

Geometry Unit 2: Angles

Bronx Early College Academy

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28 September - 7 October 2022

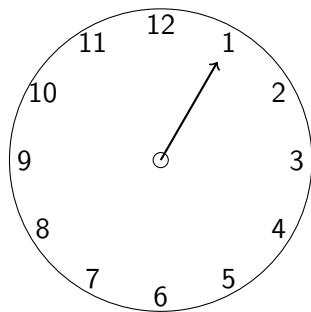
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| 2.1 Angle notation, measures | 28 September |
| 2.2 Angle addition, angle pairs | 29 September |
| 2.3 Vertical angles | 30 September |
| 2.4 Angle bisectors | 3 October |
| 2.5 Equilateral, isosceles \triangle angles | 4 October |
| 2.6 Review | 6 October |
| 2.7 Unit 2 test: Angle measures | 7 October |
| Open Middle: complementary and supplementary puzzle | |

Learning Target: I can measure angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.1 Wednesday 28 Sept

Do Now: Which takes longer, for a clock's hour hand to go from the 1 to the 4 or the 5 to the 9?



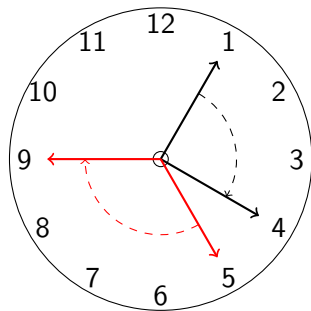
Lesson: Angle measures, internal, external, acute, obtuse, right

Learning Target: I can measure angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.1 Wednesday 28 Sept

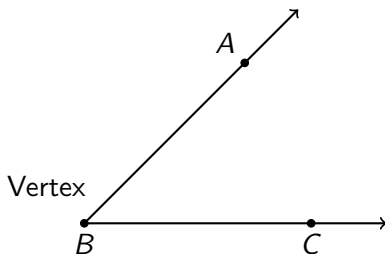
Do Now: Which takes longer, for a clock's hour hand to go from the 1 to the 4 or the 5 to the 9?



Lesson: Angle measures, internal, external, acute, obtuse, right

Two rays with a common endpoint make an *angle*

Rays \overrightarrow{BA} and \overrightarrow{BC} , vertex B .



Angle Two rays with a common endpoint, $\angle ABC$ or $\angle B$

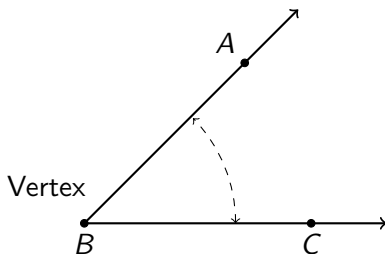
Vertex The common end point of two rays making an angle

Interior Inside, the area between the two rays

Exterior Outside, the area in the angle interior

Two rays with a common endpoint make an *angle*

Rays \overrightarrow{BA} and \overrightarrow{BC} , vertex B .



Angle Two rays with a common endpoint, $\angle ABC$ or $\angle B$

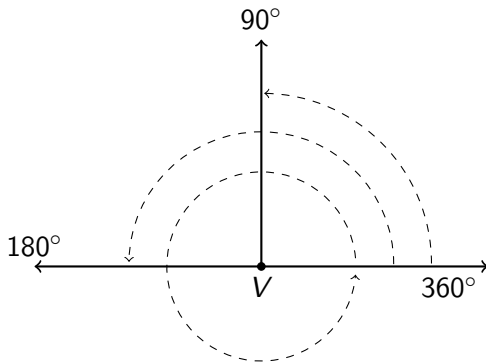
Vertex The common end point of two rays making an angle

Interior Inside, the area between the two rays

Exterior Outside, the area in the angle interior

$m\angle A$ The “measure” of angle A , how big it is

Babylonian measures: 360° in a circle



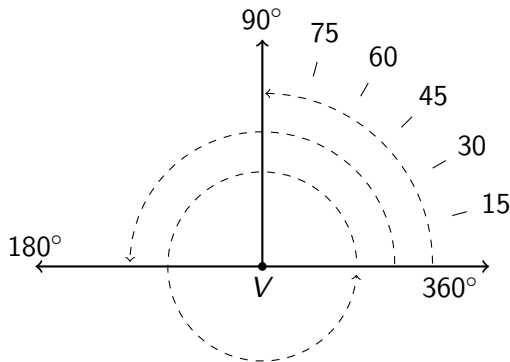
Full turn A complete rotation, 360°

Half turn A straight line, 180°

Quarter turn A *right* angle, 90°

Protractor A tool for measuring angles

Babylonian measures: 360° in a circle



Full turn A complete rotation, 360°

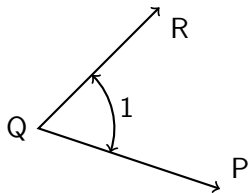
Half turn A straight line, 180°

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Protractor A tool for measuring angles

