

# Geometry Unit 2: Angles

Bronx Early College Academy

Christopher J. Huson PhD

28 September - 7 October 2022

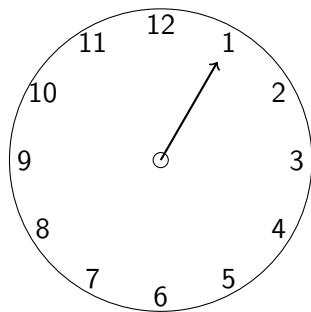
|   |              |
|---|--------------|
| 2.1 Angle notation, measures                                | 28 September |
| 2.2 Angle addition, angle pairs                             | 29 September |
| 2.3 Vertical angles   | 30 September |
| 2.4 Angle bisectors   | 3 October    |
| 2.5 Triangle sum; equilateral, isosceles $\triangle$ angles | 4 October    |
| 2.6 Review  | 6 October    |
| 2.7 Test: Angle measures                                    | 7 October    |
| Open Middle: complementary and supplementary puzzle         |              |

# Learning Target: I can measure angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.1 Wednesday 28 Sept

Do Now: Which takes longer, for a clock's hour hand to go from the 1 to the 4 or the 5 to the 9?



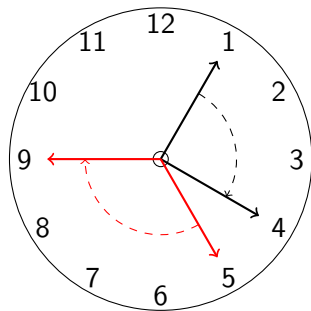
Lesson: Angle measures, internal, external, acute, obtuse, right

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2.1 Wednesday 28 Sept

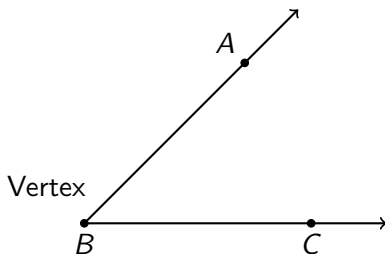
Do Now: Which takes longer, for a clock's hour hand to go from the 1 to the 4 or the 5 to the 9?



Lesson: Angle measures, internal, external, acute, obtuse, right

## Two rays with a common endpoint make an *angle*

Rays  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$ , vertex  $B$ .



**Angle** Two rays with a common endpoint,  $\angle ABC$  or  $\angle B$

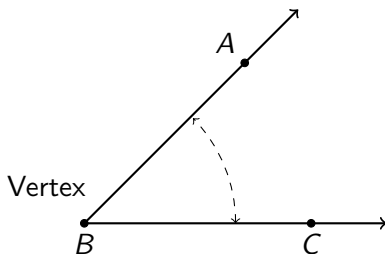
**Vertex** The common end point of two rays making an angle

**Interior** Inside, the area between the two rays

**Exterior** Outside, the area in the angle interior

## Two rays with a common endpoint make an *angle*

Rays  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$ , vertex  $B$ .



**Angle** Two rays with a common endpoint,  $\angle ABC$  or  $\angle B$

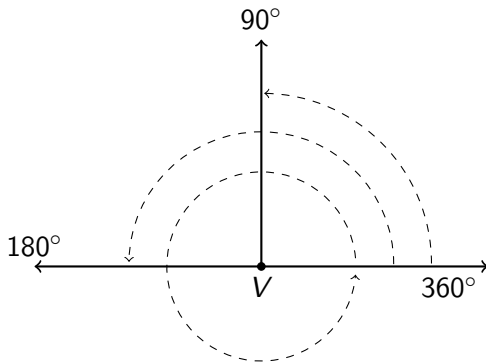
**Vertex** The common end point of two rays making an angle

**Interior** Inside, the area between the two rays

**Exterior** Outside, the area in the angle interior

**$m\angle A$**  The “measure” of angle  $A$ , how big it is

## Babylonian measures: $360^\circ$ in a circle



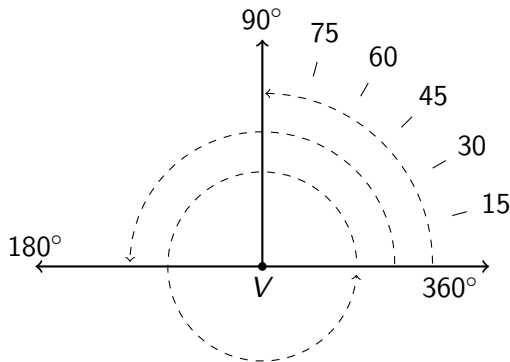
**Full turn** A complete rotation,  $360^\circ$

