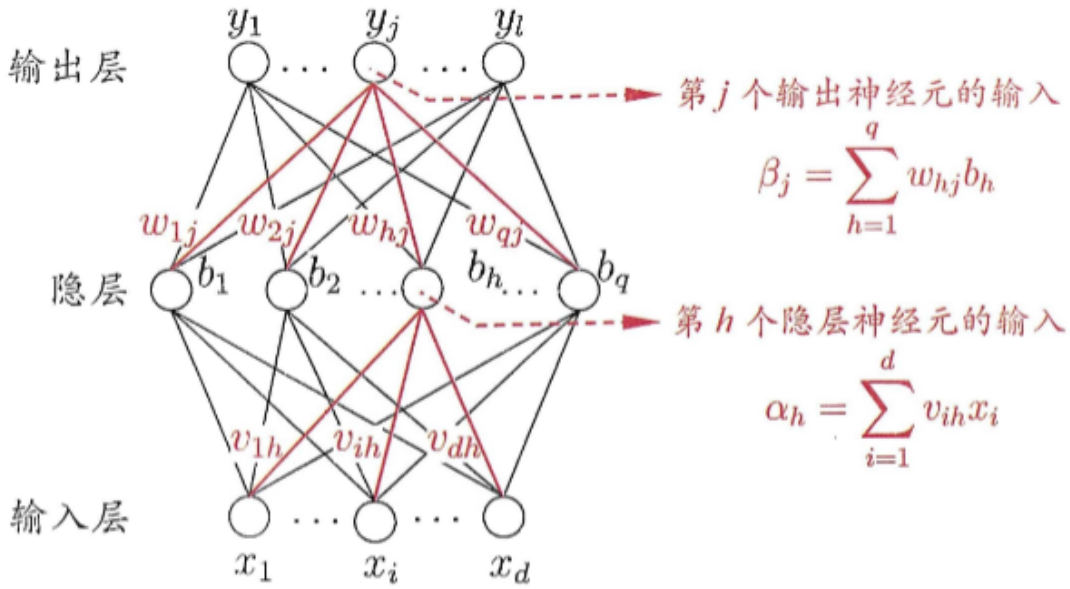

图 5.7　BP 网络及算法中的变量符号

设对于(x, y)，神经网络的输出为y'，x为d维向量（$d \times 1$），y和y'为l维向量（$l \times 1$），神经网络的中间层为q维

从输入到输出的函数为，其中$f(x)$为sigmoid函数

$$\alpha_h = \sum_{i=1}^{d} v_{ih} x_i$$
$$b_h = f(\alpha_h - \gamma_h)$$
$$\beta_j = \sum_{h=1}^{q} w_{hj} b_h$$
$$y' = f(\beta_j - \theta_j)$$

定义误差函数为

$$J = \frac{1}{2}(y' - y)^2$$

设学习率为α，基于梯度下降法，神经网络权重的更新公式为

$$v = v - \alpha \frac{\partial J}{\partial v}$$
$$w = w - \alpha \frac{\partial J}{\partial w}$$

$$\frac{\partial J}{\partial v}=\frac{\partial J}{\partial y'}\frac{\partial y'}{\partial \beta}\frac{\partial \beta}{\partial b_h}\frac{\partial b_h}{\partial \alpha_h}\frac{\partial \alpha_h}{\partial v_{ih}}$$

$$\frac{\partial J}{\partial y'}=y'-y$$

$$\frac{\partial y'}{\partial \beta}=f'(\beta_j-\theta_j)=f(\beta_j-\theta_j)(1-f(\beta_j-\theta_j))=y'(1-y')$$

$$\frac{\partial \beta}{\partial b_h}=\sum_{j=1}^{l}w_{hj}$$

$$\frac{\partial b_h}{\partial \alpha_h}=f'(\alpha_h-\gamma_h)=f(\alpha_h-\gamma_h)(1-f(\alpha_h-\gamma_h))=b_h(1-b_h)$$

$$\frac{\partial \alpha_h}{\partial v_{ih}}=x_i$$

所以

$$v=v-\alpha((y'-y)y'(1-y')\sum_{j=1}^{l}(w_{hj})b_h(1-b_h)x_i)$$