

**2.6 Test: Sequences, Regression, Interest, Functions, Logs**

1. A geometric sequence has first term 24 and common ratio  $\frac{1}{2}$ . [10 marks]
- Write down the second and third terms.
  - Write an expression for the sum of the first  $n$  terms,  $S_n$ .
  - Hence or otherwise find the sum of the first 10 terms of the sequence  $S_{10}$ .

An arithmetic sequence has a first term 150 and fourth term 111.

- Find the common difference  $d$ .
- Find the number of the first negative term in the arithmetic sequence, i.e.  $n$  such that  $u_n < 0$ .

2. Six piano students reported their average weekly practice time and their Music class grade (out of 100). The data are shown below. [7 marks]

Practice time ( $h$ )	8	3	5	3	7	9
Grade ( $G$ )	85	70	75	85	90	95

(a) Find the Pearson product-moment correlation coefficient  $r$  for these data.

(b) Based on the following guidance:

$$0 \leq |r| < 0.4: \text{weak}, \quad 0.4 \leq |r| < 0.8: \text{moderate}, \quad 0.8 \leq |r| \leq 1: \text{strong}.$$

Comment on the strength of the correlation for this data.

(c) The relationship between  $h$  and  $G$  can be modelled by a regression equation  $G = ah + b$ . Write down the values of  $a$  and  $b$ .

(d) One of the students says she should have practised more. Based on the data and assuming that she practiced 6 hours per week, estimate what her score would have been.

3. Give all numerical answers correct to two decimal places. [8 marks]

Sofia invests \$1000 in an annuity on 1 January 2020. The investment earns a fixed amount of \$60 per year.

- (a) Find the value of her investment on 1 January 2022.

Rafael also invests \$1000 on 1 January 2020. He deposits his funds in a bank account that pays a nominal annual rate of 5% compounded monthly.

- (b) Find his balance after two years.
- (c) Determine the number of complete years from 1 January 2020 until Rafael's account first has a greater balance than Sofia's investment.

4. In an experiment the area of a bacteria culture is modelled by [4 marks]

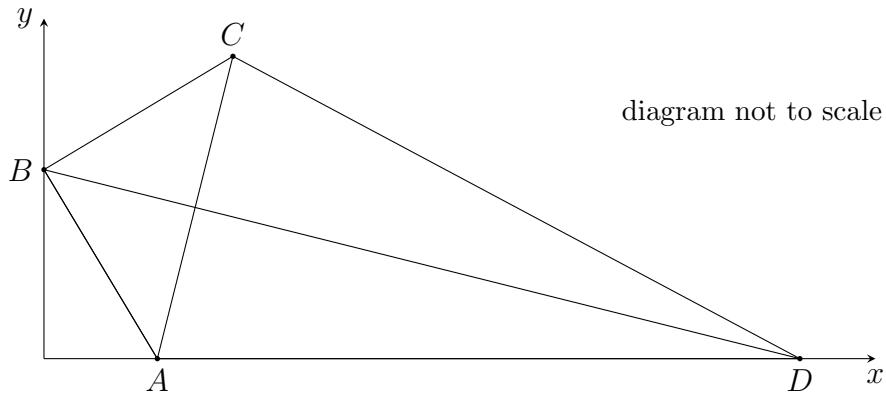
$$P(t) = Ae^{kt},$$

where  $P$  is the area in  $\text{mm}^2$  and  $t$  is the time in hours. At  $t = 0$  the area is  $80 \text{ mm}^2$ , and after 11 hours the area is  $612 \text{ mm}^2$ .

(a) Write down the value of  $A$ .

(b) Find the value of  $k$ .

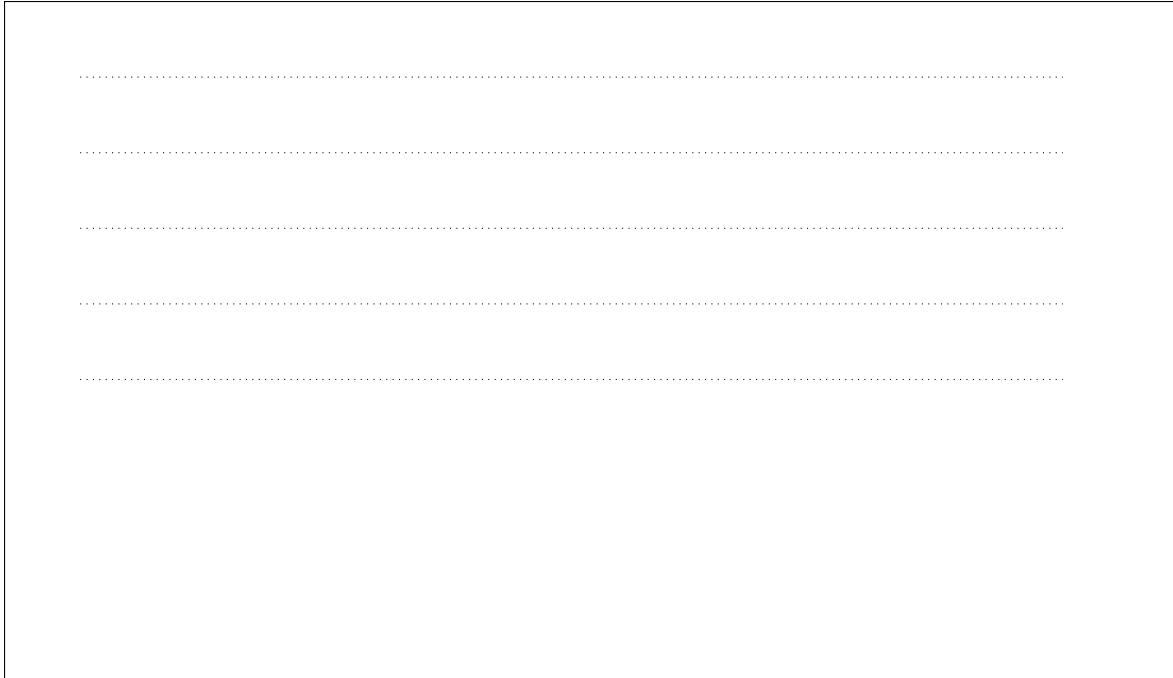
5. Camilla is designing a kite  $ABCD$  on a coordinate plane (1 unit = 10 cm). The points are  $A(3, 0)$ ,  $B(0, 5)$ ,  $C(5, 8)$ , and point  $D$  lies on the  $x$ -axis. Segment  $AC$  is perpendicular to segment  $BD$ . [6 marks]



- Find the gradient of the line through  $A$  and  $C$ .
- Hence write down the gradient of the line through  $B$  and  $D$ .
- Find the equation of line  $BD$  in the form  $ax + by + d = 0$ , where  $a, b, d$  are integers.
- Write down the  $x$ -coordinate of  $D$ .

6. Solve the following, giving exact values when possible. [6 marks]

- (a)  $3^x = 243$ .
- (b)  $\log_5(2x - 1) = 2$ .
- (c)  $\ln(4) - \ln(x) = \ln(2)$ .



The image shows a large rectangular box with a black border, occupying most of the page below the question. It is intended for students to show their working or answers.