**Half turn** A straight line,  $180^\circ$

**Quarter turn** A *right* angle,  $90^\circ$

**Protractor** A tool for measuring angles

## Babylonian measures: $360^\circ$ in a circle



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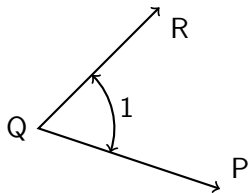
**Quarter turn** A *right* angle,  $90^\circ$

**Protractor** A tool for measuring angles



# Angle terminology and notation


Write definitions in your notebook



Angle  $Q$ , written  $\angle Q$  (also  $\angle PQR$ ,  $\angle 1$ )

Point  $Q$  is the *vertex*

The sides or *legs* are  $\overrightarrow{QR}$ ,  $\overrightarrow{QP}$

**Right angle** measuring  $90^\circ$ , mark as small square 

**Perpendicular** lines meet at right angles.  $\overline{AB} \perp \overline{CD}$

**Acute** angles measure  $< 90^\circ$

**Obtuse** angles are  $90^\circ < m\angle < 180^\circ$

**Straight angle** or a straight line measures  $180^\circ$

**Reflex angles** measure  $180^\circ < m\angle < 360^\circ$

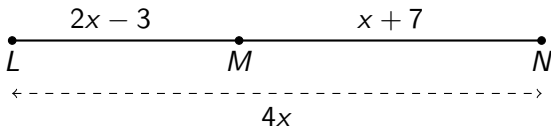
## Learning Target: I can solve for angle measures

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.2 Thursday 29 Sept

Do Now: Given  $\overline{LMN}$ ,  $LM = 2x - 3$ ,  $MN = x + 7$ ,  $LN = 4x$ .

Find  $x$ . *Don't forget to check the solution.*

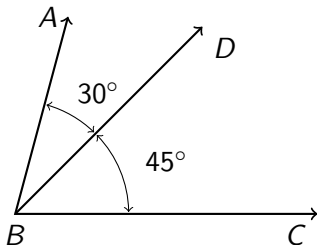


Name the geometry *postulate* that is the basis for this problem.

Lesson: Angle addition postulate, complementary, supplementary angles, linear pairs

## Angle addition postulate

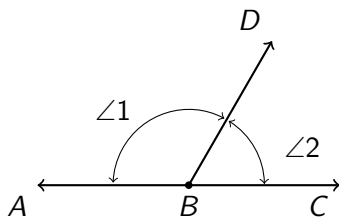
$m\angle ABD = 30^\circ$ ,  $m\angle DBC = 45^\circ$ . Find  $m\angle ABC$ .



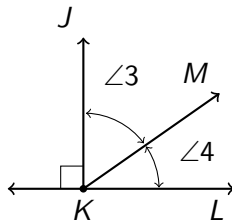
**Angle addition** The sum of the measures of *adjacent* angles is the measure of their combined angle. (postulate)

**Adjacent** “next to” each other. Adjacent angles share a common ray and are external to each other.

# Special angle pairs



Linear pair, supplementary  $\angle$ s



Complementary angles

**Linear pair** Two adjacent angles that make a straight line

**Opposite rays** collinear with a common endpoint.  $\overrightarrow{BA}$ ,  $\overrightarrow{BC}$

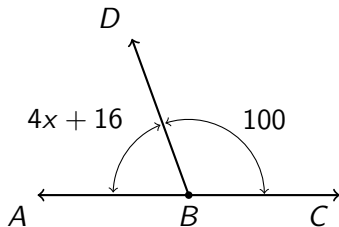
**Supplementary** Angles whose measures sum to  $180^\circ$

**Complementary** Angles whose measures sum to  $90^\circ$

**Adjacent** “next to” each other. Adjacent angles share a common ray and are external to each other.

Given two supplementary angles, a linear pair.

$m\angle ABD = 4x + 16$ ,  $m\angle CBD = 100$ . Find  $x$ .

