Solutions

Sequences and Functions: End-of-Unit Assessment

You may use a scientific calculator.

+4

1. Which formula defines the sequence $f(1) = 2$, $f(2) = 6$, $f(3) = 10$, $f(4) = 14$, $f(5) = 18$?

   A. $f(1) = 2$, $f(n) = 6 + f(n-1)$ for $n \geq 2$

   B. $f(1) = 2$, $f(n) = 4 + f(n-1)$ for $n \geq 2$

   C. $f(1) = 2$, $f(n) = 2 + f(n-1)$ for $n \geq 2$

   D. $f(1) = 6$, $f(n) = 4 + f(n-1)$ for $n \geq 2$

2. A sequence is defined by $f(1) = 3$ and $f(n) = 2 \cdot f(n-1)$ for $n \geq 2$. Which of the following statements defines the n^{th} term of f?

   A. $f(n) = 3 + 2(n-1)$ for n ≥ 1

   B. $f(n) = 3 + 2n$ for n ≥ 1

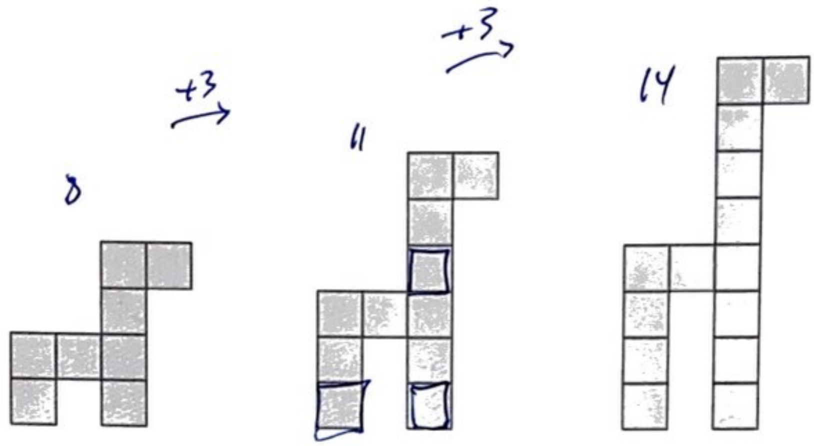
   C. $f(n) = 3 \cdot 2^{n-1}$ for n ≥ 1

   D. $f(n) = 3 \cdot 2^{n}$ for n ≥ 1

n	$f(n)$
1	3
2	6
3	12

3. Here is a growing pattern of squares:

Next page



Step 1 Step 2 Step 3

Select **all** the expressions that represent the number of squares in Step n.

A. $f(n) = 8 + 3(n-1)$ for $n \geq 1$

B. $f(n) = 3 + 8(n-1)$ for $n \geq 1$

C. $f(1) = 8$, $f(n) = 3 + f(n-1)$ for $n \geq 2$

D. $f(1) = 8$, $f(n) = 8 + f(n-1)$ for $n \geq 2$

E. $f(n) = 3 + 8n$ for $n \geq 1$

F. $f(n) = 3n + 5$ for $n \geq 1$

4. Here are some values of sequence Q. Write a recursive definition for the sequence.

$d = \frac{5}{2} = 2\frac{1}{2}$

n	$Q(n)$
1	3
3	8
7	18

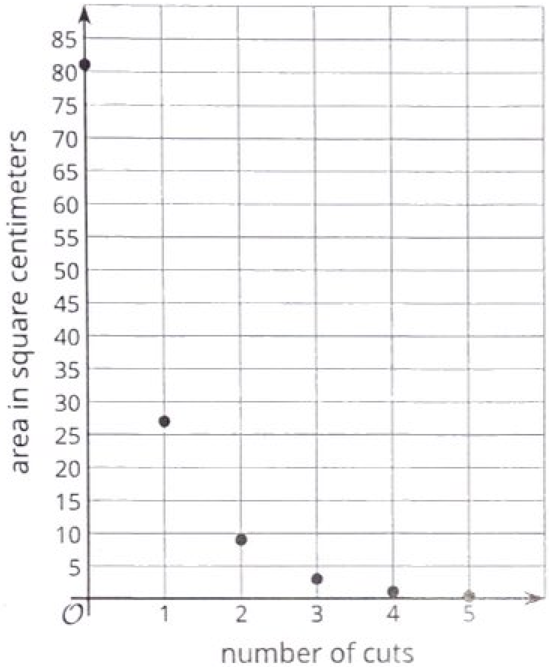
+2, +4   +5, +10

$f(1) = 3$

$f(n) = f(n-1) + 2\frac{1}{2}$

5. A piece of paper has an area of 81 cm^2. A strip is cut off that is $\frac{1}{3}$ the original area. From that strip, another strip is cut off that is $\frac{1}{3}$ the area of the first, and so on.

