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Optimizing the allocation of funds of an NFL team under the salary cap

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ABSTRACT

Every NFL team faces the complex decision of having to choose how to allocate salaries to each position while being limited by the salary cap. This paper uses regression strategies to identify which positions are worthy of greater investment, under the assumption that players are paid in an efficient market. Using a combination of univariate regression models, we identify that it is worth investing in elite players at the quarterback, guard, defensive line, and linebacker positions. In addition, through a separate set of regression models we also consider the possibility that markets are not actually efficient. We determine that the optimal way to take advantage of inefficiency is through the draft, in order to find players who can provide significant win contributions early in their careers while they are being paid on relatively low rookie contracts.

1. Introduction

The focus on analytics has been increasing within all major sports leagues in the United States since the early 2000s (Fry & Ohlmann, 2012); however, the increase in the National Football League has been relatively slow, perhaps due to the complexity of the game as an interconnected coordination of many players. As analytics is now beginning to take a stronger hold in the NFL, though (as can be seen from the Next Gen Stats program started by the league), salary cap management has appeared as one of the key applications of statistical analysis to NFL team decision-making.

Unlike some other professional sports leagues, the NFL has a strict team salary cap, meaning that teams cannot pay a luxury tax to obtain permission to have a higher player salary total. This creates a classic 'allocation of a scarce resource' decision, a topic that has generated a substantial amount of research. Radner (1972) discussed the allocation of a scarce resource in situations of uncertainty, and Borghesi (2008) applied this issue specifically to the NFL salary cap. Radner (1972) used an economic model for a problem of allocating a scarce raw material to a range of enterprises. The author assigned an output function to each enterprise and attempted to maximize the expected total output with respect to the constraint of the scarce resource. Meanwhile, Borghesi (2008) used regression to identify which NFL players were overpaid relative to their performances and to identify the impact of this overpayment on their team's performance.

When NFL executives make decisions as to which players to sign, they are aware of past performances and other observables, but can have no knowledge of the player's true future value. The literature in this area indicates that players who are paid less can earn large salary increases with improvements in their performances, while those who already receive high pay will not earn much more with improved performances (Leeds & Kowalewski, 2001).

Motivated by this uncertainty in performance, there has been some literature on how the salary structure of an NFL team can be optimized to increase individual player performances, and thus, the overall team performance. Mondello and Maxcy (2009) find that giving a player an increase in salary with incentive bonuses for performance

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within a mostly uniform salary structure (one with little spread in player salaries) will result in an improved on-field performance. Meanwhile, Jane, San, and Ou (2009) find that a uniform salary structure is optimal for team performance in the Taiwanese professional baseball league.

However, Quinn, Geier, and Berkovitz (2007) find that teams in the NFL do not have a uniform salary structure, but more of a "superstar" salary structure, with some players earning far higher salaries than others. They believe that this is due to NFL owners and managers having convex utility curves regarding wins, so that gaining a small amount of extra talent on their roster is believed to have a large impact on the utility. The paper concludes by stating, "Moreover, while there may be some rather difficult-to-detect strategies in cap allocation across players to enhance winning, teasing them out of the available data remains elusive" (Quinn et al., 2007, p. 15).

Winsberg (2015) attempted to discover some of these cap allocation strategies in order to maximize wins. He focused on only a few position groups, and concluded that paying offensive lineman and quarterbacks more than the league average leads to a decreased team performance.

The main contribution of our paper is to consider all position groups in the National Football League in order to identify the optimal percentage breakdown of the salary cap by position group that maximizes the total expected win contribution across all positions (i.e., expected wins) with the restriction of the salary cap.

It is then possible to extend our approach to add the dimension of the performance levels (or win contributions) of specific players. Not only does this identify what salary cap allocations have led teams to the most success in the past, it also provides the ability to identify the marginal win contribution that can be added by investing more money in any given position. This approach allows us to identify the positions in which an added investment will achieve the greatest increase in expected wins. Thus, when presented with limited salary cap room remaining and multiple positions to fill, a team will know which positions are worth the investment of those final dollars.

With a full consideration of all position groups and player performance levels, the goal of this analysis is to identify the best possible salary cap allocation, by which a team will maximize the win contribution per marginal cost at every position, thereby maximizing a team's expected wins.

Past results indicate that a more uniform salary structure will be found to be optimal, rather than that which currently exists in the NFL. Even though it seems as though the teams with the best quarterbacks are those that win the most, Winsberg (2015) found that it is not optimal in terms of team performance to have a highly paid quarterback. However, once player performance (in terms of win contributions) is taken into consideration, we find that the optimal allocation strategy is highly non-uniform and does allocate a relatively high salary to quarterbacks, though not as high as the top NFL quarterbacks actually receive.

