Geometry Unit 3: Transversals

Bronx Early College Academy

Christopher J. Huson PhD

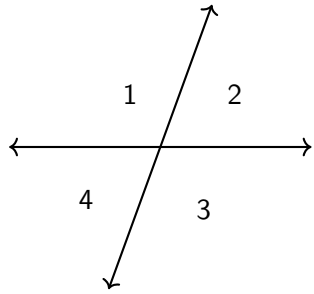
17 October - 28 October 2022

Learning Target: I can name parallel lines transversal angles

HSG.CO.C.9 Prove theorems about lines and angles 3.1 Monday 17 October

Do Now: Identify the true statements

1. $\angle 1 \cong \angle 2$
2. $\angle 2 \cong \angle 4$
3. $m\angle 1 + m\angle 4 = 180^\circ$
4. $m\angle 2 + m\angle 3 = 90^\circ$

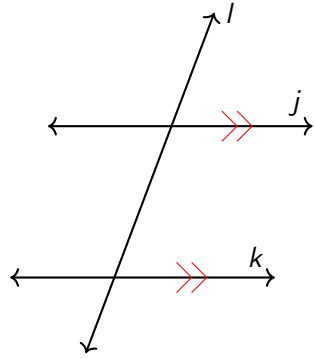


Lesson: Parallel lines crossed by a transverse line, horizontal and vertical directions

New terminology for parallel lines

Parallel lines are in the same plane and never intersect

Parallel lines	$j \parallel k$, mark with arrows
Transversal	Line l, crosses parallel lines
Interior	Inside ($\angle$s)
Exterior	Outside ($\angle$s)
Same side	On the left or right of l
Alternate	Across l from each other
Horizontal	Sideways direction
Vertical	Up and down direction



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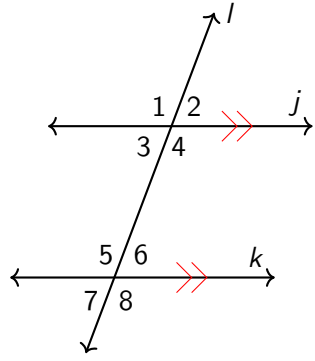


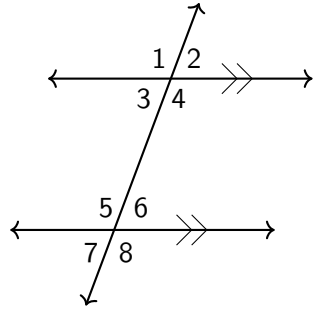
We often number the angles this way.

New theorems for parallel lines

Corresponding Having the same position. e.g. $\angle 2$ and $\angle 6$

Postulate Corresponding $\angle$s of $\parallel$ lines are congruent, $\angle 2 \cong \angle 6$

1. Alternate interior $\angle$s are $\cong$
   $\angle 4 \cong \angle 5$
2. Same-side interior $\angle$s are supplementary
   $m\angle 3 + m\angle 5 = 180$
3. Alternate exterior $\angle$s are $\cong$
   $\angle 1 \cong \angle 8$

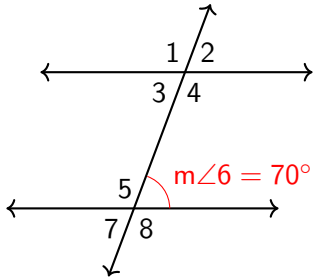


There are only two angle measures, the acute $\angle$s and the obtuse $\angle$s
And they add to $180°$, i.e. supplementary

Apply the theorems of parallel lines with a transversal

Given two parallel lines and a transversal, with $m\angle 6 = 70°$. Write down the value of each angle measure.

1. $m\angle 1 =$
2. $m\angle 2 =$
3. $m\angle 3 =$
4. $m\angle 4 =$
5. $m\angle 5 =$
6. $m\angle 6 = 70°$
7. $m\angle 7 =$
8. $m\angle 8 =$



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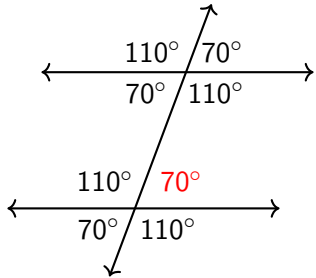


Solution

Extension: Ratios are fractions

We often state proportions as ratios

Example: Divide a distance into equal parts, i.e.

$$1 : 1$$

We say "one to one", or "in a one to one ratio."
A rectangle's length to width ratio is two to one. $2 : 1$

