Chris Elliott – Research Statement

In my research, I use ideas from derived geometry and homotopical algebra to investigate the mathematical structures inherent to quantum field theory and string theory, and their applications to algebraic geometry and geometric representation theory. In recent decades, mathematicians have made great strides in many fields of geometry and representation theory by drawing innovative ideas from supersymmetric quantum field theory (QFT). There is not yet a systematic mathematical model for supersymmetric QFTs rich enough to generate the structures arising in many applications. As a result, we cannot directly use many methods from physics to attack mathematical questions, and there are many supersymmetric theories whose mathematical applications are still unexplored. The overall aim of my work is to provide such systematic mathematical models.

There are multiple broad research programs that draw on ideas from supersymmetric QFT, which my research has the potential to inform. These include Kontsevich's homological mirror symmetry program [Kon95], the theory of symplectic duality introduced by Braden, Licata, Proudfoot and Webster [Bra+16; Bul+16] and the related development of Coulomb branches by Braverman, Finkelberg and Nakajima [BFN18], the theory of Donaldson and Seiberg–Witten invariants [SW94; Wit94], Kapustin and Witten's approach to the geometric Langlands correspondence [KW07], and Donaldson–Thomas theory of threefolds [DT98]: these programs are intimately connected to supersymmetric QFTs in 2, 3, 4 and 6 dimensions. A common thread running through many of these applications is the appearance of "higher structures": objects like $\mathbb{E}_n$, L_∞ and A_∞-algebras, higher categories, or derived stacks.

An important technique in my work, that allows me to restrict attention to mathematically salient aspects of a QFT, is the idea of "twisting". A supersymmetric field theory on a manifold M is a highly structured object. With mathematical applications in mind, we can learn a lot about a supersymmetric theory by studying its *twists*, which are obtained essentially by taking the cohomology of an operator Q coming from an odd symmetry such that $Q^2 = 0$ [1]. While the original field theory may have depended on additional structure on M, such as a metric, its twists are sensitive to less structure: "holomorphic" twists depend only on a complex structure on M, and "topological" twists only depend on M up to diffeomorphism (intermediate cases also occur).

My work models these twisted field theories using several related types of structure, each describing certain aspects of the full quantum field theory. We can model a twisted theory classically using symplectic geometry, or more precisely, a derived and stacky version of symplectic geometry where the symplectic form carries a non-zero cohomological degree [Pan+13]. At the quantum level, I can describe twisted theories locally using homotopical algebra. Finally, there are more refined structures associated to the twisted theories at the quantum level that can be described using higher categories. I will explain this work, and my proposals for future work, throughout this research statement. To summarize briefly, some of my recent results produced the following:

1. A complete classification of all twists of one of the most widely studied families of quantum field theory: supersymmetric Yang–Mills theories in any dimension [ESW20].
2. A concrete description of the algebras of local observables in topological twists in terms of higher algebra: local observables in such theories can canonically and concretely be modelled by $\mathbb{E}_n$-algebras – algebras over the operad of little n-disks [ES19].
3. A detailed understanding of the twists of gauge theory in 4 dimensions with maximal supersymmetry. In particular we have a purely algebro-geometric derivation of the dg-categories occuring in the geometric Langlands program and its quantum generalization coming from such twists [EY18; EY19; EY20].
4. A derivation of a generalization of the Hitchin system where the Higgs field is group rather than Lie algebra valued from a twist of a 5d gauge theory. We showed that this multiplicative Hitchin system on $\mathbb{CP}^1$ can be identified symplectomorphically with a moduli space of monopoles on $\mathbb{CP}^1 \times S^1$ with singularities, and by quantizing these moduli spaces we get a geometric description of representations of the Yangian quantum group [EP19].
5. A construction of a novel family of examples of quantum field theories, whose quantization we compute explicitly, that conjecturally arise as twists of a supersymmetric gravity theory. These theories have the novel property that their local observables are topological in a weak ("de Rham") sense, but not locally constant [EW20].

[1] Alternatively, twisting can be thought of in terms of symmetry breaking: I described this idea from the point of view of derived geometry in work with Gwilliam [EG20].

