Lecture 2: predictive modeling @ NYT

desc/pred/pres

descriptive:	specify x; learn $z(x)$ or $p(z\|x)$ where z is "simpler" than x
predictive:	specify x and y; learn to predict y from x
prescriptive:	specify x, y, and a; learn to prescribe a given x to maximize y

Figure 2: desc/pred/pres

- caveat: difference between observation and experiment. why?

blossom example

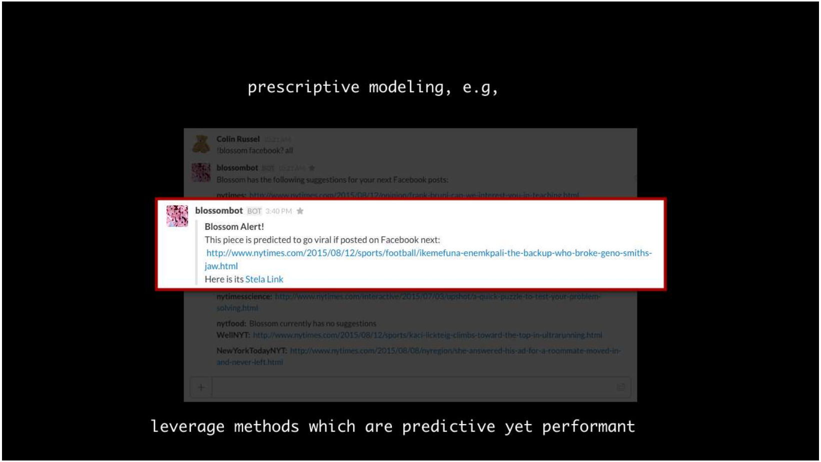

Figure 3: Reminder: Blossom

blossom + boosting ('exponential')

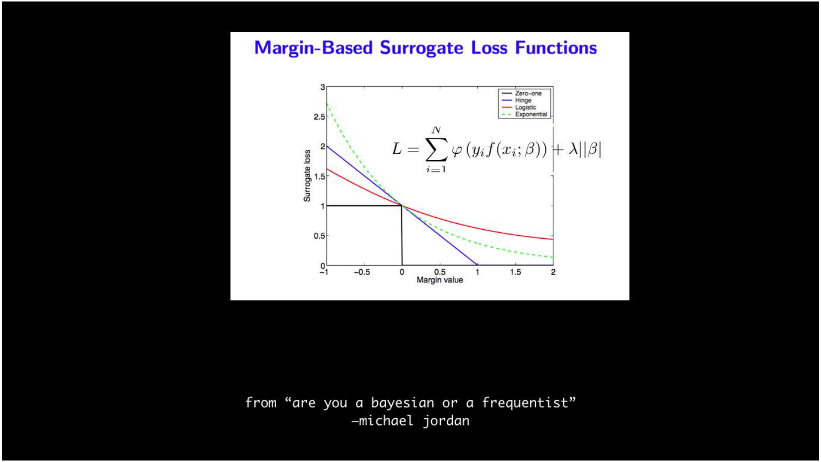

Figure 4: Reminder: Surrogate Loss Functions

tangent: logistic function as surrogate loss function

- define $f(x) \equiv \log p(y=1|x)/p(y=-1|x) \in R$

tangent: logistic function as surrogate loss function

- define $f(x) \equiv \log p(y = 1|x)/p(y = -1|x) \in R$
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- $-\log_2 p(\{y\}_1^N) = \sum_i \log_2\left(1 + \mathrm{e}^{-y_i f(x_i)}\right) \equiv \sum_i \ell(y_i f(x_i))$

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- $\ell'' > 0$, $\ell(\mu) > 1[\mu < 0]\ \forall \mu \in R$.

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- $\therefore$ maximizing log-likelihood is minimizing a surrogate convex loss function for classification (though not strongly convex, cf. Yoram's talk)

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- $\ell'' > 0$, $\ell(\mu) > 1[\mu < 0]\ \forall \mu \in R$.
- $\therefore$ maximizing log-likelihood is minimizing a surrogate convex loss function for classification (though not strongly convex, cf. Yoram's talk)
- but $\sum_i \log_2 \left(1 + \mathrm{e}^{-y_i w^T h(x_i)}\right)$ not as easy as $\sum_i \mathrm{e}^{-y_i w^T h(x_i)}$

boosting 1

L exponential surrogate loss function, summed over examples:

- $L[F] = \sum_i \exp\left(-y_i F(x_i)\right)$

boosting 1

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- $L[F] = \sum_i \exp\left(-y_i F(x_i)\right)$
- $= \sum_i \exp\left(-y_i \sum_{t'}^{t} w_{t'} h_{t'}(x_i)\right) \equiv L_t(\mathbf{w}_t)$

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- Draw $h_t \in \mathcal{H}$ large space of rules s.t. $h(x) \in \{-1, +1\}$
- label $y \in \{-1, +1\}$

boosting 1

L exponential surrogate loss function, summed over examples:

- $L_{t+1}(\mathbf{w}_t; w) \equiv \sum_i d_i^t \exp\left(-y_i w h_{t+1}(x_i)\right)$

Punchlines: sparse, predictive, interpretable, fast (to execute), and easy to extend, e.g., trees, flexible hypotheses spaces, L_1, L_∞[1], ...

