SOLUTIONS

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following table shows the probability distribution of a discrete random variable X.

X	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{3}{13}$	$\frac{1}{13}$	$\frac{4}{13}$	k

(a) Find the value of k. [3]

(b) Find $\mathrm{E}(X)$. [3]

a) $\frac{3}{13}+\frac{1}{13}+\frac{4}{13}+k=1$ (M1) (A1)

$k=\frac{5}{13}$ A1 N2

(b) $E(X)=0\frac{3}{13}+1\cdot\left(\frac{1}{13}\right)+2\left(\frac{4}{13}\right)+3\left(\frac{5}{13}\right)$ (M1) (A1)

$=\frac{24}{13}$ (≈ 1.85) A1 N2

Section A

1. (a) evidence of using $\sum p = 1$ ***(M1)***

correct working ***(A1)***

eg $\frac{3}{13}+\frac{1}{13}+\frac{4}{13}+k=1,\ 1-\frac{8}{13}$

$k=\frac{5}{13}$ ***A1 N2***

[3 marks]

(b) valid approach to find $\mathrm{E}(X)$ ***(M1)***

eg $1\times\frac{1}{13}+2\times\frac{4}{13}+3\times k$, $0\times\frac{3}{13}+1\times\frac{1}{13}+2\times\frac{4}{13}+3\times\frac{5}{13}$

correct working ***(A1)***

eg $\frac{1}{13}+\frac{8}{13}+\frac{15}{13}$

$\mathrm{E}(X)=\frac{24}{13}$ ***A1 N2***

[3 marks]

Total [6 marks]

2. (a) valid approach ***(M1)***

eg $\boldsymbol{b}=2\boldsymbol{a}$, $\boldsymbol{a}=k\boldsymbol{b}$, $\cos\theta=1$, $\boldsymbol{a}\bullet\boldsymbol{b}=-|\boldsymbol{a}||\boldsymbol{b}|$, $2p=18$

$p=9$ ***A1 N2***

[2 marks]

(b) evidence of scalar product ***(M1)***

eg $\boldsymbol{a}\bullet\boldsymbol{b}$, $(0)(0)+(3)(6)+p(18)$

recognizing $\boldsymbol{a}\bullet\boldsymbol{b}=0$ (seen anywhere) ***(M1)***

correct working ***(A1)***

eg $18+18p=0$, $18p=-18$

$p=-1$ ***A1 N3***

[4 marks]

Total [6 marks]

3. [Maximum mark: 6]

Consider the function $f(x) = \dfrac{3x+1}{x-2}$, $x \neq 2$.

(a) For the graph of f,

(i) write down the equation of the vertical asymptote;

(ii) find the equation of the horizontal asymptote. [3]

Let $g(x) = x^2 + 4$, $x \in \mathbb{R}$.

(b) Find $(f \circ g)(1)$. [3]

a) i) $x = 2$

ii) as $x \to \infty^{+/-}$ $f(x) \to \frac{3\cancel{x}}{\cancel{x}} = 3$

$y = 3$

b) $g(1) = 1^2 + 4 = 5$

$f(5) = \dfrac{3(5)+1}{5-2} = \dfrac{16}{3}$

3. (a) (i) $x = 2$ (must be an equation) ***A1*** ***N1***

(ii) valid approach ***(M1)***

eg $3 + \frac{7}{x-2}$, $x \to \infty$, $\frac{3x}{x}$, $\frac{3}{1}$, $\frac{3+\frac{1}{x}}{1-\frac{2}{x}}$, $\frac{3(x-2)+7}{x-2}$

$y = 3$ (must be an equation) ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to substitute 1 into $g(x)$ or $f(x)$ ***(M1)***

eg $1^2 + 4$, $\frac{3+1}{1-2}$

$g(1) = 5$ ***(A1)***

$(f \circ g)(1) = \frac{16}{3}$ ***A1*** ***N2***

METHOD 2

attempt to form composite function (in any order) ***(M1)***

eg $\frac{3\left(x^2+4\right)+1}{x^2+4-2}$, $\left(\frac{3x+1}{x-2}\right)^2 + 4$

correct substitution ***(A1)***

eg $\frac{3(5)+1}{5-2}$

$(f \circ g)(1) = \frac{16}{3}$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

3. [Maximum mark: 7]

Let $f(x) = \dfrac{6x-1}{2x+3}$, for $x \neq -\dfrac{3}{2}$.

