Solutions

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Point P has coordinates $(-3, 2)$, and point Q has coordinates $(15, -8)$. Point M is the midpoint of [PQ].

(a) Find the coordinates of M. [2]

Line L is perpendicular to [PQ] and passes through M.

(b) Find the gradient of L. [2]

(c) Hence, write down the equation of L. [1]

(a) $m = \left(\frac{-3+15}{2}, \frac{2+(-8)}{2}\right)$

$= (6, -3)$ A1 A1

(b) $m_{\overline{PQ}} = \frac{-8-2}{15-(-3)} = \frac{-10}{18} = -\frac{5}{9}$ (A1)

$m_{\perp} = \frac{9}{5}$ A1

(c) $y-(-3) = \frac{9}{5}(x-6)$ A1

$y+3 = \frac{9}{5}(x-6)$

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2. [Maximum mark: 7]

The function f is defined by $f(x) = \dfrac{7x+7}{2x-4}$ for $x \in \mathbb{R}$, $x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y = f(x)$, write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

(a) $f(x) = \dfrac{7x+7}{2x-4} = 0$ (M1)

$7x + 7 = 0$

$x = -1$ A1

(b) i) $x = 2$ A1 (equations)

ii) $y = 7/2$ A1

(c) $f^{-1}: \; x = \dfrac{7y+7}{2y-4}$ (M1)

$2xy - 4x = 7y + 7$

$2xy - 7y = 4x + 7$ (A1)

$f^{-1}: \quad y = \dfrac{4x+7}{2x-7} \quad x \neq \frac{7}{2}$ A1

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3. [Maximum mark: 6]

On a Monday at an amusement park, a sample of 40 visitors was randomly selected as they were leaving the park. They were asked how many times that day they had been on a ride called *The Dragon*. This information is summarized in the following frequency table.

Number of times on *The Dragon*	**Frequency**
0	6
1	16
2	13
3	2
4	3

It can be assumed that this sample is representative of all visitors to the park for the following day.

(a) For the following day, Tuesday, estimate

(i) the probability that a randomly selected visitor will ride *The Dragon*;

(ii) the expected number of times a visitor will ride *The Dragon*. [4]

It is known that 1000 visitors will attend the amusement park on Tuesday. *The Dragon* can carry a maximum of 10 people each time it runs.

(b) Estimate the minimum number of times *The Dragon* must run to satisfy demand. [2]

(a) i) $P(\text{Dragon}) = \frac{40-6}{40} = \frac{34}{40}$ (M1) A1

ii) $E(x) = (6\cdot 0 + 16\cdot 1 + 13\cdot 2 + 2\cdot 3 + 3\cdot 4)/40$ (M1)

$= (16 + 26 + 6 + 12)/40$

$= 60/40$

$= 1.5$ A1

(b) # rides $= 1000 \cdot 1.5$ (M1)

$= 1500$

runs $= \frac{1500}{10} = 150$ A1

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4. [Maximum mark: 6]

(a) Show that the equation $\cos 2x = \sin x$ can be written in the form $2\sin^2 x + \sin x - 1 = 0$. [1]

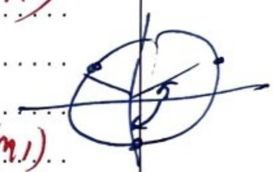
(b) Hence, solve $\cos 2x = \sin x$, where $-\pi \le x \le \pi$. [5]

(a) $\cos 2x = 1 - 2\sin^2 \cancel{x}$ (identity)

$1 - 2\sin^2 \cancel{x} = \sin x$ A1

$2\sin^2 \cancel{x} + \sin x - 1 = 0$

(b) $(2\sin x - 1)(\sin x + 1) = 0$ (M1)



$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ $\sin x = -1$ (M1)

$x = \frac{1}{6}\pi, \frac{5}{6}\pi$ A1 $x = -\frac{\pi}{2}$ A1

$x = -\frac{\pi}{2}, \frac{1}{6}\pi, \frac{5}{6}\pi$ A1

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(Question 1 continued)

(a) $P = 4+4+10$ (M1)

