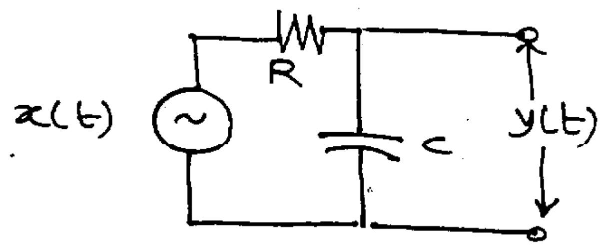
ECE 500 — EXAM #1

Consider the following linear time-invariant system



1. Show that the differential equation that describes this LTI system is [10 PTS]

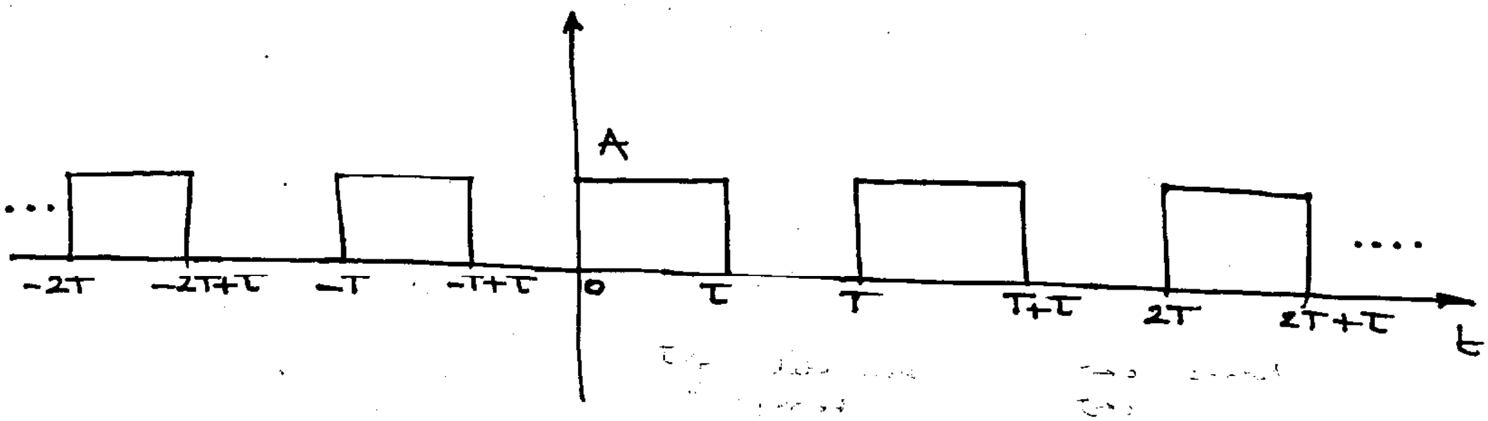
$$RC \frac{dy(t)}{dt} + y(t) = x(t)$$

2. Show that for this LTI system, the impulse response is [10 PTS]

$$h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$$

3. It turns out that this LTI system is a pretty good model of a commonly used transmission medium — the twisted pair — used for home telephones. where $x(t)$ denotes the pulses transmitted over the wire and $y(t)$ denotes the version of those pulses received. To illustrate the notion of

transmission bandwidth, consider the following $x(t)$



(a) Find the amplitude spectrum (plot of $\|C_k\|$ as a function of k) for $x(t)$ [20PTS]

(b) Suppose that $RC = 10^{-4}$, $T = 10\,\mu s$, $\tau = 5\,\mu s$, plot the amplitude spectrum of $y(t)$. [20PTS]

c) You will notice that atleast one "lobe" of the amplitude spectrum of $x(t)$ is "preserved in $y(t)$'s spectrum for part (b). Can you determine for what values of $\frac{\tau}{T}$ does $y(t)$'s spectrum not even contain one "lobe" of $x(t)$'s spectrum? [20PTS]

d) If $x(t)$ denotes the typical signal sent over your phone lines at home, and the values of RC, T are typical of that scenario. How do you relate the situation in (c) to the "clarity" of your phone line. [20 PTS]

