

# Geometry Unit 2: Angles

Bronx Early College Academy

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28 September - 7 October 2022

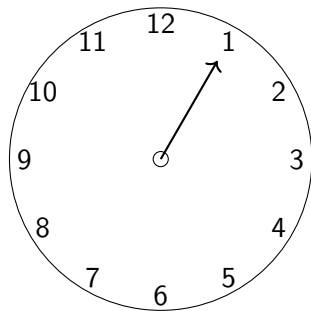
2.1 Angle notation, measures	28 September
2.2 Angle addition, angle pairs	29 September
2.3 Vertical angles	30 September
2.4 Angle bisectors	3 October
2.5 Equilateral, isosceles $\triangle$ angles	4 October
2.6 Review	6 October
2.7 Unit 2 test: Angle measures	7 October
Open Middle: complementary and supplementary puzzle	

# Learning Target: I can measure angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.1 Wednesday 28 Sept

Do Now: Which takes longer, for a clock's hour hand to go from the 1 to the 4 or the 5 to the 9?



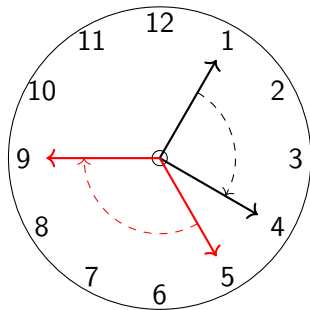
Lesson: Angle measures, internal, external, acute, obtuse, right

# Learning Target: I can measure angles

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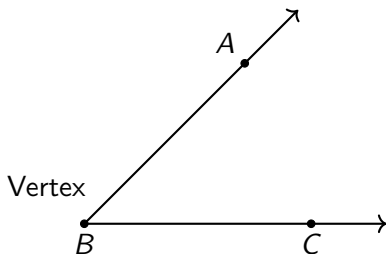
Do Now: Which takes longer, for a clock's hour hand to go from the 1 to the 4 or the 5 to the 9?



Lesson: Angle measures, internal, external, acute, obtuse, right

## Two rays with a common endpoint make an *angle*

Rays  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$ , vertex  $B$ .



**Angle** Two rays with a common endpoint,  $\angle ABC$  or  $\angle B$

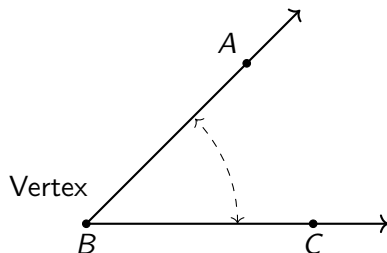
**Vertex** The common end point of two rays making an angle

**Interior** Inside, the area between the two rays

**Exterior** Outside, the area in the angle interior

# Two rays with a common endpoint make an *angle*

Rays  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$ , vertex  $B$ .



**Angle** Two rays with a common endpoint,  $\angle ABC$  or  $\angle B$

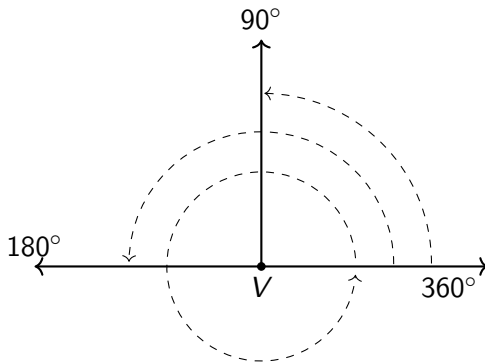
**Vertex** The common end point of two rays making an angle