Quarterbacks are the primary driver of a team's salary cap situation. As there are so few quality quarterbacks, once a team finds the QB whom they believe can bring success, they will pay whatever is necessary to keep that QB under contract. Kelly (2016) notes that there are "essentially three types of rosters: the ones that pay a quarterback big money, the ones that pay a young quarterback rookie-contract money, and the ones that don't do either". Teams with no quarterback try to find a quarterback, teams with a good quarterback try to structure their roster to win within that quarterback's prime years, and the lucky few teams with a good QB on a rookie contract enjoy both good production at quarterback and extra money to spend on other positions.

It should be noted that the optimal allocation strategies that we identify in this paper assume that players are paid efficiently (i.e., players perform to their salary: higher paid players should outperform their lesser paid peers), which is not the case in reality, especially with young players outperforming their low-pay rookie contracts. If every team had complete information on how each player would perform in the future, they could refuse to pay a player anything above the efficient salary for his performance; however, future performance is uncertain, leading to an inefficient marketplace due to varying performance expectations. Thus, the fact that one player is paid more than another does not guarantee that the higher paid player will perform better than the lower.

Thus, we will also analyze separately the ways in which the specific players' win contributions compare to their salaries, in order to identify uncompensated win contributions: win contributions beyond what would be expected given their salary. Teams that pay players low salaries and get many uncompensated wins tend to be the best teams. The success of this formula can be seen by the dominance of the Seattle Seahawks in 2013 and 2014, who earned many uncompensated wins with quarterback Russell Wilson on his rookie contract, while they also had very few players earning "superstar" salaries. In 2014, only two Seahawks players had salary cap hits of more than \$8 million.[1]

Overall, there are three questions that are fundamental to this current work. First, in what positions should teams generally invest the most money in order to maximize their expected wins? Second, what is the best way of measuring the performances (value) of players at each position? And, finally, how do players at different positions compare in terms of their additional marginal win contributions from additional investment at that position?

These three pieces of information give teams the ability to identify the available players with the highest expected performances through prediction models. Then, by considering their expected performance levels and positions, the team will be able to identify the additional marginal win contribution that will be gained by spending on one player over another, and the salary that would be efficient for each player's win contribution.

This analysis addresses this allocation problem with an optimal solution that can be the overall goal for a team, but that also offers insights to assist in individual personnel decisions. The methodology used in this analysis is applicable to any sports league with a strict salary cap.

1 http://www.spotrac.com/nfl/seattle-seahawks/cap/2014.

2. Data and methodology

This analysis requires data on NFL player compensation, NFL team performance and NFL player performance. Compensation data for the 2011-2015 seasons was obtained from spotrac.com. Though this provides only 160 team-seasons (32 teams over five years), having a data set that is focused on the recent past can be beneficial because team strategy in the NFL evolves continually. Focusing on the recent past will provide solutions that are more applicable to upcoming seasons of the NFL.

Our analysis of player salaries uses the *cap hit* of each contract as the measure of player compensation. A contract's cap hit is the amount of money that the contract counts towards the team's salary cap in a given season. The cap hit usually includes the player's salary that season plus prorated portions of any bonuses paid earlier in the contract. We chose to use the cap hit of a player's salary because the salary cap is the real constraint on NFL team spending, and thus, the dollar amount of the salary cap that a player's contract takes up (i.e., the player's cap hit) is the most important measure of a team's allocation of resources to that player. Furthermore, the sum of the cap hits at each position is a good indicator of how much a team has invested in that position. While the cap hit is the dollar amount that counts towards the salary cap, it can also be insightful to view it as a percentage of the total salary cap that teams can spend each season.

NFL team performance data on team wins were obtained from NFL.com, while data measuring player performance were gathered from Pro-Football-Reference.com. The performance measure Approximate Value (AV) is Pro Football Reference's "attempt to put a single number on each player-season since 1950" in order to measure player value.[2] Approximate Value is calculated based on linear combinations of team statistics (including points per possession), along with individual statistics including games played, games started, pro bowl selection, and production-based statistics (e.g. receiving yards for wide receivers). Since these individual statistics are cumulative, AV is a measure of the cumulative individual performance (rather than the average performance per game or per play). The team statistics are used to identify the total value to be split amongst the players on the team, while the individual statistics then identify how much of the AV is to be allocated to each player.[3]

Yurko, Ventura, and Horowitz (2018) explain that while AV is based on both objective and subjective analyses, it has the rare advantage of being available publicly and using the same scale across all positions. AV has been used in analyses by many other sources, including FiveThirtyEight (Paine & McCann, 2015).