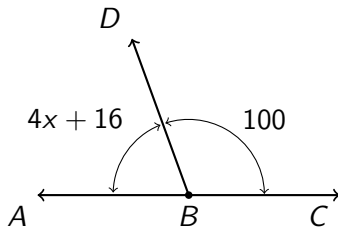


Given two supplementary angles, a linear pair.

$m\angle ABD = 4x + 16$ ,  $m\angle CBD = 100$ . Find  $x$ .

Solution:

$$m\angle ABD + m\angle CBD = 180$$



Given two supplementary angles, a linear pair.

$m\angle ABD = 4x + 16$ ,  $m\angle CBD = 100$ . Find  $x$ .

Solution:

$$m\angle ABD + m\angle CBD = 180$$

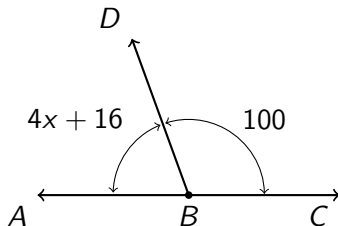
$$(4x + 16) + 100 = 180$$

...

$$x = 16$$

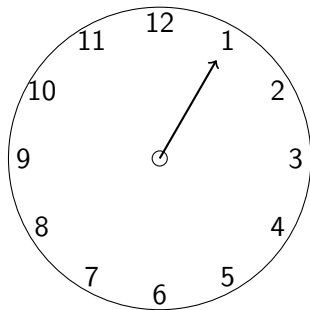
Check:

$$[4(16) + 16] + 100 = 180 \checkmark$$



## Extension (optional problems)

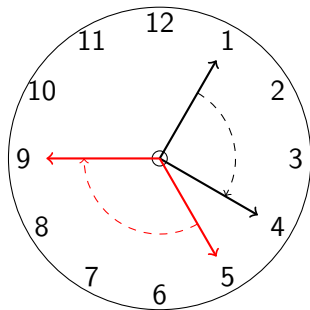
At midnight both the clock's minute hand and hour hand point at the same direction. When is the next time the clock hands coincide?





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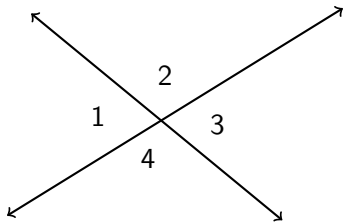


## Learning Target: I can identify vertical angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.3 Friday 30 September

Definition: *Vertical angles* are angles opposite each other when two lines intersect.  $\angle 1$  and  $\angle 3$  are vertical angles, as are  $\angle 2$  and  $\angle 4$ .

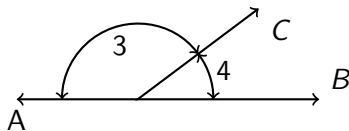
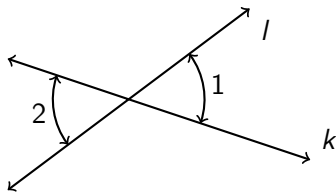


Lesson: Vertical angles

## Write down definitions in your notebook

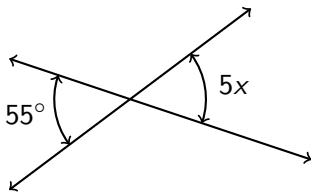
### Angle pairs

1. *Adjacent* angles share a leg (“next to each other”)
2. *Complementary* angles measures sum to  $90^\circ$
3. *Supplementary* angles sum to  $180^\circ$
4. *Vertical* or opposite angles made by intersecting lines (1, 2)
5. *Linear pairs*, adjacent angles making a straight line (3, 4)



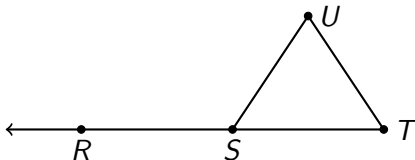
## Angle pairs: check your knowledge

1. *Complementary* angles sum to how many degrees?
2. *Supplementary* angles sum to how many degrees?
3. Given complementary angles  $\angle A$  and  $\angle B$  with  $m\angle A = 30^\circ$ . Find  $m\angle B$ .
4. Given  $m\angle A = 100^\circ$  and  $m\angle B = 2x$ . Find  $x$  such that angles  $\angle A$  and  $\angle B$  are supplementary.
5. Given vertical angles as shown. Find  $x$ .



