Simulation Based Inference: Gibbs Sampling

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Outline

Why Gibbs sampling

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- Applied widely in finance and financial econometrics.
- Sometimes are time consuming. C/C++, MATLAB call C, or R call C have to be used.

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- Gibbs sampling generates correlated time series.
- A burn in period is needed before performing inference.
- The joint distribution is not known explicitly or is difficult to sample from directly.

Gibbs Algorithm

- Suppose that we have a data set $\mathbf{y} = \{y_i, i = 1, ..., n\}$, which is generated from a distribution or a model indexed by a vector of parameters $\theta^T = (\theta_1, ..., \theta_m)$.

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- Suppose also that each parameter can be sampled conditional on other parameters in the model.
- We have generated a sample $\{\theta_i^{(1)}, ..., \theta_i^{(L)}\}$ from the conditional distribution of θ_i.
- The estimate of θ_i is

$$\hat{\theta}_i = \frac{1}{L-l} \sum_{k=l+1}^{L} \theta_i^{(k)},$$

where the first l generated numbers are discarded as burn in.

Gibbs Algorithm (cont'd)

Table: Gibbs algorithm.

Step 0.	Initialize $\{\theta_1^{(0)}, ..., \theta_m^{(0)}\}$, set $k = 0$.
Step 1.	Sample $\theta_1^{(k+1)} \vert \theta_2^{(k)}, ..., \theta_m^{(k)}$.
	Sample $\theta_2^{(k+1)} \vert \theta_1^{(k+1)}, \theta_3^{(k)}, ..., \theta_m^{(k)}$.
	.
	.
	.
	Sample $\theta_m^{(k+1)} \vert \theta_1^{(k+1)}, \theta_2^{(k+1)}, ..., \theta_{m-1}^{(k+1)}$.
Step 2.	Go to Step 1.

- Gibbs sampler generates posterior samples by sweeping through each variable to sample from its conditional distribution with the remaining variables fixed to their current values.

Example 1 – fit a univariate normal distribution

- Suppose that we have a data set $\mathbf{y} = \{y_i, i = 1, ..., n\}$, which is generated from a univariate normal distribution $y \sim N(\mu, \sigma^2)$ with pdf $p(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\big(-\frac{(y-\mu)^2}{2\sigma^2}\big)$.

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- The likelihood of the data based on the model is

$$f(\mathbf{y}|\mu, \sigma^2) = \Pi_{i=1}^n f(y_i|\mu, \sigma^2) \propto \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}} \exp\left(- \frac{\sum_{i=1}^n (y_i - \mu)^2}{2\sigma^2}\right)$$

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- The posterior distribution of the two parameters μ and σ^2 is

$$\begin{aligned} f(\mu, \sigma^2|\mathbf{y}) &\propto f(\mu|\mu_*, \sigma_*^2) \mathrm{IG}(\sigma^2|\alpha_\sigma, \beta_\sigma) \Pi_{i=1}^n f(y_i|\mu, \sigma^2) \\ &\propto \left(\frac{1}{\sigma_*^2}\right)^{\frac{1}{2}} \exp\left(-\frac{(\mu-\mu_*)^2}{2\sigma_*^2}\right) \frac{(\beta_\sigma)^{\alpha_\sigma} e^{-\beta_\sigma/\sigma^2}}{\Gamma(\alpha_\sigma)(\sigma^2)^{\alpha_\sigma+1}} \\ &\times \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}} \exp\left(-\frac{\sum_{i=1}^n (y_i - \mu)^2}{2\sigma^2}\right), \end{aligned}$$

where $f(\mu|\mu_*, \sigma_*^2)$ and $\mathrm{IG}(\sigma^2|\alpha_\sigma, \beta_\sigma)$ are two prior distributions.

Example 1 – fit a univariate normal distribution (Cont'd) – simulate μ

- The conditional distribution of μ is

$$
\begin{aligned}
f(\mu|\sigma^2, \mathbf{y}) &\propto \exp\left(-\frac{(\mu-\mu_*)^2}{2\sigma_*^2}\right)\exp\left(-\frac{\sum_{i=1}^n (y_i-\mu)^2}{2\sigma^2}\right) \\
&\propto \exp\left(-\frac{1}{2}\left[\mu^2\left(\frac{1}{\sigma_*^2}+\frac{n}{\sigma^2}\right)-2\mu\left[\frac{\mu_*}{\sigma_*^2}+\frac{\sum_{i=1}^n y_i}{\sigma^2}\right]\right]\right)
\end{aligned}
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- The conditional distribution of μ is then

$$f(\mu|\sigma^2, \mathbf{y}) \sim N\left(\frac{B}{A}, \frac{1}{A}\right)$$

where

$$B = \frac{\mu_*}{\sigma_*^2} + \frac{\sum_{i=1}^n y_i}{\sigma^2}, A = \frac{1}{\sigma_*^2} + \frac{n}{\sigma^2}$$

