Geometry Unit 2: Angles

Bronx Early College Academy

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28 September - 7 October 2022

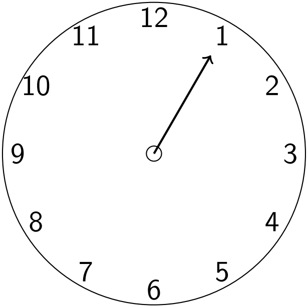
2.1 Angle notation, measures	28 September
2.2 Angle addition, angle pairs	29 September
2.3 Vertical angles	30 September
2.4 Angle bisectors	3 October
2.5 Equilateral, isosceles $\triangle$ angles	4 October
2.6 Review	6 October
2.7 Unit 2 test: Angle measures	7 October

Open Middle: complementary and supplementary puzzle

Learning Target: I can measure angles

CCSS: HSG.CO.A.1 Know precise geometric definitions 2.1 Wednesday 28 Sept

Do Now: Which takes longer, for a clock's hour hand to go from the 1 to the 4 or the 5 to the 9?



Lesson: Angle measures, internal, external, acute, obtuse, right

Learning Target: I can measure angles

CCSS: HSG.CO.A.1 Know precise geometric definitions 2.1 Wednesday 28 Sept

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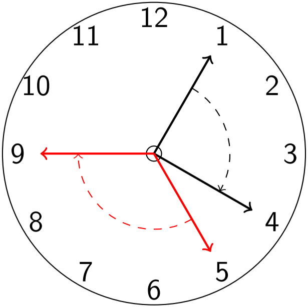
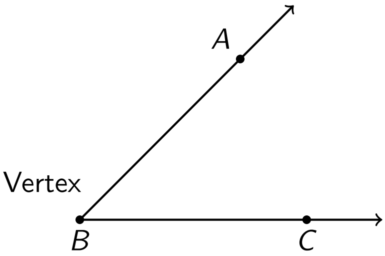


Lesson: Angle measures, internal, external, acute, obtuse, right

Two rays with a common endpoint make an *angle*

Rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$, *vertex* B.



Angle	Two rays with a common endpoint, $\angle ABC$ or $\angle B$
Vertex	The common end point of two rays making an angle
Interior	Inside, the area between the two rays
Exterior	Outside, the area in the angle interior

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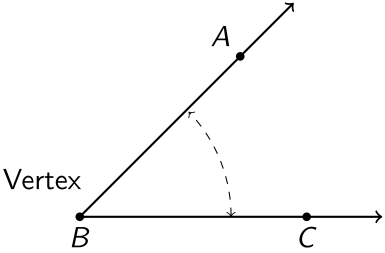
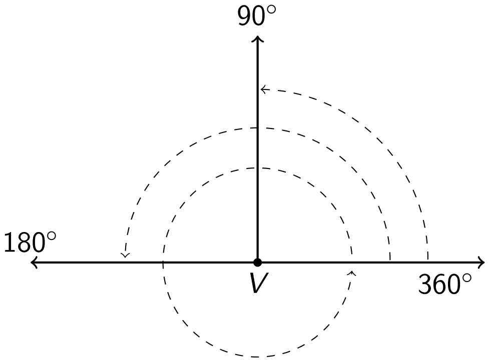
Rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$, *vertex* B.



Angle	Two rays with a common endpoint, $\angle ABC$ or $\angle B$
Vertex	The common end point of two rays making an angle
Interior	Inside, the area between the two rays
Exterior	Outside, the area in the angle interior
$m\angle A$	The "measure" of angle A, how big it is

Babylonian measures: 360° in a circle



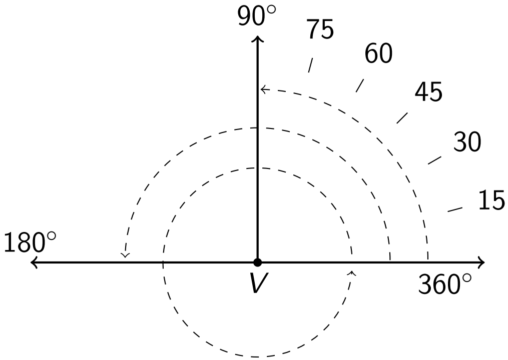
Full turn A complete rotation, 360°

Half turn A straight line, 180°

Quarter turn A *right* angle, 90°

Protractor A tool for measuring angles

Babylonian measures: 360° in a circle



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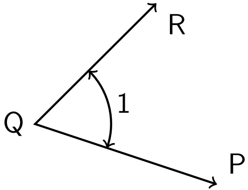
Protractor A tool for measuring angles

