Howard University

Graduate Student Seminar

April 11, 2025

Tutorial on spiking neural networks

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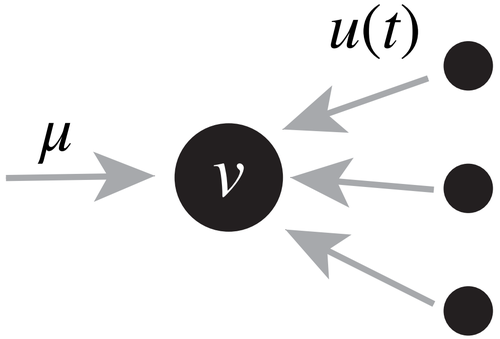
Model of a spiking neuron

Leaky integrate-and-fire neuron model



v: voltage at cell body

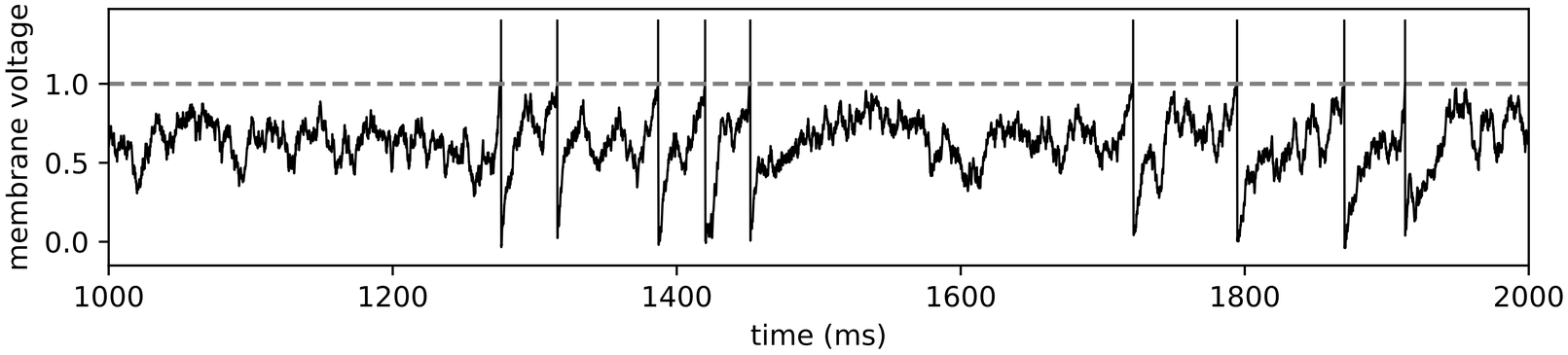
μ: constant input

u: inputs from other neurons

Leaky integrate-and-fire neuron

$$\tau_m \frac{dv}{dt} = -v + \mu + u$$

cross threshold	If $v(t) = v_{thresh}$
spike	$v(t) = \delta(t)$
reset	$v(t) = v_{reset}$