1 Twisted Gauge Theory

In joint work with Safronov and Williams [ESW20], I gave a complete classification and description of supersymmetric twists of Yang–Mills gauge theories using formal derived geometry, at the classical level. For each super Yang–Mills theory, and each square-zero odd symmetry Q, we describe a concrete model for the twist by Q: these theories are typically realized as generalizations of Chern–Simons or BF theory.

Extending these to the quantum level requires the analysis of *anomalies* for these classical theories. In dimensions ≤ 8 these twisted theories are all 1-loop exact, meaning that computing the quantization requires only a single calculation involving Feynman diagrams with 1 loop. The local observables in a quantum field theory can be described using the language of factorization algebras. In the case of a topological twist, however, in joint work with Safronov [ES19] I showed that the local observables can be defined more concretely as an $\mathbb{E}_n$-algebra: an algebra over the operad of little disks. These $\mathbb{E}_n$ algebras are mathematically interesting objects: for small n an $\mathbb{E}_n$ structure can be thought of as Koszul dual to something like a quantum group, possibly with one or more spectral parameters. More specifically, we proved the following.

Theorem 1.1 ([ES19])**.** The algebra of local operators in a topologically twisted quantum field theory in dimension n can be canonically equipped the structure of an $\mathbb{E}_n$-disk algebra whenever the natural map extending observables defined on a ball of radius r to a ball of radius $R > r$ is a quasi-isomorphism. This condition is satisfied for all twists of supersymmetric Yang–Mills theories with matter in all dimensions.

The classification of twists includes familiar entries, such as Donaldson–Witten theory in dimension 4, as well as less familiar entries. For instance, there is a topological twist in dimension 7 that depends on a G_2-structure, and a topological twist in dimension 8 defined on a Spin(7)-structure. Using the technique of factorization homology, these twisted theories will associate a filtered cochain complex to each G_2 or Spin(7)-manifold, quantizing the moduli spaces of G_2-monopoles and Spin(7)-instantons respectively. One of my short-term research aims is the following.

Objective 1.2. Where the anomaly vanishes, compute the filtered $\mathbb{E}_n$-algebra of quantum observables in all topologically twisted supersymmetric field theories in dimensions ≤ 8. Use factorization homology to describe the algebra of global observables on the most general possible class of compact n-manifold in each case.

Note that in many interesting examples the $\mathbb{E}_n$-algebra of observables will be contractible, it will only have interesting cohomology when considered as a filtered algebra, or alternatively as an explicit deformation of the algebra of observables in another twisted theory.

2 4d Gauge Theory and the Geometric Langlands Correspondence

I can extend this local understanding of twisted supersymmetric field theories considerably, focusing on a specific family of examples: twists of 4-dimensional field theories with maximal supersymmetry (referred to as $\mathcal{N} = 4$ supersymetry). This theory has a rich family of possible twists, including a $\mathbb{CP}^1$-family of topological twists whose relationship to geometric representation theory was first studied by Kapustin and Witten [KW07]. I studied these twists, and their connection to the quantum geometric Langlands correspondence, in a series of papers joint with Philsang Yoo [EY18; EY19; EY20]. We showed the following.

Theorem 2.1. The category $D_\kappa(\mathrm{Bun}_G(C))$ of κ-twisted D-modules arises by quantizing the moduli stack associated to a Kapustin–Witten twist of 4d $\mathcal{N} = 4$ gauge theory, where κ coincides with the "fundamental parameter" $\Psi \in \mathbb{CP}^1$ of Kapustin and Witten. If $\kappa \to \infty$, by considering boundary conditions compatible with a canonical choice of vacuum, the same quantization procedure produces the category $\mathrm{IndCoh}_{\mathcal{N}}(\mathrm{Flat}_G(C))$ of ind-coherent sheaves on the moduli stack of flat G-bundles with nilpotent singular support, as introduced by Arinkin and Gaitsgory.

Using this previous work, and Objective 1, I intend to develop a much more sophisticated understanding of these theories that can be applied to the geometric Langlands program. Using techniques recently developed by Gwilliam, Rabinovich and Williams [GRW20], given a stratified manifold with boundary, and the data of a classical supersymmetric field theory with boundary conditions and defects, I can compute the local observables in a twist of the whole configuration. The result will be a *stratified* factorization algebra.

