BECA / Huson / Algebra 2: Complex Numbers & Rational Exponents
15 March 2024
3.21 Unit Final Exam

Solutions

Complex Numbers and Rational Exponents: End-of-Unit Assessment

Do not use a calculator.

N.RN.2

1. Select **all** expressions that are equivalent to $64^{\frac{2}{3}}$.

A. $\left(\sqrt{64}\right)^3$

B. $\left(\sqrt[3]{64}\right)^2$

C. 4^2

D. $\sqrt[3]{64^2}$

E. $\sqrt[3]{128}$

A.REI.4

2. How many real solutions does $x^2 + 8x + 20 = 0$ have?

A. 0

B. 1

C. 2

$x^2 + 8x + 16 = -4$ $\quad \left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = 4^2 = 16$

negative

A.REI.4

3. Select **all** the solutions to $(x - 2)^2 = -16$.

A. $x = 6$

B. $x = -2$

C. $x = -6$

D. $x = 2 + 4i$

E. $x = 2 + 2i$

F. $x = 2 - 2i$

G. $x = 2 - 4i$

$x - 2 = \sqrt{-16} = 4i$

$x = 2 \pm 4i$

N.CN.2

4. Let $p = 5 - 2i$ and $q = -3 + 7i$. Write each expression in the form $a + bi$:

a. $p + q$ $= (5-2i) + (-3+7i)$

$= 2 + 5i$

b. $p - q$ $= (5-2i) - (-3+7i)$

$= 8 - 9i$

c. pq $= (5-2i)(-3+7i)$

$= -15 + 35i + 6i - 14i^2$

$= -1 + 41i$

$= +14$

A.REI.2 5. a. Show how to solve the equation $\sqrt{2x+1} - 4 = -1$.

$\sqrt{2x+1} = 3$ $+4$

$2x+1 = 9$ square

$2x = 8$ -1

$x = 4$ $\times\frac{1}{2}$

check

$\sqrt{2(4)+1} - 4 \overset{?}{=} -1$

$\sqrt{8+1} - 4 = -1$

$3 - 4 \overset{\checkmark}{=} -1$

A.REI.2

b. Explain why $\sqrt{2x+1} + 4 = -1$ has no real solution.

$\sqrt{2x+1} = -5$

$2x+1 = +25$

$2x = 24$

$x = 12$

this squaring step introduces an extraneous solution

check

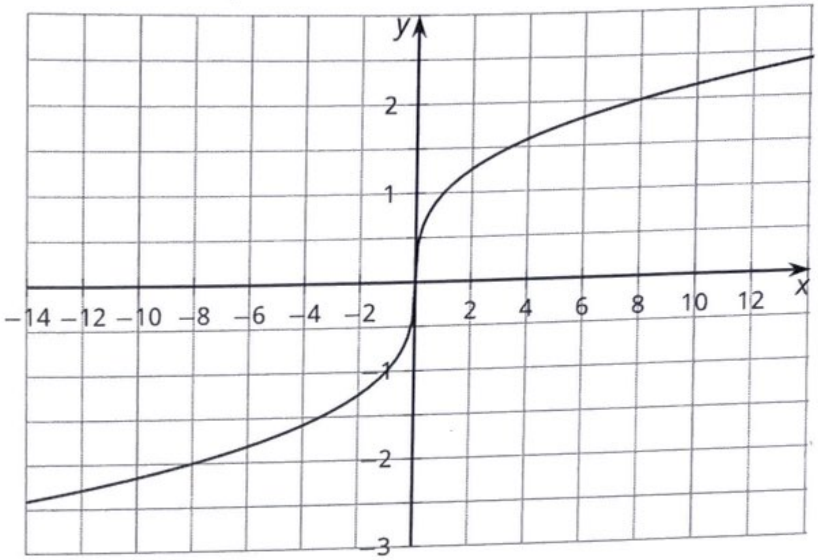
$\sqrt{2(12)+1} + 4 \overset{?}{=} -1$

$\sqrt{25} + 4 = -1$

$+5 + 4 \neq -1$ $\times$

This is the positive root

A.REI.2 6. a. Here is a graph of $g(x) = \sqrt[3]{x}$.



Use the graph of $g(x) = \sqrt[3]{x}$ to help you explain why there is only one x-intercept for every cube root function of the form $y = \sqrt[3]{x+a}$, in which a is a real number.

$\sqrt[3]{x+a} = 0$ cube

$x+a=0$

$x=-a$

The function g increases from left to right as x increases. It can only cross the x axis in one place. The $+a$ factor just sides it ~~tam~~ horizontally

b. Use the meaning of cube roots to show how to find an exact solution to the equation $\sqrt[3]{x+2} = -2$ without using a graph.

$x+2 = (-2)^3 = -8$

$x = -10$

$\sqrt[3]{(-10)+2} \stackrel{?}{=} -2$

$\sqrt[3]{-8} = -2$ ✓

c. Use the meaning of cube roots to show how to find an exact solution to the equation $\sqrt[3]{x} + 2 = -2$ without using a graph.

$\sqrt[3]{x} = -4$

$x = (-4)^3 = -64$

$\sqrt[3]{(-64)} + 2 \stackrel{?}{=} -2$

$-4 + 2 = -2$ ✓

REI. 4

7. Noah and Lin are each trying to solve the equation $x^2 - 6x + 10 = 0$. They know that the solutions to $x^2 = -1$ are i and $-i$, but they are not sure how to use this information to solve for x in their equation.

a. Here is Noah's work:

$$x^2 - 6x + 10 = 0$$
$$x^2 - 6x = -10$$
$$x^2 - 6x + 9 = -10 + 9$$
$$(x-3)^2 = -1$$

Show how Noah can finish his work using complex numbers.

$(x-3)^2 = -1$ sq. rt

$x - 3 = \pm\sqrt{-1} = \pm i$

$x = 3 \pm i$

b. Lin decides to solve the equation using the quadratic formula. Here is her work:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$
$$x = \frac{6 \pm \sqrt{36 - 40}}{2}$$

Lin knows $36 - 40$ is a negative number and isn't sure what to do next. Show how Lin can write her solution using i.

$x = \dfrac{6 \pm \sqrt{36-40}}{2}$

$= 3 \pm \frac{1}{2}\sqrt{-4}$

$= 3 \pm i$