Solutions

Lesson 8 Practice Problems

1. A pattern of dots grows exponentially. The table shows the number of dots at each step of the pattern.

step number	0	1	2	3
number of dots	1	5	25	125

   a. Write an equation to represent the relationship between the step number, n, and the number of dots, y.

$$y = 5^n$$

   b. At one step, there are 9,765,625 dots in the pattern. At what step number will that happen? Explain how you know.

$$y = 5^n = 9,765,625$$

$$n = 10$$

n	y
9	
10	9,765,625

Calculator table function

2. A bacteria population is modeled by the equation $p(h) = 10{,}000 \cdot 2^h$, where h is the number of hours since the population was measured.

   About how long will it take for the population to reach 100,000? Explain your reasoning.

$$p(h) = 10{,}000 \cdot 2^h = 100{,}000$$

$\div 10{,}000$

$$2^h = 10$$

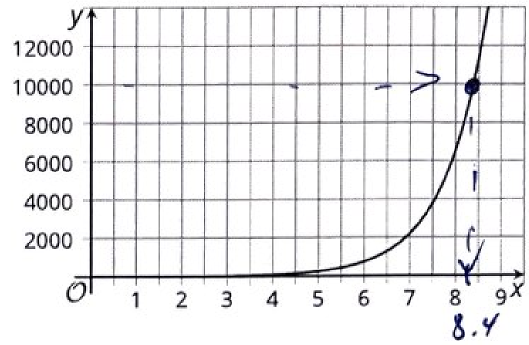
$$2^3 = 8 < 10 < 16 = 2^4$$

between 3 and four hours

3. Complete the table.

x	-4	-3	-2	0	$\frac{1}{3}$	1	3	9
10^x	$\frac{1}{10{,}000}$	$\frac{1}{1{,}000}$	$\frac{1}{100}$	1	2.15...	10	1,000	1,000,000,000

4. Here is a graph of $y = 3^x$.



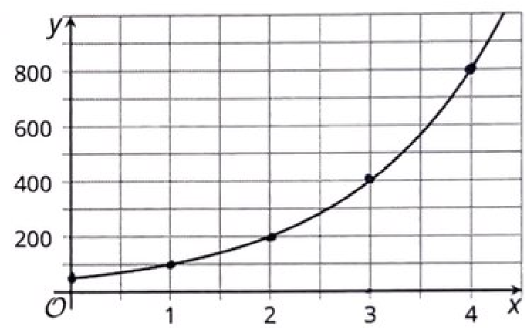
What is the approximate value of x satisfying $3^x = 10{,}000$? Explain how you know.

about 8.4, from the graph

5. One account doubles every 2 years. A second account triples every 3 years. Assuming the accounts start with the same amount of money, which account is growing more rapidly?

the account that doubles every 2 years

6. How would you describe the output of this graph for:



a. inputs from 0 to 1

y between ~50 and 100

b. inputs from 3 to 4

between 400 & 800

(From Unit 4, Lesson 1.)

7. The half-life of carbon-14 is about 5730 years.

   a. Complete the table, which shows the amount of carbon-14 remaining in a plant fossil at the different times since the plant died.

   b. About how many years will it be until there is 0.1 picogram of carbon-14 remaining in the fossil? Explain how you know.

years	picograms
0	3
5730	1.5
$2 \cdot 5730$	0.75
$3 \cdot 5730$	0.375
$4 \cdot 5730$	0.1875
$5 \cdot 5730$	0.09375

about 5 times 5730 from the table

(From Unit 4, Lesson 7.)

Solutions

Lesson 9 Practice Problems

1. For each equation in the left column, find in the right column an exact or approximate value for the unknown exponent so that the equation is true.

A. $10^y = 10$	$y = 1$	(3)	1. 0.602
B. $10^y = 20$	$y \doteq 1.3$	(5)	2. -1
C. $10^y = 2{,}000$	$y \doteq 3.3$	(6)	3. 1
D. $10^y = 900$	$y \approx 2.954$	(4)	4. 2.954
E. $10^y = 4$	$y \approx 0.6$	(1)	5. 1.301
			6. 3.301
			7. 1.999

2. Here is a logarithmic expression: $\log_{10} 100$.

   a. How do we say the expression in words?

   "log base 10 of 100"

   b. Explain in your own words what the expression means.

   what exponent value over 10 equals 100

   c. What is the value of this expression?

   2

3. The base 10 log table shows that the value of $\log_{10} 50$ is about 1.69897. Explain or show why it makes sense that the value is between 1 and 2.

