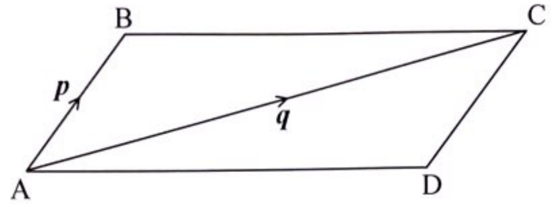
**2.** [Maximum mark: 7]

The following diagram shows the parallelogram ABCD.




Let $\overrightarrow{AB} = \boldsymbol{p}$ and $\overrightarrow{AC} = \boldsymbol{q}$. Find each of the following vectors in terms of $\boldsymbol{p}$ and/or $\boldsymbol{q}$.

(a) $\overrightarrow{CB}$ [2]

(b) $\overrightarrow{CD}$ [2]

(c) $\overrightarrow{DB}$ [3]

(a) $\overrightarrow{CB} = \cancel{q-p}\ p-q$ from C to B
from q to p

(b) $\overrightarrow{CD} = -p$ $\overrightarrow{CD}$ is negative $\overrightarrow{p}$

(c) $\overrightarrow{DB} = p-(q+(-p))$ from D to B
$= 2p - q$ $B = p$
$D = q + (-p)$
$\overrightarrow{DB} = B - D$

Geometry

pg 11 #9 [15 marks] Solutions

9 a) $\overrightarrow{PQ} = \begin{pmatrix} -11-1 \\ 8-0 \\ m-2 \end{pmatrix} = \begin{pmatrix} -12 \\ 8 \\ m-2 \end{pmatrix}$

(b) $\vec{a} \cdot \vec{b} = (1)(-3) + 1(2) + n(1) = 0$

$n = 1$

(c) i) $\begin{pmatrix} -12 \\ 8 \\ m-2 \end{pmatrix} = k\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$

$k = 4$ $\qquad \overrightarrow{PQ} = 4\vec{b}$

(ii) $m - 2 = 4$

$m = 6$

(d) i) $c = \overrightarrow{OQ} = \cancel{\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}} \begin{pmatrix} -11 \\ 8 \\ 6 \end{pmatrix}$

ii) $s = |\vec{a}|$

$= \sqrt{1^2 + 1^2 + 1^2}$

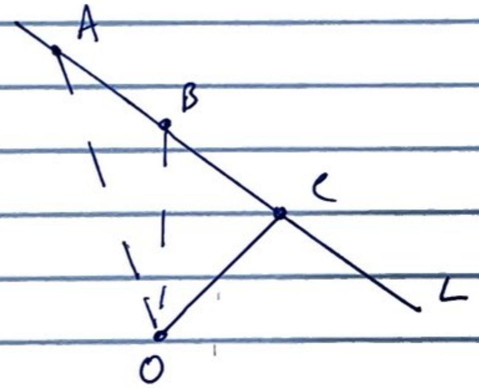
$= \sqrt{3}$ m/s

Geometry Solutions

pg 9, #8 (16 marks)

(a) i) $\vec{AB} = \vec{B} - \vec{A} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$

ii) $|\vec{AB}| = \sqrt{1^2 + (-1)^2 + (-2)^2}$
$= \sqrt{6}$




(b) $L = A + \lambda \vec{AB}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

(c) $$\begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ y \\ -1 \end{pmatrix}$$

$-2 + \lambda 1 = 0$

$\lambda = 2$

$y = 4 + \lambda(-1) = 2$

(d) i) $\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = 0 \cdot 1 + 2 \cdot (-1) + (-1)(-2)$
$= 0$

ii) $\cos\theta = \dfrac{\vec{OC} \cdot \vec{AB}}{|\vec{OC}|\,|\vec{AB}|} = \dfrac{0}{\sqrt{5}\sqrt{6}}$ $\quad |\vec{OC}| = \sqrt{0^2 + 2^2 + (-1)^2} = \sqrt{5}$

~~$\theta = \cos^{-1}\left(\frac{1}{\sqrt{30}}\right) = 1.38719\ldots$ radians~~

$\theta = \frac{\pi}{2}$ $(90°)$ ~~$\approx 1.39$ radians~~ ~~$(79.5°)$~~

(e) $A = \frac{1}{2}\sqrt{6}\left(\sqrt{0^2 + 2^2 + (-1)^2}\right)$
$= 1 \sqrt{\ }\,\circ$

Geometry

p11 # 9 [15 pts] Solutions

9)

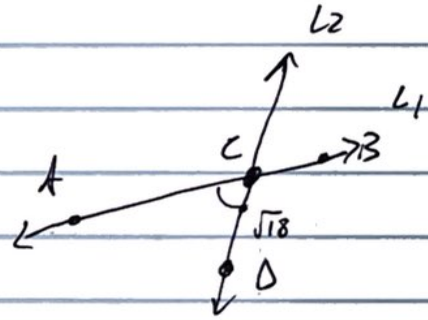
(a) i) $\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} -2-0 \\ 5-(-3) \\ 3-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$

ii)

$$\vec{r} = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + t\begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$$

(b) $\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + t\begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

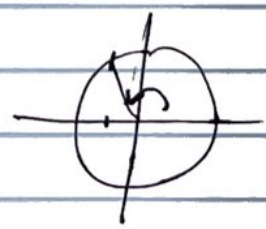
$$0 + -2t = -1$$
$$t = \tfrac{1}{2}$$
$$1 + 2t = -4 - s$$
$$1 + 2(\tfrac{1}{2}) = -4 - s$$
$$s = -6$$




$L_1$ $\vec{r} = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

$L_2$ $\vec{r} = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} - 6\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ therefore $C = (-1, 1, 2)$

(d) $|\vec{AC}| = \sqrt{(-1)^2 + (1-(-3))^2 + (2-1)^2}$
$= \sqrt{18}$



$$\cos A\hat{C}D = \frac{-9}{\sqrt{18}\sqrt{18}} = -\frac{1}{2}$$

$$A\hat{C}D = 120$$