Quantifying Uncertainty: Probability Name: Solutions
29 February 2024

1.22 PreExam: Probability, Venn diagrams

1. Given:
   $U = \{\text{the letters in the alphabet}\}$ $A = \{t, i, m, e, s\}$ $B = \{m, i, n, u, s\}$

   (a) List the members of $A \cup B$. [1 mark]

   $\{t, i, m, e, s, n, u\}$

   (b) List the elements of $A \cap B$. [1 mark]

   $\{i, m, s\}$

   (c) A letter is selected at random. What is the probability that it is a member of both sets, $(A \cap B)$? [1 mark]

   $\frac{3}{26}$

2. Events A and B are independent with $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.5$. Find each probability.

   (a) $\mathrm{P}(A \cap B)$ [2 mark]

   $0.3 \times 0.5 = 0.15$

   (b) $\mathrm{P}(A \cup B)$ [2 mark]

   $0.3 + 0.5 - 0.15 = 0.65$

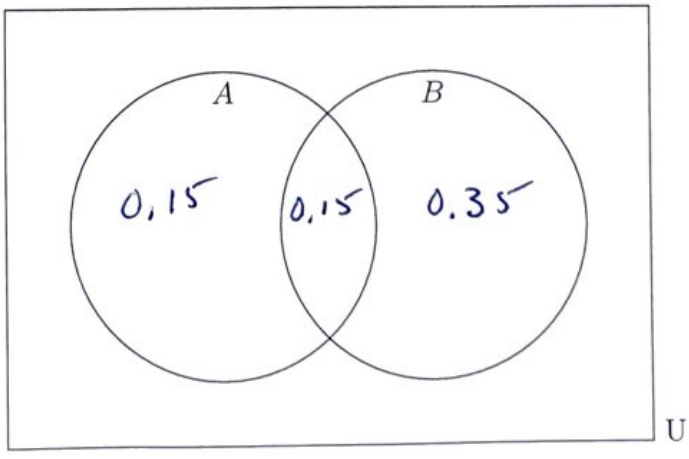
   (c) $\mathrm{P}(B' \cap A)$ [2 mark]

   $0.3 - 0.15 = 0.15$

   (d) $\mathrm{P}(A|B)$ [2 mark]

   $\frac{0.15}{0.5} = 0.3$

   (e) Mark the Venn diagram with the probabilities for each area. [2 marks]



3. The universal set U is defined as the set of positive integers less than 13.

(a) Subset is defined as $A = \{\text{multiples of three}\}$. List its elements. [1 mark]

$\{3, 6, 9, 12\}$

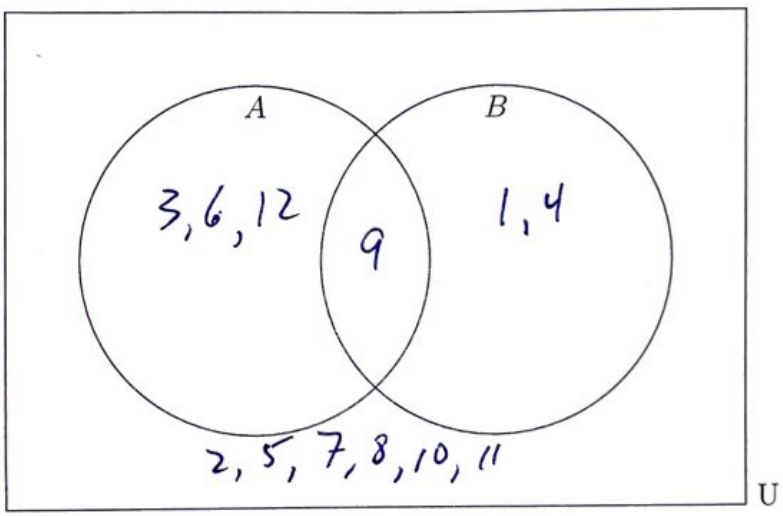
(b) Subset $B = \{\text{perfect squares}\}$. List the members of set B. [1 mark]

$\{1, 4, 9\}$

(c) List the members of $(A \cup B)'$. [1 mark]

$\{2, 5, 7, 8, 10, 11\}$

(d) Place the elements of U in the appropriate regions in the Venn diagram. [2 marks]