It is crucial to this analysis that the scale is the same across positions. The regression coefficients that our methodology uses as conversion factors from AV to the win contribution negate some of the subjectivity in the way in which AV is allocated by position. Other performance measures do exist, such as Pro Football Focus grades, but these alternatives are not available publicly, and are also subjective.

Our analysis first requires the identification of each player's win contribution each season. We observe the total AV at each position (using a split into 19 positions - QB, RB, FB, WR, TE, LT, G, C, RT, DE, DT, ILB, OLB, CB, FS, SS, K, P, and LS) for the 160 team-seasons in our data set. While there are 53 players on each team roster and each team has only 11 players on the field at any given time, these 19 positions cover all possible positions at which a player will line up (and there can be multiple players at the same position on the field - for example, a team will have two guards (G) on offense).

We used a linear regression model that predicts team wins from the total AV that the team had from each position (i.e., position 4 is WR, so $AV_{t,position4}$ is the sum of the AVs of all of the wide receivers on team t). The intercept term from this regression α_{global} is a global constant that will be used throughout our models.

$$Wins_t \sim \alpha_{global} + \sum_{i=1}^{19} \left(\beta_i * AV_{t,position\, i}\right)$$

Note that the outcome variable of this regression is not independent, as the number of games is finite, and thus the total number of wins across all teams is predetermined, since each game can only have one winner. However, given that the sole purpose of this step is to calculate conversion factors for translating AVs into win contributions, we believe that teams with greater AVs at more important positions will still tend to win more games, despite this interdependence. Therefore, this regression should still provide applicable conversion factors (denoted by β_i).

The regression performed on data from the 2011–2015 seasons has an R^2 of 0.77. The intercept and six position coefficients (QB, DE, DT, ILB, OLB, and CB) are significant at the 0.1% level. In addition, the WR and G coefficients are significant at the 5% level. In Table A.1 of the appendix, we provide estimates and standard errors for each parameter from this regression model.

Each player's win contribution for any given year can be calculated by multiplying the AV that the player obtained that year by the β_i for the player's position. In addition, a team's win contribution from any position can be calculated as the total AV from that position multiplied by the position's β_i value.

Now, knowing the win contribution that each team gained from each position, it is possible to model the salary (in terms of the cap hit) versus win contribution to identify the return-on-investment gained by spending cap space on each position.

We decided to use a univariate modeling strategy to identify these relationships. Specifically, we created a separate univariate regression for each of the 19 positions in order to identify the "compensated" (expected) win contribution from that position given the amount of the salary cap that a team invested in that position:

$$Win\ Contribution_{t,position\, i} \sim \alpha_{i,uni} + \beta_{i,uni} * \log(cap\ hit_{t,position\, i}),$$

[2] http://www.pro-football-reference.com/about/glossary.htm.

[3] Approximate value II: https://www.pro-football-reference.com/blog/index2905.html?p=466.

where $Win\ Contribution_{t,position\ i}$ and $cap\ hit_{t,position\ i}$ are the total win contribution and the cap hit for each team t at position i. We use the *uni* subscript here to differentiate the coefficients from these univariate models of cap hit versus wins from those of the earlier linear model with multiple predictor variables that converted the AV from each position into a win contribution. Table A.2 of the appendix provides parameter estimates from these univariate regression models, fit separately for each position. In these regressions, $\beta_{i,uni}$ is significant at the 5% level for 17 of the 19 positions (all but the RB and LS positions).

This univariate methodology preserves the fact that the win contribution from one position is associated with the size of the portion of the salary cap that the given team allocated to that position. Once these univariate models have been created, a team's projected (compensated) wins can be obtained through a combination of the 19 univariate regressions along with $\hat{a}_{global}$ from the earlier model:

$$Compensated\ Wins_{t,uni} = \hat{a}_{global} + \sum_{i=1}^{19} \left(\hat{a}_{i,uni} + \hat{\beta}_{i,uni} * \log(cap\ hit_{t,position\ i}) \right).$$

We call this outcome a team's "compensated wins": the number of wins that a team is expected to win based on their salary cap allocation to each position.

Once we have this formula for compensated wins, linear programming can be used to identify the salary allocation that optimizes the compensated wins given the salary cap constraint. It is important to note that this paper focuses on identifying the optimal allocation of the salary cap by *position*, not by player. Thus, we will not focus on breakdowns between starters and backups, but rather on the breakdown of the salary cap across positions.

With known value contributions per investment at each position, linear programming allocates scarce funds to these investments in order to optimize the overall value (Asher, 1962). Beginning with the rookie minimum salary for the number of players that a team must carry at each position, each additional dollar is allocated to the position that has the highest current marginal benefit (the highest partial derivative with respect to salary). Thus, this method will create a breakdown of how much should be paid to each position in order to achieve the maximal projected wins.

