BECA / IB Math 5 Exponential functions
9 March 2022

Name: SOLUTIONS

### 5.10 Quiz: Exponential functions

*Round all currency amounts to the nearest hundredth.*

1. Frank puts $2,000 into an investment account with an annual interest rate of 3.00%. Find the balance after one year.

$$FV = 2000(1+0.03)^1 = 2060$$

2. Allen invests $87,500 in an account with an annual interest rate of 2.85%. Find the balance after 4 years.

$$FV = 87{,}500(1+0.0285)^4$$
$$= \cancel{37.}\ 97{,}909.59$$

3. Sharia puts $30,000 into an investment account with an annual interest rate of 4.25%. Find the number of years required for the balance to reach $38,510.37.

$$FV = 30{,}000(1+0.0425)^t = 38{,}510.37$$
$$t = 6$$

| $t$ | |
|---|---|
| 5 | 36,940 |
| 6 | 38,510 |

4. A bond with a three year maturity and principal amount of $10,000 compounds monthly with an annual interest rate of 3.00%.

   (a) How many compounding periods are there per year?

   $k =$ 12

   (b) Find the final balance of principal and interest after three years.

$$FV = 10{,}000\left(1+\frac{0.03}{12}\right)^{12\cdot 3} = 10{,}940.51$$

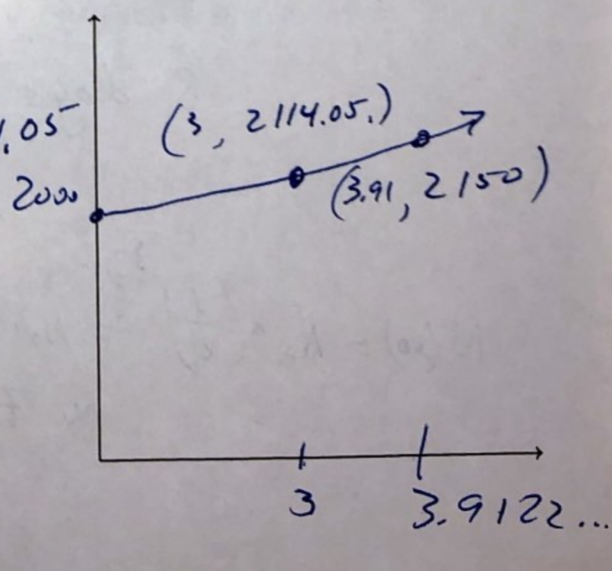
5. Lily invested SGD 2000 (Singapore dollars) in an account that pays 1.85% interest per year compounded monthly. (show your working with a labeled sketch using the axes)

   (a) Find how much Lily had in the account after 3 years.

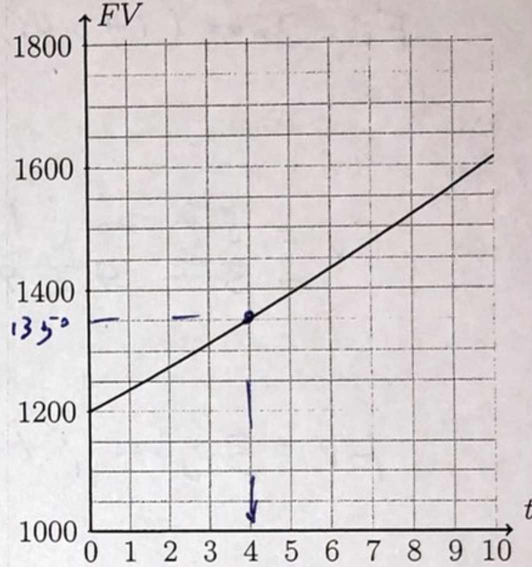
$$FV = 2000\left(1.+\frac{0.0185}{12}\right)^{12\cdot 3} = 2114.05$$

   (b) Find the number of years until she had SGD 2150 in the account.

   3.91 years



6. The graph shows the exponential function $FV = 1200 \times \left(1 + \frac{3.00}{100}\right)^t$ representing the balance of an investment account earning a fixed rate of interest over $t$ in years.




(a) Write down the initial deposit in the account.

1200

(b) Write down the annual interest rate.

3,00%

(c) How much will the account hold at the end of ten years, to the nearest hundred dollars?

\$1600

(d) When will the balance be \$1350, to the nearest year?

