BECA / IB Math 4-Polynomial and rational functions   Name: Solutions
11 February 2022

4.11 Exam: Polynomial and rational functions   CCSS.HSF.IF.C.7

1. Shown in the plot below is the function $f(x) = -x^3 + 13x - 12$.

(a) Write down the value of $f(0)$.
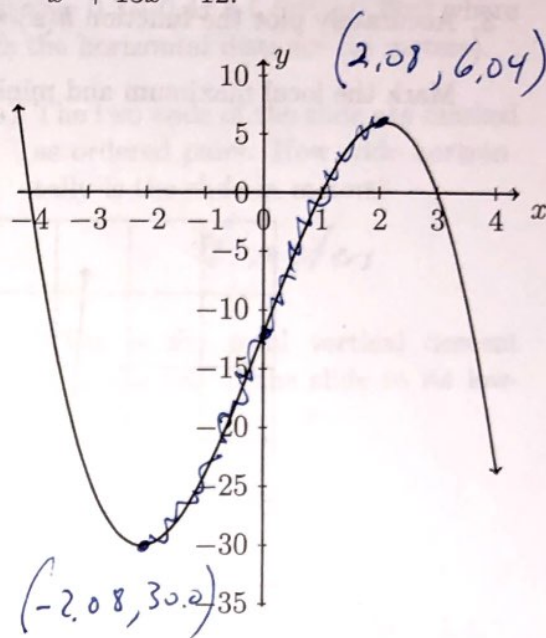
-12

(b) Write down the solutions to $f(x) = 0$.

$x = \quad -4, 1, 3$

(c) Mark the portion of the function that is *increasing* with a squiggly line.

(d) Label the local maximum and local minimum as ordered pairs.

(e) Show that 1 is an x-intercept because $x = 1$ is a solution to $f(x) = 0$.

$$f(1) = -1^3 + 13(1^2) - 12$$
$$= -1 + 13 - 12$$
$$= 0$$

$(2.08, 6.04)$

$(-2.08, 30.2)$

2. A relation composed of five points $\{(-2, 4), (j, 2), (3, 1), (3, 5), (5, k)\}$ is plotted on the below.
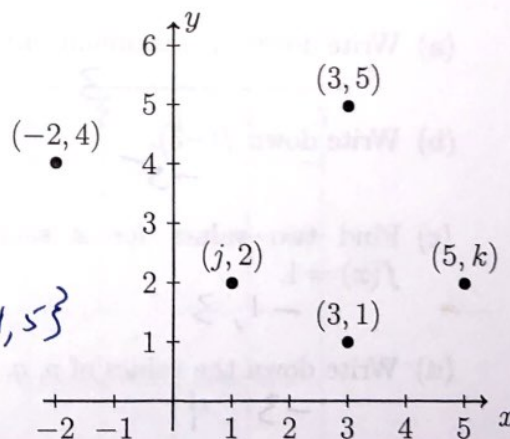
(a) Write down $j = 1$

(b) Write down $k = 2$

(c) Write down the range.