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Punchlines: sparse, predictive, interpretable, fast (to execute), and easy to extend, e.g., trees, flexible hypotheses spaces, L_1, L_∞[1], ...

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- update example weights $d_i^{t+1} = d_i^t \mathrm{e}^{\mp w}$

Punchlines: sparse, predictive, interpretable, fast (to execute), and easy to extend, e.g., trees, flexible hypotheses spaces, L_1, L_∞[1], ...

[1] Duchi + Singer "Boosting with structural sparsity" ICML '09

predicting people

- "customer journey" prediction

predicting people

- “customer journey” prediction
  - fun covariates

predicting people

- “customer journey” prediction
  - fun covariates
  - observational complication v structural models

predicting people (reminder)

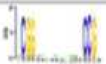
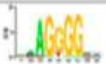
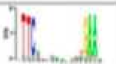
TFNAME	DB-MOTIF	MOTIF	DBNAME	d(p,q)
CBF1	CACGTG		YPD	0.032635
CGG everted repeat	CGGN*CCG		YPD	0.032821
MSN2			TRANSFAC	0.085626
HSF1	TTCNNNGAA		SCPD	0.102410
XBP1			TRANSFAC	0.140561

Figure 5: both in science and in real world, feature analysis guides future experiments

single copy (reminder)

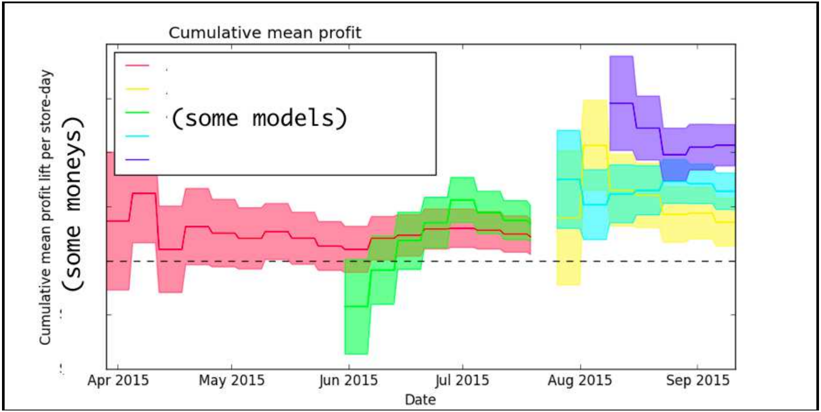


Figure 6: from Lecture 1

example in CAR (computer assisted reporting)

www.nytimes.com/2014/09/12/business/air-bag-flaw-long-known-led-to-recalls.html?_r=1

S HOME SEARCH The New York Times

BUSINESS DAY

Air Bag Flaw, Long Known to Honda and Takata, Led to Recalls

By HIROKO TABUCHI SEPT. 11, 2014

The air bag in Jennifer Griffin's Honda Civic was not among the recalled vehicles in 2008. Jim Keely

Figure 7: Tabuchi article

example in CAR (computer assisted reporting)

- cf. Friedman's "Statistical models and Shoe Leather"[2]

[2] Freedman, David A. "Statistical models and shoe leather." Sociological methodology 21.2 (1991): 291-313.

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- 2219 labeled[3] examples from 33,204 comments

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- cf. Friedman's "Statistical models and Shoe Leather"[2]
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- cf. Box's "Science and Statistics"[4]

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[3]By Hiroko Tabuchi, a Pulitzer winner

[4]Science and Statistics, George E. P. Box Journal of the American Statistical Association, Vol. 71, No. 356. (Dec., 1976), pp. 791-799.

computer assisted reporting

- Impact

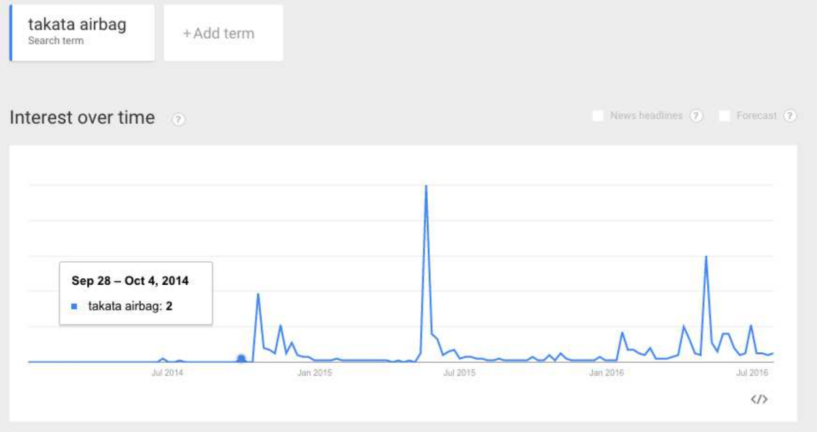

Figure 8: impact

Lecture 3: prescriptive modeling @ NYT

the natural abstraction

- operators[5] make decisions

[5] In the sense of business deciders; that said, doctors, including those who operate, also have to make decisions, cf., personalized medicines

the natural abstraction

- operators[5] make decisions
- faster horses v. cars

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the natural abstraction

- operators[5] make decisions
- faster horses v. cars
- general insights v. optimal policies

[5] In the sense of business deciders; that said, doctors, including those who operate, also have to make decisions, cf., personalized medicines

maximizing outcome

- the problem: maximizing an outcome over policies. . .

