Geometry Unit 1, part b: Area

Bronx Early College Academy

Christopher J. Huson PhD

19-23 September 2022

Learning Target: I can calculate areas

CCSS: HSG.CO.A.1 Know precise geometric definitions 1.8 Monday 19 Sept

Do Now: Practice unit conversion

1. How many days are in a week?
2. Find the number of weeks in 365 days.
   (show calculation with units)

Quiz results

Lesson: Rectangle, triangle, parallelogram area formulas

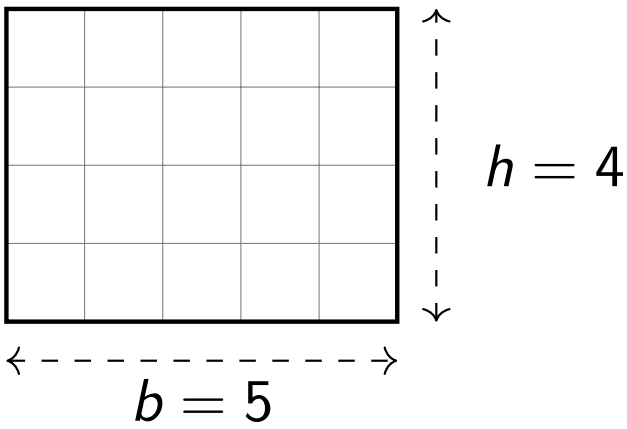
Extension: Scientific notation

The *area* of a rectangle is its base $\times$ height.

We also say "length times width"

Formula for the area of a rectangle:

$$A = b \times h$$



$$A = 5 \times 4 = 20$$

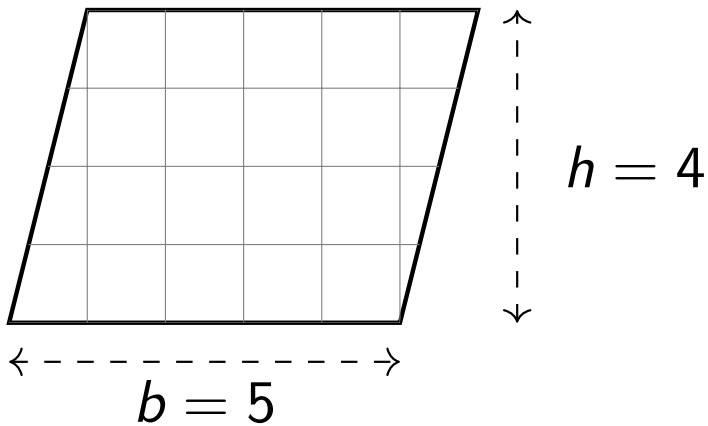
Area the quantity of unit squares that fill a shape

A parallelogram's area has the same formula as a rectangle.

Use the height, not the length of the slanted side.

Formula for the area of a parallelogram:

$$A = b \times h$$



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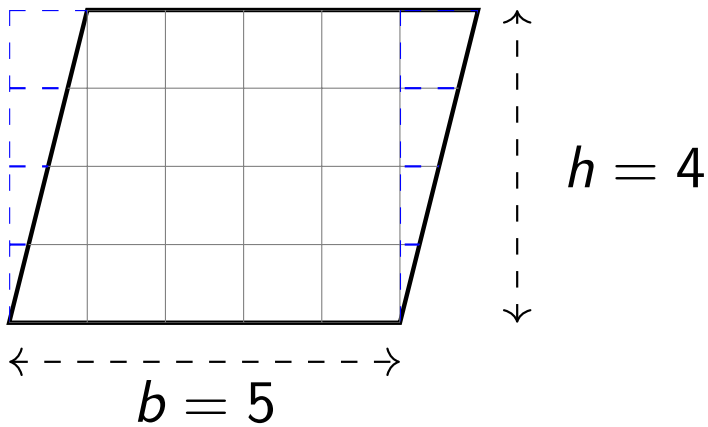


