Statistical Inference Project - Part 1: Simulation Exercise

Part 1: Simulation Exercise Instructions

In this project you will investigate the exponential distribution in R and compare it with the Central Limit Theorem.

The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter.

The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. Set lambda = 0.2 for all of the simulations.

You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

setup

```
##    dplyr    tidyr  ggplot2    knitr markdown  moments  nortest    e1071
##     TRUE     TRUE     TRUE     TRUE     TRUE     TRUE     TRUE     TRUE
```

Part 1 - Simulations

simulation inputs

settings a seed will allow us to reproduce the results

create a dataframe with column title 'individual_mean' for recording each sample distributions mean

```
##    [1] 4.477747 5.822793 4.054247 4.873564 4.609780 4.585438 4.651006
##    [8] 4.595969 3.712426 4.149294 5.157933 6.458950 3.078232 4.094112
##   [15] 5.986086 5.472690 5.419147 5.531831 4.518665 4.869121 4.133342
##   [22] 5.370928 6.321934 5.059804 5.418288 4.286439 4.486671 4.319625
##   [29] 4.219707 5.810918 5.205436 5.650520 4.473421 4.982367 5.520775
##   [36] 4.965440 6.454077 4.808679 4.626727 5.466139 4.920505 4.649314
##   [43] 4.160529 5.025663 5.858350 5.117681 3.781178 3.525809 4.439272
##   [50] 5.287117 4.048858 6.141934 4.395114 5.259441 4.761815 6.137541
##   [57] 5.071056 5.588132 4.553956 5.814644 5.581375 4.024610 5.823562
##   [64] 4.997891 5.942808 5.522696 4.459563 6.178316 6.842838 5.671803
```