maximizing outcome

- the problem: maximizing an outcome over policies. . .
- . . . while inferring causality from observation

maximizing outcome

- the problem: maximizing an outcome over policies...
- ...while inferring causality from observation
- different from predicting outcome in absence of action/policy

examples

- observation is not experiment

examples

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  - e.g., (Med.) smoking hurts vs unhealthy people smoke

examples

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  - e.g., (Med.) smoking hurts vs unhealthy people smoke
  - e.g., (Med.) affluent get prescribed different meds/treatment

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  - e.g., (Med.) smoking hurts vs unhealthy people smoke
  - e.g., (Med.) affluent get prescribed different meds/treatment
  - e.g., (life) veterans earn less vs the rich serve less[6]

[6] Angrist, Joshua D. (1990). "Lifetime Earnings and the Vietnam Draft Lottery: Evidence from Social Security Administrative Records". American Economic Review 80 (3): 313–336.

examples

- observation is not experiment
  - e.g., (Med.) smoking hurts vs unhealthy people smoke
  - e.g., (Med.) affluent get prescribed different meds/treatment
  - e.g., (life) veterans earn less vs the rich serve less[6]
  - e.g., (life) admitted to school vs learn at school?

[6] Angrist, Joshua D. (1990). "Lifetime Earnings and the Vietnam Draft Lottery: Evidence from Social Security Administrative Records". American Economic Review 80 (3): 313–336.

reinforcement/machine learning/graphical models

- key idea: model joint $p(y, a, x)$

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reinforcement/machine learning/graphical models

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- nomenclature: 'response', 'policy'/'bias', 'prior' above

in general

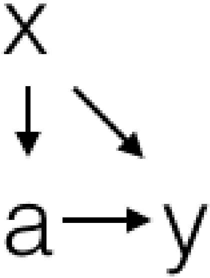

Figure 9: policy/bias, response, and prior define the distribution

also describes both the ‘exploration’ and ‘exploitation’ distributions

randomized controlled trial

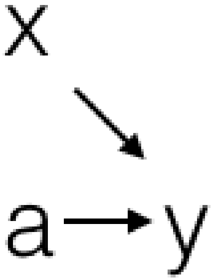

Figure 10: RCT: ‘bias’ removed, random ‘policy’ (response and prior unaffected)

also Pearl’s ‘do’ distribution: a distribution with “no arrows” pointing to the action variable.

POISE: calculation, estimation, optimization

- POISE: "policy optimization via importance sample estimation"

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- unexpected connection: personalized medicine

POISE setup and Goal

- "a causes y" $\iff \exists$ family $p_\alpha(y, a, x) = p(y|a, x)p_\alpha(a|x)p(x)$

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notation: $\{x,a,y\} \in \{X,A,Y\}$ i.e., $E_\alpha(Y)$ is not a function of y

POISE math: IS+Monte Carlo estimation=ISE

i.e, “importance sampling estimation”

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- $E_{+}(Y) \equiv \sum_{yax} y p_{+}(y, a, x)$
- $E_{+}(Y) = \sum_{yax} y p_{-}(y, a, x)(p_{+}(y, a, x)/p_{-}(y, a, x))$

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- $E_+(Y) = \sum_{yax} yp_-(y, a, x)(p_+(a|x)/p_-(a|x))$
- $E_+(Y) \approx N^{-1} \sum_i y_i(p_+(a_i|x_i)/p_-(a_i|x_i))$

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let's spend some time getting to know this last equation, the importance sampling estimate of outcome in a "causal model" ("a causes y") among $\{y, a, x\}$

Observation (cf. Bottou[7])

- factorizing $P_{\pm}(x)$: $\frac{P_{+}(x)}{P_{-}(x)} = \Pi_{\text{factors}} \frac{P_{+\text{but not}-}(x)}{P_{-\text{but not}+}(x)}$

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- factors left over are numerator ($p_{+}(a|x)$, to optimize) and denominator ($p_{-}(a|x)$, to infer if not a RCT)
- unobserved confounders will confound us (later)

[7]Counterfactual Reasoning and Learning Systems, arXiv:1209.2355

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- consider numerator for deterministic policy:
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- $\therefore$ reduces policy optimization to (weighted) classification

---

[8] Langford & Zadrozny "Relating Reinforcement Learning Performance to Classification Performance" ICML 2005

[9] Beygelzimer & Langford "The offset tree for learning with partial labels" (KDD 2009)

[10] Tutorial on "Reductions" (including at ICML 2009)

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- hidden here is a 2nd parameter, in classification, $\therefore$ harder search

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- abundant data available, under-explored IMHO

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tangent: causality as told by an economist

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IMHO underexplored

causality, as understood in marketing

- a/b testing and RCT

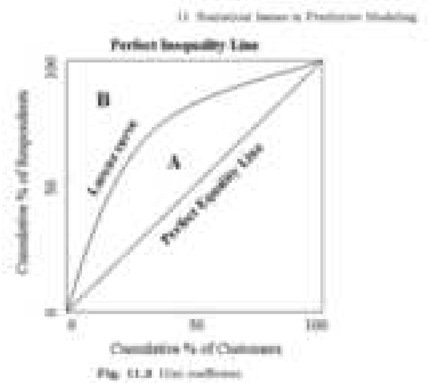

Figure 11: Blattberg, Robert C., Byung-Do Kim, and Scott A. Neslin. Database Marketing, Springer New York, 2008

