

## 3.8 Pretest: Sequences, regression

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

No calculator on this question

## Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

In an arithmetic sequence,  $u_2 = 5$  and  $u_3 = 11$ .

- (a) Find the common difference. [2]  
 (b) Find the first term. [2]  
 (c) Find the sum of the first 20 terms. [2]

$$\begin{aligned}
 \text{(a)} \quad d &= 11 - 5 = 6 & M1 A1 \\
 \text{(b)} \quad u_1 &= 5 - 6 = -1 & M1 A1 \quad 19 \\
 \text{(c)} \quad S_{20} &= \frac{20}{2} (2(-1) + (20-1)6) & A1 \quad \overline{114} \\
 &= 1120 & A1
 \end{aligned}$$

"M" - ~~Correct~~  
 Valid  
 Method points  
 "A" - Accuracy points



## 3. [Maximum mark: 7]

Let  $g(x) = x^2 + bx + 11$ . The point  $(-1, 8)$  lies on the graph of  $g$ .

- (a) Find the value of
- $b$
- .

No calculator on this question

[3]

- (b) The graph of
- $f(x) = x^2$
- is transformed to obtain the graph of
- $g$
- .

Describe this transformation.

[4]

$$\text{(a)} \quad g(-1) = (-1)^2 + b(-1) + 11 = 8 \quad A1$$

$$1 - b + 11 = 8$$

$$b = 4 \quad A1$$

$$\text{(b)} \quad g(x) = x^2 + 4x + 11$$

$$= x^2 + 4x + 4 + 7 \quad m1$$

$$= (x+2)^2 + 7 \quad A1$$

Vertex  $(-2, 7)$  Left

Transformation: ~~right~~  $\leftarrow$   $-2$   $A1$   
 $up$   $7$   $A1$

"Translate" or "shift"

do not accept

"Move"



12EP04

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**Calculator is allowed**

### Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The number of messages,  $M$ , that six randomly selected teenagers sent during the month of October is shown in the following table. The table also shows the time,  $T$ , that they spent talking on their phone during the same month.

Time spent talking on their phone ( $T$ minutes)	50	55	105	128	155	200
Number of messages ( $M$ )	358	340	740	731	800	992

The relationship between the variables can be modelled by the regression equation  $M = aT + b$ .

- (a) Write down the value of  $a$  and of  $b$ . [3]
- (b) Use your regression equation to predict the number of messages sent by a teenager that spent 154 minutes talking on their phone in October. [3]

(a)  $a = 4.30161 \dots$   $m'$   
 $\approx 4.30$   $A1$

$b = 163.331 \dots$   
 $\approx 163$   $A1$

(b)  $M_{154} = 4.30(154) + 163$   $m1$   
 $= 825.2$   $(825.779 \dots) A1$   
 $\approx 825$  messages  $\approx 826$   $A1$

must be an integer



**Calculator is allowed**

5. [Maximum mark: 7]

The first two terms of a geometric sequence are  $u_1 = 2.1$  and  $u_2 = 2.226$ .

(a) Find the value of  $r$ . [2]

(b) Find the value of  $u_{10}$ . [2]

(c) Find the least value of  $n$  such that  $S_n > 5543$ . [3]

$$(a) r = \frac{2.226}{2.1} = 1.06 \quad M1$$

$$(b) u_{10} = 2.1(1.06^{(10-1)}) \quad A1$$

$$= 3,54791... \\ \approx 3,55 \quad A1$$

$$(c) S_n = 2.1 \left( \frac{1.06^n - 1}{1.06 - 1} \right) > 5543 \quad A1$$

$$1.06^n > 5543 / 2.1 \times 0.06 + 1 \\ = 159.371...$$

$$n \geq \log_{1.06} (159.371...)$$

$$n > 87.0316 \quad A1$$

round up

$$88 \quad A1$$



8. Siân invests 50 000 Australian dollars (AUD) into a savings account which pays a nominal annual interest rate of 5.6% compounded monthly.

- (a) Calculate the value of Siân's investment after four years. Give your answer correct to two decimal places.

[3]

After the four-year period, Siân withdraws 40 000 AUD from her savings account and uses this money to buy a car. It is known that the car will depreciate at a rate of 18% per year.

The value of the car will be 2500 AUD after  $t$  years.

**Calculator is allowed**

- (b) Find the value of  $t$ .

[3]

**Working:**

$$(a) P_4 = 50,000 \left(1 + \frac{0.056}{12}\right)^{4 \times 12}$$

$$= 62,520.96616\ldots$$

$$\approx 62,520.97$$

$$(b) P_t = 40,000 \left(1 - 0.18\right)^t = 2500$$

$$0.82^t = \frac{2500}{40000} = 0.0625$$

$$t = \log_{0.82} 0.0625$$

$$= 13.9712\ldots$$

$$\approx 14.0 \text{ years}$$

**Answers:**

- (a) ..... 62,520.97  
 (b) ..... 14.0



3. [Maximum mark: 7]

**Calculator is allowed**

Let  $f(x) = x - 8$ ,  $g(x) = x^4 - 3$  and  $h(x) = f(g(x))$ .

