Quantifying Uncertainty: Probability Name:
29 February 2024

1.22 PreExam: Probability, Venn diagrams

1. Given:
   $U = \{\text{the letters in the alphabet}\}$ $A = \{t, i, m, e, s\}$ $B = \{m, i, n, u, s\}$

   (a) List the members of $A \cup B$. [1 mark]

   (b) List the elements of $A \cap B$. [1 mark]

   (c) A letter is selected at random. What is the probability that it is a member of both sets, $(A \cap B)$? [1 mark]

2. Events A and B are independent with $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.5$. Find each probability.

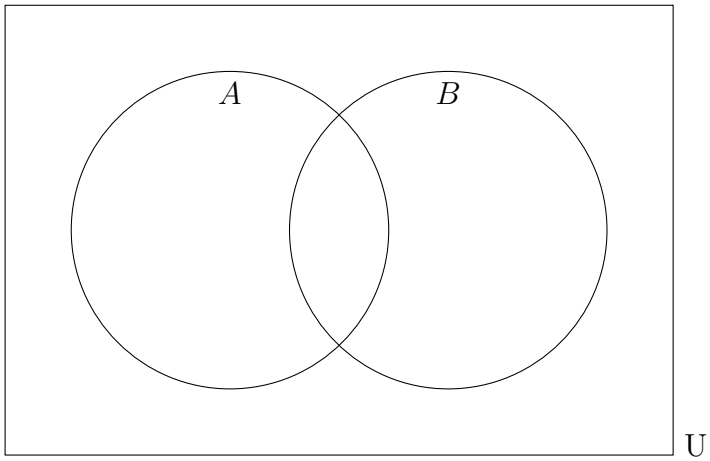
   (a) $\mathrm{P}(A \cap B)$ [2 mark]

   (b) $\mathrm{P}(A \cup B)$ [2 mark]

   (c) $\mathrm{P}(B' \cap A)$ [2 mark]

   (d) $\mathrm{P}(A|B)$ [2 mark]

   (e) Mark the Venn diagram with the probabilities for each area. [2 marks]



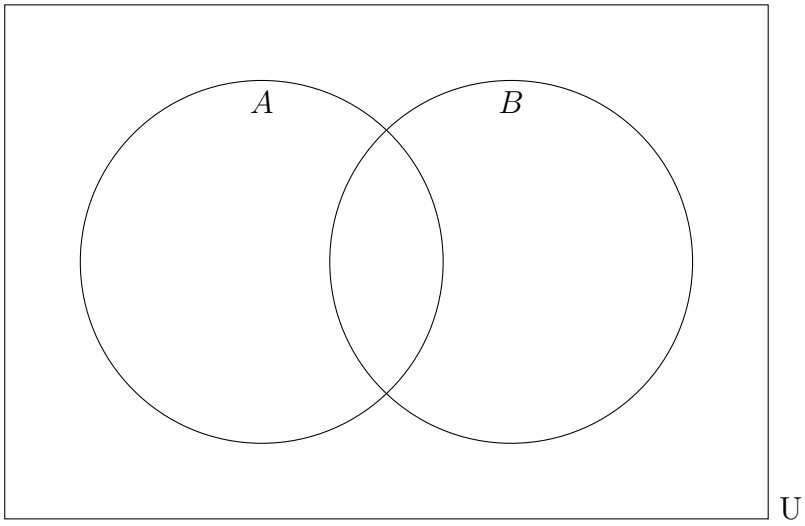
3. The universal set U is defined as the set of positive integers less than 13.

