Geometry Unit 2: Angles
Bronx Early College Academy

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3 - 14 October 2022
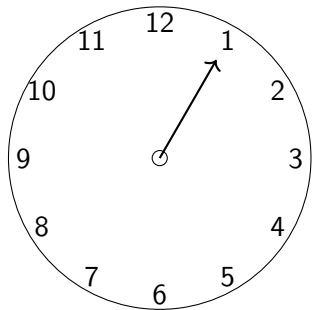
Open Middle: complementary and supplementary puzzle

Learning Target: I can measure angles
CCSS: HSG.CO.A.1 Know precise geometric definitions   2.1 Monday 3 Oct

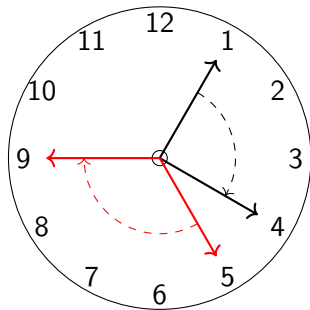Do Now: Which takes longer, for a clock's hour hand to go from the 1 to the 4 or the 5 to the 9?

Lesson: Angle measures, internal, external, acute, obtuse, right

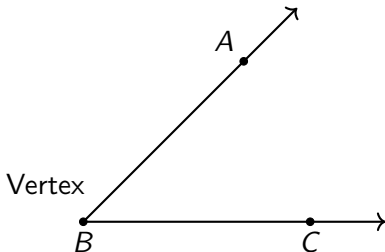## Learning Target: I can measure angles

Do Now: Which takes longer, for a clock's hour hand to go from the 1 to the 4 or the 5 to the 9?

Lesson: Angle measures, internal, external, acute, obtuse, right

Two rays with a common endpoint make an *angle*

Rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$, *vertex B*.

Angle Two rays with a common endpoint, $\angle ABC$ or $\angle B$

Vertex The common end point of two rays making an angle

Interior Inside, the area between the two rays

Exterior Outside, the area in the angle interior

Two rays with a common endpoint make an *angle*

Rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$, *vertex B*.

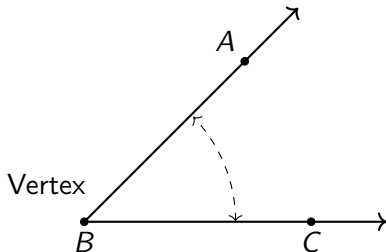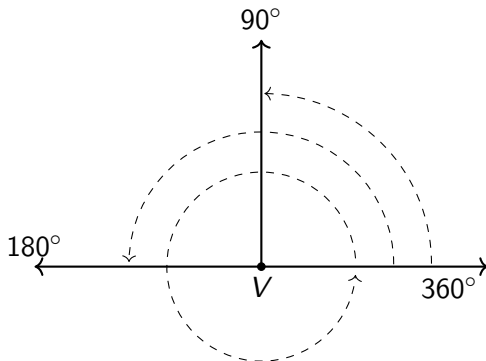Angle Two rays with a common endpoint, $\angle ABC$ or $\angle B$

Vertex The common end point of two rays making an angle

Interior Inside, the area between the two rays

Exterior Outside, the area in the angle interior

$m\angle A$ The "measure" of angle A, how big it is
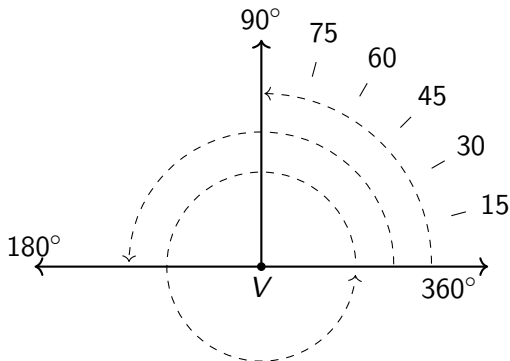
Babylonian measures: $360°$ in a circle

Full turn A complete rotation, $360°$

Half turn A straight line, $180°$

Quarter turn A *right* angle, $90°$

Protractor A tool for measuring angles

Babylonian measures: 360° in a circle

Full turn A complete rotation, 360°

Half turn A straight line, 180°
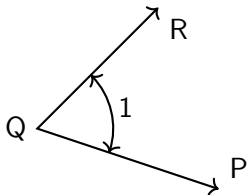
Quarter turn A *right* angle, 90°

Protractor A tool for measuring angles

Angle terminology and notation

Write definitions in your notebook

Angle Q, written $\angle Q$ (also $\angle PQR$, $\angle 1$)