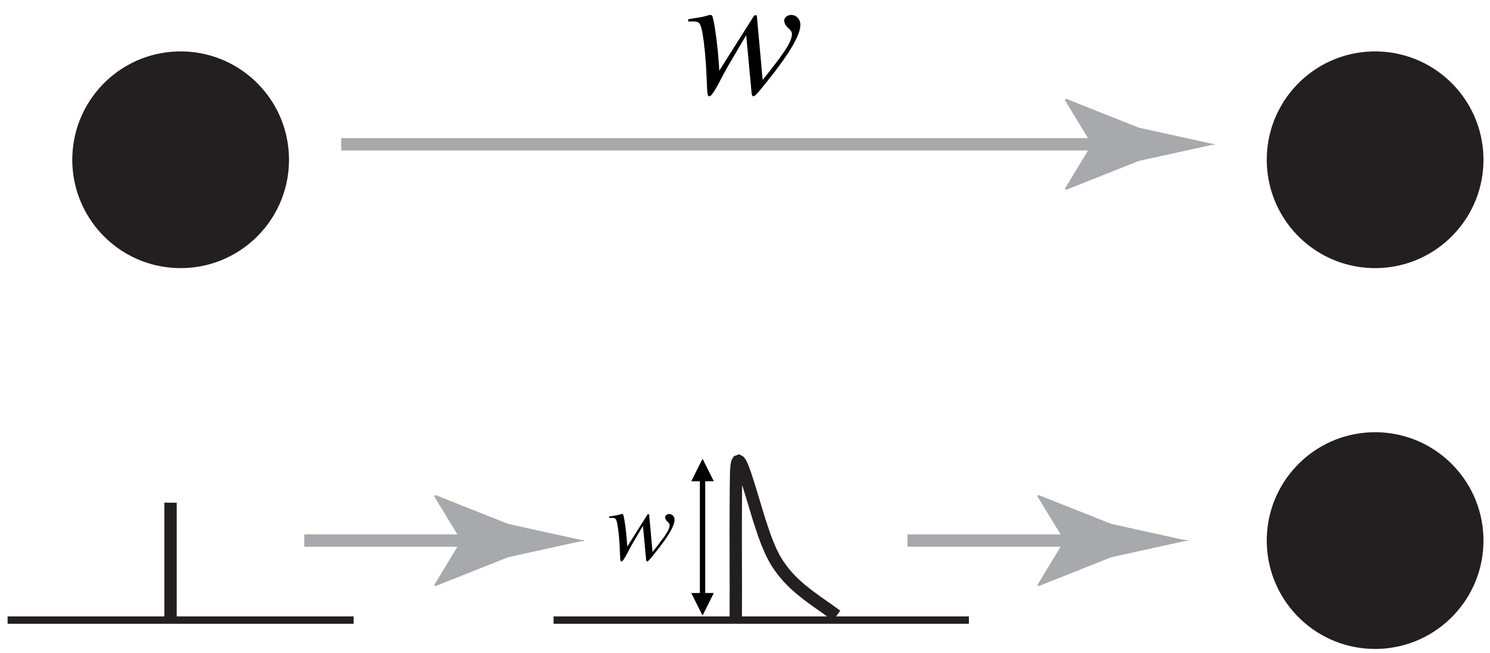


$\mu = 0.7$

$u \sim \mathcal{N}(0, \sigma^2)$

Network of two neurons

Propagation of spikes



$\delta(t)$

spike

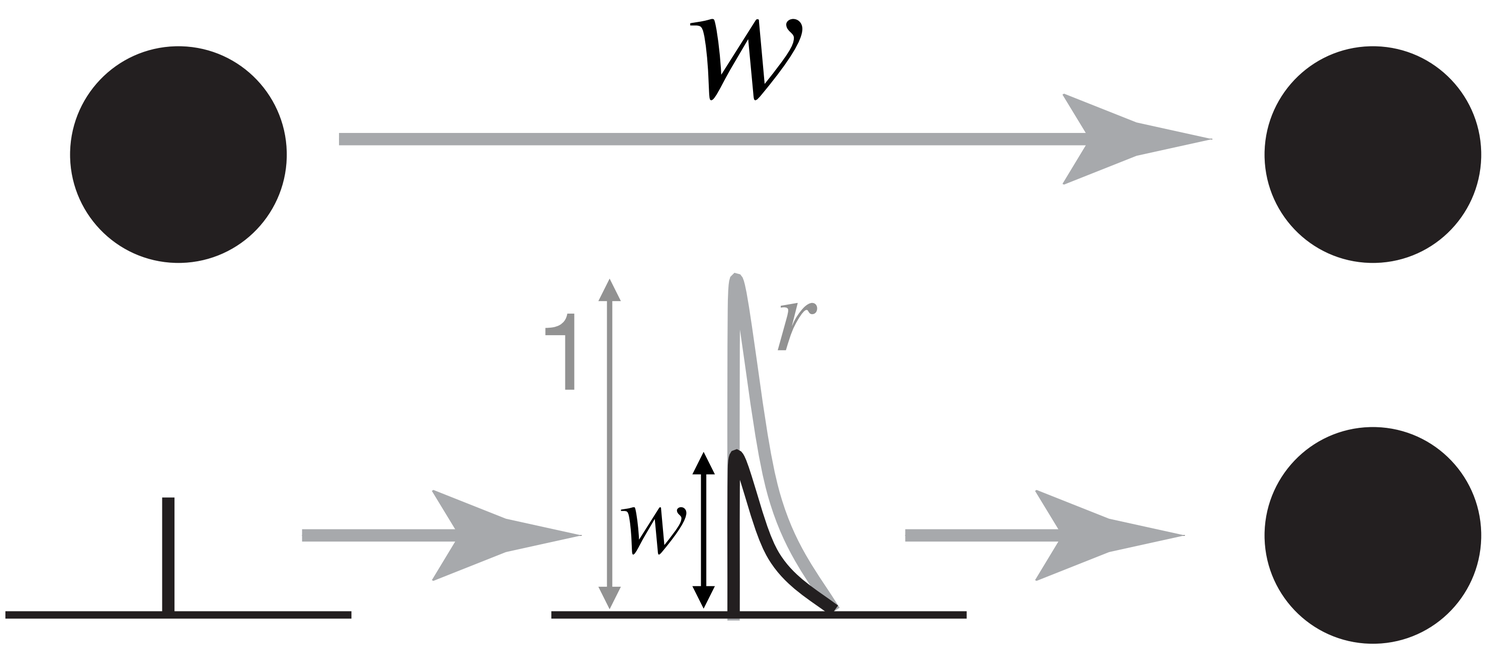
$$\tau_s \frac{du}{dt} = -u + w\delta(t)$$

