Mathematical Problems for Language Model Testing

Problem 1

Solve for x:

$$3x^2 - 7x + 2 = 0$$

Problem 2

Find the derivative of

$$f(x) = x^3 \sin(x) + e^{2x}$$

Problem 3

Evaluate the integral:

$$\int \frac{x^2 + 3x - 1}{x + 2}\, dx$$

Problem 4

Solve the system of equations:

$$\begin{aligned} 2x + 3y - z &= 7 \\ x - y + 2z &= 3 \\ 3x + 2y + z &= 12 \end{aligned}$$

Problem 5

Find all real solutions to:

$$|x - 3| + |x + 1| = 6$$

Problem 6

Prove that for any positive integer n, the sum

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Problem 7

How many ways can you arrange the letters in the word "MATHEMATICS"?

Problem 8

Find the limit:

$$\lim_{x \to 0} \frac{\sin(3x) - 3x}{x^3}$$

Problem 9

Solve for θ in the interval $[0, 2\pi]$:

$$2\sin^2(\theta) + \sin(\theta) - 1 = 0$$

Problem 10

A fair die is rolled three times. What is the probability that the sum of the three rolls is exactly 10?

Problem 11

Find the general solution to the differential equation:

$$\frac{dy}{dx} + 2y = 4x$$

Problem 12

Determine whether the series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n}$$

Problem 13

Find the eigenvalues of the matrix:

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$$

Problem 14

Solve the logarithmic equation:

$$\log_2(x) + \log_2(x-2) = 3$$

Problem 15

Find the maximum value of $f(x, y) = x^2 y$ subject to the constraint $x^2 + y^2 = 4$