(a) For the graph of f,

(i) find the y-intercept;

(ii) find the equation of the vertical asymptote;

(iii) find the equation of the horizontal asymptote. [5]

(b) Hence or otherwise, write down $\lim\limits_{x\to\infty}\left(\dfrac{6x-1}{2x+3}\right)$. [2]

a) i) $f(0) = \dfrac{6(0) \cancel{+} -1}{2(0)+3} = -\dfrac{1}{3}$

$\left(0, -\frac{1}{3}\right)$

ii) $x = -\frac{3}{2}$ $\quad\quad 2x+3=0$

$x = -\frac{3}{2}$

iii) as $x \to \infty^{+/-}$

$f(x) = \dfrac{6x-1}{2x+3} \to \dfrac{6x}{2x} = 3$

$y = 3$

b) $\lim\limits_{x\to\infty}\left(\dfrac{6x-1}{2x+3}\right) = 3$

3. (a) (i) valid method *(M1)*

eg $f(0)$, sketch of graph

y-intercept is $-\frac{1}{3}$ (exact), -0.333, $\left(0, -\frac{1}{3}\right)$ **A1 N2**

(ii) $x = -\frac{3}{2}$ (must be an equation) **A1 N1**

(iii) valid method *(M1)*

eg $\frac{6}{2}$, $f(x) = 3 - \frac{10}{2x+3}$, sketch of graph

$y = 3$ (must be an equation) **A1 N2**

[5 marks]

(b) valid approach *(M1)*

eg recognizing that $\lim\limits_{x\to\infty} f(x)$ is related to the horizontal asymptote,

table with large values of x, their y value from (a)(iii),

L'Hopital's rule $\lim\limits_{x\to\infty} f(x) = 3$.

$\lim\limits_{x\to\infty}\left(\frac{6x-1}{2x+3}\right) = 3$ **A1 N2**

[2 marks]

Total [7 marks]

4. (a) valid approach *(M1)*

eg $v(t) = 0$, sketch of graph

2.95195

$t = \log_{1.4} 2.7$ (exact), $t = 2.95$ (s) **A1 N2**

[2 marks]

(b) valid approach *(M1)*

eg $a(t) = v'(t)$, $v'(2)$

0.659485

$a(2) = 1.96\ln 1.4$ (exact), $a(2) = 0.659$ $(\mathrm{m\,s^{-2}})$ **A1 N2**

[2 marks]

(c) correct approach *(A1)*

eg $\int_0^5 |v(t)|\,dt$, $\int_0^{2.95} (-v(t))\,dt + \int_{2.95}^5 v(t)\,dt$

5.3479

distance = 5.35 (m) **A2 N3**

[3 marks]

Total [7 marks]

11. Consider the curve $y = 5x^3 - 3x$.

(a) Find $\frac{dy}{dx}$. [2]

The curve has a tangent at the point $P(-1, -2)$.

(b) Find the gradient of this tangent at point P. [2]

(c) Find the equation of this tangent. Give your answer in the form $y = mx + c$. [2]

Working:

a) $y' = 15x^2 - 3$

b) $f'(-1) = 15(-1)^2 - 3$
$= 12$

c) $y - (-2) = 12(x - (-1))$
$y = 12x + 10$

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

11. (a) $15x^2 - 3$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $15x^2$, ***(A1)*** for -3. Award at most ***(A1)(A0)*** if additional terms are seen.

[2 marks]

(b) $15(-1)^2 - 3$ ***(M1)***

Note: Award ***(M1)*** for substituting -1 into their $\frac{\mathrm{d}y}{\mathrm{d}x}$.

$= 12$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

(c) $(y - (-2)) = 12(x - (-1))$ ***(M1)***

OR

$-2 = 12(-1) + c$ ***(M1)***

Note: Award ***(M1)*** for point **and** their gradient substituted into the equation of a line.

$y = 12x + 10$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (b).

[2 marks]

Total [6 marks]

3. [Maximum mark: 6]

Consider the function $f(x) = x^2 e^{3x}$, $x \in \mathbb{R}$.

(a) Find $f'(x)$. [4]

(b) The graph of f has a horizontal tangent line at $x = 0$ and at $x = a$. Find a. [2]

a) $f'(x) = x^2(3e^{3x}) + 2x(e^{3x})$

$= (3x^2 + 2x)(e^{3x})$

b) $f'(x) = (3x+2)(x)(e^{3x}) = 0$

$x = 0$

$3x + 2 = 0$

$x = -\frac{2}{3}$

$a = -\frac{2}{3}$

3. (a) choosing product rule *(M1)*

eg $uv' + vu'$, $\left(x^2\right)'(\mathrm{e}^{3x}) + \left(\mathrm{e}^{3x}\right)' x^2$

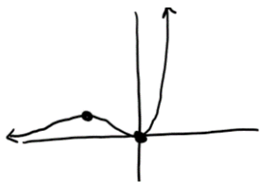
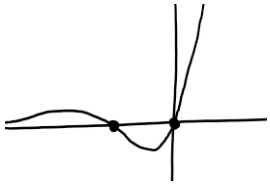
correct derivatives (must be seen in the rule) ***A1A1***

eg $2x$, $3\mathrm{e}^{3x}$

$f'(x) = 2x\mathrm{e}^{3x} + 3x^2\mathrm{e}^{3x}$ ***A1*** ***N4***

[4 marks]

(b) valid method *(M1)*

eg $f'(x) = 0$,  , 

$a = -0.667\left(= -\dfrac{2}{3}\right)$ (accept $x = -0.667$) ***A1*** ***N2***

[2 marks]

Total [6 marks]