## Angle pairs: apply your knowledge

### Triangle external angle situation



1. Given  $m\angle RSU = 115^\circ$ . Find  $m\angle TSU$
2. Given  $S$  bisects  $\overline{RT}$ ,  $RS = \frac{1}{5}(x + 8)$  and  $ST = x$ . Find  $RT$ .

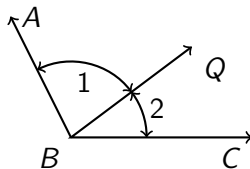
## Write down definitions in your notebook

A postulate is a fundamental statement we agree is true

1. *Scalene* triangles have three unequal sides
2. *Horizontal*, sideways or level
3. *Vertical*, straight up and down
4. An angle's *measure*, it's size, is written  $m\angle$

5. *Angle Addition Postulate*  
Measures of adjacent angles  
sum to the resulting angle

$$m\angle 1 + m\angle 2 = m\angle ABC$$

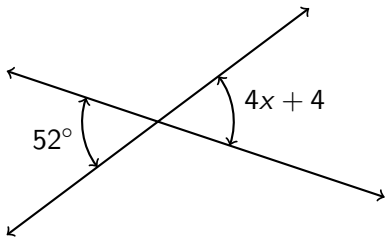


## Learning Target: I can bisect angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.4 Monday 3 October

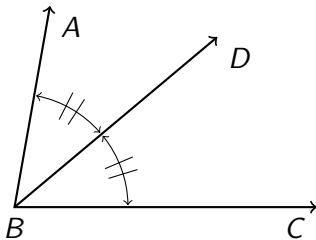
Do Now: Given vertical angles measuring  $4x + 4$  and  $52^\circ$ . Find  $x$ .



Lesson: Angle bisector situations

## Bisect an angle by dividing it exactly in half

$\overrightarrow{BD}$  bisects  $\angle ABC$  if and only if  $\angle ABD \cong \angle CBD$ .



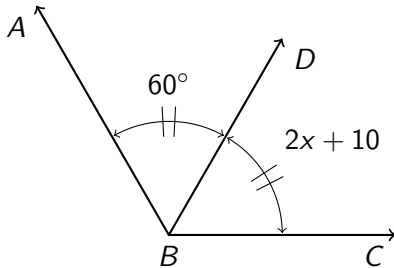
**Angle bisector** ray dividing an angle into two congruent angles

**Hash marks** mark congruent angles



## Model angle situations with algebra, then solve

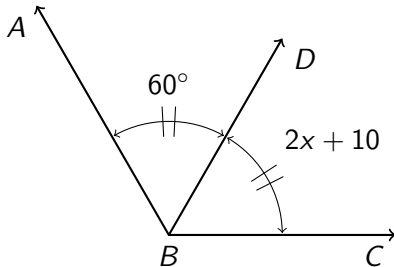
Given angle bisector  $\overrightarrow{BD}$  with  $m\angle ABD = 60^\circ$  and  $m\angle CBD = 2x + 10$ . Find  $x$ .



## Model angle situations with algebra, then solve

Given angle bisector  $\overrightarrow{BD}$  with  $m\angle ABD = 60^\circ$  and  $m\angle CBD = 2x + 10$ . Find  $x$ .

Solution:



$$\angle ABD \cong \angle CBD$$

$$2x + 10 = 60$$

$$2x = 50$$

$$x = 25$$

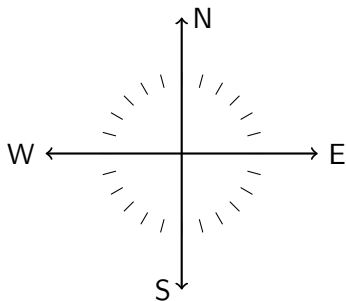
Check:

$$2(25) + 10 = 60? \checkmark$$

## Extension: Use angles for compass directions

North South East West, points of the compass

Directions are measured relative to North



**Bearing** The direction as an angle *clockwise* from north

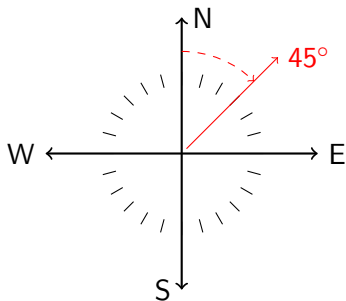
**Clockwise** The direction the clocks turn, “to the right” (tighten)