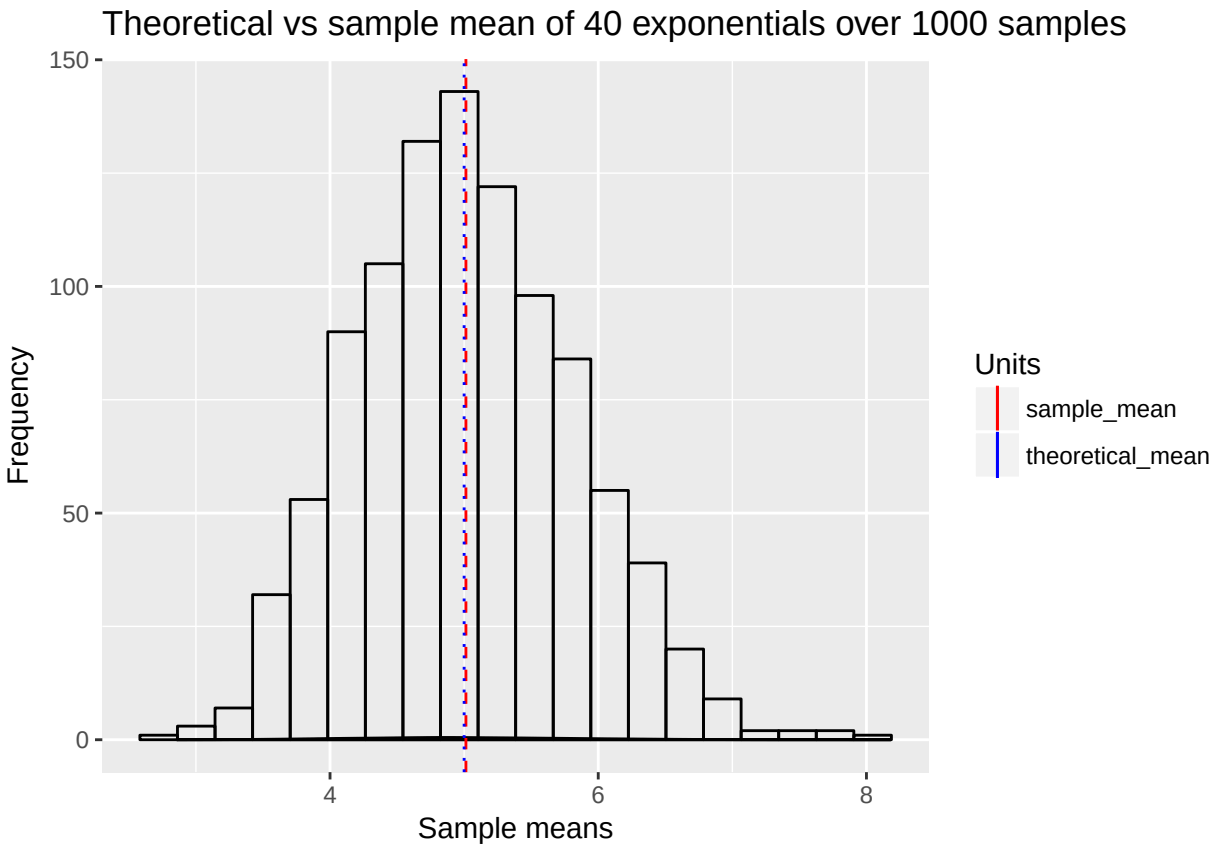
```
##   [71] 4.469114 4.801853 4.665882 4.445754 5.185193 4.125845 4.727188
##   [78] 5.067062 4.473093 5.528742 5.744555 4.635694 6.904941 4.193948
##   [85] 5.632864 4.178830 5.915544 4.502363 3.669769 4.329402 4.869422
##   [92] 4.685059 3.636841 4.480857 5.263643 4.469195 6.226405 4.741717
##   [99] 3.654736 4.509778 5.263810 5.635516 5.218723 4.096542 4.541604
##  [106] 4.534021 5.623070 4.931131 5.739006 5.122181 6.069965 4.566364
##  [113] 5.362757 3.719220 4.740278 3.962560 5.738909 5.337926 4.209345
##  [120] 4.819896 6.107608 4.827245 4.517337 4.867988 5.181967 4.549030
##  [127] 4.591865 4.430591 4.256263 4.283010 4.926699 5.304738 5.358154
##  [134] 4.795842 6.429036 4.669127 5.379337 5.436265 6.186132 5.295183
##  [141] 4.221416 5.669997 4.058217 4.649527 3.738601 4.742113 3.576317
##  [148] 6.330689 5.466280 4.131557 4.104211 4.131771 5.498004 5.178683
##  [155] 4.646368 5.689019 4.459512 4.115204 5.638382 3.870438 5.749795
##  [162] 3.855602 5.025425 6.389348 6.262231 4.758908 5.770511 5.148280
##  [169] 6.373413 4.043356 4.776938 4.573203 6.905300 5.429451 4.062134
##  [176] 6.138414 4.325707 4.142002 5.511726 5.941754 4.837658 5.515795
##  [183] 6.372840 3.553169 5.071859 3.876947 5.907754 4.925903 4.269575
##  [190] 3.066773 5.880013 6.792044 5.073425 4.867108 4.181081 5.724922
##  [197] 4.465301 4.643285 4.829955 3.171293 4.363128 5.140164 5.040522
##  [204] 7.061140 4.951941 4.512984 5.306142 5.044555 5.293588 4.525819
##  [211] 5.265364 4.719418 4.608964 6.138074 4.805210 5.350031 5.130589
##  [218] 4.140409 6.404625 4.949603 4.472438 4.161453 5.882515 4.725728
##  [225] 5.480705 4.810632 3.470458 4.684096 4.993168 5.061233 4.288307
##  [232] 4.882722 3.208555 5.239177 4.294336 3.822187 6.072706 4.725571
##  [239] 3.757056 4.893307 3.513157 3.734274 5.302279 5.551670 4.788951
##  [246] 4.930225 4.788817 4.400297 5.856831 4.613834 4.341910 4.410191
##  [253] 5.182609 4.291738 6.046860 4.864585 3.968699 4.683065 4.387079
##  [260] 6.429382 4.680842 4.588689 4.016393 4.720207 5.276618 5.784195
##  [267] 3.038445 4.637327 6.522068 5.175961 5.275567 4.987059 4.773204
##  [274] 6.263420 5.759631 5.794074 3.613251 5.450131 5.604221 4.221041
##  [281] 5.011036 4.836544 5.912570 3.899990 4.038655 3.702410 4.195832
##  [288] 4.932583 4.634887 3.991942 5.263496 5.797995 4.693850 4.254799
##  [295] 4.712065 3.533879 6.649724 3.952595 4.980208 4.994566 