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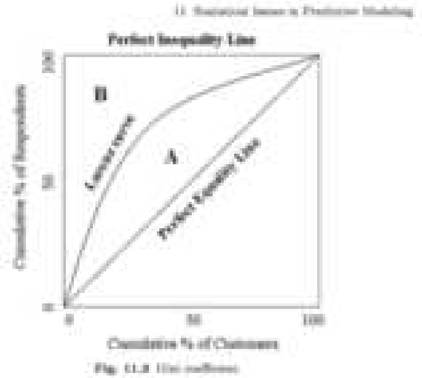

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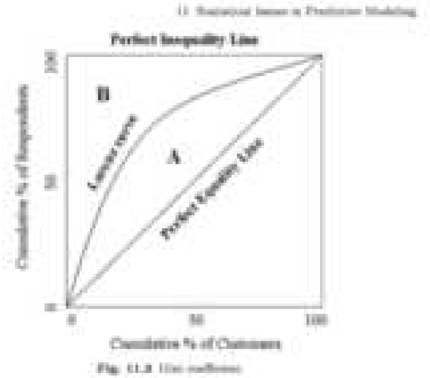

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- denominator can not be inferred, ignore at your peril

cautionary tale problem: Simpson's paradox

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- u: unobserved department they applied to

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- 'resolved' by Bickel (1975)[13] (See also Pearl[14])
- u: unobserved department they applied to
- $p(a|x) = \sum_{u=1}^{u=6} p(a|x, u)p(u|x)$

[13] P.J. Bickel, E.A. Hammel and J.W. O'Connell (1975). "Sex Bias in Graduate Admissions: Data From Berkeley". Science 187 (4175): 398–404

[14] Pearl, Judea (December 2013). "Understanding Simpson's paradox". UCLA Cognitive Systems Laboratory, Technical Report R-414.

cautionary tale problem: Simpson's paradox

- a: admissions (a=1: admitted, a=0: declined)
- x: gender (x=1: female, x=0: male)
- lawsuit (1973): $.44 = p(a = 1|x = 0) > p(a = 1|x = 1) = .35$
- 'resolved' by Bickel (1975)[13] (See also Pearl[14])
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- $p(a|x) = \sum_{u=1}^{u=6} p(a|x, u)p(u|x)$
- e.g., gender-blind: $p(a|1) - p(a|0) = p(a|u) \cdot (p(u|1) - p(u|0))$

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confounded approach: quasi-experiments + instruments

- Q: does engagement drive retention? (NYT, NFLX, . . .)

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  - US didn't directly control serving in Vietnam, either[16]
- requires **strong assumptions**, including linear model

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[17] I thank Sinan Aral, MIT Sloan, for bringing this to my attention

IV: graphical model assumption

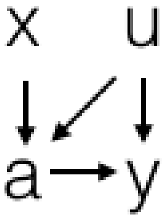

Figure 12: independence assumption

IV: graphical model assumption (sideways)

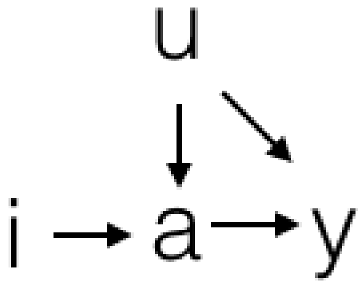

Figure 13: independence assumption

IV: review s/OLS/MOM/ (E is empirical average)

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- $E[YX_k] = E[\beta^T AX_k] + E[\epsilon]E[X_k]$
- $E[Y] = E[\beta^T A] + E[\epsilon]$ (from ansatz)
- $C(Y, X_k) = \beta^T C(A, X_k)$, not an "inversion" problem, requires "two stage regression"

IV: binary, binary case (aka “Wald estimator”)

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- $\beta = (E(Y|x=1) - E(Y|x=0))/(E(A|x=1) - E(A|x=0))$.

bandits: obligatory slide

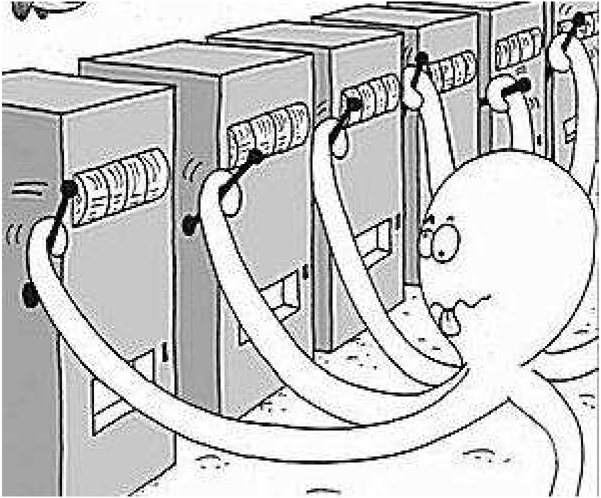

Figure 14: almost all the talks I've gone to on bandits have this image

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- wide applicability: humane clinical trials, targeting, ...

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examples

- ϵ-greedy (no context, aka 'vanilla', aka 'context-free')
- UCB1 (2002) (no context) + LinUCB (with context)
- Thompson Sampling (1933)[18],[19],[20] (general, with or without context)

---

[18] Thompson, William R. "On the likelihood that one unknown probability exceeds another in view of the evidence of two samples". Biometrika, 25(3–4):285–294, 1933.