Our procedure produces an optimal allocation of salary by position under the assumption of "efficient pay", i.e., that each player's performance will match the expected win contribution of their cap hit and that a team's actual win totals will match their compensated win totals. However, as we noted earlier, the reality is that pay is not truly efficient in this way. We can therefore quantify the "uncompensated" win contribution of individual players by comparing their actual win contributions (their AVs multiplied by the $\hat{\beta}_i$ from the win regression model) to their predicted individual compensated wins.

We obtain their predicted individual compensated wins using a player-by-player univariate regression of X = log(*cap hit*) versus Y = win contribution of individual players at each position. Then, an individual player's compensated win contribution can be calculated as the predicted win contribution for that player's cap hit from the log-curve of this new regression model. Note that this model is fitted at the player level, and therefore is distinct from our earlier regression models of win contribution (by position) on cap hit (by position) that were fitted at the position level.

An individual player's uncompensated win contribution is then their actual win contribution minus their compensated win contribution. Teams can take advantage of the inefficient marketplace by signing players whose actual win contribution they expect to be greater than their compensated win contribution based on their salary.

3. In-sample data analysis and model fit

3.1. Allocation model results

The optimal allocation strategy can be identified using linear programming, assuming the constraint of the 2016 NFL team salary cap of $155.27 million. The resulting optimal allocation can be seen in Table 1, both in dollar terms (total cap hit by position) and as a percentage of the salary cap. This allocation confirms the commonly held-belief that it is worth paying more for a quarterback, though not as high as some would expect.

Our model would probably suggest an even higher pay for quarterbacks if it were not for the extreme savings in the period considered of having several highly successful quarterbacks such as Russell Wilson and Cam Newton on rookie contracts. Relative to what top players at each position currently get paid in the league, the model suggests paying for more expensive guards, defensive lineman, and linebackers.

The optimal allocation from the univariate model pays a very low salary to running backs, which has been a trend throughout the league in recent years. However, the low salary for left tackles is the opposite of the trend in the league. Left tackles are considered as more important and valuable, as they protect the "blind side" of their quarterbacks (who are almost all right-handed). Left tackles are among the highest paid players in the NFL, but this model suggests that they are often not worth the investment. While many left tackles are being paid high salaries, not all of them deserve this, due to lackluster performances. Thus, since many left tackles with high salaries actually do not have high win contributions, the expected marginal win contribution from paying a higher salary to left tackles is lower than those of other positions. Of course, there are some talented left tackles that are exceptions to this rule.

3.2. Evaluation of team allocations

Our approach can be also be used to calculate which teams were expected to have the highest numbers of compensated wins based on their salary allocations each year. Table 2 displays the team that was projected to have the highest compensated win total each year, along with their actual win total, while Table 3 gives the same information for the team that was projected to have the lowest compensated win total in each year. For the most part, the teams with the best allocations did have successful seasons and

Table 1
Optimal allocation.

Position	Cap hit	Percent of cap
Quarterback (QB)	$13,393,874	8.6%
Running Back (RB)	$1,274,152	0.8%
Fullback (FB)	$3,238,137	2.1%
Wide Receiver (WR)[b]	$8,636,432	5.6%
Tight End (TE)	$1,035,909	0.7%
Left Tackle (LT)	$1,825,435	1.2%
Guard (G)[b]	$16,475,218	10.6%
Center (C)	$2,907,167	1.9%
Right Tackle (RT)	$3,890,938	2.5%
Defensive End (DE)[b]	$21,258,202	13.7%
Defensive Tackle (DT)[a]	$15,766,992	10.2%
Inside Linebacker (ILB)[a]	$15,589,591	10.0%
Outside Linebacker (OLB)[b]	$23,590,392	15.2%
Cornerback (CB)[b]	$11,082,839	7.1%
Free Safety (FS)	$6,761,298	4.4%
Strong Safety (SS)	$5,030,941	3.2%
Kicker (K)	$1,366,229	0.9%
Punter (P)	$1,696,254	1.1%
Long Snapper (LS)	$450,000	0.3%

[a]Some teams will have 2 starters at this position.
[b]All teams will have 2 starters at this position.
All other positions teams typically have one starter.

Table 2
Best team allocations.

Year	Team	Compensated wins	Actual
2011	PIT	10.9	12
2012	SF	9.6	11.5
2013	NO	9.2	11
2014	GB	9.0	12
2015	CIN	9.2	12

Table 3
Worst team allocations.