$$A = 5 \times 4 = 20$$

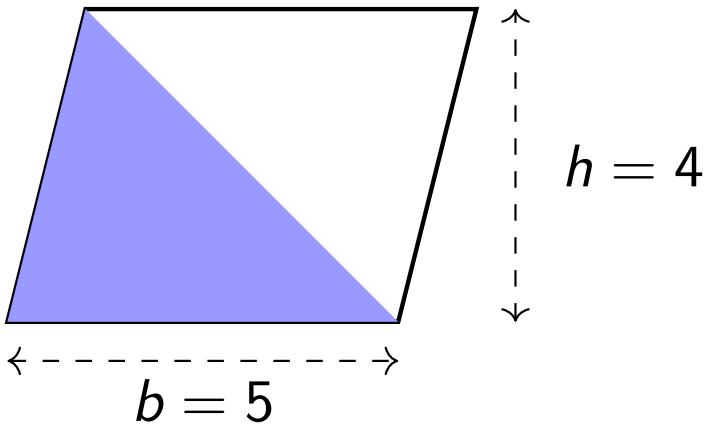
The two blue triangles match

A triangle has half the area of its base times height.

Use the triangle's height or *altitude*, not the side length.

Formula for the area of a triangle:

$$A = \frac{1}{2} b \times h$$

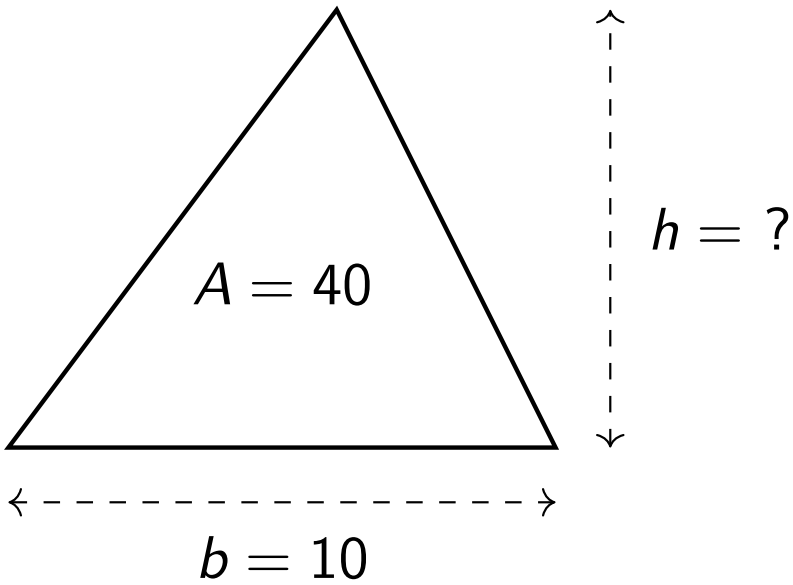


$$A = \frac{1}{2}(5 \times 4) = 10$$

Altitude The height of a triangle (distance $\perp$ to its base)

Find a missing dimension using the area formula

Given the area of a triangle is 40 and its base is 10, find its height.



$$A = \frac{1}{2}(10 \times h) = 40$$

Write formulas in notebook

Rectangle $A = b \times h$ (base times height or length times width)

Parallelogram $A = b \times h$

Triangle $A = \frac{1}{2}(b \times h)$

Area the quantity of unit squares that fill a shape

Units We say "square units", i.e. square inches (abbreviated in^2), square miles, etc.

Altitude Height (distance $\perp$ to the baseline)

Extension (optional): *Scientific notation*

Use for very large or small numbers instead of decimals

Exponents mean repeated multiplication:

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000$$

1. The distance to the sun is 150,000,000,000 meters $= 1.5 \times 10^{11}$
2. The population of NYC is $8,000,000 =$

3. The area of the earth is 2×10^8 square miles $=$

Scientific notation Compact notation for big numbers, $a \times 10^k$

Exponent Repeated multiplication. The number of decimal places in base 10

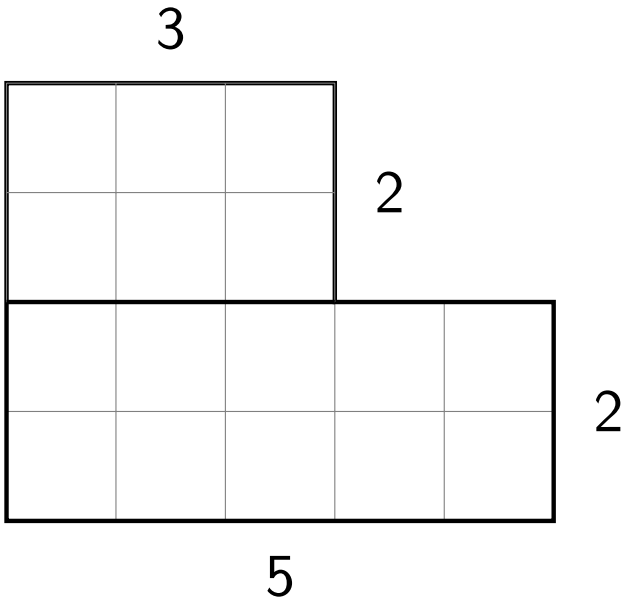
Base 10 The system of place value we use for numbers

Mantissa The coefficient in scientific notation

Learning Target: I can calculate the area of a circle

CCSS: HSG.CO.A.1 Know precise geometric definitions 1.9 Tuesday 20 Sept

Do Now: Two rectangles are shown. Calculate the area of each and the combined total area.



Lesson: Area of a circle, π, rounding

Extension: Significant figures

The *area* and *circumference* of a circle are multiples of π.

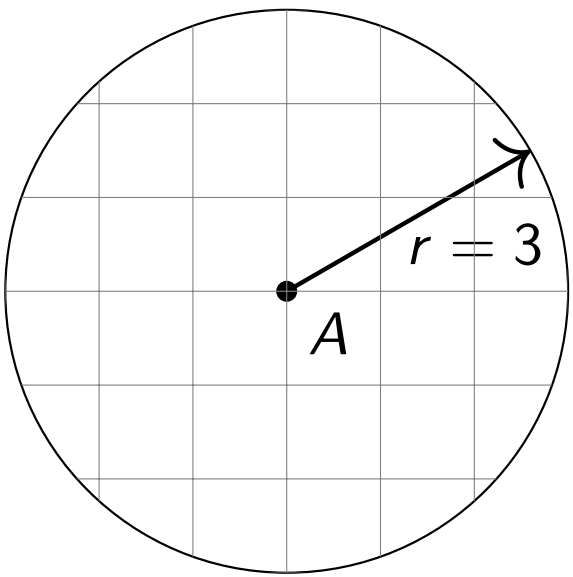
π is an *irrational number*

Area of a circle:

$$A = \pi r^2$$

Circumference (distance around):

$$C = 2\pi r$$



Circle A with radius $r = 3$

$$A = \pi \times 3^2 = 9\pi = 28.2743...$$

$$C = 2\pi \times 3 = 6\pi = 18.8495...$$

Radius Segment from the center to the edge of a circle, r

Diameter Segment/length across the whole circle, $D = 2r$

Numbers don't always need to be exact

Round up when the next digit is 5 or more

Round down otherwise

Is π closer to three or four?

$\pi = 3.1415926...$

Numbers don't always need to be exact

Round up when the next digit is 5 or more
Round down otherwise

$\pi = 3.1415926... \approx 3$ to the nearest whole number

Numbers don't always need to be exact

Round up when the next digit is 5 or more
Round down otherwise

$\pi = 3.1415926... \approx 3$ to the nearest whole number
$\pi = 3.1415926... \approx 3.1$ to the nearest tenth
$\pi = 3.1415926... \approx 3.14$ to the nearest hundredth
$\pi = 3.1415926... \approx 3.142$ to the nearest thousandth

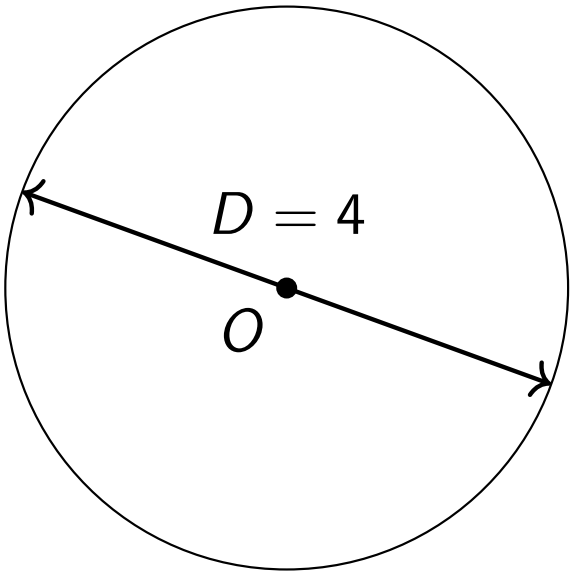
Numbers don't always need to be exact

Round up when the next digit is 5 or more
Round down otherwise

$\pi = 3.1415926... \approx 3$ to the nearest whole number
$\pi = 3.1415926... \approx 3.1$ to the nearest tenth
$\pi = 3.1415926... \approx 3.14$ to the nearest hundredth
$\pi = 3.1415926... \approx 3.142$ to the nearest thousandth

