La Scuola d'Italia / Huson / IB Math: Sequences First & last name:
5 November 2025 Grade:

2.5 Test: Draft

1. Eight piano students reported their average weekly practice time and their diploma score (out of 150). The data are shown below.

Practice time h (h)	28	13	45	33	17	29	39	36
Diploma score D	115	82	120	116	79	101	110	121

The relationship between h and D is modelled by a regression line $D = ah + b$.

(a) Find the Pearson product–moment correlation coefficient r for these data.

(b) Write down the values of a and b from your GDC regression output.

(c) One of the students says she would have practised 5 more hours per week. Using the model, estimate how her score might have changed.

2. A runner collects data to see whether the time to run 5000 m depends on the runner's age. For eight male runners he records:

Age x (years)	18	24	28	36	40	46	52	62
Time t (minutes)	29.4	29.2	31.1	33.6	32.2	33.1	35.2	40.4

(There is also a scatter diagram showing time increasing with age.)

(a) Find the Pearson correlation coefficient r.

(b) A sports science book gives the following guidance:

$$0 \le |r| < 0.4\text{: weak}, \quad 0.4 \le |r| < 0.8\text{: moderate}, \quad 0.8 \le |r| \le 1\text{: strong}.$$

Comment on the strength of the correlation for this data.

(c) Write down the regression line of t on x in the form $t = ax + b$.

(d) Estimate the time for a 57-year-old runner using your regression line.

3. In an experiment the area of a mould patch is modelled by

$$P(t) = Ae^{kt},$$

where P is the area in mm^2 and t is the time in days. At $t = 0$ the area is 112 mm^2, and after 5 days the area is 360 mm^2.

(a) Write down the value of A.

(b) Find the value of k.

4. Dilara is designing a kite $ABCD$ on a coordinate plane (1 unit = 10 cm). The points are

$$A(2,0), \quad B(0,4), \quad C(4,6),$$

and point D lies on the x–axis. Segment AC is perpendicular to segment BD.

   (a) Find the gradient of the line through A and C.

   (b) Hence write down the gradient of the line through B and D.

   (c) Find the equation of line BD in the form $ax + by + d = 0$, where a, b, d are integers.

   (d) Write down the x–coordinate of D.

5. A new university records the number of applications in its first two years:

Year n	1	2
Applications u_n	12 300	12 669

   (a) Calculate the percentage increase in applications from year 1 to year 2.

   (b) Assume that the applications follow a geometric sequence (u_n).

      i. Write down the common ratio.

      ii. Find a formula for u_n.

      iii. Find the number of applications expected in year 11, giving your answer to the nearest integer.

   (c) In year 1 there are 10 380 places available. The number of places increases by 600 each year. Let (v_n) be the number of places in year n. Write down a formula for v_n.

   (d) For the first 10 years every place is filled. Each student who takes a place pays an $80 acceptance fee. Find the total amount of acceptance fees received in the first 10 years.

   (e) Let $n = k$ be the first year in which the number of places available exceeds the number of applications. Find k.

   (f) State whether for all $n > k$ the university will have places for all applicants. Justify your answer briefly.

6. Give all numerical answers correct to two decimal places.

   A person places $30 000 in an account on 1 January 2005. The account pays *simple* interest at a fixed annual rate. On 1 January 2007 the balance is $31 650.

(a) Find the annual simple interest rate.

(b) A second person also invests \$30 000 on 1 January 2005, but in an account that pays a nominal annual rate of 2.5% compounded annually. Find the balance after two years.

(c) Determine the number of complete years from 1 January 2005 until the compound-interest account first has a greater balance than the simple-interest account.

(d) On 1 January 2007 the first person reinvests 80% of the money from the simple-interest account into a new account paying 3% per year, compounded quarterly.

i. Calculate the amount reinvested on 1 January 2007.

ii. Find the number of complete years it will take for the balance in this new account to exceed \$30 000.

7. In a geometric sequence, the fourth term is 8 times the first term. The sum of the first 10 terms is 2557.5. Find the 10th term of this sequence.

8. A population of rare birds, P_t, can be modelled by the equation

$$P_t = P_0 e^{kt},$$

where P_0 is the initial population and t is measured in decades. After one decade it is estimated that

$$\frac{P_1}{P_0} = 0.9.$$

(a) i. Find the value of k.

ii. Interpret the meaning of the value of k in the context of the population.

(b) Find the least number of whole years for which

$$\frac{P_t}{P_0} < 0.75.$$

9. The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

Distance, x km	11500	7500	13600	10800	9500	12200	10400
Price, y dollars	15000	21500	12000	16000	19000	14500	17000

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

(a) i. Find the correlation coefficient.

ii. Write down the value of a and of b.

(b) On 1 January 2010, Lina buys a car which has travelled 11000 km. Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest \$100.

(c) The price of a car decreases by 5% each year. Calculate the price of Lina's car after 6 years.

(d) Lina will sell her car when its price reaches \$10 000. Find the year when Lina sells her car.

2. Sequences: geometric and arithmetic

Part A (geometric sequence)

A geometric sequence has first term 1024 and fourth term 128.

(a) Show that the common ratio is $\frac{1}{2}$.

(b) Find the eleventh term of the sequence.

(c) Find the sum of the first eight terms.

(d) Find the smallest number of terms for which the sum of the sequence first exceeds 2047.968.

Part B (arithmetic sequence)

Consider the arithmetic sequence

$$1,\ 4,\ 7,\ 10,\ 13,\ \ldots$$

(a) Find the eleventh term.

(b) The sum of the first n terms of this sequence is given by

$$S_n = \frac{n(3n-1)}{2}.$$

i. Find the sum of the first 100 terms.

ii. The sum of the first n terms is 477.

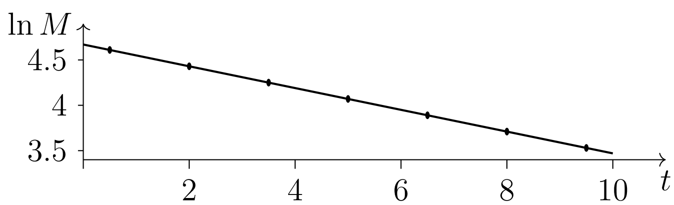
A. Show that $3n^2 - n - 954 = 0$.

La Scuola d'Italia / Huson / IB Math: Sequences
5 November 2025

First & last name:
Grade:

B. Hence find the value of n. You may use your GDC.

10. The mass M of a decaying substance is measured at one–minute intervals. The points $(t, \ln M)$ are plotted for $0 \leq t \leq 10$, where t is in minutes. The line of best fit is drawn. This is shown in the diagram.



The correlation coefficient for this linear model is $r = -0.998$.

(a) State two words that describe the linear correlation between $\ln M$ and t.

(b) The equation of the line of best fit is

$$\ln M = -0.12t + 4.67.$$

Given that $M = a \times b^t$, find the value of b.