Point Q is the *vertex*

The sides or *legs* are $\overrightarrow{QR}$, $\overrightarrow{QP}$

Right angle measuring $90°$, mark as small square $\llcorner$

Perpendicular lines meet at right angles. $\overline{AB} \perp \overline{CD}$

Acute angles measure $< 90°$

Obtuse angles are $90° < m\angle < 180°$

Straight angle or a straight line measures $180°$

Reflex angles measure $180° < m\angle < 360°$

Extension: Solving absolute value problems with algebra

There are two possible values for the absolute value of an unknown expression

$$|x + 5| = 7 \qquad\qquad 3 \times |2x| + 5 = 17$$

Learning Target: I can solve for angle measures

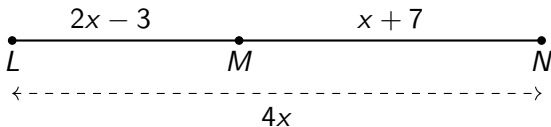CCSS: HSG.CO.A.1 Know precise geometric definitions   2.2 Tuesday 4 Oct

Do Now: Given $\overline{LMN}$, $LM = 2x - 3$, $MN = x + 7$, $LN = 4x$.
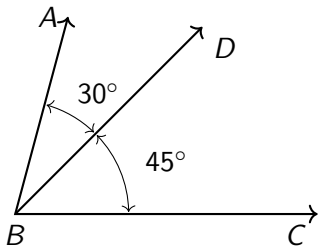Find x.   *Don't forget to check the solution.*

Name the geometry *postulate* that is the basis for this problem.

Lesson: Angle addition postulate, complementary, supplementary angles, linear pairs

Angle addition postulate

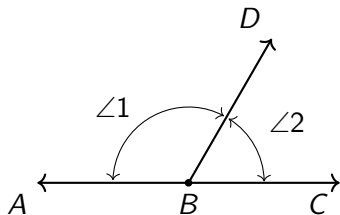m$\angle ABD = 30°$, m$\angle DBC = 45°$. Find m$\angle ABC$.

Angle addition The sum of the measures of *adjacent* angles is the
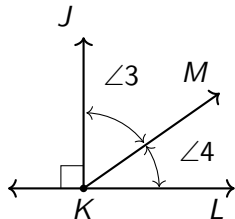measure of their combined angle. (postulate)

$$m\angle ABD + m\angle DBC = m\angle ABC$$

Adjacent "next to" each other. Adjacent angles share a
common ray and are external to each other.

Special angle pairs

Linear pair, supplementary ∠s

Complementary angles

Linear pair   Two adjacent angles that make a straight line

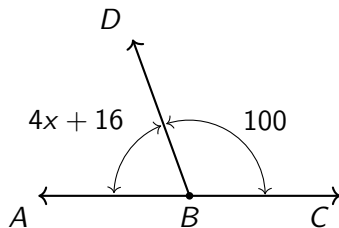Opposite rays   collinear with a common endpoint. e.g. $\overrightarrow{BA}$, $\overrightarrow{BC}$

Supplementary   Angles whose measures sum to $180°$

Complementary   Angles whose measures sum to $90°$

Adjacent   "next to" each other. Adjacent angles share a common ray and are external to each other.

Given two supplementary angles, a linear pair.

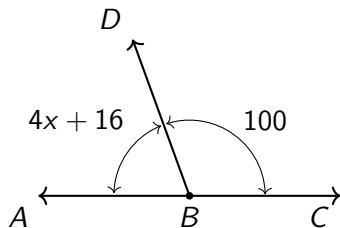m$\angle ABD = 4x + 16$, m$\angle CBD = 100$. Find x.

Given two supplementary angles, a linear pair.

m$\angle ABD = 4x + 16$, m$\angle CBD = 100$. Find x.
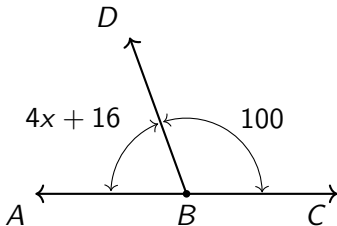
Solution:

$$m\angle ABD + m\angle CBD = 180$$

Given two supplementary angles, a linear pair.

m$\angle ABD = 4x + 16$, m$\angle CBD = 100$. Find x.