synaptic filtering

$$\tau_m \frac{dv}{dt} = -v + u + \mu$$

post-synaptic neuron voltage

Express u in terms of w



$\delta(t)$

spike

$$\tau_s \frac{dr}{dt} = -r + \delta(t)$$

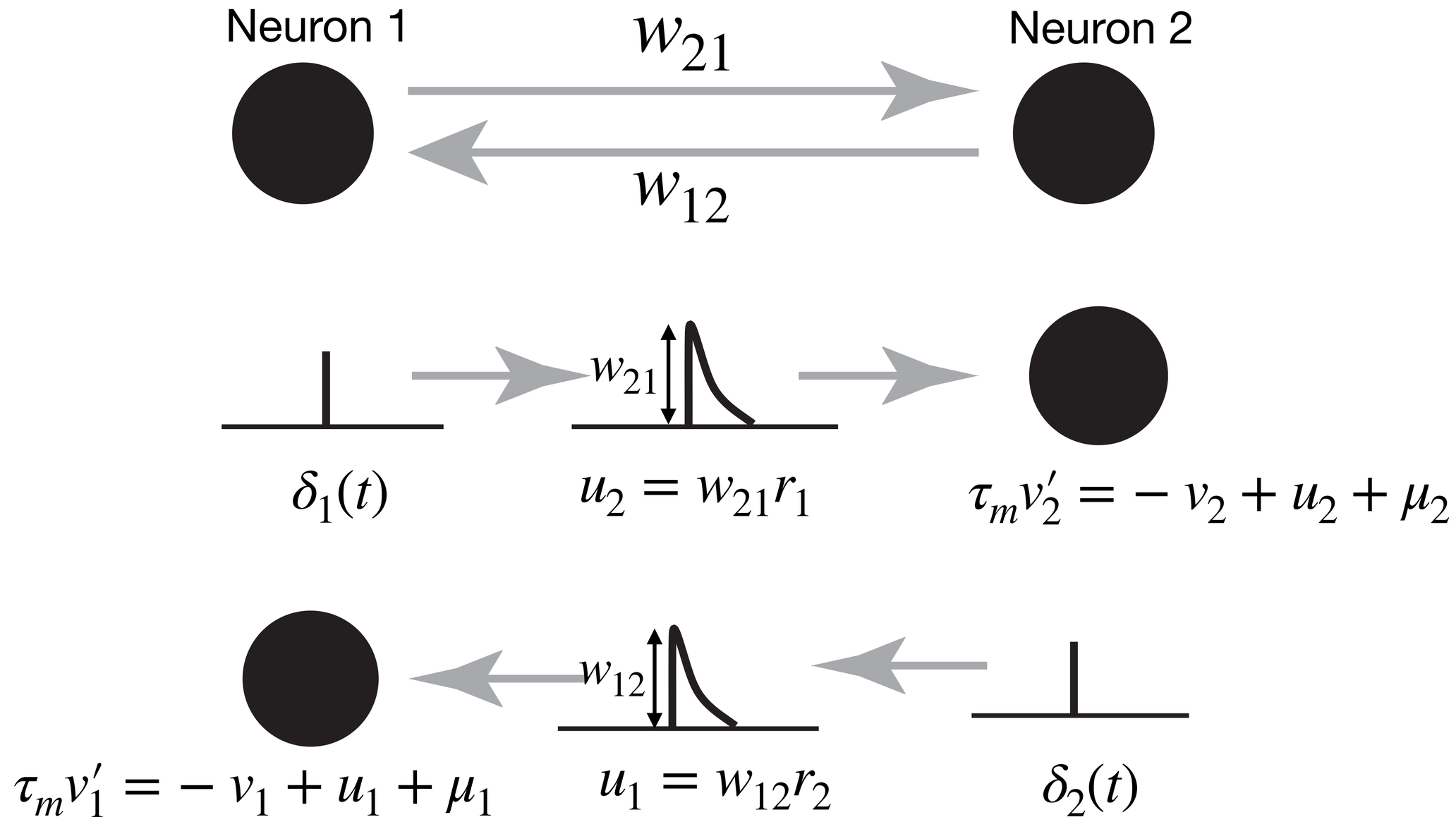
$$u = wr$$