Year	Team	Compensated wins	Actual
2011	STL	0.0	2
2012	NYG	3.6	9
2013	OAK	5.2	4
2014	SD	6.7	9
2015	CHI	6.2	6

Table 4
Total 2011–15 uncompensated win contribution.

Rank	Name	Uncomp. Win Cont. Total
1	Russell Wilson	15.8
2	Cam Newton	12.3
3	Tom Brady	10.4
4	Aaron Rodgers	9.9
5	Andy Dalton	9.2
6	Drew Brees	8.6
7	Matt Ryan	6.7
8	Richard Sherman	6.0
9	Alex Smith (QB)	5.9
10	Ryan Tannehill	5.7

Table 5
Average 2011–15 uncompensated win contribution.

Rank	Name	Uncomp. Win Cont. Average
1	Russell Wilson	4.0
2	Cam Newton	2.5
3	Tom Brady	2.1
4	Teddy Bridgewater	2.0
5	Aaron Rodgers	2.0
6	Andy Dalton	1.8
7	Drew Brees	1.7
8	Jameis Winston	1.7
9	Derek Carr	1.6
10	Ryan Tannehill	1.4

those with the worst allocations did not, but it is important to note that these were the teams which were projected to have the most/fewest compensated wins, not actual wins. It is also interesting to note that while the top teams are not projected to be too far above the league average of eight wins, some of the worst teams are projected to have very few wins. This indicates that while allocating closer to optimally will not provide separation from other well-allocated teams, allocating heavily towards the wrong positions will greatly hurt a team's chances of winning many games (though the 2012 Giants had an impressive number of actual wins given their sub-optimal allocation).

One interesting team in Table 2 is the 2013 New Orleans Saints. When considering their allocation, three of the five highest cap hits (including the two highest) are a quarterback (Drew Brees) and two guards (Jahri Evans and Ben Grubbs). Thus, the Saints were focusing their allocation primarily on the optimal offensive positions from the model and were able to win eleven games in the regular season, before being eliminated from the playoffs in the divisional round.

3.3. Uncompensated wins

While the New Orleans Saints were the best team in 2013 in terms of compensated wins, the Seattle Seahawks (which eliminated the Saints and won the Super Bowl) had the highest uncompensated wins of any team in 2013. Uncompensated wins account for the difference between a team's actual wins and their compensated wins. Thus, a team will have high uncompensated wins if players outperform the cap hit of their contract.

Based on their compensated wins, the Seahawks were expected to win less than half of their games that year. However, with impressive production from many players on rookie contracts with cap hits under $1 million (Russell Wilson, Richard Sherman, Bobby Wagner, Golden Tate, Doug Baldwin, Malcolm Smith, K.J. Wright, Byron Maxwell, Walter Thurmond, J.R. Sweezy, etc.), the Seahawks were able to achieve more uncompensated wins than any other team in the league that year (see Table 7 for average uncompensated wins for 2011 to 2015). More generally, the overpayment or underpayment of specific players is a major contributor to the difference between compensated and actual wins, with rookie contracts being a specific underpayment that is mandated by the salary structure of the National Football League.

Fig. 1 shows the regression log-curve for the relationship between the player cap hit and the player win contribution for each position. The coefficient for the log(*cap hit*)

Table 6
Average 2011-15 Non-QB uncompensated win contribution.

Rank	Name	Uncomp. Win Cont. Average
1	Richard Sherman	1.2
2	J.J. Watt	1.0
3	Patrick Peterson	0.9
4	Marcus Peters	0.8
5	Justin Houston	0.8
6	Aaron Donald	0.8
7	Luke Kuechly	0.8
8	Bobby Wagner	0.8
9	Von Miller	0.8
10	Lavonte David	0.7

Table 7
Team rankings of average uncompensated wins per season.

Rank	Team	Uncompensated win Avg	Rank	Team	Uncompensated win Avg
1	NE	4.0	17	PHI	−0.3
2	DEN	3.9	18	CHI	−0.4
3	SEA	3.0	19	SD	−0.4
4	GB	2.6	20	ATL	−0.5
5	BAL	2.0	21	STL	−0.7
6	ARI	1.7	22	MIA	−0.7
7	CIN	1.7	23	NYJ	−0.8
8	NYG	1.3	24	DET	−0.8
9	CAR	1.1	25	BUF	−1.2
10	SF	1.0	26	MIN	−1.3
11	IND	0.8	27	OAK	−1.8
12	PIT	0.6	28	WAS	−2.2
13	NO	0.5	29	TB	−3.1
14	KC	0.2	30	TEN	−3.1
15	HOU	0.1	31	CLE	−3.3
16	DAL	0.1	32	JAC	−4.0

Table 8
2016 Projected Wins.

