Geometry Unit 1, part b: Area
Bronx Early College Academy

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19-23 September 2022

Learning Target: I can calculate areas
CCSS: HSG.CO.A.1 Know precise geometric definitions · · · · · · 1.8 Monday 19 Sept

Do Now: Practice unit conversion

1. How many days are in a week?
2. Find the number of weeks in 365 days.
   (show calculation with units)

Quiz results

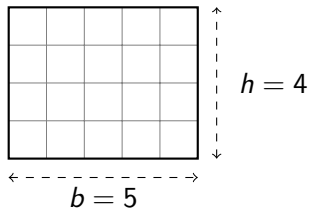Lesson: Rectangle, triangle, parallelogram area formulas

Extension: Scientific notation

The *area* of a rectangle is its base × height.
We also say "length times width"

Formula for the area of a rectangle:

$$A = b \times h$$
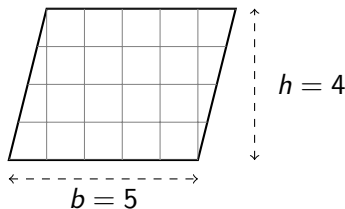

$h = 4$

$A = 5 \times 4 = 20$

$b = 5$

Area the quantity of unit squares that fill a shape

A parallelogram's area has the same formula as a rectangle.

Use the height, not the length of the slanted side.

Formula for the area of a parallelogram:

$$A = b \times h$$

$h = 4$

$b = 5$

$A = 5 \times 4 = 20$

A parallelogram's area has the same formula as a rectangle.

Use the height, not the length of the slanted side.

Formula for the area of a parallelogram:
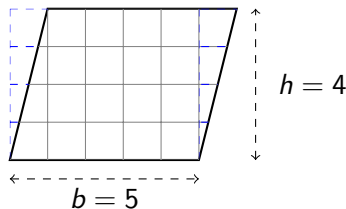
$$A = b \times h$$

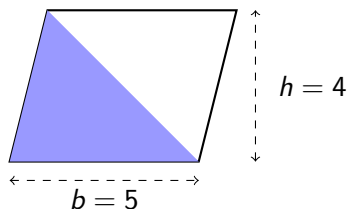

$$A = 5 \times 4 = 20$$

The two blue triangles match

A triangle has half the area of its base times height.

Use the height, not the side length.

Formula for the area of a triangle:

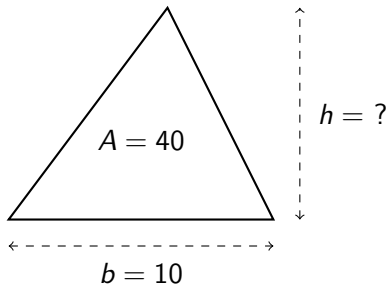$$A = \frac{1}{2}b \times h$$

$h = 4$

$b = 5$

$$A = \frac{1}{2}(5 \times 4) = 10$$

Find a missing dimension using the area formula

Given the area of a triangle is 40 and its base is 10, find its height.

$$A = \frac{1}{2}(10 \times h) = 40$$

Write formulas in notebook

Rectangle $A = b \times h$ (base times height or length times width)

Parallelogram $A = b \times h$

Triangle $A = \frac{1}{2}(b \times h)$

Area the quantity of unit squares that fill a shape

Units We say "square units", i.e. square inches (abbreviated in^2), square miles, etc.

Extension (optional): *Scientific notation*

Use for very large or small numbers instead of decimals

Exponents mean repeated multiplication:

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000$$

1. The distance to the sun is 150,000,000,000 meters $= 1.5 \times 10^{11}$
2. The population of NYC is $8,000,000 =$
3. The area of the earth is 2×10^8 square miles $=$

Scientific notation Compact notation for big numbers, $a \times 10^k$

   Exponent Repeated multiplication. The number of decimal places in base 10

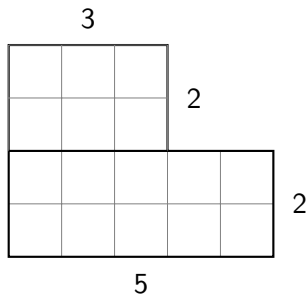   Base 10 The system of place value we use for numbers

   Mantissa The coefficient in scientific notation

Learning Target: I can calculate the area of a circle
CCSS: HSG.CO.A.1 Know precise geometric definitions   1.9 Tuesday 20 Sept

Do Now: Two rectangles are shown. Calculate the area of each and the combined total area.