Angle terminology and notation

Write definitions in your notebook



Angle Q, written $\angle Q$ (also $\angle PQR$, $\angle 1$)

Point Q is the *vertex*

The sides or *legs* are $\overrightarrow{QR}$, $\overrightarrow{QP}$

Right angle measuring 90°, mark as small square 

Perpendicular lines meet at right angles. $\overline{AB} \perp \overline{CD}$

Acute angles measure $< 90^\circ$

Obtuse angles are $90^\circ < m\angle < 180^\circ$

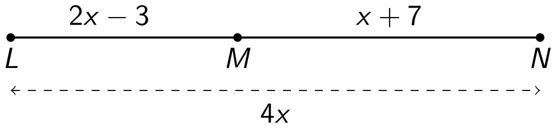
Straight angle or a straight line measures 180°

Reflex angles measure $180^\circ < m\angle < 360^\circ$

Learning Target: I can solve for angle measures

CCSS: HSG.CO.A.1 Know precise geometric definitions 2.2 Thursday 29 Sept

Do Now: Given $\overline{LMN}$, $LM = 2x - 3$, $MN = x + 7$, $LN = 4x$.
Find x. *Don't forget to check the solution.*

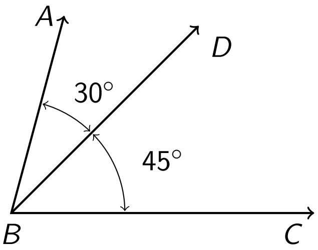


Name the geometry *postulate* that is the basis for this problem.

Lesson: Angle addition postulate, complementary, supplementary angles, linear pairs

Angle addition postulate

m$\angle ABD = 30^\circ$, m$\angle DBC = 45^\circ$. Find m$\angle ABC$.

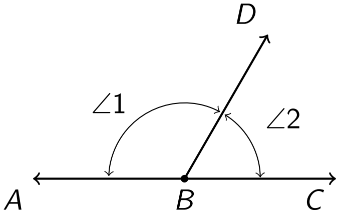


Angle addition: The sum of the measures of *adjacent* angles is the measure of their combined angle. (postulate)

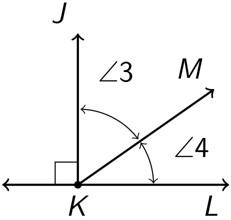
$$m\angle ABD + m\angle DBC = m\angle ABC$$

Adjacent: "next to" each other. Adjacent angles share a common ray and are external to each other.

Special angle pairs



Linear pair, supplementary ∠s



Complementary angles

Linear pair Two adjacent angles that make a straight line

Opposite rays collinear with a common endpoint. e.g. $\overrightarrow{BA}$, $\overrightarrow{BC}$

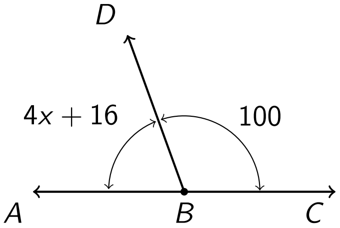
Supplementary Angles whose measures sum to 180°

Complementary Angles whose measures sum to 90°

Adjacent "next to" each other. Adjacent angles share a common ray and are external to each other.

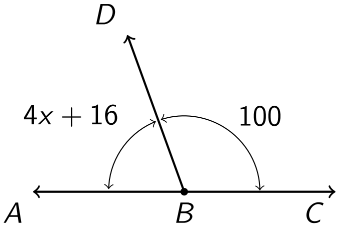
Given two supplementary angles, a linear pair.

m$\angle ABD = 4x + 16$, m$\angle CBD = 100$. Find x.



Given two supplementary angles, a linear pair.

$m\angle ABD = 4x + 16$, $m\angle CBD = 100$. Find x.

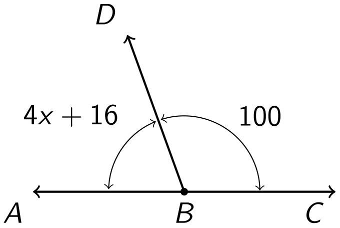


Solution:

$$m\angle ABD + m\angle CBD = 180$$

Given two supplementary angles, a linear pair.

$m\angle ABD = 4x + 16$, $m\angle CBD = 100$. Find x.



