

4.15 PreExam: Exponential Functions

Construct an exponential function symbolically given a description of the relationship F.LE.2.ii

1. A colony of insects grows exponentially with a growth factor of 3 each day. By what growth factor does the population change each 12 hours? Express your answer two ways: as a radical and a fractional exponent.

$$\frac{12}{24} = \frac{1}{2} \quad \& \quad 3^{\frac{1}{2}} = \sqrt{3}$$

2. A bacteria population, in thousands, is represented by the function $B(t) = 100 \times 1.15^t$, where t is the time in hours.

- (a) What is the initial number of bacteria?

$$100,000$$

- (b) What is the growth factor per hour?

$$1.15$$

- (c) What is the growth factor for ten hours?

$$1.15^{10} = 4.04555\dots$$

- (d) What is the population after 10 hours?

$$404.555\dots \text{ thousand}$$

3. An investment of \$1,000 doubles in value after 6 years. Write an exponential function $V(t)$ to model the investment value, with t in years. Express your answer two ways: as a radical and a fractional exponent.

$$\begin{aligned} V(t) &= 1,000 \times 2^{\frac{t}{6}} \\ &= 1,000 \times (\sqrt[6]{2})^t \end{aligned}$$

4. The graph shows the exponential function $f(x)$.

- (a) Write down the initial value of the function.

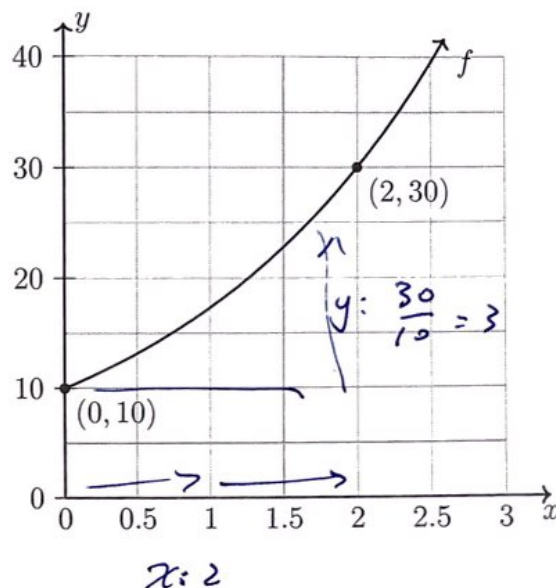
$$10$$

- (b) By what factor do the values of f increase each time x increases by 1?

$$\sqrt{3} = 3^{\frac{1}{2}}$$

- (c) Write an expression for the function $f(x)$.

$$\begin{aligned} f(x) &= 10 \cdot 3^{\frac{x}{2}} \\ &= 10 (\sqrt{3})^x \end{aligned}$$



5. A sample of radioactive material has a half-life of 8 years. Initially there are 7.5 grams of the material.

- (a) How much of the material remains after 8 years?

$$\frac{7.5}{2} = 3.75$$

- (b) How much of the material remains after 4 years?

$$\frac{4}{8} = \frac{1}{2} \quad 7.5 \cdot \sqrt{\left(\frac{1}{2}\right)} = 5.303 \dots$$

- (c) Write an exponential function $A(t)$ to model the amount of material remaining, with t in years.

$$A(t) = 7.5 \cdot \left(\frac{1}{2}\right)^{\frac{t}{8}}$$