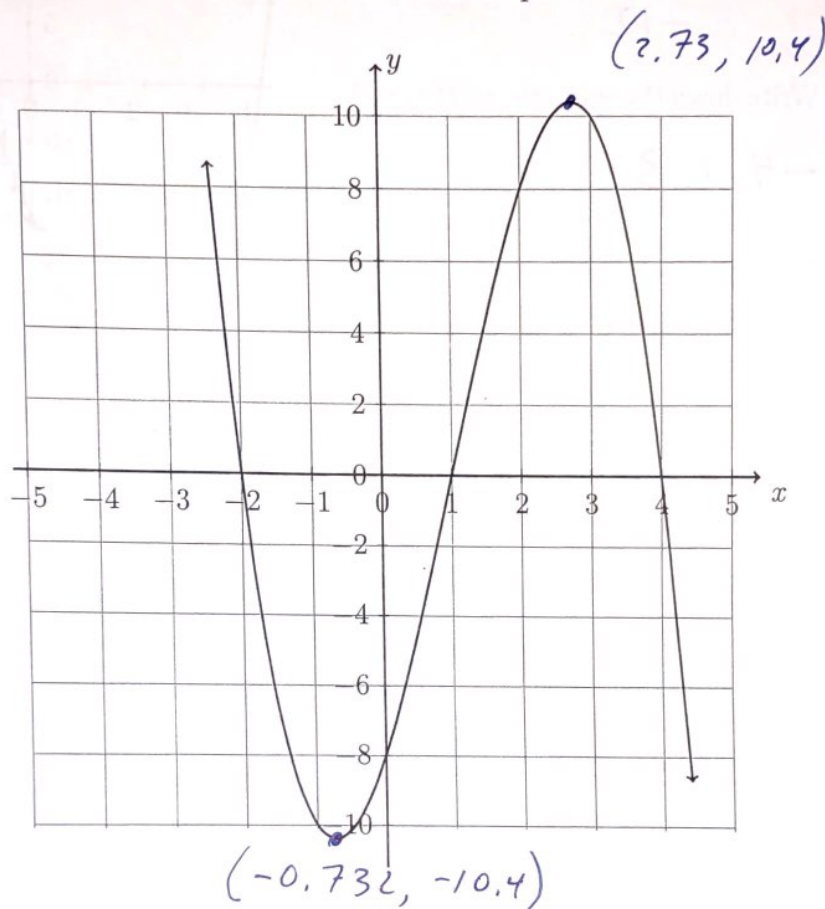Range: $\{4, 2, 1, 5, \cancel{2}\} = \{1, 2, 4, 5\}$

(d) Name a point that, if removed, would make the relation a function.

$(3, 1) \quad$ or $\quad (3, 5)$

$(-2, 4)$
$(3, 5)$
$(j, 2)$ $(5, k)$
$(3, 1)$

3. Accurately plot the function $h(x) = -x^3 + 3x^2 + 6x - 8$.

Mark the local maximum and minimums as ordered pairs.

$(2.73, 10.4)$

$(-0.732, -10.4)$

4. The function $f(x) = ax^2 + bx + c$ is graphed below over its domain, $p \leq x < q$.

(a) Write down the maximum value of f.

3

(b) Write down $f(-3)$.

-5
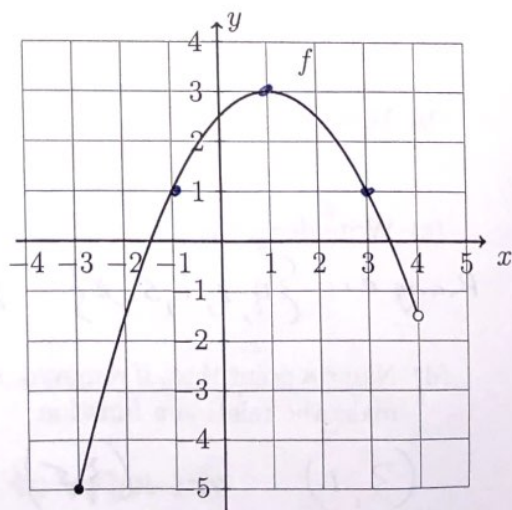
(c) Find two values for x such that $f(x) = 1$.
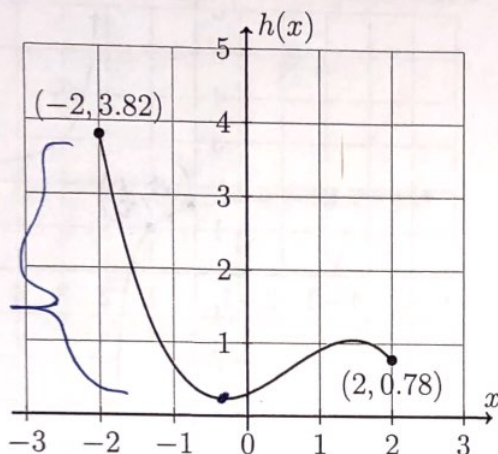
$-1, 3$

(d) Write down the values of p, q.