Solution:

$$m\angle ABD + m\angle CBD = 180$$

$$(4x + 16) + 100 = 180$$

$$\cdots$$

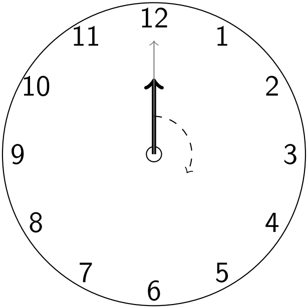
$$x = 16$$

Check:

$$[4(16) + 16] + 100 = 180 \checkmark$$

Extension (optional problems)

At midnight both the clock's minute hand and hour hand point in the same direction. When is the next time the clock hands coincide?

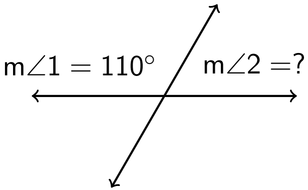


Learning Target: I can identify vertical angles

CCSS: HSG.CO.A.1 Know precise geometric definitions 2.3 Friday 30 September

Do Now: Check your knowledge of angle pairs

1. *Complementary* angles sum to how many degrees?
2. *Supplementary* angles sum to how many degrees?
3. Given complementary angles $m\angle A = 30^\circ$. Find $m\angle B$.
4. Given intersecting lines. $m\angle 1 = 110^\circ$. Find $m\angle 2$.

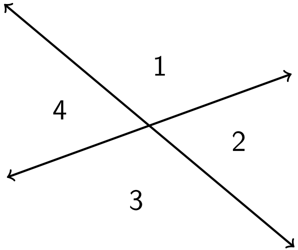


Lesson: Vertical angles

Intersecting lines make two pairs of congruent angles

Angles *opposite* each other match:

$$\angle 1 \cong \angle 3,\ \angle 2 \cong \angle 4$$

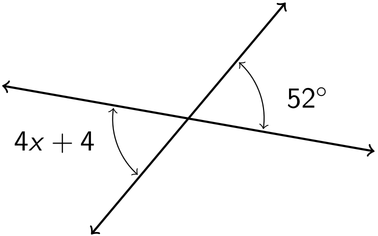


Vertical angles Opposite each other when two lines intersect. $\angle 1$ and $\angle 3$ are vertical angles, as are $\angle 2$ and $\angle 4$.

Opposite Across from each other. (opposite angles and vertical angles means the same thing)

Use vertical angles to solve for x

Given vertical angles measuring $4x + 4$ and 52°. Find x.



Use vertical angles to solve for x

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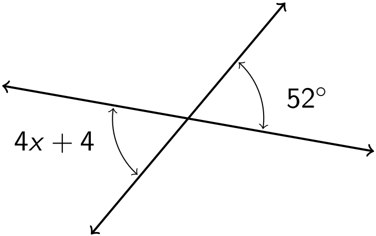


Solution:

$$4x + 4 = 52$$

$$x = 12$$

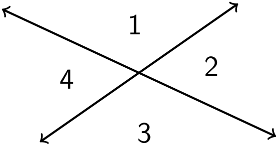
Check:

$$4(12) + 4 = 52 \checkmark$$

Extension: Use logic to show vertical angles are congruent

Given intersecting lines making angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$.

Prove $\angle 2 \cong \angle 4$.



Linear pairs are supplementary

$$m\angle 2 + m\angle 1 = 180$$

$$m\angle 4 + m\angle 1 = 180$$

Both equal 180, so they are equal (*transitive property* of equality)

$$m\angle 2 + m\angle 1 = m\angle 4 + m\angle 1$$

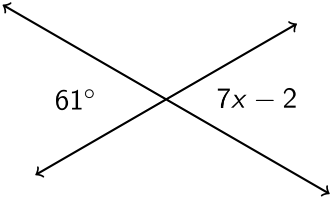
Subtract $m\angle 1$ from both sides (*cancellation* law) link

$$\angle 2 \cong \angle 4 \quad Q.E.D.$$

Learning Target: I can bisect angles

CCSS: HSG.CO.A.1 Know precise geometric definitions 2.4 Monday 3 October

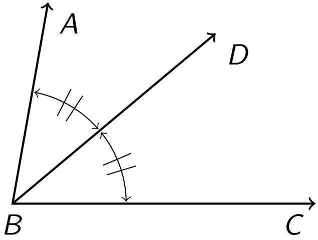
Do Now: Given vertical angles measuring $7x - 2$ and 61°. Find x.



