Unit 11: Circle angles, sectors, arcs Name:
27 February 2023

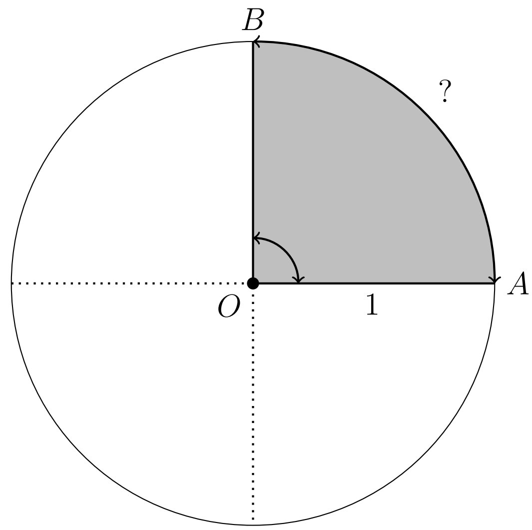
11.1 Extension: Radian measure

1. A *unit* circle with a radius $r = 1$ is divided in quarters. One sector, AOB, is shaded as shown.

   (a) Find the circumference in terms of π.
   ($C = 2\pi r$)

   (b) Write down $m\angle AOB$ in *degrees*.

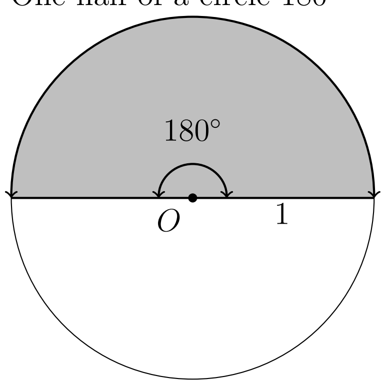
   (c) Find the *length* of the arc $\widehat{AB}$ in terms of π.



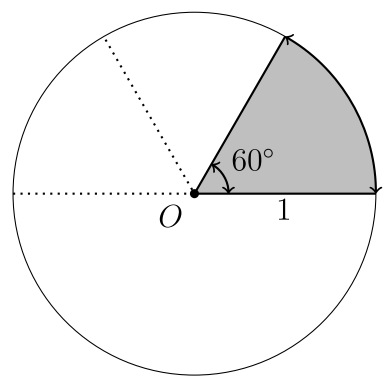
2. The length of the arc of a unit circle is a measure of the central angle called *radians*. The circumference of the full circle is $2\pi = 360°$.

   Mark each angle with its radian measure.

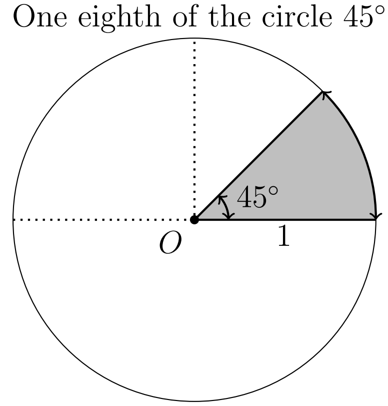
   (a) One half of a circle 180°



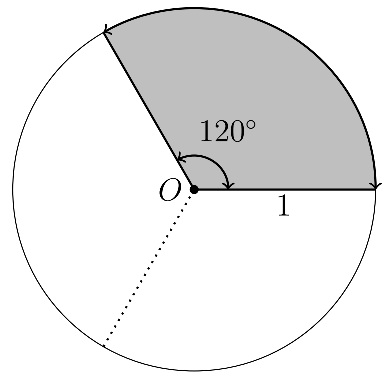
   (b) One sixth of the circle 60°



   (c) One eighth of the circle 45°



(d) One third of a circle 120°



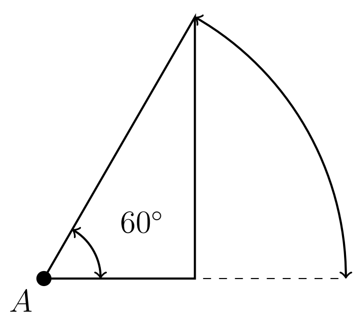
3. Algebra view of *radians* to *degrees* using the formula $2\pi = 360°$ or $\pi = 180°$.

   Apply the appropriate formula.