   $10^1 = 10 < 50 < 100 = 10^2$

4. Here is a table of some logarithm values.

x	$\log_{10}(x)$
200	2.3010
300	2.4771
400	2.6021 ←
500	2.6990
600	2.7782
700	2.8451
800	2.9031
900	2.9542
1,000	3 ←

a. What is the approximate value of $\log_{10}(400)$?

2.6021

b. What is the value of $\log_{10}(1000)$? Is this value approximate or exact? Explain how you know.

3 exact

$10^3 = 1000$ exactly

5. What is the value of $\log_{10}(1{,}000{,}000{,}000)$? Explain how you know.

9 : $10^9 = 1{,}000{,}000{,}000$
Count the zeros, move the decimal point

6. A bank account balance, in dollars, is modeled by the equation $f(t) = 1{,}000 \cdot (1.08)^t$, where t is time measured in years.

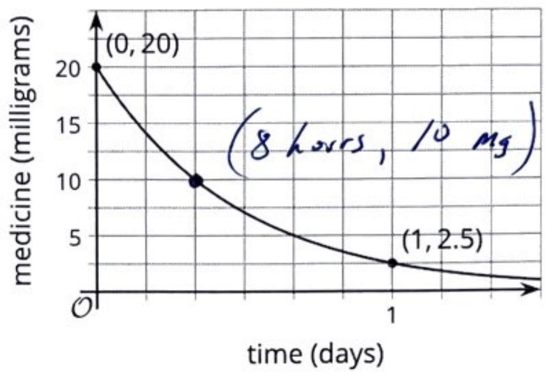
About how many years will it take for the account balance to double? Explain or show how you know. "rule of 72" $8 \cdot 9 = 72$

9 years

check

(From Unit 4, Lesson 8.) $1.08^9 = 1.999\ldots$

7. The graph shows the number of milligrams of a chemical in the body, d days after it was first measured.



a. Explain what the point $(1, 2.5)$ means in this situation.

after one Day the ~~concentration is~~ amount of medicine is 2.5 mg

b. Mark the point that represents the amount of medicine left in the body after 8 hours.

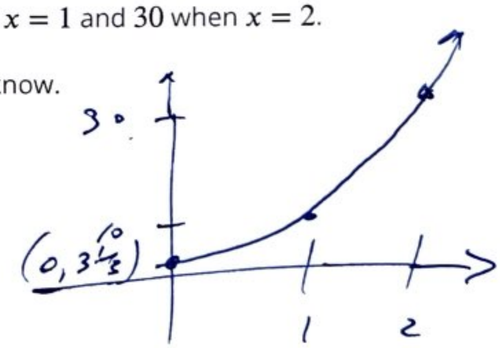
(From Unit 4, Lesson 3.)

8. The exponential function f takes the value 10 when $x = 1$ and 30 when $x = 2$.

a. What is the y-intercept of f? Explain how you know.

$r = \frac{30}{10} = 3$

$r_0 = \frac{10}{3} = 3\frac{1}{3}$



b. What is an equation defining f?

(From Unit 4, Lesson 6.)

$f(x) = 3\frac{1}{3} \cdot \cancel{10}\, 3^x$

Lesson 10 Practice Problems

1. a. Use the base-2 log table (printed in the lesson) to approximate the value of each exponential expression.

   i. 2^5 32

   ii. $2^{3.7}$ 12.996...

   iii. $2^{4.25}$ 19.0273...

   b. Use the base-2 log table to find or approximate the value of each logarithm.

   i. $\log_2 4$ 2

   ii. $\log_2 17$ 4.0874...

   iii. $\log_2 35$ 5.12528...

2. Here is a logarithmic expression: $\log_2 64$.

   a. How do we say the expression in words?

   "log base two of 64"

   b. Explain in your own words what the expression means.

   the exponent value that would raise the base two to be equal to 64.

   c. What is the value of this expression?

   6

3. a. What is $\log_{10}(100)$? What about $\log_{100}(10)$?

   2 , 1/2

   b. What is $\log_2(4)$? What about $\log_4(2)$?

   2 1/2

   c. Express b as a power of a if $a^2 = b$.

   $a = b^{\frac{1}{2}}$

4. In order for an investment, which is increasing in value exponentially, to increase by a factor of 5 in 20 years, about what percent does it need to grow each year? Explain how you know.

