

2.5 Test: Draft

1. A sequence is defined by $u_1 = 4$ and $u_{n+1} = 3u_n - 2$ for $n \geq 1$.
 - (a) Write down u_2 and u_3 .
 - (b) Show that $u_n = 3^n + 1$ satisfies the recurrence.
 - (c) Hence find u_6 .
2. A geometric sequence has first term 12 and common ratio $\frac{5}{6}$.
 - (a) Write down the second and third terms.
 - (b) Find the sum of the first n terms, S_n .
 - (c) Find the least value of n such that $S_n > 50$.
3. A quantity P decreases according to $P(t) = 1800e^{-kt}$.
 - (a) Given that $P(2) = 1500$, find k .
 - (b) Find the time when $P(t) = 900$.
 - (c) State whether the graph of P is linear, quadratic or exponential.
4. Solve the following, giving exact values when possible.
 - (a) $3^x = 27$.
 - (b) $\log_5(2x - 1) = 2$.
 - (c) $\ln(4) - \ln(x) = \ln(2)$.
5. Consider the function $f(x) = 2^x - 3$.
 - (a) Find $f(0)$ and $f(2)$.
 - (b) Solve $2^x - 3 = 5$.
 - (c) Describe the transformation that maps $y = 2^x$ to $y = f(x)$.
6. An arithmetic sequence has first term $a_1 = 950$ and common difference $d = 25$.
 - (a) Write an expression for a_n .
 - (b) Find the smallest n such that $a_n \geq 1400$.
 - (c) Find the sum of the first n terms when n is the value from part (b).
7. A quadratic function is given by $g(x) = x^2 - 6x + 5$.
 - (a) Find the coordinates of the vertex.

- (b) Solve $g(x) = 0$.
- (c) The line $y = mx$ intersects the graph of g at two distinct points. Find the range of m for which this happens.

Paper 2 style (GDC allowed)

1. A music teacher records the number of hours h students practise each week and the mark M each student receives on a test.
 - (a) Use your GDC to find the product-moment correlation coefficient r .
 - (b) The regression line of M on h is $M = ah + b$. Write down a and b from your GDC output.
 - (c) A student currently practises 10 hours per week. Use the regression line to estimate how many marks the student might get if they increase practice to 14 hours per week.
2. A school enrolment in year 1 is 1200 students and increases each year by 3.5%.
 - (a) Show that the number of students in year n can be modelled by $E_n = 1200(1.035)^{n-1}$.
 - (b) Find the number of students in year 8.
 - (c) A scholarship fund pays 75 euros to each enrolled student each year. Find the total amount paid in the first 6 years.
3. The number of seats available at the school in year n is given by $S_n = 1100 + 40(n - 1)$.
 - (a) Write down S_1 and S_{10} .
 - (b) Find the least value of n such that $S_n \geq E_n$ from question 9.
 - (c) Explain, with reference to the models, whether the school will always have enough seats for all applicants for all future years.
4. The sum of the first n terms of a geometric sequence is $S_n = 9(1 - (4/5)^n)$.
 - (a) Find the first term and the common ratio.
 - (b) Find S_∞ .
 - (c) Find the least value of n such that $S_\infty - S_n < 0.0008$.
5. An investment of €5000 grows at a nominal annual rate of 2.4% compounded monthly.
 - (a) Write a formula for the value V after t years.
 - (b) Use your GDC to find the value after 5 years, correct to the nearest euro.

- (c) Determine the time needed for the investment to reach €6000.
6. Consider the function $f(x) = \ln(x + 2) - 1/2$ for $x \geq -1$.
- (a) Find $f(-1)$ and $f(0)$.
- (b) Solve $\ln(x + 2) - 1/2 = 0$.
- (c) Sketch the graph of $y = f(x)$ on $-1 \leq x \leq 4$.
7. A researcher records the age of used cars (years) and their selling price (in thousands of euros).
- (a) Find the correlation coefficient.
- (b) Write down the regression equation of price on age.
- (c) Comment on whether it is sensible to use this regression equation to predict the price of a brand new car.
8. Define $f(x) = 2x + 3$ and $g(x) = 5e^{0.2x}$.
- (a) Find $(f \circ g)(x)$.
- (b) Find $(g \circ f)(x)$.
- (c) Solve $(f \circ g)(x) = 33$ using your GDC.

Markscheme (outline)

1. (a) $u_2 = 3 \cdot 4 - 2 = 10$, $u_3 = 3 \cdot 10 - 2 = 28$. (b) Substitute: $3(3^n + 1) - 2 = 3^{n+1} + 1$. (c) $u_6 = 3^6 + 1 = 730$.
2. (a) 12, 10, etc. (b) $S_n = 72(1 - (5/6)^n)$. (c) Solve $72(1 - (5/6)^n) > 50$.
3. (a) $1500 = 1800e^{-2k} \Rightarrow e^{-2k} = 5/6 \Rightarrow k = \frac{1}{2} \ln(6/5)$. (b) $900 = 1800e^{-kt} \Rightarrow e^{-kt} = 1/2 \Rightarrow t = \ln 2/k$. (c) Exponential.
4. (a) $x = 3$. (b) $2x - 1 = 25 \Rightarrow x = 13$. (c) $\ln 4 - \ln x = \ln 2 \Rightarrow x = 2$.
5. (a) $f(0) = -3$, $f(2) = 1$. (b) $2^x = 8 \Rightarrow x = 3$. (c) Translation down 3 units.
6. (a) $a_n = 950 + 25(n - 1)$. (b) $950 + 25(n - 1) \geq 1400 \Rightarrow n = 19$. (c) $S_{19} = \frac{19}{2}(1900 + 450) = \frac{19}{2} \times 2350$.
7. (a) Vertex $(3, -4)$. (b) Roots $x = 1, 5$. (c) Discriminant > 0 gives $m^2 < 4(1)(5)$.
8. Use GDC: r positive, a, b from regression output, substitution gives estimate.
9. (a) Shown algebraically. (b) $E_8 = 1200(1.035)^7$. (c) $6 \times 75 \times E_{avg}$.
10. (a) $S_1 = 1100$, $S_{10} = 1460$. (b) Solve $1100 + 40(n - 1) \geq 1200(1.035)^{n-1}$. (c) Geometric growth eventually exceeds linear.
11. (a) $u_1 = 9/5$, $r = 4/5$. (b) $S_\infty = 45$. (c) $9(4/5)^n < 0.0008$.
12. (a) $V = 5000(1 + 0.024/12)^{12t}$. (b) Evaluate with GDC. (c) Solve for t .
13. (a) $f(-1) = -1/2$, $f(0) = \ln 2 - 1/2$. (b) $x = e^{1/2} - 2$. (c) Standard log curve.
14. Negative correlation, regression not suitable beyond data range.
15. (a) $(f \circ g)(x) = 10e^{0.2x} + 3$. (b) $(g \circ f)(x) = 5e^{0.2(2x+3)}$. (c) Solve $10e^{0.2x} + 3 = 33$.