4.667310
##  [302] 3.633684 3.972221 6.197083 4.298184 4.741036 4.601320 4.813853
##  [309] 6.168023 4.754357 4.535436 5.064495 5.523006 5.361631 5.326654
##  [316] 6.220913 4.829723 5.004350 4.539446 4.824659 5.157739 4.860073
##  [323] 6.363575 5.043780 4.857918 5.169988 5.099879 5.616262 3.668756
##  [330] 5.595927 6.176703 5.248422 4.904656 6.116093 4.426638 4.698453
##  [337] 4.983272 5.714761 4.780819 5.111644 3.508864 4.943474 4.989637
##  [344] 4.882901 5.233404 5.894025 6.696226 3.763661 5.691752 4.360740
##  [351] 5.182130 4.464654 5.047331 5.362107 5.792441 4.938366 4.352097
##  [358] 4.985860 5.388543 4.159870 3.453201 4.432070 4.174557 3.694752
##  [365] 6.241594 5.100268 5.763008 4.397284 4.673909 6.084358 3.768166
##  [372] 3.562913 4.658418 5.364576 5.999158 5.426150 5.518400 5.127671
##  [379] 5.240800 5.871354 5.334936 5.242126 4.865186 6.206548 4.317952
##  [386] 4.901590 4.520377 5.044043 4.589646 4.706063 4.990059 4.956417
##  [393] 5.788356 4.489123 5.161494 3.893473 4.734369 5.170372 5.101158
##  [400] 5.496167 4.563370 5.000997 4.966623 5.227250 6.115488 5.362927
##  [407] 4.613273 5.511576 4.715607 5.380889 4.456061 4.502880 5.496374
##  [414] 4.798677 4.842253 4.455561 5.442711 5.728878 3.968698 4.836183
##  [421] 6.374506 6.092238 3.725423 4.749322 5.881284 4.645986 3.440139
##  [428] 5.527435 3.964469 4.853024 5.651641 6.148513 4.679216 5.175982
##  [435] 3.824224 4.242490 4.277092 4.561974 5.392557 6.083951 4.701939
##  [442] 5.501492 5.224103 5.061754 3.892543 3.520397 5.679169 4.991979
```

```
##  [449] 5.061649 3.554232 5.897533 6.061054 6.292199 4.897514 3.951472
##  [456] 5.072400 4.934352 4.739600 5.624493 4.984889 6.736274 5.320495
##  [463] 5.101484 4.798725 3.464178 6.136682 4.642252 6.309471 5.294252
##  [470] 4.251434 4.738161 4.411612 4.299202 4.658648 3.883301 3.554120
##  [477] 5.458336 3.921185 5.707558 4.516157 5.891252 4.747727 4.898291
##  [484] 5.572317 5.791519 5.239739 4.796976 4.311430 3.821789 4.138058
##  [491] 6.541622 4.283524 4.848950 4.794439 5.917761 4.202179 4.168637
##  [498] 4.890398 4.619871 6.019120 4.905601 4.746729 4.869200 4.527614
##  [505] 5.539640 4.065573 4.887316 3.752372 6.071505 6.061408 6.854201
##  [512] 6.326465 4.653098 4.236720 4.283393 5.321177 5.068602 5.138423
##  [519] 3.590435 7.504604 6.465339 4.414801 6.564753 3.893277 4.578836
##  [526] 5.242132 5.611620 3.354495 6.136239 5.382540 5.578820 5.493164
##  [533] 