

4 Dec

Solutions

3.8 Pretest: Sequences, regression

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

No calculator on this question

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

In an arithmetic sequence,  $u_2 = 5$  and  $u_3 = 11$ .

- (a) Find the common difference. [2]
- (b) Find the first term. [2]
- (c) Find the sum of the first 20 terms. [2]

(a)  $d = 11 - 5 = 6$  m1 A1

(b)  $u_1 = 5 - 6 = -1$  m1 A1 19

(c)  $S_{20} = \frac{20}{2} (2 \cdot (-1) + (20-1)6)$  A1 114

$= 1120$  A1 114 - 2 = 112

"m" - Valid method points

"A" - Accuracy points



3. [Maximum mark: 7]

Let  $g(x) = x^2 + bx + 11$ . The point  $(-1, 8)$  lies on the graph of  $g$ .

(a) Find the value of  $b$ .

No calculator on this question

[3]

(b) The graph of  $f(x) = x^2$  is transformed to obtain the graph of  $g$ .

Describe this transformation.

[4]

$$(a) \quad g(-1) = (-1)^2 + b(-1) + 11 = 8$$

$$1 - b + 11 = 8$$

$$b = 4$$

$$(b) \quad g(x) = x^2 + 4x + 11$$

$$= x^2 + 4x + 4 + 7$$

$$= (x+2)^2 + 7$$

Vertex  $(-2, 7)$  LeftTransformation: ~~right~~ 2 up 7

$$\begin{pmatrix} -2 \\ 7 \end{pmatrix}$$

"Translate" or "shift"  
do not accept  
"move"



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

Calculator is allowed

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The number of messages,  $M$ , that six randomly selected teenagers sent during the month of October is shown in the following table. The table also shows the time,  $T$ , that they spent talking on their phone during the same month.

Time spent talking on their phone ( $T$ minutes)	50	55	105	128	155	200
Number of messages ( $M$ )	358	340	740	731	800	992

The relationship between the variables can be modelled by the regression equation  $M = aT + b$ .

- (a) Write down the value of  $a$  and of  $b$ . [3]
- (b) Use your regression equation to predict the number of messages sent by a teenager that spent 154 minutes talking on their phone in October. [3]

(a)  $a = 4.30161...$   $m^1$   
 $\approx 4.30$   $A1$

$b = 163.331...$   
 $\approx 163$   $A1$

(b)  $M_{154} = 4.30(154) + 163$   $m1$   
 $= 825.2$   
 $\approx 825$  messages  $(825.779...) A1$   
 $\approx 826$   $A1$   
 must be an integer





Calculator is allowed

5. [Maximum mark: 7]

The first two terms of a geometric sequence are  $u_1 = 2.1$  and  $u_2 = 2.226$ .(a) Find the value of  $r$ . [2](b) Find the value of  $u_{10}$ . [2](c) Find the least value of  $n$  such that  $S_n > 5543$ . [3]

$$(a) \quad r = \frac{2.226}{2.1} = 1.06 \quad \text{M1} \quad \text{A1}$$

$$(b) \quad u_{10} = 2.1(1.06^{(10-1)}) \quad \text{A1}$$

$$= 3.54791...$$

$$\approx 3.55 \quad \text{A1}$$

$$(c) \quad S_n = 2.1 \left( \frac{1.06^n - 1}{1.06 - 1} \right) > 5543 \quad \text{A1}$$

$$1.06^n > 5543 / 2.1 \times 0.06 + 1$$

$$= 159.371...$$

$$n \geq \log_{1.06} (159.371...)$$

$$n > 87.0316 \quad \text{A1}$$

round up

$$88$$

A1



8. Siân invests 50 000 Australian dollars (AUD) into a savings account which pays a nominal annual interest rate of 5.6% **compounded monthly**.

- (a) Calculate the value of Siân's investment after four years. Give your answer correct to two decimal places. [3]

After the four-year period, Siân withdraws 40 000 AUD from her savings account and uses this money to buy a car. It is known that the car will depreciate at a rate of 18% per year.

The value of the car will be 2500 AUD after  $t$  years.

**Calculator is allowed**

- (b) Find the value of  $t$ . [3]

Working:

$$\begin{aligned} (a) \quad P_4 &= 50,000 \left(1 + \frac{0.056}{12}\right)^{4 \times 12} \\ &= 62,520.96616... \\ &\approx 62,520.97 \end{aligned}$$

$$\begin{aligned} (b) \quad P_t &= 40,000 (1 - 0.18)^t = 2500 \\ 0.82^t &= \frac{2500}{40000} = 0.0625 \\ t &= \log_{0.82} 0.0625 \\ &= 13.9712... \\ &\approx 14.0 \text{ years} \end{aligned}$$

Answers:

- (a) ..... 62,520.97  
(b) ..... 14.0



3. [Maximum mark: 7]

Calculator is allowed

Let  $f(x) = x - 8$ ,  $g(x) = x^4 - 3$  and  $h(x) = f(g(x))$ .