This expression of u will be used for network training

$$\tau_m \frac{dv}{dt} = -v + u + \mu$$

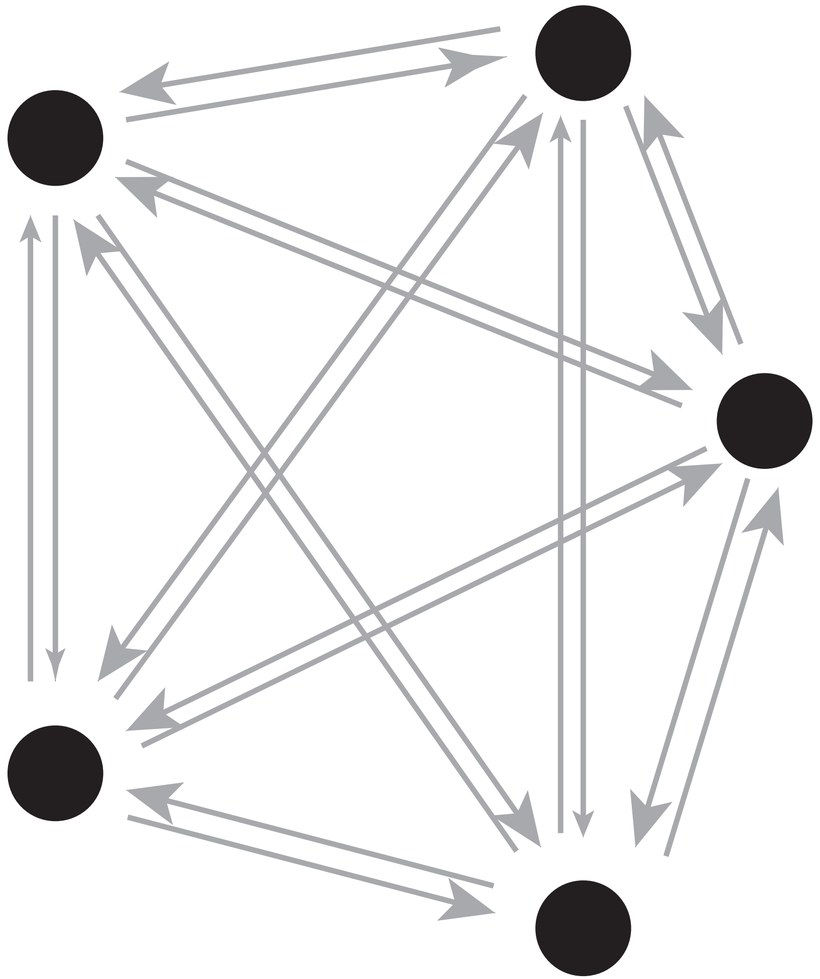
post-synaptic neuron voltage

Two interacting neurons

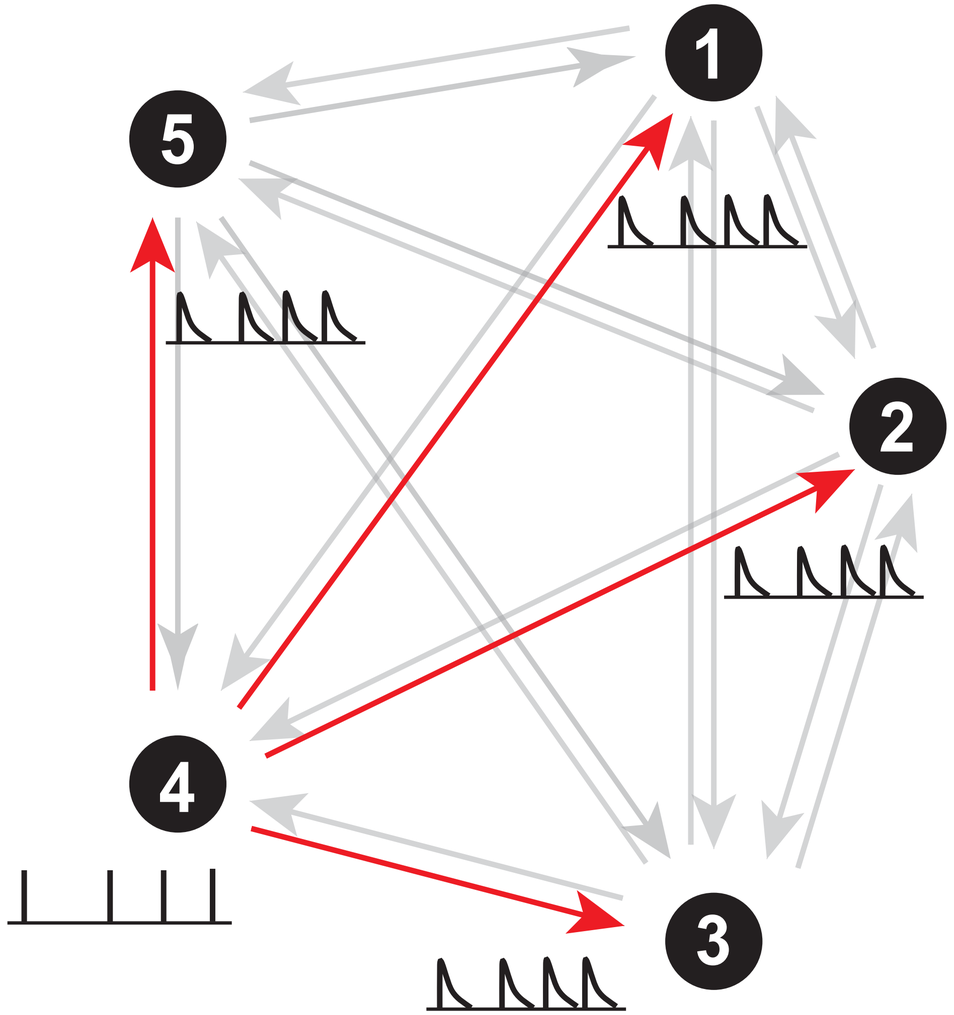