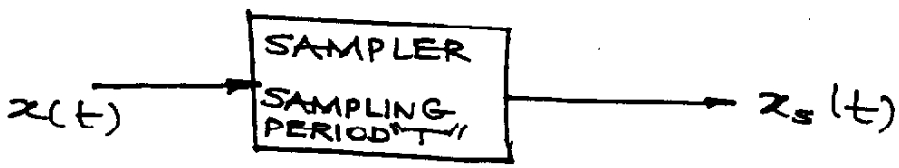
4. Let $x(t)$ be a signal such that

$$X(\omega) = \begin{cases} 1 - \left|\frac{\omega}{\omega_0}\right| & |\omega| < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

change to time

$\omega_0 = \frac{2\pi}{T}$

Let $x(t)$ be sampled as such

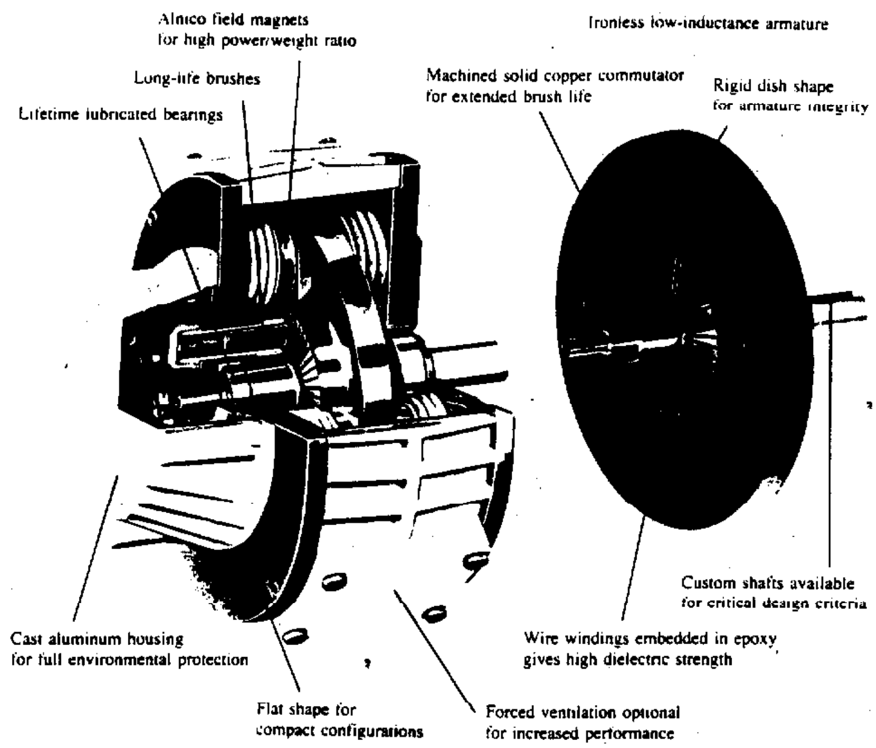


(a) What's the Nyquist rate of sampling for $x(t)$ [20 PTS]

(b) If $x_s(t)$ denotes the input to the RC circuit in page 1, and $y_s(t)$ denotes the corresponding output, for what values of R.C and T does $y_s(t)$ resemble $x(t)$ as close as possible. Is $x(t)$ and $y_s(t)$ identical? [30 PTS]

5. A motor of the type given in the next page is used in many practical applications. This includes electric vehicles too! Unfortunately, these motors cannot "accelerate" very quickly.

point you would have noted in the press that Electric cars are unable to respond as quickly as normal IC-engine based vehicles. We are going to illustrate this point

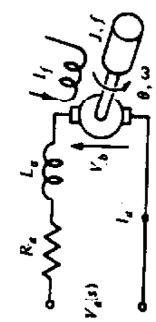


OUTPUT — ANGULAR POSITION

$$\frac{\theta(s)}{V_a(s)} = \frac{K_m}{s[(R_a + L_a s)(Js + f) + K_b K_m]}$$

Transfer function

INPUT — VOLTAGE



dc-motor, armature controlled

$\frac{d\theta(t)}{dt}$ should follow $u(t)$ or else its ...

(a) What's the output $\theta(t)$ when the input $v_a(t) = u(t)$ the unit-step function. Assume that $k_b = k_m = 1$, $\frac{R_a}{L_a} = 10^9$, $\frac{R_a f + 1}{R_a J} = 10$ [25 PTS]

(b) Why do you think such motors are labeled as "sluggish"? [25 PTS]

[TOTAL: 200 PTS]

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