Team	Compensated wins projected	Uncomp win average 2011–15	Projected wins	Actual wins
DEN	7.8	3.9	11.7	9
GB	9.0	2.6	11.6	10
SEA	8.6	3.0	11.6	10.5
CIN	9.4	1.7	11.0	6.5
NE	6.7	4.0	10.7	14
BAL	8.3	2.0	10.3	8
PIT	9.2	0.6	9.8	11
HOU	9.6	0.1	9.7	9
ARI	8.0	1.7	9.7	7.5
NYG	7.9	1.3	9.2	11
KC	8.8	0.2	9.1	12
DAL	8.9	0.1	9.0	13
SF	7.9	1.0	8.9	2
STL	9.3	−0.7	8.7	4
SD	9.0	−0.4	8.6	5
CAR	7.4	1.1	8.5	6
IND	7.7	0.8	8.5	8
ATL	8.6	−0.5	8.1	11
CHI	7.9	−0.4	7.5	3
NO	6.9	0.5	7.4	7
BUF	8.4	−1.2	7.2	7
OAK	9.0	−1.8	7.2	12
MIN	8.5	−1.3	7.2	8
NYJ	8.0	−0.8	7.2	5
PHI	7.4	−0.3	7.1	7
DET	7.9	−0.8	7.0	9
MIA	7.3	−0.7	6.6	10
TEN	8.6	−3.1	5.5	9
TB	8.3	−3.1	5.2	9
WAS	4.5	−2.2	2.4	8.5
JAC	6.0	−4.0	2.1	3
CLE	5.3	−3.3	2.0	1

in each of these player-level models shown in Fig. 1 is significant at the 0.1% level (with the exception of LS, which is significant at the 1% level). The expected (compensated) win contribution for a player is the y-coordinate of the log-curve for their x-coordinate (the player's cap hit). The player's uncompensated win contribution is their actual win contribution (AV multiplied by $\hat{\beta}_i$ for the position from the original win model) minus this expected win contribution. As was noted earlier, it is optimal for teams to attempt to sign players that they believe will be above the line (i.e., their actual win contribution will be greater than the compensated win contribution for their salary, or, equivalently, their uncompensated win contribution will be greater than zero).

When observing Fig. 1, it is important to pay attention to differences in scale. For example, the top of the y-axis of the quarterback plot reaches a far higher win contribution than the other charts. The win contribution does favor quarterbacks heavily, which makes sense, as it is the position that is widely considered to have the highest impact on the quality of a team.

Accordingly, quarterbacks dominate the chart when observing the highest uncompensated win contributions. Table 4 shows the best ten cumulative uncompensated win contributions (2011-15 seasons), while Table 5 shows the best ten average uncompensated win contributions per season (2011-15 seasons). Note that providing both rankings is not redundant, as some players were not in the league all five seasons. With the exception of Richard Sherman, who is only on the cumulative chart not the average chart, every player in the top ten is a quarterback. Also, unsurprisingly, seven of the ten quarterbacks on the average uncompensated win contribution chart are players who were on their rookie contract for most of this five-year span. For comparison, Tables 4 and 5 also shows the worst five quarterbacks in terms of total and average uncompensated win contributions. These tend to be well-compensated quarterbacks who were injured for large parts of the 2011-15 seasons.

The same pattern is also evident when we exclude quarterbacks. Table 6 shows the top ten non-quarterbacks in average uncompensated wins per season. Again, every player in this chart was on their rookie contract for at least part of the sample 2011 to 2015. Interestingly, every player is a defensive player, probably due to the fact that the quarterback dominates a team's win contribution from its offense, while there is no single position that dominates the defensive win contribution.

These results indicate that to achieve high uncompensated wins, teams must be skilled in selecting the best players in the NFL Draft, because with the exception of top tier quarterbacks, it is these recently drafted players with low rookie salaries that contribute the most uncompensated wins. It is generally accepted that the reason why the Seahawks had sustained success during this time period is that they succeeded in finding successful players in the NFL Draft. This is evident in Table 7, as the Seahawks have one