[19] AKA "probability matching", "posterior sampling"

[20] cf., "Bayesian Bandit Explorer" (link)

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In Thompson sampling we will generate 1 datum at a time, by

- asserting a parameterized generative model for $p(y|a, x, \theta)$
- using a deterministic but averaged policy

TS: connecting w/"generative causal modeling" 1

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- hold up:
  - Q1: what's $p(\theta|D)$?
  - Q2: how am I going to evaluate this integral?

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  - (now you're not only generative, you're Bayesian.)

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TS: connecting w/"generative causal modeling" 2

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  - take 1 sample "θ_t" of θ from $p(\theta|D)$
  - $a_t = h(x_t; \theta_t) = \mathrm{argmax}_a Q(a, x, \theta_t)$

That sounds hard.

No, just general. Let's do toy case:

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- $= \Pi_a \theta^{\alpha+S_a-1}(1-\theta_a)^{\beta+F_a-1}$
- $\therefore \theta_a \sim \mathrm{Beta}(\alpha+S_a, \beta+F_a)$

Thompson sampling: results (2011)

An Empirical Evaluation of Thompson Sampling

Olivier Chapelle
Yahoo! Research
Santa Clara, CA
chap@yahoo-inc.com

Lihong Li
Yahoo! Research
Santa Clara, CA
lihong@yahoo-inc.com

Figure 15: Chaleppe and Li 2011

TS: words

In the realizable case, the reward is a stochastic function of the action, context and the unknown, true parameter θ^*. Ideally, we would like to choose the action maximizing the expected reward, $\max_a \mathbb{E}(r|a, x, \theta^*)$.

Of course, θ^* is unknown. If we are just interested in maximizing the immediate reward (exploitation), then one should choose the action that maximizes $\mathbb{E}(r|a, x) = \int \mathbb{E}(r|a, x, \theta)P(\theta|D)d\theta$.

But in an exploration / exploitation setting, the probability matching heuristic consists in randomly selecting an action a according to its probability of being optimal. That is, action a is chosen with probability

$$\int \mathbb{I}\left[\mathbb{E}(r|a, x, \theta) = \max_{a'} \mathbb{E}(r|a', x, \theta)\right] P(\theta|D)d\theta,$$

where $\mathbb{I}$ is the indicator function. Note that the integral does not have to be computed explicitly: it suffices to draw a random parameter θ at each round as explained in Algorithm 1. Implementation of the algorithm is thus efficient and straightforward in most applications.

Figure 16: from Chaleppe and Li 2011

TS: p-code

Algorithm 1 Thompson sampling

$D = \emptyset$
for $t = 1, \ldots, T$ **do**
  Receive context x_t
  Draw θ^t according to $P(\theta|D)$
  Select $a_t = \arg\max_a \mathbb{E}_r(r|x_t, a, \theta^t)$
  Observe reward r_t
  $D = D \cup (x_t, a_t, r_t)$
end for

Figure 17: from Chaleppe and Li 2011

TS: Bernoulli bandit p-code[22]

Algorithm 2 Thompson sampling for the Bernoulli bandit

Require: α, β prior parameters of a Beta distribution
  $S_i = 0, F_i = 0, \ \forall i$. {Success and failure counters}
  for $t = 1, \ldots, T$ **do**
    for $i = 1, \ldots, K$ **do**
      Draw θ_i according to Beta$(S_i + \alpha, F_i + \beta)$.
    end for
    Draw arm $\hat{\imath} = \arg\max_i \theta_i$ and observe reward r
    if $r = 1$ **then**
      $S_{\hat{\imath}} = S_{\hat{\imath}} + 1$
    else
      $F_{\hat{\imath}} = F_{\hat{\imath}} + 1$
    end if
  end for

Figure 18: from Chaleppe and Li 2011

TS: Bernoulli bandit p-code (results)

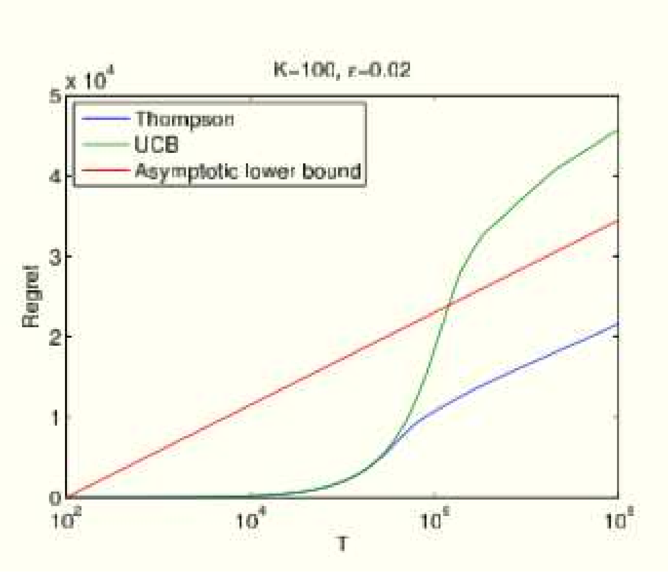

Figure 19: from Chaleppe and Li 2011

UCB1 (2002), p-code

Deterministic policy: UCB1.
Initialization: Play each machine once.
Loop:

- Play machine j that maximizes $\bar{x}_j + \sqrt{\frac{2 \ln n}{n_j}}$, where $\bar{x}_j$ is the average reward obtained from machine j, n_j is the number of times machine j has been played so far, and n is the overall number of plays done so far.

