1.22 PreExam: Probability, Venn diagrams

1. Given:
   $U = \{\text{the letters in the alphabet}\}$ $A = \{t, i, m, e, s\}$ $B = \{m, i, n, u, s\}$
   (a) List the members of $A \cup B$. [1 mark]

   (b) List the elements of $A \cap B$. [1 mark]

   (c) A letter is selected at random. What is the probability that it is a member of both sets, $(A \cap B)$? [1 mark]

2. The events A and B are independent with $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.5$. Find each probability.
   (a) $\mathrm{P}(A \cap B)$? [2 mark]

   (b) $\mathrm{P}(A \cup B)$? [2 mark]

   (c) $\mathrm{P}(B' \cap A)$? [2 mark]

   (d) $\mathrm{P}(A|B)$? [2 mark]

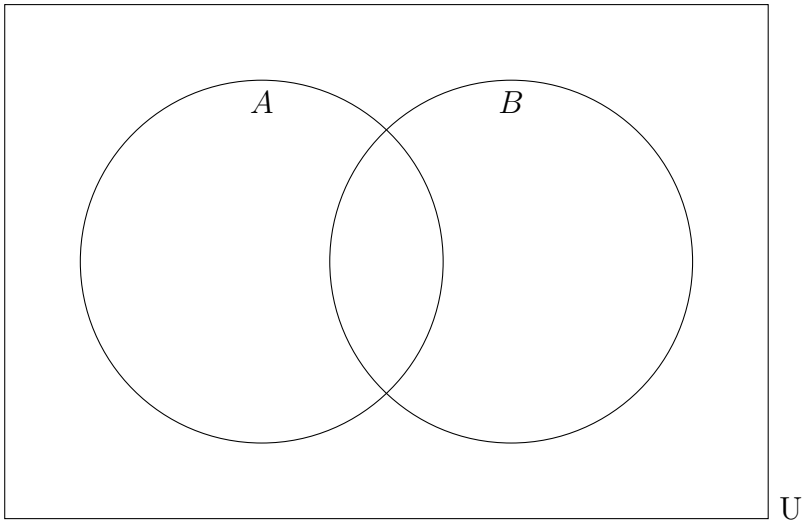
3. The universal set U is defined as the set of positive integers less than 10. The subsets A and B are defined as follows:

   $A = \{\text{the odd numbers}\}$ $\quad$ $B = \{\text{prime numbers}\}$

   (a) List the members of A'. [1 mark]

   (b) List the members of $(A \cup B)'$. [1 mark]

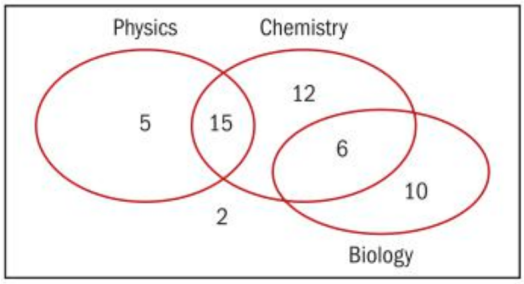
   (c) Place the elements of A and B in the appropriate regions in the Venn diagram below. [2 marks]



   (d) List the items in $A \cap B$. [1 mark]

   (e) If an element is selected at random, what is the probability that it is a member of both sets, $(A \cap B)$? [1 mark]

4. : The Venn diagram illustrates the number of students taking each of the three sciences: physics, chemistry and biology.



A student is randomly chosen from the group.

Find the probability that

a the student studies chemistry or biology (2 marks)

b the student studies neither physics nor biology (2 marks)

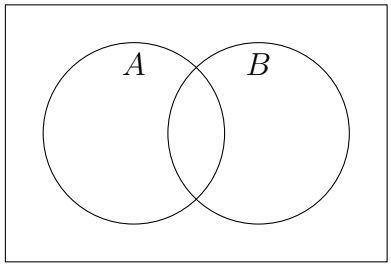
c the student studies physics, given that they study chemistry (2 marks)

d the student studies biology, given that they study physics (2 marks)

e the student studies physics, given that they do not study biology. (2 marks)

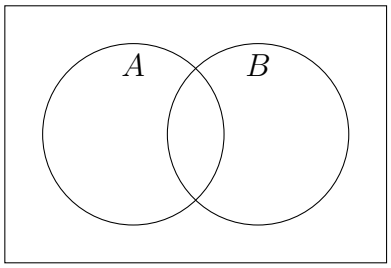
5. For each Venn diagram, shade the area representing the expression. Use pencil.

