1.23 Exam: Probability, Venn diagrams

1. Given:
   $U = \{\text{the letters in the alphabet}\}$ $A = \{s, o, u, t, h\}$ $B = \{b, r, o, n, x\}$

   (a) List the members of $A \cap B$. [1 mark]

   (b) List the elements of $A \cup B$. [1 mark]

   (c) A letter is selected at random. What is the probability that it is a member of either or both sets, $(A \cup B)$? [1 mark]

2. Events A and B are independent with $\mathrm{P}(A) = 0.8$, $\mathrm{P}(B) = 0.25$. Find each probability.

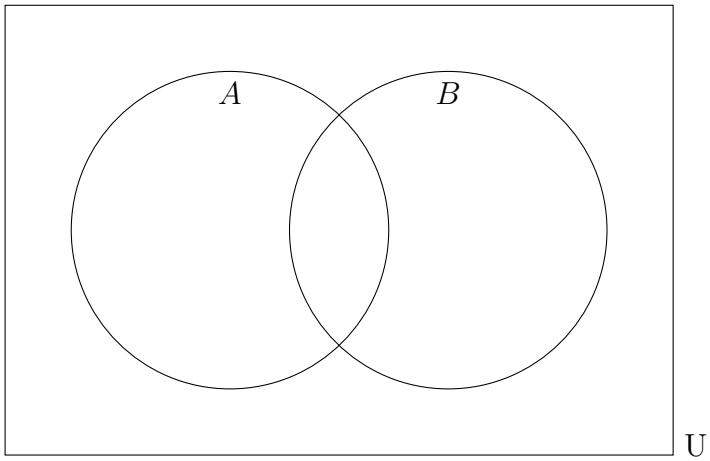
   (a) $\mathrm{P}(A \cap B)$ [2 mark]

   (b) $\mathrm{P}(A \cup B)$ [2 mark]

   (c) $\mathrm{P}(A \cap B')$ [2 mark]

   (d) $\mathrm{P}(B|A)$ [2 mark]

   (e) Mark the Venn diagram with the probabilities for each area. [2 marks]

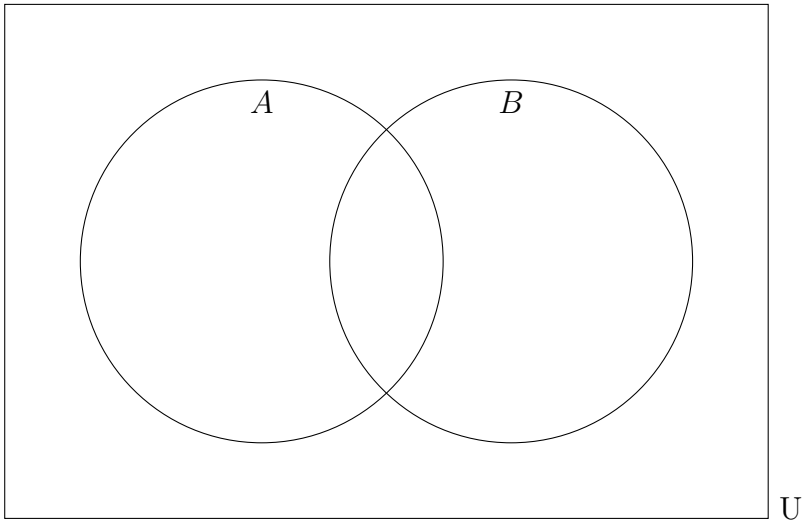


3. The universal set U is defined as the set of positive integers less than 9.

   (a) Subset is defined as $A = \{\text{multiples of two}\}$. List its elements. [1 mark]

   (b) Subset $B = \{\text{prime numbers}\}$. List the members of set B. [1 mark]

   (c) Place the elements of U in the appropriate regions in the Venn diagram. [2 marks]



   (d) List the members of $(A' \cap B)$. [1 mark]

   (e) If an element is selected at random, what is the probability that it is a member of both sets, $(A \cap B)$? [1 mark]

   (f) If a member of set A is selected at random, what is the probability that it is also a member of set B, i.e. the conditional probability $(B|A)$? [2 marks]

4. A jar contains 20 marbles, 12 of which are red, 5 are blue, and 3 are green.

   (a) A marble is selected at random. Find the probability it is *not* red. [1 mark]

   (b) The marble is replaced and a second marble is selected. Given that the second marble is not red, find the probability it is green. [1 mark]

   (c) The marbles are returned to the jar and two marbles are selected at random. Find the probability that both are blue. [2 mark]