$-3, 4$

(e) Write down the range of f.

$-5 \leq y < 3$

5. A pool slide is modeled by the cubic function $h(x) = 0.3 + 0.4x + 0.5x^2 - 0.29x^3$ where h is the height in meters above ground and x is the horizontal distance (in meters).

$(-2, 3.82)$

3.59

$(2, 0.78)$

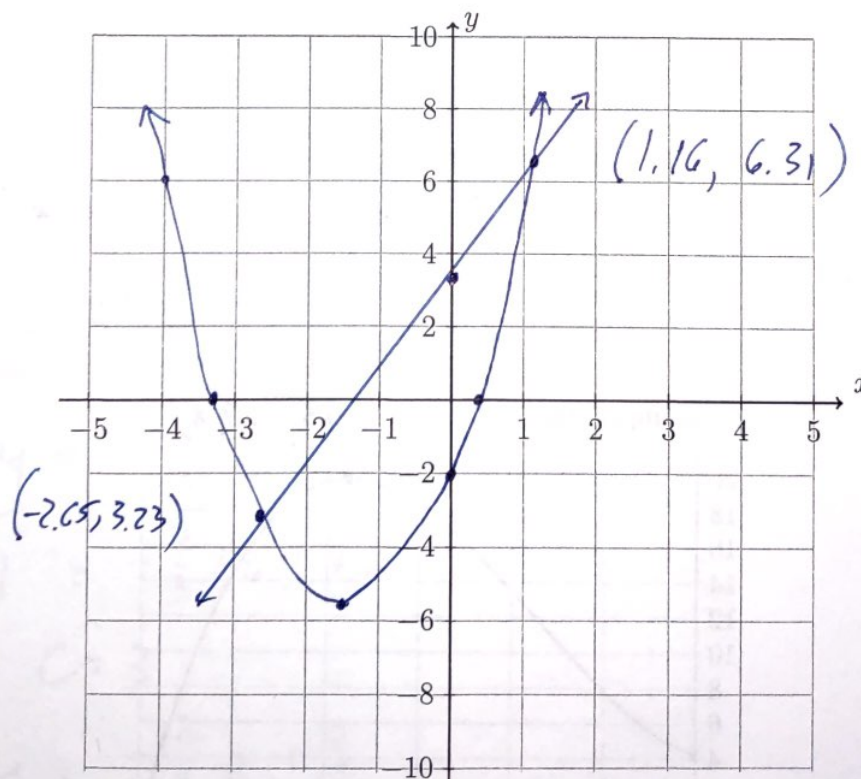(a) The two ends of the slide are marked as ordered pairs. How wide horizontally is the slide in meters?

4 meters

(b) What is the total vertical descent from the top of the slide to its lowest point?

3.59 m

6. Accurately plot the two functions, $f(x) = 1.75x^2 + 5.1x - 2$ and $g(x) = 2.5x + 3.4$.

Mark and label the two intersections, $f(x) = g(x)$, as ordered pairs. Round to the nearest hundredth.

$(1.16, 6.31)$

$(-2.65, 3.23)$

7. A rational function of the form $f(x) = \dfrac{1}{x - p} + q$ is shown on the grid below.

(a) Write down the equation of the horizontal asymptote.

$$y = 2$$

(b) Write down the equation of the vertical asymptote.