(a) $A \cup B$



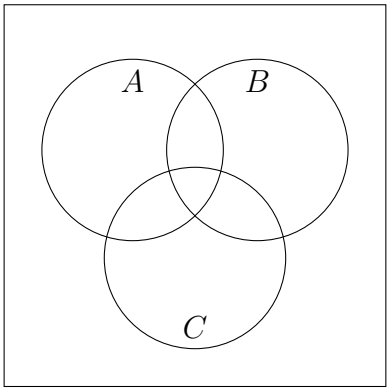
[2 marks]

(b) $A' \cap B$



[2 marks]

(c) $(A \cap B) \cup C$



[2 marks]

6. The events A and B are mutually exclusive with $\mathrm{P}(A) = 0.7$ and $\mathrm{P}(B) = 0.2$.

(a) Write down $\mathrm{P}(A \cup B)$. [1 mark]

(b) Find $\mathrm{P}(A' \cup B)$. [1 mark]

7. The events A and B are independent with $\mathrm{P}(A) = 0.5$ and $\mathrm{P}(B) = 0.8$.

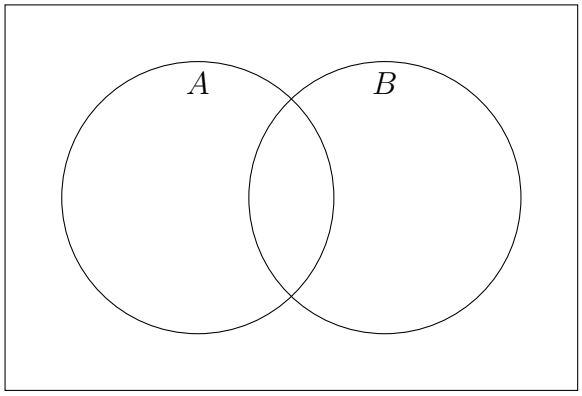
   (a) Find $\mathrm{P}(A \cap B)$. [2 marks]

   (b) Find $\mathrm{P}(A \cup B)$. [2 marks]

   (c) Find $\mathrm{P}(B|A)$. [2 marks]

8. Given events A and B with $\mathrm{P}(A) = 0.4$, $\mathrm{P}(B) = 0.5$, $\mathrm{P}(A \cap B) = 0.25$.

   (a) Completely mark the Venn diagram with probabilities for each area. [2 marks]



   (b) Find $\mathrm{P}(A \cup B)$. [2 marks]

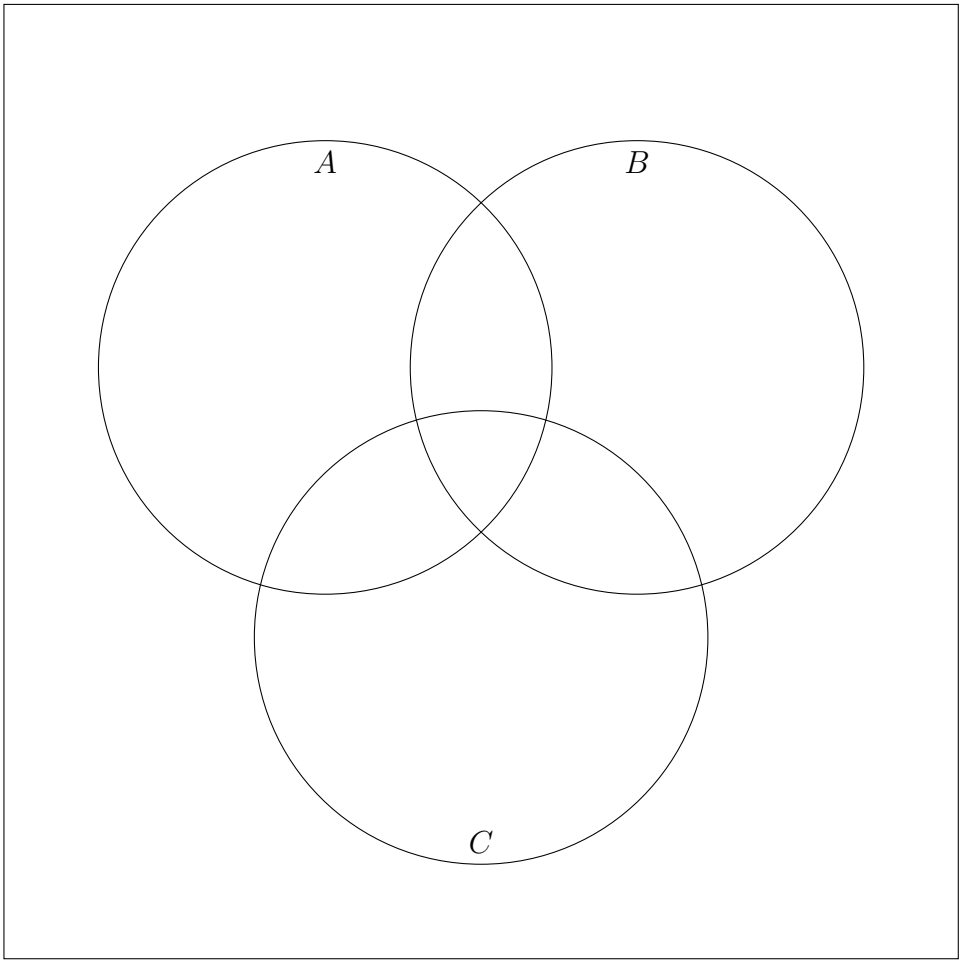
   (c) State whether events A and B are independent. Justify your answer. [3 marks]