Solution:

$$\text{m}\angle ABD + \text{m}\angle CBD = 180$$

$$(4x + 16) + 100 = 180$$

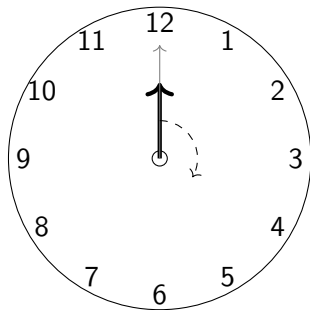$$\cdots$$

$$x = 16$$

Check:

$$[4(16) + 16] + 100 = 180 \checkmark$$

Extension (optional problems)

At midnight both the clock's minute hand and hour hand point in the same direction. When is the next time the clock hands coincide?
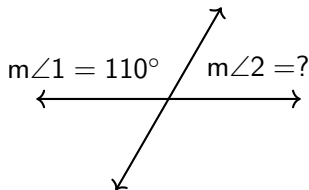
Learning Target: I can identify vertical angles
CCSS: HSG.CO.A.1 Know precise geometric definitions   2.3 Friday 7 Oct

Do Now: Check your knowledge of angle pairs

1. *Complementary* angles sum to how many degrees?
2. *Supplementary* angles sum to how many degrees?
3. Given complementary angles m$\angle A = 30°$. Find m$\angle B$.
4. Given intersecting lines. m$\angle 1 = 110°$. Find m$\angle 2$.
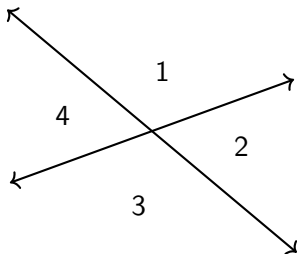
$$m\angle 1 = 110° \quad m\angle 2 = ?$$

Lesson: Vertical angles

Intersecting lines make two pairs of congruent angles

Angles *opposite* each other match:

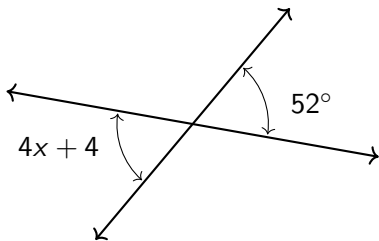$$\angle 1 \cong \angle 3, \ \angle 2 \cong \angle 4$$

Vertical angles Opposite each other when two lines intersect.
$\qquad$ $\angle 1$ and $\angle 3$ are vertical angles, as are $\angle 2$ and $\angle 4$.

$\quad$ Opposite Across from each other. (opposite angles and vertical
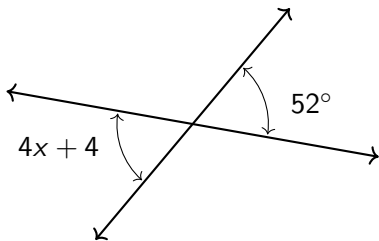$\qquad$ angles means the same thing)

Use vertical angles to solve for x

Given vertical angles measuring $4x + 4$ and $52°$. Find x.

Use vertical angles to solve for x

Given vertical angles measuring $4x + 4$ and $52°$. Find x.

Solution:

$$4x + 4 = 52$$

$$x = 12$$

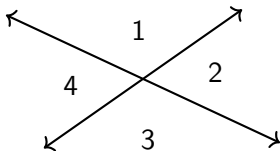Check:

$$4(12) + 4 = 52 \checkmark$$

Extension: Use logic to show vertical angles are congruent

Given intersecting lines making angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$.

Prove $\angle 2 \cong \angle 4$.

Linear pairs are supplementary

$$m\angle 2 + m\angle 1 = 180$$

$$m\angle 4 + m\angle 1 = 180$$

Both equal 180, so they are equal (*transitive property* of equality)

$$m\angle 2 + m\angle 1 = m\angle 4 + m\angle 1$$

Subtract $m\angle 1$ from both sides (*cancellation* law)   link
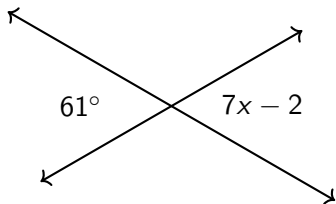
$$\angle 2 \cong \angle 4 \ Q.E.D.$$

Learning Target: I can bisect angles
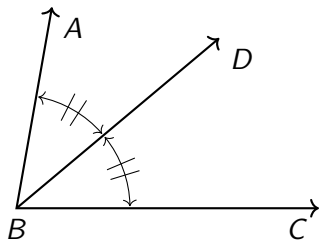CCSS: HSG.CO.A.1 Know precise geometric definitions   2.4 Tuesday 11 October

Do Now: Given vertical angles measuring $7x - 2$ and $61°$. Find x.

