# 1

Let $\alpha > 0$. We compute, term-by-term in the power series expansion

$$\begin{aligned}
\frac{d}{dx}x^{-\alpha}J_\alpha(x) &= \sum_{m\geq 0}\frac{(-1)^m}{m!\,\Gamma(m+\alpha+1)}2^{-2m-\alpha}\cdot 2m\cdot x^{2m-1}\\
&= -x^{-\alpha}\sum_{m\geq 0}\frac{(-1)^{m-1}}{m!\,\Gamma(m+\alpha+1)}2^{-2m-\alpha}\cdot 2m\cdot x^{2m-1+\alpha}\\
&= -x^{-\alpha}\sum_{m\geq 1}\frac{(-1)^{m-1}}{(m-1)!\,\Gamma(m+\alpha+1)}2^{-2m-\alpha+1}x^{2m-1+\alpha}\\
&= -x^{-\alpha}\sum_{n\geq 0}\frac{(-1)^{n}}{n!\,\Gamma(n+\alpha+2)}2^{-2n-\alpha-1}x^{2n+1+\alpha}\\
&= -x^{-\alpha}J_{\alpha+1}(x)
\end{aligned}$$

as required. On the third line we noted that the $m = 0$ term was equal to zero, and on the fourth line we re-indexed by setting $n = m - 1$.