```
In [1]: p1, p2, f1, f2 = symbols('p1:3,f1:3', real=True, positive=True)
        r11, r12, r21, r22 = symbols('r_11,r_12,r_21,r_22', real=True, positive
```

```
In [2]: p2expr = (f2 - p1 * r12) / (1 - r21)
        p1expr = (f1 - p2 * r21) / (1 - r12)
        p1expr, p2expr
```

Out[2]: $$\left(\frac{f_1 - p_2 r_{21}}{-r_{12} + 1}, \quad \frac{f_2 - p_1 r_{12}}{-r_{21} + 1} \right)$$

```
In [3]: p1expr.subs(p2, p2expr) - p1
```

Out[3]: $$-p_1 + \frac{f_1 - \frac{r_{21}\left(f_2 - p_1 r_{12}\right)}{-r_{21} + 1}}{-r_{12} + 1}$$

```
In [4]: solve(_, p1)[0]
```

Out[4]: $$\frac{f_1 r_{21} - f_1 + f_2 r_{21}}{r_{12} + r_{21} - 1}$$

```
In [5]: _.subs(f2, 1-f1)
```

Out[5]: $$\frac{f_1 r_{21} - f_1 + r_{21}\left(-f_1 + 1\right)}{r_{12} + r_{21} - 1}$$

```
In [6]: simplify(_)
```

Out[6]: $$\frac{-f_1 + r_{21}}{r_{12} + r_{21} - 1}$$