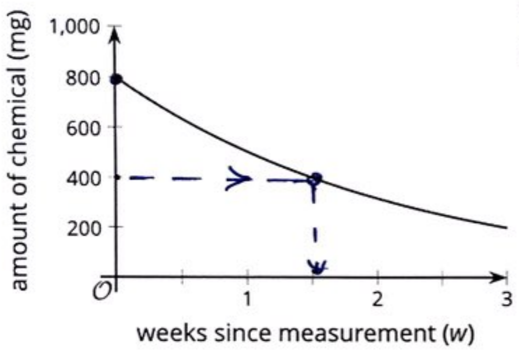
$(1+r)^{20} = 5$

$1+r = \sqrt[20]{5} = 5^{1/20} = 1.083798...$

$r \approx 8.38\%$

(From Unit 4, Lesson 4.)

5. Here is the graph of the amount of a chemical remaining after it was first measured. The chemical decays exponentially.



What is the approximate half-life of the chemical? Explain how you know.

$\frac{800}{400} = 2$

1.5 weeks

(From Unit 4, Lesson 7.)

6. Find each missing exponent.

a. $10^? = 100$ 2

b. $10^? = 0.01$ −2

c. $\left(\frac{1}{10}\right)^? = \frac{1}{1{,}000}$ 3

d. $2^? = \frac{1}{2}$ −1

e. $\left(\frac{1}{2}\right)^? = 2$ −1

(From Unit 4, Lesson 8.)

7. Explain why $\log_{10} 1 = 0$.

because $10^0 = 1$

(From Unit 4, Lesson 9.)

8. How are the two equations $10^2 = 100$ and $\log_{10}(100) = 2$ related?

they are equivalent
(definition of logarithms)

(From Unit 4, Lesson 9.)

Lesson 11 Practice Problems

1. Select **all** expressions that are equal to $\log_2 8$. $= 3$

   A. $\log_5 20$ ✗

   B. $\log_5 125$

   C. $\log_{10} 100$ ✗

   D. $\log_{10} 1{,}000$

   E. $\log_3 27$

   F. $\log_{10} 0.001$ ✗

2. Which expression has a greater value: $\log_{10} \frac{1}{100}$ or $\log_2 \frac{1}{8}$? Explain how you know.

   $= -2 \qquad = -3$

   $-2 > -3$

   $\log_{10} \frac{1}{100} > \log_2 \frac{1}{8}$

3. Andre says that $\log_{10}(55) = 1.5$ because 55 is halfway between 10 and 100. Do you agree with Andre? Explain your reasoning.

   No. Exponential functions are not linear.

   $\log_{10} 55 = 1.740\ldots$

4. An exponential function is defined by $k(x) = 15 \cdot 2^x$.

   a. Show that when x increases from 1 to 1.25 and when it increases from 2.75 to 3, the value of k grows by the same factor.

   $$\frac{k(1.25)}{k(1)} = \frac{15 \cdot 2^{1.25}}{15 \cdot 2^{1}} = 2^{0.25} \qquad \frac{k(2.75)}{k(3)} = \frac{15 \cdot 2^{3}}{15 \cdot 2^{2.75}} = 2^{0.25}$$

   b. Show that when x increases from t to $t + 0.25$, $k(t)$ also grows by this same factor.

   $$\frac{k(t)}{k(t+0.25)} = \frac{15 \cdot 2^{t+0.25}}{15 \cdot 2^{t}} = 2^{0.25}$$

(From Unit 4, Lesson 5.)

5. How many times does \$1 need to double in value to become \$1,000,000? Explain how you know.

   A little less than 20 times

   $2^{20} = 1,048,576$

(From Unit 4, Lesson 8.)

6. What values could replace the "?" in these equations to make them true?

   a. $\log_{10} 10{,}000 = ?$ 4

   b. $\log_{10} 10{,}000{,}000 = ?$ 7

   c. $\log_{10} ? = 5$ 100,000

   d. $\log_{10} ? = 1$ 10

(From Unit 4, Lesson 9.)

7. a. What value of t would make the equation $2^t = 6$ true?

between $t = 2$ and $t = 3$

b. Between which two whole numbers is the value of $\log_2 6$? Explain how you know.

$2^2 = 4 < 6 < 8 = 2^3$

$2 < \log_2 6 < 3$

(From Unit 4, Lesson 10.)

8. For each exponential equation, write an equivalent equation in logarithmic form.

a. $3^4 = 81$ $\log_3 81 = 4$

b. $10^0 = 1$ $\log_{10} 1 = 0$

c. $4^{\frac{1}{2}} = 2$ $\log_4 2 = \frac{1}{2}$

d. $2^t = 5$ $\log_2 5 = t$

e. $m^n = C$ $\log_m C = n$

(From Unit 4, Lesson 10.)