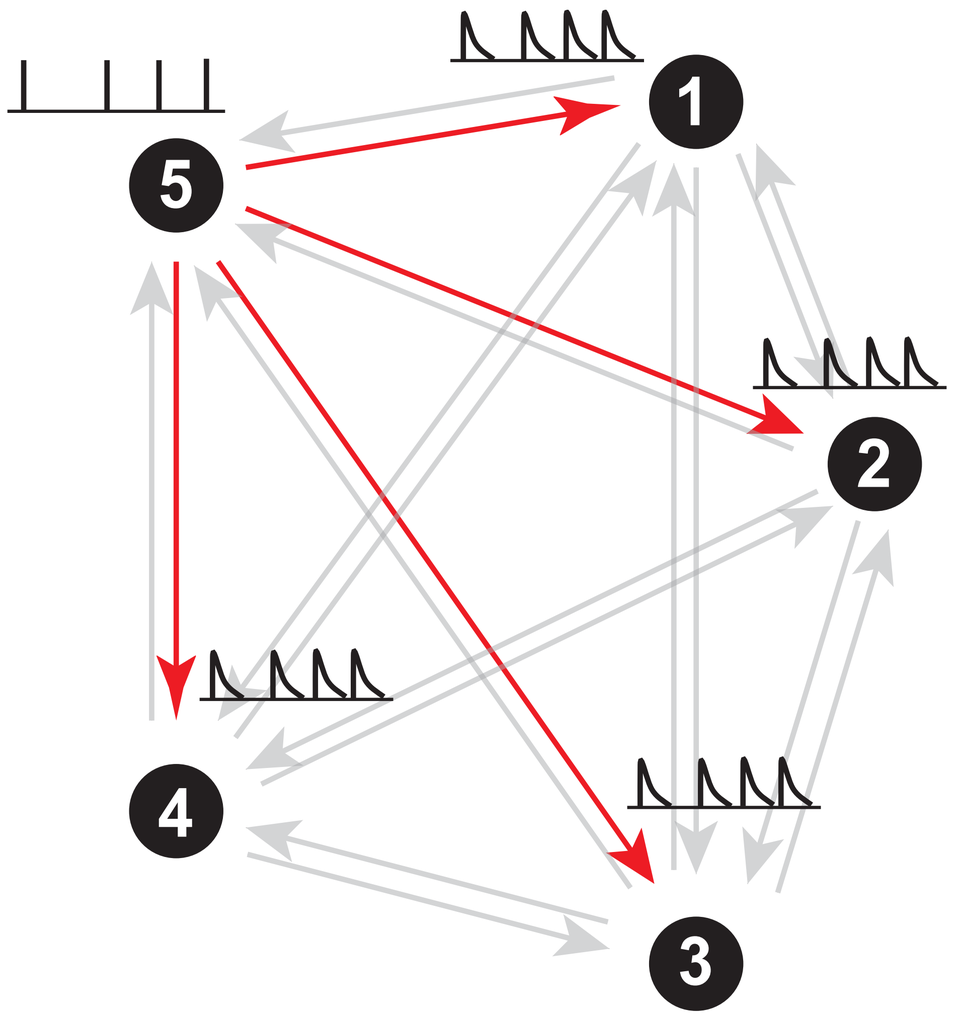
Network of N neurons

All-to-all connectivity

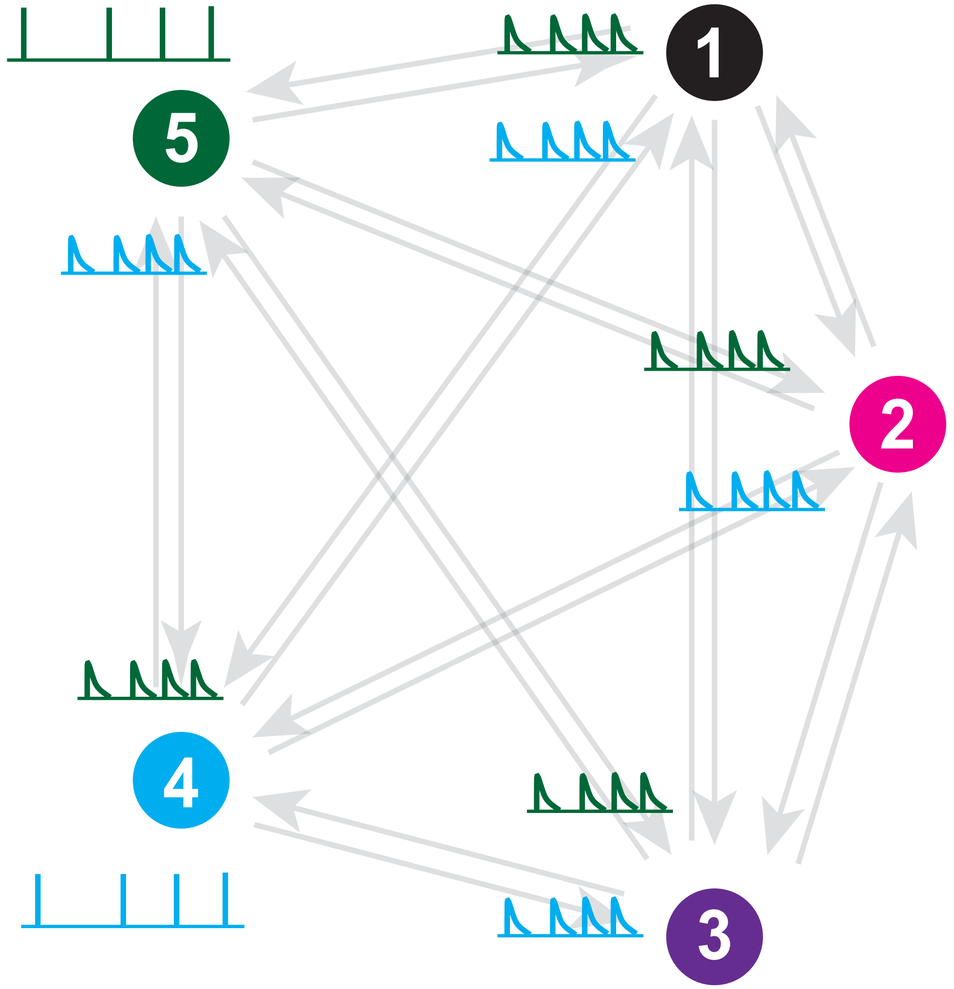


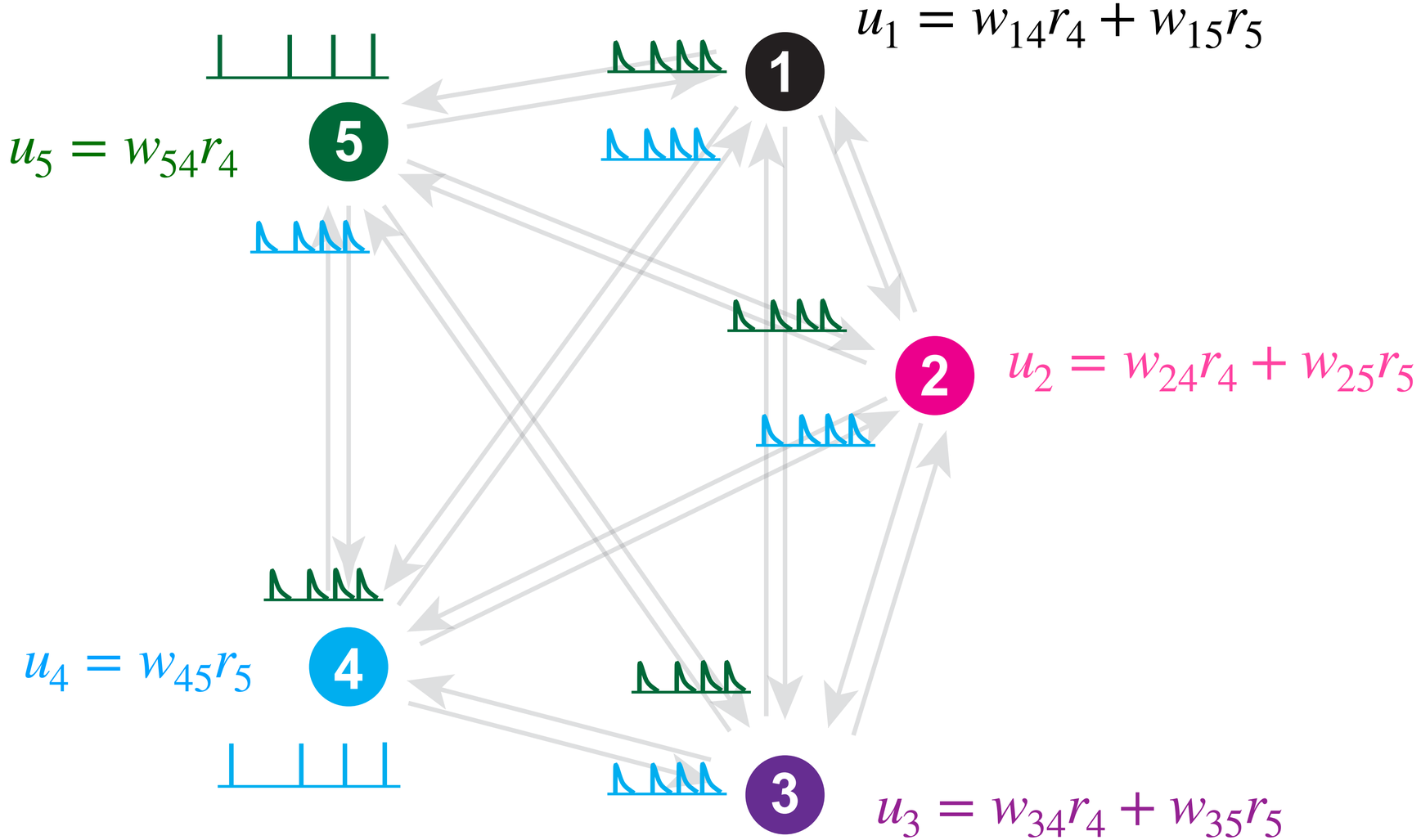
Spikes propagate to post synaptic neurons



