Geometry Unit 4: Volume

Bronx Early College Academy

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31 October - 18 November 2022

Learning Target: I can fold nets into 3-dimensional solids

HSG.CO.C.9 Prove theorems about lines and angles 4.1 Monday 31 October

Do Now

1. Review your Deltamath assignments
2. Check your Jumprope scores
3. Set a study goal
4. Answer survey in Google Classroom, “Mark as Done”

Lesson: Nets, Deltamath classwork practice
Homework: Area formulas review problem set

Learning Target: I can calculate the volume of a *rectangular prism*

HSG.CO.C.9 Prove theorems about lines and angles 4.2 Tuesday 1 November

Do Now

1. Find the area of a rectangle 4 inches by 6 inches
2. Find the length of a rectangle 7 inches wide with an area of 63 square inches

Lesson: Prism definitions, volume formula
Homework: Deltamath practice

A prism is a polyhedron, a 3-dimensional shape

Solid A 3-dimensional object

Face A flat surface of a geometric solid

Edge A line segment where two faces meet

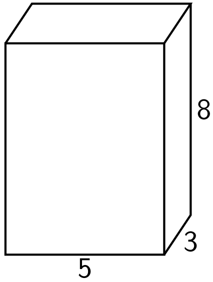
Vertex A point where edges meet

Prism A solid with two identical, parallel, bases and uniform cross section

Base Flat shapes that form the top and bottom or ends of a prism

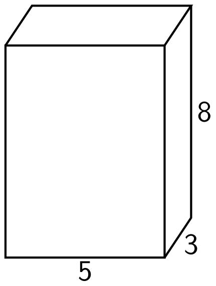
Lateral face The sides of a prism, which are parallelograms

Cross section The shape of a plane's intersection with a solid

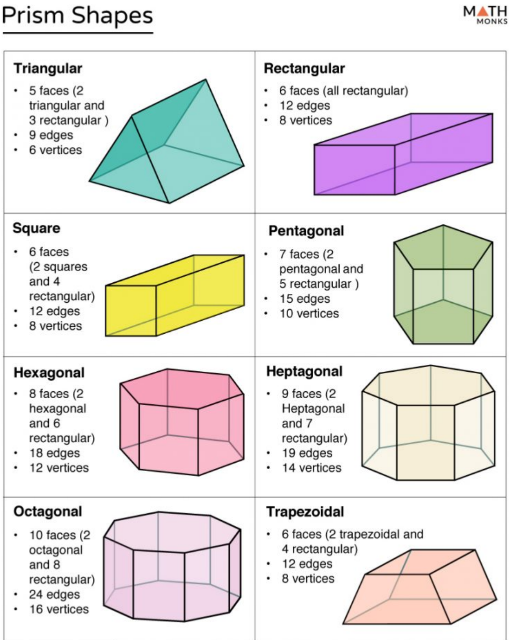


Common types of prisms, named by their base

Rectangular	Bases are rectangles (or squares)
Triangular	Triangular base
Hexagonal	Six-sided base, a hexagon
Cylinder	Solid with two parallel circles as bases
Right	Lateral faces are a right angles to the base
Oblique	Slanted



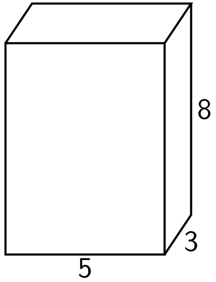
Math Monks prisms page


Prism Shapes
MATH MONKS
Triangular
• 5 faces (2 triangular and 3 rectangular)
• 9 edges
• 6 vertices
Rectangular
• 6 faces (all rectangular)
• 12 edges
• 8 vertices
Square
• 6 faces (2 squares and 4 rectangular)
• 12 edges
• 8 vertices
Pentagonal
• 7 faces (2 pentagonal and 5 rectangular)
• 15 edges
• 10 vertices
Hexagonal
• 8 faces (2 hexagonal and 6 rectangular)
• 18 edges
• 12 vertices
Heptagonal
• 9 faces (2 Heptagonal and 7 rectangular)
• 19 edges
• 14 vertices
Octagonal
• 10 faces (2 octagonal and 8 rectangular)
• 24 edges
• 16 vertices
Trapezoidal
• 6 faces (2 trapezoidal and 4 rectangular)
• 12 edges
• 8 vertices

Volume is a measure of space, the number of unit cubes a solid contains

Given the area of the base B and height h,
the volume of a prism is $V = B \times h$

Rectangular	$V = l \times w \times h$
Square	$V = s^2 \times h$
Triangular	$V = \frac{1}{2}(l \times w \times h)$
Cylinder	$V = \pi r^2 \times h$



Learning Target: I can solve for a missing parameter

HSG.CO.C.9 Prove theorems about lines and angles 4.3 Thursday 3 November

Do Now

1. Find the area of a circle with radius $r = 10$, in terms of π
2. Find the radius of a circle with area $A = 49\pi$

Lesson: Using algebra to solve problems, Deltamath practice
Homework: Handout practice with volume calculations

Muhammad ibn Musa al-Khwarizmi - the “father” of algebra

Persian 780 - 847 AD worked in Baghdad during the “Islamic golden age”