Example 1 – fit a univariate normal distribution (Cont'd) – simulate σ^2

- The conditional distribution of σ^2 is

$$
\begin{aligned}
f(\sigma^2|\mu, \mathbf{y}) &\propto \frac{(\beta_\sigma)^{\alpha_\sigma} e^{-\beta_\sigma/\sigma^2}}{\Gamma(\alpha_\sigma)(\sigma^2)^{\alpha_\sigma+1}} \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}} \exp\left(-\frac{\sum_{i=1}^n (y_i-\mu)^2}{2\sigma^2}\right) \\
&\propto \frac{e^{-\beta_\sigma/\sigma^2}}{(\sigma^2)^{\alpha_\sigma+1}} \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}} \exp\left(-\frac{\sum_{i=1}^n (y_i-\mu)^2}{2\sigma^2}\right) \\
&\propto \left(\frac{1}{\sigma^2}\right)^{(\alpha_\sigma+\frac{n}{2})+1} \exp\left(-\frac{\beta_\sigma + \frac{1}{2}\sum_{i=1}^n (y_i-\mu)^2}{\sigma^2}\right)
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\end{aligned}
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- The conditional distribution of σ^2 is

$$f(\sigma^2|\mu,\mathbf{y}) \sim \mathrm{IG}(\hat{\alpha}_\sigma, \hat{\beta}_\sigma)$$

where

$$\hat{\alpha}_\sigma = \alpha_\sigma + \frac{n}{2}, \hat{\beta}_\sigma = \beta_\sigma + \frac{1}{2}\sum_{i=1}^n (y_i-\mu)^2$$

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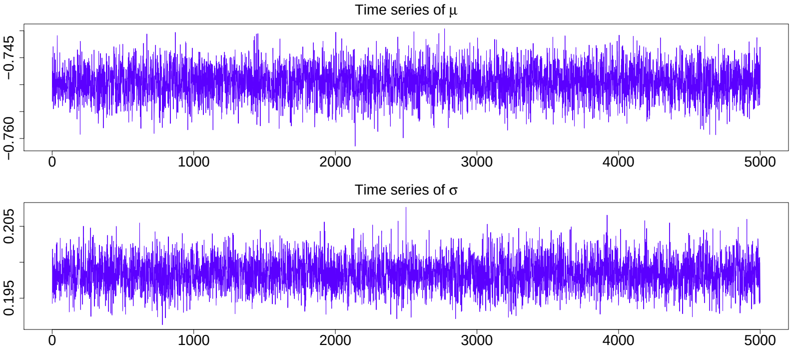

The algorithm was iterated 5000 times to fit a data set artificially generated from $N(-0.75, 0.2^2)$.

Example 1 – fit a univariate normal distribution (Cont'd)

Table: Estimate μ and σ of a normal distribution $N(\mu, \sigma^2)$.

	Case 1			Case 2		
Parameter	True	Average	SD	True	Average	SD
μ	**-0.75**	-0.7495	0.00307	**0.5**	0.48144	0.03935
σ	**0.2**	0.1984	0.00223	**2.5**	2.51564	0.02814

The Gibbs algorithm was iterated 5000 times. After the first 500 iterations were discarded, parameters were estimated by sample means.

Example 2 – fit a mixture of two normal distributions

- Suppose that we have a data set $\mathbf{y} = \{y_i, i = 1, ..., n\}$, which is generated from a mixture of two normal distributions.

$$f(y) = \rho \times f_1(y|\mu_1, \sigma_1^2) + (1 - \rho) \times f_2(y|\mu_2, \sigma_2^2),$$

where $0 < \rho < 1$. The comment functions $f_i(y|\mu_i, \sigma_i^2), i = 1, 2$, are normal pdfs.

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- For the distribution to be identifiable, we assume that $\mu_1 < \mu_2$ and $\sigma_1^2 < \sigma_2^2$.
- The likelihood of the proposed model takes a complicated form.
- To simplify this, we introduce latent variables $\mathbf{z} = (z_1, ..., z_n)$, which are defined below,

$$z_i = \begin{cases} 0, & \text{with probability } \rho, \\ 1, & \text{with probability } 1 - \rho, \end{cases}$$

for $i = 1, ..., n$.