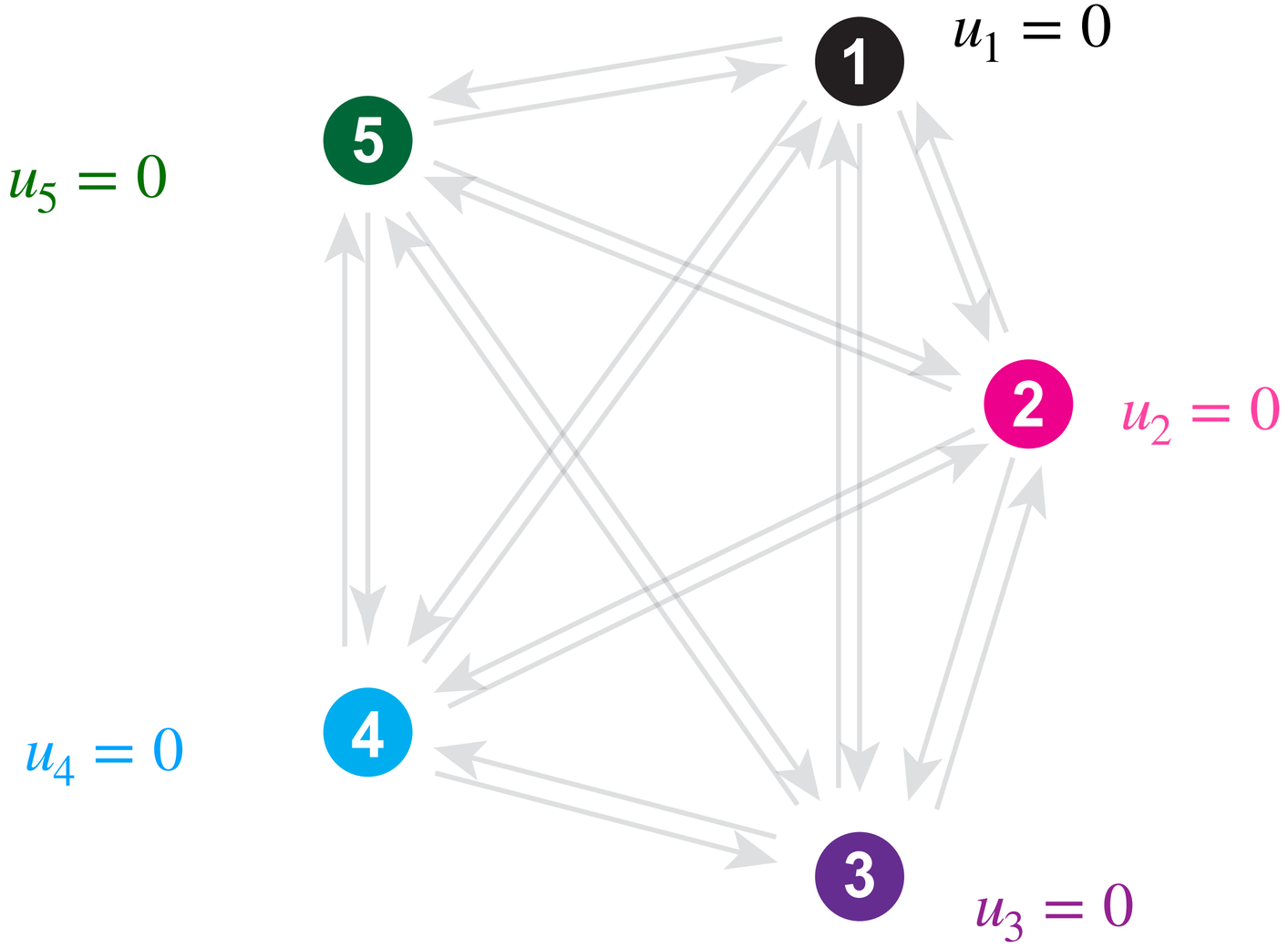

1
2
3
4
5

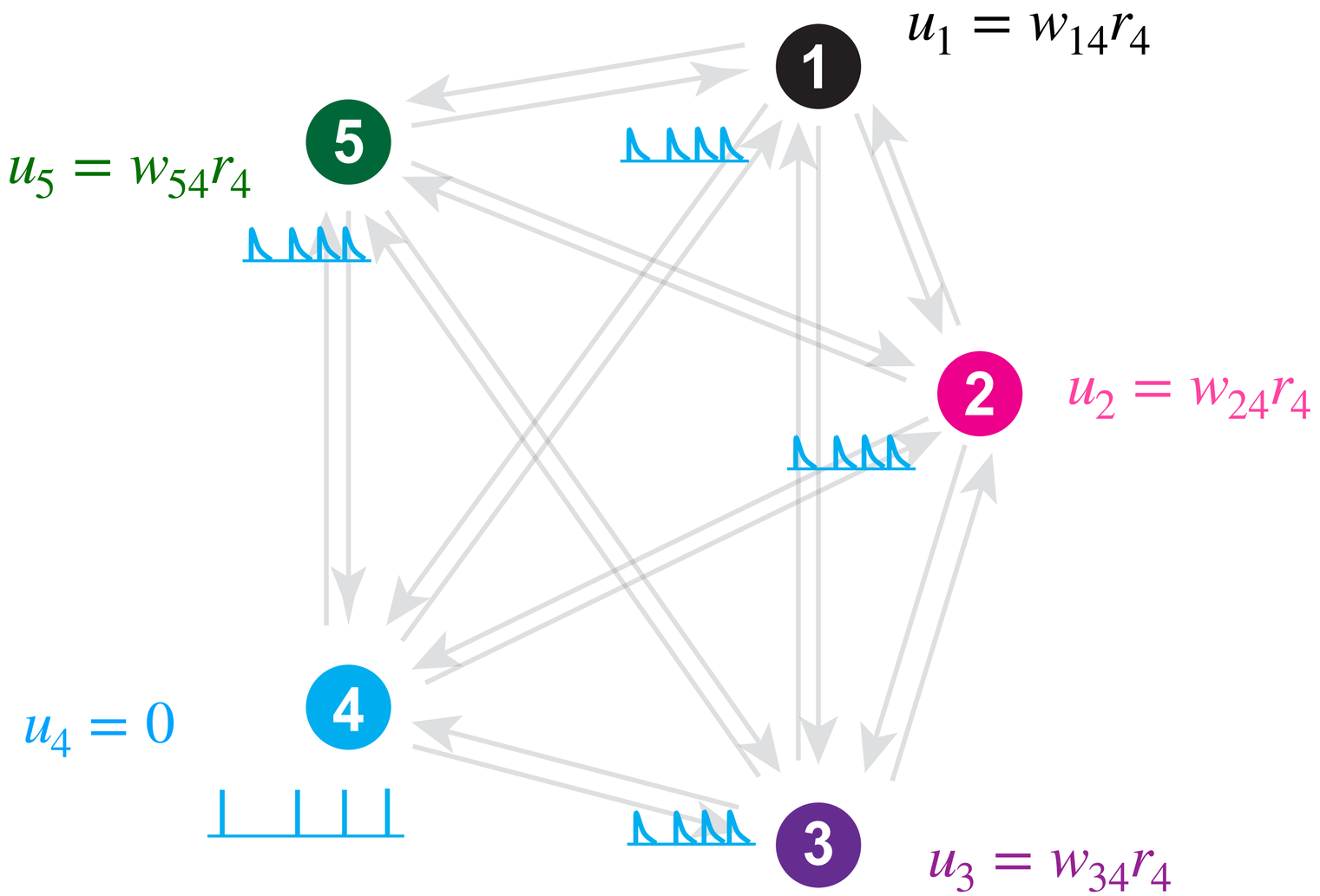
Each neuron adds up the synaptic inputs it received from neurons that spiked