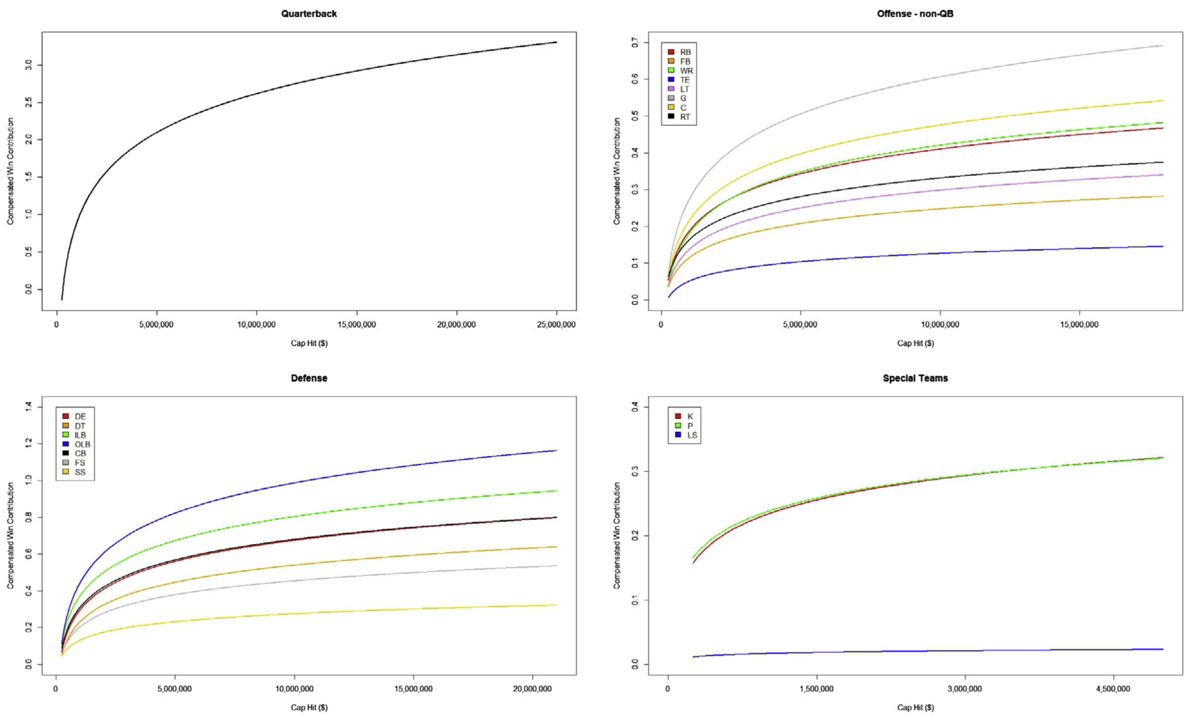

Fig. 1. Relationship between cap hit and win contribution by position.

of the highest uncompensated win averages of any team from 2011 to 2015.

It is also noteworthy that the top franchises in uncompensated wins per season in Table 7 are, in fact, the best franchises over the sample from 2011 to 2015. A high uncompensated win average indicates that, over this period, the franchise succeeded in identifying players that would outperform their contracts. Over this five-year sample, average uncompensated wins per season has a correlation of 0.9 with average actual wins per season. Drafting well is the key not only to increasing uncompensated wins, but also to leading a team to the top of the league standings.

4. Out-of-sample forecasting of the 2016 NFL season

The models developed in this analysis were then used to project the 2016 season out-of-sample based on the teams' salary cap allocations that season. The 2016 projections from our model for compensated wins can be seen in Table 8. To obtain a projected win total, we added each team's average uncompensated wins from 2011-15 to its compensated win projection, which assumes some consistency in a franchise's skill at acquiring talent.

Interestingly, the team with the highest projection for compensated wins is the Houston Texans. The Texans famously signed quarterback Brock Osweiler to a very high-paying contract prior to the 2016 season, despite his lack of experience, and were paying defensive end J.J. Watt a high salary as well. Thus, they had high salaries at two positions that should be allocated high amounts, though in the event Osweiler under-performed and Watt missed the majority of the season due to injury. Meanwhile, the Washington Redskins were projected to achieve the fewest compensated wins for 2016. This result was influenced heavily by the Redskins having allocations that were far higher than optimal at three offensive positions: left tackle (Trent Williams), wide receiver (primarily Pierre Garcon and Desean Jackson), and tight end (Jordan Reed and Vernon Davis).

As can be seen from Table 8, the uncompensated win average is the primary driver of projected wins, when calculated in this manner. Thus, the teams that had success in 2011-15 continued to be the best projected for 2016.

In the end, compensated wins had a correlation of 0.3 with actual wins in 2016, while projected wins had a correlation of 0.41 with actual wins (the win totals can be seen in the right column of Table 8). The teams with the largest differences between projected and actual wins were the Washington Redskins, Oakland Raiders, and Dallas Cowboys. This can be accounted for by a large upward swing in uncompensated wins driven by contributions from rookie contract players for the Washington Redskins, and good play by young quarterbacks Derek Carr for the Oakland Raiders and Dak Prescott for the Dallas Cowboys.