$= 18$ cm A1

(b) $\theta = \frac{10}{4} = 5/2$ (A1) A1

(c) $A = \left(\frac{1}{2}\right)\frac{5}{2}\left(4^2\right)$ (A1)

$= \frac{5}{\cancel{8}}\ \cancel{40}\ 20$ cm^2 A1

3. [Maximum mark: 5]

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}$, $x \neq 2$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

(i) the y-axis;

(ii) the x-axis. [2]

(a) i) $x = 2$ A1
ii) $y = 1$ A1

(b) i) $f(0) = 1 - \frac{1}{0-2} = \frac{3}{2}$ A1
$\left(0, \frac{3}{2}\right)$

ii) $f(x) = 1 - \frac{1}{x-2} = 0$
$x - 2 = 1$
$x = 3$
$(3, 0)$ A1

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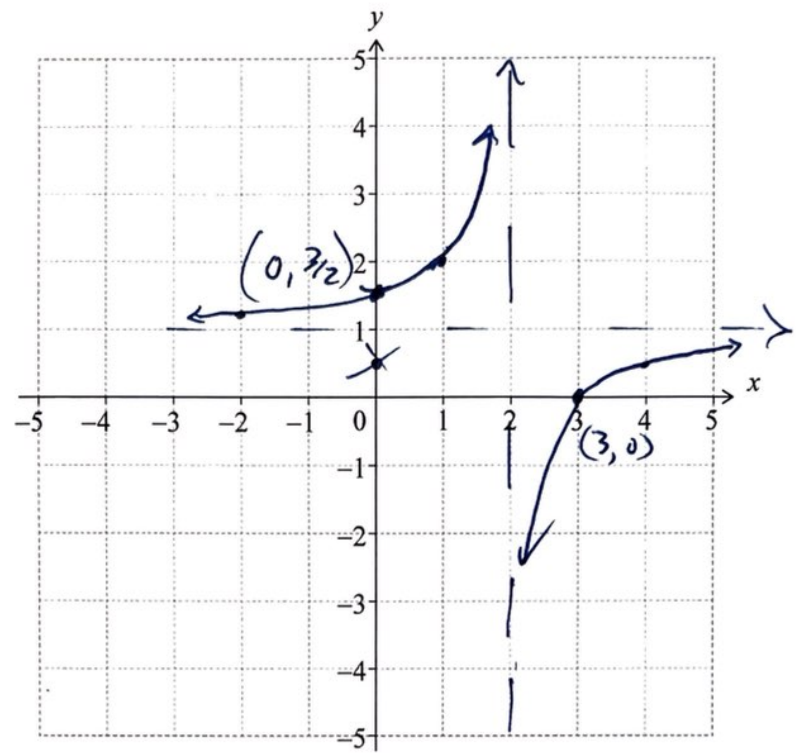
(Question 3 continued)

(c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b). [1]



5. [Maximum mark: 6]

Find the range of possible values of k such that $e^{2x} + \ln k = 3e^{x}$ has at least one real solution.

$$e^{2x} - 3e^{x} + \ln k = 0$$

$u = e^{x}$ (M1)

$a = 1$
$b = -3$
$c = \ln k$

$$e^{x} = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(\ln k)}}{2}$$

$$\frac{3 \pm \sqrt{9 - 4\ln k}}{2} > 0$$

$e^{x} > 0$ AND
$\Delta \geq 0$ (M1)

$$3 > \sqrt{9 - 4\ln k}$$

$$9 > 9 - 4\ln k$$

$$4\ln k > 0$$

$$k > 0$$

$$\Delta = 9 - 4\ln k \geq 0 \quad \text{(A1)}$$

$$\frac{9}{4} \geq \ln k \quad \text{(A1)}$$

$$k \leq e^{\frac{9}{4}} \quad \text{A1}$$

$$0 < k \leq e^{9/4} \quad \text{A1}$$

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Paper 1 TZ1

SOLUTIONS

7. (a) $h(0) = 2(0)e^0 + 3$ (M1)

$y = 3$ A1

(b) $h'(x) = 2xe^x + 2e^x$ (M1) A1

(c) $2xe^x + 2e^x = 0$ (M1) (A1)