(a) Find  $h(x)$ .

[2]

Let  $C$  be a point on the graph of  $h$ . The tangent to the graph of  $h$  at  $C$  is parallel to the graph of  $f$ .

(b) Find the  $x$ -coordinate of  $C$ .

[5]

$$(a) \quad h(x) = (x^4 - 3) - 8$$

m1

$$= x^4 - 11$$

A1

$$(b) \quad \text{slope } f = 1$$

A1

$$h'(x) = 4x^3 = 1$$

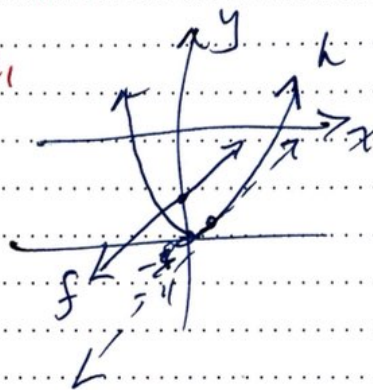
$$x = \sqrt[3]{\frac{1}{4}}$$

A1

$$= 0.629961...$$

$$\approx 0.630$$

(exact is sufficient)



5. [Maximum mark: 6]

No calculator on this question

Consider the function  $f$ , with derivative  $f'(x) = 2x^2 + 5kx + 3k^2 + 2$  where  $x, k \in \mathbb{R}$ .(a) Show that the discriminant of  $f'(x)$  is  $k^2 - 16$ . [2](b) Given that  $f$  is an increasing function, find all possible values of  $k$ . [4]

$$(a) \quad f'(x) \quad \begin{array}{l} a=2 \\ b=5k \\ c=3k^2+2 \end{array}$$

$$\Delta = (5k)^2 - 4(2)(3k^2+2) \quad A1$$

$$= 25k^2 - 24k^2 - 16 \quad A1$$

$$= k^2 - 16$$

$$(b) \quad f'(x) > 0 \quad m1 \quad \text{No solutions,} \\ \Delta < 0 \quad m1$$

$$k^2 - 16 < 0$$

$$k^2 < 16$$

$$-4 < k < 4 \quad A1 \quad A1$$





4 Dec 2025

# 3.8 PreTest Sequences, regression

Solutions

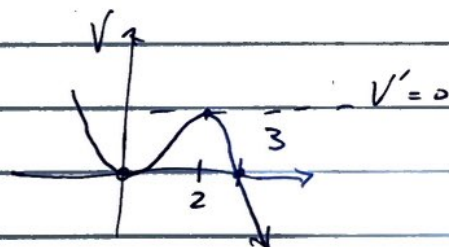
(a)  $3x + x + y = 12$

$y = 12 - 4x$  AI

(b)  $V = 3x \cdot x \cdot y$  AI

$= 3x^2(12 - 4x)$

$= 36x^2 - 12x^3$  AI



(c)  $\frac{dV}{dx} = -36x^2 + 72x$  AI AI

(d)(i)  $\frac{dV}{dx} = -36x^2 + 72x = 0$  AI MI

$(x-2)(x) = 0$  AI  $x \neq 0$

$x = 2$  cm

(e)  $V = 2 \cdot (3 \cdot 2) \cdot (12 - 4(2))$  AI

$= 2(6)(4)$

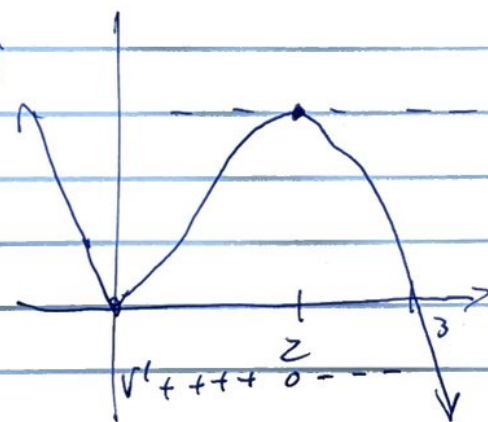
$= 48$  cm<sup>3</sup> AI

(d)(ii)  $V'(2) = -36(2^2) + 72(2) = 0$  V

$V'(\text{less than } 2) > 0$  MI

$V'(\text{more than } 2) < 0$  AI

Therefore  $x=2$  is max for V AI



or  $\frac{d^2V}{dx^2} \Big|_{x=2} = -72x + 72 = -72 < 0$

therefore V has a local max at  $x=2$  (concave down)