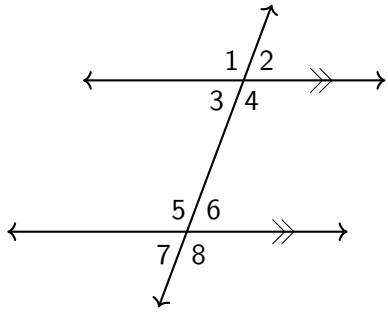
Learning Target: I can calculate transversal angles

HSG.CO.C.9 Prove theorems about lines and angles

3.2 Tuesday 18 October

Do Now: Identify each angle

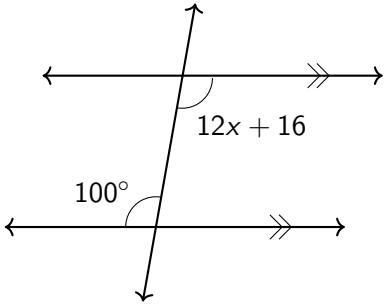
1. Opposite $\angle 4$
2. Corresponding to $\angle 3$
3. Alternate exterior to $\angle 8$
4. Same side interior to $\angle 5$
5. Alternate interior to $\angle 4$



Lesson: Solve for angle measures

Parallel lines intersected by a transversal. Find x.

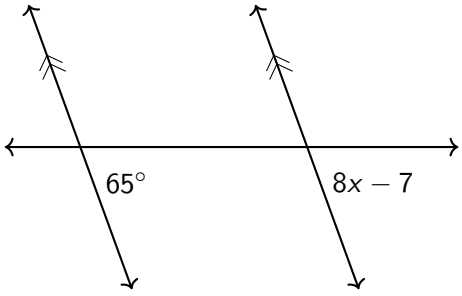
Alternate interior angles measure 100° and $12x + 16$, as shown.



Are the angles congruent or supplementary?

Parallel lines intersected by a transversal. Find x.

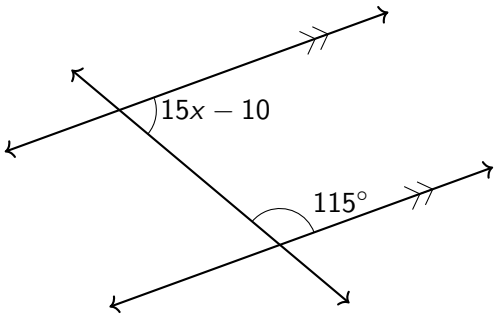
Parallel lines do not have to be horizontal.



State the postulate or theorem you are employing.

Parallel lines intersected by a transversal. Find x.

Given: Same side interior angles measure 115° and $15x - 10$.



Remember the check.

Extension: *Partitioning* a segment or angle in a ratio

Point B divides $\overline{AC}$ in a $2:1$ ratio, i.e. $AB = 2BC$
Ray $\overrightarrow{BD}$ divides $\angle ABC$ in a $2:1$ ratio. Find x.

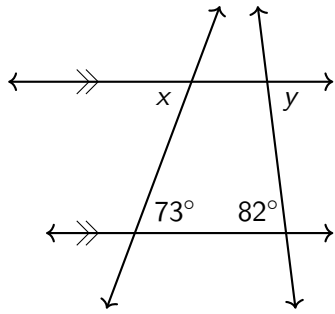
Learning Target: I can calculate triangle angles

HSG.CO.C.9 Prove theorems about lines and angles

3.3 Thursday 20 October

Do Now:

1. Given two parallel lines, two transversals
2. Find x, y
3. What relationship are you using? (e.g. vertical angles, same-side exterior angles, alternate interior angles)



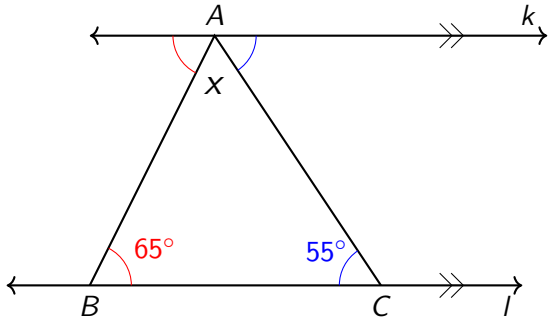
Lesson: The sum of a triangle's *interior* angles is 180°

Triangle sum theorem

Triangle sum theorem

Given parallel lines $k \parallel l$, $\triangle ABC$, $\text{m}\angle B = 65°$, $\text{m}\angle C = 55°$.

Find $\text{m}\angle BAC = x$.