   (d) Find $\mathrm{P}(A|B)$. [2 marks]

9. There are 80 athletes playing the following sports:

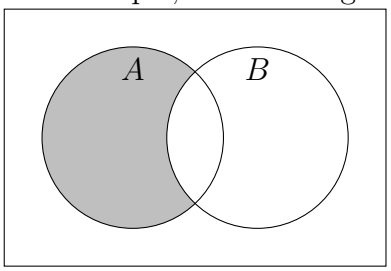
   - 35 play Archery
   - 44 play Badminton
   - 39 play Cricket
   - 16 play Archery and Badminton
   - 15 play Archery and Cricket
   - 10 play Badminton and Cricket
   - 3 play all three of these sports

   Complete the Venn diagram below with the number of students in each region to represent the situation. [4 marks]



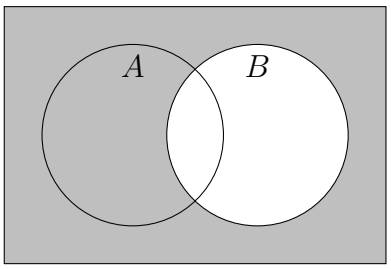
10. For each Venn diagram, write an expression representing the shaded area.

(a) For example, for this diagram



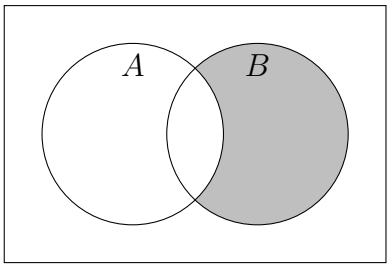
Expression: $A \cap B'$

(b)



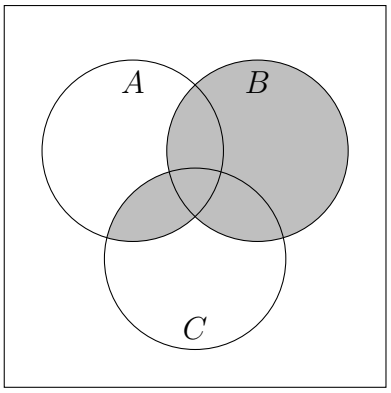
Expression:

(c)



Expression:

(d)



Expression:

11. Given:
$U = \{\text{the letters in the alphabet}\}$
$A = \{a, b, c, d, e, f, g, h, i, j\}$ $B = \{h, i, j, k, l, m, n, o, p, q\}$

(a) What is $A \cap B$?

(b) What is $(A \cup B)'$?

12. Forty IB high school students range in age from 15 to 18 years old. The following table shows the frequencies of each age.

Age (years)	15	16	17	18
Frequency	5	k	15	7

(a) Calculate the value of k. [1 mark]

(b) Write down the mode. [1 mark]

(c) Find the value of the range. [1 marks]

(d) Find the median. [1 marks]

(e) Find the mean. [2 marks]

(f) Find the standard deviation. [2 marks]

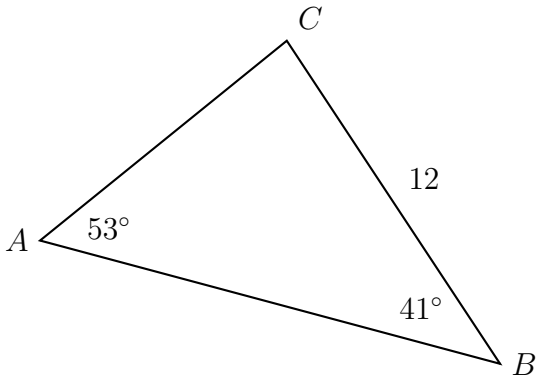
13. A runner records her pace in terms of distance run (d) in miles over time (t) in minutes during a 4.5 mile run. She models her pace with a linear regression equation $d = at + b$.

