4 Dec

Solutions

La Scuola d'Italia / Huson / IB Mathematics: Statistics

~~17 November~~ 2025

N19/5/MATME/SP1/ENG/TZ0/XX

3.8 Pretest: Sequences, regression

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

No calculator on this question

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

(a) Find the common difference. [2]

(b) Find the first term. [2]

(c) Find the sum of the first 20 terms. [2]

(a) $d = 11 - 5 = 6$ M1 A1

(b) $u_1 = 5 - 6 = -1$ M1 A1

$\begin{array}{r} 19 \\ 6 \\ \hline 114 \end{array}$

(c) $S_{20} = \frac{20}{2}\left(2\cdot(-1) + (20-1)6\right)$ A1

$114 - 2 = 112$

$= 1120$ A1

"M" – ~~Correct~~ Valid method points

"A" – Accuracy points

3. [Maximum mark: 7]

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of g.

x y

(a) Find the value of b. **No calculator on this question** [3]

(b) The graph of $f(x) = x^2$ is transformed to obtain the graph of g.

Describe this transformation. [4]

(a) $g(-1) = (-1)^2 + b(-1) + 11 = 8$ (M1) A1

$1 - b + 11 = 8$

$b = 4$ A1

(b) $g(x) = x^2 + 4x + 11$

$= x^2 + 4x + 4 + 7$ M1

$= (x+2)^2 + 7$ A1

Vertex $(2, 7)$

Transformation: ~~Right~~ Left 2 A1

up 7 A1

$\begin{pmatrix} -2 \\ 7 \end{pmatrix}$

"Translate" or "shift"

do not accept

"move"

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Calculator is allowed

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The number of messages, M, that six randomly selected teenagers sent during the month of October is shown in the following table. The table also shows the time, T, that they spent talking on their phone during the same month.

Time spent talking on their phone (T minutes)	50	55	105	128	155	200
Number of messages (M)	358	340	740	731	800	992

The relationship between the variables can be modelled by the regression equation $M = aT + b$.

(a) Write down the value of a and of b. [3]

(b) Use your regression equation to predict the number of messages sent by a teenager that spent 154 minutes talking on their phone in October. [3]

(a) $a = 4.30161...$
≈ 4.30

$b = 163.331...$
≈ 163

(b) $M_{154} = 4.30(154) + 163$
$= 825.2$
≈ 825 messages

$(825.779...$
$\approx 826)$

5. [Maximum mark: 7] **Calculator is allowed**

The first two terms of a geometric sequence are $u_1 = 2.1$ and $u_2 = 2.226$.

(a) Find the value of r. [2]

(b) Find the value of u_{10}. [2]

(c) Find the least value of n such that $S_n > 5543$. [3]

(a) $r = \frac{2.226}{2.1} = 1.06$

(b) $u_{10} = 2.1\left(1.06^{(10-1)}\right)$

$= 3.54791...$

≈ 3.55

(c) $S_n = 2.1\left(\frac{1.06^n - 1}{1.06 - 1}\right) > 5543$

$1.06^n > 5543/2.1 \times 0.06 + 1$

$= 159.371...$

$n > \log_{1.06}(159.371...)$

$n > 87.0316$

round up

88

8. Siân invests 50 000 Australian dollars (AUD) into a savings account which pays a nominal annual interest rate of 5.6 % **compounded monthly**.

(a) Calculate the value of Siân's investment after four years. Give your answer correct to two decimal places. **[3]**

After the four-year period, Siân withdraws 40 000 AUD from her savings account and uses this money to buy a car. It is known that the car will depreciate at a rate of 18 % per year.

The value of the car will be 2500 AUD after t years.

Calculator is allowed

(b) Find the value of t. [3]

Working:

(a) $P_4 = 50{,}000\left(1+\frac{0.056}{12}\right)^{4\cdot 12}$

$= 62{,}520.96616\ldots$

$\approx 62{,}520.97$

(b) $P_t = 40{,}000(1-0.18)^t = 2500$

$0.82^t = \frac{2500}{40000} = 0.0625$

$t = \log_{0.82} 0.0625$

$= 13.9712\ldots$

≈ 14.0 years

Answers:

(a) 62,520.97

(b) 14.0

3. [Maximum mark: 7]

Calculator is allowed

Let $f(x) = x - 8$, $g(x) = x^4 - 3$ and $h(x) = f(g(x))$.