Objective 2.2. In each of the Kapustin–Witten twists, compute:

1. The 4d stratified factorization algebras associated to Gaiotto corner configurations on $\Sigma \times \mathbb{R}^2_{\geq 0}$.
2. The category of line operators in the 3d boundary theories, and natural functors from these categories to the category of modules for the vertex algebra at the corner.

Let us elaborate on the meaning of these constructions, and their significance for geometric representation theory. Recent work of Gaiotto and Frenkel–Gaiotto [FG20] has analyzed the Kapustin–Witten twists of boundary conditions for 4d supersymmetric gauge theory, and applied this to the quantum geometric Langlands program at irrational level. I will first compute the stratified factorization algebra on a 4-manifold with boundary describing the observables in one of Kapustin–Witten's twists in the interior, and the observables in a twist of a supersymmetric boundary theory on the boundary.

Next, consider a 4-manifold with corners of the form $\Sigma \times \mathbb{R}^2_{\geq 0}$, where Σ is a Riemann surface. Work of Gaiotto–Rapčák [GR19] and Frenkel–Gaiotto describes field theories on such a corner configuration, where we consider a 4d twist of $\mathcal{N} = 4$ supersymmetric gauge theory on the interior, a twisted 3d boundary theory on each face of the boundary, and a 2d interface theory at the corner. This corner theory is not topological, but only holomorphic. When $\Sigma = \mathbb{R}^2$, the stratified factorization algebra will comprise an $\mathbb{E}_4$ algebra in the interior, a pair of $\mathbb{E}_3$-modules associated to the boundary walls, and a vertex algebra with the structure of a bimodule for the $\mathbb{E}_3$-algebras at the corner as indicated in Figure 1.

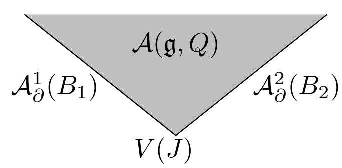

Figure 1: Illustration of the stratified factorization algebra data associated to a corner configuration on $\mathbb{R}^2 \times \mathbb{R}^2_{\geq 0}$. Here $\mathcal{A}$ is an $\mathbb{E}_4$-algebra constructed as the quantum algebra of observables in the interior, depending on a reductive Lie algebra $\mathfrak{g}$ and a topological twist Q; $\mathcal{A}^1_\partial$ and $\mathcal{A}^2_\partial$ are $\mathbb{E}_3$ algebras with an action of $\mathcal{A}$ associated to a pair of 3d boundary theories (B_1 and B_2); and V is a vertex algebra with the structure of a $(\mathcal{A}^1_\partial, \mathcal{A}^2_\partial)$-bimodule associated to a holomorphic junction J between the two boundary conditions.

Finally, as well as local operators, I can compute *line operators* in each of the 3d boundary theories. The collection of line operators in a 3d topological field theory has the structure of a braided monoidal dg-category.

Frenkel and Gaiotto proposed a description of integral kernels for the quantum geometric Langlands correspondence at irrational level using these ideas. Using my work with Yoo [EY19], I will also be able to address the rational level case. As my prior work shows, in the rational level case we obtain the correct categories of sheaves for the geometric Langlands program by considering the action of the algebra of local observables on the category of boundary conditions. The moduli space of *vacua* is the spectrum of the algebra of local observables. In the geometric Langlands program there is a correction term to the most straightforward statement, due to Arinkin and Gaitsgory [AG15]. I showed that this correction term arises in gauge theory by considering the localization at a point in the moduli space of vacua.

