2.5 Review Compound Interest — Solutions

Daniel

2\. (a) $P_2 = 30{,}000(1+2r) = 31{,}650$

$r = 2.75\%$

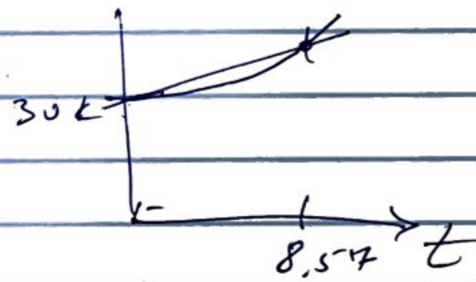
Rebecca

(b) $P_2' = 30{,}000(1+0.025)^2$

$= 31{,}518.80$ Ans

(c) $30{,}000(1+0.025)^t > 30{,}000\cancel{(1+2r)^t}(1+0.0275t)$

9 years



(d) i) $31{,}650 \cdot 0.80 = 25{,}320$ Ans

ii) $P_t = 25{,}320\,\cancel{1.03}\left(1+\frac{0.03}{4}\right)^{4t} > 30{,}000$

$$\left[\left(1+\frac{0.03}{4}\right)^4\right]^t > \frac{30{,}000}{25{,}320} = 1.18483\ldots$$

$$(1.03034)^t > 1.18483$$

$$t > \log_{1.03034} 1.18483 = 5.67448\ldots$$

6 years

6. [Maximum mark: 6]

In a geometric sequence, the fourth term is 8 times the first term. The sum of the first 10 terms is 2557.5. Find the 10th term of this sequence.

$u_4 = u_1 r^{(4-1)} = 8u_1$

$r^3 = 8$

$r = 2$

$S_{10} = u_1\left(\frac{2^{10}-1}{2-1}\right) = 2557.5$

$u_1 = 2.5$

$u_{10} = \cancel{2.5}\ 2.5 \cdot 2^{(10-1)}$

$= 1280$

7. [Maximum mark: 8]

Note: One decade is 10 years

A population of rare birds, P_t, can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$.

(a) (i) Find the value of k.

(ii) Interpret the meaning of the value of k. [3]

(b) Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.75$. [5]

a) i) $\frac{P_1}{P_0} = e^{1K} = 0.9$

$K = \ln 0.9 = -0.105361...$

≈ -0.105

ii) The population is decreasing by about 10% per decade

(b) $\frac{P_t}{P_0} = e^{-0.105361t} < 0.75$

$0.9^t < 0.75$

$t = \log_{0.9} 0.75 = 2.73045...$

3 years

2.5 Review

Solutions

#8 Price of a used car

(a)i) $r = -0.9943$

(a)ii) $a = -1.58096\ldots \approx -1.58$

$b = \cancel{0.988728\ldots \approx 0.989}$

$33,480.30$

(b) $y = -1.58(11,000) + 33,480.30$

$= 16,089.70$

$\approx \$16,100$

(c) $FV = 16,100(1-0.05)^6$

$= \$11,835$

(d) $FV = 16,100(0.95)^t = 10,000$

$t = \log_{0.95}\left(\frac{10,000}{16,100}\right) = 9.28453\ldots$

2019

2.5 Review

Solutions

#3 Part A

Geometric sequence

(a) $u_4 = 1024\, r^{(4-1)} = 128$

$r^3 = \frac{1}{8}$

$r = 1/2$

(b) $u_{11} = 1024\left(\frac{1}{2}\right)^{(11-1)} = 1$

(c) $S_8 = 1024\left(\frac{1-\left(\frac{1}{2}\right)^8}{1-1/2}\right) = 2040$

(d) $S_n = 1024\left(\frac{1-(1/2)^n}{1-\frac{1}{2}}\right) > 2047.968$

$\left(\frac{1}{2}\right)^n < \left(\frac{2047.968}{1024}\right)\left(\frac{1}{2}\right) - 1 = \cancel{0.954168...}$ 0.000016

$n > \log_{1/2} \cancel{0.954168...}\ 0.000016 = 15.9658$

16

Part B

(a) $u_{11} = 1 + 3(11-1) = 31$

(b) i) $S_{100} = \frac{100}{2}(3(100)-1) = 14,950$

ii) $S_n = \frac{n}{2}(3n-1) = 477$

$3n^2 - n = 954$

$3n^2 - n - 954 = 0$

$n = 18$ polyRoots$(3n^2-n-954=0, n)$

$\left\{-\frac{53}{3}, 18\right\}$

2.5 Review SOLUTIONS

5. The Mass M of a decaying substance

(a) strong negative

(b) $\ln(a \times b^t) = -0.12t + 4.67$

$\ln a + \ln b^t = -0.12t + 4.67$

$\ln a + t \ln b = -0.12t + 4.67$ since a is constant $\ln a = 4.67$

$\ln b = -0.12$

$b = e^{-0.12}$

$= 0.88692\ldots$

≈ 0.887