

2. [Maximum mark: 4]

Given that  $\frac{dy}{dx} = \cos\left(x - \frac{\pi}{4}\right)$  and  $y = 2$  when  $x = \frac{3\pi}{4}$ , find  $y$  in terms of  $x$ .

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3. [Maximum mark: 5]

The function  $f$  is defined by  $f(x) = \frac{2x+4}{3-x}$ , where  $x \in \mathbb{R}$ ,  $x \neq 3$ .

(a) Write down the equation of

(i) the vertical asymptote of the graph of  $f$ ;

(ii) the horizontal asymptote of the graph of  $f$ .

[2]

(b) Find the coordinates where the graph of  $f$  crosses

(i) the  $x$ -axis;

(ii) the  $y$ -axis.

[2]

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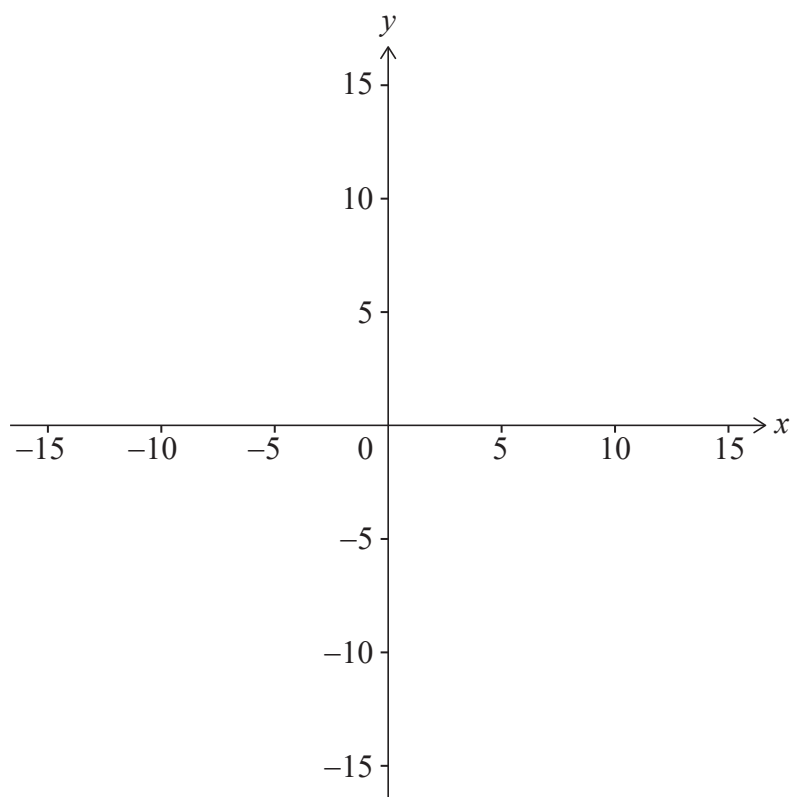
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(Question 3 continued)

(c) Sketch the graph of  $f$  on the axes below.

[1]



**6.** [Maximum mark: 7]

(a) Show that  $2x - 3 - \frac{6}{x-1} = \frac{2x^2 - 5x - 3}{x-1}$ ,  $x \in \mathbb{R}$ ,  $x \neq 1$ . [2]

(b) Hence or otherwise, solve the equation  $2\sin 2\theta - 3 - \frac{6}{\sin 2\theta - 1} = 0$  for  $0 \leq \theta \leq \pi$ ,  $\theta \neq \frac{\pi}{4}$ . [5]

[illegible]

Do **not** write solutions on this page.

## Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle  $P$  moves along the  $x$ -axis. The velocity of  $P$  is  $v \text{ m s}^{-1}$  at time  $t$  seconds, where  $v(t) = 4 + 4t - 3t^2$  for  $0 \leq t \leq 3$ . When  $t = 0$ ,  $P$  is at the origin  $O$ .

- (a) (i) Find the value of  $t$  when  $P$  reaches its maximum velocity.
  - (ii) Show that the distance of  $P$  from  $O$  at this time is  $\frac{88}{27}$  metres. [7]
- (b) Sketch a graph of  $v$  against  $t$ , clearly showing any points of intersection with the axes. [4]
- (c) Find the total distance travelled by  $P$ . [5]

8. [Maximum mark: 15]

Consider the function  $f(x) = a^x$  where  $x, a \in \mathbb{R}$  and  $x > 0, a > 1$ .