   (a) Subset is defined as $A = \{\text{multiples of three}\}$. List its elements. [1 mark]

   (b) Subset $B = \{\text{perfect squares}\}$. List the members of set B. [1 mark]

   (c) List the members of $(A \cup B)'$. [1 mark]

   (d) Place the elements of U in the appropriate regions in the Venn diagram. [2 marks]



   (e) If an element is selected at random, what is the probability that it is a member of both sets, $(A \cap B)$? [1 mark]

   (f) If a member of set B is selected at random, what is the probability that it is also a member of set A, i.e. the conditional probability $(A|B)$? [2 mark]

4. A jar contains 30 marbles, 12 of which are red, 8 are blue, and 10 are green.
   (a) A marble is selected at random. Find the probability it is red. [1 mark]

   (b) The marble is replaced and a second marble is selected. Given that the second marble is not red, find the probability it is blue. [1 mark]

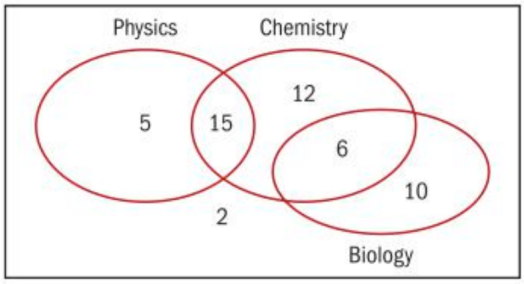
   (c) The marbles are returned to the jar and two marbles are selected at random. Find the probability that both are green. [2 mark]

5. Draw a tree diagram to represent the taxi cab problem in the textbook. First, there are two cab companies, 85% are black and the rest are yellow. Then, the witness identifies the color of the cab correctly 80% of the time. [3 marks]
   (a) Label the branches with the probabilities. [1 marks]

   (b) Calculate the probabilities of each four outcomes. [2 marks]

   (c) Given that the witness identified the cab as yellow, find the probability that it was black, i.e. that she was wrong. [3 marks]

6. : The Venn diagram illustrates the number of students taking each of the three sciences: physics, chemistry and biology.



A student is randomly chosen from the group.

Find the probability that

a the student studies chemistry or biology (2 marks)

b the student studies neither physics nor biology (2 marks)

c the student studies physics, given that they study chemistry (2 marks)

d the student studies biology, given that they study physics (2 marks)

e the student studies physics, given that they do not study biology. (2 marks)

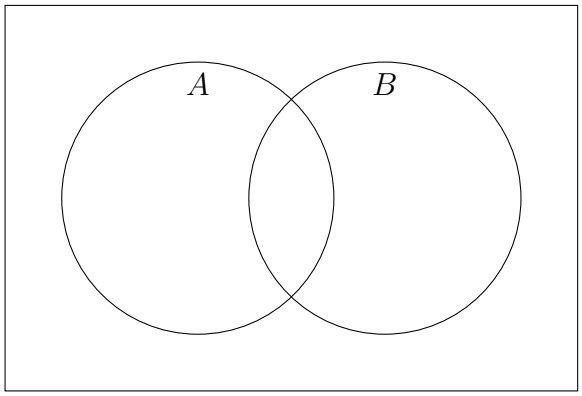
7. The events A and B are mutually exclusive with $\mathrm{P}(A) = 0.30$ and $\mathrm{P}(B) = 0.15$.

(a) Write down $\mathrm{P}(A \cap B)$. [1 mark]

(b) Write down $\mathrm{P}(A \cup B)$. [1 mark]

8. Given events A and B with $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.6$, $\mathrm{P}(A \cap B) = 0.35$.

   (a) Completely mark the Venn diagram with probabilities for each area. [2 marks]

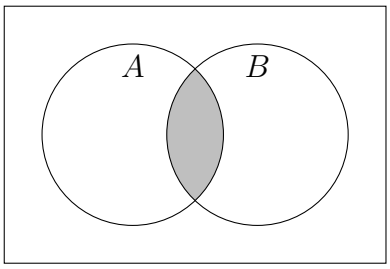


   (b) Find $\mathrm{P}(A \cup B)$. [2 marks]

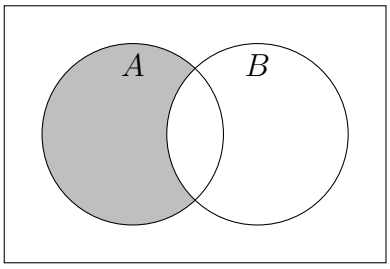
   (c) State whether events A and B are independent. Justify your answer. [3 marks]