Example 2 – fit a mixture of two normal distributions (Cont'd)

- Conditional on these latent variables, we have

$$y_i|\theta \sim \begin{cases} \mathcal{N}(\mu_1, \sigma_1^2), & \text{if } z_i = 0, \\ \mathcal{N}(\mu_2, \sigma_2^2), & \text{if } z_i = 1. \end{cases}$$

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- The likelihood of the data $\mathbf{y}$ given the model separates into two parts, each of them corresponding to each of the two mixture components,

$$l(\theta|\mathbf{y}, \mathbf{z}) \propto \prod_{i=1, z_i=0}^{n} \frac{\rho}{\sigma_1} \exp\left\{ -\frac{1}{2} \frac{(y_i - \mu_1)^2}{\sigma_1^2} \right\}$$
$$\prod_{i=1, z_i=1}^{n} \frac{(1-\rho)}{\sigma_2} \exp\left\{ -\frac{1}{2} \frac{(y_i - \mu_2)^2}{\sigma_2^2} \right\}$$

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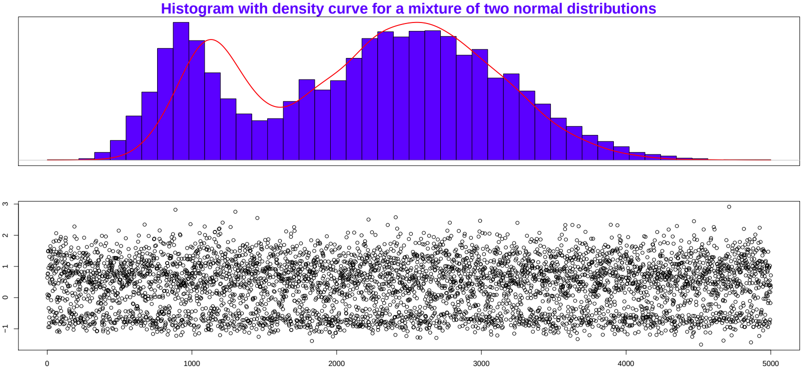

Figure: The data was generated from a mixture of two normal distributions with pdf: $f(y) = 0.25 \times f_1(y|-0.75, 0.2^2) + 0.75 \times f_2(y|0.75, 0.6^2)$

Gibbs algorithm

Table: The Gibbs estimation procedure.

Step 0.	Initialize $\mu_1, \mu_2, \sigma_1, \sigma_2$ and ρ.
Step 1.	Sample $z_i, i = 1, ..., n$.
Step 2.	Sample ρ.
Step 3.	Sample μ_1, σ_1^2.
Step 4.	Sample μ_2, σ_2^2.
Step 5.	Go to **Step 1**.

Example 2 – fit a mixture of two normal distributions (Cont'd)

Step 1. The conditional probability that the observed y_i has been generated by the first mixture component is

$$f(z_i = 0|\mathbf{y}, \theta) = \frac{\frac{\rho}{\sigma_1} \exp\left\{ -\frac{1}{2}\frac{(y_i-\mu_1)^2}{\sigma_1^2} \right\}}{\frac{\rho}{\sigma_1} \exp\left\{ -\frac{1}{2}\frac{(y_i-\mu_1)^2}{\sigma_1^2} \right\} + \frac{(1-\rho)}{\sigma_2} \exp\left\{ -\frac{1}{2}\frac{(y_i-\mu_2)^2}{\sigma_2^2} \right\}}.$$

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- The simulation of the latent variables $z_i, i = 1, ..., n$, can be carried out by simulating Bernoulli distributions.

Example 2 – fit a mixture of two normal distributions (Cont'd)

Step 2. The conditional posterior density of $f(\rho|\theta_{-\rho}, \mathbf{y}, \mathbf{z})$, where $\theta_{-\rho}$ denotes the remaining parameters except ρ, has the following kernel,

$$f(\rho|\mathbf{y}, \theta_{-\rho}, \mathbf{z}) \propto \rho_1^{T_0}(1-\rho)^{T_1},$$

where $T_k = \#\{z_i = k\}, k = 0, 1$, the number of observations assigned to the k-th component.

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where $T_k = \#\{z_i = k\}, k = 0, 1$, the number of observations assigned to the k-th component.

- It is obvious that $\rho \sim \text{Beta}(T_0 + 1, T_1 + 1)$, and the simulation of ρ is easier to carry out.

Example 2 – fit a mixture of two normal distributions (Cont'd)

Step 3. The conditional distribution of μ_1 is

$$
\begin{aligned}
& f(\mu_1|\mathbf{y}, \theta_{-\mu_1}, \mathbf{z}) \\
& \propto \prod_{i=1, z_i=0}^{n} \exp\left\{ -\frac{1}{2} \frac{(y_i - \mu_1)^2}{\sigma_1^2} \right\} \\
& \propto \exp\left\{ -\frac{1}{2} \left[\frac{T_0}{\sigma_1^2} \mu_1^2 - 2 \frac{\mu_1}{\sigma_1^2} \sum_{i=1, z_i=0}^{n} y_i \right] \right\}.
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\end{aligned}
$$

