La Scuola d'Italia / Huson / IB Math: Sequences First & last name:
31 October 2025 Grade:

2.3 Classwork: Review; due Monday 3 November

Paper 1 style (no calculator)

1. **1.** A sequence is defined by

$$u_1 = 4, \qquad u_{n+1} = 3u_n - 2 \quad (n \geq 1).$$

(a) Write down u_2 and u_3.

(b) Show that $u_n = 3^n + 1$ satisfies the recurrence.

(c) Hence find u_6.

2. A geometric sequence has first term 12 and common ratio $\frac{5}{6}$.

(a) Write down the second and third terms.

(b) Find the sum of the first n terms, S_n.

(c) Find the least value of n such that $S_n > 50$.

3. A quantity P decreases according to $P(t) = 1800e^{-kt}$.

(a) Given that $P(2) = 1500$, find k.

(b) Find the time when $P(t) = 900$.

(c) State whether the graph of P is linear, quadratic or exponential.

4. Solve the following, giving exact values when possible.

(a) $3^x = 27$.

(b) $\log_5(2x - 1) = 2$.

(c) $\ln(4) - \ln(x) = \ln(2)$.

5. Consider the function

$$f(x) = 2^x - 3.$$

(a) Find $f(0)$ and $f(2)$.

(b) Solve $2^x - 3 = 5$.

(c) Describe the transformation that maps $y = 2^x$ to $y = f(x)$.

6. An arithmetic sequence has first term $a_1 = 950$ and common difference $d = 25$.

(a) Write an expression for a_n.

(b) Find the smallest n such that $a_n \geq 1400$.

(c) Find the sum of the first n terms when n is the value from part (b).

7. A quadratic function is given by $g(x) = x^2 - 6x + 5$.

(a) Find the coordinates of the vertex.

(b) Solve $g(x) = 0$.

(c) The line $y = mx$ intersects the graph of g at two distinct points. Find the range of m for which this happens.

Paper 2 style (GDC allowed)

8. A music teacher records the number of hours h students practise each week and the mark M each student receives on a test. The data for eight students are entered into a GDC.

(a) Use your GDC to find the product–moment correlation coefficient r.

(b) The regression line of M on h is $M = ah + b$. Write down a and b from your GDC output.

(c) A student currently practises 10 hours per week. Use the regression line to estimate how many marks the student might get if they increase practice to 14 hours per week.

9. A school enrolment in year 1 is 1200 students and increases each year by 3.5%.

(a) Show that the number of students in year n can be modelled by $E_n = 1200(1.035)^{n-1}$.

(b) Find the number of students in year 8.

(c) A scholarship fund pays \$75 to each enrolled student each year. Find the total amount paid in the first 6 years.

10. The number of seats available at the school in year n is given by $S_n = 1100+40(n-1)$.

(a) Write down S_1 and S_{10}.

(b) Find the least value of n such that $S_n \geq E_n$ from question 9.

(c) Explain, with reference to the models, whether the school will always have enough seats for all applicants for all future years.

11. The sum of the first n terms of a geometric sequence is

$$S_n = 9\left(1 - \left(\frac{4}{5}\right)^n\right).$$

(a) Find the first term and the common ratio.

(b) Find S_∞.

(c) Find the least value of n such that $S_\infty - S_n < 0.0008$.

12. An investment of €5000 grows at a nominal annual rate of 2.4% compounded monthly.

(a) Write a formula for the value V after t years.

(b) Use your GDC to find the value after 5 years, correct to the nearest euro.

(c) Determine the time needed for the investment to reach €6000.

13. Consider the function $f(x) = \ln(x+2) - \dfrac{1}{2}$ for $x \geq -1$.

(a) Find $f(-1)$ and $f(0)$.

(b) Solve $\ln(x+2) - \dfrac{1}{2} = 0$.

(c) Sketch the graph of $y = f(x)$ on $-1 \leq x \leq 4$.

14. A researcher records the age of used cars (years) and their selling price (in thousands of euros). The data are entered into a GDC.

(a) Find the correlation coefficient.