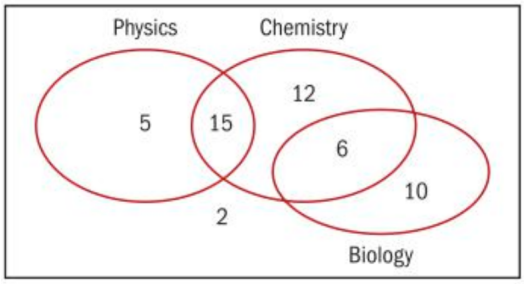
5. Draw a tree diagram to represent the taxi cab problem in the textbook. First, there are two cab companies, 85% are black and the rest are yellow. Then, the witness identifies the color of the cab correctly 80% of the time. [3 marks]

   (a) Label the branches with the probabilities. [1 marks]

   (b) Calculate the probabilities of each four outcomes. [2 marks]

   (c) Given that the witness identified the cab as yellow, find the probability that it was black, i.e. that she was wrong. [3 marks]

6. : The Venn diagram illustrates the number of students taking each of the three sciences: physics, chemistry and biology.



A student is randomly chosen from the group.

Find the probability that

a the student studies chemistry or biology (2 marks)

b the student studies neither physics nor biology (2 marks)

c the student studies physics, given that they study chemistry (2 marks)

d the student studies biology, given that they study physics (2 marks)

e the student studies physics, given that they do not study biology. (2 marks)

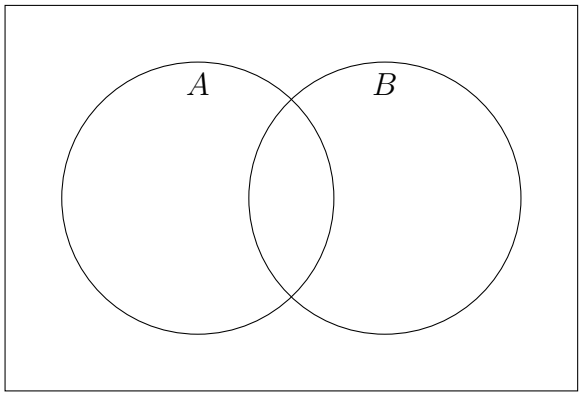
7. The events A and B are mutually exclusive with $\mathrm{P}(A) = 0.70$ and $\mathrm{P}(B) = 0.2$.

(a) Write down $\mathrm{P}(A \cup B)$. [1 mark]

(b) Write down $\mathrm{P}(A \cap B)$. [1 mark]

8. Given events A and B with $\mathrm{P}(A) = 0.7$, $\mathrm{P}(B) = 0.5$, $\mathrm{P}(A \cap B) = 0.35$.

   (a) Completely mark the Venn diagram with probabilities for each area. [2 marks]

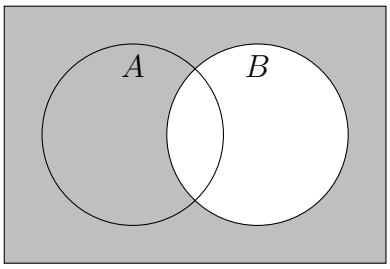


   (b) Find $\mathrm{P}(A \cup B)$. [2 marks]

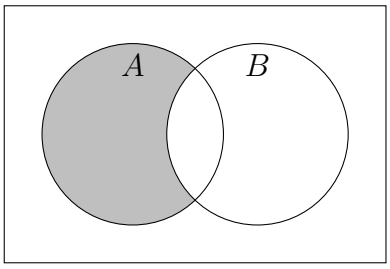
   (c) State whether events A and B are independent. Justify your answer. [3 marks]

9. For each Venn diagram, write an expression representing the shaded area. [5 marks]

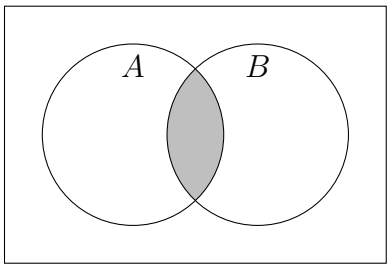
(a)



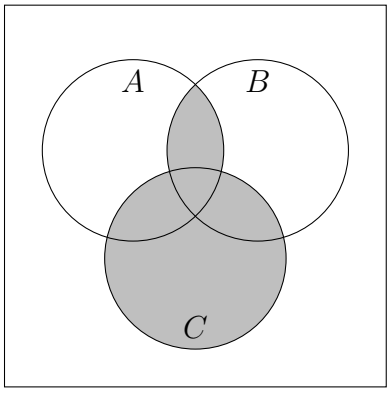
(b)



(c)



(d)



10. A survey of fruit lovers is taken, all of whom like at least one of the three fruits: apples, bananas, and cherries. The following information is gathered:

    - 50 people like apples
    - 40 like bananas
    - 30 like cherries
    - 25 like apples and bananas
    - 20 like apples and cherries
    - 15 like bananas and cherries
    - 10 like all three fruits

    Complete the Venn diagram below with the number of individuals in each region to represent the situation. How many people in total were surveyed? [4 marks]