La Scuola d'Italia / Huson / IB Mathematics
18 December 2025
8.2 Functions - Calculator (May 2021 HL Paper 1 TZ2).

2221−7211

5. [Maximum mark: 5]

Roger buys a new laptop for himself at a cost of £495. At the same time, he buys his daughter Chloe a higher specification laptop at a cost of £2200.

It is anticipated that Roger's laptop will depreciate at a rate of 10% per year, whereas Chloe's laptop will depreciate at a rate of 15% per year.

(a)   Estimate the value of Roger's laptop after 5 years.   [2]

Roger and Chloe's laptops will have the same value k years after they were purchased.

(b)   Find the value of k.   [2]

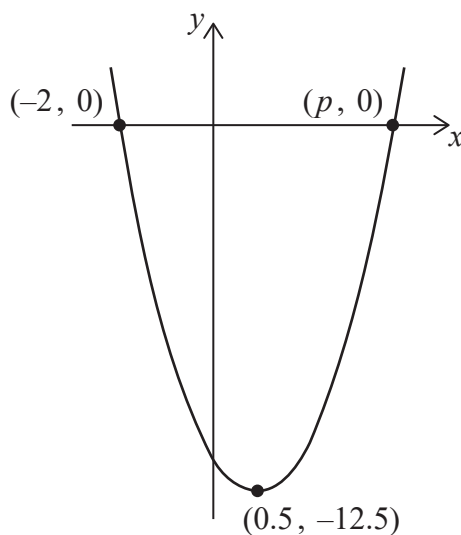(c)   Comment on the validity of your answer to part (b).   [1]

6. [Maximum mark: 7]

Consider the function $f(x) = ax^2 + bx + c$. The graph of $y = f(x)$ is shown in the diagram. The vertex of the graph has coordinates $(0.5, -12.5)$. The graph intersects the x-axis at two points, $(-2, 0)$ and $(p, 0)$.

diagram not to scale

$(-2, 0)$ $(p, 0)$

$(0.5, -12.5)$

(a) Find the value of p. [1]

(b) Find the value of

    (i) a.

    (ii) b.

    (iii) c. [5]

(c) Write down the equation of the axis of symmetry of the graph. [1]

(This question continues on the following page)

7. [Maximum mark: 6]

A meteorologist models the height of a hot air balloon launched from the ground. The model assumes the balloon travels vertically upwards and travels 450 metres in the first minute.

Due to the decrease in temperature as the balloon rises, the balloon will continually slow down. The model suggests that each minute the balloon will travel only 82% of the distance travelled in the previous minute.

(a) Find how high the balloon will travel in the first 10 minutes after it is launched. [3]

(b) The balloon is required to reach a height of at least 2520 metres.

Determine whether it will reach this height. [2]

(c) Suggest a limitation of the given model. [1]

2.   [Maximum mark: 7]

A function is defined by $f(x) = 2 - \dfrac{12}{x+5}$ for $-7 \le x \le 7$, $x \ne -5$.

(a)   Find the range of f.   [3]

(b)   Find an expression for the inverse function $f^{-1}(x)$. The domain is not required.   [3]

(c)   Write down the range of $f^{-1}(x)$.   [1]

5.   (a) $£495 \times 0.9^5 = £292 \; (£292.292...)$ **(M1)A1**

[2 marks]

(b) $495 \times 0.9^k = 2200 \times 0.85^k$ **(M1)**
      $k = 26.1 \; (26.0968...)$ **A1**

> **Note:** Award **M1A0** for $k-1$ in place of k.

[2 marks]

(c)   depreciation rates unlikely to be constant (especially over a long time period)
                              R1

> **Note:** Accept reasonable answers based on the magnitude of k or the fact
> that "value" depends on factors other than time.

[1 mark]

Total [5 marks]

6.   (a)   3   *A1*

> **Note:** Accept $(3, 0)$ seen.

                                                                                                          [1 mark]

   (b)   **METHOD 1**

   $0 = 4a - 2b + c$, $0 = 9a + 3b + c$, $-\dfrac{25}{2} = \dfrac{1}{4}a + \dfrac{1}{2}b + c$ *(M1)(A1)*

   (i)   2   *A1*

   (ii)   –2   *A1*

   (iii) –12   *A1*

> **Note:** Award the *(M1)(A1)* if at least one correct value is seen.
>   Do not apply *FT* form part (a) if workings are not shown.

   METHOD 2

   $-12.5 = a\,(0.5 + 2)(0.5 - 3)$ *(M1)*

   (i) $a = 2$ *A1*

   $0 = 2x\,(3)^2 + 3b + c$
   $0 = 2x\,(-2)^2 + (-2)b + c$ *(M1)*

   (ii) $b = -2$ *A1*

   (iii) $c = -12$ *A1*

                                                                                                          [5 marks]

   (c) $x = 0.5$ *A1*

> **Note:** Do not *FT* from their part (b), this is a contradiction with the diagram.

                                                                                                          [1 mark]

                                                                                                   Total [7 marks]

7. (a) recognition of geometric sequence *eg* $r = 0.82$ **(M1)**

$$S_{10} = \frac{450\left(1 - 0.82^{10}\right)}{1 - 0.82}$$ **(A1)**

$$= 2160 \text{ m} \ (2156.37...)$$ **A1**

[3 marks]

(b) $$S_\infty = \frac{450}{1 - 0.82}$$ **(M1)**

$$= 2500 < 2520 \text{ so the balloon will not reach the required height.}$$ **A1**

[2 marks]

(c) horizontal motion not taken into account,
rate of cooling will not likely be linear,
balloon is considered a point mass / size of balloon not considered,
effects of wind/weather unlikely to be consistent,
a discrete model has been used, whereas a continuous one may
offer greater accuracy   **R1**

Note: Accept any other sensible answer.

[1 mark]

Total [6 marks]

2. (a) $\left(f(-7) = \right) \ 8$ and $\left(f(7) = \right) \ 1$ *(A1)*

range is $f(x) \leq 1, \ f(x) \geq 8$ *A1A1*

> **Note:** Award at most **A1A1A0** if strict inequalities are used.

[3 marks]

(b) interchanging x, y at any stage *(A1)*

$$y = 2 - \frac{12}{x+5}$$

$$\frac{12}{x+5} = 2 - y$$

$$\frac{12}{2-y} = x + 5$$ *(A1)*

$$\frac{12}{2-y} - 5 = x$$

$$\left(f^{-1}(x) = \right) \ \frac{12}{2-x} - 5 \ \left(= \frac{2+5x}{2-x} \right)$$ *A1*

[3 marks]

(c) range is $-7 \leq f^{-1}(x) \leq 7, \ f^{-1}(x) \neq -5$ *A1*

[1 mark]

Total [7 marks]