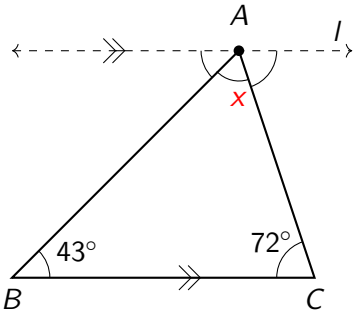
Interior The three angles that are *inside* the triangle

Theorem The sum of the measures of the three internal angles of a triangle is 180°

Mark 3 missing angle measures to make a straight angle

An *auxilary* line l is drawn through A, parallel to triangle base $\overline{BC}$.

Find m$\angle BAC$.



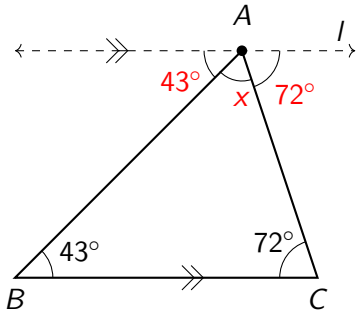
Auxilary An extra line added to a diagram

Linear triple Three adjacent angles that make a straight line

Mark 3 missing angle measures to make a straight angle

An *auxilary* line l is drawn through A, parallel to triangle base $\overline{BC}$.

Find $\text{m}\angle BAC$.



$$43 + x + 72 = 180$$

$$x = 65°$$

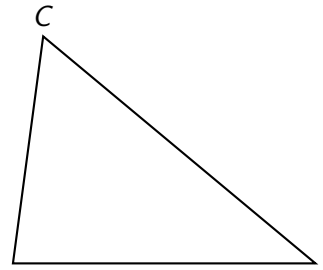
Theorem:
$\text{m}\angle A + \text{m}\angle B + \text{m}\angle C = 180°$
for any triangle

Auxilary An extra line added to a diagram

Linear triple Three adjacent angles that make a straight line

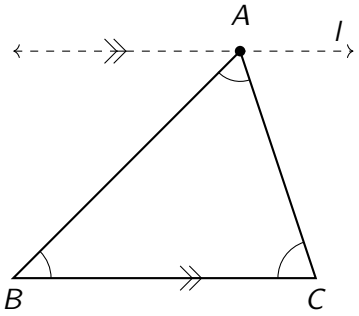
Find the missing angle measure

Given $\triangle ABC$, $\text{m}\angle A = 82°$, $\text{m}\angle C = 59°$. Find $\text{m}\angle B$.



Triangle sum theorem (180°)

Check your notes



Auxilary line An extra line added to a diagram

Linear triple Three adjacent angles that make a straight line

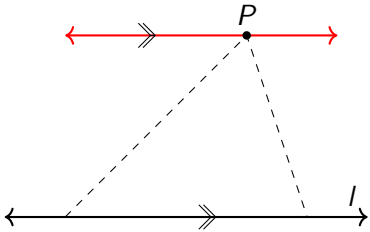
Interior angles The three angles that are inside the triangle

Theorem The sum of a triangle's angles is 180°

$$m\angle A + m\angle B + m\angle C = 180°$$

Extension: Euclid's fifth postulate (the Parallel Postulate)

Given a line and a point, there exists one line through the point parallel to the line.



Euclid Greek author of the most successful math book of all time, *The Elements*

Postulate A statement we assume is true as the basis of all further mathematical theorems and proofs

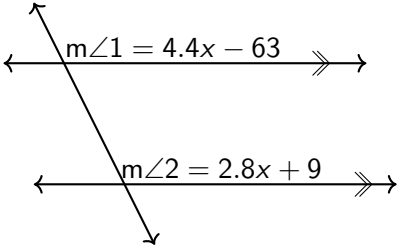
Non-Euclidean geometries Alternative mathematics not using the Parallel Postulate. Lobachevsky (1826 Russian), Bolyai (1832 Hungarian), Einstein (1916 German)

Learning Target: I can find the angles of a parallelogram

HSG.CO.C.9 Prove theorems about lines and angles 3.4 Friday 21 October

Do Now: Two parallel lines intersect a transversal. Given corresponding angles $m\angle 1 = 4.4x - 63$ and $m\angle 2 = 2.8x + 9$.

Find the measure of $\angle 1$.