Whole	The ones place, e.g. 3, 14, -15
tenths	First digit after the decimal, 0.3, 6.8
hundredths	Second decimal digit, 5.45
thousandths	Third decimal place, 18.123
Rounding	Writing an approximation of a number
Approximate	About equal to, not exact, $\approx$

Use the symbol π for an *exact* answer



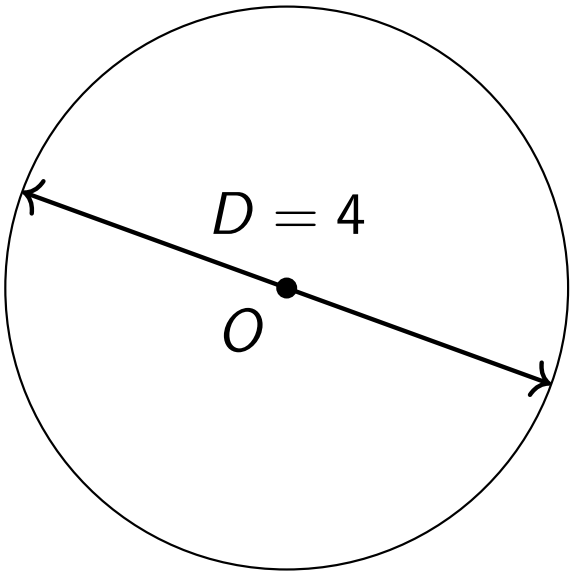
Circle O with diameter $D = 4$

1. Find the radius of the circle.

2. Find the exact circumference.

3. Round to the nearest hundredth.

Use the symbol π for an *exact* answer



Circle O with diameter $D = 4$

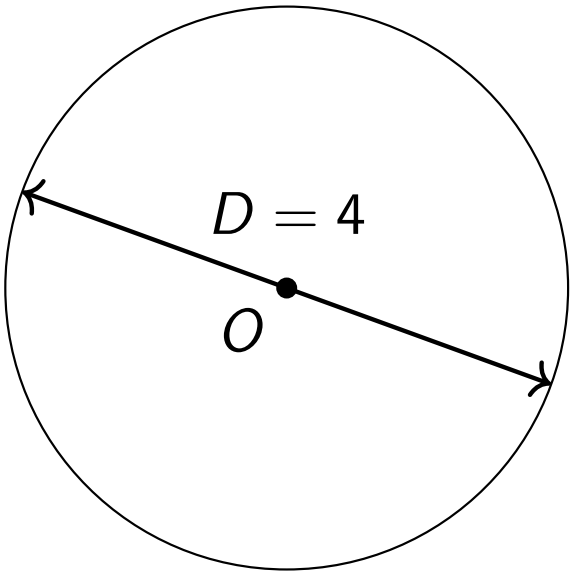
1. Find the radius of the circle.

   $r = \frac{1}{2}D = \frac{4}{2} = 2$

2. Find the exact circumference.

3. Round to the nearest hundredth.

Use the symbol π for an *exact* answer



Circle O with diameter $D = 4$

1. Find the radius of the circle.

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   $C = 2\pi r = 2\pi 2 = 4\pi$

