

**Prep #16 Polynomials and algebra**

1. Simplify each expression.

(a)  $x^{\frac{2}{3}} \cdot x^{\frac{1}{3}} =$

(d)  $(x^{\frac{3}{2}}y^3)^2 =$

(b)  $x^{\frac{4}{5}} \cdot x^{\frac{6}{5}} =$

(e)  $(x^{\frac{2}{3}}y^4)^{\frac{1}{2}} =$

(c)  $\frac{\sqrt[3]{8x^2}}{\sqrt{16x}} =$

(f)  $\frac{x^{\frac{3}{4}}}{x^{\frac{1}{4}}} =$

2. Write the expression as a polynomial in standard form.

(a)  $(x - 3)(x + 3)$

(b)  $(x + y)(x^2 - xy + y^2)$

3. Simplify each complex expression to the form  $a + bi$ , with real numbers  $a$  and  $b$ .

(a)  $(2 + 3i)(3 - 4i) =$

(c)  $(2xi + 4)^2 =$

(b)  $(xi - 5)^2 =$

(d)  $-2i(\sqrt{-3} + 4i) - 5i^3$

The quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

4. Solve each equation. Expression the answer in  $a + bi$  form.

(a)  $2x^2 + 5x + 8 = 0$

(b)  $3x^2 + 7x + 5 = 0$

5. Determine the solution of each equation algebraically.

(a)  $\sqrt{3x + 7} = x - 1$

(b)  $\sqrt{4x + 1} = 11 - x$

Geometric Series:

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \text{ where } r \neq 1$$

6. Write a recursive formula for the sequence 16, 8, 0,  $-8$ ,  $\dots$

7. A sequence is defined by the recursive formula

$$\begin{aligned} a_1 &= 30 \\ a_n &= a_{n-1} + 5 \end{aligned}$$

Write an explicit formula for the sequence.

8. The sum of the first  $n$  terms of the geometric sequence beginning 1, 1.5, 2.25,  $\dots$  is 171, rounded to *the nearest integer*. Find  $n$ .

9. Complete the table for the geometric sequence  $a$ .

$n$	1	2	3	4	5
$a_n$	100	80			

Model the sequence with an exponential function.

10. A survey was conducted to compare the dietary habits of American and Japanese families. Families were asked which they had eaten for dinner most recently, meat or fish. The proportions of each answer are shown in the table below.

Nationality	Meat	Fish	Total
Americans	0.78	0.22	1.00
Japanese	0.43	0.57	1.00

- (a) Does the survey data indicate that Americans and Japanese families have similar dietary habits? Justify your answer.

- (b) 200 American families and 100 Japanese families participated in the survey. Calculate the number of each category of response and enter it in the appropriate cell in the table below.

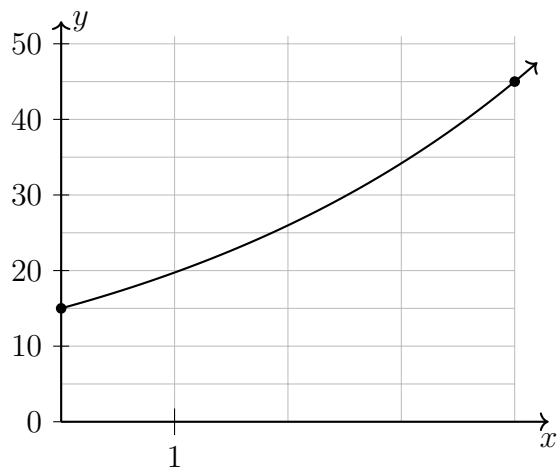
Nationality	Meat	Fish	Total
Americans			200
Japanese			100

- (c) The survey was conducted in Kansas City (an inland city) and Tokyo (a city on the Pacific Ocean). How might that affect the survey's findings?

11. An exponential function  $f(x)$  is graphed.

- (a) Write down an equation for  $f(x)$ .