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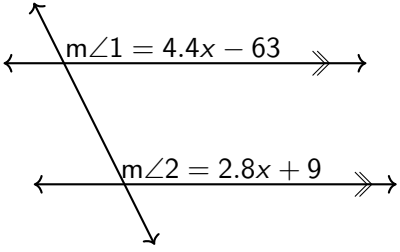


Corresponding angles are $\cong$

$$4.4x - 63 = 2.8x + 9$$
$$1.6x = 72$$
$$x = 45$$

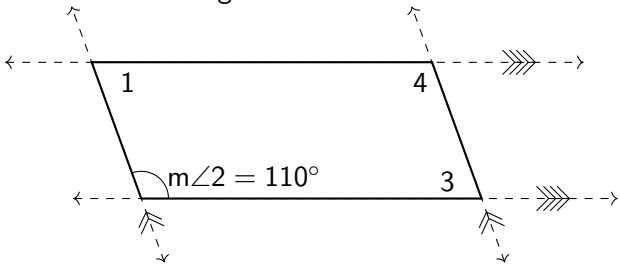
$m\angle 1 = 4.4(45) - 63 = 135^\circ$

Check: $m\angle 2 = 2.8(45) + 9 = 135$

A parallelogram's opposite sides are parallel and congruent

Consecutive angles are supplementary. Opposite angles are congruent.

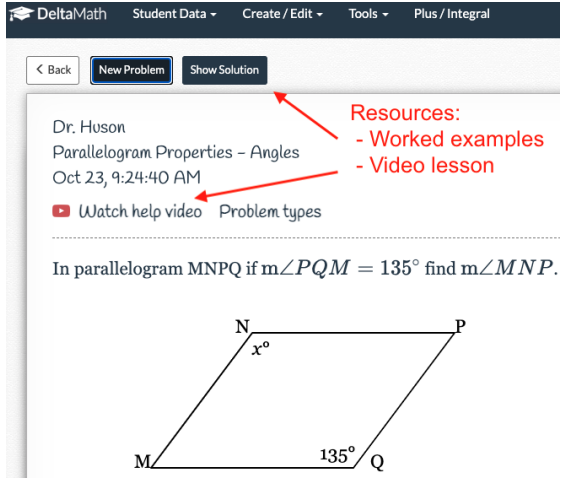
Find the other angle measures.



Classwork practice using Deltamath

Join Deltamath class and complete account setup if you haven't already

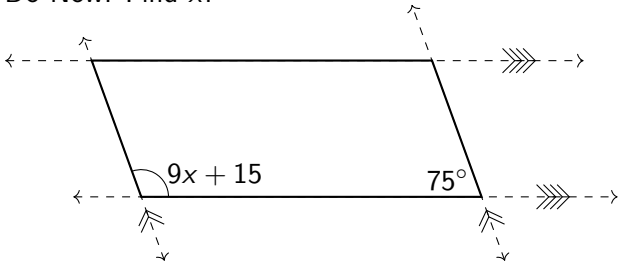
1. 3.3 Deltamath introduction
2. 3.4 Parallelogram angles
3. Forgot your Chromebook? Complete problems on paper. Finish Deltamath at home.



Learning Target: I can calculate external triangle angles

HSG.CO.C.9 Prove theorems about lines and angles 3.5 Monday 24 October

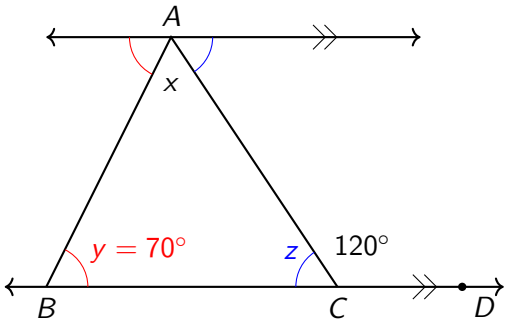
Do Now: Find x.



Lesson: Triangle external angle theorem

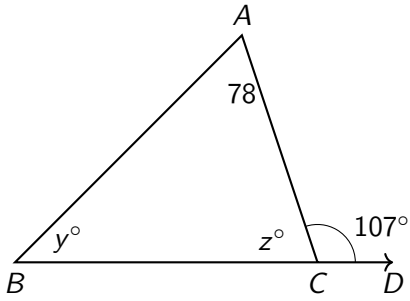
Triangle external angles

Given $\triangle ABC$, $\text{m}\angle B = 70°$, $\text{m}\angle ACD = 120°$. Find x, z.



External angle Angles with measures x, y, and z are the triangle's *internal angles*. Its *external angle* is $\text{m}\angle ACD = 120°$.

