

1.7 Quiz: Sequences, Open book: notes and calculator allowed

1. A sequence is defined as follows: 3, 7, 11, 15, ...

(a) Write down the first term u_1 . [1]

(b) Is the sequence arithmetic, geometric, or neither? [1]

(c) Find the value of the next term in the sequence. [1]

(d) Find a general expression for u_n , the n^{th} term. [3]

(a) 3

(b) arithmetic

(c) $15 + 4 = 19$

(d) $u_n = 3 + (n-1)4$

3. The fourth term of a geometric sequence $u_4 = 108$ and the fifth term $u_5 = 162$.

(a) Find the common ratio r . [1]

(b) Find the first term in the sequence. [3]

(c) Hence, find a general expression for u_n , the n^{th} term. [2]

$$(a) \quad r = \frac{162}{108} = \frac{3}{2}$$

$$(b) \quad u_4 = u_1 \cdot \left(\frac{3}{2}\right)^{4-1} = 108$$

$$u_1 = 32$$

$$(c) \quad u_n = 32 \cdot \left(\frac{3}{2}\right)^{n-1}$$

2. The first three terms of a geometric sequence are 27, 9, 3, ...

(a) Find the common ratio r . [2]

(b) Find the next two terms in the sequence. [2]

(c) Find a general expression for u_n , the n^{th} term. [2]

$$(a) \quad r = \frac{9}{27} = \frac{1}{3}$$

$$(b) \quad u_4 = 3 \cdot \frac{1}{3} = 1$$

$$u_5 = 1 \cdot \frac{1}{3} = \frac{1}{3}$$

$$(c) \quad u_n = 27 \cdot \left(\frac{1}{3}\right)^{n-1}$$

4. In an arithmetic sequence $u_5 = 38$ and $u_{13} = 86$.

(a) Find the common difference. [2]

(b) Find u_1 , the first term of the sequence. [2]

(c) Find the largest term in the sequence that is less than 200. [2]

method 1

$$\begin{aligned} (a) \quad u_5 &= u_1 + (5-1)d = 38 \\ u_{13} &= u_1 + (13-1)d = 86 \end{aligned} \quad \begin{array}{l} \uparrow \\ \text{subtract} \end{array}$$

$$8d = 48$$

$$d = 6$$

method 2

$$d = \frac{86-38}{13-5} = \frac{48}{8} = 6$$

(b)

$$u_5 = u_1 + (5-1)6 = 38$$

$$u_1 = 14$$

(c)

$$u_n = 14 + (n-1)6 < 200$$

$$n-1 < \frac{200-14}{6} = 31$$

$$n < 32$$

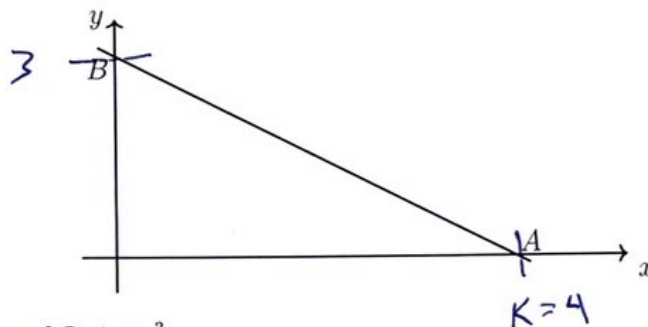
$$n = 31$$

$$u_{31} = 14 + (31-1)6 = 194$$

Challenge: Linear equations and quadratic functions

5. The diagram shows the straight line L_1 , which intersects the x -axis at $A(k, 0)$ and the y -axis at $B(0, 3)$.

diagram is not to scale



The gradient of L_1 is $-\frac{3}{4}$.

- (a) Write down the equation of the line L_1 . [1]
 (b) Find the value of k . [2]
 (c) The line L_2 is perpendicular to L_1 and passes through $(2, 1)$.
 i. Write down the gradient of the line L_2 . [1]
 ii. Hence, write down the equation of L_2 . Leave your answer in the form $y - a = m(x - b)$. [2]

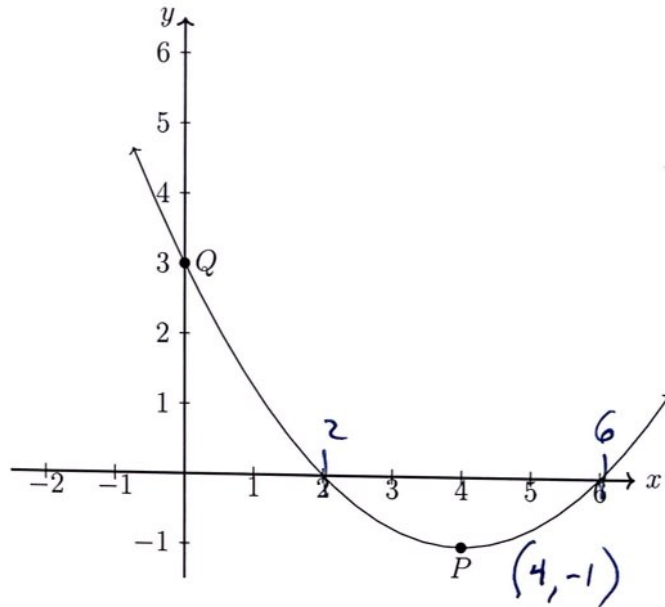
(a) $y = -\frac{3}{4}x + 3$

(b) $y = -\frac{3}{4}x + 3 = 0$
 $x = 4, k = 4$

(c) i) $m_{\perp} = \frac{4}{3}$

ii) ~~$y - 3 = -\frac{4}{3}(x - 4)$~~
 $y - 1 = \frac{4}{3}(x - 2)$

6. Let f be a quadratic function. Part of the graph of f is shown below. The vertex is at $P(4, -1)$ and the y -intercept is at $Q(0, 3)$.



- (a) The function f can be written in the form $f(x) = a(x - h)^2 + k$. Write down the value of h and of k .
- (b) Find a .
- (c) Find the zeros of the function f , such that $f(x) = 0$.

$$(a) \quad h = 4, \quad k = -1$$

$$(b) \quad f(0) = a(0 - 4)^2 - 1 = 3$$

$$16a = 4$$

$$a = \frac{1}{4}$$

$$(c) \quad f(x) = \frac{1}{4}(x - 4)^2 - 1$$

$$= \frac{1}{4}x^2 - 2x + 4 - 1$$

$$= \frac{1}{4}(x^2 - 8x + 12)$$

$$= \frac{1}{4}(x - 2)(x - 6) \quad x = 2, 6$$