In [1]:
```python
f1, f2, f3 = symbols('f1:4', real=True, positive=True)
p1, p2, p3 = symbols('p1:4', real=True, positive=True)
r11, r12, r13 = symbols('r11,r12,r13', real=True, positive=True)
r21, r22, r23 = symbols('r21,r22,r23', real=True, positive=True)
r31, r32, r33 = symbols('r31,r32,r33', real=True, positive=True)
```

In [2]:
```python
p3expr = (f3 - p1 * r13 - p2 * r23) / r33 - p3
p2expr = (f2 - p1 * r12 - p3 * r32) / r22 - p2
p1expr = (f1 - p2 * r21 - p3 * r31) / r11 - p1
p1expr, p2expr, p3expr
```

Out[2]:
$$\left(-p_1 + \frac{f_1 - p_2 r_{21} - p_3 r_{31}}{r_{11}},\quad -p_2 + \frac{f_2 - p_1 r_{12} - p_3 r_{32}}{r_{22}},\quad -p_3 + \frac{f_3 - p_1 r_{13} - p_2 r_{23}}{r_{33}}\right)$$

In [3]:
```python
sols = solve((p1expr, p2expr, p3expr), (p1, p2, p3))
p1sol = sols[p1]
p2sol = sols[p2]
p3sol = sols[p3]
p1sol.collect((f1,f2,f3))
```

Out[3]:
$$\frac{f_1 (r_{22} r_{33} - r_{23} r_{32}) + f_2 (-r_{21} r_{33} + r_{23} r_{31}) + f_3 (r_{21} r_{32} - r_{22} r_{31})}{r_{11} r_{22} r_{33} - r_{11} r_{23} r_{32} - r_{12} r_{21} r_{33} + r_{12} r_{23} r_{31} + r_{13} r_{21} r_{32} - r_{13} r_{22} r_{31}}$$

In [4]:
```python
p1sol = p1sol.subs(r11, 1-r12-r13).subs(r22, 1-r21-r23).subs(r33, 1-r31-r32)
p2sol = p2sol.subs(r11, 1-r12-r13).subs(r22, 1-r21-r23).subs(r33, 1-r31-r32)
p3sol = p3sol.subs(r11, 1-r12-r13).subs(r22, 1-r21-r23).subs(r33, 1-r31-r32)
```

In [5]:
```python
simplify(p1sol).collect((f1,f2,f3))
```

Out[5]:
$$\frac{f_1 (r_{21} r_{31} + r_{21} r_{32} - r_{21} + r_{23} r_{31} - r_{23} - r_{31} - r_{32} + 1) + f_2 (r_{21} r_{31} + r_{21} r_{32} - r_{21} + r_{23} r_{31}) + f_3 (r_{21} r_{31} + r_{21} r_{32} + r_{23} r_{31} - r_{31})}{r_{12} r_{23} + r_{12} r_{31} + r_{12} r_{32} - r_{12} + r_{13} r_{21} + r_{13} r_{23} + r_{13} r_{32} - r_{13} + r_{21} r_{31} + r_{21} r_{32} - r_{21} + r_{23} r_{31} - r_{23} - r_{31} - r_{32} + 1}$$

In [6]:
```python
_.subs(f1, 1-f2-f3).simplify().collect((f2,f3))
```

Out[6]:
$$\frac{f_2 (r_{23} + r_{31} + r_{32} - 1) + f_3 (r_{21} + r_{23} + r_{32} - 1) + r_{21} r_{31} + r_{21} r_{32} - r_{21} + r_{23} r_{31} - r_{23} - r_{31} - r_{32} + 1}{r_{12} r_{23} + r_{12} r_{31} + r_{12} r_{32} - r_{12} + r_{13} r_{21} + r_{13} r_{23} + r_{13} r_{32} - r_{13} + r_{21} r_{31} + r_{21} r_{32} - r_{21} + r_{23} r_{31} - r_{23} - r_{31} - r_{32} + 1}$$

Show that this is consistent with the $n = 2$ case, when all terms involving a_3 are set to zero.

In [7]:
```python
_.subs({f3: 0, r13: 0, r23: 0, r31: 0, r32: 0})
```

Out[7]:
$$\frac{-f_2 - r_{21} + 1}{-r_{12} - r_{21} + 1}$$

In [8]:
```python
_.subs(f2, 1-f1)
```

Out[8]:
$$\frac{f_1 - r_{21}}{-r_{12} - r_{21} + 1}$$