2. The graph approaches $y = 3$.
3. The graph approaches $y = 2x + 3$.
4. The graph approaches $y = x^2 + x + 1$.
5. The graph approaches $y = 0$.

B. $$\begin{array}{r} 2 \\ x-1 \,\big)\, 2x \\ 2x-2 \\ \hline +2 \end{array}$$

C. $$\begin{array}{r} 2 \\ x-1 \,\big)\, 2x+3 \\ 2x-2 \\ \hline 5 \end{array}$$

D. $$\begin{array}{r} 2x+3 \\ x-1 \,\big)\, 2x^2+x+3 \\ 2x^2-2x \\ \hline 3x+3 \\ 3x-3 \\ \hline 6 \end{array}$$

E. $$\begin{array}{r} x^2+x+1 \\ x-1 \,\big)\, x^3 \\ x^3-x^2 \\ \hline x^2 \\ x^2-x \\ \hline x \\ x-1 \\ \hline 1 \end{array}$$

4. Let the function P be defined by $P(x) = x^3 + 2x^2 - 13x + 10$. Mai divides $P(x)$ by $x + 5$ and gets:

$$
\begin{array}{r}
x^2 - 3x + 2 \\
x + 5 \overline{\smash{\big)}\, x^3 + 2x^2 - 13x + 10} \\
\underline{-x^3 - 5x^2} \qquad\qquad\quad \\
-3x^2 - 13x \qquad\quad \\
\underline{3x^2 + 15x} \qquad\quad \\
2x + 10 \\
\underline{-2x - 10} \\
0
\end{array}
$$

How could we tell by looking at the remainder that $(x + 5)$ is a factor?

remainder $= 0$

$\Rightarrow (x+5)$ is factor

(From Unit 2, Lesson 13.)

5. For the polynomial function $f(x) = x^4 + 3x^3 - x^2 - 3x$ we have $f(-3) = 0$, $f(-2) = -6$, $f(-1) = 0$, $f(0) = 0$, $f(1) = 0$, $f(2) = 30$, $f(3) = 144$. Rewrite $f(x)$ as a product of linear factors.

$f(x) = (x+3)(x+1)(x-1)x$

$(+3) * (+1) * (-1) = -3$

no leading coefficient

(From Unit 2, Lesson 15.)

6. There are many cones with a volume of 60π cubic inches. The height $h(r)$ in inches of one of these cones is a function of its radius r in inches where $h(r) = \frac{180}{r^2}$.

   a. What is the height of one of these cones if its radius is 2 inches?

   $h(2) = \frac{180}{2^2} = 45$ inches

   b. What is the height of one of these cones if its radius is 3 inches?

   $h(3) = \frac{180}{3^2} = 20$ inches

   c. What is the height of one of these cones if its radius is 6 inches?

   $h(6) = \frac{180}{6^2} = 5$ inches

   (From Unit 2, Lesson 16.)

7. A cylindrical can needs to have a volume of 10 cubic inches. There needs to be a label around the side of the can. The function $S(r) = \frac{20}{r}$ gives the area of the label in square inches where r is the radius of the can in inches.

   a. As r gets closer and closer to 0, what does the behavior of the function tell you about the situation? $r \to 0$ $S(r) \to +\infty$

   the surface area gets larger and larger

   b. As r gets larger and larger, what does the end behavior of the function tell you about the situation?

   $r \to +\infty$

   $S(r) \to 0$

   the area of the label would get smaller and smaller

   (From Unit 2, Lesson 17.)

8. Match each rational function with a description of its end behavior as x gets larger and larger.

A. $9x$ (5)

B. $\frac{9}{x}$ (1)

C. $\frac{99x}{x}$ (4)

D. $\frac{99+x}{x}$ (2)

E. $\frac{99x+9}{x}$

F. $\frac{99+9x}{x}$ (3)

1. The value of the expression gets closer and closer to 0.
2. The value of the expression gets closer and closer to 1.
3. The value of the expression gets closer and closer to 9.
4. The value of the expression is 99.
5. The value of the expression gets larger and larger in the positive direction.
6. The value of the expression gets larger and larger in the negative direction.

(From Unit 2, Lesson 18.)

Solutions

Lesson 20 Practice Problems

1. A local office supply store charges $18 to set up their business card printing machine with the design and $0.15 in materials per business card to print. Select **all** equations that could represent an expression for the average cost $A(x)$ of printing a batch of x business cards.

   A. $A(x) = \frac{18+x}{0.15}$

   B. $A(x) = \frac{18+0.15x}{x}$

   C. $A(x) = \frac{0.15+18x}{x}$

   D. $A(x) = \frac{0.15}{18+x}$

   E. $A(x) = \frac{18+0.15x}{18+x}$

   F. $A(x) = \frac{18}{x} + 0.15$

2. The school band is in charge of a new set of uniforms made with a new logo. A local business charges $140 to set up the logo with the design and $0.25 in materials per logo printed. The function $C(x) = \frac{140+0.25x}{x}$ represents the average cost per logo if x uniforms are printed by this business.

   a. What is the average cost per uniform to get the logo printed on 25 uniforms?

   $C(25) = \frac{140 + 0.25(25)}{25} = \cancel{0.306} \quad 5.85$

   b. What is the average cost per uniform to get the logo printed on 100 uniforms?

   $C(100) = \frac{140 + 0.25(100)}{100} = \cancel{0.264} \quad 1.65$

   c. How many uniforms should be printed to have an average cost of $1 per logo?

   $C(x) = \frac{140+0.25x}{x} = 1 \qquad 140+0.25x = x$
   $x = 186\frac{2}{3}$

   d. What will happen to the price as the number of uniforms printed increases?

   it decreases, approaching ~~0.3~~ 0.25 per shirt

3. Two competing sports equipment suppliers sell footballs at different prices. Supplier A charges \$85 in shipping, and charges \$2.59 per football. Supplier B charges \$50 shipping, and charges \$4.29 per football. A school wants to buy 40 balls. Which supplier has the lowest average cost per ball?

$A = 85 + 2.59(40) = 188.60$ A is cheaper

$B = 50 + 4.29(40) = 221.60$

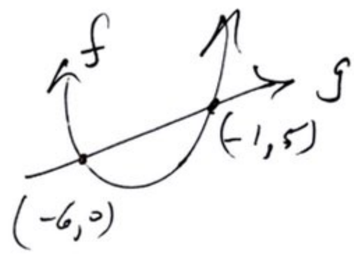
4. What is one point of intersection between the graphs of the functions $f(x) = (x+6)(x+2)$ and $g(x) = x+6$?

A. $(0, 6)$

B. $(-1, 5)$

C. $(-2, 0)$

D. $(-4, -4)$



(From Unit 2, Lesson 11.)

5. The graph of a polynomial $f(x) = (5x-3)(x+4)(x+a)$ has x-intercepts at -4, $\frac{3}{5}$, and 6. What is the value of a?

3/5 -4

$a = -6$

(From Unit 2, Lesson 15.)

6. The function $f(x) = \frac{3x-4}{x+6}$ can be rewritten in the form $f(x) = 3 + \frac{-22}{x+6}$. What is the end behavior of $y = f(x)$?

$x \to +\infty \quad y \to 3$

$x \to -6, \quad y \to \infty$

(From Unit 2, Lesson 19.)