9. For each Venn diagram, write an expression representing the shaded area. [5 marks]

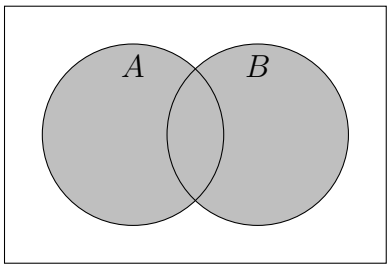
(a)



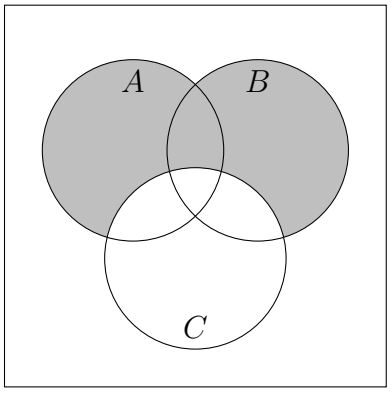
(b)



(c)



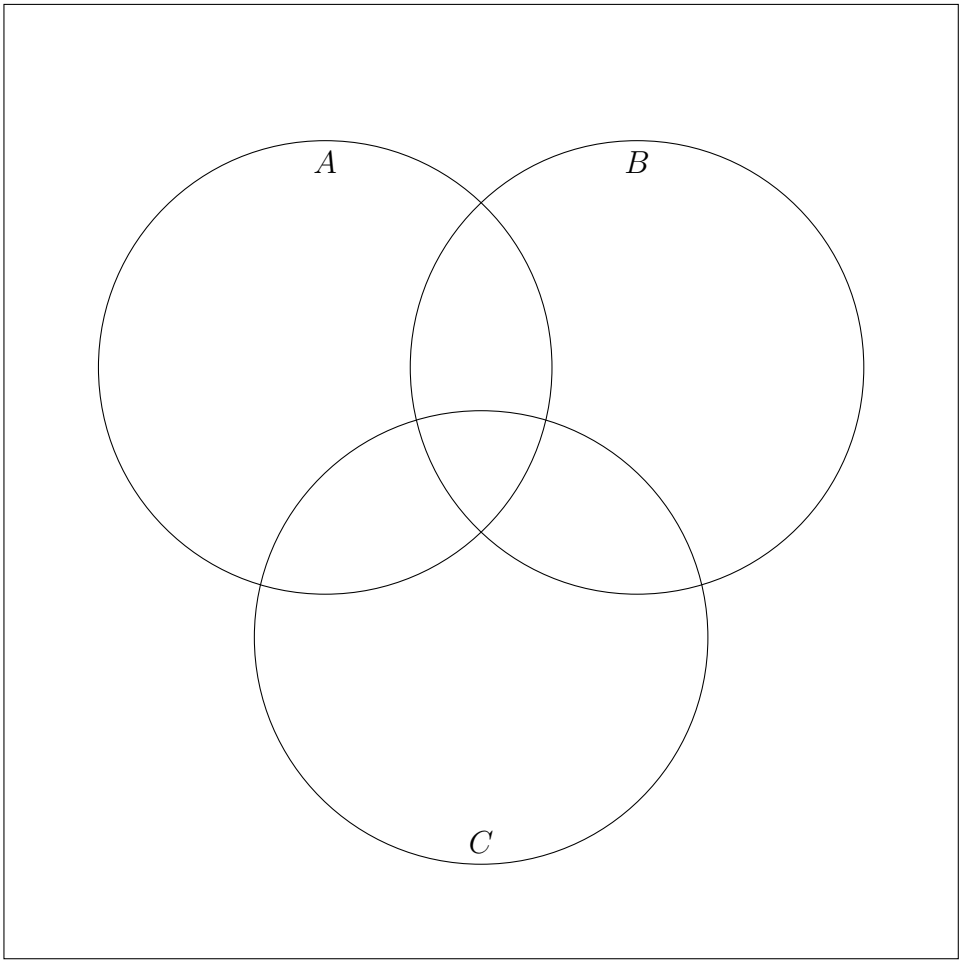
(d)