$$x = 2$$

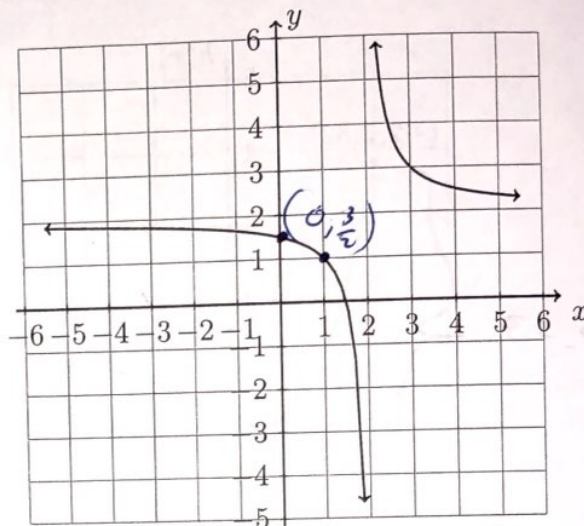(c) Hence, write down p and q.

$$p = 2, \quad q = 2$$

(d) Find $f(0)$.

$$f(0) = \frac{1}{0-2} + 2 = \frac{3}{2}$$

(e) Solve for x such that $f(x) = 1$.

$$\frac{1}{x-2} + 2 = 1$$

$$x = 1$$

8. The temperature $(C°)$ over a 24 hour day starting at midnight is modeled by the function $f(t) = -0.0073t^3 + 0.15t^2 + 0.43t + 4.2$.
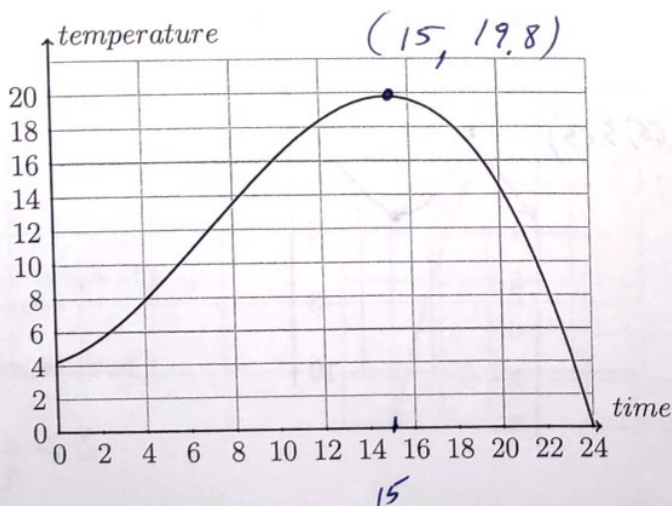
(a) Write down the temperature at midnight, when $t = 0$.

$$4.2 \ \ °C$$

(b) Over what interval is the temperature increasing?

$$[0, 15) \quad \text{or} \quad 0 \le t \# < 15$$

(c) Find the maximum temperature during the day.

$$19.8 \ °C$$

$$(15, 19.8)$$

$$15$$

Linear functions

CCSS.8.F.B.4

9. A linear function f is graphed below.

   (a) Write down it's slope.

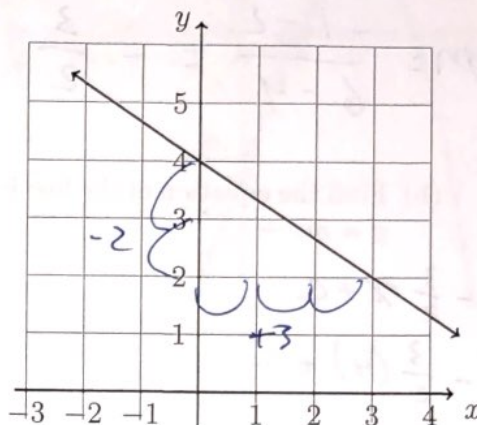 $m = \quad -\dfrac{2}{3}$ ✓

   (b) Write down it's y-intercept.

   $b = \quad 4$ ✓

   (c) Write down the equation of the line.

