BECA / Dr. Huson / Geometry
Unit 1: Segments, length, and area
20 Sept 2022

Name: Solutions

1.8 Extension: Scientific notation

By "scientific notation" we mean in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.

1. Convert each value to scientific notation.

   (a) 5000 $= 5 \times 10^3$

   (b) 12,000 $= 1.2 \times 10^4$

   (c) 450 $= 4.5 \times 10^2$

   (d) 1,060,000 $= 1.06 \times 10^6$

2. Expand each value to regular numeric form. (i.e. an integer)

   (a) $9 \times 10^2 = 900$

   (b) $1.5 \times 10^5 = 150{,}000$

   (c) $6.22 \times 10^3 = 6220$

   (d) $1.41 \times 10^2 = 141$

3. Calculate each product. Leave in exponential form.

   (a) $10^2 \times 10^2 = 10^4$

   (b) $10^3 \times 10^5 = 10^8$

4. Calculate and write as scientific notation.

   (a) $22.5 \times 14^2 - 700$

   $= 3710$

   $= 3.71 \times 10^3$

   (b) The mean distance of the earth to the moon, 384,000 kilometers.

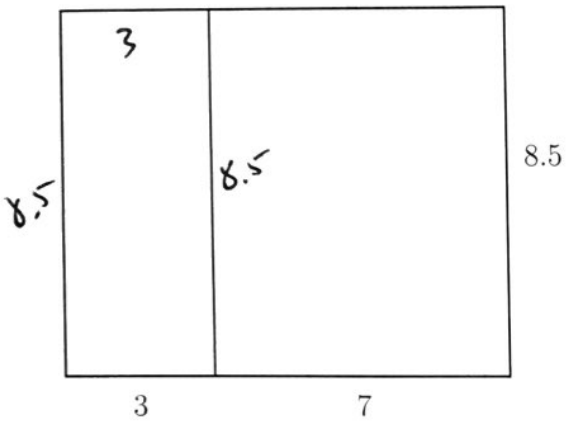
   3.84×10^5 km

5. The dimensions of an American football field are 360 feet by 160 feet. Express the area of a football field in square feet in scientific notation.

   $A = 360 \times 160 = 57600$

   $= 5.76 \times 10^5$

6. A compound shape composed of two rectangles is shown with dimensions marked, both having heights of 8.5 and with base lengths of 3 and 7 respectively.



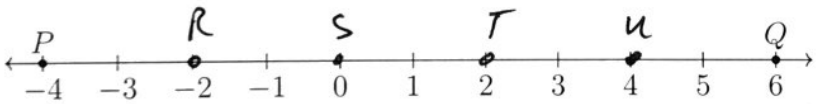
(a) Find the perimeter of the smaller rectangle on the left.

$P = 2(3) + 2(8.5)$
$= 23$

(b) Find the total area of the combined rectangles

$A = 8.5(3+7)$
$= 85$

7. Given $\overleftrightarrow{PQ}$ as shown on the number line. Divide segment $\overline{PQ}$ into five congruent segments by marking and labeling the points R, S, T, and U on the numberline.

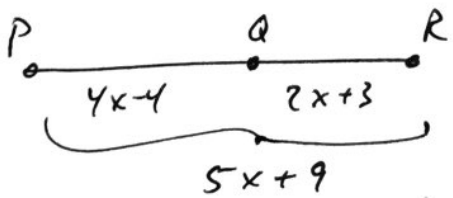


$PQ = 6-(-4) = 10$

$\frac{10}{5} = 2$

8. Given $\overline{PQR}$, with $PQ = 4x - 4$, $QR = 2x + 3$, and $PR = 5x + 9$. Find PR.

(show the check)



$(4x-4) + (2x+3) = 5x+9$
$6x - 1 = 5x + 9$
$x = 10$

$PR = 5(10) + 9$
$= 59$

$PQ = 4(10) - 4$
$= 36$

$QR = 2(10) + 3$
$= 23$

$36 + 23 = 59$ ✓

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1.9 Extension: Significant figures

Significant figures are the digits in a number that are meaningful for accuracy, as opposed to zeros for place value. See MathIsFun definitions Significant Digits