Figure 20: UCB1

from Auer, Peter, Nicolo Cesa-Bianchi, and Paul Fischer. "Finite-time analysis of the multiarmed bandit problem." Machine learning 47.2-3 (2002): 235-256.

TS: with context

Algorithm 3 Regularized logistic regression with batch updates

Require: Regularization parameter $\lambda > 0$.
  $m_i = 0,\ q_i = \lambda$. {Each weight w_i has an independent prior $\mathcal{N}(m_i, q_i^{-1})$}
  for $t = 1, \dots, T$ **do**
    Get a new batch of training data $(\mathbf{x}_j, y_j),\ \ j = 1, \dots, n$.
    Find $\mathbf{w}$ as the minimizer of: $\displaystyle \frac{1}{2}\sum_{i=1}^{d} q_i (w_i - m_i)^2 + \sum_{j=1}^{n} \log(1 + \exp(-y_j \mathbf{w}^\top \mathbf{x}_j))$.
    $m_i = w_i$
    $\displaystyle q_i = q_i + \sum_{j=1}^{n} x_{ij}^2 p_j (1 - p_j),\ \ p_j = (1 + \exp(-\mathbf{w}^\top \mathbf{x}_j))^{-1}$ {Laplace approximation}
  end for

Figure 21: from Chaleppe and Li 2011

LinUCB: UCB with context

Algorithm 1 LinUCB with disjoint linear models.

Inputs: $\alpha \in \mathbb{R}_+$
for $t = 1, 2, 3, \ldots, T$ **do**
  Observe features of all arms $a \in \mathcal{A}_t$: $\mathbf{x}_{t,a} \in \mathbb{R}^d$
  for all $a \in \mathcal{A}_t$ **do**
    if a is new **then**
      $\mathbf{A}_a \leftarrow \mathbf{I}_d$ (d-dimensional identity matrix)
      $\mathbf{b}_a \leftarrow \mathbf{0}_{d \times 1}$ (d-dimensional zero vector)
    end if
    $\hat{\boldsymbol{\theta}}_a \leftarrow \mathbf{A}_a^{-1}\mathbf{b}_a$
    $p_{t,a} \leftarrow \hat{\boldsymbol{\theta}}_a^\top \mathbf{x}_{t,a} + \alpha\sqrt{\mathbf{x}_{t,a}^\top \mathbf{A}_a^{-1}\mathbf{x}_{t,a}}$
  end for
  Choose arm $a_t = \arg\max_{a \in \mathcal{A}_t} p_{t,a}$ with ties broken arbitrarily, and observe a real-valued payoff r_t
  $\mathbf{A}_{a_t} \leftarrow \mathbf{A}_{a_t} + \mathbf{x}_{t,a_t}\mathbf{x}_{t,a_t}^\top$
  $\mathbf{b}_{a_t} \leftarrow \mathbf{b}_{a_t} + r_t\mathbf{x}_{t,a_t}$
end for

Figure 22: LinUCB

TS: with context (results)

Table 2: CTR regrets on the display advertising data.

Method	TS			LinUCB			ε-greedy			Exploit	Randor
Parameter	0.25	0.5	1	0.5	1	2	0.005	0.01	0.02		
Regret (%)	4.45	3.72	3.81	4.99	4.22	4.14	5.05	4.98	5.22	5.00	31.95

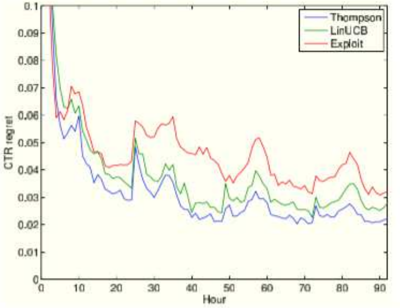

Figure 4: CTR regret over the 4 days test period for 3 algorithms: Thompson sampling with $\alpha = 0.5$, LinUCB with $\alpha = 2$, Exploit-only. The regret in the first hour is large, around 0.3, because the algorithms predict randomly (no initial model provided).

Figure 23: from Chapelle and Li 2011

Bandits: Regret via Lai and Robbins (1985)

THEOREM 2. *Assume that $I(\theta, \lambda)$ satisfies* (1.6) *and* (1.7) *and that Θ satisfies* (1.9). *Fix $j \in \{1, \ldots, k\}$, and define Θ_j and Θ_j^* by* (2.1). *Let φ be any rule such that for every $\boldsymbol{\theta} \in \Theta_j^*$, as $n \to \infty$*

$$\sum_{i \neq j} E_{\boldsymbol{\theta}} T_n(i) = o(n^a) \qquad \text{for every } a > 0, \tag{2.2}$$

where $T_n(i)$, defined in (1.2), *is the number of times that the rule φ samples from Π_i up to stage n. Then for every $\boldsymbol{\theta} \in \Theta_j$ and every $\epsilon > 0$,*

$$\lim_{n \to \infty} P_{\boldsymbol{\theta}}\left\{T_n(j) \geq (1 - \epsilon)(\log n)/I(\theta_j, \theta^*)\right\} = 1, \tag{2.3}$$

where θ^* is defined in (1.4), *and hence*

$$\liminf_{n \to \infty} E_{\boldsymbol{\theta}} T_n(j)/\log n \geq 1/I(\theta_j, \theta^*).$$