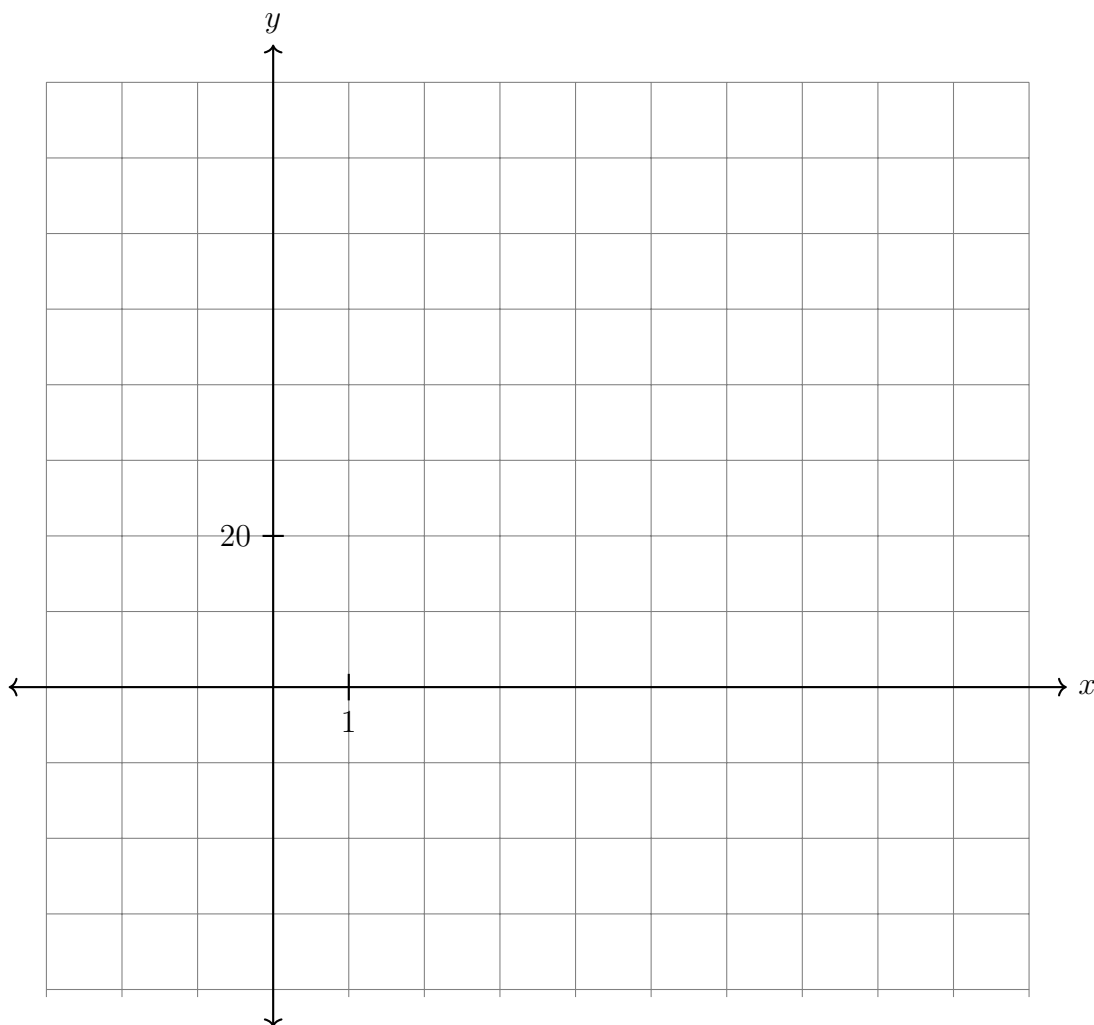
- (b) Find the average rate of change of the function over the interval  $0 < x < 4$ .



12. A manufacture of portable speakers finds that its profit varies depending on the number of speakers it makes. The profit,  $p(x)$ , in thousands of dollars as a function of the number of speakers,  $x$ , in thousands is modeled by

$$p(x) = -x^3 + 7x^2 + 6x - 25$$

- (a) Graph  $y = p(x)$ , over the interval  $0 \leq x \leq 8$  on the set of axes below.
- (b) Over the given interval, state the coordinates of the maximum of  $p$  rounding all values to the *nearest integer*. Explain what the point means in terms of the number of speakers and profit.