**Interior** Inside, the area between the two rays

**Exterior** Outside, the area in the angle interior

**$m\angle A$**  The “measure” of angle  $A$ , how big it is

## Babylonian measures: $360^\circ$ in a circle



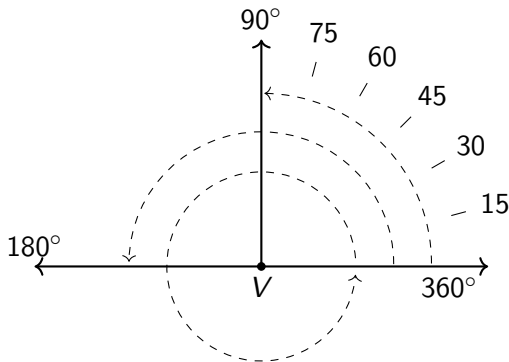
**Full turn** A complete rotation,  $360^\circ$

**Half turn** A straight line,  $180^\circ$

**Quarter turn** A *right* angle,  $90^\circ$

**Protractor** A tool for measuring angles

## Babylonian measures: $360^\circ$ in a circle



**Full turn** A complete rotation,  $360^\circ$

**Half turn** A straight line,  $180^\circ$

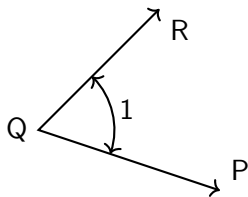
**Quarter turn** A *right* angle,  $90^\circ$

**Protractor** A tool for measuring angles



# Angle terminology and notation


Write definitions in your notebook



Angle  $Q$ , written  $\angle Q$  (also  $\angle PQR$ ,  $\angle 1$ )

Point  $Q$  is the *vertex*

The sides or *legs* are  $\overrightarrow{QR}$ ,  $\overrightarrow{QP}$

**Right angle** measuring  $90^\circ$ , mark as small square 

**Perpendicular** lines meet at right angles.  $\overline{AB} \perp \overline{CD}$

**Acute** angles measure  $< 90^\circ$

**Obtuse** angles are  $90^\circ < m\angle < 180^\circ$

**Straight angle** or a straight line measures  $180^\circ$

**Reflex angles** measure  $180^\circ < m\angle < 360^\circ$

## Learning Target: I can solve for angle measures

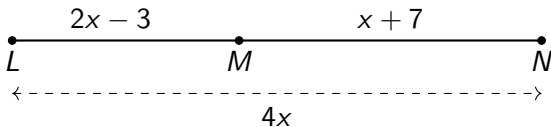
CCSS: HSG.CO.A.1 Know precise geometric definitions

2.2 Thursday 29 Sept

Do Now: Given  $\overline{LMN}$ ,  $LM = 2x - 3$ ,  $MN = x + 7$ ,  $LN = 4x$ .

Find  $x$ .

*Don't forget to check the solution.*

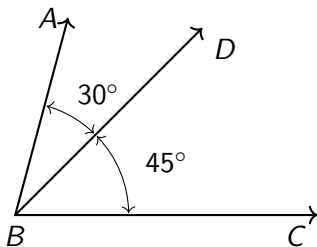


Name the geometry *postulate* that is the basis for this problem.

Lesson: Angle addition postulate, complementary, supplementary angles, linear pairs

## Angle addition postulate

$m\angle ABD = 30^\circ$ ,  $m\angle DBC = 45^\circ$ . Find  $m\angle ABC$ .

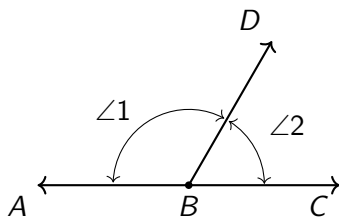


**Angle addition** The sum of the measures of *adjacent* angles is the measure of their combined angle. (postulate)

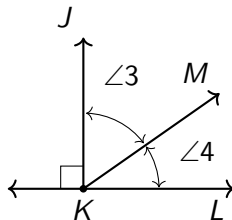
$$m\angle ABD + m\angle DBC = m\angle ABC$$