(e) If an element is selected at random, what is the probability that it is a member of both sets, $(A \cap B)$? [1 mark]

$\frac{1}{12}$

(f) If a member of set B is selected at random, what is the probability that it is also a member of set A, i.e. the conditional probability $(A|B)$? [2 mark]

$\frac{1}{3}$

4. A jar contains 30 marbles, 12 of which are red, 8 are blue, and 10 are green.

   (a) A marble is selected at random. Find the probability it is red. [1 mark]

   $P(\text{red}) = \frac{12}{30} = \frac{2}{5}$

   (b) The marble is replaced and a second marble is selected. Given that the second marble is not red, find the probability it is blue. [1 mark]

   $P(\text{blue} \mid \text{red}') = \frac{8}{18} = \frac{4}{9}$

   (c) The marbles are returned to the jar and two marbles are selected at random. Find the probability that both are green. [2 mark]

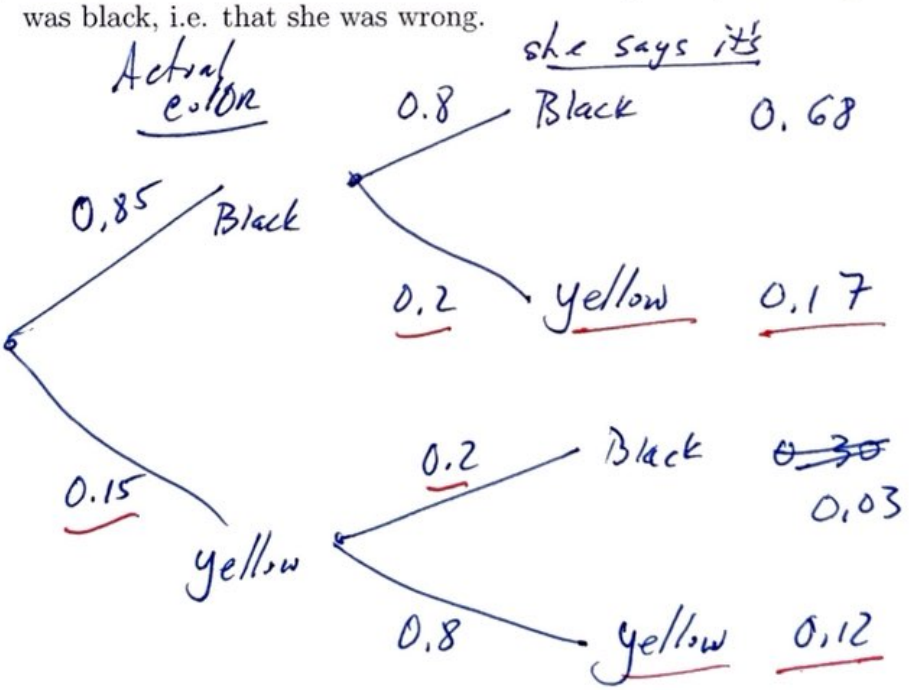
   $P(\text{green, green}) = \frac{10}{30} \cdot \frac{9}{29} = \frac{3\cancel{0}}{29\cancel{0}}$

5. Draw a tree diagram to represent the taxi cab problem in the textbook. First, there are two cab companies, 85% are black and the rest are yellow. Then, the witness identifies the color of the cab correctly 80% of the time. [3 marks]

   (a) Label the branches with the probabilities. [1 marks]

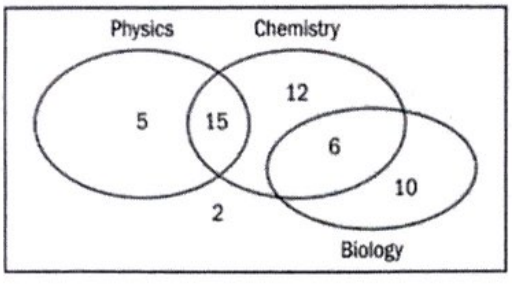
   (b) Calculate the probabilities of each four outcomes. [2 marks]

   (c) Given that the witness identified the cab as yellow, find the probability that it was black, i.e. that she was wrong. [3 marks]



$P(\text{Black} \mid \text{yellow}) = \frac{0.17}{0.17 + 0.12} = 0.586\ldots$

6. : The Venn diagram illustrates the number of students taking each of the three sciences: physics, chemistry and biology.