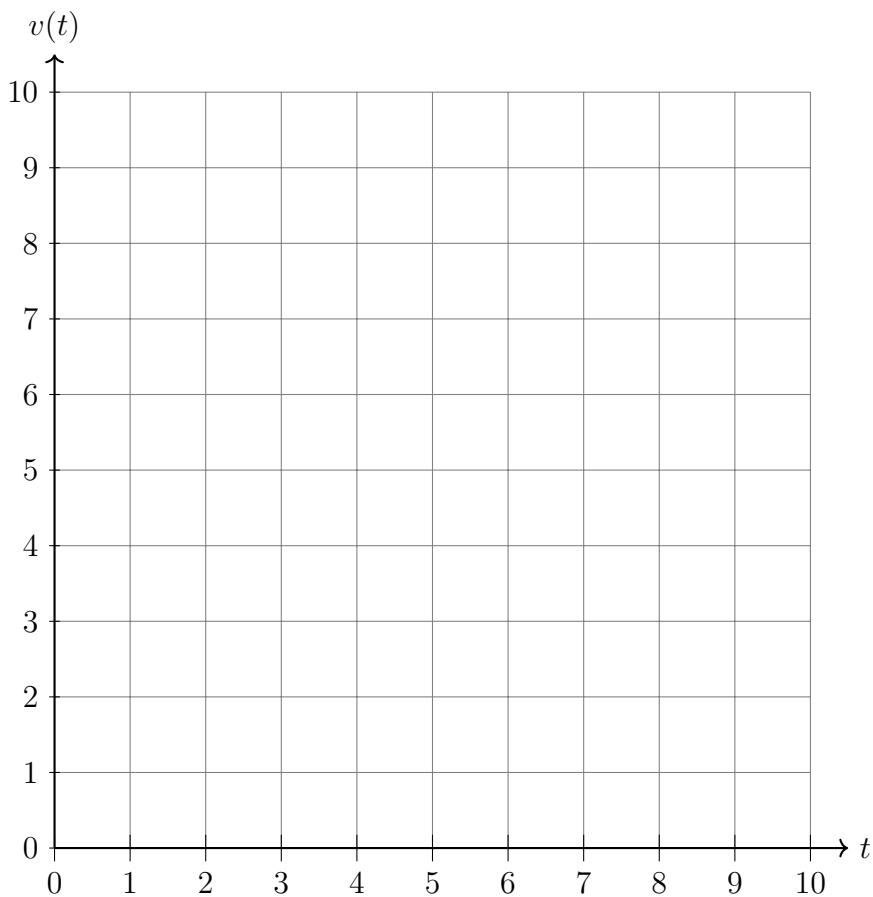
13. Over the set of integers, factor the function  $f(x) = x^4 - 5x^2 + 4$ .

14. Given the function  $P(x) = x^3 - 3x^2 - 2x + 4$ , find the value of  $P(-1)$  and  $P(1)$ .

Now identify the correct statement.

- (a)  $(x - 1)$  is a factor because  $P(-1) = 2$ .
- (b)  $(x + 1)$  is a factor because  $P(-1) = 2$ .
- (c)  $(x + 1)$  is a factor because  $P(1) = 0$ .
- (d)  $(x - 1)$  is a factor because  $P(1) = 0$ .

15. An investment of \$4000 earns a continuous interest rate of 8%. On the axes below, graph the value of the investment  $v(t)$  in thousands of dollars versus time  $t$  in years.



- (a) Find the value of the investment after 5 years rounding to the *nearest dollar*.
- (b) Find the time it will take the investment to double in value, rounded to the *nearest tenth of a year*.

16. Biologists are studying a new bacterium. They create a culture with 100 of the bacteria and anticipate that the number of bacteria will double every 30 hours. Write an equation for the number of bacteria,  $B$ , in terms of the number of hours,  $t$ , since the experiment began.
  
17. During the summer, Adam saved \$4000 and Betty saved \$3500. Adam deposited his money in Bank A at an annual rate of 2.4% compounded monthly. Betty deposited her money in Bank B at an annual rate of 4% compounded quarterly. Write two functions that represent the value of each account after  $t$  years if no other deposits or withdrawals are made, where Adam's account value is represented by  $A(t)$ , and Betty's by  $B(t)$ .

Using technology, determine, to the *nearest tenth* of a year, how long it will take for the two accounts to have the same amount of money in them. Justify your answer.