SIE 431 - Simulation Modeling and Analysis
Fall 2018
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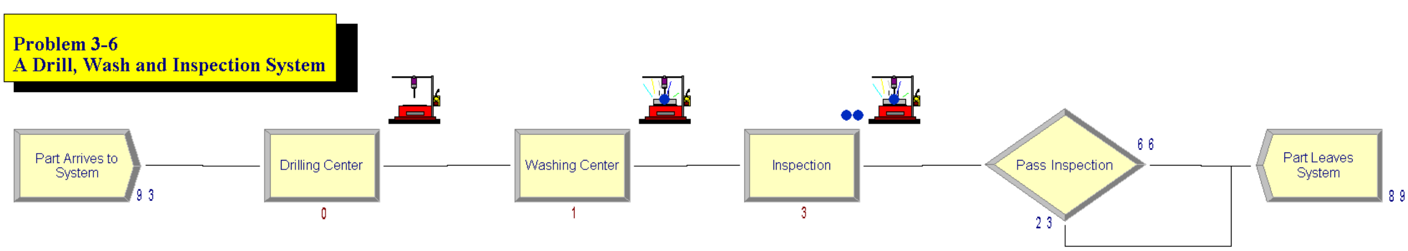
Homework 2

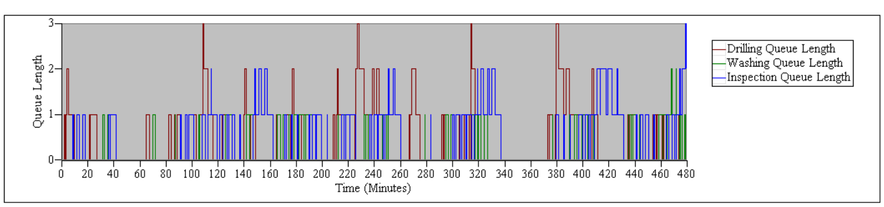
1. Problem 3-6 in the textbook;

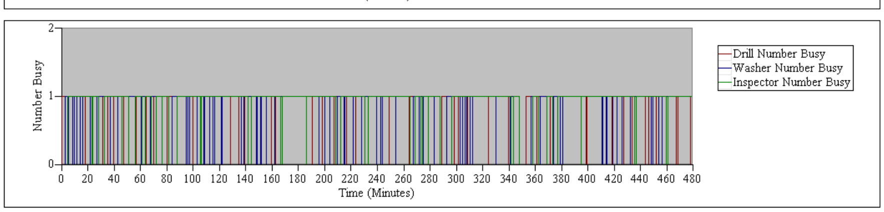
3-6 Modify Model 3-1 with all of the following changes in the same single model (not three separate models). Put a text box in your model with your results on the number of parts that both pass and fail inspection (which you can just read off of your final "frozen" animation), and at the inspection center, the average and maximum time in queue, the average and maximum queue length, and the utilization of the inspector resource. To help format your text box, choose the Courier New font in it; you might want to use Notepad or another plan-text editor (not word-processiong software) to compose your text-box contents, then copy/paste them into the Arena text box.

- Add a second machine to which all parts go immediately after exiting the first machine for a separate kind of processing (for example, the first machine is drilling and the second machine is washing). Processing times at the second machine are independent of those from the first machine but drawn from the same distribution as for the first machine. Gather all the statistics as before, plus the time in queue, queue length, and utilization at the second machine.
- Immediately after the second machine, there's a pass/fail inspection that takes a constant 4.5 minutes to carry out and has a 75% chance of a passing result; queueing is possible for the single inspector, and the queue is first-in, first-out. All parts exit the system regardless of whether they pass the test. Count the number that fail and the number that pass, and gather statistics on the time in queue, queue length, and utilization at the inspection center. (HINT: Try the Decide flowchart module.)
- Include plots to track the queue length and number busy at all three stations; put all queue lengths together in the same plot, and put all numbers busy in the same plot (you'll need to turn on the plot Legends to identify the three curves). Configure them as needed.
- Run the simulation for 480 minutes instead of 20 minutes.