(b) Write down the regression equation of price on age.

(c) Comment on whether it is sensible to use this regression equation to predict the price of a brand new car.

15. Define $f(x) = 2x + 3$ and $g(x) = 5e^{0.2x}$.

(a) Find $(f \circ g)(x)$.

(b) Find $(g \circ f)(x)$.

(c) Solve $(f \circ g)(x) = 33$ using your GDC.

Markscheme (outline)

1. (a) $u_2 = 3 \cdot 4 - 2 = 10$, $u_3 = 3 \cdot 10 - 2 = 28$. (b) Substitute: $3(3^n + 1) - 2 = 3^{n+1} + 1$. (c) $u_6 = 3^6 + 1 = 729 + 1 = 730$.

2. (a) $12, 12 \cdot \frac{5}{6} = 10$, etc. (b) $S_n = 12\frac{1-(5/6)^n}{1-5/6} = 72\big(1 - (5/6)^n\big)$. (c) Solve $72(1 - (5/6)^n) > 50$.

3. (a) $1500 = 1800e^{-2k} \Rightarrow e^{-2k} = \frac{5}{6} \Rightarrow k = \frac{1}{2}\ln\frac{6}{5}$. (b) $900 = 1800e^{-kt} \Rightarrow e^{-kt} = \frac{1}{2} \Rightarrow t = \frac{\ln 2}{k}$. (c) Exponential.

4. (a) $x = 3$. (b) $2x - 1 = 25 \Rightarrow x = 13$. (c) $\ln 4 - \ln x = \ln 2 \Rightarrow \ln\frac{4}{x} = \ln 2 \Rightarrow \frac{4}{x} = 2 \Rightarrow x = 2$.

5. (a) $f(0) = -3$, $f(2) = 1$. (b) $2^x = 8 \Rightarrow x = 3$. (c) Vertical translation down 3 units.

6. (a) $a_n = 950 + 25(n - 1)$. (b) $950 + 25(n - 1) \geq 1400 \Rightarrow 25(n - 1) \geq 450 \Rightarrow n - 1 \geq 18 \Rightarrow n = 19$. (c) $S_{19} = \frac{19}{2}(2 \cdot 950 + 18 \cdot 25)$.

7. (a) Vertex at $(3, -4)$. (b) Roots $x = 1, 5$. (c) Discriminant of $x^2 - 6x + 5 - mx = 0$ positive.

8. Use GDC: r close to the one in your source; slope $a > 0$; increase of 4 h multiplies slope by 4.

9. (a) Standard geometric growth statement. (b) $E_8 = 1200(1.035)^7$. (c) Arithmetic sum with 6 terms.

10. (a) $S_1 = 1100$, $S_{10} = 1100 + 40 \cdot 9 = 1460$. (b) Solve $1100 + 40(n-1) \geq 1200(1.035)^{n-1}$ numerically. (c) Geometric will eventually outgrow linear.

11. (a) $u_1 = 9(1 - 4/5) = 9/5$, $r = 4/5$. (b) $S_\infty = \frac{9}{1-4/5} = 45$. (c) $45 - 9(1 - (4/5)^n) < 0.0008 \Rightarrow 9(4/5)^n < 0.0008$.

12. (a) $V(t) = 5000\left(1 + \frac{0.024}{12}\right)^{12t}$. (b), (c) Use solver.

13. (a) $f(-1) = \ln 1 - \frac{1}{2} = -\frac{1}{2}$, $f(0) = \ln 2 - \frac{1}{2}$. (b) $\ln(x + 2) = \frac{1}{2} \Rightarrow x + 2 = e^{1/2} \Rightarrow x = e^{1/2} - 2$. (c) Standard log sketch.

14. Positive correlation, negative slope (if age vs price with usual data), regression usable only inside data range.

15. (a) $(f \circ g)(x) = 2\big(5e^{0.2x}\big) + 3 = 10e^{0.2x} + 3$. (b) $(g \circ f)(x) = 5e^{0.2(2x+3)}$. (c) Solve $10e^{0.2x} + 3 = 33$.