Week 9A Splines, Extension Material

Problem of Bezier Curves

- **Local control**: Moving one control point affects the entire curve.
- **Incomplete**: No circle, elipses, conic sections and etc.

Local control These curves suffer from non-local control. Moving one control point will also affect the entire curve.

Additionally, each berstein polynomial is active (non-zero) over the entire interval [0,1]. The curve is a blend of these functions so every control point has an effect on the curve for all t from [0,1]

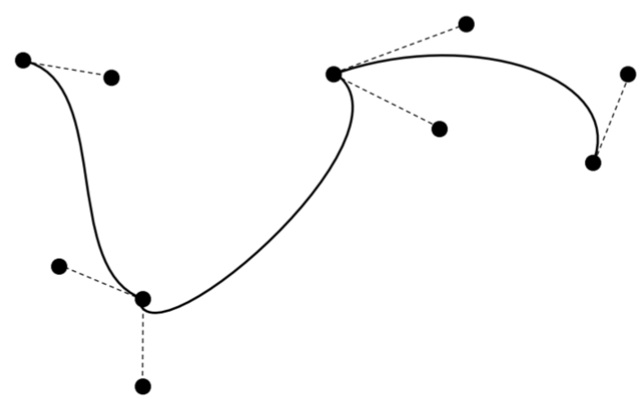
Splines

> A spline is a smooth piecewise-polynomial function (for some measurement of smoothness). The place where the polynomials join are called **knots**

A joined sequence of Bezier curves is an example of a spline.

Local control A spline provides local control. A control point only affects the curve within a limited neighbourhood.

Bezier splines We can draw longer curves as sequences of Bezier sections with common endpoints:



Parametric Continuity A curve is said to have $\mathbf{C}^n$ continuity if the *nth derivative* is continuous for all t:

$$v_n(t) = \frac{d^n P(t)}{dt^n}$$

$\mathbf{C}^0$: the curve is connected. $\mathbf{C}^1$: a point travelling along the curve doesn't have any instantaneous changes in velocity. $\mathbf{C}^2$: no instantaneous changes in acceleration

Geometric Continuity A curve is said to have $\mathbf{G}^n$ continuity if the *normalised derivative* is continuous for all t:

$$\hat{\mathbf{v}}_n(t) = \frac{\mathbf{v}_n(t)}{|\mathbf{v}_n(t)|}$$

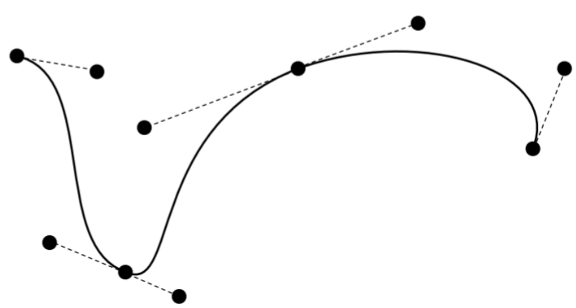
$\mathbf{G}^1$: means tangents to the curve are continuous. $\mathbf{G}^2$: means the curve has continuous curvature.

Summary on continuity

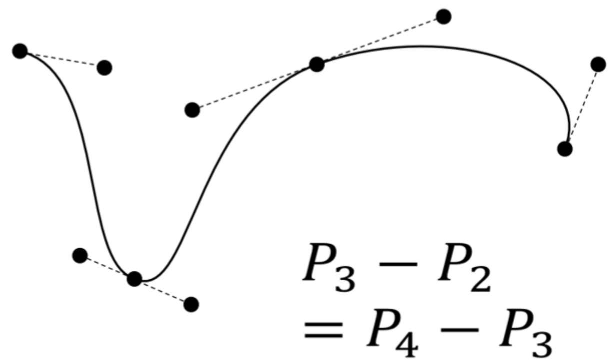
- Geometric continuity is important if we are drawing a curve
- Parametric continuity is important if we are using a curves as a guide for motion

Cont. Bezier splines

- If the control points are *collinear*, the curve has $\mathbf{G}^1$ continuity.



- If the control points are collinear and *equally spaced*, then the curves has $\mathbf{C}^1$



continuity:

B-splines

Bezier splines can be generalised into a large class called *basis splines* or *B-splines*

A B-spline of degree m has equation:

$$P(t) = \sum_{k=0}^{L} N_k^m(t) P_k$$

where L is the number of control points, with $L > m$

Coefficient function The $N_k^m(t)$ funtion is defined recursively:

$$N_k^m(t) = \left(\frac{t - t_k}{t_{m+k} - t_k}\right) N_k^{m-1}(t)$$
$$+ \left(\frac{t_{m+k+1} - t}{t_{m+k+1} - t_{k+1}}\right) N_{k+1}^{m-1}(t)$$
$$N_k^0(t) = 1 \text{ if } t_k < t \leq t_{k+1}$$
$$N_k^0(t) = 0 \text{ otherwise}$$

Knot vector

The sequence $(t_0, t_1, \dots, t_{m+L})$ is called the *knot vector*. The knots are ordered so $t_k \leq t_{k+1}$. Knots mark the limits of the influence of each control point. Control point P_k affects the curve between knots t_k and t_{k+m+l}

Number of Knots The number of knots in the knot vector is always equal to the nu,ber of control points plus the order of the curve.

For example, a cubic (m = 3) with five control points have 9 items in the knot vector:

(0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1)

Uniform/Non-uniform

Uniform B-splines have *equally spaced* knots. Non-uniform B-splines allow knots to be positioned arbitrarily even and repeat.

A *multiple knot* is a knot value that is repeated several times. Mutiple knots created discontinuities in the derivatives.