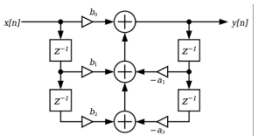
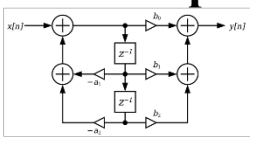
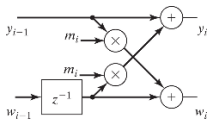
EE 419 - Project 6 - Project Report

Efficient IIR Filter Implementations

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Bench #: 9 **Section: 2**

Learning Objectives:

- Observe the effects of coefficient precision on the frequency response, pole/zero locations and stability of IIR filters.
- Develop and verify filter implementations that reduce the effects of coefficient rounding on filter performance.
- Observe "limit cycles" and find the relationship of their severity to filter coefficient values and quantization precision

1) Modeling the Effects of Coefficient Quantization on an IIR Double Notch Filter

Ideal (Full Precision) Filter Design and Analysis:

Filter Design Specifications:

Notch filter (two notches)
Notch Center Frequencies: f_1 = 1000 ± 5% Hz, f_2 = 2000 ± 5% Hz,
Sampling Rate S = 48 kHz
Pole radius = 0.95
Peak Passband Gain $|H(f)|_{max}$ = 1.0 (+/- 1%)

a) What are the pole and zero locations? (Using polar coordinates: Radius ∠ Angle)

Poles:	0.95∠15°	0.95∠–15°	0.95∠7.5°	0.95∠–7.5°
Zeros:	1∠15°	1∠–15°	1∠7.5°	1∠–7.5°

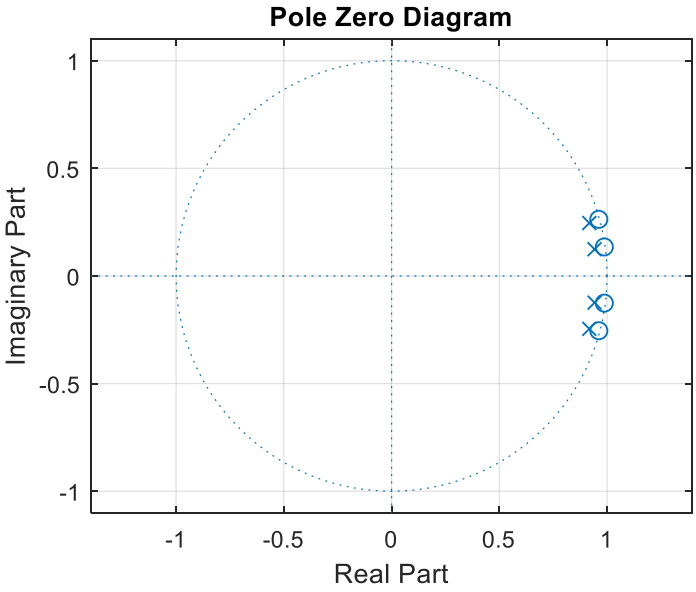
b) Report your difference equation filter coefficients (floating point values):

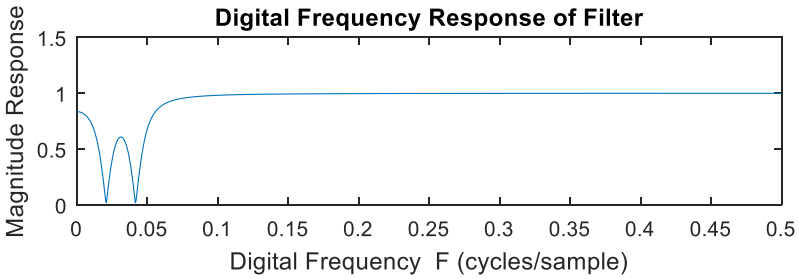
Overall Gain Constant K = 0.9037008
Difference Equation Coefficients:

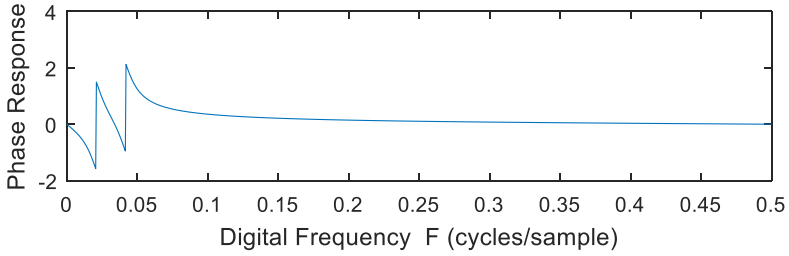
	k=0	k=1	k=2	k=3	k=4
A_k:	**1**	**-3.71900430655947**	**5.26216053096260**	**-3.3564013866699**	**0.8145062500**
B_k:	**0.9037008**	**-3.53775491267499**	**5.26916197402696**	**-3.53775491267499**	**0.9037008**

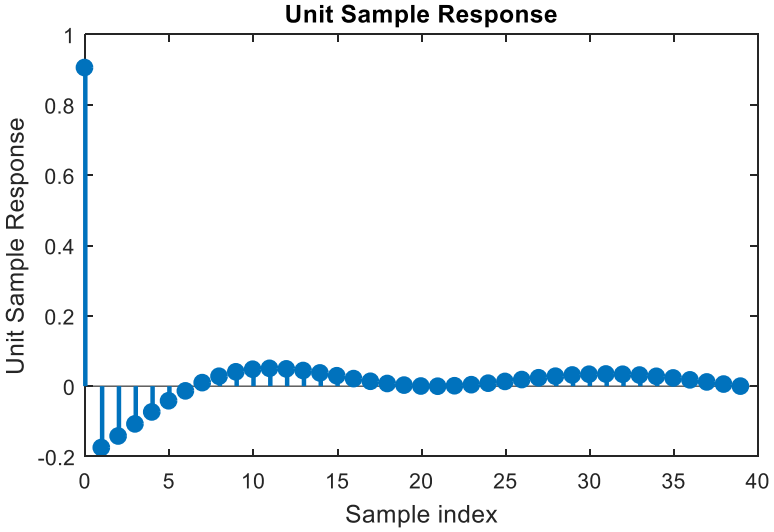
Filter Analysis Results:

Notch Center Frequencies: (Specification: f_1 = 1000 ± 50 Hz, f_2 = 2000 ± 100 Hz)
f_1 = **1000.1 Hz** % Error f_1 = **0.01%**
f_2 = **1999.9 Hz** % Error f_2 = **0.005%**


Pole Zero Diagram
Imaginary Part
Real Part
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1


Digital Frequency Response of Filter
Magnitude Response
Digital Frequency F (cycles/sample)
1.5
1
0.5
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5


Phase Response
Digital Frequency F (cycles/sample)
4
2
0
-2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5


Unit Sample Response
Unit Sample Response
Sample index
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
20
25
30
35
40

Scaled Integer Filter Implementation Modeling – 4th-order Direct Form II Transposed Filter:

4th Order Direct Form IIR Filter - Scale Factor = 512

Overall Gain Constant K = 0.904296875
Difference Equation Coefficients:

	k=0	k=1	k=2	k=3	k=4
A_k (integer):	512	-1904	2694	-1718	417
B_k (integer):	463	-1811	2698	-1811	463
A_k (effective):	1	-3.7187500	5.26171875	-3.35546875	0.814453125
B_k (effective):	0.904296875	-3.537109375	5.26953125	-3.537109375	0.904296875
% Error A_k	0%	-0.006838028%	0.0083954292158%	-0.0277868039%	0.0065223563355%
% Error B_k	0.06595933079%	-0.0182470999%	0.0070082486525%	-0.0182470999%	0.06595933079%

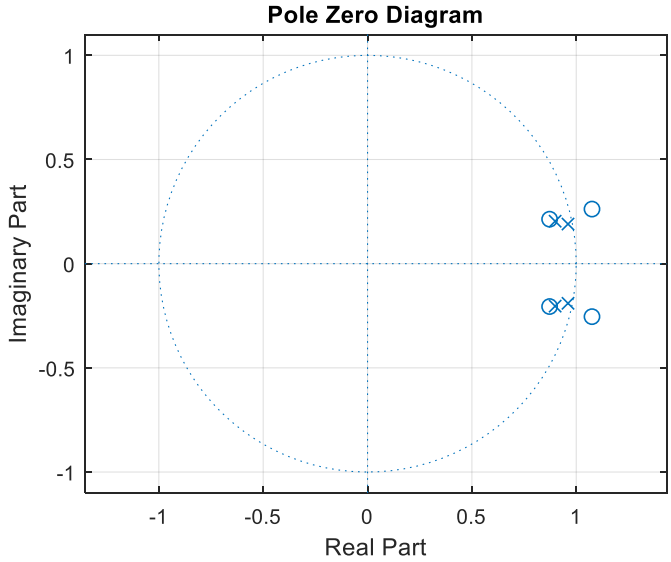
Bits Required for integer coefficients = 13

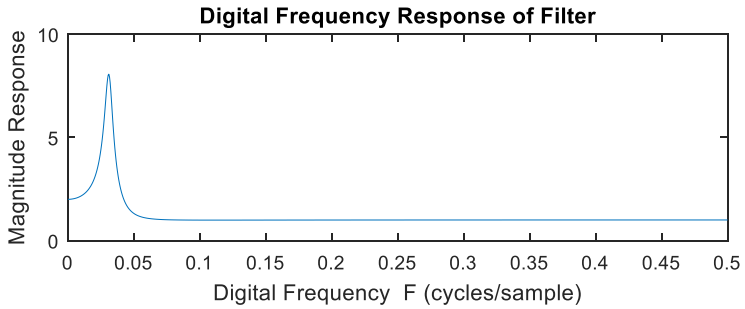
Notch Center Frequencies: (Specification: f_1 = 1000 ± 50 Hz, f_2 = 2000 ± 100 Hz)

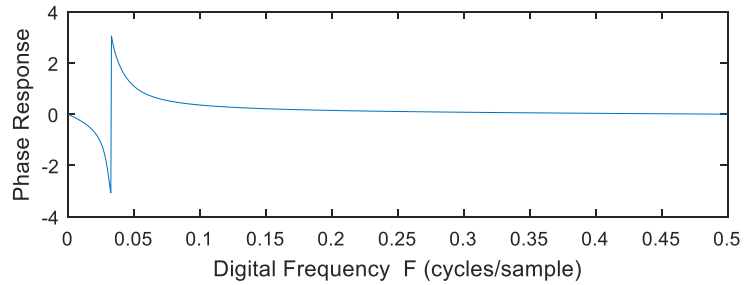
f_1 = **NA Hz** % Error f_1 = **100%**
f_2 = **NA Hz** % Error f_2 = **100%**

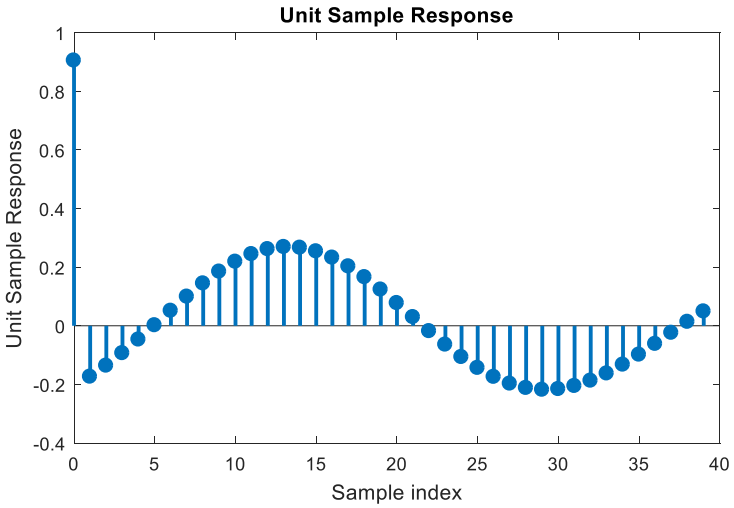
Pole and Zero Locations: (Using polar coordinates: Radius ∠ Angle)

