Jumprope
1) Quadratics - A.REI.4 # 4
2) Rational expr. - A.APR.6 # 1, 7
3) Geometric sum - F.BF.7 # 2, 5
4) Graphing - F.IF.7d # 3, 6

Illustrative Mathematics

Solutions

# 2.28 Polynomials and Rational Functions: End-of-Unit Assessment

You may use a four-function or scientific calculator, but not a graphing calculator.

A.APR.6 Rewrite rational expressions (with #7)

1. Select the expression that is equivalent to $\frac{2x^3+11x^2-21x}{x^2+3x}$ for $x \neq -3$ or $0$.

$$\begin{array}{r} 2x+5 \\ x^2+3x \overline{)\,2x^3+11x^2-21x} \\ 2x^3+6x^2 \\ \hline 5x^2-21x \\ 5x^2+15x \\ \hline -36x \end{array}$$

$\frac{-36}{x+3}$

A. $2x+5-\frac{6}{x+3}$

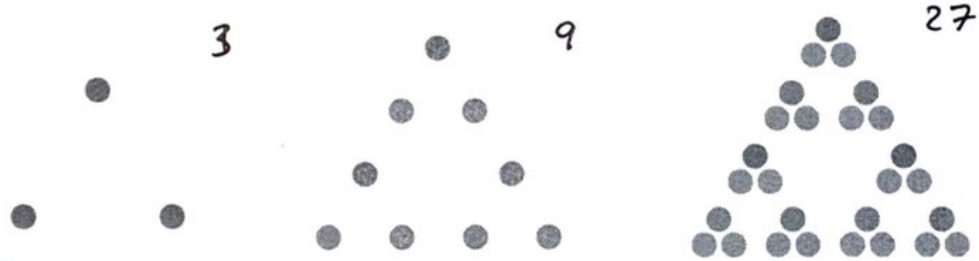
B. $2x+17-\frac{20}{x+3}$

C. $2x+17-\frac{36}{x^2+3x}$

D. $2x+5-\frac{36}{x+3}$

F.BF.7 Solve problems using the Geometric sum formula (with #5)

2. Here are the first three stages of a sequence of dots. If we drew each of the first 15 stages, how many dots would we have to draw? Select **all** of the expressions representing this number.




A. $3(1+3+3^2+\ldots+3^{14})$

B. $\frac{1-3^{15}}{1-3}$

C. $(1-3)(1+3+3^2+\ldots+3^{14})$

D. $3\frac{1-3^{15}}{1-3}$

E. $3(1-3)(1+3+3^2+\ldots+3^{14})$

$a = 3$

$r = 3$

$n = 15$

$S = 3\left(\frac{1-3^{15}}{1-3}\right)$

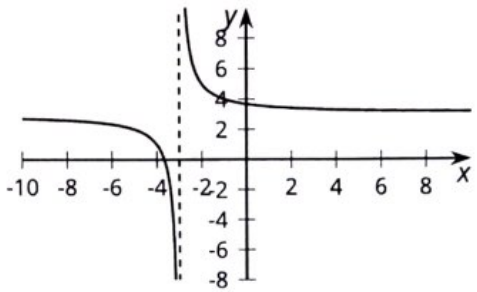
$S = 3+9+27+\ldots+3^{15}$

$= 3(1+3+3^2+\ldots+3^{14})$

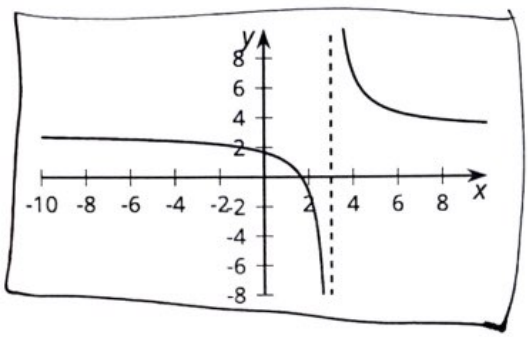
F.IF.7d Graph rational functions (with #6)

3. Which figure below is a graph of $r(x) = 2 + \frac{x+1}{x-3}$? $x \neq 3$

A.

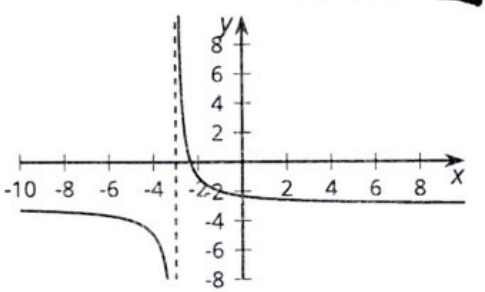



B.

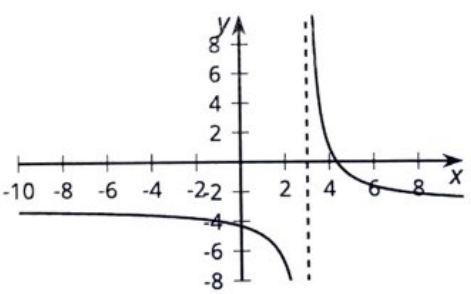



Vertical asymptote $x = 3$ (horizontal $y = 3$)

C.