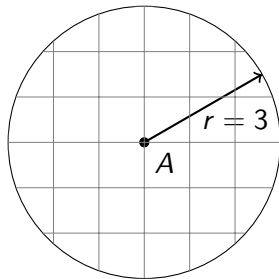Lesson: Area of a circle, π, rounding

Extension: Significant figures

The *area* and *circumference* of a circle are multiples of π.

π is an irregular number

Area of a circle:

$$A = \pi r^2$$

Circumference (distance around):

$$C = 2\pi r$$

Circle A with radius $r = 3$

$$A = \pi \times 3^2 = 9\pi = 28.2743...$$

$$C = 2\pi \times 3 = 6\pi = 18.8495...$$

Radius   Segment from the center to the edge of a circle, r

Diameter   Segment/length across the whole circle, $D = 2r$

Numbers don't always need to be exact

Round up when the next digit is 5 or more
Round down otherwise

Is π closer to three or four?

$\pi = 3.1415926...$

Numbers don't always need to be exact

Round up when the next digit is 5 or more
Round down otherwise

$\pi = 3.1415926... \approx 3$ to the nearest whole number

Numbers don't always need to be exact

Round up when the next digit is 5 or more
Round down otherwise

$\pi = 3.1415926... \approx 3$ to the nearest whole number
$\pi = 3.1415926... \approx 3.1$ to the nearest tenth
$\pi = 3.1415926... \approx 3.14$ to the nearest hundredth
$\pi = 3.1415926... \approx 3.142$ to the nearest thousandth

Numbers don't always need to be exact

Round up when the next digit is 5 or more
Round down otherwise

$\pi = 3.1415926... \approx 3$ to the nearest whole number
$\pi = 3.1415926... \approx 3.1$ to the nearest tenth
$\pi = 3.1415926... \approx 3.14$ to the nearest hundredth
$\pi = 3.1415926... \approx 3.14\textbf{2}$ to the nearest thousandth

Whole The ones place, e.g. 3, 14, -15

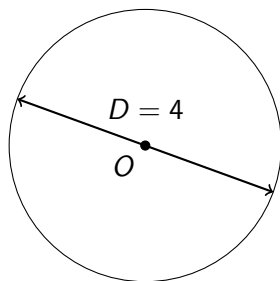tenths First digit after the decimal, 0.3, 6.8

hundredths Second decimal digit, 5.45

thousandths Third decimal place, 18.123

Rounding Writing an approximation of a number

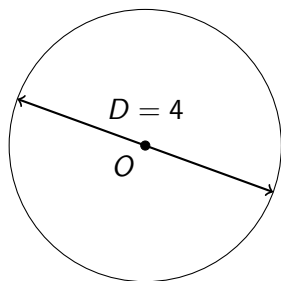Approximate About equal to, not exact, $\approx$