Angle terminology and notation


Write definitions in your notebook



Angle Q , written $\angle Q$ (also $\angle PQR$, $\angle 1$)

Point Q is the *vertex*

The sides or *legs* are \overrightarrow{QR} , \overrightarrow{QP}

Right angle measuring 90° , mark as small square 

Perpendicular lines meet at right angles. $\overline{AB} \perp \overline{CD}$

Acute angles measure $< 90^\circ$

Obtuse angles are $90^\circ < m\angle < 180^\circ$

Straight angle or a straight line measures 180°

Reflex angles measure $180^\circ < m\angle < 360^\circ$

Learning Target: I can solve for angle measures

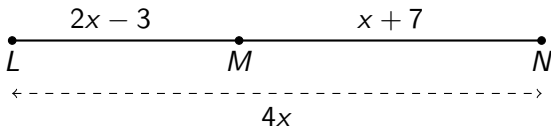
CCSS: HSG.CO.A.1 Know precise geometric definitions

2.2 Thursday 29 Sept

Do Now: Given \overline{LMN} , $LM = 2x - 3$, $MN = x + 7$, $LN = 4x$.

Find x .

Don't forget to check the solution.

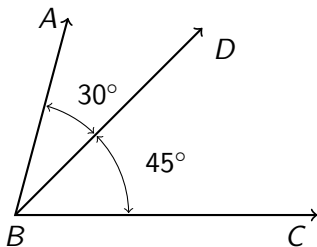


Name the geometry *postulate* that is the basis for this problem.

Lesson: Angle addition postulate, complementary, supplementary angles, linear pairs

Angle addition postulate

$m\angle ABD = 30^\circ$, $m\angle DBC = 45^\circ$. Find $m\angle ABC$.

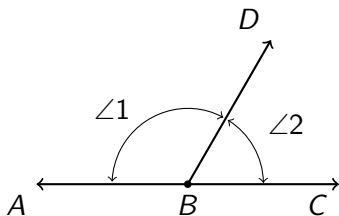


Angle addition The sum of the measures of *adjacent* angles is the measure of their combined angle. (postulate)

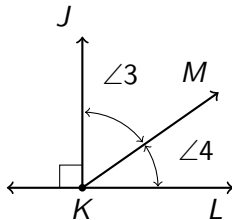
$$m\angle ABD + m\angle DBC = m\angle ABC$$

Adjacent “next to” each other. Adjacent angles share a common ray and are external to each other.

Special angle pairs



Linear pair, supplementary \angle s



Complementary angles

Linear pair Two adjacent angles that make a straight line

Opposite rays collinear with a common endpoint. e.g. \overrightarrow{BA} , \overrightarrow{BC}

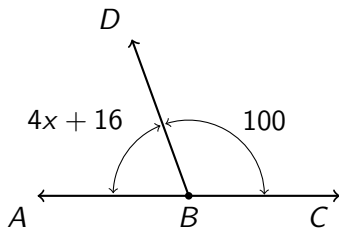
Supplementary Angles whose measures sum to 180°

Complementary Angles whose measures sum to 90°

Adjacent “next to” each other. Adjacent angles share a common ray and are external to each other.

Given two supplementary angles, a linear pair.

$m\angle ABD = 4x + 16$, $m\angle CBD = 100$. Find x .

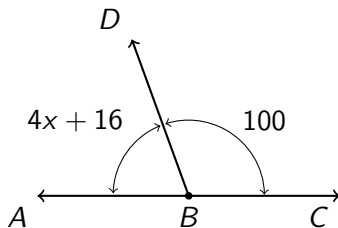


Given two supplementary angles, a linear pair.

$m\angle ABD = 4x + 16$, $m\angle CBD = 100$. Find x .

Solution:

$$m\angle ABD + m\angle CBD = 180$$



Given two supplementary angles, a linear pair.

$m\angle ABD = 4x + 16$, $m\angle CBD = 100$. Find x .

