SIE 431 Simulation Modeling and Analysis
Fall 2018
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Homework 1 (due on Sept 12th, Wednesday, 10am)

Consider a simple system with a single arrival stream and a single server. The following are data recorded for 10.0 minutes.

Time t	Event
0.0	Customer 1 arrives and enters service
2.3	Customer 1 departs
4.0	Customer 2 arrives and enters service
5.4	Customer 3 arrives and waits
6.1	Customer 4 arrives and waits
7.3	Customer 2 departs
7.3	Customer 3 enters service
8.4	Customer 5 arrives and waits
9.5	Customer 3 departs
10.0	Simulation ends

Question 1:
Based on the above information compute the following (show the intermediate steps):
1. The average waiting time in queue;
2. The average waiting time in system;
3. The average number of customers in queue;
4. The average number of customers in system;
5. The server utilization.

Answer 1:

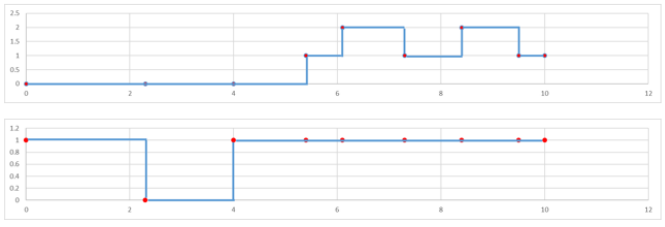
System 	Clock 10.00	$B(t)$ 1	$Q(t)$ 1	Arrival times of custs. in queue (8.4)	Event calendar
Number of completed waiting times in queue 4	Total of waiting times in queue 5.3		Area under $Q(t)$ 6.9		Area under $B(t)$ 8.3
$Q(t)$ graph $B(t)$ graph	 Time (Minutes)				
Interarrival times	~~4.0~~, ~~1.4~~, ~~0.7~~, ~~2.3~~				
Service times	~~2.3~~, ~~3.5~~, ~~2.2~~				

Figure 1. This shows the final event of the simulation.

1. $The\ average\ waiting\ time\ in\ the\ queue = \frac{Total\ of\ times\ in\ queue}{\#\ of\ times\ in\ queue}$

Number of completed waiting times in queue	Total of waiting times in queue
4	5.3

$$\frac{Total\ of\ times\ in\ queue}{\#\ of\ times\ in\ queue} = \frac{5.3\ minutes}{4\ customers} = 1.325\ minutes\ per\ customer$$

2. $The\ average\ waiting\ time\ in\ the\ system = \frac{Total\ of\ times\ in\ system}{\#\ of\ times\ in\ system}$

num completed system = 3			
customer	waiting time in system (departure time - arrival time)		
1	2.3		
2	3.3		
3	4.1		
total	9.7		

$$\frac{Total\ of\ times\ in\ system}{\#\ of\ times\ in\ system} = \frac{9.7\ minutes}{3\ customers} = 3.233\ minutes\ per\ customer$$

3. $The\ average\ \#\ of\ customers\ in\ the\ queue = \frac{1}{10-0}\int_0^{10} Q(t)dt$

$$\frac{1}{10-0}\int_0^{10} Q(t)dt = \frac{1}{10}\big(Area\ under\ Q(t)\big) = \frac{1}{10}(6.9) = 0.69\ customers$$

4. $The\ average\ \#\ of\ customers\ in\ the\ system = \frac{1}{10-0}\int_0^{10} n(t)dt$

$$n(t) = Q(t) + B(t), so\ \frac{1}{10-0}\int_0^{10} n(t)dt = \frac{1}{10-0}\int_0^{10} Q(t)dt + \frac{1}{10-0}\int_0^{10} B(t)dt$$

$$= \frac{1}{10-0}\int_0^{10} Q(t)dt + \frac{1}{10-0}\int_0^{10} B(t)dt = \frac{1}{10}\big(Area\ under\ Q(t)\big) + \frac{1}{10}\big(Area\ under\ B(t)\big)$$

$$= \frac{1}{10}(6.9 + 8.3) = \frac{15.2}{10} = 1.52\ customers$$

5. $server\ utilization = \frac{Area\ under\ B(t)}{Final\ Clock\ Value} = \frac{8.3}{10} = 0.83 = 83\%\ utilization$

Question 2:
Make a plot for n(t) vs. t, where n(t) is the number of customers in the system at t.

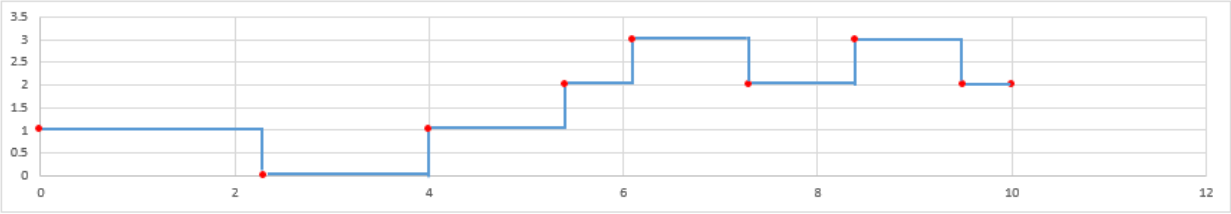
Answer 2:

$$n(t) = Q(t) + B(t)$$

# of customers in the system = # of customers in the queue + # of customers being served

part	time	event	Q(t)	B(t)	n(t)
1	0	arrives	0	1	1
1	2.3	dep	0	0	0
2	4	arr	0	1	1
3	5.4	arr	1	1	2
4	6.1	arr	2	1	3
2	7.3	dep	1	1	2
5	8.4	arr	2	1	3
3	9.5	dep	1	1	2
_	10	End	1	1	2

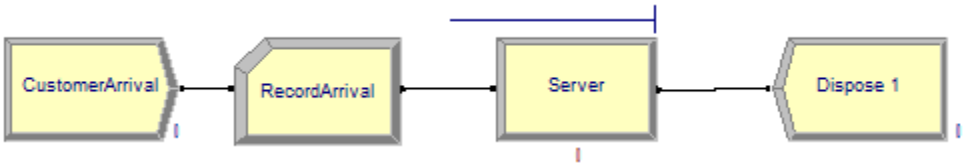
N(t) vs t



Question 3:
Create an ARENA model for the above problem. Suppose interarrival times are uniformly distributed between 1 and 5 minutes and service times are uniformly distributed between 2 and 3 minutes. Run the simulation model for 100 minutes and provide a summary for the following:

1. The average waiting time in queue;
2. The average waiting time in system;
3. The average number of customers in queue;
4. The average number of customers in system;
5. The server utilization;
6. Please upload this ARENA .doe file in Dropbox.

Answer 3:



1. The average waiting time in the queue = 0.2553 minutes
2. The average waiting time in the system = 2.8016 minutes
3. The average # of customers in the queue = 0.07915317 customers
4. The average # of customers in the system =

   avg. # queue + avg. # served = 0.7894 + 0.07915317 = 0.86855317 customers
5. The server utilization = 0.7894 = 78.94% utilized
6. ARENA file name = 431 hwk1.doe