Another of my research objectives extends this idea further by using an interpretation that is also applicable to the topological field theories studied in the homological mirror symmetry community. Work of Johnson-Freyd and Scheimbauer [JFS17] gave a functorial model for relative topological field theories. It is well-known that the B-model with target $\mathcal{X}$, viewed as a functorial field theory assigning a category $\mathrm{QCoh}(\mathcal{X})$ (or the category of ind-coherent sheaves $\mathrm{IndCoh}(\mathcal{X})$), only defines a fully extended functorial field theory if $\mathcal{X}$ is smooth and proper – this excludes the 2d B-model coming from compactification of the 4d Kapustin–Witten B-twist along a Riemann surface C, where $\mathcal{X}$ is the moduli stack $\mathrm{Flat}_G(C)$ of flat G-bundles on C. On the other hand, we *can* associate a fully extended functorial field theory to this 2d B-model, which we call the theory of observables. This is the functorial field theory valued in a Morita category of $\mathbb{E}_2$ algebras that assigns the algebra of local observables to the point. I will use this to construct the Kapustin–Witten B-twist as a relative field theory. This can then be extended to more general Kapustin–Witten twists using the category of renormalized D-modules developed by Gaitsgory.

Objective 2.3. For a general class of derived stacks $\mathcal{X}$, show that the category $\mathrm{IndCoh}(\mathcal{X})$ defines an extended TQFT *relative* to its theory of observables. That is, the following natural transformation is relatively dualizable:

$$\begin{tikzcd} \mathrm{Bord}_2 \arrow[r, "\mathrm{Obs}"] \arrow[rr, bend right, "\mathrm{triv}"'] & \mathbb{E}_2\text{-Alg} \arrow[r, "\text{-mod}"] \arrow[d, phantom, "\Downarrow \mathrm{IndCoh}(\mathcal{X})"] & \mathrm{dgCat}. \\ & {} & \end{tikzcd}$$

Show that in the case $\mathcal{X} = \mathrm{Flat}_G(C)$ arising by dimensional reduction from the 4d Kapustin–Witten B-twist, the choice of a vacuum defines a second relative field theory, and therefore by composing the two we can define a fully extended functorial TQFT. Prove a corresponding statement for more general 4d twists, starting with a category of renormalized D-modules.

3 5 and 6d Twisted Gauge Theory and Integrable Systems

In joint work with Vasily Pestun [EP19], I studied moduli spaces arising either from twists of 5 dimensional super Yang–Mills theories, or as the Coulomb branches of 4-dimensional quiver gauge theories of ADE type. Our work concerns the following moduli space (versions of which have been studied previously by Hurtubise-Markman [HM02], Bouthier [Bou14] and Frenkel-Ngô [FN11]).

Definition 3.1. The moduli space of *multiplicative G-Higgs bundles* on a curve C consists of pairs (P, ϕ) where P is a principal G-bundle on C and ϕ is a meromorphic automorphism of P. We fix the locations of the poles of ϕ at a divisor $D = \{z_1, \ldots, z_k\}$. We can also fix the local behaviour near the poles – controlled by a dominant coweight $\omega_{z_i}^\vee$ of G at each puncture. Denote these moduli spaces by $\mathrm{mHiggs}_G(C, D)$ (without fixed local behaviour) and $\mathrm{mHiggs}_G(C, D, \omega^\vee)$ (with fixed local behaviour where $\omega^\vee$ denotes a k-tuple of dominant coweights) respectively.

We focus our attention on the following rational/trigonometric/elliptic trichotomy:

- (Rational) $C = \mathbb{CP}^1$ and we fix a framing at the point ∞.
- (Trigonometric) $C = \mathbb{CP}^1$ and we fix a B_+ reduction at ∞ and a B_--reduction at 0 so that the respective induced T-reductions coincide.
- (Elliptic) $C = E$ is an elliptic curve (with no additional decorations).

In these cases the moduli space – like the ordinary Hitchin system – has the structure of an algebraic integrable system which we can naturally describe using the theory of shifted Poisson and coisotropic structures [Cal+17; MS18]. In particular it has an algebraic symplectic structure. If one doesn't fix dominant coweights at the punctures the full infinite-type moduli space has a Poisson structure and the moduli spaces with fixed coweights are symplectic leaves.

A theorem of Charbonneau and Hurtubise [CH10] (for GL_n) and Smith [Smi16] (for general G) tell us that the moduli space of multiplicative G-Higgs bundles (or rather its polystable locus) is analytically isomorphic to the moduli space of G-monopoles on $C \times S^1$. In the rational case this moduli space of periodic monopoles can be realized as a hyperkähler quotient. In particular it is holomorphic symplectic. In this case Pestun and I prove the following.