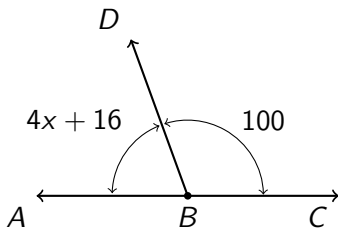
Solution:

$$m\angle ABD + m\angle CBD = 180$$

$$(4x + 16) + 100 = 180$$

...

$$x = 16$$

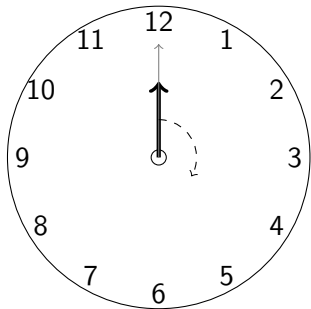


Check:

$$[4(16) + 16] + 100 = 180 \checkmark$$

Extension (optional problems)

At midnight both the clock's minute hand and hour hand point in the same direction. When is the next time the clock hands coincide?



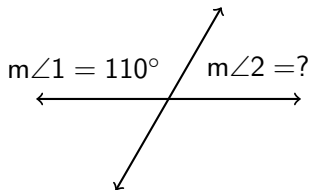
Learning Target: I can identify vertical angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.3 Friday 30 September

Do Now: Check your knowledge of angle pairs

1. *Complementary* angles sum to how many degrees?
2. *Supplementary* angles sum to how many degrees?
3. Given complementary angles $m\angle A = 30^\circ$. Find $m\angle B$.
4. Given intersecting lines. $m\angle 1 = 110^\circ$. Find $m\angle 2$.

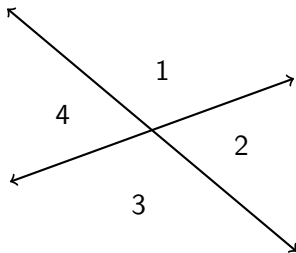


Lesson: Vertical angles

Intersecting lines make two pairs of congruent angles

Angles *opposite* each other match:

$$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$$

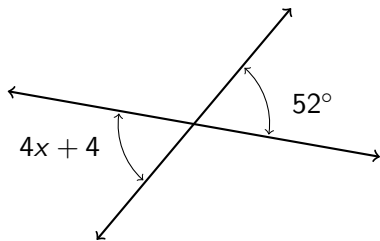


Vertical angles Opposite each other when two lines intersect.
 $\angle 1$ and $\angle 3$ are vertical angles, as are $\angle 2$ and $\angle 4$.

Opposite Across from each other. (opposite angles and vertical angles means the same thing)

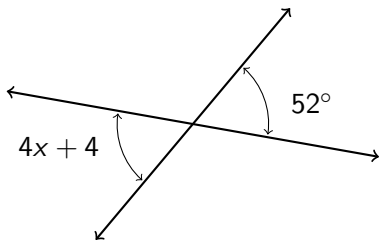
Use vertical angles to solve for x

Given vertical angles measuring $4x + 4$ and 52° . Find x .



Use vertical angles to solve for x

Given vertical angles measuring $4x + 4$ and 52° . Find x .



Solution:

$$4x + 4 = 52$$

$$x = 12$$

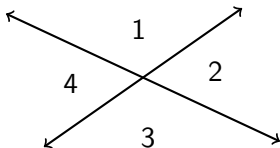
Check:

$$4(12) + 4 = 52 \checkmark$$

Extension: Use logic to show vertical angles are congruent

Given intersecting lines making angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$.

Prove $\angle 2 \cong \angle 4$.



Linear pairs are supplementary

$$m\angle 2 + m\angle 1 = 180$$

$$m\angle 4 + m\angle 1 = 180$$

Both equal 180, so they are equal (*transitive property* of equality)

$$m\angle 2 + m\angle 1 = m\angle 4 + m\angle 1$$

Subtract $m\angle 1$ from both sides (*cancellation law*)

[link](#)

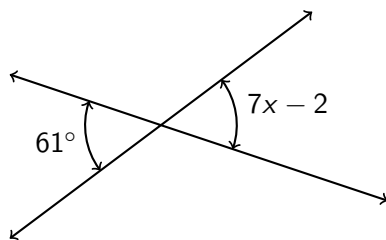
$$\angle 2 \cong \angle 4 \text{ Q.E.D.}$$

Learning Target: I can bisect angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.4 Monday 3 October

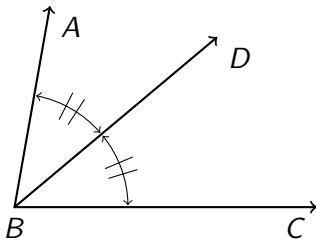
Do Now: Given vertical angles measuring $7x - 2$ and 61° . Find x .