Figure 24: Lai Robbins

Thompson sampling (1933) and optimality (2013)

Theorem 2. *For any instance $\Theta = \{\mu_1, ..., \mu_N\}$ of Bernoulli MAB,*

$$R(T,\Theta) \leq (1+\epsilon) \sum_{i \neq I^*} \frac{\ln(T)\Delta_i}{KL(\mu_i, \mu^*)} + O(N/\epsilon^2)$$

Recall that we have $\lim_{T\to\infty} \frac{R(T,\Theta)}{\ln(T)} \geq \sum_{i\neq I^*} \frac{\Delta_i}{KL(\mu_i,\mu^*)}$. Above theorem says that Thompson Sampling matches this lower bound. We also have the following problem independent regret bound for this algorithm.

Theorem 3. *For all Θ,*

$$R(T) = \max_{\Theta} R(T,\Theta) \leq O(\sqrt{NT\log T} + N)$$

For proofs of above theorems, refer to [2].

Figure 25: TS result

from S. Agrawal, N. Goyal, ”Further optimal regret bounds for Thompson Sampling”, AISTATS 2013.; see also Agrawal, Shipra, and Navin Goyal. “Analysis of Thompson Sampling for the Multi-armed Bandit Problem.” COLT. 2012 and Emilie Kaufmann, Nathaniel Korda, and R´emi Munos. Thompson sampling: An asymptotically optimal finite-time analysis. In Algorithmic Learning Theory, pages 199–213. Springer, 2012.

other ‘Causalities’: structure learning

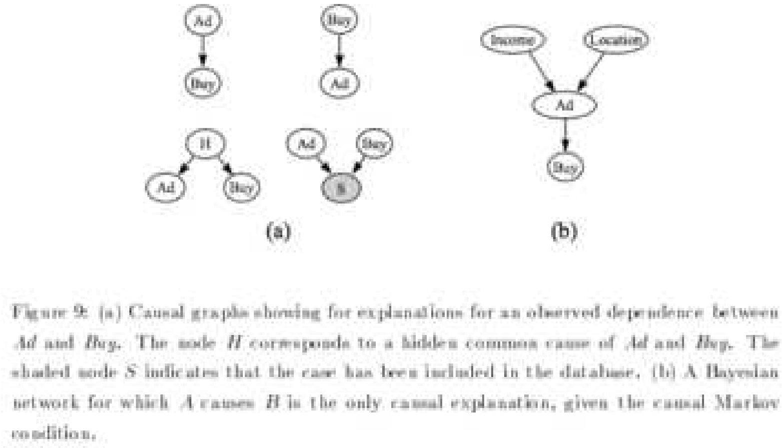

Figure 26: from heckerman 1995

D. Heckerman. A Tutorial on Learning with Bayesian Networks. Technical Report MSR-TR-95-06, Microsoft Research, March, 1995.

other 'Causalities': potential outcomes

- model distribution of $p(y_i(1), y_i(0), a_i, x_i)$

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- see Morgan + Winship[23] for connections between frameworks

[23] Morgan, Stephen L., and Christopher Winship. *Counterfactuals and causal inference* Cambridge University Press, 2014.

Lecture 4: descriptive modeling @ NYT

review: (latent) inference and clustering

- what does kmeans mean?

review: (latent) inference and clustering

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  - maximize $p(x|\theta)$

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  - given $d : R^D \to R^1$
  - assign z_i
- generative modeling gives meaning
  - given $p(x|z, \theta)$
  - maximize $p(x|\theta)$
  - output assignment $p(z|x, \theta)$

actual math

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- alternate optimization on θ and on q
  - NB: q step gives $q(z) = p(z|x, \theta)$

actual math

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  - NB: q step gives $q(z) = p(z|x, \theta)$
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actual math

- define $P \equiv p(x, z|\theta)$
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  - e.g., LDAs: word counts are sufficient statistics

tangent: more math on GMMs, part 1

Energy U (to be minimized):

- $-U \equiv E_q \log P = \sum_z \sum_i q_i(z) \log P(x_i, z_i) \equiv U_x + U_z$

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  $\Rightarrow -U_x = \sum_i r_{ik} \left(-\frac{1}{2}(x_i - \mu_k)^2 \lambda_k + \frac{1}{2} \ln \lambda_k - \frac{1}{2} \ln 2\pi\right)$

[24] math is simpler if you work with $\lambda_k \equiv \sigma^{-2}$

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simple to minimize for parameters $\vartheta = \{\mu_k, \lambda_k\}$

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tangent: more math on GMMs, part 2

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tangent: Gaussians $\in$ exponential family[26]

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  - $\exp(B(x)) = (2\pi)^{-1/2}$
- note that in a mixture model, there are separate η (and thus $A(\eta)$) for each value of z

[25] Choosing $\eta(\theta) = \eta$ called 'canonical form'

[26] NB: Gaussians $\in$ exponential family, GMM $\notin$ exponential family! (Thanks to Eszter Vértes for pointing out this error in earlier title.)