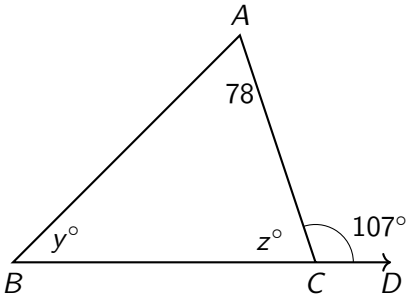
External angle $m\angle ACD = 107^\circ$, $m\angle A = 78^\circ$



External angle is a linear pair with the triangle's adjacent internal angle

Linear pair Supplementary angles that make a straight line

External angle $m\angle ACD = 107°$, $m\angle A = 78°$



$$107 + z = 180$$

$$78 + y + z = 180$$

$$z = 180 - 107 = 73°$$
$$y = 180 - 73 - 78 = 29°$$

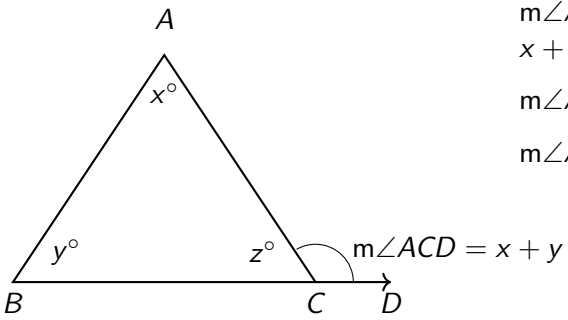
Note: $29 + 78 = 107$ Why?

External angle is a linear pair with the triangle's adjacent internal angle

Linear pair Supplementary angles that make a straight line

Triangle external angle theorem

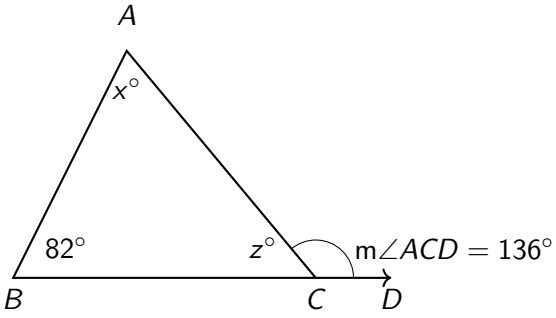
The measure of a triangle's external angle is the sum of its opposite internal angles.



$m\angle ACD + z = 180$	Linear pair
$x + y + z = 180°$	$\triangle$ sum theorem
$m\angle ACD + z = x + y + z$	Transitive prop
$m\angle ACD = x + y$	Cancel z

Apply the triangle external angle theorem

Find x.



Learning Target: I can calculate transversal angles (algebra review)

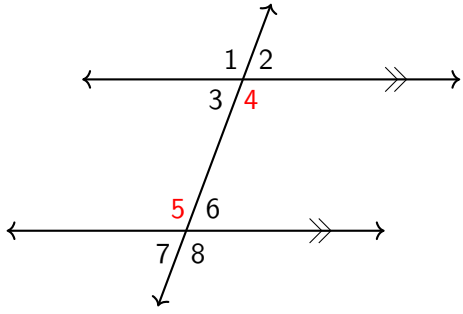
HSG.CO.C.9 Prove theorems about lines and angles

3.6 Tuesday 25 October

Given two parallel lines and a transversal,

$m\angle 4 = 3x$ and $m\angle 5 = x + 70$.

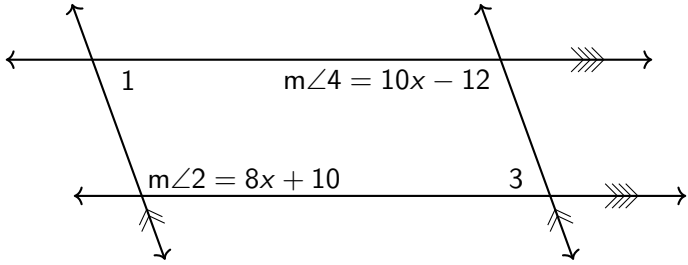
Write an equation, then solve for x.



Learning Target: I can review with my classmates

HSG.CO.C.9 Prove theorems about lines and angles 3.7 Thursday 27 October

Two parallel lines intersect a second set of parallel lines. Given $m\angle 2 = 8x + 10$ and $m\angle 4 = 10x - 12$, find the measure of $\angle 1$.



Learning Target: I can demonstrate mastery of parallel lines and transversal situations

HSG.CO.C.9 Prove theorems about lines and angles

3.8 Friday 28 October

Unit 3 Test: Parallel lines and transversals