$x = -1$ A1

$h(-1) = 2(-1)e^{-1} + 3$ (M1)

$= -2e^{-1} + 3$

$(-1, -2e^{-1} + 3)$ A1

(d) $h'(x) = 2xe^x + 2e^x$

(i) $h''(x) = (2xe^x + 2e^x) + 2e^x$ A1 A1

$= (2x+4)e^x$

(ii) $h''(x) = (2x+4)e^x > 0$ (M1)

$2x + 4 > 0$

$x > -2$ A1

P1 TZ1 SOLUTIONS

8 (a)

(i) $S_5 = 5^2 + 4(5)$ (M1)

$= 45$ A1

(ii) $u_6 = S_6 - S_5$ (M1)

$= 60 - 45$

$= 15$ A1

(b) $S_6 = \frac{6}{2}(u_1 + 15) = 60$ (M1)

$u_1 = 5$ A1

(c) $u_6 = 5 + d(6-1) = 15$ (M1)

$d = 2$ (A1)

$u_n = 5 + 2(n-1)$ A1

(d) $v_2 = 5, \ v_4 = 15$

$r^2 = \frac{15}{5} = 3$ (M1)(A1)

$r = \pm\sqrt{3}$ A1

(e) $v_{99} = 5 \cdot r^{(99-2)} < 0$

$r^{97} < 0$

$r = -\sqrt{3}$ (M1)

$v_5 = 15 \cdot (-\sqrt{3})$

$= -15\sqrt{3}$ A1

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Paper 1 TZ2 Solutions

7. (a) $x = -2$ A1

(b) $h = -2$ A1 A1
$k = -5$

(c) $f(0) = \frac{1}{4}(0+2)^2 - 5$ (M1)
$y = -4$ A1

(d) $f'(x) = \frac{1}{4}(2)(x+2)$
$= \frac{1}{2}x + 1$ (A1)
$f'(0) = \frac{1}{2}(0) + 1 = 1$ (M1)
$m_{\perp} = -1$ A1
$y - (-4) = -1(x - 0)$
$y = \cancel{-x+4}\ -x - 4$ A1

(e) Q: $\frac{1}{4}(x+2)^2 - 5 = -x - 4$ (M1)
$x^2 + 4x + 4 = -4x + 4$
$x^2 + 8x = 0$ (A1)
$x = 0, -8$ A1
$y = -(-8) - 4 = 4$ (M1)
$Q(-8, 4)$ A1

$d_{PQ} = \sqrt{(0-(-8))^2 + (-4-4)^2}$ (A1)
$= 8\sqrt{2}$ A1

9 TZ 2 Paper 1 SOLUTIONS

9. (a) $Q: x^2+y^2=9$

$y^2 = 9-x^2$ A1

$y = \sqrt{9-x^2}$ $y>0$ given 1st Quadrant

(b) $A_0 = \frac{1}{2}(2)\left|(-3-x)\right|\sqrt{9-x^2}$ (A1)(M1)

$= (x+3)\sqrt{9-x^2}$ A1

(c) $\frac{dA}{dx} = (x+3)\left(\frac{1}{2}\right)\left(9-x^2\right)^{-\frac{1}{2}}(-2x)+(1)\sqrt{9-x^2}$ (M1)(M1)

$= \frac{-x(x+3)}{\sqrt{9-x^2}} + \frac{\left(\sqrt{9-x^2}\right)^2}{\sqrt{9-x^2}}$ A1

$= \frac{-x^2-3x+9-x^2}{\sqrt{9-x^2}}$ A1

$= \frac{9-3x-2x^2}{\sqrt{9-x^2}}$ (4)

(d) $A_{max}: \frac{dA}{dx} = \frac{9-3x-2x^2}{\sqrt{9-x^2}} = 0$ (M1)

(M1)

$(3+x)(3-2x)=0$ (A1) disregard $x=-3$

$x = 3/2$ A1

$y = \pm\sqrt{9-(3/2)^2}$ (M1)

$R_y: = -\frac{3}{2}\sqrt{3}$ A1