1. Write down the number of significant digits in each value.

   (a) 8 — 1

   (b) 27 — 2

   (c) 60 — 2

   (d) 120 — 3

   (e) 105.5 — 4

   (f) 1.7320 — 5

2. Round each value to three sig figs

   (a) 1,472,654 $\approx$ 1,470,000
   (population of the Bronx)

   (b) π $\approx$ 3.14

   (c) 8,804,190 $\approx$ 8,804,190 * ambiguous
   (population of NYC)

   (d) $\sqrt{2}$ $\approx$ 1.41

3. Do the calculation two ways: round each value to three sig figs before calculating versus round only at the end.

   (a) $39.37^2 - 1510$
   $39.4^2 - 1510$
   $\approx 1552 - 1510$
   $= 42$

   (b) $1.2548^2\pi$
   $\approx 1.25^2\pi$
   $= 4.9087\ldots \approx 4.91$

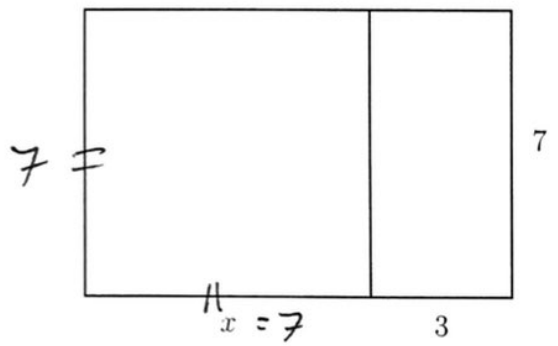
   (c) $39.37^2 - 1510$
   $= 39.9969$
   ≈ 40.0

   (d) $1.2548^2\pi$
   $= 4.94651\ldots$
   ≈ 4.95

   What do you notice. In calculations, when should values be rounded?

   The values turn out somewhat different. Round only at the end.

4. A large rectangle is divided by a vertical line into a square and a smaller rectangle, as shown. Its height is 7 and base $x + 3$, as marked.



(a) Find the area of the square.

$A_{sq} = 7^2 = 49$

(b) Find the perimeter of the large rectangle.

$P = 10 + 7 + 10 + 7$

$= 34$

5. Find the *area* of the shape shown below composed of a rectangle and two semi-circular caps. Round the result to three sig figs.

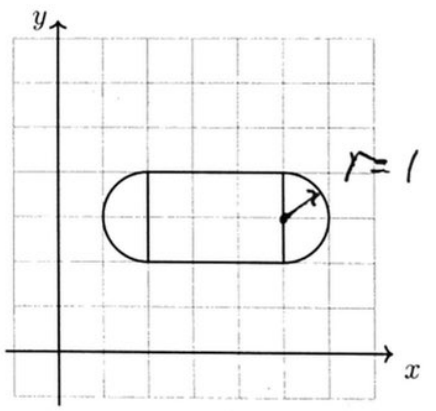
$A_{rect} = 2 \times 3 = 6$

$A_C = \pi 1^2 = \pi$

$A_{TOTAL} = \pi + 6$

$= 9.14159\ldots$

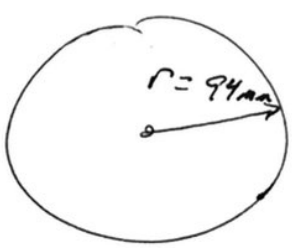
≈ 9.14



6. Given that the distance from the earth to the sun is 94.297 million miles. Find the circumference of the earth's orbit around the sun. (leave your result in scientific notation rounded to three sig figs)

$C = 2\pi\ 9.4297 * 10^7$

$=$



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1.10 Extension: Confidence intervals and the margin of error

Learn to use and intepret common notation for confidence intervals (see MathBootCamps)

- Plus or minus a *margin of error*. e.g. $v = 24.8 \pm 4.5$
- As an interval or range, $(20.3, 29.3)$. (brackets are also used, i.e. $[20.3, 29.3]$)
- As an inequality, $20.3 \leq v \leq 29.3$

1. The height of a Christmas tree rounded to the nearest foot is 7 feet. What is the shortest the tree could be? The tallest? Express your answer as an interval or range, with parenthesis.