Algebra	Mathematics with symbols (named after al-Khwarizmi’s book, al-jabra)
Algorithm	Logical steps to solve a problem (comes from his name)
Unknown	A symbol or letter representing a number, x, y, a, π, θ
“reduction”	Cancellation of like terms on opposite sides of the equation

"Solve for x" or "isolate the variable"

The algorithm developed by al-Khwarizmi

Operation Combine two numbers (multiplication or addition, for example)

Identity 0 for addition, 1 for multiplication.

$$a + 0 = a \text{ and } a \times 1 = a$$

Inverse Two values that make the identity for an operation.

$$a + (-a) = 0 \text{ and } a \times \frac{1}{a} = 1$$

$$a = b \Longleftrightarrow a + c = b + c$$

Multiplying and dividing fractions

Rational numbers those that can be expressed as fractions, $\dfrac{p}{q} \in \mathbb{Q}$

Numerator The top number in a fraction, *dividend*, p

Denominator *Divisor*, bottom number in a fraction, q

Reciprocal The multiplicative inverse

Division Means to multiply by the reciprocal. $a \div b = \frac{a}{b} = a \times \frac{1}{b}$

To multiply fractions, multiply the numerators and denominators

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

To divide fractions, multiply by the reciprocal

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$$

Learning Target: I can calculate the surface area of a rectangular prism

HSG.CO.C.9 Prove theorems about lines and angles 4.4 Friday 4 November

Do Now: Lumber used in construction called a "two-by-four" is actually $3\frac{1}{2}$ inches by $1\frac{1}{2}$ inches by 8 feet long.

1. Find the area of the rectangular cross section, $3\frac{1}{2}$ inches by $1\frac{1}{2}$ inches
2. Find the area of a triangular wedge cut from a two-by-four that is $3\frac{1}{2}$ inches by one foot long.

Lesson: Surface area definition, formula; adding fractions
Homework: Deltamath practice
Extension: Deltamath absolute value, percent error

Surface area is the combined total area of the faces of a polyhedron

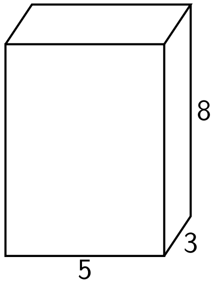
Surface area The total area of the outside of a solid

Given a rectangular prism with dimensions l, w, and h the surface area is the sum of the six faces:

$$S.A. = 2lw + 2lh + 2wh$$

$$= 2(5 \times 3) + 2(5 \times 8) + 2(3 \times 8)$$

$$= 158 \text{ square units}$$



Adding and subtracting fractions

To add fractions with the same denominator, add the numerators.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Equivalent fractions Fractions that are equal.

$$\frac{a}{b} = \frac{a}{b} \times \frac{c}{c} = \frac{ac}{bc}$$

LCM Lowest Common Multiple, for two fractions, multiples of the denominators that are equal.

Mixed fraction A whole number and a fraction. e.g. $3\frac{1}{2}$

Adding fractions with different denominators

First convert to equivalent fractions with a common denominator. e.g. find

$$\frac{1}{3}+\frac{1}{2}$$

Convert to sixths

$$\frac{1}{3}\times\frac{2}{2}=\frac{2}{6} \text{ and } \frac{1}{2}\times\frac{3}{3}=\frac{3}{6}$$

Add these equivalent fractions instead:

$$\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$$

Learning Target: I can calculate the volume of spheres, cones, and pyramids

HSG.CO.C.9 Prove theorems about lines and angles 4.5 Thursday 10 November

Do Now: Find the volume of a $3\frac{1}{2}$ inch long scrap of a "two-by-four".
(remember, the actual cross section is $3\frac{1}{2}$ inches by $1\frac{1}{2}$ inches)

Lesson: More volume formulas; exponent review
Homework: Deltamath practice
Extension: Deltamath exponent rules

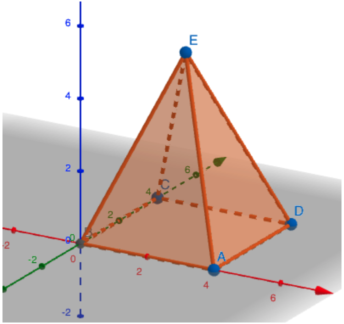
Volume of a cone or pyramid is one-third of a prism

Given a base with area B and a height h,
the volume of a cone or pyramid is $V = \frac{1}{3}B \times h$

Rectangular $V = \frac{1}{3}(l \times w \times h)$

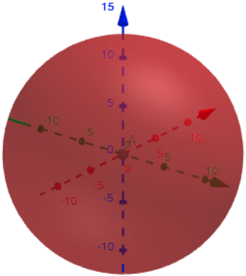
Square $V = \frac{1}{3}(s^2 \times h)$

Cone $V = \frac{1}{3}(\pi r^2 \times h)$



Volume and surface area of a sphere is a function of π

Given a sphere with radius r



Sphere	A ball or globe shape
Volume	$V = \frac{4}{3}\pi r^3$
Surface area	$S.A. = 4\pi r^2$

Exponents mean repeated multiplication

Superscript "Writing above," used for exponentiation. x^2

Subscript "Writing below," used for labeling or naming. x_2

Multiplying exponents with the same base $\underbrace{\overbrace{a \times a \times a}^{3} \times \overbrace{a \times a}^{2}}_{5} = a^5$