Lesson: Angle bisector situations

Bisect an angle by dividing it exactly in half

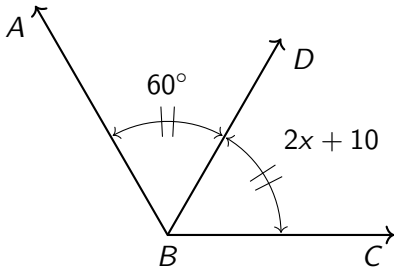$\overrightarrow{BD}$ bisects $\angle ABC$ if and only if $\angle ABD \cong \angle CBD$.

Angle bisector ray dividing an angle into two congruent angles

 Hash marks mark congruent angles
              (conventions differ for marking angles)
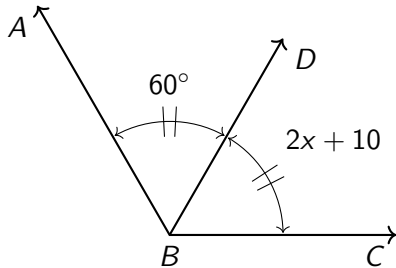
Model angle situations with algebra, then solve

Given angle bisector $\overrightarrow{BD}$ with m$\angle ABD = 60°$ and
m$\angle CBD = 2x + 10$. Find x.

Model angle situations with algebra, then solve

Given angle bisector $\overrightarrow{BD}$ with m$\angle ABD = 60°$ and
m$\angle CBD = 2x + 10$. Find x.

Solution:

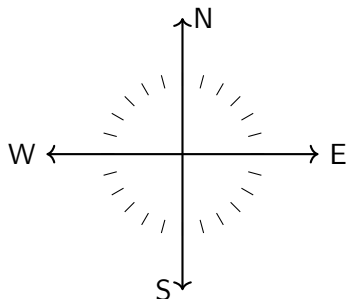$$\angle ABD \cong \angle CBD$$

$$2x + 10 = 60$$

$$2x = 50$$

$$x = 25$$

Check:
$2(25) + 10 = 60$? ✓

Extension: Use angles for compass directions

North South East West, points of the compass

Directions are measured relative to North

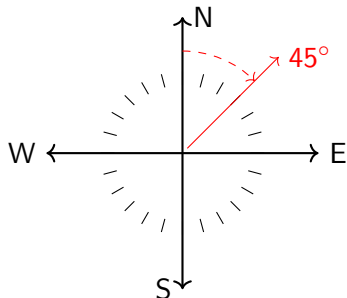

Bearing The direction as an angle *clockwise* from north

Clockwise The direction the clocks turn, "to the right" (tighten)

Counterclockwise Opposite of clocks, "to the left" (loosen)

Extension: Use angles for compass directions
North South East West, points of the compass

Directions are measured relative to North

"Northeast," half way between north and east, i.e. bearing $45°$

north is $0°$
east is $90°$
south is $180°$
west is $270°$

Bearing   The direction as an angle *clockwise* from north
Clockwise   The direction the clocks turn, "to the right" (tighten)
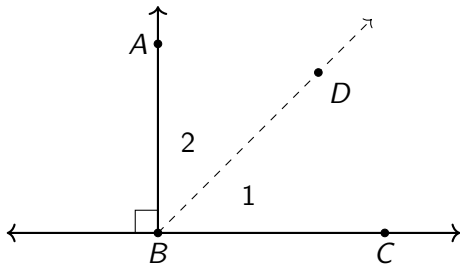Counterclockwise   Opposite of clocks, "to the left" (loosen)

LT: I can work with equilateral and isosceles-right △s
CCSS: HSG.CO.A.1 Know precise geometric definitions   2.5 Wednesday 12 October

Do Now: Given perpendiculars $\overrightarrow{AB} \perp \overleftrightarrow{BC}$, and that the ray $\overrightarrow{BD}$ bisects $\angle ABC$, making two angles, $\angle 1$ and $\angle 2$.
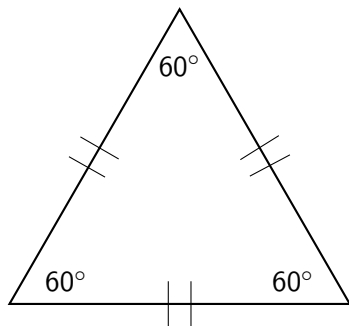
Find the measures of $\angle 1$, $\angle 2$.
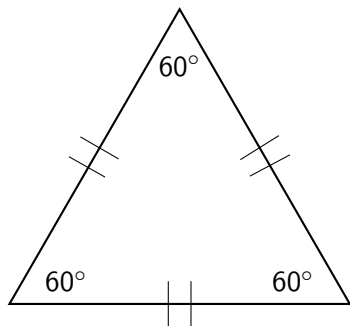