**Counterclockwise** Opposite of clocks, “to the left” (loosen)

## Extension: Use angles for compass directions

North South East West, points of the compass

Directions are measured relative to North



“Northeast,” half way between north and east, i.e. bearing  $45^\circ$

north is  $0^\circ$

east is  $90^\circ$

south is  $180^\circ$

west is  $270^\circ$

**Bearing** The direction as an angle *clockwise* from north

**Clockwise** The direction the clocks turn, “to the right” (tighten)

**Counterclockwise** Opposite of clocks, “to the left” (loosen)

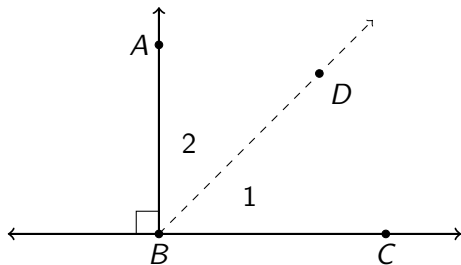
# LT: I can work with equilateral and isosceles-right $\triangle$ s

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.5 Tuesday 4 October

Do Now: Given perpendiculars  $\overrightarrow{AB} \perp \overrightarrow{BC}$ , and that the ray  $\overrightarrow{BD}$  bisects  $\angle ABC$ , making two angles,  $\angle 1$  and  $\angle 2$ .

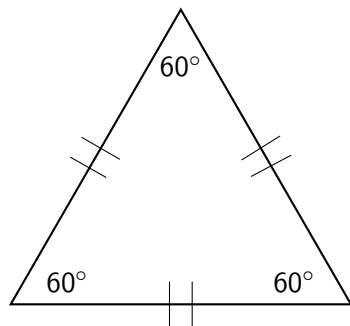
Find the measures of  $\angle 1$ ,  $\angle 2$ .



Lesson: Isosceles base theorem, special triangles

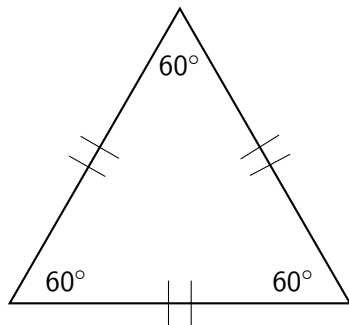
$60^\circ - 60^\circ - 60^\circ$ ,  $30^\circ - 60^\circ - 90^\circ$ ,  $45^\circ - 45^\circ - 90^\circ$

# Equilateral $\triangle$ , special relationships and measures

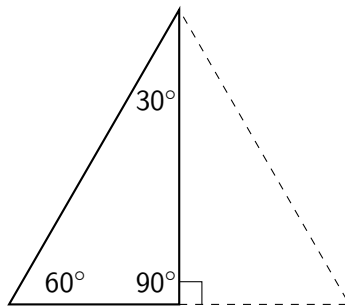


$$60^\circ - 60^\circ - 60^\circ$$

## Equilateral $\triangle$ , special relationships and measures



$$60^\circ - 60^\circ - 60^\circ$$

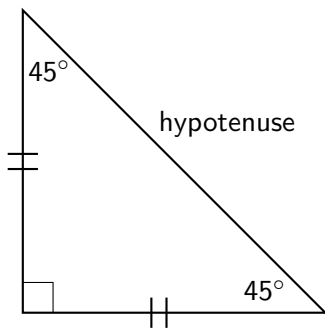


$$30^\circ - 60^\circ - 90^\circ$$

**Equiangular** means having equal angles

**Equilateral** having equal sides

## Isosceles-right triangles' angles measure $45^\circ - 45^\circ - 90^\circ$



**Hypotenuse** the longest side of a right triangle, opposite the  $90^\circ$  angle



# Angle relationships

Review: Angle postulates and theorems you have learned.

1.  $\perp$  lines and complementary  $\angle$ s make  $90^\circ$
2. linear pairs add to  $180^\circ$
3. vertical  $\angle$ s are  $\cong$
4. definition of an angle bisector

# Open Middle problem (fun)

Use digits from 0 to 9. Using a digit no more than once.

The first two angle measures are complementary. The second two angles supplementary. (degrees)

|  |  |  |  |  |  |
|--|--|--|--|--|--|
|  |  |  |  |  |  |
|  |  |  |  |  |  |