3. Round to the nearest hundredth.

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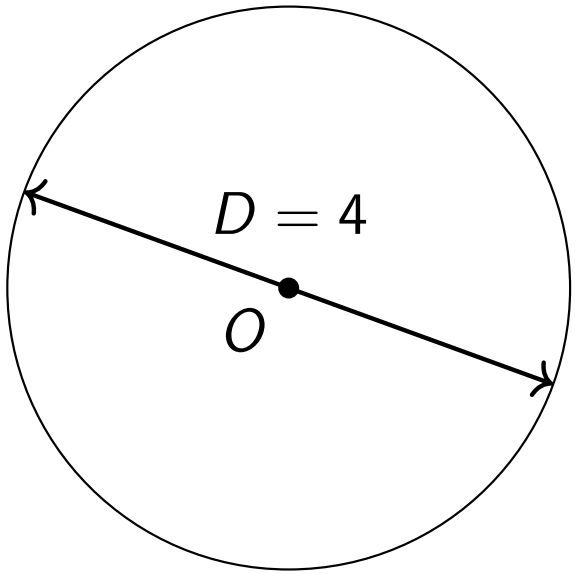


Circle O with diameter $D = 4$

1. Find the radius of the circle.

   $r = \frac{1}{2}D = \frac{4}{2} = 2$

2. Find the exact circumference.

   $C = 2\pi r = 2\pi 2 = 4\pi$

3. Round to the nearest hundredth.

   $C = 4\pi = 12.5663706... \approx 12.57$

Exact solution Written with symbols or an ellipse (...).
Also said as "give your answer *in terms of* π".

Write formulas in notebook

Circle	All points with equal distance from the circle center
Radius	Distance from the circle center to its edge, r
Diameter	Length across the whole circle, $D = 2r$
Circle area	Formula $A = \pi r^2$
Circumference	The distance around a circle (i.e. perimeter), $C = 2\pi r$
Semi-circle	Half of a circle
π	A special number, $\pi = 3.14159265358...$
Irrational	Number that can not be written as a fraction, π, $\sqrt{2}$
Exact solution	Written with symbols or an ellipse (...). Also said as "give your answer *in terms of* π".

Extension: Three digits is usually exact enough

Scientists and engineers say *significant figures*, or in IB, "sig figs"

Round to three digits

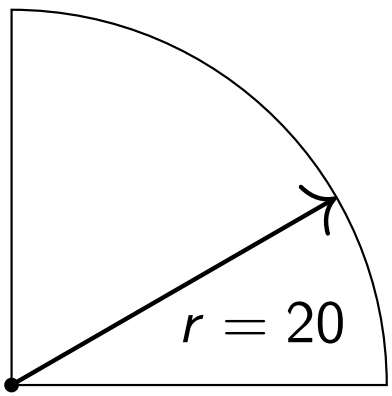
- $\pi = 3.14159265358... \approx 3.14$
- $\sqrt{2} = 1.4142135... \approx 1.41$
- Dr. Huson's height $h \approx 67.5$ inches
- 365 days in a year (actually 365.2421897, source)
- Avogadro's number $N_A \approx 6.02 \times 10^{23}$

Sig figs Significant figures, the number of digits required for the desired precision. In IB mathematics and most practical matters, the convention is 3 sig figs.

Learning Target: I can quantify error in calculations

CCSS: HSG.CO.A.1 Know precise geometric definitions 1.10 Wednesday 21 Sept

Do Now: Find the area of a quarter circle with radius $r = 20$ cm, rounding to the nearest whole number.



Lesson: Percent error formula

Extension: Confidence intervals

Learning Target: I can quantify error in calculations

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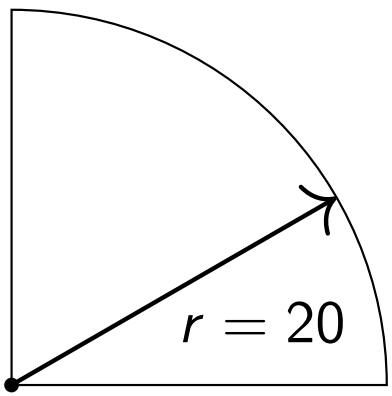

$$A = \frac{1}{4}\pi \times 20^2 = 100\pi$$

$$= 314.15926... \approx 314 \text{ square units}$$

Lesson: Percent error formula

Extension: Confidence intervals

Quantify measurement and rounding inaccuracy as a percent

Convention: Treat all errors as a positive amount

Given v_A = Approximate value, v_E = Exact value

Percent error

$$\epsilon = \left| \frac{v_A - v_E}{v_E} \right| \times 100\%$$

Which is more accurate?

$\pi \approx 3.14$ $\pi \approx \frac{22}{7}$ (Archimedes c. 250 B.C.)