Use the symbol π for an *exact* answer

Circle O with diameter $D = 4$

$D = 4$

O

1. Find the radius of the circle.

2. Find the exact circumference.

3. Round to the nearest hundredth.

Use the symbol π for an *exact* answer

$D = 4$
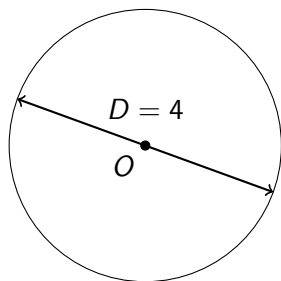
O

Circle O with diameter $D = 4$

1. Find the radius of the circle.

   $r = \frac{1}{2}D = \frac{4}{2} = 2$

2. Find the exact circumference.

3. Round to the nearest hundredth.

Use the symbol π for an *exact* answer

$D = 4$

O

Circle O with diameter $D = 4$

1. Find the radius of the circle.

   $r = \frac{1}{2}D = \frac{4}{2} = 2$

2. Find the exact circumference.

   $C = 2\pi r = 2\pi 2 = 4\pi$

3. Round to the nearest hundredth.

Use the symbol π for an *exact* answer

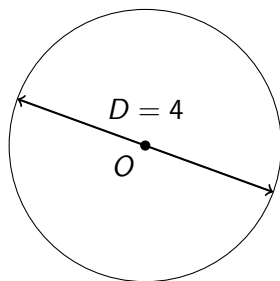Circle O with diameter $D = 4$

1. Find the radius of the circle.

   $r = \frac{1}{2}D = \frac{4}{2} = 2$

2. Find the exact circumference.

   $C = 2\pi r = 2\pi 2 = 4\pi$

3. Round to the nearest hundredth.

   $C = 4\pi = 6.2831853... \approx 6.28$

Exact solution   Written with symbols or an ellipse (...).
           Also said as "give your answer *in terms of* π".

Write formulas in notebook

Circle   All points with equal distance from the circle center

Radius   Distance from the circle center to its edge, r

Diameter   Length across the whole circle, $D = 2r$

Circle area   Formula $A = \pi r^2$

Circumference   The distance around a circle (i.e. perimeter),
$$C = 2\pi r$$

Semi-circle   Half of a circle

π A special number, $\pi = 3.14159265358...$

Irrational   Number that can not be written as a fraction, π, $\sqrt{2}$

Exact solution   Written with symbols or an ellipse (...).
Also said as "give your answer *in terms of* π".

Extension: Three digits is usually exact enough

Scientists and engineers say *significant figures*, or in IB, "sig figs"

Round to three digits

- $\pi = 3.14159265358... \approx 3.14$
- $\sqrt{2} = 1.4142135... \approx 1.41$
- Dr. Huson's height $h \approx 67.5$ inches
- 365 days in a year (actually 365.2421897, source)
- Avogadro's number $N_A \approx 6.02 \times 10^{23}$

Sig figs   Significant figures, the number of digits required for
          the desired precision. In IB mathematics and most
          practical matters, the convention is 3 sig figs.

Learning Target: I can quantify error in calculations
CCSS: HSG.CO.A.1 Know precise geometric definitions   1.10 Wednesday 21 Sept

Do Now: Find the area of a quarter circle with radius $r = 20$ cm, rounding to the nearest whole number.
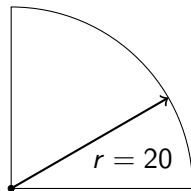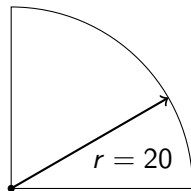

Lesson: Percent error formula

Extension: Confidence intervals

Learning Target: I can quantify error in calculations
CCSS: HSG.CO.A.1 Know precise geometric definitions   1.10 Wednesday 21 Sept

Do Now: Find the area of a quarter circle with radius $r = 20$ cm, rounding to the nearest whole number.

$A = \frac{1}{4}\pi \times 20^2 = 100\pi$

$= 314.15926... \approx 314$ square units

Lesson: Percent error formula

Extension: Confidence intervals

Quantify measurement and rounding inaccuracy as a percent

Convention: Treat all errors as a positive amount

Given v_a = approximate value, v_e = exact value

Percent error

$$\epsilon = \left| \frac{v_a - v_e}{v_e} \right| \times 100\%$$

Which is more accurate?

$\pi \approx 3.14$ $\pi \approx \frac{22}{7}$

Relative error decimal format (i.e. 5% versus 0.05)

ϵ The Greek letter epsilon, meaning error

Quantify measurement and rounding inaccuracy as a percent
Convention: Treat all errors as a positive amount

Given $v_a =$ approximate value, $v_e =$ exact value

Percent error
$$\epsilon = \left| \frac{v_a - v_e}{v_e} \right| \times 100\%$$

Which is more accurate?

$\pi \approx 3.14$

$\epsilon = \left| \frac{3.14 - \pi}{\pi} \right| \times 100\%$

$= 0.05069...\%$

$\pi \approx \frac{22}{7}$

$\epsilon = \left| \frac{22/7 - \pi}{\pi} \right| \times 100\%$

$= 0.04024...\%$

Relative error decimal format (i.e. 5% versus 0.05)

ϵ The Greek letter epsilon, meaning error

Unit conversions are often approximate
39.3701 inches is a more exact value

There are approximately 39 inches in a meter.

$$1 \text{ meter} \approx 39 \text{ inches}$$

Find the percent error in this conversion ratio.