Lesson: Angle bisector situations

Bisect an angle by dividing it exactly in half

$\overrightarrow{BD}$ bisects $\angle ABC$ if and only if $\angle ABD \cong \angle CBD$.

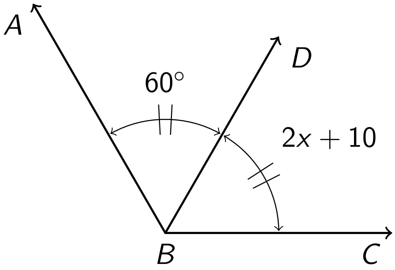


Angle bisector ray dividing an angle into two congruent angles

Hash marks mark congruent angles

Model angle situations with algebra, then solve

Given angle bisector $\overrightarrow{BD}$ with $m\angle ABD = 60^\circ$ and $m\angle CBD = 2x + 10$. Find x.



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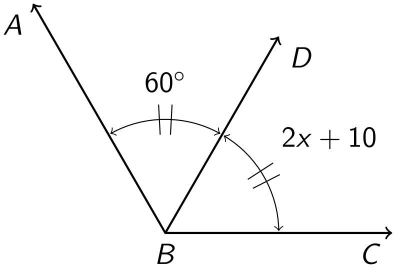


Solution:

$$\angle ABD \cong \angle CBD$$

$$2x + 10 = 60$$

$$2x = 50$$

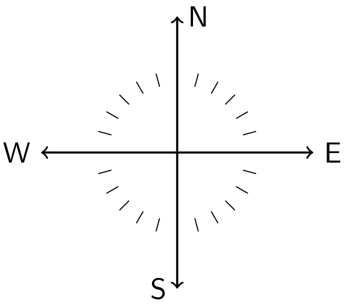
$$x = 25$$

Check:
$2(25) + 10 = 60?$ ✓

Extension: Use angles for compass directions

North South East West, points of the compass

Directions are measured relative to North



Bearing The direction as an angle *clockwise* from north

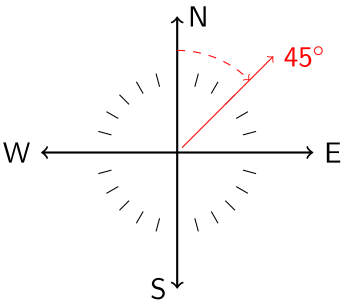
Clockwise The direction the clocks turn, “to the right” (tighten)

Counterclockwise Opposite of clocks, “to the left” (loosen)

Extension: Use angles for compass directions

North South East West, points of the compass

Directions are measured relative to North



"Northeast," half way between north and east, i.e. bearing 45°

north is 0°
east is 90°
south is 180°
west is 270°

Bearing The direction as an angle *clockwise* from north

Clockwise The direction the clocks turn, "to the right" (tighten)

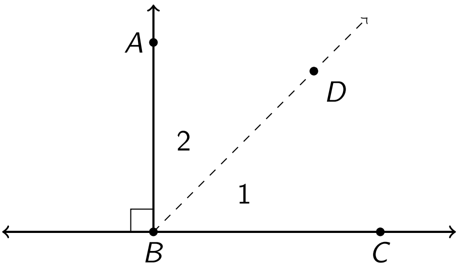
Counterclockwise Opposite of clocks, "to the left" (loosen)

LT: I can work with equilateral and isosceles-right △s

CCSS: HSG.CO.A.1 Know precise geometric definitions 2.5 Tuesday 4 October

Do Now: Given perpendiculars $\overrightarrow{AB} \perp \overleftrightarrow{BC}$, and that the ray $\overrightarrow{BD}$ bisects $\angle ABC$, making two angles, $\angle 1$ and $\angle 2$.

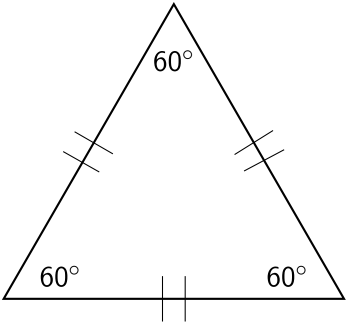
Find the measures of $\angle 1$, $\angle 2$.



