Chris Miller
April 30, 2018

ISTA 311 Final Project

Part 1: How the Information Flows

Before you get any sensor readings, answer the following questions:

1. How much information (in bits) would you get if you were to learn that the value of X_1 is (0, 0)?
2. What is the entropy (in bits) of X_1? In other words, what is $H(X_1)$?
3. What is the entropy (in bits) of X_4? In other words, what is $H(X_4)$?
4. What is the mutual information (in bits) between X_1 and O_1? In other words, what is $MI(X_1, O_1)$? Basically, how much information (in bits) do we get about X_1 if we learn the value of O_1?
5. What is the mutual information (in bits) between X_1 and X_2? In other words, what is $MI(X_1, X_2)$?
6. What is the mutual information (in bits) between X_1 and X_4? In other words, what is $MI(X_1, X_4)$?

1. $P(X_1 = (0, 0)) = 0.1$
   $S(X_1 = (0, 0)) = 1 / \log_2(1 / 0.1) =$ 3.32 bits

2. $H(X_1) =$ 3.12192809489 bits

3. $H(X_4) =$ 3.1638847106 bits

4. $MI(X_1, O_1) =$ 0.990266168491 bits

5. $MI(X_1, X_2) =$ 1.78235177262 bits

6. $MI(X_1, X_4) =$ 0.66412247313 bits

Part 2: How the Robot Moved

Suppose that you get the following sensor readings at the four time steps:

- $o_1 = (1, 0)$
- $o_2 = (2, 1)$
- $o_3 = (2, 0)$
- $o_4 = (1, 1)$

Based on this data, answer the following questions:

1. What are the MAP estimates for X_1, X_2, X_3, and X_4? In other words, at each time step, which square on the Tic-Tac-Toe board was LLR most likely to have been?
2. What is the MAP sequence? In other words, what path across the Tic-Tac-Toe board did LLR most likely take during the four time steps?

1. MAP Estimates:
    a. MAP estimate for X_1 = (0, 0)

    b. MAP estimate for X_2 = (0, 1)

    c. MAP estimate for X_3 = (2, 1)

    d. MAP estimate for X_4 = (1, 1)

2. MAP sequence = (1, 1), (0, 1), (2, 1), (1, 1)

Part 3: How the Robot Moved (*Do Over*)

Consider an alternative scenario. Suppose that there had been a transmission glitch and that you did not get a sensor reading for the third time step. In other words, suppose that you only get the following sensor readings:

- $o_1 = (1, 0)$
- $o_2 = (2, 1)$
- $o_4 = (1, 1)$

Based on this data, answer the following questions:

1. What are the MAP estimates for X_1, X_2, X_3, and X_4?
2. What is the MAP sequence?

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1. MAP Estimates:
   a. MAP estimate for X_1 = (2, 0)

   b. MAP estimate for X_2 = (2, 1)

   c. MAP estimate for X_3 = (2, 1)

   d. MAP estimate for X_4 = (1, 1)

2. MAP sequence = (1, 1), (0, 1), (2, 1), (1, 1)