   $(6.5, 7.5)$

2. Express the value $v = 10 \pm 1.5$ as an inequality.

   $8.5 \leq v \leq 11.5$

3. A person's weight is estimated as 125 lbs. plus or minus 5 lbs. Express that as a percent, i.e. in the form $125 \pm x\%$.

   $\% = \frac{5}{125} = 4\%$ $\qquad 125 \pm 4\%$

4. The radius of a circle rounded to the nearest foot is 10 feet. Find the possible values for the area of the circle. Express your answer as an interval / range.

   $9.5 \leq r < 10.5$

   $A = \pi 9.5^2 = 283.528\ldots$, $\pi 10.5^2 = 346.3605\ldots$ $\qquad (284, 346)$

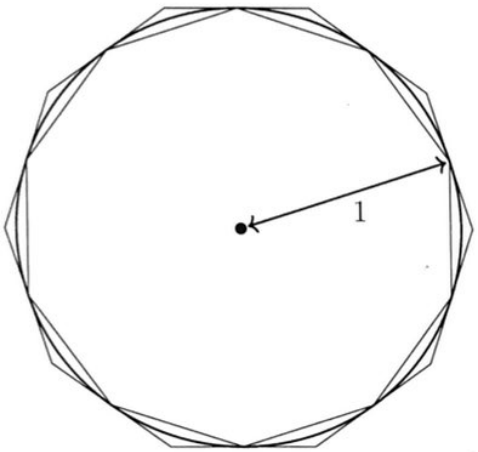
5. The length of a rectangular field is between 20 and 21 meters, and its width is between 8 and 9 meters. Find the area of the field, expressed as an inequality.

   $A_- = 20 \times 8 = 160$

   $A_+ = 21 \cdot 9 = 189$

   $160 \leq A \leq 189 \ m^2$

6. The diagram shows a circle sandwiched between two decagons (10-sided polygons). The area of the smaller, inscribed decagon is $A_{inner} \approx 2.9389$ and the larger, circumscribed decagon's area is $A_{outer} \approx 3.2492$. Use these bounds to approximate π plus or minus a margin of error.



$A = \pi 1^2 = \pi$

$2.9389 < \pi < 3.2492$

Average $= \dfrac{2.9389 + 3.2492}{2}$

$= 3.09405$

$\epsilon = 3.2492 - 3.09405 = 0.15515$

$\pi = 3.09405 \pm 0.15515$

7. Find the area of the $\triangle ABC$ is shown below with $A(3,2)$, $B(7,4)$, and $C(4,8)$.

(a) First find the area of the red rectangle with sides $b = 4$, $h = 6$.

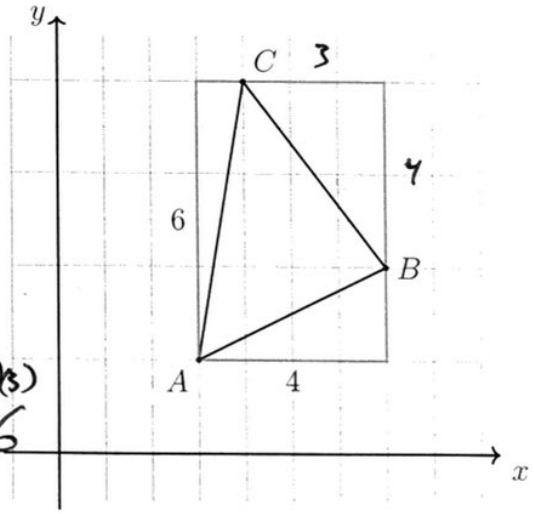
$A = 4 \cdot 6 = 24$

(b) Find the area of the three triangles surrounding $\triangle ABC$ in the rectangle.

$A_1 = \frac{1}{2}(6)(1) = 3$

$A_2 = \frac{1}{2}(4)(2) = 4$ $A_3 = \frac{1}{2}(4)(3) = 6$

(c) Subtract their areas from the rectangle to find $A_{\triangle ABC}$



$A_{\triangle ABC} = 24 - 3 - 4 - 6$

$= 11$