PUMP2: Problem Set 1

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1 In-Set-tion

(a) $\{\varnothing\}, \{1\}, \{2\}, \{cat\}, \{dog\}, \{1, 2\}, \{1, cat\}, \{1, dog\}, \{2, cat\}, \{2, dog\}, \{cat, dog\}, \{1, 2, cat\}, \{1, 2, dog\}$ $\{\varnothing\}, \{\varnothing, cat\}, \{\varnothing, dog\}, \{\varnothing, cat, dog\}, \{\{\}\}, \{\{\}\}, \{\{\}\},$

(b) $\sum_{k=1}^{n} \binom{n}{k} = 2^n$

(c) *Proof.* We check the base case. When the set S has zero element, the powerset of S has only one element, the null set which is $2^0 = 1$. As a result, the base case is true. As discovered above, we know that $\sum_{k=1}^{n} \binom{n}{k} = 2^n$ We induct on n. We have to show that $2 \sum_{k=1}^{n} \binom{n}{k} = 2^{n+1}$. We see that $\sum_{k=1}^{n} \binom{n}{k} + \sum_{k=1}^{n} \binom{n}{k} = 2^n + 2^n = 2(2^n) = 2^{n+1}$ Thus the proposition stays true. $\square$