minutes (t)	0	8	15	22	30
miles (d)	0	1.8	2.7	3.7	4.5

(a) Find the values of a, b, and the correlation r. [3 marks]

(b) Explain what the value of a represents in the context of the situation. [2 marks]

14. The following diagram shows triangle ABC (not drawn to scale).



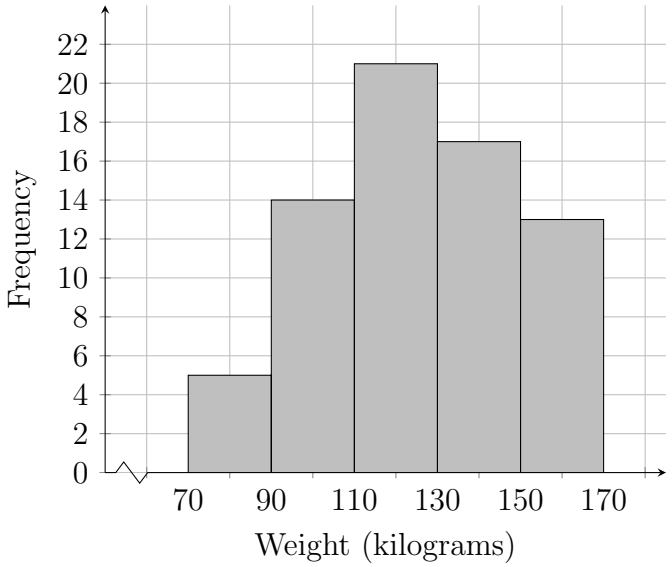
$BC = 12$, $C\hat{A}B = 53°$, and $A\hat{B}C = 41°$

(a) Find the measure of $A\hat{C}B$. [1 mark]

(b) Find AC. [3 marks]

(c) Find the area of triangle ABC. [3 marks]

15. The histogram below shows the weight w in kilograms for 70 professional football players.



The following is the frequency table for the distribution of w.

HR (x)	$70 \leq x < 90$	$90 \leq x < 110$	$110 \leq x < 130$	$130 \leq x < 150$	$150 \leq x < 170$
Freq	5	14	21	p	13

(a) Write down the value of p. [1 mark]

(b) Write down the modal class. [2 marks]

(c) A player is selected at random. Find the probability that the athlete weighs less than 110 kilograms. [2 marks]

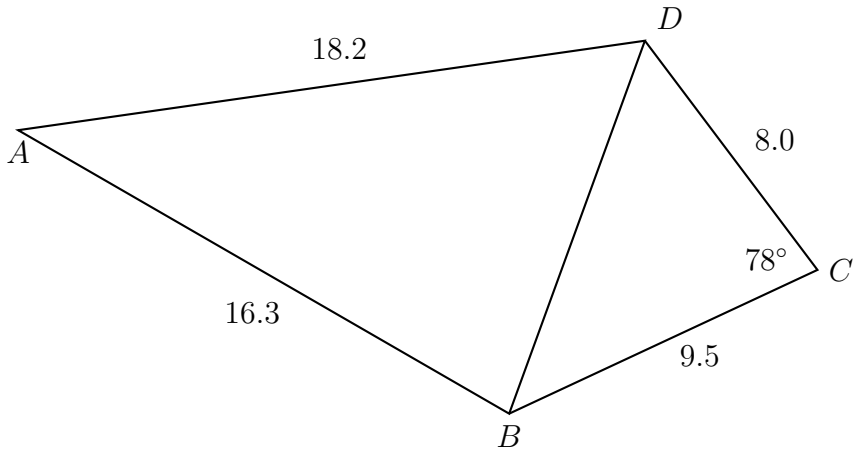
(d) Write down the mid-interval value for the class $110 \leq x < 130$. [1 mark]

(e) Hence find an estimate for the

i. mean; [2 marks]

ii. standard deviation. [2 marks]

16. The following diagram shows quadrilateral $ABCD$ (not drawn to scale).



$AB = 16.3$, $BC = 9.5$, $CD = 8.0$, $AD = 18.2$, and $B\hat{C}D = 78°$

(a) Find BD. [3 marks]

(b) Find $A\hat{B}D$. [3 marks]

17. A survey question has three possible responses: A, B, and C. Among 100 surveys, the frequency of the answers collected were as follows: $n(A) = 10, n(B) = 35,$ and $n(C) = 55$.

    (a) If a survey is selected at random, what this the probability the response was B or C?

    (b) What is the probability a survey selected at random was an answer other than B or C?