BECA / Huson / Alg 2
8 May 2024
4.17 Exam

Solutions

Exponential Functions and Equations: Mid-Unit Assessment

Do not use a calculator.

1. A bacteria population is growing exponentially with a growth factor of $\frac{1}{6}$ each hour. By what growth factor does the population change each half hour? Select all that apply.

   A. $\frac{1}{12}$

   B. $\sqrt{\frac{1}{6}}$

   C. $\frac{1}{3}$

   D. $\sqrt{6}$

   E. $\left(\frac{1}{6}\right)^{\frac{1}{2}}$

2. Let the function P represent the population $P(d)$, in thousands, of a colony of insects d days after first being measured. A model for P is $P(d) = 10 \cdot (1.08)^d$. Select all the statements that are true about this situation.

   A. There were 1,080 insects when the colony was first counted.

   B. One week after the colony was first counted, there were 10,800 insects.

   C. The growth factor per day is 1.08.

   D. The growth factor per week is $1.08 \cdot 7$.

   E. The growth factor per hour is $1.08^{\frac{1}{24}}$.

3. Scientists measure a bacteria population and find that it is 10,000. Five days later, they find that the population has doubled. Which function f could describe the bacteria population d days after the scientists first measured it, assuming it grows exponentially?

A. $f(d) = 10{,}000 \cdot 2^d$

B. $f(d) = 10{,}000 \cdot (\sqrt[5]{2})^d$

C. $f(d) = 10{,}000 \cdot \left(\frac{1}{\sqrt[5]{2}}\right)^d$

D. $f(d) = 10{,}000 \cdot 2^{5d}$

4. The value of a collectible toy is increasing exponentially. The two points on the graph show the toy's initial value and its value 3 weeks afterward.

$\frac{10}{5} = 2$

$3 - 0 = 3$

a. Express the toy's value t, in dollars, as a function of time w, in weeks, after purchase.

$t = 5 \cdot 2^{t/3}$

b. Write an expression to represent the toy's value 10 days after purchase.

$t(10) = 5 \cdot 2^{10/3}$

5. A sample of radium has a weight of 1.5 mg and a half-life of approximately 6 years.

   a. How much of the sample will remain after 6 years? 3 years? 1 year?

   6: 0.75

   $3: 1.5 \cdot 1/\sqrt{2} = 1.0606\ldots \approx 1.06$

   $1: 1.5 \cdot \frac{1}{\sqrt[6]{2}} = 1.3363\ldots \approx 1.34$

   b. Find a function f which models the amount of radium $f(t)$, in mg, remaining after t years.

   $f(t) = 1.5 \cdot \left(\frac{1}{2}\right)^{t/6}$

6. a. The area, in square meters, of a pond covered by an algae bloom decreases exponentially after a treatment is applied. Fill out the table, giving the area covered by the algae in square meters d days after the treatment is applied.

   +2

days	0	1	2	3	4
area	150	106	75	53	37.5

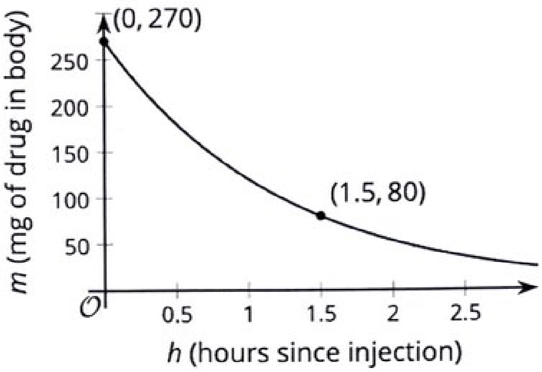
   $\frac{75}{150} = \frac{1}{2}$ $\quad r = \left(\frac{1}{2}\right)^{1/2}$

   b. Another pond has an algae bloom that is also decreasing exponentially. The area of this bloom in square meters is given by the function $B(d) = 100 \cdot 2^{-\frac{d}{7}}$, where d is days since the first measurement of the bloom. Which of the two algae blooms was larger initially? Which is decreasing more quickly? Explain how you know.

   The first pond is larger initially: 150 > 100

   The first pond is also decreasing more quickly, halfing each two days versus seven days

7. The graph shows the amount of a medicine m, in milligrams, remaining in a patient's body h hours after receiving an injection. The amount of the medicine decreases exponentially.



a. By what factor did the medicine decrease in the first hour and a half? Explain how you know.

$$\frac{80}{270} = 0.\overline{296296}$$

b. By what factor did the medicine decrease in the first half hour? What about in the first hour? Explain how you know.

half hour: $\sqrt[3]{\frac{80}{270}} = \left(\frac{80}{270}\right)^{\frac{1}{3}} = \frac{2}{3}$

hour: $\left(\frac{2}{3}\right)^2 = 4/9 = 0.4444...$

c. Write an equation relating m, the number of milligrams of the drug in the patient's body, and h, the number of hours since the injection.

$$m = 270 \cdot \left(\frac{4}{9}\right)^h$$