5.740477 6.504287 4.580708 3.873859 6.435901 3.954076 3.846098
##  [540] 4.276177 3.345978 5.092901 5.185938 4.490627 5.407109 7.011434
##  [547] 5.202242 4.454697 4.892080 4.367424 4.492683 5.212965 6.554220
##  [554] 4.577367 4.743780 5.080222 5.211727 5.444413 4.291443 4.853056
##  [561] 4.630321 5.539254 5.236185 4.828844 4.618248 5.530703 5.957565
##  [568] 5.578294 6.015347 4.537794 4.298652 5.580827 4.835162 5.759225
##  [575] 5.168847 5.266744 5.324653 4.387931 5.163712 4.125640 4.477525
##  [582] 3.747911 5.143150 5.003039 4.932372 6.094689 4.623277 5.807622
##  [589] 6.203238 5.869480 5.591491 4.662491 5.197290 5.043625 5.656121
##  [596] 4.182846 5.409931 5.072883 5.084881 6.159595 4.767353 5.732997
##  [603] 5.775053 4.286567 3.657520 5.405702 5.121611 5.069974 4.513671
##  [610] 6.936890 5.235590 5.348315 4.653865 5.614751 4.444038 5.532906
##  [617] 5.927265 5.429617 4.051481 3.806394 6.162959 4.866095 6.605226
##  [624] 4.981015 4.264633 5.318596 5.668851 4.463554 4.621976 4.153423
##  [631] 4.818440 4.476730 4.750898 6.577281 4.593291 3.395923 4.345696
##  [638] 4.887947 7.158041 4.827355 4.878246 5.996550 4.384831 3.728125
##  [645] 4.491725 5.735386 3.959715 6.161134 4.908355 3.883489 4.780960
##  [652] 3.694795 4.192615 5.437152 4.312696 4.273212 4.926011 5.762921
##  [659] 3.749239 5.171213 4.194477 4.583727 4.658810 5.572814 6.822411
##  [666] 5.638156 4.477625 4.506593 5.741876 4.510563 3.605504 4.644458
##  [673] 6.862343 6.652800 6.057681 4.450379 5.609990 5.423132 5.539745
##  [680] 4.610807 4.745065 4.912782 5.103121 4.340517 4.690703 4.994462
##  [687] 4.515350 4.638734 3.923666 3.711721 5.971237 5.896926 4.796163
##  [694] 4.543238 5.236295 5.005100 3.785963 3.969587 4.645283 5.037105
##  [701] 5.284682 3.783895 4.820856 5.497989 4.396230 6.406011 5.202607
##  [708] 4.976005 5.442225 5.509046 6.092741 6.031377 4.055405 5.923368
##  [715] 4.939050 4.023327 4.463944 4.855765 5.290758 4.504613 5.503977
##  [722] 6.316373 4.383938 5.377641 4.558542 4.904459 5.070459 4.366099
##  [729] 6.812654 5.670670 4.081834 4.156038 5.057307 4.068375 5.607176
##  [736] 5.682891 4.779732 4.852555 4.201290 5.214789 5.134449 4.901751
##  [743] 2.954696 4.519285 4.440303 5.847399 3.564119 5.014556 5.657918
##  [750] 5.106643 5.757282 4.518732 4.054912 5.902931 4.292346 3.632577
##  [757] 3.242516 4.105855 5.308046 5.001726 4.506080 5.118768 4.735058
##  [764] 5.349266 6.957045 5.935728 4.668147 