**Adjacent** “next to” each other. Adjacent angles share a common ray and are external to each other.

# Special angle pairs



Linear pair, supplementary  $\angle$ s



Complementary angles

**Linear pair** Two adjacent angles that make a straight line

**Opposite rays** collinear with a common endpoint. e.g.  $\overrightarrow{BA}$ ,  $\overrightarrow{BC}$

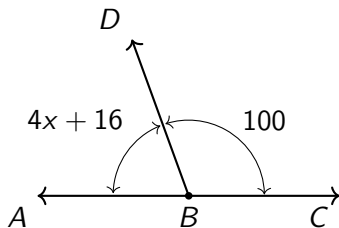
**Supplementary** Angles whose measures sum to  $180^\circ$

**Complementary** Angles whose measures sum to  $90^\circ$

**Adjacent** “next to” each other. Adjacent angles share a common ray and are external to each other.

Given two supplementary angles, a linear pair.

$m\angle ABD = 4x + 16$ ,  $m\angle CBD = 100$ . Find  $x$ .

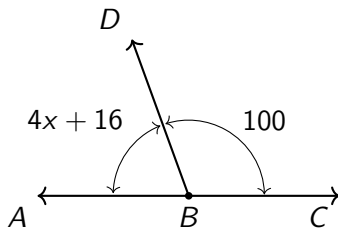


Given two supplementary angles, a linear pair.

$m\angle ABD = 4x + 16$ ,  $m\angle CBD = 100$ . Find  $x$ .

Solution:

$$m\angle ABD + m\angle CBD = 180$$



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$m\angle ABD = 4x + 16$ ,  $m\angle CBD = 100$ . Find  $x$ .

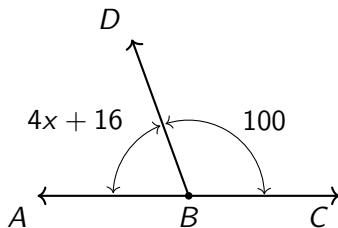
Solution:

$$m\angle ABD + m\angle CBD = 180$$

$$(4x + 16) + 100 = 180$$

...

$$x = 16$$

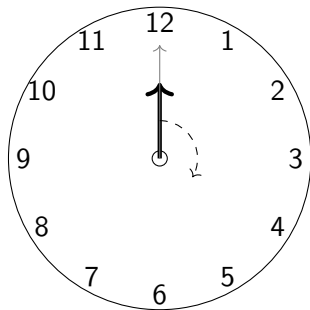


Check:

$$[4(16) + 16] + 100 = 180 \checkmark$$

## Extension (optional problems)

At midnight both the clock's minute hand and hour hand point in the same direction. When is the next time the clock hands coincide?





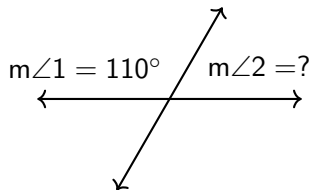
# Learning Target: I can identify vertical angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.3 Friday 30 September

Do Now: Check your knowledge of angle pairs

1. *Complementary* angles sum to how many degrees?
2. *Supplementary* angles sum to how many degrees?
3. Given complementary angles  $m\angle A = 30^\circ$ . Find  $m\angle B$ .
4. Given intersecting lines.  $m\angle 1 = 110^\circ$ . Find  $m\angle 2$ .

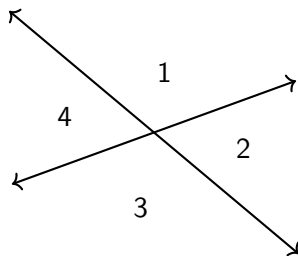


Lesson: Vertical angles

# Intersecting lines make two pairs of congruent angles

Angles *opposite* each other match:

$$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$$

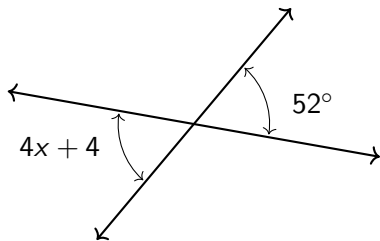


**Vertical angles** Opposite each other when two lines intersect.  
 $\angle 1$  and  $\angle 3$  are vertical angles, as are  $\angle 2$  and  $\angle 4$ .