$n(u) = 50$

A student is randomly chosen from the group.

Find the probability that

a the student studies chemistry or biology (2 marks)

$P(C \cup B) = \frac{43}{50}$

b the student studies neither physics nor biology (2 marks)

$P((P \cup B)') = \frac{2+12}{50} = \frac{7}{25}$

c the student studies physics, given that they study chemistry (2 marks)

$P(P|C) = \frac{15}{33} = \frac{5}{11}$

d the student studies biology, given that they study physics (2 marks)

$P(B|P) = 0$

e the student studies physics, given that they do not study biology. (2 marks)

$P(P|B') = \frac{20}{37}$

7. The events A and B are mutually exclusive with $P(A) = 0.30$ and $P(B) = 0.15$.

(a) Write down $P(A \cap B)$. [1 mark]

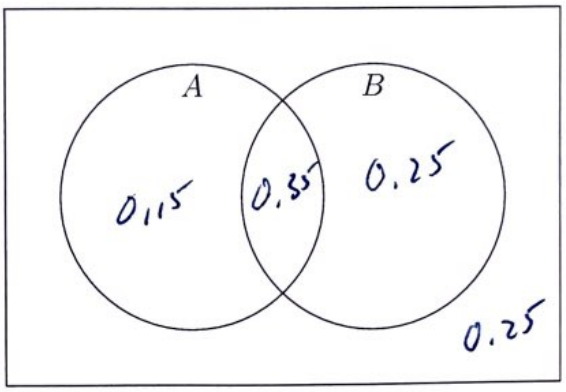
0

(b) Write down $P(A \cup B)$. [1 mark]

0.45

7. Given events A and B with $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.6$, $\mathrm{P}(A \cap B) = 0.35$.

   (a) Completely mark the Venn diagram with probabilities for each area. [2 marks]



   (b) Find $\mathrm{P}(A \cup B)$. [2 marks]

$0.5 + 0.6 - 0.35 =$ 0.75

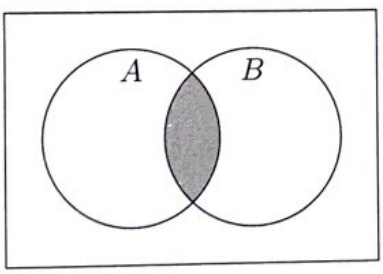
   (c) State whether events A and B are independent. Justify your answer. [3 marks]

No $0.5 \times 0.6 = 0.3 \neq 0.35$

$P(A) \times P(B) \neq P(A \cap B)$

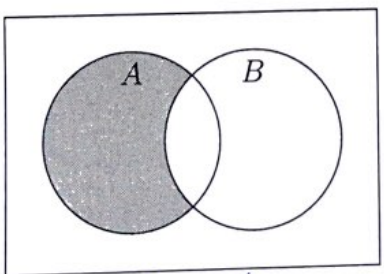
8. For each Venn diagram, write an expression representing the shaded area. [5 marks]

(a)



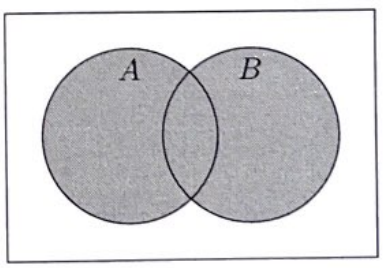
$A \cap B$

(b)



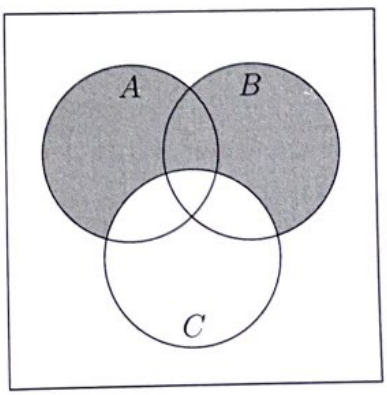
$A \cap B'$

(c)



$A \cup B$

(d)



$(A \cup B) \cap C'$

10. There are 60 students enrolled in the following courses:

- 28 take Archery
- 30 take Biology
- 22 take Calculus
- 8 take Archery and Biology
- 7 take Archery and Calculus
- 10 take Biology and Calculus
- 5 take all three classes

Complete the Venn diagram below with the number of students in each region to represent the situation. [4 marks]