(a) Find  $h(x)$ . [2]

Let C be a point on the graph of  $h$ . The tangent to the graph of  $h$  at C is parallel to the graph of  $f$ .

(b) Find the  $x$ -coordinate of C. [5]

$$(a) h(x) = (x^4 - 3) - 8 \quad m1$$

$$= x^4 - 11 \quad A1$$

$$(b) \text{ slope of } f = 1 \quad A1$$

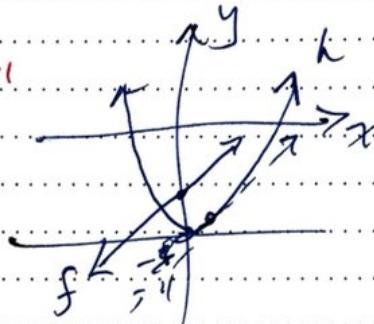
$$h'(x) = 4x^3 = 1 \quad A1$$

$$x = \sqrt[3]{\frac{1}{4}} \quad A1$$

$$= 0.629961 \dots$$

$$\approx 0.630$$

*(exact is sufficient)*



5. [Maximum mark: 6]

No calculator on this question

Consider the function  $f$ , with derivative  $f'(x) = 2x^2 + 5kx + 3k^2 + 2$  where  $x, k \in \mathbb{R}$ .

(a) Show that the discriminant of  $f'(x)$  is  $k^2 - 16$ . [2]

(b) Given that  $f$  is an increasing function, find all possible values of  $k$ . [4]

$$(a) f'(x)$$

$$a = 2$$

$$b = 5k$$

$$c = 3k^2 + 2$$

$$\Delta = (5k)^2 - 4(2)(3k^2 + 2) \quad A1$$

$$= 25k^2 - 24k^2 - 16 \quad A1$$

$$= k^2 - 16$$

$$(b) f'(x) > 0 \text{ m1} \quad \text{No solutions, } \Delta < 0 \text{ m1}$$

$$k^2 - 16 < 0$$

$$k^2 < 16$$

$$-4 < k < 4 \quad A1 \quad A1$$



4 Dec 2025

### 3.8 Pre-test Sequences, regression

Solutions

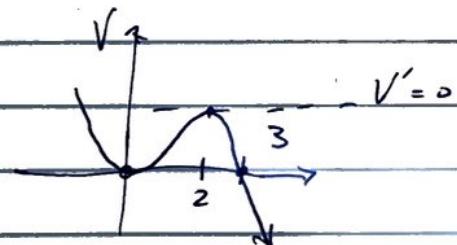
$$(a) 3x + x + y = 12$$

$$y = 12 - 4x \quad A1$$

$$(b) V = 3x \cdot x \cdot y \quad A1$$

$$= 3x^2(12 - 4x)$$

$$= 36x^3 - 12x^4 \quad A1$$



$$(c) \frac{dV}{dx} = -36x^2 + 72x \quad A1 \quad A1$$

$$(d)/i) \frac{dV}{dx} = -36x^2 + 72x = 0 \quad A1 \\ (x-2)(x)=0 \quad A1 \quad x \neq 0 \\ x=2 \quad cm$$

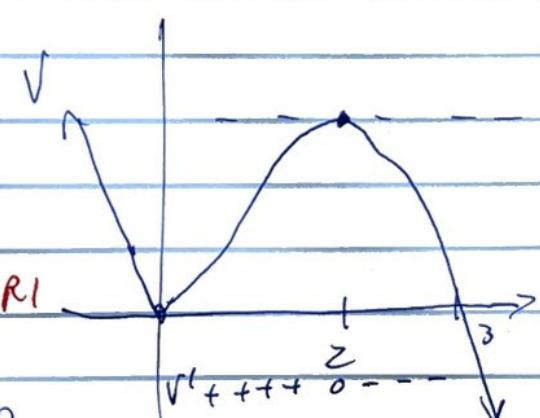
$$(e) V = 2 \cdot (3 \cdot 2) \cdot (12 - 4(2)) \quad A1 \\ = 2(6)(4) \\ = 48 \quad cm^3 \quad A1$$

$$\text{Q. ii) } V'(2) = -36(2^2) + 72(2) = 0$$

$$V'(\text{less than } 2) > 0 \quad M1$$

$$V'(\text{more than } 2) < 0 \quad A1$$

Therefore  $x=2$  is max for  $V$  R1



$$\text{or } \left. \frac{d^2V}{dx^2} \right|_{x=2} = -72x + 72 = -72 < 0$$

therefore  $V$  has a local max at  $x=2$  (concave down)