Lesson: Angle bisector situations

Bisect an angle by dividing it exactly in half

\overrightarrow{BD} bisects $\angle ABC$ if and only if $\angle ABD \cong \angle CBD$.

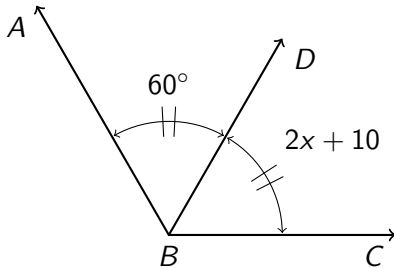


Angle bisector ray dividing an angle into two congruent angles

Hash marks mark congruent angles

Model angle situations with algebra, then solve

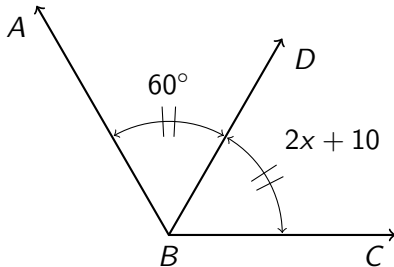
Given angle bisector \overrightarrow{BD} with $m\angle ABD = 60^\circ$ and $m\angle CBD = 2x + 10$. Find x .



Model angle situations with algebra, then solve

Given angle bisector \overrightarrow{BD} with $m\angle ABD = 60^\circ$ and $m\angle CBD = 2x + 10$. Find x .

Solution:



$$\angle ABD \cong \angle CBD$$

$$2x + 10 = 60$$

$$2x = 50$$

$$x = 25$$

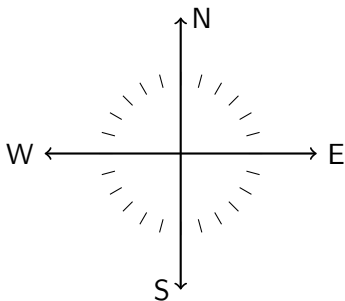
Check:

$$2(25) + 10 = 60? \checkmark$$

Extension: Use angles for compass directions

North South East West, points of the compass

Directions are measured relative to North



Bearing The direction as an angle *clockwise* from north

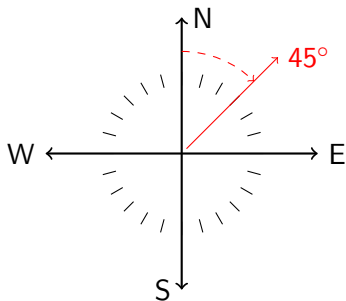
Clockwise The direction the clocks turn, “to the right” (tighten)

Counterclockwise Opposite of clocks, “to the left” (loosen)

Extension: Use angles for compass directions

North South East West, points of the compass

Directions are measured relative to North



“Northeast,” half way between north and east, i.e. bearing 45°

north is 0°

east is 90°

south is 180°

west is 270°

Bearing The direction as an angle *clockwise* from north

Clockwise The direction the clocks turn, “to the right” (tighten)

Counterclockwise Opposite of clocks, “to the left” (loosen)

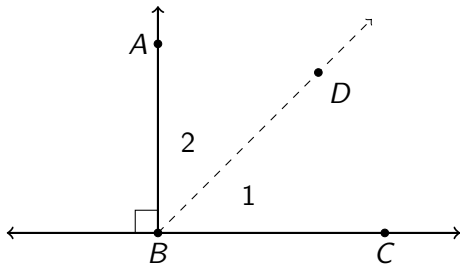
LT: I can work with equilateral and isosceles-right \triangle s

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.5 Tuesday 4 October

Do Now: Given perpendiculars $\overrightarrow{AB} \perp \overrightarrow{BC}$, and that the ray \overrightarrow{BD} bisects $\angle ABC$, making two angles, $\angle 1$ and $\angle 2$.

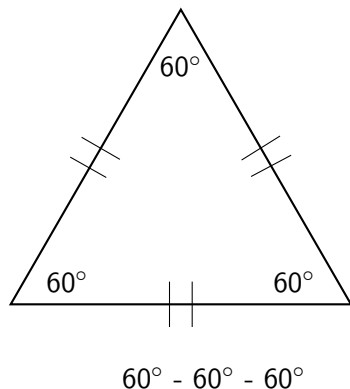
Find the measures of $\angle 1$, $\angle 2$.