Poles (effective):	0.97917588∠11.1988°	0.97917588∠−11.1988°	0.9216633966∠12.7765°	0.9216633966∠−12.7765°
Zeros (effective):	1.1098513141∠13.4497°	1.1098513141∠−13.4497°	0.9010215939∠13.4497°	0.9010215939∠−13.4497°









4th Order Direct Form IIR Filter – Sufficient Precision - Scale Factor =2^13 = 8192

Overall Gain Constant K = 0.9036865234375
Difference Equation Coefficients:

	k=0	k=1	k=2	k=3	k=4
A_k (integer):	8192	-30466	43108	-27496	6672
B_k (integer):	7403	-28981	43165	-28981	7403
A_k (effective):	1	-3.71899414063	5.26220703125	-3.3564453125	0.814453125
B_k (effective):	0.9036865234375	-3.53771972656	5.2691650390625	-3.53771972656	0.9036865234375
% Error A_k	0	0.000273351%	0.00088367291584%	0.001308718%	0.0065223563355%
% Error B_k	0.0015797886314%	0.000994588754%	0.0005816931725%	0.000994588754%	0.0015797886314%

Bits Required for integer coefficients = 18

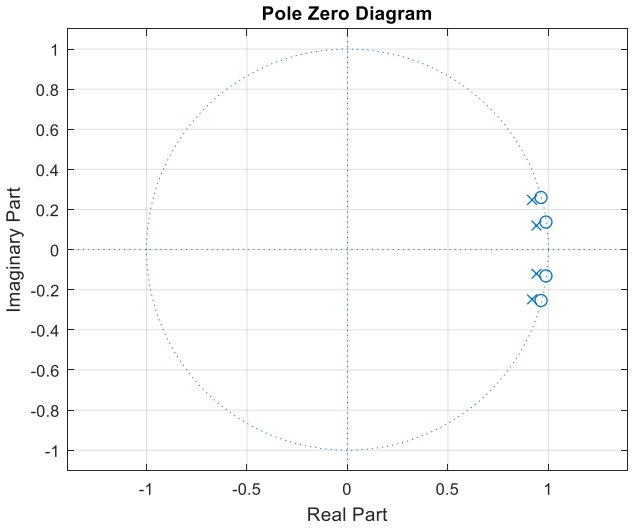
Notch Center Frequencies: (Specification: $f_1 = 1000 \pm 50$ Hz, $f_2 = 2000 \pm 100$ Hz)

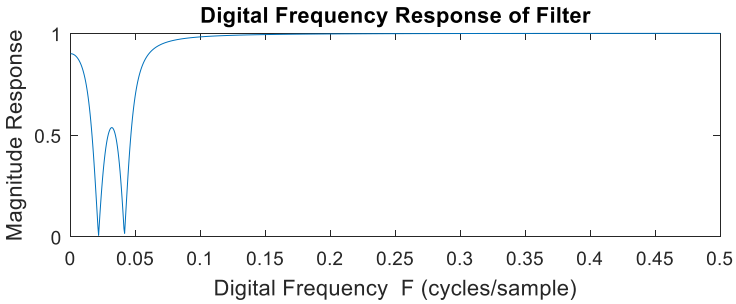
$f_1 =$ **1033Hz** % Error $f_1 =$ **3.3%**

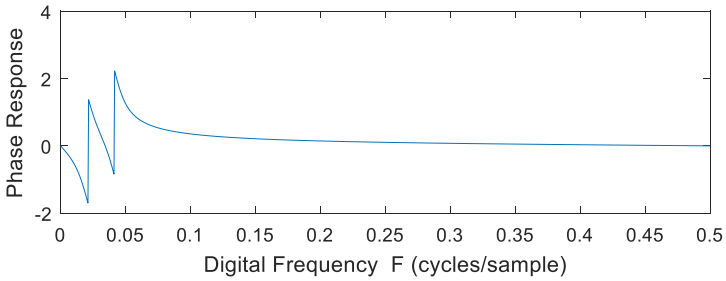
$f_2 =$ **1994Hz** % Error $f_2 =$ **0.3%**

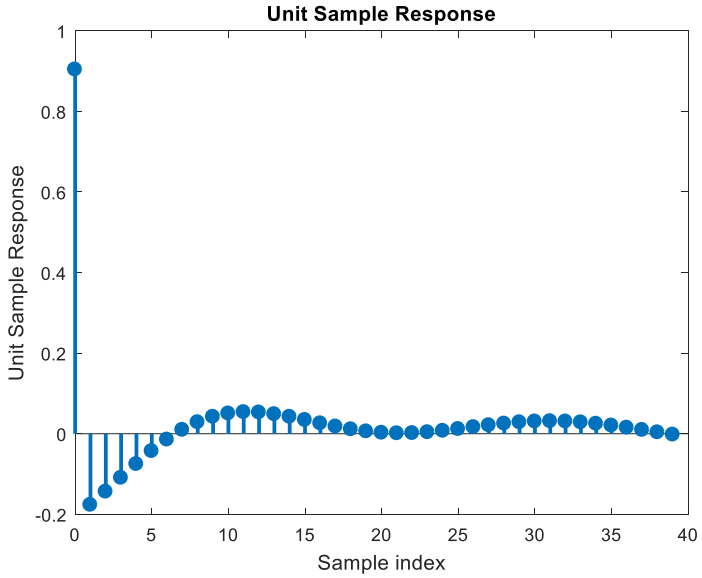
Pole and Zero Locations: (Using polar coordinates: Radius ∠ Angle)

Poles (effective):	0.9516884992∠15.095°	0.9516884992∠−15.095°	0.9482835698∠7.2766°	0.9482835698∠7.2766°
Zeros (effective):	0.999999999999992∠14.8862°	0.999999999999992∠−14.8862°	1.000000000000007∠7.7167°	1.000000000000007∠−7.7167°









Scaled Integer Filter Implementation – Cascaded 2nd-order Filters:

Cascaded 2nd Order IIR Filters – Scale Factor = 512

Difference Equation Coefficients and Pole/Zero Locations:

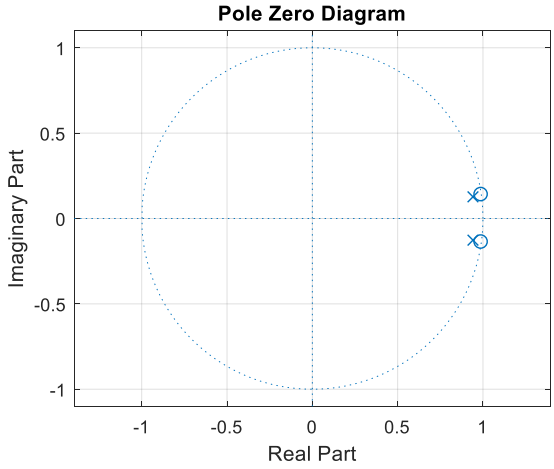
	First 2nd Order Section			Second 2nd Order Section		
	k=0	k=1	k=2	k=0	k=1	k=2
A_k (exact):	1	-1.883745237	0.90250	1	-1.83525907	0.90250
B_k (exact):	0.9037008	-1.791939029	0.9037008	0.9037008	-1.745815884	0.9037008
A_k (integer):	512	-964	462	512	-940	462
B_k (integer):	463	-917	463	463	-894	463
A_k (effective):	1	-1.8828125	0.90234375	1	-1.8359375	0.90234375
B_k (effective):	0.904296875	-1.791015625	0.904296875	0.904296875	-1.74609375	0.904296875
Poles (effective):	0.949918∠7.6758°	0.949918∠−7.6758°		0.9499178∠14.9021°	0.9499178∠−14.9021°	
Zeros (effective):	1∠7.9948°	1∠−7.9948°		1∠15.1066°	1∠−15.1066°	

Notch Center Frequencies: (Specification: f_1 = 1000 ± 50 Hz, f_2 = 2000 ± 100 Hz)

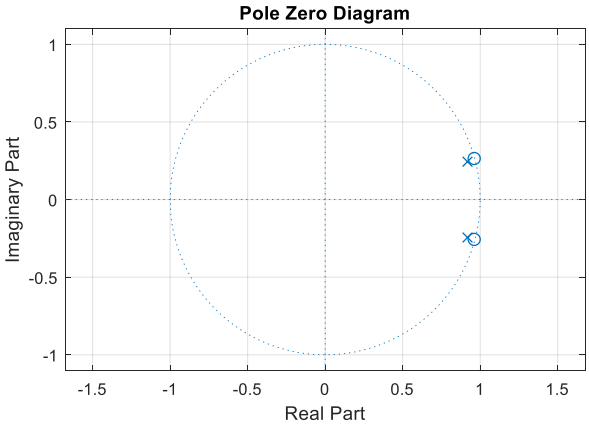
f_1 = **1057Hz** % Error f_1 = **5.7%** 2nd Order Section: (First or Second)

f_2 = **2018Hz** % Error f_2 = **0.9%** 2nd Order Section: (First or Second)

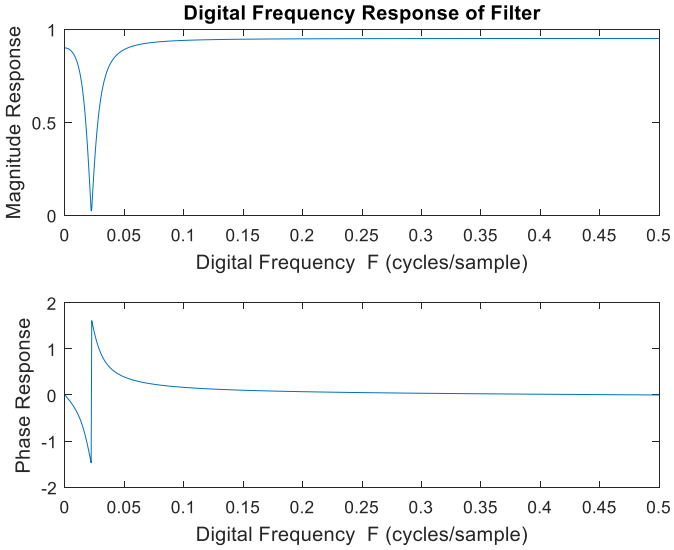
First 2nd-Order Section Pole/Zero Diagram



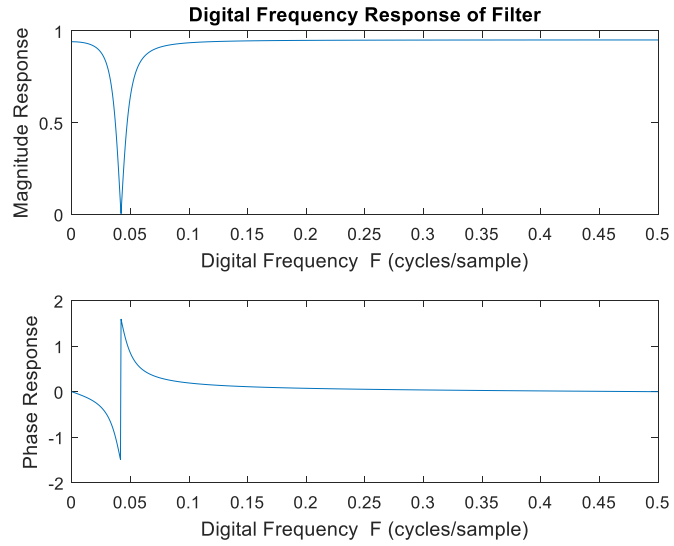
Second 2nd-Order Section Pole/Zero Diagram