Relative error decimal format (i.e. 5% versus 0.05)

ϵ The Greek letter epsilon, meaning error

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Percent error

$$\epsilon = \left| \frac{v_A - v_E}{v_E} \right| \times 100\%$$

Which is more accurate?

$\pi \approx 3.14$

$\epsilon = \left| \frac{3.14 - \pi}{\pi} \right| \times 100\%$

$= 0.05069...\%$

$\pi \approx \frac{22}{7}$ (Archimedes c. 250 B.C.)

$\epsilon = \left| \frac{22/7 - \pi}{\pi} \right| \times 100\%$

$= 0.04024...\%$

Relative error decimal format (i.e. 5% versus 0.05)

ϵ The Greek letter epsilon, meaning error

Unit conversions are often approximate

39.3701 inches is a more exact value

There are approximately 39 inches in a meter.

$$1 \text{ meter} \approx 39 \text{ inches}$$

Find the percent error in this conversion ratio.

Unit conversions are often approximate

39.3701 inches is a more exact value

There are approximately 39 inches in a meter.

$$1 \text{ meter} \approx 39 \text{ inches}$$

Find the percent error in this conversion ratio.

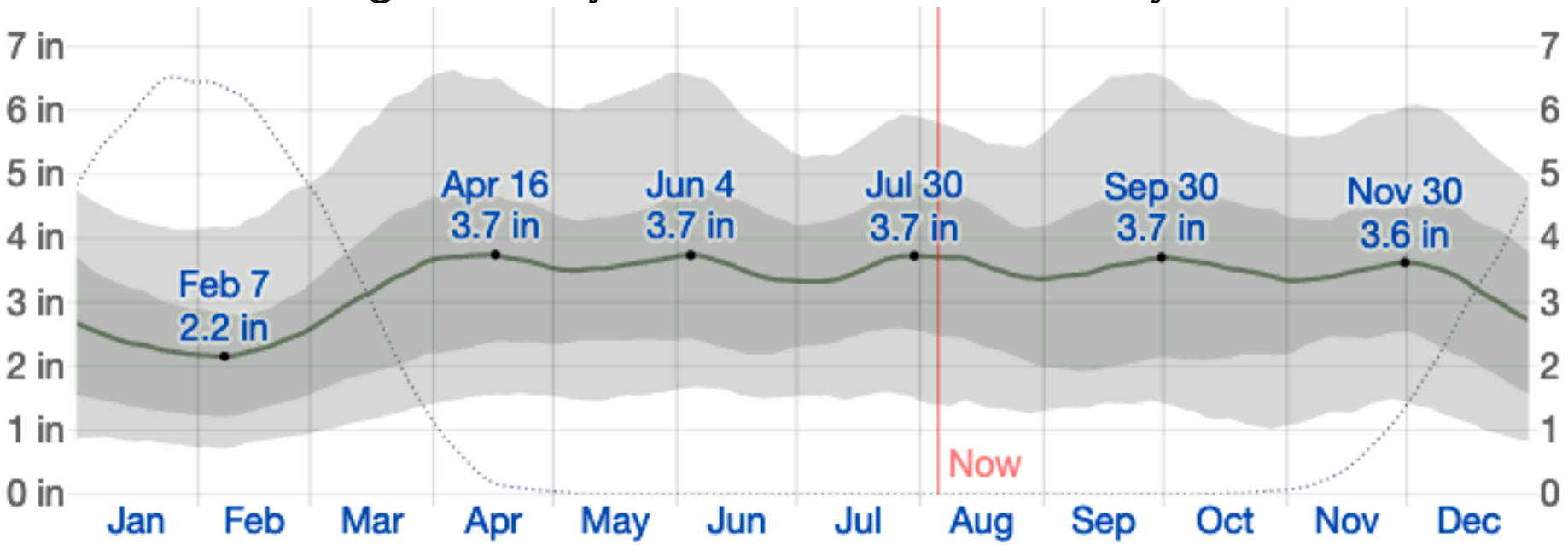
$$\epsilon = \left| \frac{39 - 39.3701}{39.3701} \right| \times 100\%$$

$$= 0.940053492...\% \approx 1\% \text{ error}$$

Quantify an error as interval around the best guess

- What is a typical retirement age? 65 ± 5 years
- SUNY New Paltz SAT scores are between 1070 and 1260.
- How much does it rain in New York City? (WeatherSpark)

