Function Operations   SOLUTIONS
5 January 2025

1 a) $f(3) = 2(3) - 7 = -1$
  b) $f(x) = 2x - 7 = 0$
       $x = 7/2$
  c) $f(1-x) = 2(1-x) - 7$
        $= -2x - 5$
  d) $f^{-1}:$ $x = 2y - 7$
        $y = \dfrac{x+7}{2}$

2. a) $g(x-3) = (x-3)^2 - 4$
        $= x^2 - 6x + 5$
   b) $g^{-1}:$ $x = y^2 - 4$
        $y = \sqrt{x+4}$ $x > -4$

3. a) $g(3) = 2(3) - 5 = 1$
   b) $f \circ g(3) = f(1) = 2 - 1^2 = 1$
   c) $(f \circ g)(x) = 2 - (2x-5)^2$
        $= -4x^2 + 20x - 23$

4. $f(x) = \dfrac{4x-2}{5}$

   $f^{-1}:$ $x = \dfrac{4y-2}{5}$

   $f^{-1}:$ $y = \dfrac{5x+2}{4}$

5. $g(x) = \frac{1}{3}x + 2$

$g(0) = \frac{1}{3}(0) + 2 = 2 \quad (0, 2)$
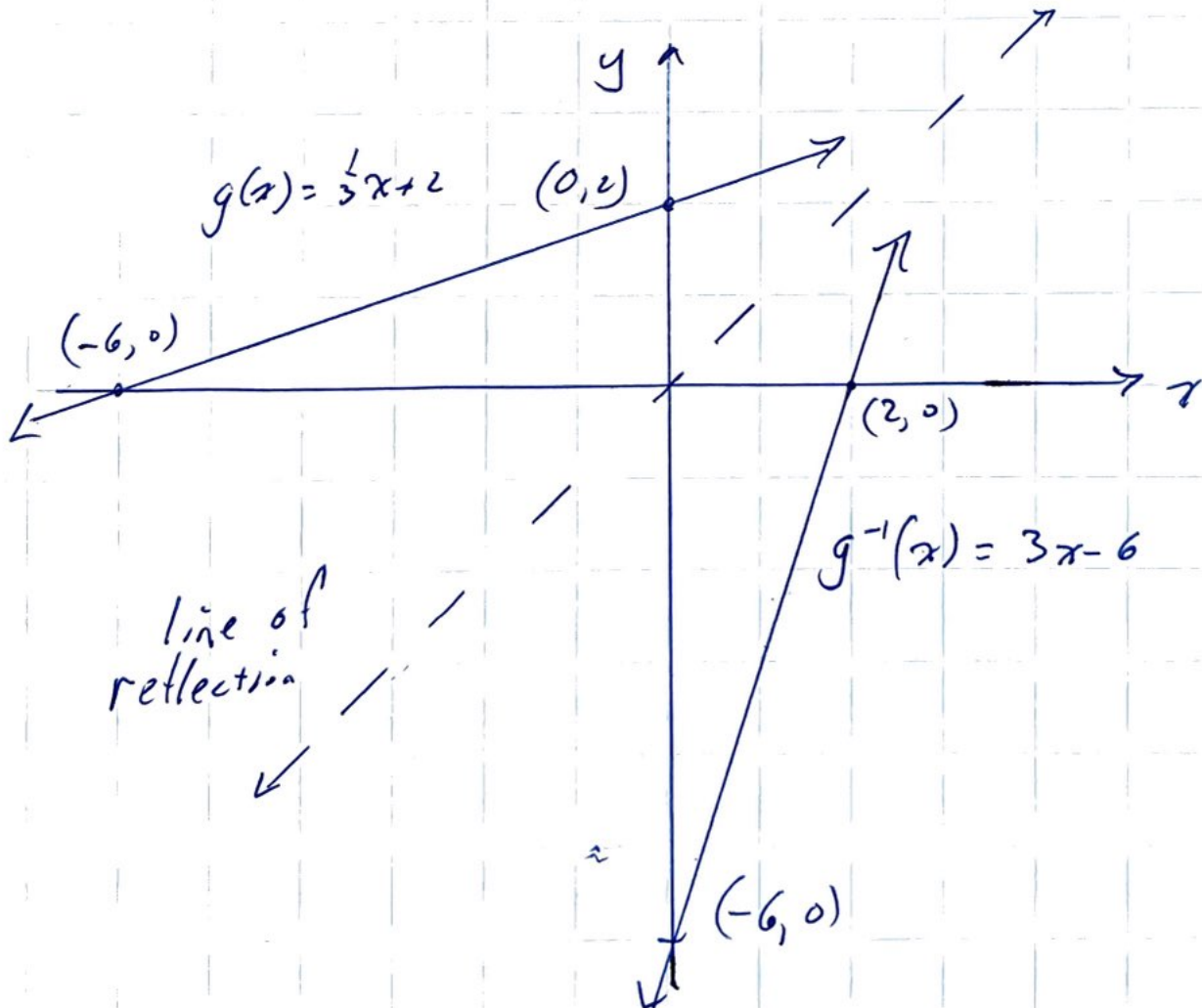
$g(x) = \frac{1}{3}x + 2 = 0$

