

## 2.5 Test: Draft

1. A sequence is defined by  $u_1 = 4$  and  $u_{n+1} = 3u_n - 2$  for  $n \geq 1$ .
  - (a) Write down  $u_2$  and  $u_3$ .
  - (b) Show that  $u_n = 3^n + 1$  satisfies the recurrence.
  - (c) Hence find  $u_6$ .
2. A geometric sequence has first term 12 and common ratio  $\frac{5}{6}$ .
  - (a) Write down the second and third terms.
  - (b) Find the sum of the first  $n$  terms,  $S_n$ .
  - (c) Find the least value of  $n$  such that  $S_n > 50$ .
3. A quantity  $P$  decreases according to  $P(t) = 1800e^{-kt}$ .
  - (a) Given that  $P(2) = 1500$ , find  $k$ .
  - (b) Find the time when  $P(t) = 900$ .
  - (c) State whether the graph of  $P$  is linear, quadratic or exponential.
4. Solve the following, giving exact values when possible.
  - (a)  $3^x = 27$ .
  - (b)  $\log_5(2x - 1) = 2$ .
  - (c)  $\ln(4) - \ln(x) = \ln(2)$ .
5. Consider the function  $f(x) = 2^x - 3$ .
  - (a) Find  $f(0)$  and  $f(2)$ .
  - (b) Solve  $2^x - 3 = 5$ .
  - (c) Describe the transformation that maps  $y = 2^x$  to  $y = f(x)$ .
6. An arithmetic sequence has first term  $a_1 = 950$  and common difference  $d = 25$ .
  - (a) Write an expression for  $a_n$ .
  - (b) Find the smallest  $n$  such that  $a_n \geq 1400$ .
  - (c) Find the sum of the first  $n$  terms when  $n$  is the value from part (b).
7. A quadratic function is given by  $g(x) = x^2 - 6x + 5$ .
  - (a) Find the coordinates of the vertex.

- (b) Solve  $g(x) = 0$ .
- (c) The line  $y = mx$  intersects the graph of  $g$  at two distinct points. Find the range of  $m$  for which this happens.

## Paper 2 style (GDC allowed)

1. A music teacher records the number of hours  $h$  students practise each week and the mark  $M$  each student receives on a test.
  - (a) Use your GDC to find the product-moment correlation coefficient  $r$ .
  - (b) The regression line of  $M$  on  $h$  is  $M = ah + b$ . Write down  $a$  and  $b$  from your GDC output.
  - (c) A student currently practises 10 hours per week. Use the regression line to estimate how many marks the student might get if they increase practice to 14 hours per week.
2. A school enrolment in year 1 is 1200 students and increases each year by 3.5%.
  - (a) Show that the number of students in year  $n$  can be modelled by  $E_n = 1200(1.035)^{n-1}$ .
  - (b) Find the number of students in year 8.
  - (c) A scholarship fund pays 75 euros to each enrolled student each year. Find the total amount paid in the first 6 years.
3. The number of seats available at the school in year  $n$  is given by  $S_n = 1100 + 40(n - 1)$ .
  - (a) Write down  $S_1$  and  $S_{10}$ .
  - (b) Find the least value of  $n$  such that  $S_n \geq E_n$  from question 9.
  - (c) Explain, with reference to the models, whether the school will always have enough seats for all applicants for all future years.
4. The sum of the first  $n$  terms of a geometric sequence is  $S_n = 9(1 - (4/5)^n)$ .
  - (a) Find the first term and the common ratio.
  - (b) Find  $S_\infty$ .
  - (c) Find the least value of  $n$  such that  $S_\infty - S_n < 0.0008$ .
5. An investment of €5000 grows at a nominal annual rate of 2.4% compounded monthly.
  - (a) Write a formula for the value  $V$  after  $t$  years.
  - (b) Use your GDC to find the value after 5 years, correct to the nearest euro.