**Opposite** Across from each other. (opposite angles and vertical angles means the same thing)

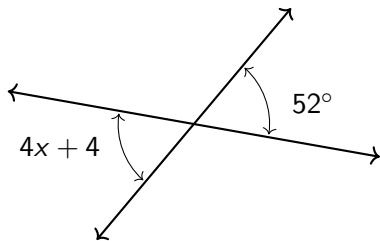
## Use vertical angles to solve for $x$

Given vertical angles measuring  $4x + 4$  and  $52^\circ$ . Find  $x$ .



## Use vertical angles to solve for $x$

Given vertical angles measuring  $4x + 4$  and  $52^\circ$ . Find  $x$ .



Solution:

$$4x + 4 = 52$$

$$x = 12$$

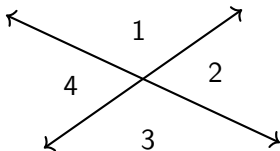
Check:

$$4(12) + 4 = 52 \checkmark$$

## Extension: Use logic to show vertical angles are congruent

Given intersecting lines making angles  $\angle 1$ ,  $\angle 2$ ,  $\angle 3$ ,  $\angle 4$ .

Prove  $\angle 2 \cong \angle 4$ .



Linear pairs are supplementary

$$m\angle 2 + m\angle 1 = 180$$

$$m\angle 4 + m\angle 1 = 180$$

Both equal 180, so they are equal (*transitive property* of equality)

$$m\angle 2 + m\angle 1 = m\angle 4 + m\angle 1$$

Subtract  $m\angle 1$  from both sides (*cancellation law*)

[link](#)

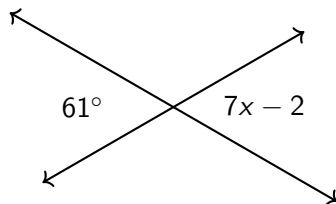
$$\angle 2 \cong \angle 4 \text{ Q.E.D.}$$

# Learning Target: I can bisect angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.4 Monday 3 October

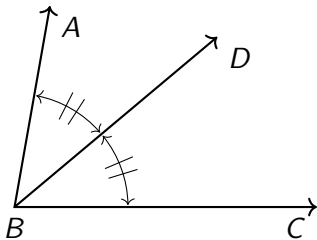
Do Now: Given vertical angles measuring  $7x - 2$  and  $61^\circ$ . Find  $x$ .



Lesson: Angle bisector situations

## Bisect an angle by dividing it exactly in half

$\overrightarrow{BD}$  bisects  $\angle ABC$  if and only if  $\angle ABD \cong \angle CBD$ .

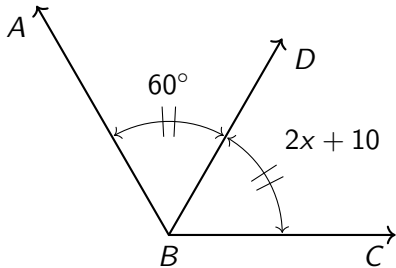


**Angle bisector** ray dividing an angle into two congruent angles

**Hash marks** mark congruent angles  
(conventions differ for marking angles)

## Model angle situations with algebra, then solve

Given angle bisector  $\overrightarrow{BD}$  with  $m\angle ABD = 60^\circ$  and  $m\angle CBD = 2x + 10$ . Find  $x$ .

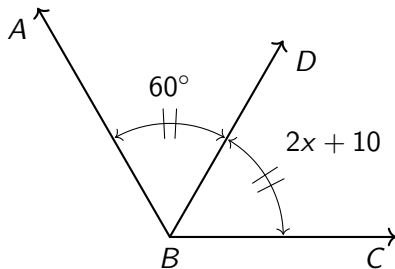




## Model angle situations with algebra, then solve

Given angle bisector  $\overrightarrow{BD}$  with  $m\angle ABD = 60^\circ$  and  $m\angle CBD = 2x + 10$ . Find  $x$ .

