CZ2003 Computer Graphics and Visualization

Report for Lab 3

In this report creation of surfaces and solid objects based on parametric functions is described with respect to the attached file as will as other topics which bears relevance to the lab.

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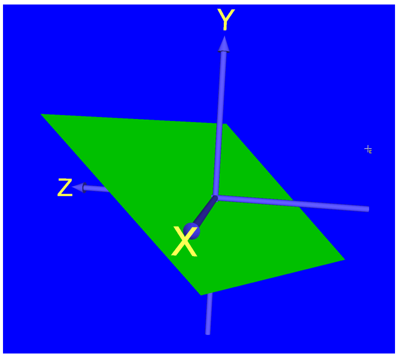
Problem 1: Define surface shapes

Plane

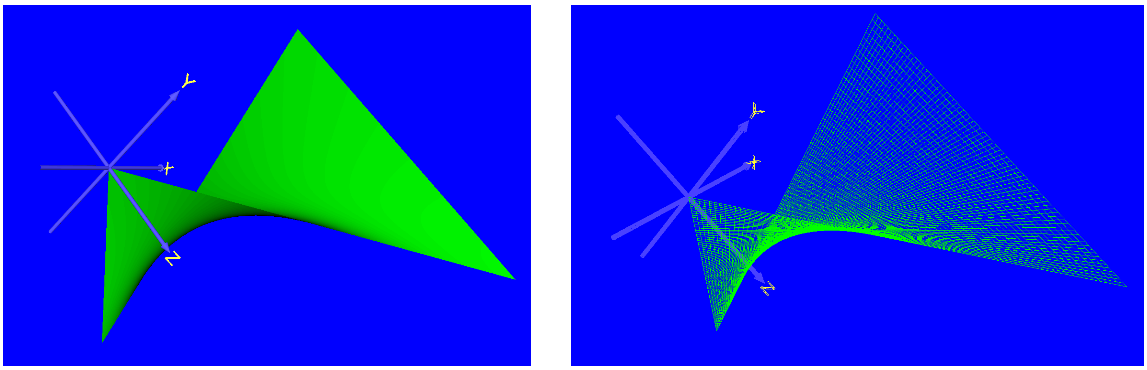
To describe a plane I chose to convert $z = x + y$ into a parametric form: $x = u, \quad y = v, \quad z = u + v$. The graphical representation of the plane is shown bellow.



A plane using the domain $[-0.5, 0.5]$ *on both* u *and* v

Bilinear surface

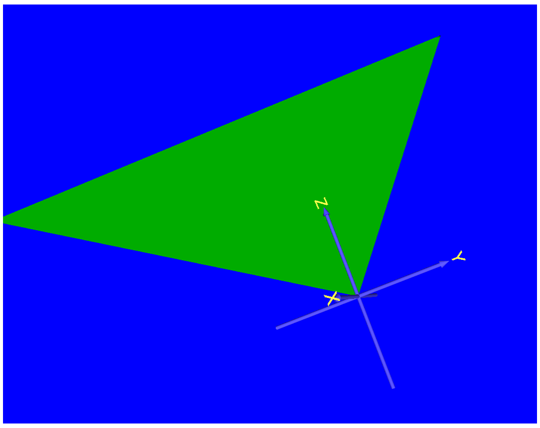
Bilinear surfaces are unique by having a uv term in the parametric function of the shape. This results in potential curvature of the plane in comparison with the linear counterpart. By selecting some random vectors the following bilinear surface is expressed by $x = u + v - uv$, $y = -2u + 2v + 2uv$, $z = u + 3v - 4uv$.