10. There are 60 students enrolled in the following courses:

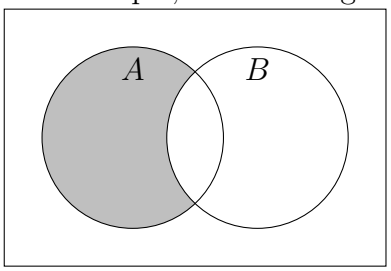
    - 28 take Archery
    - 30 take Biology
    - 22 take Calculus
    - 8 take Archery and Biology
    - 7 take Archery and Calculus
    - 10 take Biology and Calculus
    - 5 take all three classes

    Complete the Venn diagram below with the number of students in each region to represent the situation. [4 marks]



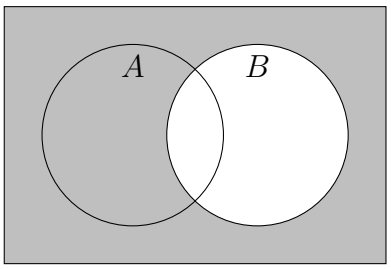
11. For each Venn diagram, write an expression representing the shaded area.

(a) For example, for this diagram



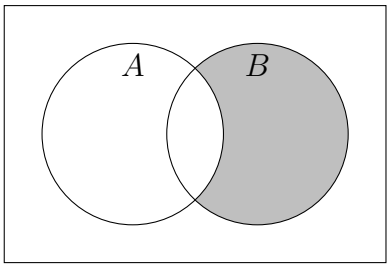
Expression: $A \cap B'$

(b)



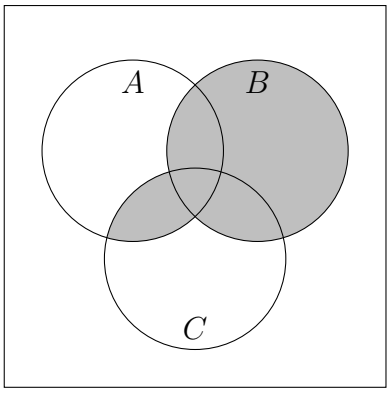
Expression:

(c)



Expression:

(d)



Expression:

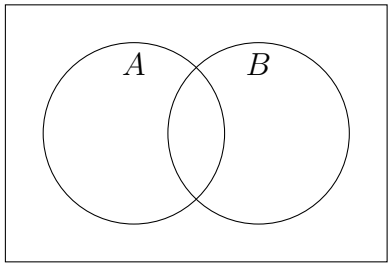
12. Given:
$U = \{\text{the letters in the alphabet}\}$
$A = \{a, b, c, d, e, f, g, h, i, j\}$ $B = \{h, i, j, k, l, m, n, o, p, q\}$

(a) What is $A \cap B$?

(b) What is $(A \cup B)'$?

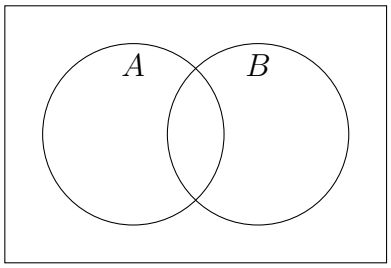
13. For each Venn diagram, shade the area representing the expression. Use pencil.

(a) $A \cup B$



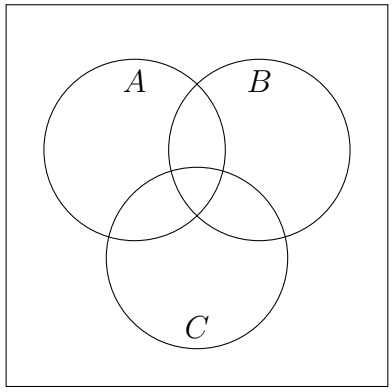
[2 marks]

(b) $A' \cap B$



[2 marks]

(c) $(A \cap B) \cup C$



[2 marks]

14. Forty IB high school students range in age from 15 to 18 years old. The following table shows the frequencies of each age.