Solution:



$$\angle ABD \cong \angle CBD$$

$$2x + 10 = 60$$

$$2x = 50$$

$$x = 25$$

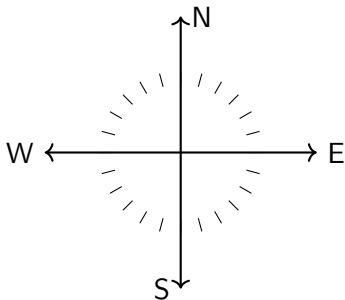
Check:

$$2(25) + 10 = 60? \checkmark$$

## Extension: Use angles for compass directions

North South East West, points of the compass

Directions are measured relative to North



**Bearing** The direction as an angle *clockwise* from north

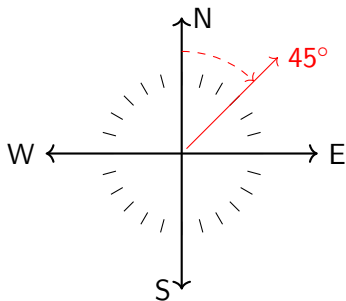
**Clockwise** The direction the clocks turn, “to the right” (tighten)

**Counterclockwise** Opposite of clocks, “to the left” (loosen)

## Extension: Use angles for compass directions

North South East West, points of the compass

Directions are measured relative to North



“Northeast,” half way between north and east, i.e. bearing  $45^\circ$

north is  $0^\circ$   
east is  $90^\circ$   
south is  $180^\circ$   
west is  $270^\circ$

**Bearing** The direction as an angle *clockwise* from north

**Clockwise** The direction the clocks turn, “to the right” (tighten)

**Counterclockwise** Opposite of clocks, “to the left” (loosen)

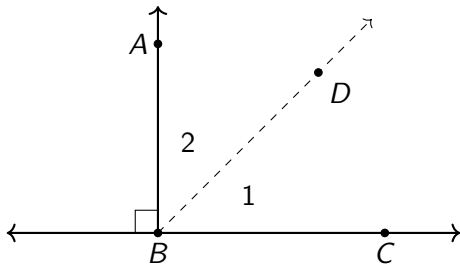
## LT: I can work with equilateral and isosceles-right $\triangle$ s

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.5 Tuesday 4 October

Do Now: Given perpendiculars  $\overrightarrow{AB} \perp \overrightarrow{BC}$ , and that the ray  $\overrightarrow{BD}$  bisects  $\angle ABC$ , making two angles,  $\angle 1$  and  $\angle 2$ .

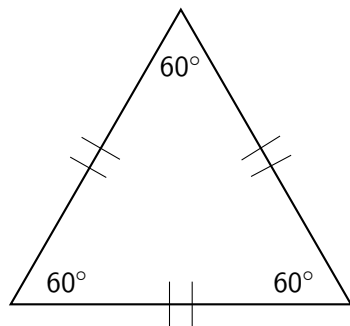
Find the measures of  $\angle 1$ ,  $\angle 2$ .



Lesson: Isosceles base theorem, special triangles

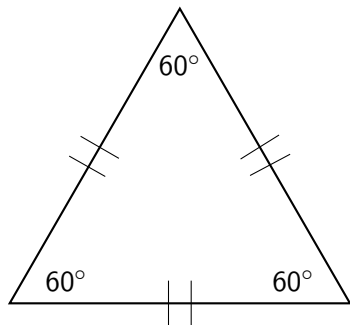
$60^\circ - 60^\circ - 60^\circ$ ,  $30^\circ - 60^\circ - 90^\circ$ ,  $45^\circ - 45^\circ - 90^\circ$