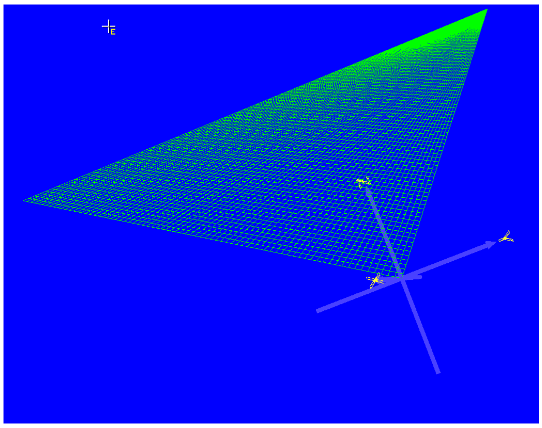


A bilinear surface using domain $[0, 1]$ *on both* u *and* v

Triangle

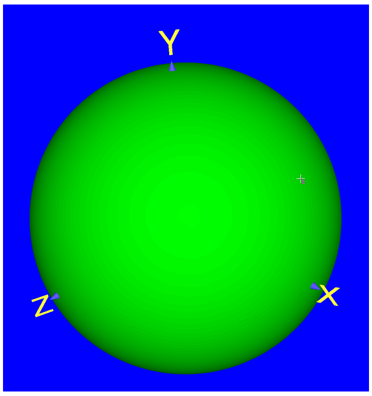
Since ordinary bilinear surfaces are derived from vectors of four points, a triangle can be expressed by allowing two of these points be identical. Thus by using these three points and utilising the formula provided by the lecture slides we can convert these three points into a bilinear surface which reaches the points. For this example the resulting triangle is defined parametrically by $x = -2u - v + uv$, $y = 2u - 5v + 5uv$, $z = 3u + 3v - 3uv$.



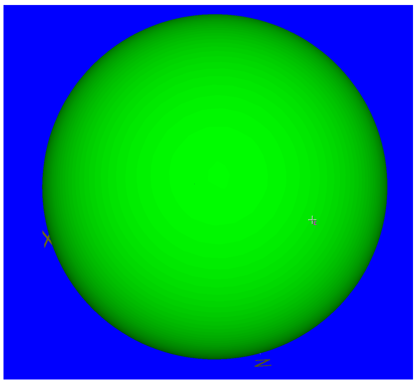


Sphere

The sphere has the equation in parametric form $x = \cos(u)\ \sin(v)$, $y = \sin(u)\ \sin(v)$, $z = \cos(v)$ where $0 \leq u \leq 2\pi$ and $0 \leq v \leq \pi$. To render the sphere without black spots however we need to limit both upper bounds and lower bounds by a small decimal value, 0.00001 for example and preferably remove multiple decimals from the approximation of π. The bounds chosen were thus $[0.000001,\ 6.283]$ for u and $[0.0000001,\ 3.141]$ for v. This removed all black spots present in the original representation. In further representations based trigonometric functions contained in this report this has been utilised as well.



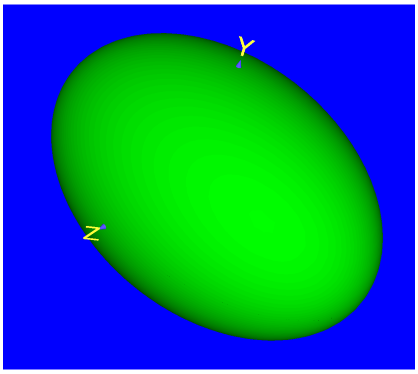
The sphere



Backside of sphere without black spots

Ellipsoid

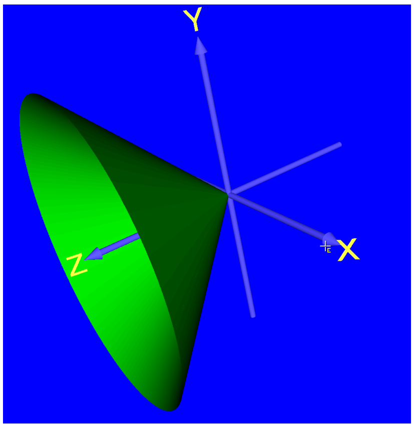
The parametric representation of an ellipsoid can be very similar to a sphere since the only difference is to stretch the sphere out among specific directions. The parametric function us thus $x = a\cos(u)\sin(v)$, $y = b\sin(u)\sin(v)$, $z = c\cos(v)$ with similar bounds as previously to prevent black spots.



An ellipsoid where $a = 1.5,\ b = c = 1.$

Cone

The parametric representation of a cone can be similar to that of a circle except that it has an additional coefficient for each axis. The equation is thus $x = v\cos(u)$, $y = v\sin(u)$, $z = v$ where $0 \le u \le 2\pi$ but for the sake of black spot prevention will be $[0.000001, 6.283]$.