Lesson: Isosceles base theorem, special triangles
$60°$ - $60°$ - $60°$, $30°$ - $60°$ - $90°$, $45°$ - $45°$ - $90°$

Equilateral △, special relationships and measures

60° - 60° - 60°

Equilateral △, special relationships and measures
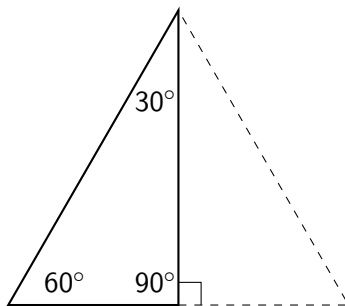
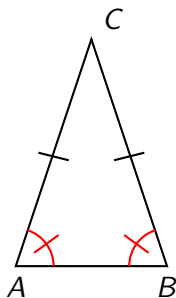$60° - 60° - 60°$ $30° - 60° - 90°$

Equiangular   means having equal angles

Equilateral   having equal sides

The *base* angles of an isosceles triangle are congruent

Isosceles base theorem: If $\overline{AC} \cong \overline{BC}$ then $\angle A \cong \angle B$
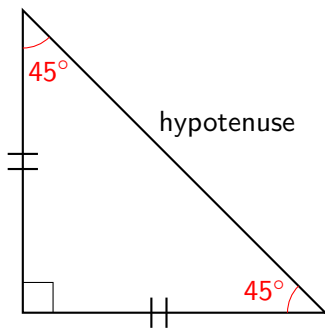

Base angles ∠s opposite the congruent sides in an isosceles △

Included angle The angle between two given sides of a triangle
($\angle C$ is included between $\overline{AC}$ and $\overline{BC}$)
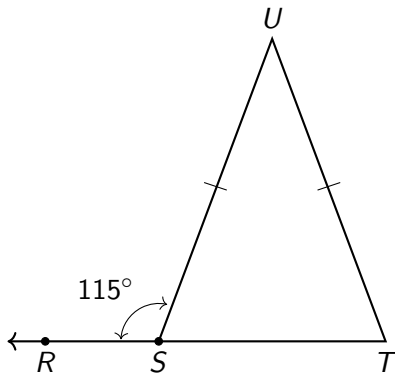
Theorem Something we can prove using logic

Isosceles-right triangles' angles measure $45°$ - $45°$ - $90°$

Hypotenuse the longest side of a right triangle, opposite the $90°$ angle

Multiple step problem: apply your knowledge

Given isosceles triangle with $\overline{SU} \cong \overline{TU}$, m$\angle RSU = 115°$.
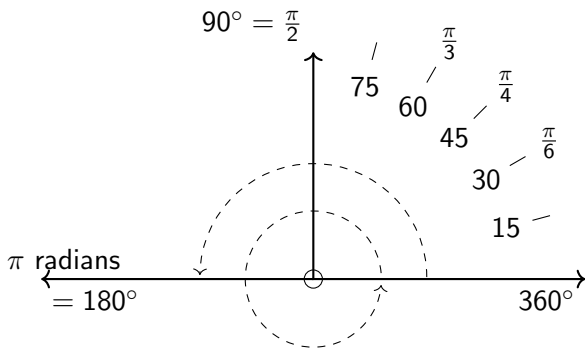

1. Find m$\angle TSU$

2. Find m$\angle T$

Extension: Radian units for angle measures

Mathematicians use radians because calculations are simpler

Convert *units*: $360° = 2\pi$ radians:

     Degree   One 360th of a full turn

     Radian   A full circle is 2π radians. 1 radian $\approx 57°$

     Gradian   One 400th of a full turn

LT: I can review length and angle measures with peers

CCSS: HSG.CO.A.1 Know precise geometric definitions   2.6 Thursday 13 October

Angle concepts and theorems you have learned

1. Angle addition situations
2. Angle pairs
   2.1 $\perp$ lines and complementary angles make $90°$
   2.2 Vertical $\angle$s are $\cong$
3. Angle bisectors
4. Isosceles base angle theorem, special triangles

Learning Target: I can quantify angles
CCSS: HSG.CO.A.1 Know precise geometric definitions   2.7 Friday 14 October

Unit test

Open Middle problem (fun)

Use digits from 0 to 9. Using a digit no more than once.

The first two angle measures are complementary. The second two angles supplementary. (degrees)