(a) Find $h(x)$. **[2]**

Let C be a point on the graph of h. The tangent to the graph of h at C is parallel to the graph of f.

(b) Find the x-coordinate of C. **[5]**

(a) $h(x) = (x^4 - 3) - 8$

$= x^4 - 11$

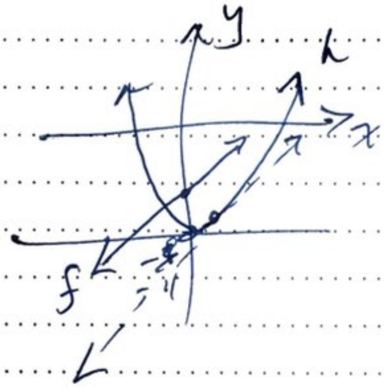
(b) $\text{slope}_f = 1$

$h'(x) = 4x^3 = 1$

$x = \sqrt[3]{\frac{1}{4}}$

$= 0.629961\ldots$

≈ 0.630



5. [Maximum mark: 6] **No calculator on this question**

Consider the function f, with derivative $f'(x) = 2x^2 + 5kx + 3k^2 + 2$ where x, $k \in \mathbb{R}$.

(a) Show that the discriminant of $f'(x)$ is $k^2 - 16$. [2]

(b) Given that f is an increasing function, find all possible values of k. [4]

(a) $f'(x)$ $a = 2$

$b = 5k$

$c = 3k^2 + 2$

$\Delta = (5k)^2 - 4(2)(3k^2+2)$ A1

$= 25k^2 - 24k^2 - 16$ A1

$= k^2 - 16$

(b) $f'(x) > 0$ M1 NO SOLUTIONS,

$\Delta < 0$ M1

$k^2 - 16 < 0$

$k^2 < 16$

$-4 < k < 4$ A1 A1

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3.8 Pretest Sequences, regression Solutions

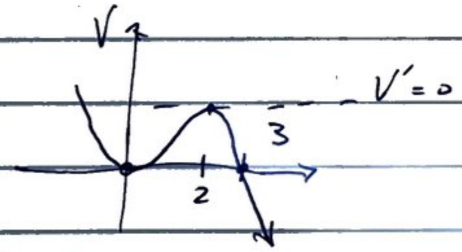
(a) $3x + x + y = 12$

$y = 12 - 4x$ A1

(b) $V = 3x \cdot x \cdot y$ A1

$= 3x^2(12 - 4x)$

$= 36x^2 - 12x^3$ A1



(c) $\frac{dV}{dx} = -36x^2 + 72x$ A1 A1

(d)(i) $\frac{dV}{dx} = -36x^2 + 72x = 0$ ~~A~~ M1

$(x-2)(x) = 0$ A1 $x \neq 0$

$x = 2$ cm

(e) $V = 2x(3 \cdot 2)(12 - 4(2))$ A1

$= 2(6)(4)$

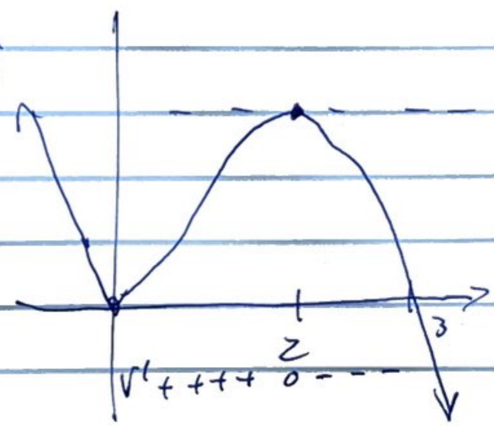
$= 48$ cm^3 A1

d ii) $V'(2) = -36(2^2) + 72(2) = 0$

$V'(\text{less than } 2) > 0$ M1

$V'(\text{more than } 2) < 0$ A1

Therefore $x = 2$ is max for V R1



or $\left.\frac{d^2V}{dx^2}\right|_{x=2} = -72x + 72 = -72 < 0$

therefore V has a local max at $x = 2$ (concave down)