   $y = -\dfrac{2}{3}x + 4$ ✓

10. Write the linear equation $y + 5 = 3(x - 2)$ in the form $y = mx + c$.

$$y + 5 = 3x - \cancel{6}$$
$$y = 3x - \cancel{5}\, 11$$

11. A line has a gradient (slope) of $-\dfrac{2}{3}$ and passes through the point $(6, -1)$. Find the equation of the line in the form $y = mx + c$.

$$y = -\frac{2}{3}x + c$$
$$-1 = -\frac{2}{3}(6) + c$$
$$c = 3$$
$$y = -\frac{2}{3}x + 3$$

12. A line goes through the points $(4, 2)$ and $(6, -1)$.

(a) Find the gradient of the line.
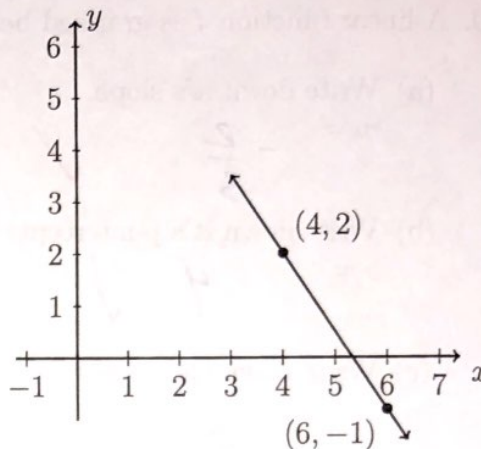
$$m = \frac{-1-2}{6-4} = -\frac{3}{2}$$

(b) Find the equation of the line in the form $y = mx + c$.
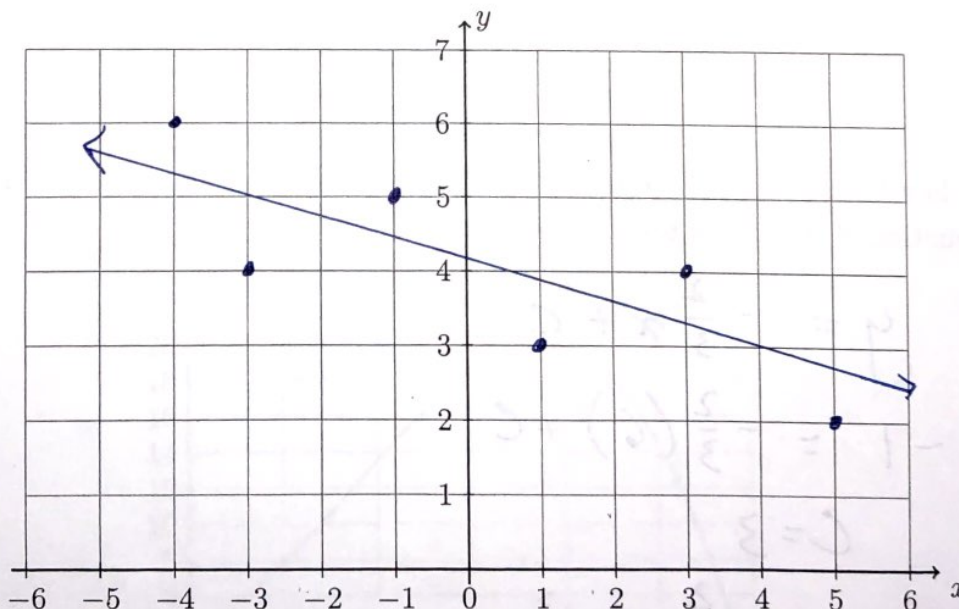
$$y = -\frac{3}{2}x + c$$
$$2 = -\frac{3}{2}(4) + c$$
$$c = 8$$
$$y = -\frac{3}{2}x + 8$$

13. A linear equation is desired to model a set of data.

(a) Plot the following points on the grid: $(-4, 6)$, $(-3, 4)$, $(-1, 5)$, $(1, 3)$, $(3, 4)$, $(5, 2)$

(b) Draw a line of best fit through the data. (use a straight edge for full credit)

14. Anything you'd like me to know?