Answer 1:





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Results:

number of parts that pass inspection: 66
number of parts that fail inspection: 23

Inspection Center
time in queue: Average=2.1083, Max=8.4438
queue length: Average=0.4062,  Max=3.0000
utilization of inspector resource: 0.8363

Washing Center
time in queue: Average= 0.8335, Max= 4.2153
queue length: Average= 0.1615, Max= 2.0000
utilization of washer resource: 0.6656
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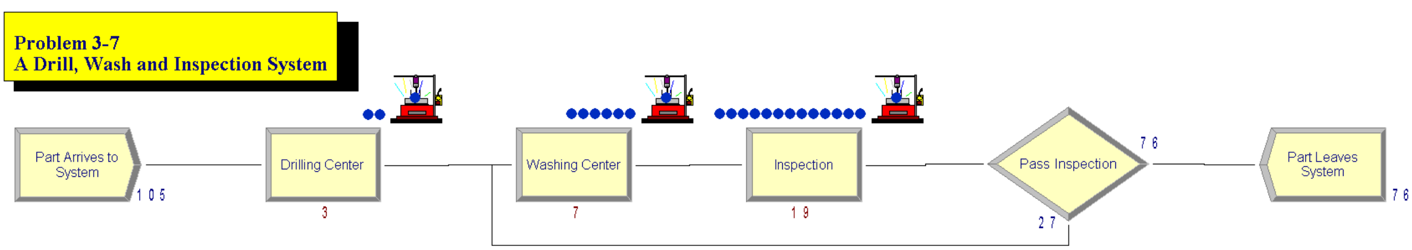
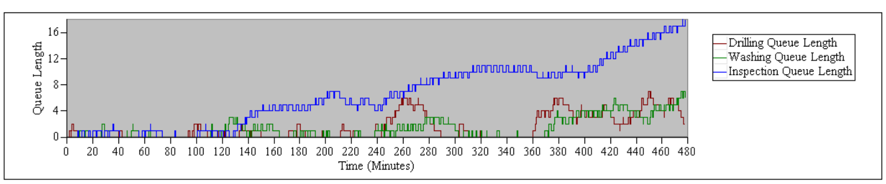
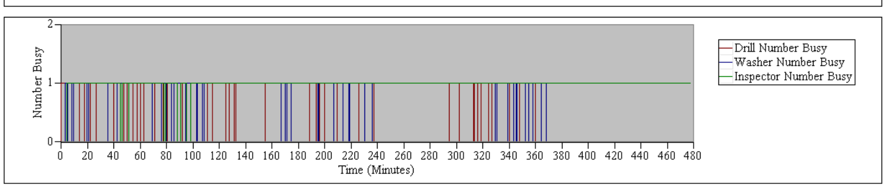
2. Problem 3-7 in the textbook;

3-7 In Exercise 3-6, suppose that parts that fail inspection after being washed are sent back and re-washed, instead of leaving; such re-washed parts must then undergo the same inspection, and have the same probability of failing (as improbable as that might seem). There's no limit on how many times a given part might have to loop back through the washer and inspector. Run this model under the same conditions as Exercise 3-6, and compare the results for the time in queue, queue length, and utilization at the inspection center. Of course, this time there's no need to count the number of parts that fail and pass, since they all eventually pass (or do they?). You may have to allow for more room in some of the queue animations. Put a text box in your model comparing the same inspection-center results with those requested in Exercise 3-6. Do a paper-and-pencil, that is, no-simulation, analysis of what's happening in the long run, that is, infinite run-length, at both the inspection center and the washing center; figure out the "effective" arrival rates to both of those places, and compare with their service rates—are your simulation results consistent with this? Maybe extend your simulation run as far past the 480 minutes as you can.

Answer 2:





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Results:

number of parts that pass inspection: 76
number of parts that fail inspection: 27

3-6 Inspection Center
time in queue: Average=2.1083, Max=8.4438
queue length: Average=0.4062,  Max=3.0000
utilization of inspector resource: 0.8363

3-7 Inspection Center
time in queue: Average=25.2774, Max=56.0850
queue length: Average=6.5748,  Max=18.0000
utilization of inspector resource: 0.9737
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Long-run Analysis:

The rate that parts enter the system (0.2 parts per minute) is greater than the rate that parts leave the system (0.17 parts per minute), therefore as the simulation time reaches infinity, more parts will be in the systems (Drilling Center, Washing Center and Inspection) and they will each reach a steady utilization point. This means that for each system, the effective arrival rate will be approximately equal to the service rate offset by the utilization of the server. The Washing Center follows a triangular distribution with the following average arrival:

$$\mu = \frac{1}{3}(a+b+c) = \frac{1}{3}(1+3+6) = \frac{10}{3} \textit{minute average arrival time}$$

This means that the service rate for the Washing Center is the reciprocal, or 0.3 parts per minute. Similarly, the Inspection has an average arrival every 4.5 minutes, so the service rate for the Inspection should be 1/4.5 or 0.22 parts per minute.