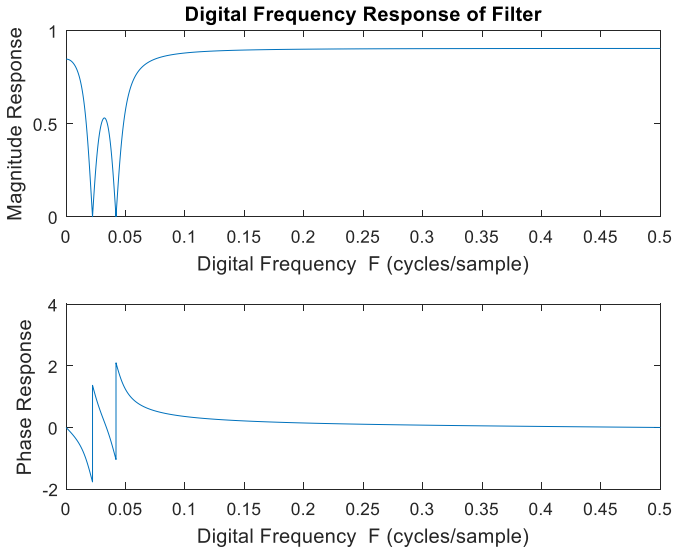
First 2nd-Order Section Frequency Response



Second 2nd-Order Section Frequency Response



Two Cascaded 2nd-Order Sections Overall Frequency Response



Compare the performance of the Cascaded 2nd-Order Section to that of the 4th-0rder Direct Form implementation with the same scale factor.

The cascaded 2nd order performs as well as the sufficient precision 4th order implementation without sacrificing the number of bits required to quantize. This allows for quicker computation and less memory. In contrast, the 4th order of same scale factor was ineffective and did not even produce notches near the respective frequencies (large quantization error).

Scaled Integer Filter Implementation – Lattice-Ladder Filter Structure

4th Order Lattice-Ladder Filter – Scale Factor = 512

Reflection and Ladder Coefficients:

	k=0	k=1	k=2	k=3	k=4
m_k (exact):	-0.98740016714	0.9897933530542	-0.9722786694	0.8145062500	
C_k (exact):	0.00013140236803	0.000511412468	-0.0040208042	-0.176887745634	0.903700800
m_k (integer):	-506	507	-498	417	
C_k (integer):	0	0	-2	-91	463
m_k (effective):	-0.9882812500	0.99023437500	-0.9726562500	0.81445312500	
C_k (effective):	0	0	-0.0039062500	-0.17773437500	0.90429687500

Notch Center Frequencies: (Specification: $f_1 = 1000 \pm 50$ Hz, $f_2 = 2000 \pm 100$ Hz)

Exact Filter Coefficients:

$f_1 =$ **1001.5Hz** % Error $f_1 =$ **0.15%**

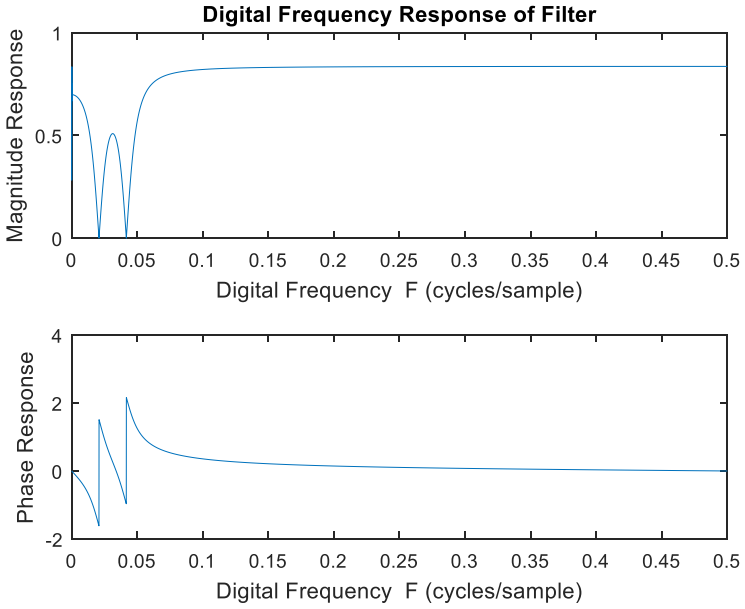
$f_2 =$ **2001.6Hz** % Error $f_2 =$ **0.08%**

Quantized (Reduced Precision) Filter Coefficients:

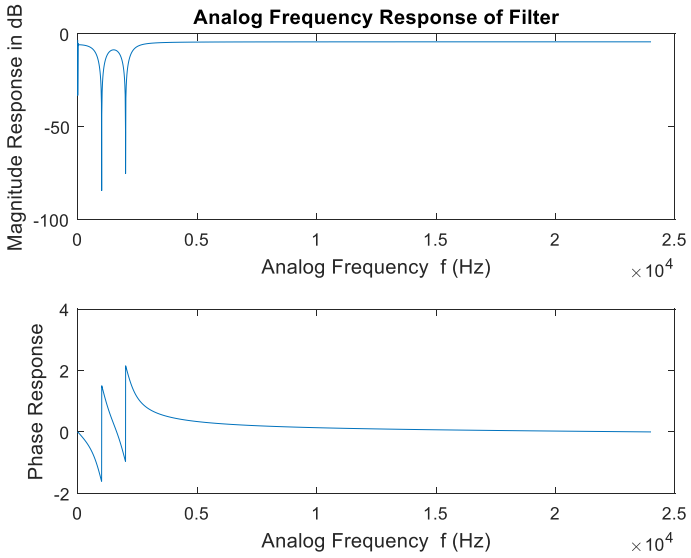
$f_1 =$ **1003.2Hz** % Error $f_1 =$ **0.32%**

$f_2 =$ **2003.2Hz** % Error $f_2 =$ **0.16%**

Exact Coefficient Lattice-Ladder Filter Frequency Response



Quantized Coefficient Lattice-Ladder Filter Frequency Response



Compare the frequency response results of these two filters (compared the full precision and quantized filters to each other), and compare these results to those from the 4th-0rder Direct Form, and cascaded 2nd-order Direct Form implementations that used the same scale factors. Did the lattice-ladder implementation achieve similar or better/worse results? [Explain the filter performance metrics by which you reached your conclusions.]