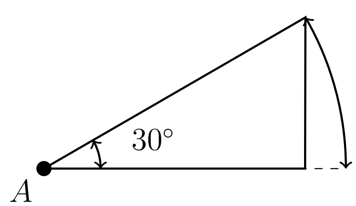
   $r = d \times \dfrac{\pi}{180}$

   $d = r \times \dfrac{180}{\pi}$

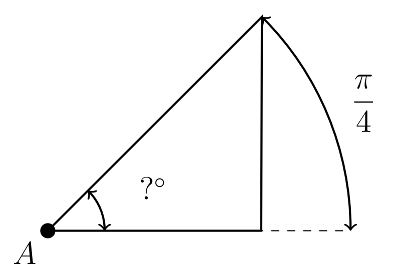
   (a) $60° = ?$ radians

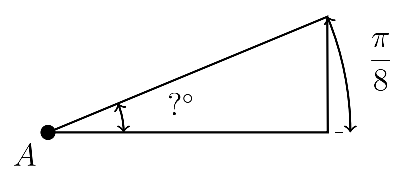
   (b) $30° = ?$ radians

   (c) $\dfrac{\pi}{4} = ?$ degrees

   (d) $\dfrac{\pi}{8} = ?$ degrees

4. Do Now: Convert each set of units. One inch $= \frac{1}{12}$ foot or one foot $= 12$ inches.
   (a) How many feet are 30 inches?
   (b) How many inches are 8.25 feet?

   Example Greek letters are π, θ, α, Δ, β, σ, Σ, ϵ

5. Practice: Convert between units.

   General method: if $A = B$ multiply by $\frac{A}{B}$ or $\frac{B}{A}$. For example, π radians $= 180$ degrees so
   $r = d \times \frac{\pi}{180}$ and $d = r \times \frac{180}{\pi}$

   (a) $40^\circ = ?$ radians
   (b) $\frac{\pi}{7} = ?$ degrees
   (c) 1 foot = 12 inches

       3.5 feet =
   (d) 54 inches =
   (e) 1 euro = 1.21 dollars

       20 euro =
   (f) 100 dollars =
   (g) 1 mile = 5,280 feet

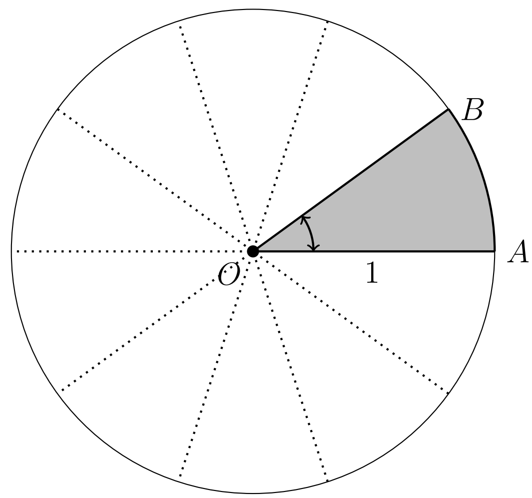
       10,000 feet =
   (h) $\frac{1}{2}$ mile =

6. The shaded sector of the unit circle is *one tenth* of the whole circle, as shown.

   (a) Write down the circumference in terms of π. ($C = 2\pi r$)

   (b) Find $m\angle AOB$ in *degrees*.

   (c) Find $m\angle AOB$ in *radians*.



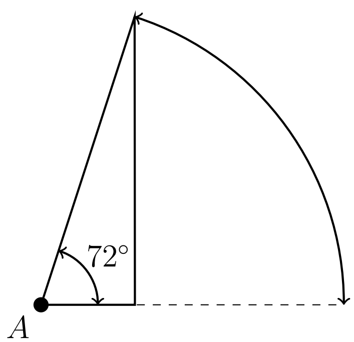
7. Convert equivalent angle measures between *radians* and *degrees* ($2\pi = 360°$, $\pi = 180°$).

   Apply the appropriate formula.

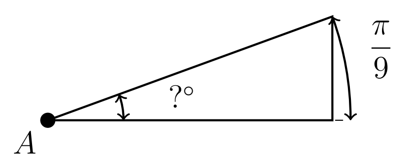
   $r = d \times \dfrac{\pi}{180}$

   $d = r \times \dfrac{180}{\pi}$

   (a) $72° = ?$ radians
   Express in terms of π



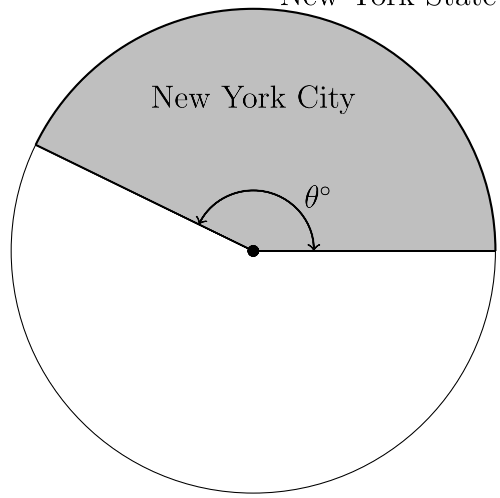
   (b) $\dfrac{\pi}{9} = ?$ degrees



8. Lesson: *Pie charts* represent proportions using sector areas and central angles.

   Population of NY City is 8,340,000
   Population of NY State is 19,500,000

   New York State

   (a) Find the fraction of New Yorkers, x, who reside in NYC as a percentage.