3.895769 4.535237 5.102497
##  [771] 4.310592 3.938283 4.126013 3.852711 4.138734 5.606984 6.035594
##  [778] 3.567138 4.141862 4.821682 4.868641 5.307269 5.379678 4.882561
##  [785] 4.852300 4.710748 4.722812 4.216816 4.750527 6.061939 5.253589
##  [792] 5.044351 5.231250 4.752823 5.489209 4.942084 4.421773 5.349520
##  [799] 4.224268 5.360239 4.449251 4.884072 6.039860 4.084703 4.990998
##  [806] 3.146932 4.528028 4.432593 4.442897 6.174298 7.002486 5.508463
##  [813] 5.125758 4.677978 5.269065 6.025973 4.676071 6.506003 4.793703
##  [820] 5.375697 5.935360 4.409179 5.907521 5.025884 5.030626 4.669713
```

```
##  [827] 4.423117 5.238939 5.203819 6.012558 4.989242 5.899154 3.947181
##  [834] 4.977896 5.039147 4.847321 4.989667 5.841969 4.768444 4.604721
##  [841] 4.238668 3.417163 4.324409 3.604881 6.009194 5.156613 4.990983
##  [848] 6.725242 4.984345 4.951376 5.255504 4.296999 5.880931 4.063275
##  [855] 4.871584 5.262350 3.948572 4.214408 4.637993 5.117635 4.221434
##  [862] 5.444820 4.961525 5.751924 4.622110 6.191520 4.780517 4.822940
##  [869] 5.160174 6.296330 5.271327 3.306434 4.809731 5.253143 4.395803
##  [876] 3.868722 5.960459 4.070624 4.432135 6.238343 4.694527 4.774069
##  [883] 4.540402 3.858785 6.075365 5.345174 3.728951 5.435282 6.324202
##  [890] 4.010521 4.486403 4.565843 3.613974 6.790007 5.449002 4.358349
##  [897] 4.265322 3.934556 5.800707 4.472908 4.003084 6.659780 3.558281
##  [904] 4.735158 5.368375 3.848311 4.631142 6.201723 3.865159 4.727346
##  [911] 5.392299 4.604678 4.941461 5.356088 4.463160 5.447044 4.180540
##  [918] 5.730145 5.063693 4.701625 4.413578 5.359979 5.372060 4.651471
##  [925] 4.457331 3.776663 5.072917 3.698343 6.389875 3.550352 3.841411
##  [932] 4.475090 4.395080 3.964174 4.955963 4.066199 4.551732 3.380941
##  [939] 5.160395 4.324008 5.179249 3.652153 4.516456 5.977623 4.637730
##  [946] 5.270218 5.866953 4.916347 4.763201 4.886284 3.744452 4.756629
##  [953] 4.302548 5.346065 5.164238 4.716826 4.433024 4.742462 4.667845
##  [960] 6.417788 5.858835 4.246867 5.975394 4.746015 5.180167 4.086013
##  [967] 5.157983 5.079510 4.255219 4.710702 6.547870 5.525139 7.537365
##  [974] 5.092021 6.853127 5.201422 4.767741 3.870819 4.126253 4.784860
##  [981] 3.856164 6.803727 6.546472 5.605184 6.854520 6.595731 4.414702
##  [988] 4.153311 5.463051 4.459573 4.530181 6.155396 6.247589 4.499640
##  [995] 4.574150 5.863842 5.029003 4.885902 6.010222 4.312497
```