5. Summary and discussion

This paper has presented an optimization of salary cap allocations for NFL teams based on a regression approach that combines a linear regression for connecting a player's win contribution to his approximate value with a series of additional linear regressions for connecting win contribution by position to cap hit by position.

The optimal allocation found by our approach suggests that, relative to current NFL salaries, it is optimal to pay for high-end players at guard, defensive line, and linebacker, rather than at left tackle as is commonly believed. The model also supports the current trend throughout the league of paying lower salaries to running backs.

One shortcoming of our modeling approach is that it assumes that all teams will achieve the same win contribution returns from an investment at each given position; i.e., every player is paid exactly efficiently according to their win contribution. However, this is not actually the case, as teams have varying abilities in terms of player evaluation, as well as incomplete information, leading to an inefficient market. We have also created univariate models by player at each position, in order to investigate which players produce more (or less) than the win contribution that would be expected based on their salary. Thus, we can observe which teams are getting higher returns than expected (i.e., a greater uncompensated win contribution) from the players that they are paying.

Through these models, we identified that the Seattle Seahawks were among the teams that achieved the highest uncompensated wins from 2011 to 2015, due to the fact that the Seahawks were able to make many successful draft picks and had productive players playing on low rookie-contract salaries (especially Russell Wilson). In addition, we find that a team's uncompensated win total is correlated very closely with the team's actual win total. This implies that the key way to ensure that teams will be among the premier organizations is to draft players who will achieve high win contributions while still playing on their rookie contracts (which typically last four years).

Overall, we believe that if a team focuses their salary allocation on positions that have higher optimal salaries in our univariate model (unless they have players on their rookie contracts in those positions) and is able to draft players who can make an impact in the league quickly, that team will be expected to win more games. Optimally, a team can create prediction models for player win contributions, use those projections to observe the expected efficient salary for each player, and attempt to sign players whose salaries implied by the existing free agent market are lower than what were determined to be their expected efficient salaries. If a team is able to sign many players for salaries that are below their efficient value, they will achieve more uncompensated wins and then have the salary cap space to invest more money in key positions where a high return of compensated wins is expected, thus achieving the maximal expected wins.

Appendix

See Tables A.1 and A.2.

Table A.1
Parameter estimates from the linear regression model of wins as a function of the approximate value at each position.

Position	Estimate	Std. Error	t-statistic	p-value
(Intercept)	-9.021	1.077	-8.374	**0.000**
QB	0.285	0.061	4.646	**0.000**
RB	0.053	0.038	1.413	0.160
FB	0.136	0.115	1.187	0.237
WR	0.056	0.028	2.002	**0.047**
TE	0.025	0.053	0.477	0.634
LT	0.036	0.039	0.930	0.354
G	0.070	0.033	2.151	**0.033**
C	0.055	0.042	1.333	0.185
RT	0.043	0.041	1.055	0.293
DE	0.084	0.020	4.172	**0.000**
DT	0.072	0.021	3.428	**0.001**
ILB	0.097	0.024	4.054	**0.000**
OLB	0.128	0.023	5.524	**0.000**
CB	0.109	0.031	3.521	**0.001**
FS	0.062	0.038	1.627	0.106
SS	0.042	0.037	1.147	0.253
K	0.087	0.099	0.882	0.379
P	0.115	0.141	0.816	0.416
LS	0.015	0.235	0.065	0.949

Notes: We provide estimates, standard errors, test statistics and p-values for the intercept (α_{global}) and slope (β_i) for each position i. Gray shading is used to indicate estimates that are significantly different from zero.

Table A.2
Parameter estimates from the univariate linear regression models of the win contribution as a function of the log (cap hit), fitted separately for each position.

Results from Univariate Models			
Position	Alpha Estimate	Beta Estimate	Beta Std. Error
QB	-1.584	0.318	0.138
RB	0.249	0.030	0.033
FB	-0.930	0.077	0.020
WR	-2.175	0.205	0.062
TE	-0.210	0.025	0.011
LT	-0.329	0.043	0.014
G	-4.973	0.390	0.060
C	-0.549	0.069	0.023
RT	-1.028	0.092	0.015
DE	-6.542	0.504	0.054
DT	-4.891	0.374	0.042
ILB	-4.314	0.369	0.076
OLB	-6.700	0.559	0.089
CB	-2.414	0.263	0.080
FS	-1.905	0.160	0.019
SS	-1.449	0.119	0.014
K	-0.187	0.032	0.010
P	-0.311	0.040	0.012
LS	-0.010	0.002	0.002

Notes: Estimates of the intercept (α alpha) and slope (β beta) are given, along with the standard error of the estimated slope. Gray shading is used to indicate estimates that are significantly different from zero.

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