tangent: variational joy $\in$ exponential family

- as before, $-U = \sum_i r_{ik} \left(\eta_k^T T(x_i) - A(\eta_k) + B(x_i) \right)$

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- $\therefore \partial_{\eta_{k,\alpha}} A(\eta_k) \leftarrow E[T_{k,\alpha}|k]$ (canonical)
- nice connection w/physics, esp. mean field theory[27]

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[27] read MacKay, David JC. *Information theory, inference and learning algorithms*, Cambridge university press, 2003 to learn more. Actually you should read it regardless.

clustering and inference: GMM/k-means case study

- generative model gives meaning and optimization

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general framework: E+M/variational

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  - EDHMM: edhmm.github.io

example application: LDA+topics

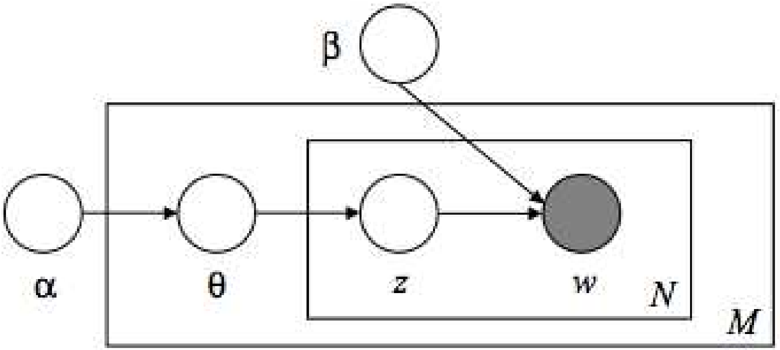

Figure 27: From Blei 2003

rec engine via CTM [28]

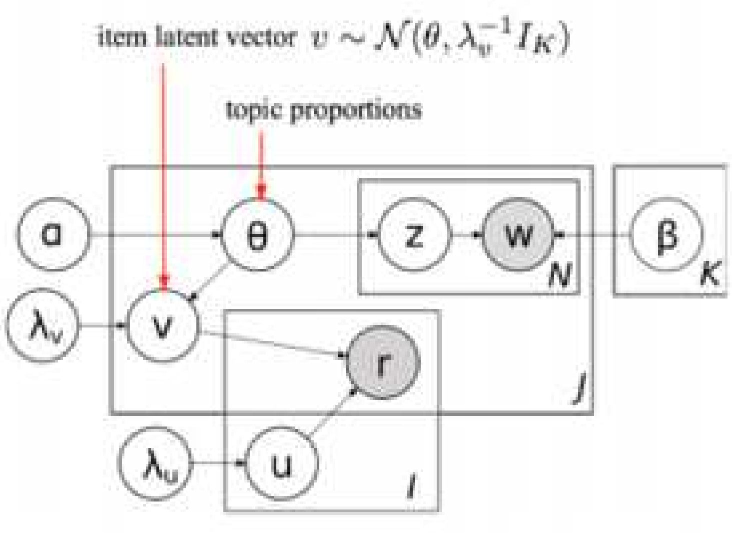

Figure 28: From Blei 2011

recall: recommendation via factoring

$$\min_{U,V} \sum_{i,j} (r_{ij} - u_i^T v_j)^2 + \lambda_u ||u_i||^2 + \lambda_v ||v_j||^2,$$

Figure 29: From Blei 2011

CTM: combined loss function

Maximization of the posterior is equivalent to maximizing the complete log likelihood of U, V, $\theta_{1:J}$, and R given λ_u, λ_v and β,

$$\mathcal{L} = -\frac{\lambda_u}{2}\sum_i u_i^T u_i - \frac{\lambda_v}{2}\sum_j (v_j - \theta_j)^T (v_j - \theta_j) \tag{7}$$
$$+ \sum_j \sum_n \log\left(\sum_k \theta_{jk}\beta_{k,w_{jn}}\right) - \sum_{i,j} \frac{c_{ij}}{2}(r_{ij} - u_i^T v_j)^2 .$$

Figure 30: From Blei 2011

CTM: updates for factors

$$u_i \leftarrow (VC_iV^T + \lambda_u I_K)^{-1} VC_i R_i \tag{8}$$

$$v_j \leftarrow (UC_jU^T + \lambda_v I_K)^{-1} (UC_j R_j + \lambda_v \theta_j). \tag{9}$$

Figure 31: From Blei 2011

CTM: (via Jensen's, again) bound on loss

$$
\begin{aligned}
\mathcal{L}(\theta_j) &\geq -\frac{\lambda_v}{2}(v_j - \theta_j)^T (v_j - \theta_j) \\
&\quad + \sum_n \sum_k \phi_{jnk} \left(\log \theta_{jk}\beta_{k,w_{jn}} - \log \phi_{jnk}\right) \\
&= \mathcal{L}(\theta_j, \phi_j). \qquad (10)
\end{aligned}
$$

Figure 32: From Blei 2011

Lecture 5 data product

data science and design thinking

- knowing customer

data science and design thinking

- knowing customer
- right tool for right job

data science and design thinking

- knowing customer
- right tool for right job
- practical matters:

data science and design thinking

- knowing customer
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- knowing customer
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  - munging
  - data ops
  - ML in prod

Thanks!

Thanks MLSS students for your great questions; please contact me @chrishwiggins or chris.wiggins@{nytimes,gmail}.com with any questions, comments, or suggestions!