Of the quantized filter implementations, the lattice ladder produces the closest result to the full precision filter. The error in the notch frequencies are 0.32% and 0.16% respectively with a scale of 512. In comparison, the 2nd order cascaded had errors of 5.7% and 0.9% respectively. Lastly the 4th order direct form produced only a single notch and was nothing like the original filter. Therefore it is clear that the lattice ladder implementation is the most accurate for the same scale factor.

Matlab code used for computing the lattice-ladder filter coefficients, and for plotting the frequency response plots:

```
%get exact Ak and Bk coefficients for desired notch
[Ak, Bk, Hf, Fd] = show_filter_responses_pz([0.95*exp(j*pi*15/180), 0.95*exp(-j*pi*15/180),
0.95*exp(j*pi*7.5/180), 0.95*exp(-j*pi*7.5/180)],[1*exp(j*pi*15/180), 1*exp(-j*pi*15/180),
1*exp(j*pi*7.5/180), 1*exp(-j*pi*7.5/180)], 0.9037008, 48e3,100e3, 40, 1);

%get lattice ladder coefficients, then compute the exact lattice ladder
[M, C] = tf2latc(Bk, Ak);
[Y] = latcfilt(M, C, Hf);

plot_freq_responses(Fd, Y, 48e3, 9)

ScaleFactor = 512;

%compute scaled coefficients and round to integer
scaled_M0 = round(M(1)*ScaleFactor);
scaled_M1 = round(M(2)*ScaleFactor);
scaled_M2 = round(M(3)*ScaleFactor);
scaled_M3 = round(M(4)*ScaleFactor);

%get the quantized version of the coefficients
unscaled_M0 = scaled_M0/ScaleFactor;
unscaled_M1 = scaled_M1/ScaleFactor;
unscaled_M2 = scaled_M2/ScaleFactor;
unscaled_M3 = scaled_M3/ScaleFactor;

%compute scaled coefficients and round to integer
scaled_C0 = round(C(1)*ScaleFactor);
scaled_C1 = round(C(2)*ScaleFactor);
scaled_C2 = round(C(3)*ScaleFactor);
scaled_C3 = round(C(4)*ScaleFactor);
scaled_C4 = round(C(5)*ScaleFactor);
```

```
%get the quantized version of the coefficients
unscaled_C0 = scaled_C0/ScaleFactor;
unscaled_C1 = scaled_C1/ScaleFactor;
unscaled_C2 = scaled_C2/ScaleFactor;
unscaled_C3 = scaled_C3/ScaleFactor;
unscaled_C4 = scaled_C4/ScaleFactor;

%create arrays for coefficients
scaled_M = [scaled_M0, scaled_M1, scaled_M2, scaled_M3];
scaled_C = [scaled_C0, scaled_C1, scaled_C2, scaled_C3, scaled_C4];

unscaled_M = [unscaled_M0, unscaled_M1, unscaled_M2, unscaled_M3];
unscaled_C = [unscaled_C0, unscaled_C1, unscaled_C2, unscaled_C3, unscaled_C4];

%compute rounded version of lattice ladder
[Y_q] = latcfilt(unscaled_M, unscaled_C, Y);
plot_freq_responses(Fd, Y_q, 48e3, 11)
```

3) Limit Cycles from Quantization of Multiplier Outputs

Compose a Matlab function that will compute and plot the unit sample responses for two filters with the following difference equations:

$$y[n] = x[n] - A_1 y[n-1] \quad \text{(unquantized)}$$

$$y_q[n] = x[n] - Q\{ A_1 y[n-1] \} \quad \text{(quantized multiplier output)}$$

where $Q\{ w \}$ is a quantized, finite bit precision version of w,

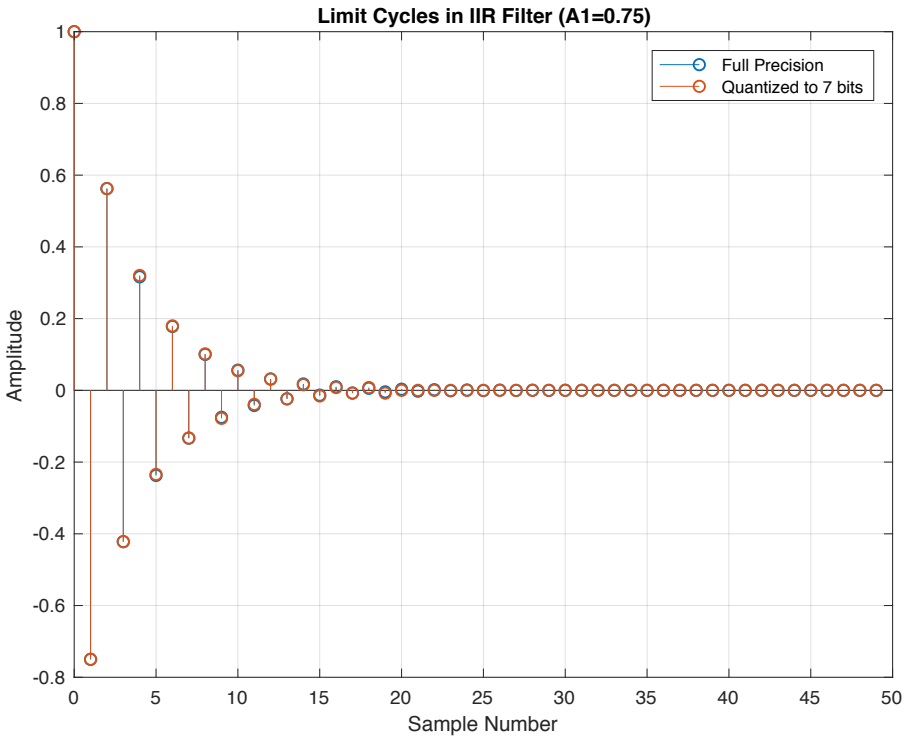
$Q\{ w \} =$ `round[w *(2^# bits)] /(2^# bits)`

The function should accept the following input parameters:

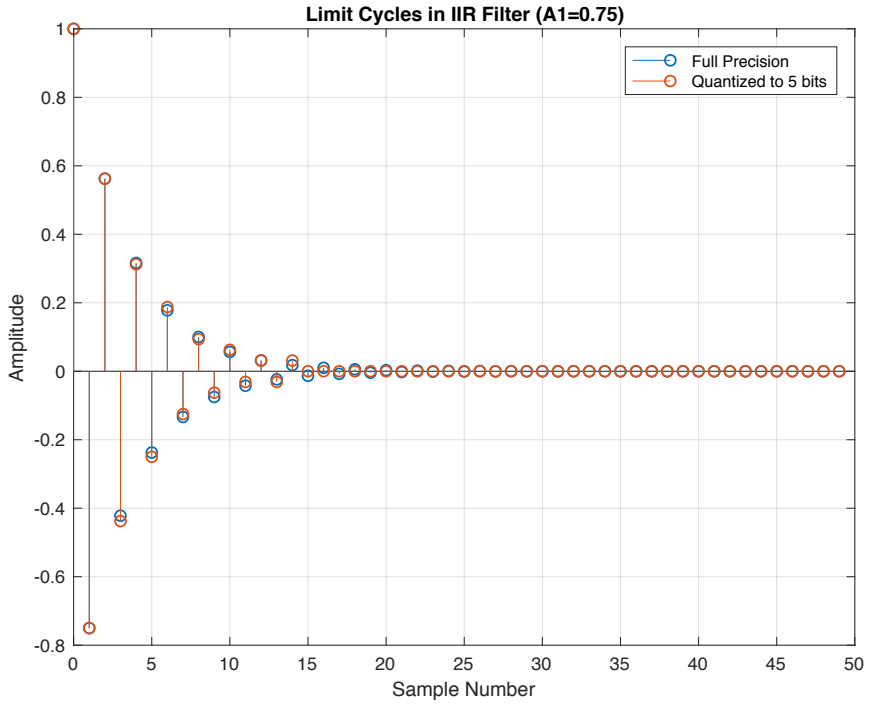
1) Feedback loop filter coefficient: A_1
2) The number of bits for the multiplier output quantization: # bits
3) The length of the unit sample response sequence to compute & plot.

Test Results:

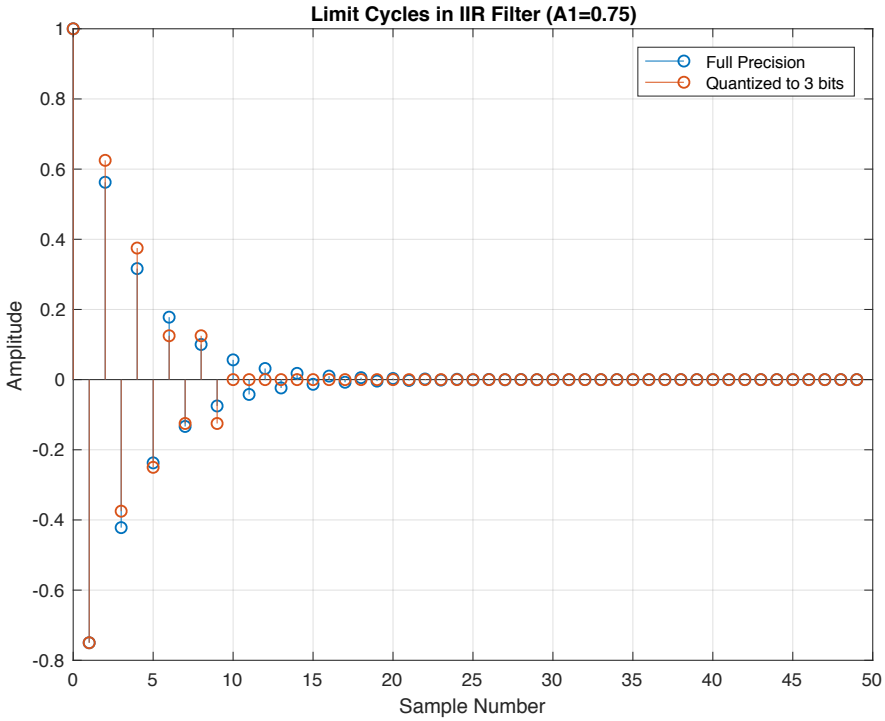
Test Case: $A_1 = 0.75$; # bits = 7



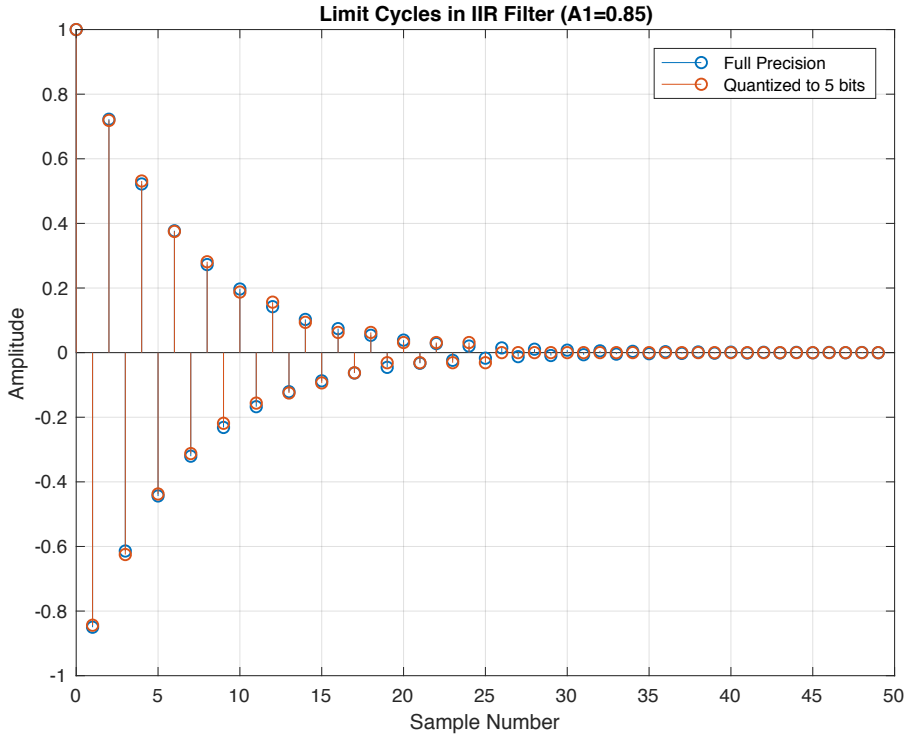
Test Case: $A_1 = 0.75$; # bits = 5



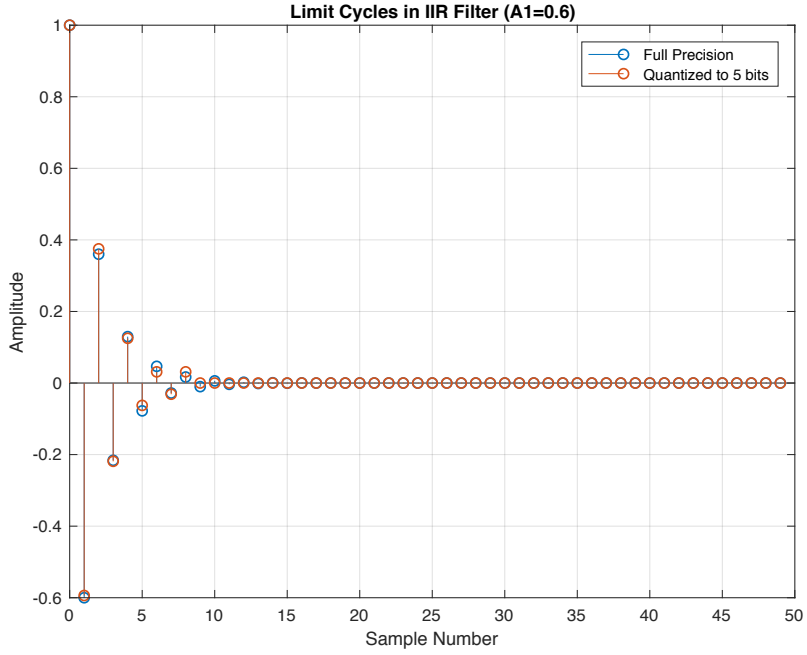
Test Case: $A_1 = 0.75$; # bits = 3



Test Case: $A_1 = 0.85$; # bits = 5



Test Case: $A_1 = 0.60$; # bits = 5



Describe your observations about the effects of changing the number of quantization bits and the A_1 filter coefficient on the limit cycle oscillations.

By increasing the number of quantization bits, you effectively decrease the quantization error but increase the computation time and cycles. Therefore, there is a tradeoff. Furthermore, increasing the A1 coefficient will increase the length of the unit sample response, but the error between points in the response is les than that of lower coefficients.

Matlab m-file function code:

```
function [y_n, y_q, n] = limit_cycles_unit_sample_response(A_coeff, bits, size)
%this function takes in a feedback loop coefficient, # of bits to quantize,
%and length of response to compute both the full precision output and
%and quantized output based of the difference equation:
%y[n] = x[n] - A1y[n-1]
%the function will plot the two on a single plot for comparison

%initialize sequences
x_n = zeros(1,size);
y_n = zeros(1,size);
y_q = zeros(1,size);

x_n(1) = 1;                                 %unit sample
n = 0:size-1;                              %sample integer
A_1 = A_coeff;                              %full precision coefficient
A_q = round(A_coeff*2^bits);                %quantized coefficient


%loop transfer function
for i=1:length(x_n)
    if i == 1                               %first point
        %assume zero initial conditions
        y_n(i) = x_n(i);
        y_q(i) = x_n(i);
    else
        y_n(i) = x_n(i) - A_1*y_n(i-1);     %calc full precision output
        %calc quantized output
        y_q(i) = x_n(i) - round(A_coeff*2^bits*y_n(i-1))/(2^bits);
    end
end

%plot both output sequences on same plot
figure

stem(n, y_n)
hold on
stem(n, y_q)
legend_2 = ['Quantized to ', num2str(bits), ' bits'];
legend('Full Precision', legend_2)
grid
title1 = strcat('Limit Cycles in IIR Filter (A1=',num2str(A_coeff), ')');
xlabel('Sample Number')
ylabel('Amplitude')
title(title1)
end
```

Conclusions:

Summarize one or two learnings about the effects of numeric quantization in digital filter implementations that this project helped you understand better. Also describe any particular challenges that you had to overcome, and at least one suggestion for improvement of this lab in the future.

Name: Aiku Shintani

Conclusions: Through completion of this project, an understanding of the effects of numeric quantization was established. The effectiveness of numeric quantization was accessed for several test cases including: adjustment of the scale factor, adjustment of the filter architecture to a cascaded one, and adjustment of the filter architecture to a lattice-ladder one. Increasing the scale factor results in an increase in the precision of the quantization and results in the best results. Splitting a 4^{th} order architecture into two individual cascaded 2^{nd} order filters, while using the same scale factor for both, proved the effectiveness of cascading filters. The adjustment of the architecture to a lattice-ladder one proved to be the most effective in obtaining the desired frequency response. The most challenging part of the lab was ensuring that the peak gain would be 1.00000,

exactly. If one uses an inadequate K value, the rest of the computations (especially quantization's), for the remainder of the lab, will be affected negatively.

Name: Chris Adams

Conclusions: By completing this project, a better understanding of how quantization effects digital filter was achieved. While quantization can cause substantial error, there are methods such as the 2^{nd} order cascaded direct form and the lattice ladder implementations that minimize error. Quantization allows for lower complexity hardware and faster computation which is why it is important to understand the tradeoffs associated with it. One of the harder parts of this experiment was to visually tell the difference between some of the quantization implementations. Some of the methods produce closely accurate results such that it is hard to see graphically. However, by calculating the percent error, it is far easier to compare. I thought this lab was well structured, but if anything could change, I would want to learn about the benefits and drawbacks of different quantization methods on the hardware side.