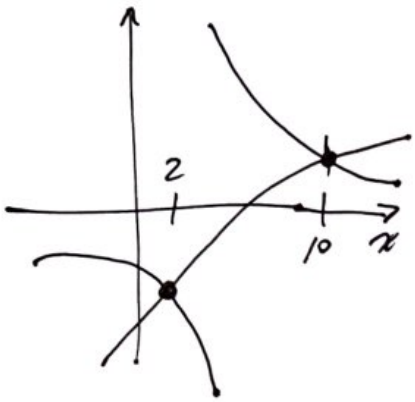



D.



A.REI, 4 Solve quadratic equations

4. Find all values of $x$ that make the equation true.




$$\frac{3}{x-4} = \frac{x-5}{x}$$

$3x = (x-4)(x-5)$

$= x^2 - 9x + 20$

$x^2 - 12x + 20 = 0$

$(x-2)(x-10) = 0$

$x = 2, 10$

Check

$\frac{3}{(2)-4} = \frac{2-5}{2}$ ?

$-\frac{3}{2} = -\frac{3}{2}$ ✓

$\frac{3}{10-4} = \frac{10-5}{10}$

$1/2 = 1/2$ ✓

5. Kiran and Mai are trying to figure out if this equation is an identity:

F.BF.7 derive/explore the geometric sum formula

$$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

Kiran picks several different values for $a$ and $b$ to test. For each pair he tries, the left side and the right side have the same value. Since his tests all worked, Kiran thinks the equation is an identity.

Mai started by using the distributive property on the expression on the left side. She then arranged the result to look the same as the right side. Since she can write both sides the same way, Mai thinks the equation is an identity.

Who do you think had a better argument for why the equation is an identity? Explain your reasoning.

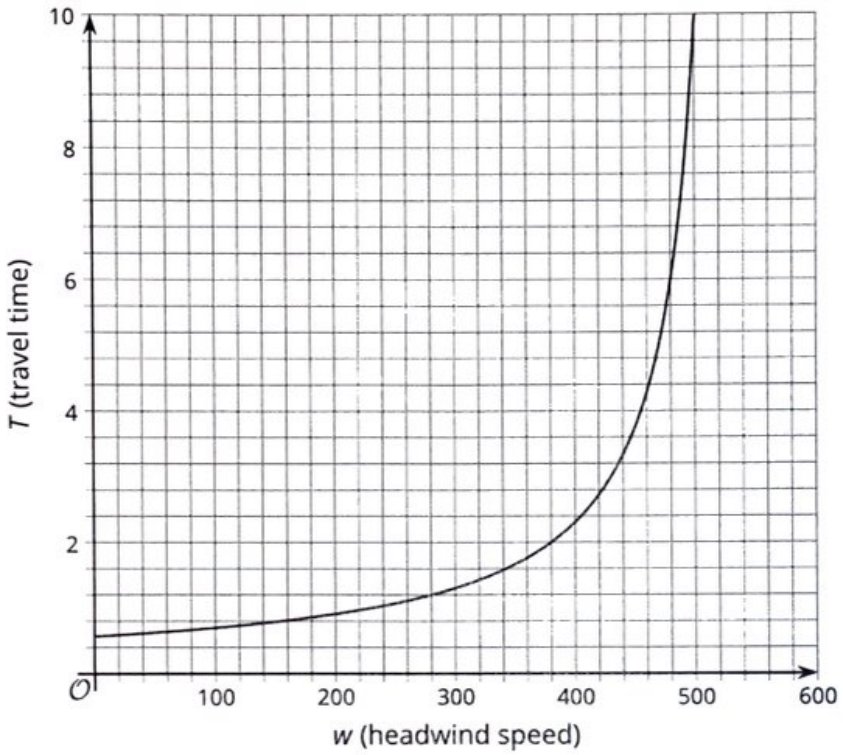
Mai's argument is better. It is more general because it applies to all possible values of a & b. Kiran's method may have missed certain values which would violate the equality.

F.IF.7d Graph rational expressions

6. A plane is going to fly 300 miles at a planned speed of 530 mph. The flight will have an average headwind of $w$ miles per hour the entire time, meaning the plane is flying directly against the wind. The time $T$, in hours, of the flight is a function of the speed of the headwind $w$, in mph, and can be modeled by

$$T(w) = \frac{300}{530 - w}$$

Here is the graph of $y = T(w)$:




a. What does $T(110)$ mean in this situation?

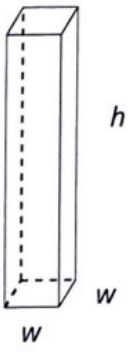
The time to fly 300 miles if the wind speed is 110 mph.

b. At what value of $w$ does the graph have a vertical asymptote? Explain how you know and what this asymptote means in the situation.

w = 530 mph (wind speed)
Algebraically at that value the denominator is zero and the function undefined. OR
With the plane's flight speed equal to the wind speed, it would never arrive

A.APR.6 Rewrite rational expressions

7. There are many square prisms with volume 125 in$^3$. Let $w$ represent the side length of the square base and $h$ represent the height in inches. Then the formula for the volume $V$, in cubic inches, of these prisms is $V = w^2h$ and the equation for the surface area $S$, in square inches, is $S = 2w^2 + 4wh$.




$V = w^2h = 125$

$h = \frac{125}{w^2}$

Write an equation for $S$ as a function of $w$ when the volume of the prism is 125 in$^3$. Explain or show your work.

$S(w) = 2w^2 + 4w\left(\frac{125}{w^2}\right)$

$= 2w^2 + \frac{500}{w}$