   (b) Find the central angle of the shaded area, $\theta = x \times 360°$

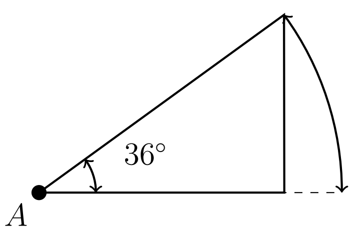
9. Practice: Convert between *radians* to *degrees* knowing $2\pi = 360°$ or $\pi = 180°$.

   Apply the appropriate formula. Leave radians in terms of π.

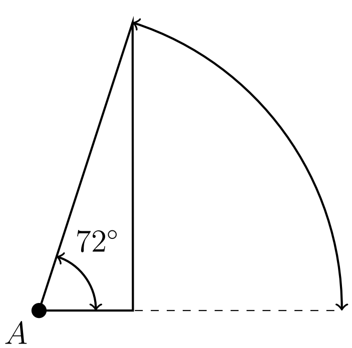
   $r = d \times \dfrac{\pi}{180}$

   $d = r \times \dfrac{180}{\pi}$

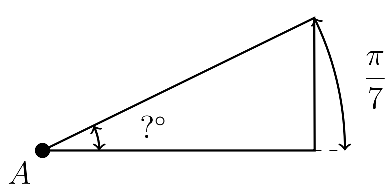
   (a) $36° = ?$ radians

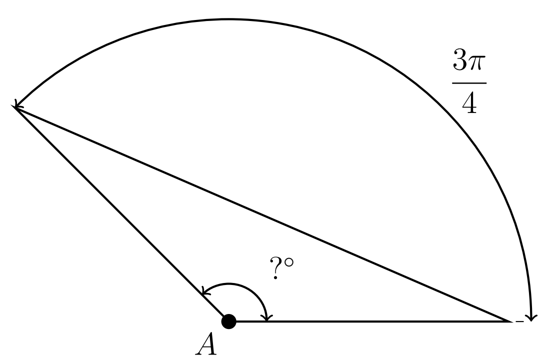
   (b) $72° = ?$ radians

   (c) $\dfrac{\pi}{7} = ?$ degrees

   (d) $\dfrac{3\pi}{4} = ?$ degrees

10. Convert between units.

General method: if $A = B$ multiply by $\frac{A}{B}$ or $\frac{B}{A}$. For example, π radians = 180 degrees, therefore
$r = d \times \frac{\pi}{180}$ and $d = r \times \frac{180}{\pi}$

(a) $135° = ?$ radians

(b) $\frac{3\pi}{5} = ?$ degrees

(c) 1 mile = 5,280 feet

14,520 feet =

(d) $\frac{1}{4}$ mile =

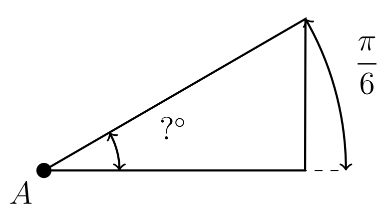
11. Convert units of *radians* and *degrees* ($2\pi = 360°$, $\pi = 180°$).

Apply the appropriate formula.

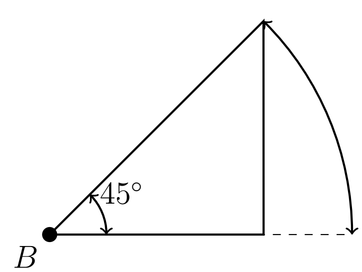
$d = r \times \frac{180}{\pi}$

$r = d \times \frac{\pi}{180}$

(a) $m\angle A = \frac{\pi}{6} = ?$ degrees



(b) $m\angle B = 45° = ?$ radians
(in terms of π)



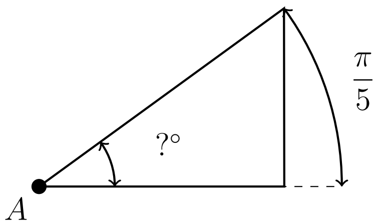
12. Convert units of *radians* and *degrees* ($2\pi = 360°$, $\pi = 180°$).

Apply the appropriate formula.