Lesson: Isosceles base theorem, special triangles

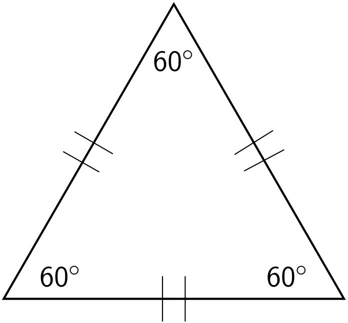
60° - 60° - 60°, 30° - 60° - 90°, 45° - 45° - 90°

Equilateral △, special relationships and measures

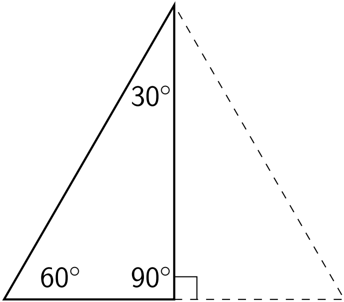


60° - 60° - 60°

Equilateral △, special relationships and measures



60° - 60° - 60°



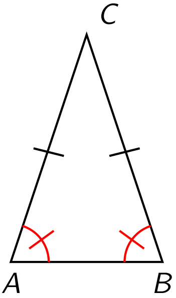
30° - 60° - 90°

Equiangular means having equal angles

Equilateral having equal sides

The *base* angles of an isosceles triangle are congruent

Isosceles base theorem: If $\overline{AC} \cong \overline{BC}$ then $\angle A \cong \angle B$

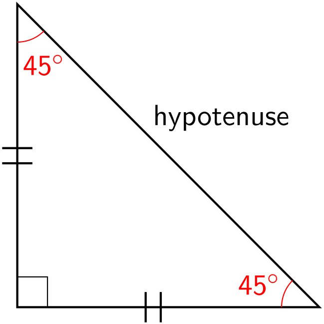


Base angles $\angle$s opposite the congruent sides in an isosceles △

Included angle The angle between two given sides of a triangle ($\angle C$ is included between $\overline{AC}$ and $\overline{BC}$)

Theorem Something we can prove using logic

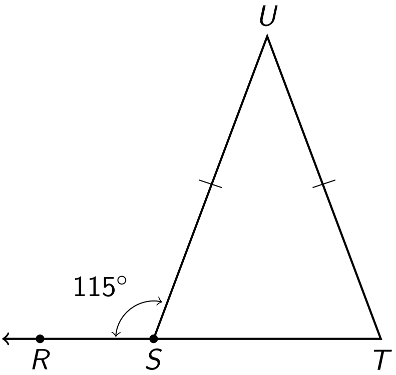
Isosceles-right triangles' angles measure 45° - 45° - 90°



Hypotenuse the longest side of a right triangle, opposite the 90° angle

Multiple step problem: apply your knowledge

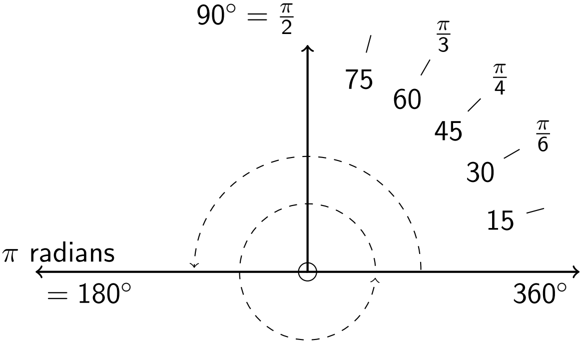
Given isosceles triangle with $\overline{SU} \cong \overline{TU}$, $m\angle RSU = 115^\circ$.



1. Find $m\angle TSU$
2. Find $m\angle T$

Extension: Radian units for angle measures

Mathematicians use radians because calculations are simpler



Convert *units*: $360^\circ = 2\pi$ radians:

Degree One 360th of a full turn

Radian A full circle is 2π radians. 1 radian $\approx 57^\circ$

Gradian One 400th of a full turn

LT: I can review length and angle measures with peers

CCSS: HSG.CO.A.1 Know precise geometric definitions 2.6 Thursday 6 October

Angle concepts and theorems you have learned

1. Angle addition situations
2. Angle pairs
   2.1 $\perp$ lines and complementary angles make 90°
   2.2 Vertical $\angle$s are $\cong$
3. Angle bisectors
4. Isosceles base angle theorem, special triangles

Learning Target: I can quantify angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.7 Friday 7 October

Unit test

Open Middle problem (fun)

Use digits from 0 to 9. Using a digit no more than once.

The first two angle measures are complementary. The second two angles supplementary. (degrees)