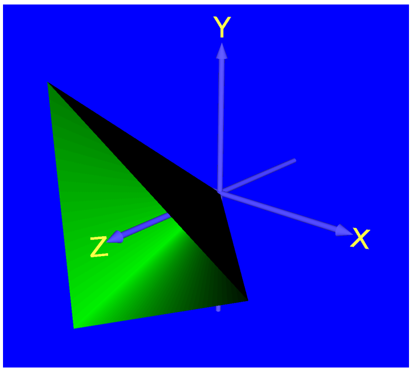


A cone

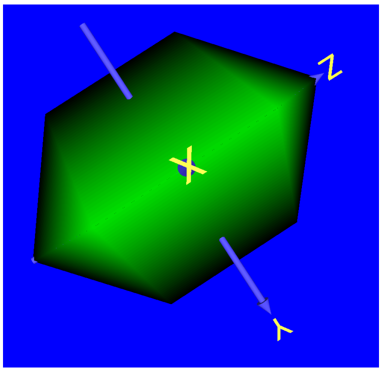
Problem 2: Remark regarding sampling resolution

With regard to the given shapes, it is very easy to set a resolution well above the limit of what is required to express the shape properly. To put it into perspective: the starting resolution given by the source code was 75 for each parameter which means that 75 points along straight lines in the bounding box will be used to represent the function with respect to the domain. The result of this is that if a line is parallel with an axis the required resolution is 1, however if it is curved the resolution have to be sufficiently higher such that the user does not see the sampling effects.

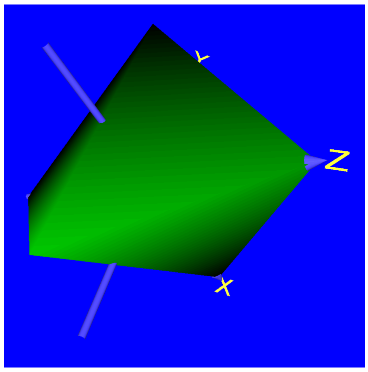
This behaviour can be seen if one for example limits the resolution of the angle on a cone to 3. Instead of showing what we saw in previous section we instead see a pyramid-shaped cone. Similarly we can adjust the resolution to create seemingly indefinite amount of shapes.



A cone with resolution 3 on the angle

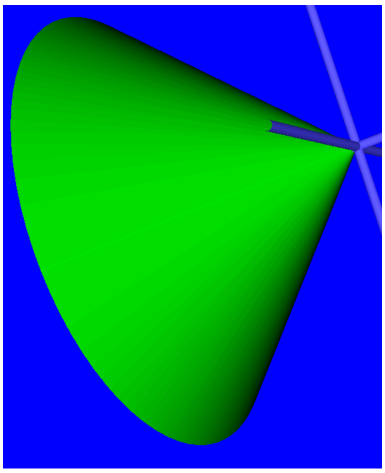


A sphere with limited resolution

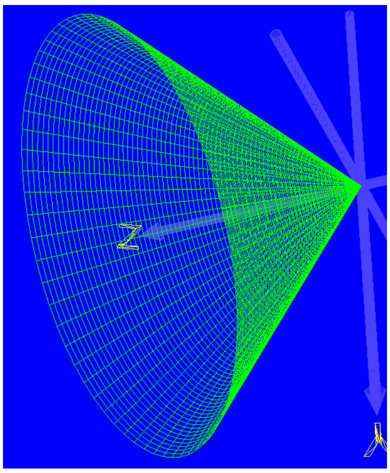


An ellipsoid with limited resolution

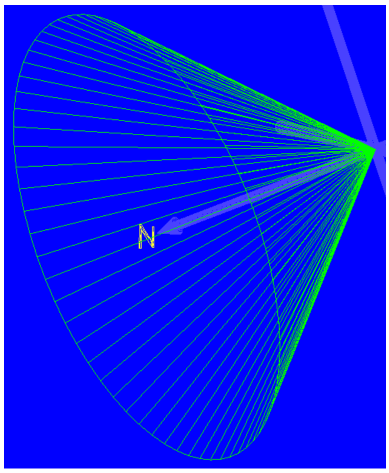
However this behaviour can also be used to limit computation since there is no relevance in setting a higher resolution than what is required for the shape. Using this we may lower the resolution of the straight vector of a cone from 75 to 1. Setting the graphics mode to wireframe allows an observer see the difference.



No visible difference on surface



Wireframe view with resolution 75



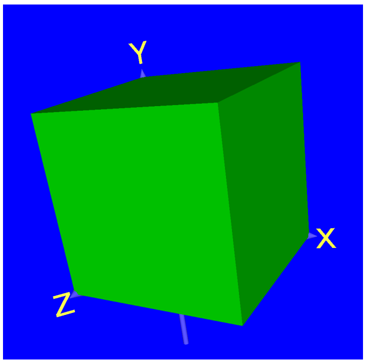
Wireframe view with resolution 1

As we can see there is no apparent difference on the surface despite its resolution.