$x = -6 \quad (-6, 0)$

$g^{-1}: \quad x = \frac{1}{3}y + 2$

$y = 3x - 6$

$g^{-1}(0) = 3(0) - 6 = -6 \quad (0, -6)$

$g^{-1}(x) = 3x - 6 = 0$

$x = 2 \quad (2, 0)$

$g(x) = \frac{1}{3}x + 2 \quad (0, 2)$

$(-6, 0)$

$(2, 0)$

$g^{-1}(x) = 3x - 6$

line of reflection

$(-6, 0)$

5. $g(x) = \frac{1}{3}x + 2$

   $g^{-1}:$ $x = \frac{1}{3}y + 2$ (see graph page)

            $y = 3x - 6$

6. a) $(f \circ g)(x) = 2(x+4) = 2x+8$

   b) $(g \circ f)(x) = (2x) + 4$

   c) $2x+8 = 2x+4$

           No Solution

7. $f(x) = x^2 - 6$

   Domain: $\mathbb{R}$

   Range $y \geq -6$

8. $f(x) = \dfrac{3x+2}{x+1}$ $x \neq -1$

   a) $x = -1$

        $y = \dfrac{2}{3}$ 3

   b)   Domain $\mathbb{R}, x \neq -1$

        Range: $\mathbb{R}$ $y \neq 3$

9. Domain: $\{x = -1, 0, 1, 3, 4\}$

   Range: $\{y = 0, 1, 2, 3\}$

10. $x = 2, y = 1$

     Domain: $\mathbb{R}, x \neq 2$

     Range: $\mathbb{R}, y \neq 1$

Graph accurately in pencil using a straight edge or smooth curve.

11. Given the graph of the function $f(x)$ shown in Figure 3

   (a) Label points on the function representing $f(-1) = -2$ and $f(4) = -1$

   (b) Graph the inverse of $f(x)$ on the same axes. Label the inverses of
       the points named in part (a)
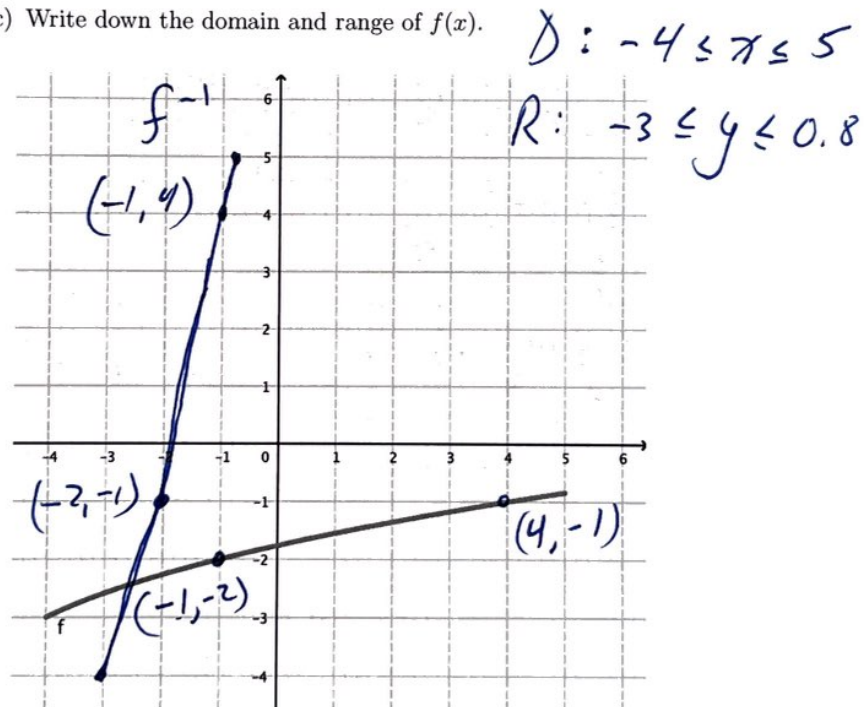
   (c) Write down the domain and range of $f(x)$.

$D: -4 \leq x \leq 5$

$R: -3 \leq y \leq 0.8$

Figure 3: Label given points and plot inverse.

2. [Maximum mark: 5]

Two functions, f and g, are defined in the following table.

x	-2	1	3	6
$f(x)$	6	3	1	-2
$g(x)$	-7	-2	5	9

(a) Write down the value of $f(1)$. [1]

(b) Find the value of $(g \circ f)(1)$. [2]

(c) Find the value of $g^{-1}(-2)$. [2]

a) $f(1) = 3$

b) $f(1) = 3$
      $g(3) = 5$

c) $g^{-1}(-2) = 1$ $\left(g(1) = -2 \right)$

Turn over