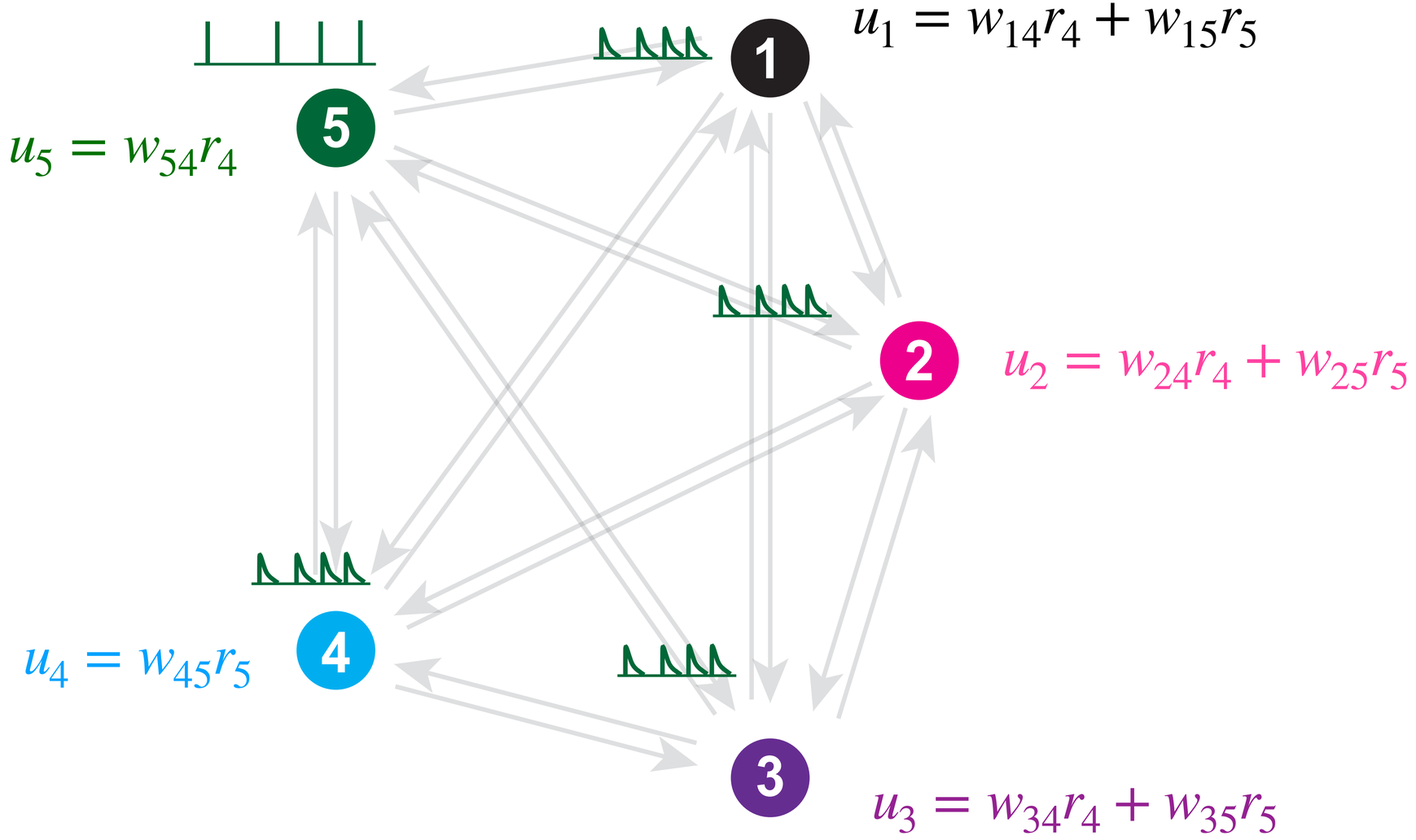



$u_1 = w_{14}r_4 + w_{15}r_5$
1
$u_5 = w_{54}r_4$
5
2
$u_2 = w_{24}r_4 + w_{25}r_5$
$u_4 = w_{45}r_5$
4
3
$u_3 = w_{34}r_4 + w_{35}r_5$

Implementation of spike propagation




1
$u_1 = w_{14} r_4$
5
$u_5 = w_{54} r_4$
2
$u_2 = w_{24} r_4$
4
$u_4 = 0$
3
$u_3 = w_{34} r_4$


1
$u_1 = w_{14}r_4 + w_{15}r_5$
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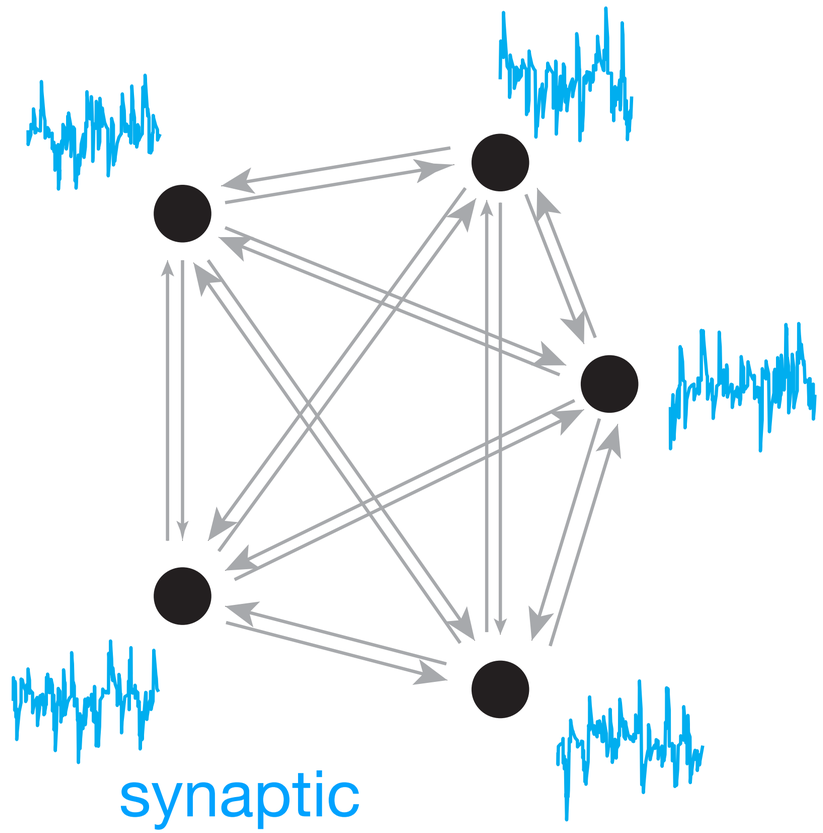
$u_2 = w_{24}r_4 + w_{25}r_5$
3
$u_3 = w_{34}r_4 + w_{35}r_5$
4
$u_4 = w_{45}r_5$
5
$u_5 = w_{54}r_4$