Here is a graph and table representing sequence k, where $k(n)$ is the area in square centimeters of the strip of paper after n cuts.



number of cuts	area in square centimeters
0	81
1	27
2	9
3	3
4	1

$\cdot\frac{1}{3}$

$\cdot\frac{1}{3}$

a. Is sequence k geometric or arithmetic? Explain how you know.

Geometric. The terms are multiplied by $\frac{1}{3}$.

b. Write an equation to define sequence k recursively.

$f(0) = 81$ $\qquad$ $k(0) = 81$

$f(n) = \frac{1}{3}f(n-1)$ $\qquad$ $k(n) = \frac{1}{3}k(n-1)$ $n \geq 1$

c. For term $k(n)$, what are some values of n that make sense to use? What are some values of n that don't make sense to use? Explain your reasoning.

$n = 0, 1, 5, 10$ makes sense. Not $\frac{1}{3}, -5, \pi$.

n counts the terms of the sequence and must be a whole number. Also, practically at a certain point the paper will be too small to cut.

6. The first two numbers in a sequence h are $h(1) = 2$ and $h(2) = 6$.

a. If h is an arithmetic sequence, write a definition for the n^{th} term of h. Explain or show your reasoning.

$d = 6 - 2 = 4$

$h(n) = 2 + 4(n-1)$

THE FIRST term is 2. FOR EACH n greater than 1, 4 is added.

b. If h is a geometric sequence, write a definition for the n^{th} term of h. Explain or show your reasoning.

$r = \frac{6}{2} = 3$

$h(n) = 2 \cdot 3^{(n-1)}$

The first term is 2 and for each n after n=1, another 3 is multiplied.

7. Here are two sequences:

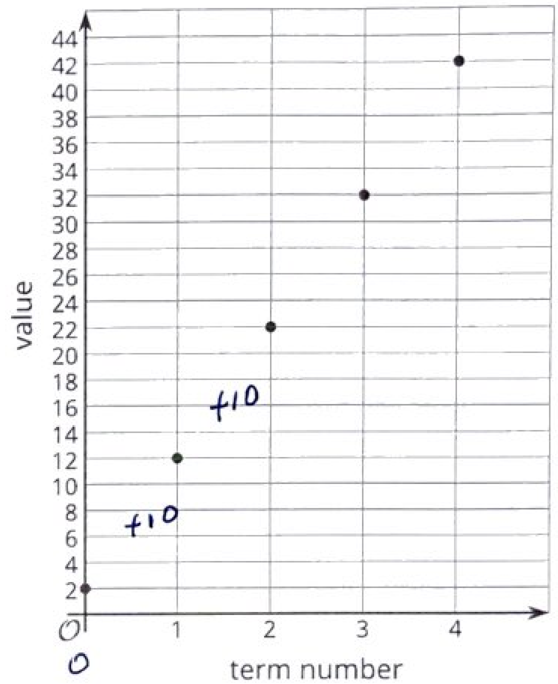
Sequence A

term number	value
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	1
3	2
4	4

$*2$

$*2$

Sequence B



a. For sequence A, describe a way to produce each new term from the previous term.

multiply by two

b. For sequence B, describe a way to produce each new term from the previous term.

add 10

c. Write a definition for the n^{th} term of sequence A.

$A(n) = \frac{1}{4} \cdot 2^n$

d. Write a definition for the n^{th} term of sequence B.

$B(n) = 2 + 10n$

e. If these sequences continue, then which is greater, $A(9)$ or $B(9)$? Explain or show how you know.

$A(9) = 128 > B(9) = 92$

A is geometric so it will eventually be greater. (geometric always increase more) $A(9) = 128 > B(9) = 92$