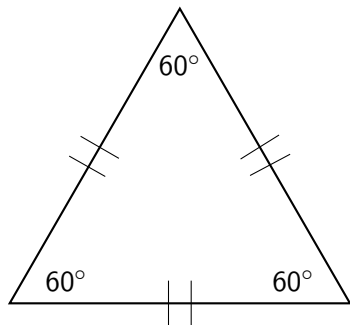
Lesson: Isosceles base theorem, special triangles

$60^\circ - 60^\circ - 60^\circ$, $30^\circ - 60^\circ - 90^\circ$, $45^\circ - 45^\circ - 90^\circ$

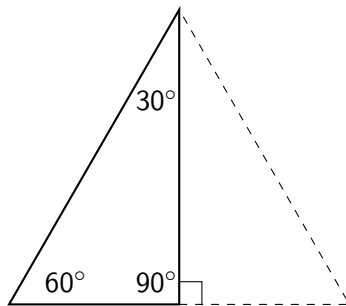
Equilateral \triangle , special relationships and measures



Equilateral \triangle , special relationships and measures



$$60^\circ - 60^\circ - 60^\circ$$



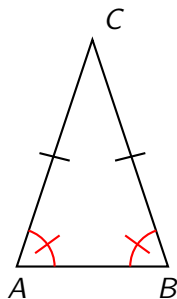
$$30^\circ - 60^\circ - 90^\circ$$

Equiangular means having equal angles

Equilateral having equal sides

The *base* angles of an isosceles triangle are congruent

Isosceles base theorem: If $\overline{AC} \cong \overline{BC}$ then $\angle A \cong \angle B$

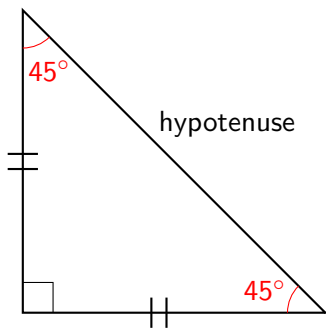


Base angles \angle s opposite the congruent sides in an isosceles \triangle

Included angle The angle between two given sides of a triangle
($\angle C$ is included between \overline{AC} and \overline{BC})

Theorem Something we can prove using logic

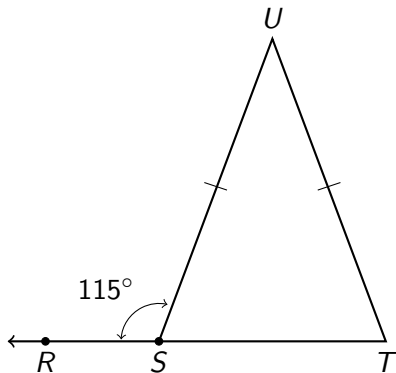
Isosceles-right triangles' angles measure $45^\circ - 45^\circ - 90^\circ$



Hypotenuse the longest side of a right triangle, opposite the 90° angle

Multiple step problem: apply your knowledge

Given isosceles triangle with $\overline{SU} \cong \overline{TU}$, $m\angle RSU = 115^\circ$.

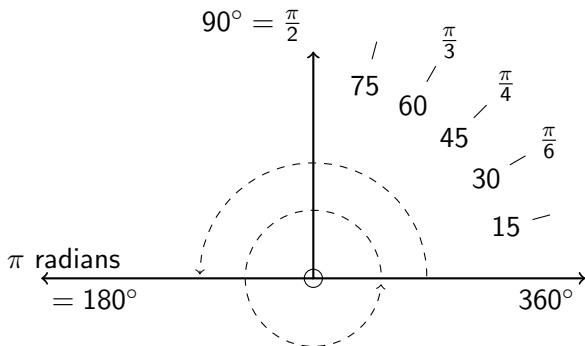


1. Find $m\angle TSU$

2. Find $m\angle T$

Extension: Radian units for angle measures

Mathematicians use radians because calculations are simpler



Convert *units*: $360^\circ = 2\pi$ radians:

Degree One 360th of a full turn

Radian A full circle is 2π radians. $1 \text{ radian} \approx 57^\circ$

Gradian One 400th of a full turn

LT: I can review length and angle measures with peers

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.6 Thursday 6 October

Angle concepts and theorems you have learned

1. Angle addition situations
2. Angle pairs
 - 2.1 \perp lines and complementary angles make 90°
 - 2.2 Linear pairs and supplementary \angle s sum to 180°
 - 2.3 Vertical \angle s are \cong
3. Angle bisectors
4. Isosceles base angle theorem, special triangles

Learning Target: I can quantify angles

CCSS: HSG.CO.A.1 Know precise geometric definitions

2.7 Friday 7 October

Unit test

Open Middle problem (fun)

Use digits from 0 to 9. Using a digit no more than once.

The first two angle measures are complementary. The second two angles supplementary. (degrees)

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