Average Monthly Rainfall in New York City



Interval A range, e.g. from 10 to 12

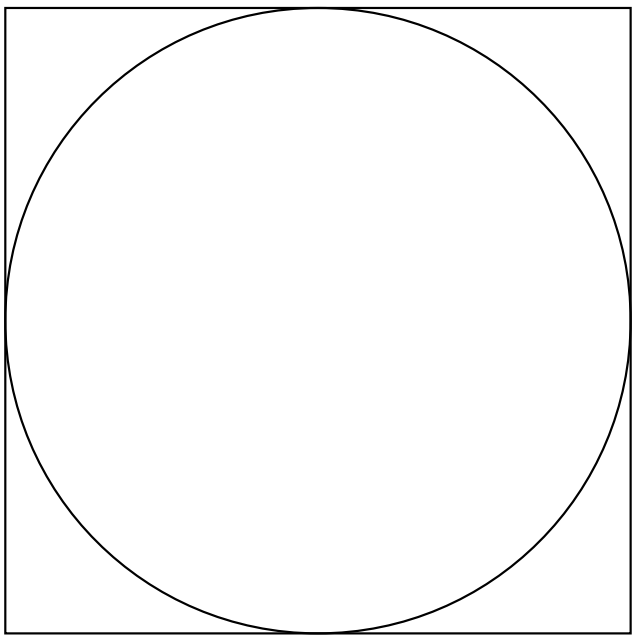
Confidence Not certain, but most likely range of values

$\pm$ Plus or minus

Learning Target: I can study together with my classmates

CCSS: HSG.CO.A.1 Know precise geometric definitions 1.11 Thursday 22 Sept

Do Now: Estimate the percentage of the square's area covered by the circe. (then calculate your percent error)



Lesson: Peer review, notebook check, homework inventory due

Unit test tomorrow

Learning Target: I can study together with my classmates

CCSS: HSG.CO.A.1 Know precise geometric definitions 1.11 Thursday 22 Sept

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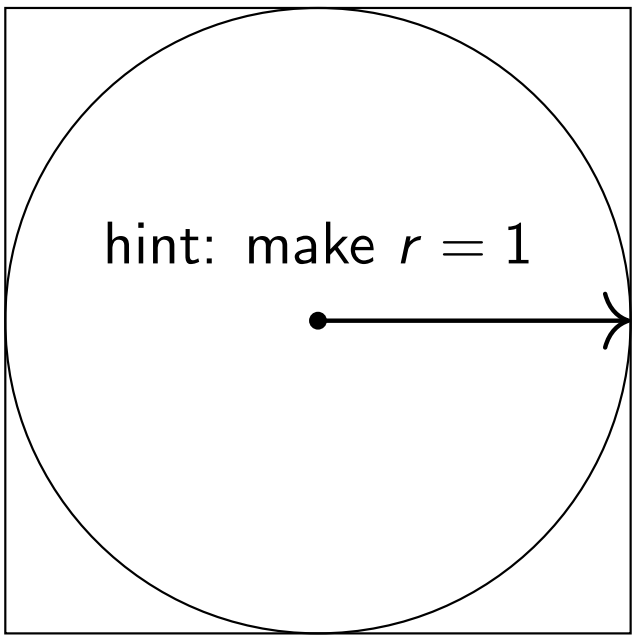

Guestimating three quarters, or 75%

$$A_{square} = 2 \times 2 = 4$$

$$A_{circle} = \pi \times 1^2 = \pi = 3.14159...$$

$$\% \text{ coverage} = \frac{\pi}{4} = 0.78539... \approx 78.5\%$$

$$\epsilon = \left|\frac{75 - 78.539...}{78.539...}\right| \times 100\%$$

$$= 4.5070...\% \approx 4.5\% \text{ error}$$

Lesson: Peer review, notebook check, homework inventory due

Unit test tomorrow

Groupwork review for test tomorrow

"Roundtable" of four students, with four topics assigned

Geometry skills to study / teach

1. Line segments, length, number lines
2. Perimeter and area
3. Precision, percent error
4. Modeling situations and solving with algebra

Learning Target: I can quantify length and area

CCSS: HSG.CO.A.1 Know precise geometric definitions 1.12 Friday 23 Sept

Unit test