Learn to generate target activity patterns

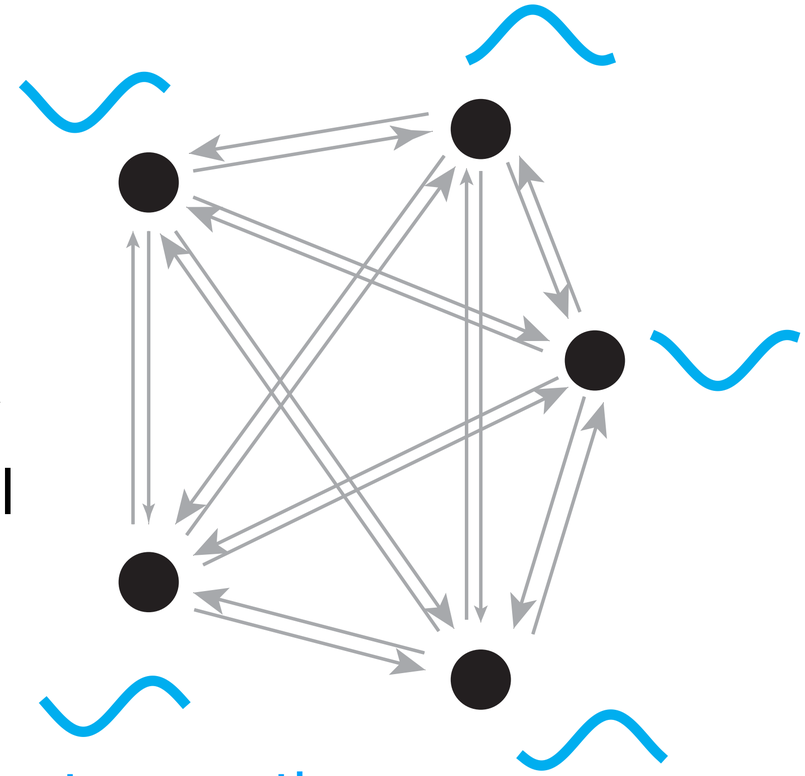
Before learning



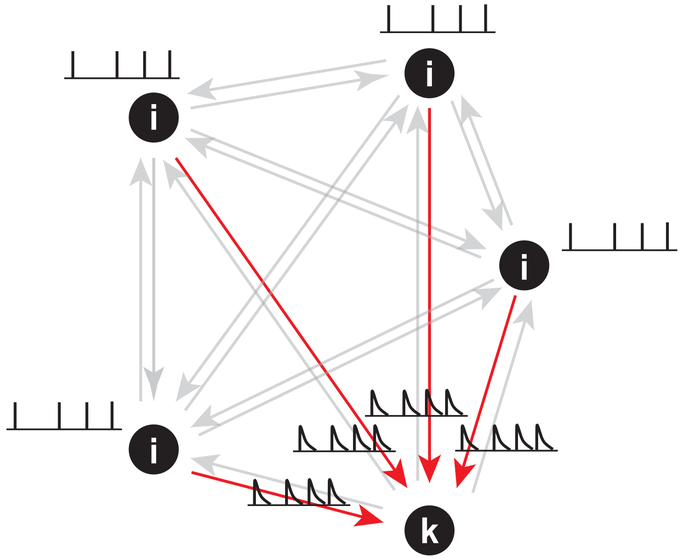
synaptic
activity (u)

external
cue

After learning



target synaptic
activity pattern



Minimize the following cost function by optimizing $\mathbf{w}$:

$$C = \frac{1}{2}\sum_{t=1}^{T}[u_t - f_t]^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

synaptic drive *u* should follow the target pattern *f*

Express u in terms of $\mathbf{w}$ and $\mathbf{r}$.

$$C = \frac{1}{2}\sum_{t=1}^{T}[\mathbf{w}\cdot\mathbf{r}_t - f_t]^2 + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

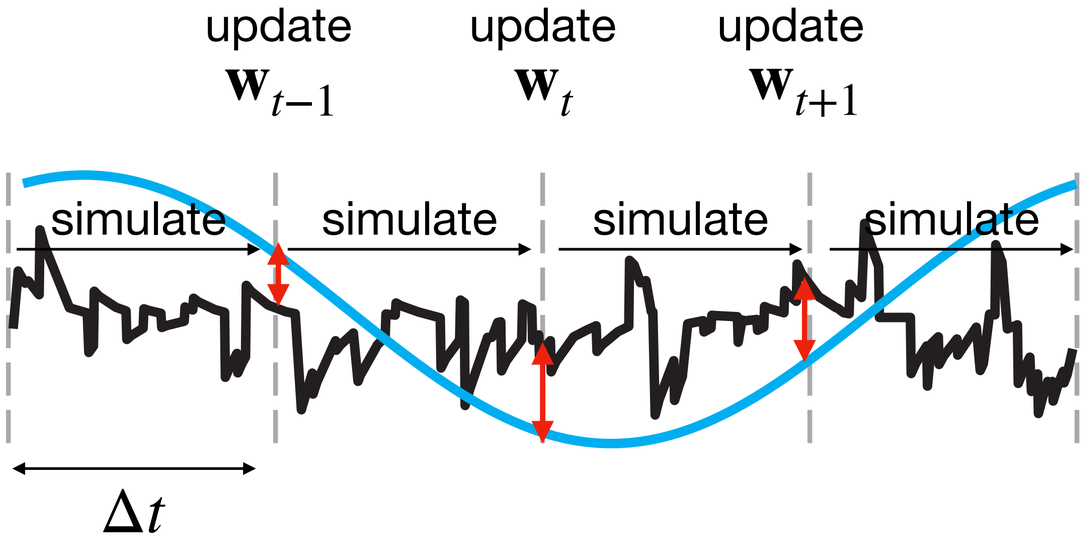
Synaptic drive to neuron k

$$u_k = \sum_{i=1, i\neq k}^{N} w_{ki} r_i$$

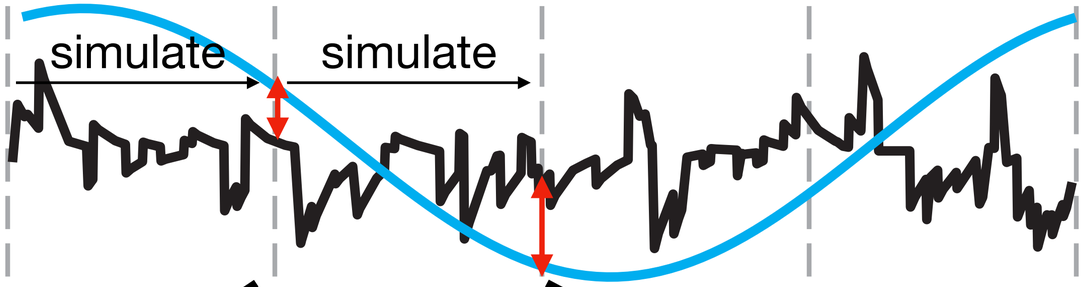
Simplify the notation

$$u = \sum_{i=1}^{N-1} w_i r_i = \mathbf{w}\cdot\mathbf{r}$$

Derive RLS (recursive least squares) algorithm that optimizes $\mathbf{w}$ every Δt (=20ms) to reduce C.



update $\mathbf{w}_{n-1}$ update $\mathbf{w}_n$



$$C(\mathbf{w}_{n-1}) = \frac{1}{2}\sum_{t=1}^{n-1}(\mathbf{w}_{n-1}\cdot\mathbf{r}_t - f_t)^2 + \frac{\lambda}{2}\|\mathbf{w}_{n-1}\|^2$$

$$C(\mathbf{w}_n) = \sum_{t=1}^{n}(\mathbf{w}_n\cdot\mathbf{r}_t - f_t)^2 + \lambda\|\mathbf{w}_n\|^2$$

$$\nabla_{\mathbf{w}_{n-1}} C(\mathbf{w}_{n-1}) = \sum_{t=1}^{n-1}(\mathbf{w}_{n-1}\cdot\mathbf{r}_t - f_t)\mathbf{r}_t + \lambda\mathbf{w}_{n-1} = \mathbf{0}$$

$$\nabla_{\mathbf{w}_n} C(\mathbf{w}_n) = \sum_{t=1}^{n}(\mathbf{w}_n\cdot\mathbf{r}_t - f_t)\mathbf{r}_t + \lambda\mathbf{w}_n = \mathbf{0}$$

subtract two eqn
to get an update rule

$$\mathbf{w}_n = \mathbf{w}_{n-1} - e_n P_n \mathbf{r}_n$$

$$e_n = \mathbf{w}_{n-1}\cdot\mathbf{r}_n - f_n$$

$$P_n = \left(\sum_{t=1}^{n}\mathbf{r}_t\mathbf{r}_t' + \lambda I\right)^{-1}$$

Update P_n recursively:

$$P_n = P_{n-1} - \frac{P_{n-1}\mathbf{r}_n\mathbf{r}_n' P_{n-1}}{1 + r_n' P_{n-1} r_n}$$