Theorem 3.2 ([EP19])**.** In the rational case, the isomorphism identifying the moduli space of periodic monopoles and the moduli space of multiplicative Higgs bundles is compatible with the holomorphic symplectic structures on both sides. The holomorphic symplectic structure on the multiplicative Higgs moduli space can be identified with the pullback of the Poisson Lie structure under the map $\mathrm{mHiggs}_G^{\mathrm{fr}}(\mathbb{CP}^1, D) \to G_1[[z^{-1}]]$ given by restriction to a formal neighbourhood of the framed point ∞. For $G = \mathrm{GL}_n$ the symplectic leaves coincide with the symplectic leaves classified by Shapiro [Sha16].

Our equivalence promotes the holomorphic symplectic structure on $\mathrm{mHiggs}_G^{\mathrm{fr}}(\mathbb{CP}^1, D, \omega^\vee)$ to a hyperkähler structure. We can identify the holomorphic symplectic space obtained by rotating to a point q in the twistor sphere with the moduli space of *q-connections*: principal G-bundles P equipped with a meromorphic isomorphism $P \to q^*P$ from P to its translate.

This work is motivated in part by the work of Nekrasov and Pestun [NP12], which implies that moduli spaces of multiplicative Higgs bundles arise as the Seiberg-Witten integrable system associated to ADE quiver gauge theories. In particular they should admit natural hyperkähler structures. Nekrasov and Pestun also conjectured that the quantization of these moduli spaces should be closely related to the Yangian – this conjecture is verified by our Poisson map to the Poisson Lie group that quantizes to give the Yangian algebra. The moduli spaces $\mathrm{mHiggs}_G^{\mathrm{fr}}(\mathbb{CP}^1, D, \omega^\vee)$ quantize to give representations of the Yangian first studied by Gerasimov, Kharchev, Lebedev and Oblezin [Ger+05].

The ordinary Hitchin system and the multiplicative Hitchin system themselves comprise two parts of another rational/trigonometric/elliptic trichotomy; one can additionally consider *elliptic* Hitchin systems, given by maps into the moduli stack $\mathrm{Bun}_G^{\mathrm{ss}}(E)$ of semistable G-bundles on an elliptic curve. These moduli stacks arise either from twists of 6-dimensional super Yang–Mills theories, or as the Coulomb branches of 4d quiver gauge theories of affine ADE type. Let us consider the case where $G = \mathrm{GL}_n$. In complex structure J, via the Nahm transform we can identify the component of the elliptic Hitchin system of degree k with a moduli space of flat bundles on a punctured torus $\mathrm{Flat}_{\mathrm{GL}_k}(T^2, \{p_1, \ldots, p_n\})$. These will describe symplectic leaves in the double loop group $\widehat{\widehat{\mathrm{GL}}}_n$. The following research goal is interesting even if $n = 2$.

Objective 3.3. Quantize this description in order to obtain a new geometric description of the quantum toroidal algebra $U_{q,t}(\widehat{\widehat{\mathfrak{gl}}}_n)$, generalizing Schiffmann-Vasserot's identification for $n = 1$ of the quantum toroidal algebra and the stable limit of the spherical DAHA [SV13].

4 Twisted Supergravity and Twisted Holography

While widely studied in physics, the mathematical study of supersymmetric gravity theories is much less developed than that of gauge theory; until now, almost all supergravity theories have been viewed as mathematically intractible. My techniques, however, allow me to take a similar approach to the one discussed for gauge theories above, and compute the supersymmetric twists of these sophisticated supergravity theories. While the interpretation of this procedure in the context of gravity is slightly different, many of the same techniques can be applied.

There is a maximal supergravity theory defined on flat 11-manifolds. Supersymmetric theories in 11-dimensions admit several inequivalent twists, which I can classify using a result of Igusa [Igu70], including one defined on the product of a Calabi-Yau 5-fold and a line, and one defined on the product of a G_2-manifold and a complex surface. The former twist has been conjecturally described by Costello and Li [CL16] in terms of Kodaira–Spencer, or BCOV, theory.