Show the sample mean and compare it to the theoretical mean of the distribution.

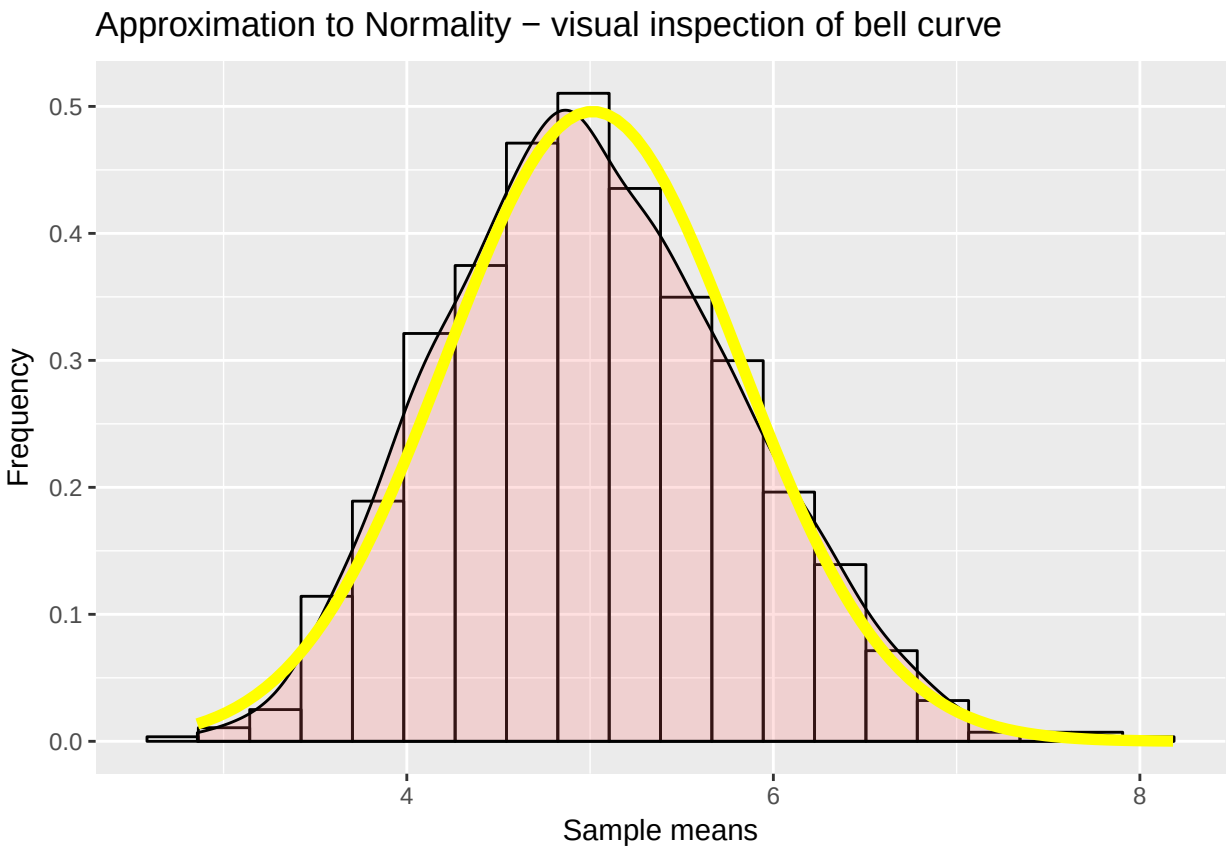
plot samples with theoretical mean vs sample mean



Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.

Show that the distribution is approximately normal.

visually inspect bell curve



https://en.wikipedia.org/wiki/Normal_probability_plot

nortest package to the rescue

http://stats.stackexchange.com/questions/52293/r-qqplot-how-to-see-whethe 52295

Test 1 - skewness and kurtosis, they should be around (0,3)

```
## [1] 0.2887447
## [1] -0.0225037
```

Test 2 - Shapiro-Wilks test

```
##
##  Shapiro-Wilk normality test
##
## data:  data
## W = 0.99399, p-value = 0.0004827
```

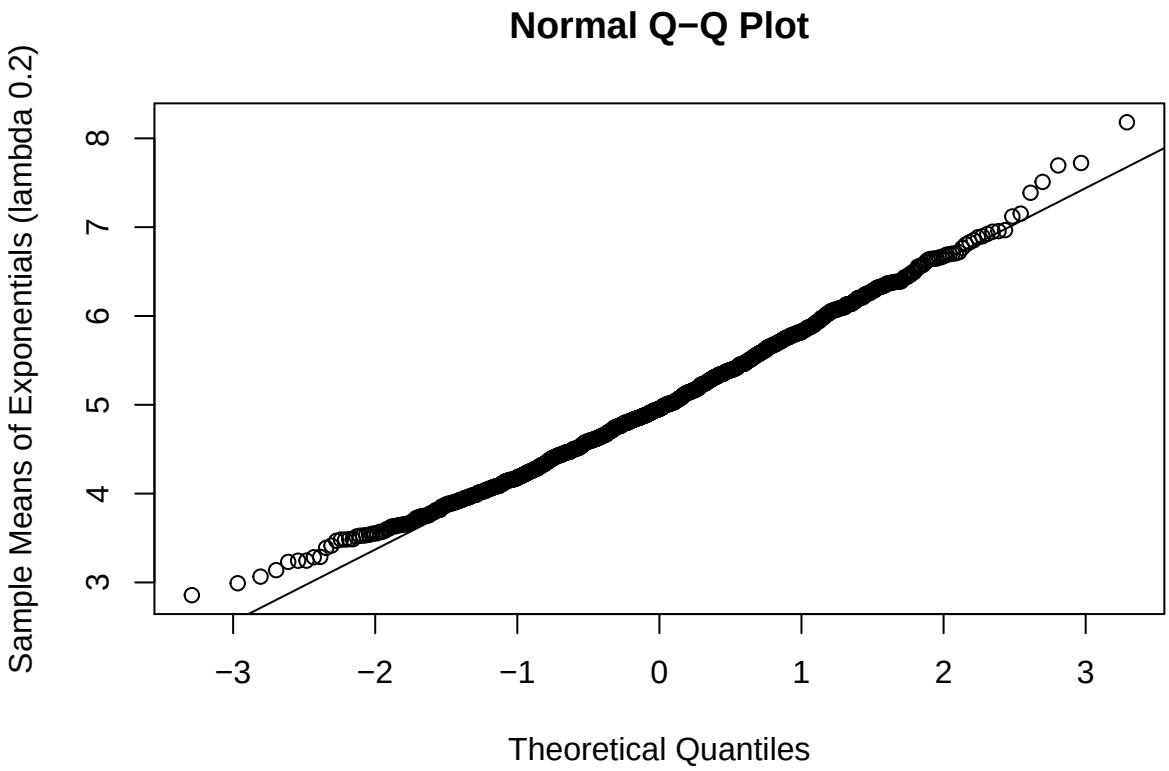
Test 3 - Kolmogorov-Smirnov test

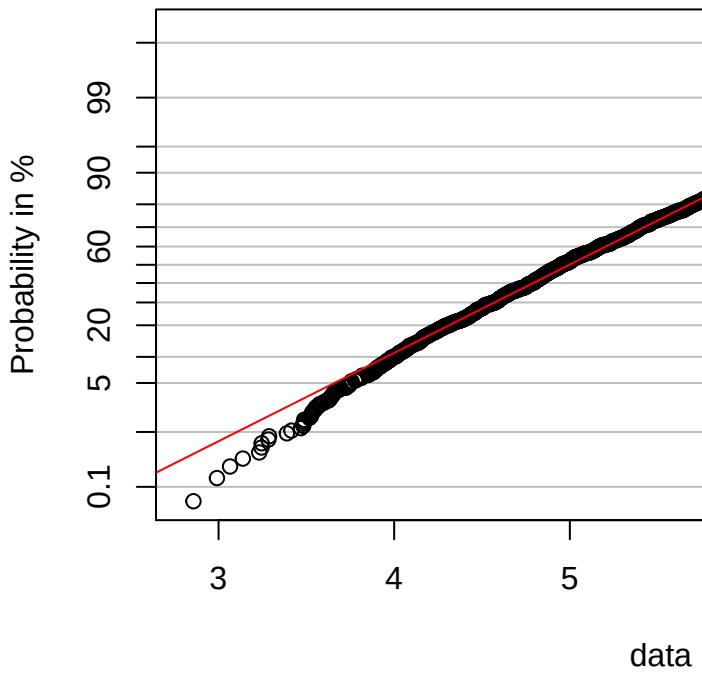
```
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  data
## D = 0.033966, p-value = 0.1988
## alternative hypothesis: two-sided
```

Test 4 - Anderson-Darling test

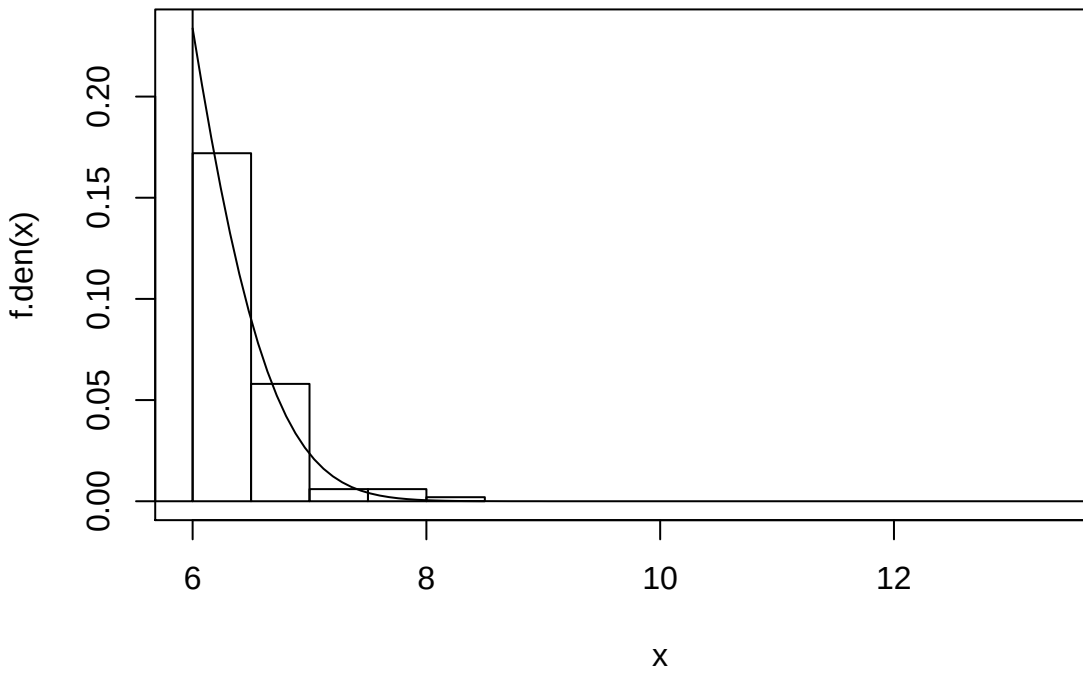
```
##
##  Anderson-Darling normality test
##
## data:  data
## A = 1.2078, p-value = 0.003775
```

Test 5 - qq-plot: you should observe a good fit of the straight line





Test 6 - p-plot: you should observe a good fit of the straight line



Test 7 - fitted normal density