Age (years)	15	16	17	18
Frequency	5	k	15	7

(a) Calculate the value of k. [1 mark]

(b) Write down the mode. [1 mark]

(c) Find the value of the range. [1 marks]

(d) Find the median. [1 marks]

(e) Find the mean. [2 marks]

(f) Find the standard deviation. [2 marks]

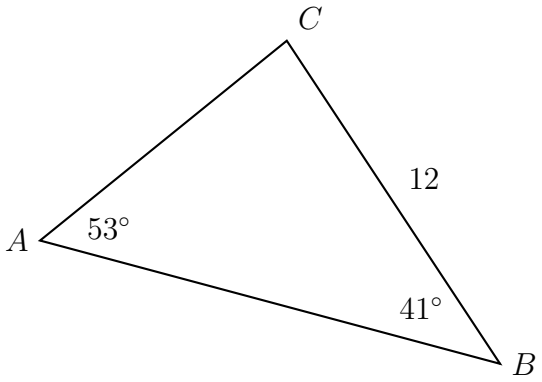
15. A runner records her pace in terms of distance run (d) in miles over time (t) in minutes during a 4.5 mile run. She models her pace with a linear regression equation $d = at + b$.

minutes (t)	0	8	15	22	30
miles (d)	0	1.8	2.7	3.7	4.5

(a) Find the values of a, b, and the correlation r. [3 marks]

(b) Explain what the value of a represents in the context of the situation. [2 marks]

16. The following diagram shows triangle ABC (not drawn to scale).



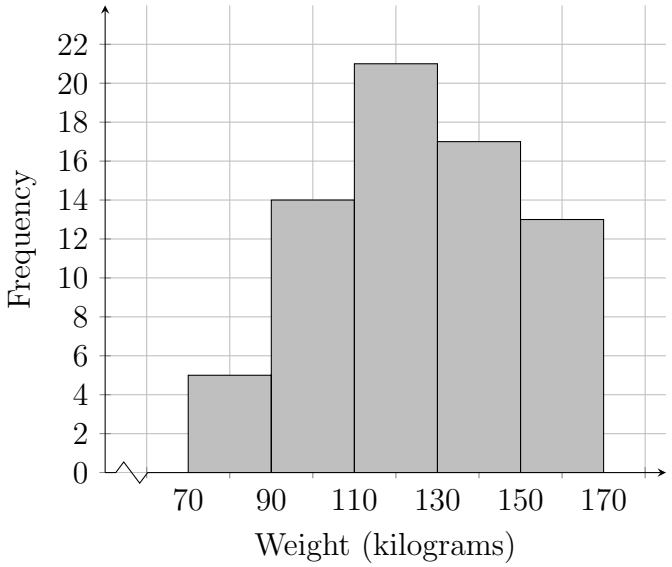
$BC = 12$, $C\hat{A}B = 53°$, and $A\hat{B}C = 41°$

(a) Find the measure of $A\hat{C}B$. [1 mark]

(b) Find AC. [3 marks]

(c) Find the area of triangle ABC. [3 marks]

17. The histogram below shows the weight w in kilograms for 70 professional football players.



The following is the frequency table for the distribution of w.

HR (x)	$70 \leq x < 90$	$90 \leq x < 110$	$110 \leq x < 130$	$130 \leq x < 150$	$150 \leq x < 170$
Freq	5	14	21	p	13

(a) Write down the value of p. [1 mark]

(b) Write down the modal class. [2 marks]

(c) A player is selected at random. Find the probability that the athlete weighs less than 110 kilograms. [2 marks]

(d) Write down the mid-interval value for the class $110 \leq x < 130$. [1 mark]

(e) Hence find an estimate for the

i. mean; [2 marks]

ii. standard deviation. [2 marks]

18. A survey question has three possible responses: A, B, and C. Among 100 surveys, the frequency of the answers collected were as follows: $n(A) = 10, n(B) = 35,$ and $n(C) = 55$.

    (a) If a survey is selected at random, what this the probability the response was B or C?

    (b) What is the probability a survey selected at random was an answer other than B or C?