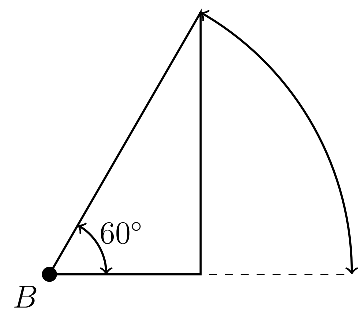
$d = r \times \frac{180}{\pi}$

$r = d \times \frac{\pi}{180}$

(a) $m\angle A = \frac{\pi}{5} = ?$ degrees



(b) $m\angle B = 60° = ?$ radians
(in terms of π)

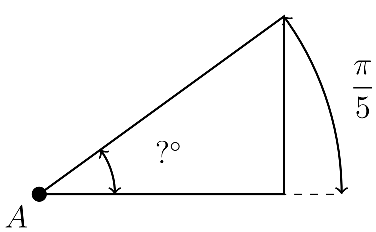


13. Convert units of *radians* and *degrees* ($2\pi = 360°$, $\pi = 180°$).

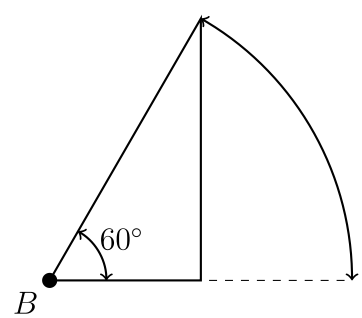
    Apply the appropriate formula.

    $d = r \times \dfrac{180}{\pi}$ $\qquad$ $r = d \times \dfrac{\pi}{180}$

    (a) $m\angle A = \dfrac{\pi}{5} = ?$ degrees

    (b) $m\angle B = 60° = ?$ radians
    (in terms of π)

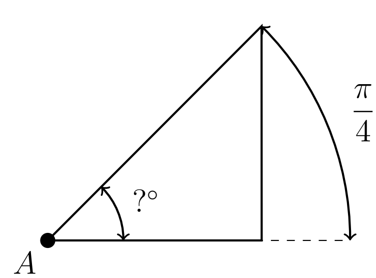
    

14. Do Now: Convert units of *radians* and *degrees* ($2\pi = 360°$, $\pi = 180°$).

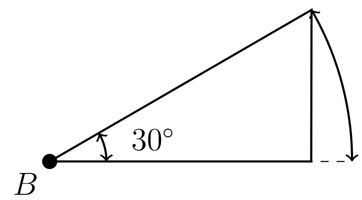
    Apply the appropriate formula.

    $d = r \times \dfrac{180}{\pi}$ $\qquad$ $r = d \times \dfrac{\pi}{180}$

    (a) $m\angle A = \dfrac{\pi}{4} = ?$ degrees

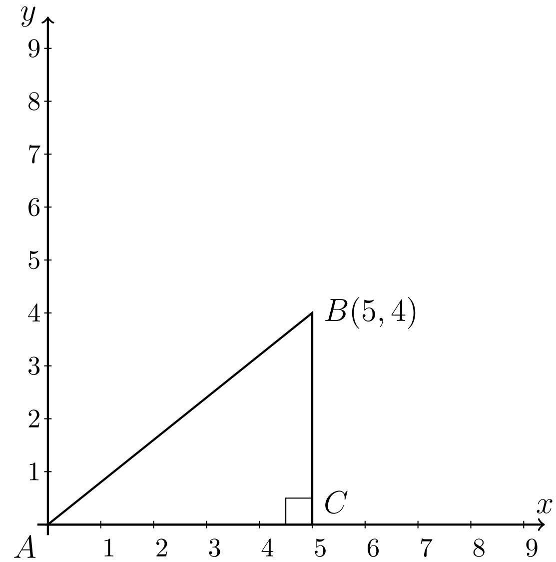
(b) $m\angle B = 30^\circ = ?$ radians
(in terms of π)



15. Right $\triangle ABC$ is drawn in *standard position* with vertex A on the origin and right $\angle C$ on the x-axis, as shown.

   (a) Find the length of the hypotenuse AB using the Pythagorean Theorem $a^2 + b^2 = c^2$. (leave as a radical)

   (b) Find the slope of the line segment $\overline{AB}$ as a decimal.



16. Right $\triangle ABC$ is drawn in *standard position* with vertex A on the origin and right $\angle C$ on the x-axis, as shown.

   (a) Find the length of the hypotenuse AB using the Pythagorean Theorem $a^2 + b^2 = c^2$. (leave as a radical)

   (b) Find the slope of the line segment $\overline{AB}$ as a decimal.