- (c) Determine the time needed for the investment to reach €6000.
6. Consider the function  $f(x) = \ln(x + 2) - 1/2$  for  $x \geq -1$ .
- (a) Find  $f(-1)$  and  $f(0)$ .
- (b) Solve  $\ln(x + 2) - 1/2 = 0$ .
- (c) Sketch the graph of  $y = f(x)$  on  $-1 \leq x \leq 4$ .
7. A researcher records the age of used cars (years) and their selling price (in thousands of euros).
- (a) Find the correlation coefficient.
- (b) Write down the regression equation of price on age.
- (c) Comment on whether it is sensible to use this regression equation to predict the price of a brand new car.
8. Define  $f(x) = 2x + 3$  and  $g(x) = 5e^{0.2x}$ .
- (a) Find  $(f \circ g)(x)$ .
- (b) Find  $(g \circ f)(x)$ .
- (c) Solve  $(f \circ g)(x) = 33$  using your GDC.

## Markscheme (outline)

1. (a)  $u_2 = 3 \cdot 4 - 2 = 10$ ,  $u_3 = 3 \cdot 10 - 2 = 28$ . (b) Substitute:  $3(3^n + 1) - 2 = 3^{n+1} + 1$ . (c)  $u_6 = 3^6 + 1 = 730$ .
2. (a) 12, 10, etc. (b)  $S_n = 72(1 - (5/6)^n)$ . (c) Solve  $72(1 - (5/6)^n) > 50$ .
3. (a)  $1500 = 1800e^{-2k} \Rightarrow e^{-2k} = 5/6 \Rightarrow k = \frac{1}{2} \ln(6/5)$ . (b)  $900 = 1800e^{-kt} \Rightarrow e^{-kt} = 1/2 \Rightarrow t = \ln 2/k$ . (c) Exponential.
4. (a)  $x = 3$ . (b)  $2x - 1 = 25 \Rightarrow x = 13$ . (c)  $\ln 4 - \ln x = \ln 2 \Rightarrow x = 2$ .
5. (a)  $f(0) = -3$ ,  $f(2) = 1$ . (b)  $2^x = 8 \Rightarrow x = 3$ . (c) Translation down 3 units.
6. (a)  $a_n = 950 + 25(n - 1)$ . (b)  $950 + 25(n - 1) \geq 1400 \Rightarrow n = 19$ . (c)  $S_{19} = \frac{19}{2}(1900 + 450) = \frac{19}{2} \times 2350$ .
7. (a) Vertex  $(3, -4)$ . (b) Roots  $x = 1, 5$ . (c) Discriminant  $> 0$  gives  $m^2 < 4(1)(5)$ .
8. Use GDC:  $r$  positive,  $a, b$  from regression output, substitution gives estimate.
9. (a) Shown algebraically. (b)  $E_8 = 1200(1.035)^7$ . (c)  $6 \times 75 \times E_{avg}$ .
10. (a)  $S_1 = 1100$ ,  $S_{10} = 1460$ . (b) Solve  $1100 + 40(n - 1) \geq 1200(1.035)^{n-1}$ . (c) Geometric growth eventually exceeds linear.
11. (a)  $u_1 = 9/5$ ,  $r = 4/5$ . (b)  $S_\infty = 45$ . (c)  $9(4/5)^n < 0.0008$ .
12. (a)  $V = 5000(1 + 0.024/12)^{12t}$ . (b) Evaluate with GDC. (c) Solve for  $t$ .
13. (a)  $f(-1) = -1/2$ ,  $f(0) = \ln 2 - 1/2$ . (b)  $x = e^{1/2} - 2$ . (c) Standard log curve.
14. Negative correlation, regression not suitable beyond data range.
15. (a)  $(f \circ g)(x) = 10e^{0.2x} + 3$ . (b)  $(g \circ f)(x) = 5e^{0.2(2x+3)}$ . (c) Solve  $10e^{0.2x} + 3 = 33$ .