In work with Williams [EW20]I studied 4d and 5d theories, which we conjecture are given by suitable twists of minimal supergravity in these dimensions. These theories can be obtained from Costello and Li's conjectural description by compactification. These theories are very interesting in their own right. For example, we proved the following result.

Theorem 4.1 ([EW20])**.** There is an interacting QFT that can be defined on $\mathbb{C}^{2n} \times \mathbb{R}^m$ for any n and m, involving the natural holomorphic symplectic form on $\mathbb{C}^{2n}$. This theory is 1-loop exact, and it has a 1-loop anomaly given by a class in the Gelfand–Fuchs cohomology of the Lie algebra of Hamiltonian vector fields on $\mathbb{C}^{2n}$. This class vanishes if $m \leq 6$. The resulting factorization algebra of quantum local observables is not locally constant, but the action of the group $\mathbb{R}^{4n+m}$ of all translations is homotopically trivialized.

In my research going forward, I intend to investigate the problem of the classification and computation of all possible twists of supergravity theories, using the same techniques as used in my work with Safronov and Williams [ESW20]. While supergravity theories have much more complicated action functionals than Yang–Mills theories, performing the twist of the underlying free theory is a matter of homological algebra: one can write down the BV complex of the underlying free theory starting from the data of the quadratic term in the Taylor expansion of the action functional around a fixed classical solution. While the higher order terms in this Taylor expansion are typically very complicated for supergravity theories, after twisting, I expect to be able write down a "smaller" quasi-isomorphic complex with much fewer fields. The higher order terms can be computed using homotopy transfer along this quasi-isomorphism, using explicit formulas as given, for example, by Loday and Vallette [LV12, Theorem 10.3.9] to show that many terms in the complicated supergravity interaction no longer contribute after twisting.

Objective 4.2. Prove the Costello–Li conjecture, as well as our 4- and 5-dimensional conjectures, by computing all twists of 11-dimensional supergravity.

With both descriptions for twisted gauge theory and twisted supergravity in hand, I will be able to investigate twisted examples of the AdS/CFT, or holographic, duality between such pairs of theories. A proposal for how this duality behaves after twisting was recently made by Costello [Cos17; CG18; CP20], in terms of the idea of Koszul duality. Consider $\mathbb{R}^n \subseteq \mathbb{R}^{10}$, where $\mathbb{R}^n$ is thought of as the worldsheet of a stack of N D_{n-1}-branes. Using what the research described so far, I will then be able to construct two $\mathbb{E}_n$ algebras.

1. The algebra $\mathcal{A}_{\text{gauge}}$ of local operators in a topological or partially topological twist of supersymmetric gauge theory on $\mathbb{R}^n$ for the gauge group GL_N, in the limit $N \to \infty$.
2. The algebra $\mathcal{A}_{\text{gravity}}$ of local operators in a topological or partially topological twist of supergravity on $\mathbb{R}^{10} \setminus \mathbb{R}^n \cong \mathbb{R}^n \times \mathbb{R}_{>0} \times S^{9-n}$.

Costello's formulation of twisted holography proposes that the Koszul dual $\mathcal{A}^!_{\text{gauge}}$ is a deformation of $\mathcal{A}_{\text{gravity}}$.

Objective 4.3. Construct twisted holography dualities in the case $n = 4$ by realizing the algebra of local operators in twisted supergravity theory as a deformation of the Koszul dual to the algebra of local operators in 4d $\mathcal{N} = 4$ gauge theory.

This result will have applications to the theory of quantum groups. For example, there is a 4d partially topological twist, where the Koszul dual to the algebra of local observables can be identified with a quantum deformation of the enveloping algebra $U(\mathfrak{gl}_N[\varepsilon][z])$, where ε is a parameter of degree 1. This story parallels work of Costello, and later Costello, Witten and Yamazaki, who studied the Koszul dual to the algebra of local operators in a twist of 4d $\mathcal{N} = 1$ gauge theory, and proved that it recovered the Yangian quantum group. Using twisted holography, we will identify a deformation of this quantum group, in the large N limit, with the observables in twisted supergravity theory. This gives the quantum group a lot of additional structure, for example, it will canonically act on the category of line operators in the twisted supergravity theory, or on the algebra of observables in the twist of any supersymmetric boundary theory.

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