# Equilateral $\triangle$ , special relationships and measures

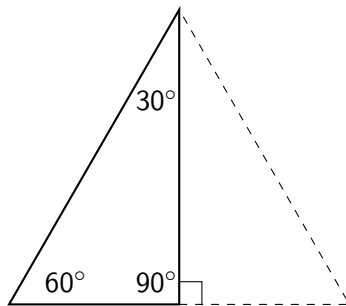


$$60^\circ - 60^\circ - 60^\circ$$

## Equilateral $\triangle$ , special relationships and measures



$$60^\circ - 60^\circ - 60^\circ$$



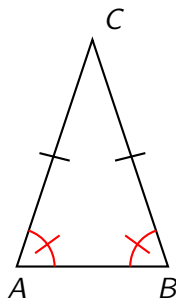
$$30^\circ - 60^\circ - 90^\circ$$

**Equiangular** means having equal angles

**Equilateral** having equal sides

# The *base* angles of an isosceles triangle are congruent

*Isosceles base theorem:* If  $\overline{AC} \cong \overline{BC}$  then  $\angle A \cong \angle B$

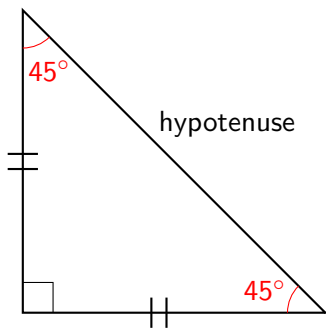


**Base angles**  $\angle$ s opposite the congruent sides in an isosceles  $\triangle$

**Included angle** The angle between two given sides of a triangle  
( $\angle C$  is included between  $\overline{AC}$  and  $\overline{BC}$ )

**Theorem** Something we can prove using logic

## Isosceles-right triangles' angles measure $45^\circ - 45^\circ - 90^\circ$

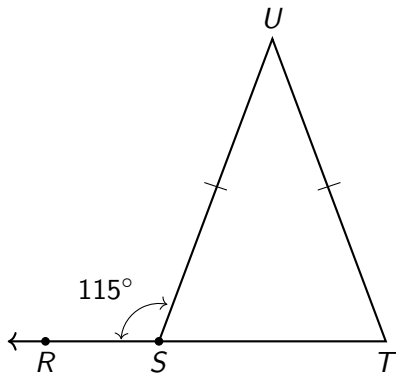


**Hypotenuse** the longest side of a right triangle, opposite the  $90^\circ$  angle



## Multiple step problem: apply your knowledge

Given isosceles triangle with  $\overline{SU} \cong \overline{TU}$ ,  $m\angle RSU = 115^\circ$ .

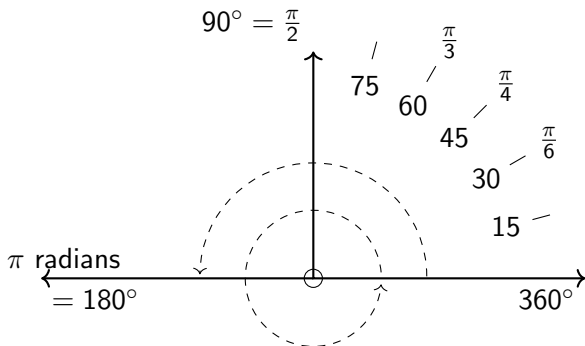


1. Find  $m\angle TSU$

2. Find  $m\angle T$

## Extension: Radian units for angle measures

Mathematicians use radians because calculations are simpler



Convert *units*:  $360^\circ = 2\pi$  radians:

**Degree** One 360th of a full turn

**Radian** A full circle is  $2\pi$  radians.  $1 \text{ radian} \approx 57^\circ$

**Gradian** One 400th of a full turn

# LT: I can review length and angle measures with peers

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.6 Thursday 6 October

Angle concepts and theorems you have learned

1. Angle addition situations
2. Angle pairs
  - 2.1  $\perp$  lines and complementary angles make  $90^\circ$
  - 2.2 Vertical  $\angle$ s are  $\cong$
3. Angle bisectors
4. Isosceles base angle theorem, special triangles

# Learning Target: I can quantify angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.7 Friday 7 October

Unit test

# Open Middle problem (fun)

Use digits from 0 to 9. Using a digit no more than once.

The first two angle measures are complementary. The second two angles supplementary. (degrees)