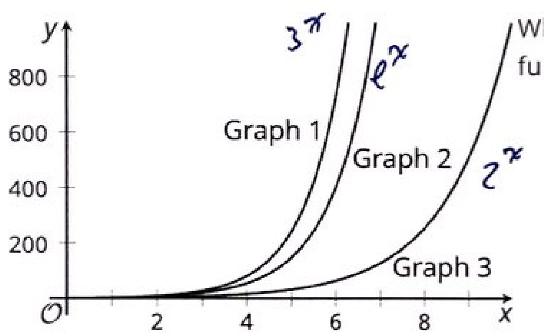
Lesson 12 Practice Problems

1. Put the following expressions in order from least to greatest.

   - e^3 ~25
   - 2^2 4
   - e^2 ~8
   - $2e$ ~6
   - e^e ~20

   2^2, $2e$, e^2, e^e, e^3

2. Here are graphs of three functions: $f(x) = 2^x$, $g(x) = e^x$, and $h(x) = 3^x$.



   Which graph corresponds to each function? Explain how you know.

   larger base → more rapid increase

3. Which of the statements are true about the function f given by $f(x) = 100 \cdot e^{-x}$? Select **all** that apply.

   A. The y-intercept of the graph of f is at $(0, 100)$.

   B. The values of f increase when x increases. X

   C. The value of f when $x = -1$ is a little less than 40. X

   D. The value of f when $x = 5$ is less than 1.

   E. The value of f is never 0.

4. Suppose you have $1 to put in an interest-bearing account for 1 year and are offered different options for interest rates and compounding frequencies (how often interest is calculated), as shown in the table. The highest interest rate is 100%, calculated once a year. The lower the interest rate, the more often it gets calculated.

   a. Complete the table with expressions that represent the amount you will have after one year, and then evaluate each expression to find its value in dollars (round to 5 decimal places).

interest rate	frequency per year	expression	value in dollars after 1 year
100%	1	$1 \cdot (1+1)^1$	2
10%	10	$1 \cdot (1+0.1)^{10}$	2.5937
5%	20	$1 \cdot (1+0.05)^{20}$	2.6532
1%	100	$1 \cdot (1+0.01)^{100}$	2.7048
0.5%	200	$1 \cdot (1+0.005)^{200}$	2.7115
0.1%	1,000	$1 \cdot (1+0.001)^{1000}$	2.7169
0.01%	10,000	$1 \cdot (1+0.0001)^{10{,}000}$	2.7181
0.001%	100,000	$1 \cdot (1+.00001)^{100{,}000}$	2.7182

   b. Predict whether the account value will be greater than $3 if there is an option for a 0.0001% interest rate calculated 1 million times a year. Check your prediction.

   No.

   c. What do you notice about the values of the account as the interest rate gets smaller and the frequency of compounding gets larger?

   The value converges to $e \approx 2.7182...$

5. The function f is given by $f(x) = (1+x)^{\frac{1}{x}}$. How do the values of f behave for small positive and large positive values of x?

as $x \to 0^+$, $f(x) \to e$

as $x \to \infty^+$, $f(x) \to 1$

6. Since 1992, the value of homes in a neighborhood has doubled every 16 years. The value of one home in the neighborhood was \$136,500 in 1992.

   a. What is the value of this home, in dollars, in the year 2000? Explain your reasoning.

   $V(t) = 136{,}500 \cdot 2^{\left(\frac{2000-1992}{16}\right)}$

   $= \$193{,}040\ldots$

   b. Write an equation that represents the growth in housing value as a function of time in t years since 1992.

   $V(t) = 136{,}500 \cdot 2^{\left(\frac{t - \cancel{1992}}{16}\right)}$

   c. Write an equation that represents the growth in housing value as a function of time in d decades since 1992.

   $V(d) = 136{,}500 \cdot 2^{d/1.6}$

   d. Use one of your equations to find the value of the home, in dollars, 1.5 decades after 1992.

   $V(1.5) = 136{,}500 \cdot 2^{1.5/1.6}$

   $\approx \$216{,}425.70..$

(From Unit 4, Lesson 4.)

7. Write two equations—one in exponential form and one in logarithmic form—to represent each question. Use "?" for the unknown value.

   a. "To what exponent do we raise the number 5 to get 625?"

   $5^{?} = 625$ $? = \log_5 625$

   b. "What is the log, base 3, of 27?"

   $\log_3 27 = ?$ $3^{?} = 27$

(From Unit 4, Lesson 10.)

8. Clare says that $\log 0.1 = -1$. Kiran says that $\log(-10) = -1$. Do you agree with either one of them? Explain your reasoning.

yes, clare is correct because $10^{-1} = 0.1$

Kiran is not correct. $10^{-1} \neq -10$

(From Unit 4, Lesson 11.)