When the simulation runs for a long time (4025 minutes), we can estimate the effective arrival rates using Little's Formula:

$$L = \lambda w, \text{ or}$$

the average # of parts in the system = effective arrival rate * average time in the system

	Average # in Queue	Average # being served	Average # in system (queue + server)	Average time in system	Effective Arrival rate (# / time)	Effective Arrival rate offset by utilization
Inspection	63.0774	0.9969	64.0743	244.38	0.26219126	0.2614
Washing Center	1.8979	0.8503	2.7482	10.7083	0.256642044	0.2951

Figure 1. Data and calculations from long-run simulation

The estimations from the simulation are not completely accurate for steady-state approximations because the simulation time was limited to a maximum of 4025 minutes.

3. Problem 3-14 in the textbook. You need to:

a) solve the problem using the parameters specified in the problem description;

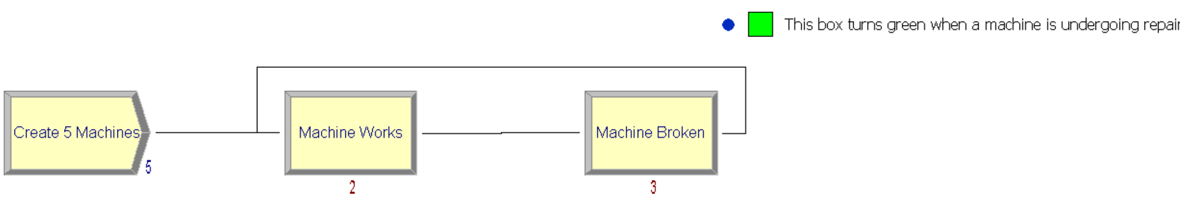
b) solve the problem by changing the parameters into: 1) each machine is up for an average of 8.5 hours (exponentially distributed) and 2) the service time is on average 2.5 hours (exponentially distributed).

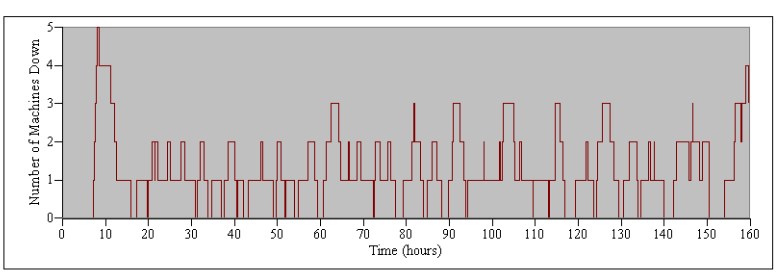
c) solve problem 3-14 with the parameters specified in b) by using the rate diagram.

d) Compare the results from a), b) and c). Explain why they are similar or different.

> **3-14** Five identical machines operate independently in a small shop. Each machine is up (that is, works) for between 7 and 10 hours (uniformly distributed) and then breaks down. There are two repair technicians available, and it takes one technician between 1 and 4 hours (uniformly distributed) to fix a machine; only one technician can be assigned to work on a broken machine even if the other technician is idle. If more than two machines are broken down at a given time, they form a (virtual) FIFO "repair" queue and wait for the first available technician. A technician works on a broken machine until it is fixed, regardless of what else is happening in the system. All uptimes and downtimes are independent of each other. Starting with all machines at the beginning of an "up" time, simulate this for 160 hours and observe the time-average number of machines that are down (in repair or in queue for repair), as well as the utilization of the repair technicians as a group; put your results in a Text box in your model. Animate the machines when they're either undergoing repair or in queue for a repair technician, and plot the total number of machines down (in repair plus in queue) over time. (HINT: Think of the machines as "customers" and the repair technicians as "servers" and note that there are always five machines floating around in the model and they never leave.)

Answer 3a:



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Results:

time-average number of machines that are down: 1.2349
utilization of the repair technicians: 0.5541
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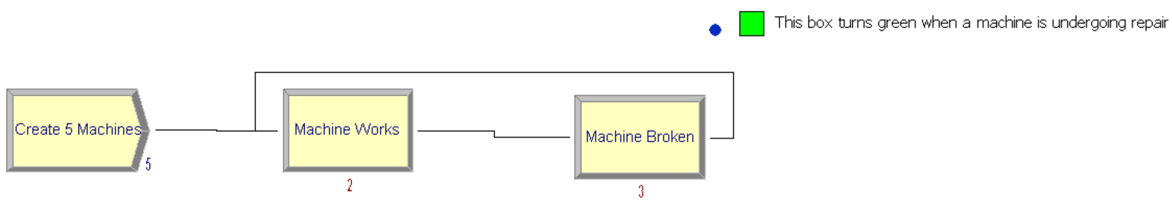
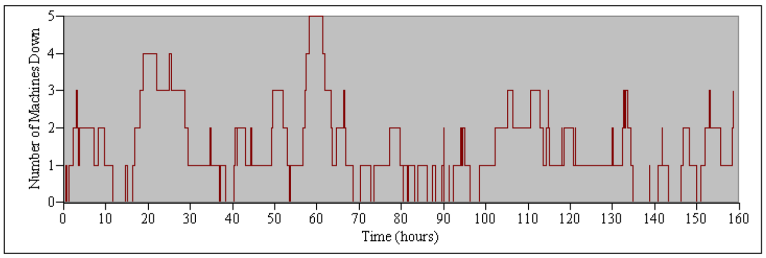
Answer 3b:



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Results:

time-average number of machines that are down: 1.4432
utilization of the repair technicians: 0.6046
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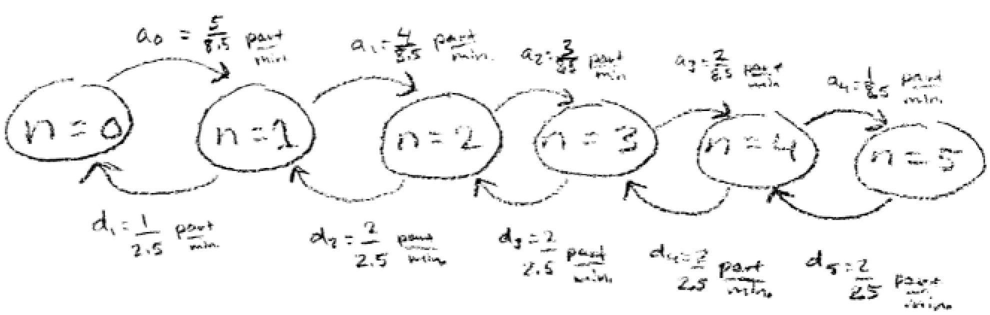
Answer 3c:

Rate Diagram:

n = # of machines down



$$P_0 = \left[1 + \frac{a_0}{d_1} + \frac{a_0 a_1}{d_1 d_2} + \frac{a_0 a_1 a_2}{d_1 d_2 d_3} + \frac{a_0 a_1 a_2 a_3}{d_1 d_2 d_3 d_4} + \frac{a_0 a_1 a_2 a_3 a_4}{d_1 d_2 d_3 d_4 d_5}\right]^{-1} = 0.26$$

$$P_1 = \frac{a_0}{d_1} P_0 = 0.38$$

$$P_2 = \frac{a_0 a_1}{d_1 d_2} P_0 = 0.22$$

$$P_3 = \frac{a_0 a_1 a_2}{d_1 d_2 d_3} P_0 = 0.10$$

$$P_4 = \frac{a_0 a_1 a_2 a_3}{d_1 d_2 d_3 d_4} P_0 = 0.03$$

$$P_5 = \frac{a_0 a_1 a_2 a_3 a_4}{d_1 d_2 d_3 d_4 d_5} P_0 = 0.004$$

Results:

time-average # of machines that are down =

$0P_0 + 1P_1 + 2P_2 + 3P_3 + 4P_4 + 5P_5 = \underline{1{,}268094926}$

Utilization of repair technician =

$\frac{0}{2}P_0 + \frac{1}{2}P_1 + \frac{2}{2}P_2 + \frac{2}{2}P_3 + \frac{2}{2}P_4 + \frac{2}{2}P_5 = \underline{0.5488095697}$

Answer 3d:

The differences in the results are due to the different distributions and simulation times used:

a – Uniform distributions (160 hours)
b – Exponential distributions (160 hours)
c – Exponential distributions (steady-state)

The uniform distribution has a lower variance than the exponential distribution in this problem:

$$\text{Uniform distribution: } \sigma^2 = \frac{1}{12}(b-a)^2 = \frac{1}{12}(10-7)^2 = \frac{9}{12} = 0.75$$

$$\text{Exponential distribution: } \sigma^2 = \mu^2 = 8.5^2 = 72.25$$

This explains the difference between a from b and c. The reason why b is different from c is because problem 3c deals with a steady-state simulation. As the simulation runs forever, the variance from the distributions matters less and less because the simulation statistics will have enough time to converge. This means that problem 3c should have the most reliable results, followed by problem 3a and lastly problem 3b.