The graph of  $f$  contains the point  $\left(\frac{2}{3}, 4\right)$ .

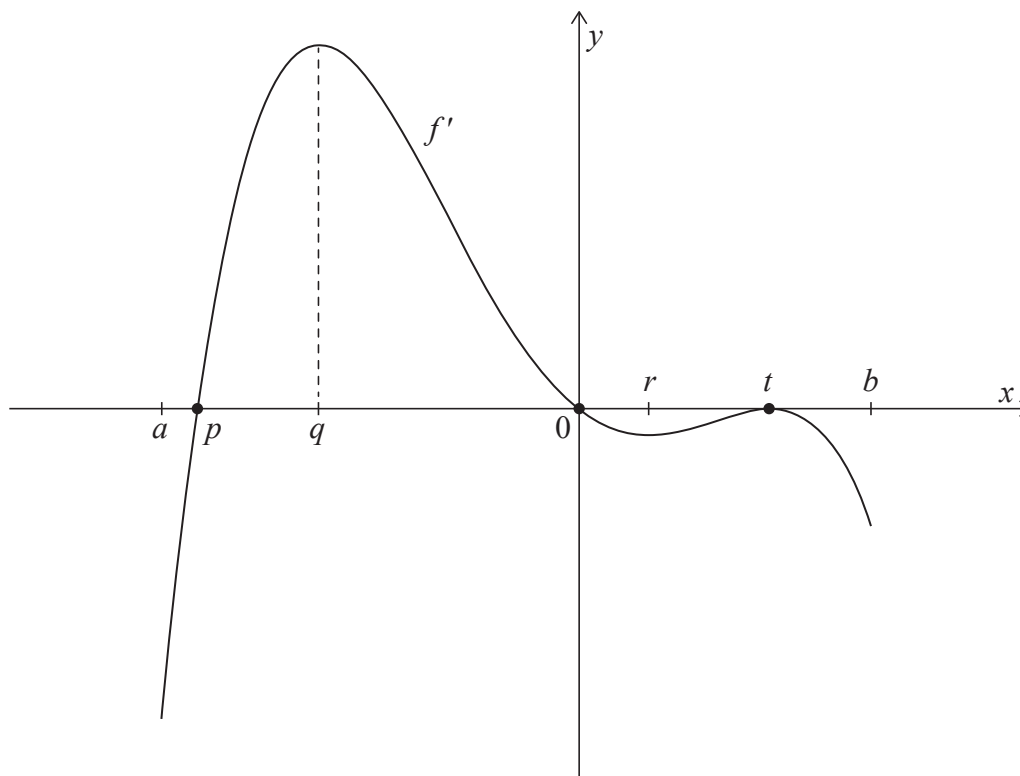
- (a) Show that  $a = 8$ . [2]
- (b) Write down an expression for  $f^{-1}(x)$ . [1]
- (c) Find the value of  $f^{-1}(\sqrt{32})$ . [3]
- (d) Consider the arithmetic sequence  $\log_8 27, \log_8 p, \log_8 q, \log_8 125$ , where  $p > 1$  and  $q > 1$ .
  - (i) Show that  $27, p, q$  and  $125$  are four consecutive terms in a geometric sequence.
  - (ii) Find the value of  $p$  and the value of  $q$ . [9]



Do **not** write solutions on this page.

9. [Maximum mark: 14]

Consider a function  $f$  with domain  $a < x < b$ . The following diagram shows the graph of  $f'$ , the derivative of  $f$ .



The graph of  $f'$ , the derivative of  $f$ , has  $x$ -intercepts at  $x = p$ ,  $x = 0$  and  $x = t$ . There are local maximum points at  $x = q$  and  $x = t$  and a local minimum point at  $x = r$ .

- (a) Find all the values of  $x$  where the graph of  $f$  is increasing. Justify your answer. [2]
- (b) Find the value of  $x$  where the graph of  $f$  has a local maximum. [1]
- (c) (i) Find the value of  $x$  where the graph of  $f$  has a local minimum. Justify your answer.
- (ii) Find the values of  $x$  where the graph of  $f$  has points of inflexion. Justify your answer. [5]
- (d) The total area of the region enclosed by the graph of  $f'$ , the derivative of  $f$ , and the  $x$ -axis is 20.

Given that  $f(p) + f(t) = 4$ , find the value of  $f(0)$ . [6]

References:

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