- It is easier to see that $\mu_1 \sim \mathcal{N}(\frac{b_1}{a_1}, \frac{1}{a_1})$, where

$$
a_1 = \frac{T_0}{\sigma_1^2}, \quad b_1 = \frac{1}{\sigma_1^2}\sum_{i=1, z_i=0}^{n} y_i.
$$

Example 2 – fit a mixture of two normal distributions (Cont'd)

Step 4. Given an Inverse Gamma prior distribution $\sigma_1^2 \sim \mathcal{IG}(\alpha, \delta)$, the conditional distribution of σ_1^2 is

$$f(\sigma_1^2|\mathbf{y}, \mathbf{z}, \theta_{\sigma_1^2}) \propto \frac{1}{(\sigma_1^2)^{\alpha+1}} \exp\left\{ -\frac{\delta}{\sigma_1^2} \right\} \prod_{i=1,z_i=0}^{n} \frac{1}{\sigma_1^2} \exp\left\{ -\frac{1}{2} \frac{(y_i - \mu_1)^2}{\sigma_1^2} \right\}$$

$$\propto \frac{1}{(\sigma_1^2)^{\alpha+1+T_0/2}} \exp\left\{ -\frac{\delta + \frac{1}{2}\sum_{i=1,z_i=0}^{n}(y_i - \mu_1)^2}{\sigma_1^2} \right\}.$$

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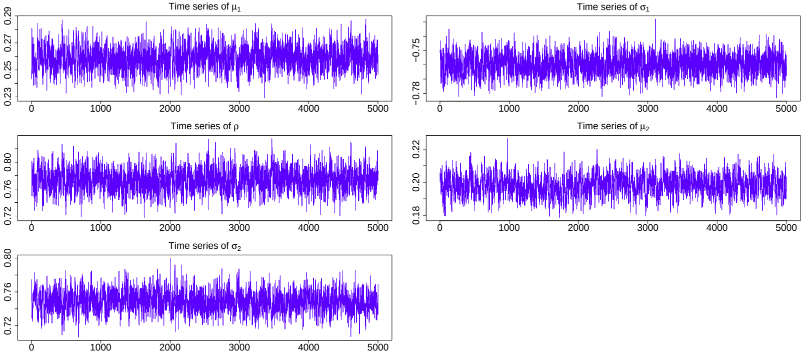
$$\propto \frac{1}{(\sigma_1^2)^{\alpha+1+T_0/2}} \exp\left\{-\frac{\delta+\frac{1}{2}\sum_{i=1,z_i=0}^{n}(y_i-\mu_1)^2}{\sigma_1^2}\right\}.$$

- It is seen that $\sigma_1^2 \sim \mathcal{IG}(c, d)$, where $c = \alpha + T_0/2$ and $d = \delta + \frac{1}{2}\sum_{i=1,z_i=0}^{n}(y_i-\mu_1)^2$, respectively.

Example 2 – fit a mixture of two normal distributions – sampled time series



The algorithm was iterated 5000 times to fit a data set artificially generated from $0.25 \times N(-0.75, 0.2^2) + (1 - 0.25) \times N(0.75, 0.6^2)$.

Example 2 – fit a mixture of two normal distributions (Cont'd)

Table: Fit a mixture distribution: $\rho \times N(\mu_1, \sigma_1^2) + (1-\rho) \times N(\mu_2, \sigma_2^2)$.

	Case 1			Case 2		
Parameter	True	Average	SD	True	Average	SD
μ_1	**-0.75**	-0.7624	0.0075	**0.25**	0.2451	0.0048
σ_1	**0.2**	0.2023	0.0059	**0.2**	0.1943	0.0045
ρ	**0.25**	0.2439	0.0077	**0.5**	0.4913	0.0100
μ_2	**0.75**	0.7241	0.0128	**1.25**	1.2096	0.0271
σ_2	**0.6**	0.6187	0.0098	**1.2**	1.2061	0.0169

The Gibbs algorithm was iterated 5000 times. After the first 500 iterations were discarded, parameters were estimated by sample means.

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- The conditional or posterior distribution of each individual parameter is easier to simulate.
- Gibbs algorithms are usually easier to implement.
- Gibbs methods are widely used in solving time series models.
- Mixture distributions can be imposed to time series and non-time series models.

Thank you!

References

[1] Robert, C. and G., Casella. 2013. Monte Carlo Statistical Methods: Edition 2. London: Chapman and Hall.

[2] Gilks, W. R., Richardson, S., and Spiegelhalter, D. J. 1995. Markov Chain Monte Carlo in Practice. London: Chapman and Hall.

[3] Paul Glasserman. 2013. Monte Carlo Methods in Financial Engineering (Stochastic Modelling and Applied Probability) . (Springer).