Problem 3: Define solid shapes

Solid box

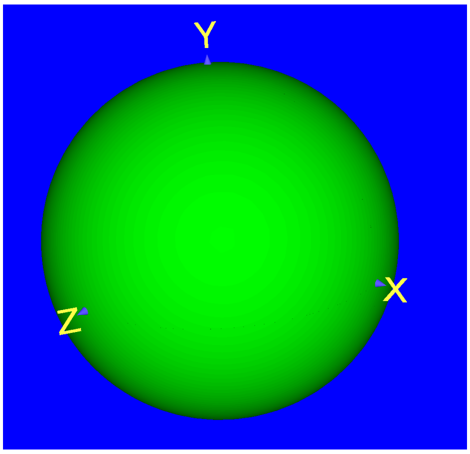
The parametric representation of a solid box is very simple: $x = u, \quad y = v, \quad z = w$. We then let each parameter have similar domains to give the appearance of a box. Since we use 3 parameters we can reach each point in the body and thus it is solid.



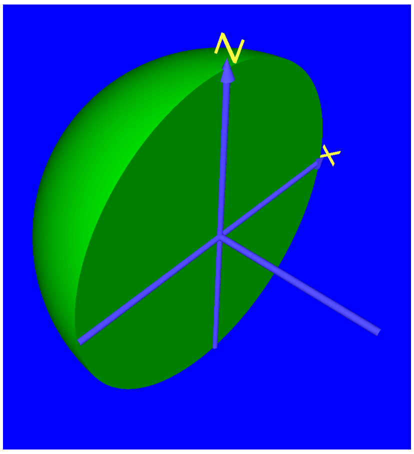
A box

Solid sphere

The parametric representation of a solid sphere almost identical to a hollow sphere: $x = \cos(u)\ \sin(v)$, $y = \sin(u)\ \sin(v), \quad z = w\ \cos(v)$. As mentioned the difference is only an additional parameter added to z. This allows the parametric function touch all point within the sphere and thus the sphere is solid.



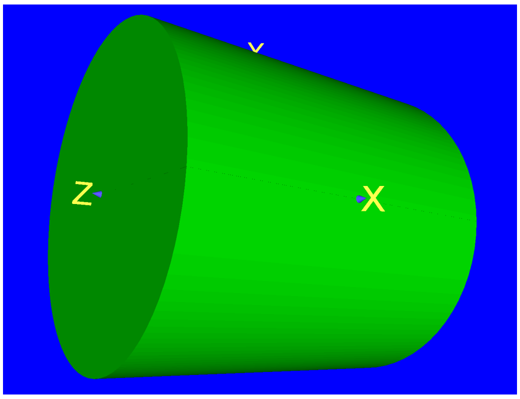
A solid sphere



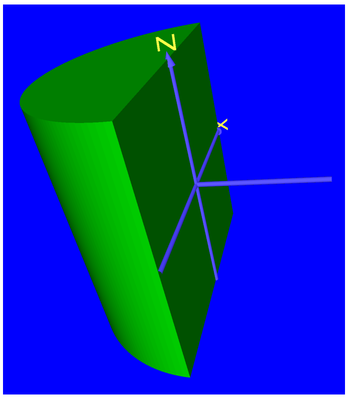
A sphere cut in half to show it solidity

Solid Cylinder

The parametric representation of a cylinder is $x = \cos(u)$, $y = \sin(u)$, $z = w$. However, then it will be hollow. If we want a solid cylinder we need to add a parameter to x and z such that $x = v\ \cos(u)$, $y = v\,\sin(u)$.



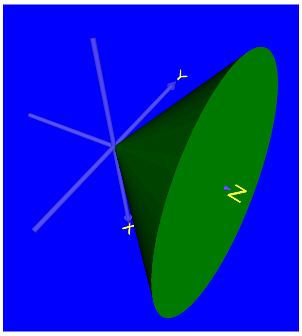
A solid cylinder



A cylinder cut in half to show it solidity

Solid Cone

Similarly as in the case of the cylinder, if we want to solidify the cone we need only to add a additional parameter such that $x = v\ w\ \cos(u)$, $y = v\ w\,\sin(u)$, $z = w$.



A solid cone

Problem 4: Remark on solid objects

As we can see, to solidify objects there seems to be a need to add a third parameter. Mathematically it is logical since their hollow representations are simply curved surfaces. The fact that the surfaces becomes solid when using a third parameter makes sense since the object goes from being 2D object to 3D objects with regard to the amount of vectors needed to express the shape. All 2D objects need 2 vectors. All 3D objects need 3 vectors.