Unit conversions are often approximate
39.3701 inches is a more exact value

There are approximately 39 inches in a meter.

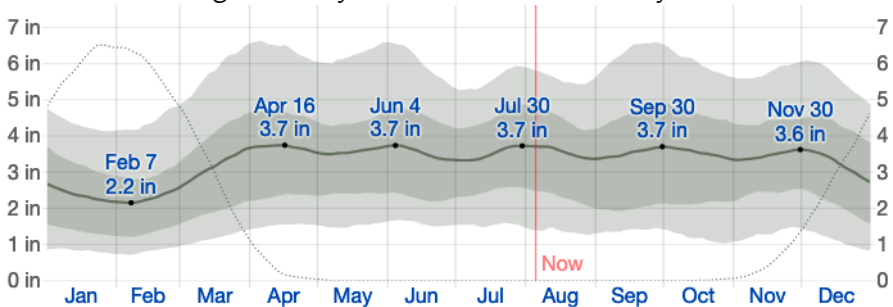$$1 \text{ meter} \approx 39 \text{ inches}$$

Find the percent error in this conversion ratio.

$$\epsilon = \left| \frac{39 - 39.3701}{39.3701} \right| \times 100\%$$

$$= 1.945994...\% \approx 2\% \text{ error}$$

Quantify an error as interval around the best guess

- ▶ What is a typical retirement age? 65 ± 5 years
- ▶ SUNY New Paltz SAT scores are between 1070 and 1260.
- ▶ How much does it rain in New York City? (WeatherSpark)

Average Monthly Rainfall in New York City

Interval   A range, e.g. from 10 to 12

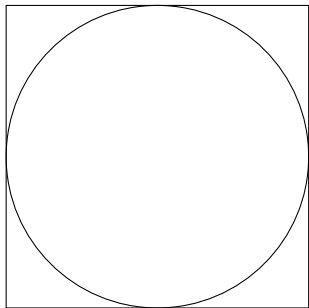Confidence   Not certain, but most likely range of values

$\pm$ Plus or minus

Learning Target: I can study together with my classmates
CCSS: HSG.CO.A.1 Know precise geometric definitions   1.11 Thursday 22 Sept

Do Now: Estimate the percentage of the square's area covered by the circe. (then calculate your percent error)

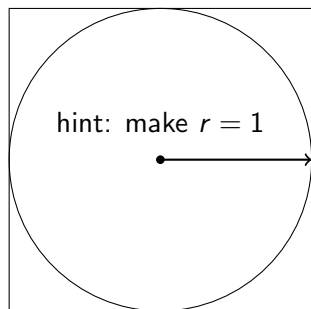Lesson: Peer review, notebook check, homework inventory due

Unit test tomorrow

Learning Target: I can study together with my classmates
CCSS: HSG.CO.A.1 Know precise geometric definitions   1.11 Thursday 22 Sept

Do Now: Estimate the percentage of the square's area covered by the circe. (then calculate your percent error)

hint: make $r = 1$

Guestimating three quarters, or 75%

$$A_{square} = 2 \times 2 = 4$$

$$A_{circle} = \pi \times 1^2 = \pi = 3.14159...$$

% coverage $= \frac{\pi}{4} = 0.78539... \approx 78.5\%$

$$\epsilon = \left| \frac{75 - 78.539...}{78.539...} \right| \times 100\%$$

$$= 4.5070...\% \approx 4.5\% \text{ error}$$

Lesson: Peer review, notebook check, homework inventory due

Unit test tomorrow

Groupwork review for test tomorrow

"Roundtable" of four students, with four topics assigned

Geometry skills to study / teach

1. Line segments, length, number lines
2. Perimeter and area
3. Precision, percent error
4. Modeling situations and solving with algebra

Learning Target: I can quantify length and area
CCSS: HSG.CO.A.1 Know precise geometric definitions   1.12 Friday 23 Sept

Unit test