La Scuola d'Italia / Huson / IB Math: Bivariate Statistics

29 October 2025

2.1 Classwork: Stats intro; markdown solutions due 31 October

1. Given a geometric sequence with $u_1 = 9$ and $r = \frac{4}{3}$

    1. Find u_8.

    Solution: $u_8 = 9 \cdot \left(\frac{4}{3}\right)^{8-1}$

    $= \frac{16384}{243} = 67.42386\ldots \approx 67.4$

    2. Find S_8, the sum of the first eight terms of the sequence.

    Solution: $S_8 = 9 \cdot \dfrac{\left(\frac{4}{3}\right)^8 - 1}{\frac{4}{3} - 1}$

    $= \frac{58975}{243} = 242.695\ldots \approx 243$

    3. $S_k \approx 825.37$. Find k algebraically.

    Solution:

    $$S_k = 9 \cdot \frac{\left(\frac{4}{3}\right)^k - 1}{\frac{4}{3} - 1} = 825.37$$

    $$\left(\frac{4}{3}\right)^k = 36.5693\ldots$$

    $$k = \log_{\frac{4}{3}} 36.5693 \cdots \approx 12$$

2. Consider the following set of data:

x	2	4	6	8	10
y	3	7	5	11	14

    1. Write down the coordinates of the mean point $(\bar{x}, \bar{y})$.

    Solution: $(6, 8)$

    2. A linear regression of y on x gives the equation $y = ax + b$. Write down the values of a and b.

    Solution: $a = 1.3, b = 0.2$

    3. Write down the value of r , the Pearson's product-moment correlation coefficient for this set of data.

    Solution: $r = 0.919239\ldots \approx 0.919$

    4. Characterize the correlation coefficient by choosing one of the following: strong positive correlation, weak positive correlation, no correlation, weak negative correlation, strong negative correlation.

    Solution: Strong positive correlation.

3. Find each value as an integer (no calculator).

   1. $\log_3 27 = 3$
   2. $\log_3 9 + \log_3 3 = 2 + 1 = 3$
   3. $\log_3 9 - \log_3 81 = 2 - 4 = -2$

4. Consider the following set of data:

x	15	25	35	50	65	80
y	480	440	420	360	310	270

   1. Write down the coordinates of the mean point $(\bar{x}, \bar{y})$.

   Solution: $(45, 380)$

   2. A linear regression of y on x gives the equation $y = ax + b$. Write down the values of a and b.

   Solution: $a = -3.2623\ldots \approx -3.26$, $b = 526.803\ldots \approx 527$

   3. Write down the value of r , the Pearson's product-moment correlation coefficient for this set of data.

   Solution: $r = -0.9978\ldots \approx -0.998$

   4. Characterize the correlation of the data.

   Solution: Strong negative correlation.

5. Three consecutive terms of a geometric sequence are $x - 1$, 4, and $x + 5$. Find the possible values of x.

   Solution:
   $$r = \frac{4}{x - 1} = \frac{x + 5}{4}$$
   $$16 = (x - 1)(x + 5)$$
   $$0 = x^2 + 4x - 21$$
   $$0 = (x - 3)(x + 7)$$
   $$x = 3 \text{ or } x = -7$$

6. Solve $\log_2 x + \log_2(x - 6) = 4$ for $x > 6$.

   Solution:
   $$\log_2 x + \log_2(x - 6) = 4$$
   $$\log_2[x(x - 6)] = 4$$
   $$x(x - 6) = 16$$
   $$x^2 - 6x - 16 = 0$$
   $$(x - 8)(x + 2) = 0$$
   $$x = 8 \text{ (since } x > 6, \text{ discard } x = -2)$$