4 years

7. The half life of radioactive iodine 131 is eight days. That is, one half of this isotope decays over this period of time. Given an intial amount of $I_{131}$ of $N_0$, use this formula for the amount remaining $N(t)$ as a function of time $t$ in days:

$$N(t) = N_0 \times \left(\frac{1}{2}\right)^{t/8}$$

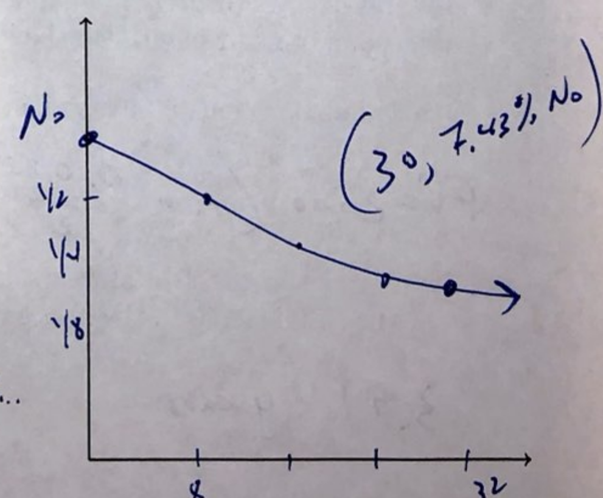
(a) How long does it take for half of a given amount of $I_{131}$ to decay?

8 days

(b) Find the fraction of iodine 131 that would remain after 30 days.

$N(30) = N_0 \times \left(\frac{1}{2}\right)^{\frac{30}{8}} = N_0 \cdot 7.4325\ldots$

$\approx 7.43\%$




Sketch a labeled graph as working.

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8. A fruit fly population doubles every 5 days. There are currently ten fruit flies in a laboratory container. With $t$ representing time, in days, then the population of flies can be modeled by

$$P(t) = A \times b^{t/5}$$

(a) Write down the value of $A$

$10$

(b) Write down the value of $b$

$2$

(c) About how many flies will there be in two weeks?

$P(14) = 10 \cdot 2^{14/5} = 69.644\ldots$

$\approx 70$ flies

(d) Find the time needed to reach a population of 160.

$P(20) = 160$

| $t$ | $P$ |
|---|---|
| 20 | 160 |

9. Graph $f(t) = 75,000\,(1 - 0.25)^t$, representing the depreciation of an asset over $t$ years.

(a) Write down the initial cost of the asset.

~~17,~~ 75,000

(b) Write down the percentage value lost each year.

25%

(c) Find the value of the investment after one year.

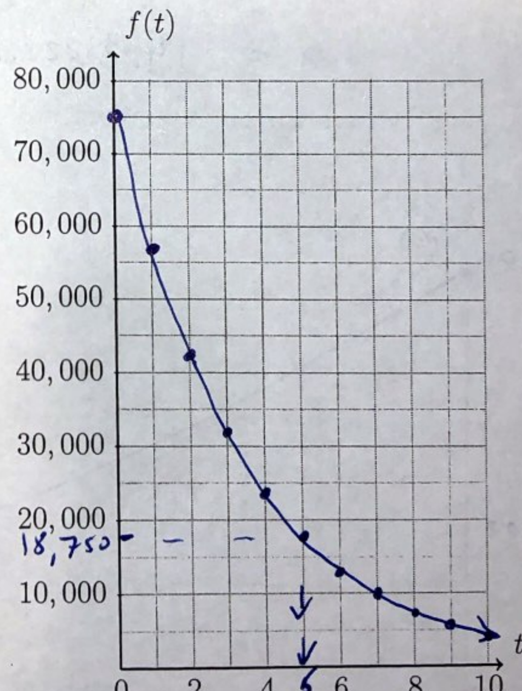
$f(1) = 75{,}000\,(0.75)^1$

$= 56{,}250$

(d) Find the number of years to depreciate three quarters of the value.

$75{,}000 \times 25\% = 18750$

$\approx 5$ years



10. The spread of a virus in the lungs is modeled by $y = 15e^x$, with $x$ the time in hours.

(a) Find the quantity of the viruses after two hours.

$y = 15e^2 = 110.8358\ldots \approx 111$

(b) Find the number of hours for viruses to spread to 45,000.

about 8 hours

| $x$ | $y$ |
|---|---|
| 7 | 16,449 |
| 8 | 44,714 |
| 9 | 121,546 |

11. The temperature of hot metal bar as it cools is modeled by the function

$$T(x) = 150e^{-0.07x} + 45$$

where $T(x)$ is the temperature in degrees Celsius and $x$ is the time in hours.

(a) Write down the initial temperature at time zero.

$T(0) = 150 + 45 = 195$

(b) Find the temperature after 24 hours.

$T(24) = 150e^{-0.07(24)} + 45 = 72.956\ldots \approx 73.0$

(c) Find the time to cool to 100°C.

$x = 14.3328\ldots \approx 14.3$ hours

(d) Graph the bar's temperature. Label each of your answers with their values.