Notations. Let G be a simple Lie group of rank r. Let $i = 1 \dots r$ label the nodes of Dynkin graph. Let $\lambda_i^\vee : \mathbb{C}^\times \to T_G$ be fundamental coweights, and $\lambda_i : T_G \to \mathbb{C}^\times$ be fundamental weights. Let $\rho_i : G \to \mathrm{End}(V_i)$ be irreducible representation of G, i.e. V_i is irreducible G-module with highest weight λ_i. Let χ_i be the character of ρ_i, that is $\rho_i(g) = \mathrm{tr}_{V_i} \rho_i(g)$.

Conjecture 1. The moduli space $M_{G,\mathbf{n},g_\infty}$ of smooth G-monopoles on $\mathbb{C}_x \times S^1$ with fixed charge $\mathbf{n} = (n_1, \dots, n_r) \in \mathbb{Z}^r_{>=0}$ at infinity $x = \infty$ in the asymptotic sector $g_\infty \in T_G/W$ is the subspace of analytic maps $g : \mathbb{C}_x \to G$ restricted by the conditions

1) The matrix elements of $\rho_i(g(x))$ are polynomials of x of degree n_i

2) The coefficient at x^n of $\chi_i(g(x))$ is fixed to be $\chi_i(g_\infty)$.

Conjecture 2. The $\dim_{\mathbb{C}} M_{G,\mathbf{n},g_\infty} = 2\sum_{i=1}^r n_i$

Conjecture 3. The coefficients $u_{i,k}$ for $k = 1, \dots n_i$ in the character polynomials

$$\chi_i(g(x)) = \chi_i(g_\infty)(x^{n_i} + u_{i,1}x^{n_i-1} + \dots u_{i,n_i}) \tag{0.1}$$

are algebraically independent Poisson commuting functions on $M_{G,\mathbf{n},\mathfrak{g}_\infty}$ which provide a non-degenerate map $u : M_{G,\mathbf{n},\mathfrak{g}_\infty} \to U$ where $\dim_{\mathbb{C}} U = \sum_{i=1}^r n_i$.

Hence $M_{G,\mathbf{n},\mathfrak{g}_\infty}$ is the phase space of algebraic completely integrable system with the base U.

Conjecture 4. The $M_{G,\mathbf{n},\mathfrak{g}_\infty}$ is a holomorphic symplectic leaf of holomorphic Poisson-Lie group $G[\mathbb{C}]$ with quasi-triangular rational r-matrix type Poisson bracket.

Conjecture 5. The holomorphic symplectic structure on M is isomorphic to the holomorphic symplectic structure defined by the hyperKahler definition of M as the moduli space of solutions of the Bogomolny equationson $\mathbb{R}^2 \times S^1$ of the form

$$d_A\Phi = \star F_A \tag{0.2}$$

with a certain fixed asymptotics of the solutions at ∞ (fix the topological sector n) and the asymptotics of the field Φ at $x \to \infty$ such that $\exp(\Phi) = g_\infty$.

Conjecture 6. (Example). For $G = SL(2)$ the complex variables $(p_i, \phi_i)|_{i=1,\dots,n}$ with $p_i \in \mathbb{C}$ and $\phi_i \in \mathbb{C}/(2\pi\sqrt{-1}\mathbb{Z})$ are Darboux coordinates on $M_{G,n,1}$ with the parametrization

$$g(x) = \prod_{i=1}^n \begin{pmatrix} p_i - x & e^{\phi_i} \\ e^{-\phi_i} & 0 \end{pmatrix} \tag{0.3}$$