Double Mass Spring-Damper Adaptive Control

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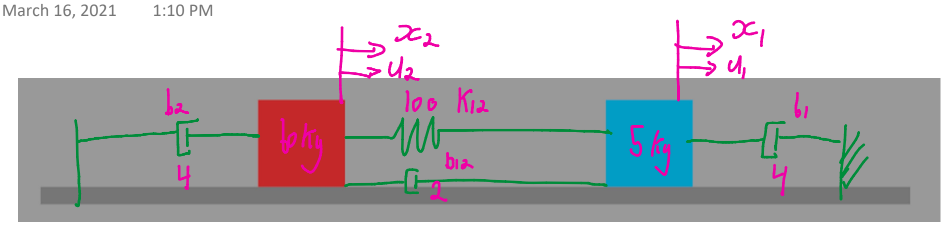

$$60\ddot{x}_2 = u_2 - 4\dot{x}_2 - (\dot{x}_2 - \dot{x}_1)2 - 100(x_2 - x_1)$$

$$\ddot{x}_2 = \frac{1}{60}\left[-4\dot{x}_2 - 2\dot{x}_2 + 2\dot{x}_1 - 100x_2 + 100x_1 + u_2\right]$$

$$= \frac{1}{60}\left[-6\dot{x}_2 + 2\dot{x}_1 - 100x_2 + 100x_1 + u_2\right]$$

$$= \frac{100}{60}x_1 - \frac{100}{60}x_2 + \frac{2}{60}\dot{x}_1 - \frac{6}{60}\dot{x}_2 + \frac{u_2}{60}$$

$$5\ddot{x}_1 = u_1 - 4\dot{x}_1 - 2(\dot{x}_1 - \dot{x}_2) - 100(x_1 - x_2)$$

$$\ddot{x}_1 = \frac{1}{5}\left[-4\dot{x}_1 - 2\dot{x}_1 + 2\dot{x}_2 - 100x_1 + 100x_2 + u_1\right]$$

$$= \frac{1}{5}\left[-6\dot{x}_1 + 2\dot{x}_2 - 100x_1 + 100x_2 + u_1\right]$$

$$= \frac{-100x_1}{5} + \frac{100x_2}{5} - \frac{6}{5}\dot{x}_1 + \frac{2}{5}\dot{x}_2 + \frac{u_1}{5}$$

$$m_2\ddot{x}_2 = u_2 - b_{12}(\dot{x}_2 - \dot{x}_1) - K_{12}(x_2 - x_1) - b_2\dot{x}_2$$

$$= u_2 - b_{12}\dot{x}_2 + b_{12}\dot{x}_1 - K_{12}x_2 + K_{12}x_1 - b_2\dot{x}_2$$

$$= K_{12}x_1 - K_{12}x_2 + b_{12}\dot{x}_1 - (b_{12} + b_2)\dot{x}_2 + u_2$$

$$m_1\ddot{x}_1 = u_1 - b_1\dot{x}_1 - b_{12}(\dot{x}_1 - \dot{x}_2) - K_{12}(x_1 - x_2)$$

$$= u_1 - b_1\dot{x}_1 - b_{12}\dot{x}_1 + b_{12}\dot{x}_2 - K_{12}x_1 + K_{12}x_2$$

$$= u_1 - K_{12}x_1 + K_{12}x_2 - (b_1 + b_{12})\dot{x}_1 + b_{12}\dot{x}_2$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} K_{12} & -K_{12} \\ -K_{12} & K_{12} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} (b_1 + b_{12}) & -b_{12} \\ -b_{12} & (b_{12} + b_2) \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$H\ddot{q} + Kq + B\dot{q} = \tau$$

$$\hookrightarrow H\ddot{q} = \tau - Kq - B\dot{q}$$

Define Sliding Surface

$$S = \dot{\tilde{q}} - A\tilde{q} = \dot{q} - \dot{q}_r$$

$$= \dot{q} - (\dot{q}_d + A\tilde{q})$$

negative Eigs

$$\dot{S} = \ddot{q} - \ddot{q}_r$$

Define Lyapunov function Candidate

$$V = \tfrac{1}{2} S^T H S$$

$$\dot{V} = S^T H \dot{S} + \tfrac{1}{2} S^T \dot{H} S$$

$$= S^T H(\ddot{q} - \ddot{q}_r) + \tfrac{1}{2} S^T \dot{H} S$$

$$= S^T(H\ddot{q} - H\ddot{q}_r) + \tfrac{1}{2} S^T \dot{H} S$$

$$= S^T(\tau - Kq - B\dot{q} - H\ddot{q}_r) + \tfrac{1}{2} S^T \dot{H} S$$

Need to eliminate this term

$$= S^T(\tau - Kq - B(S + \dot{q}_r) - H\ddot{q}_r) + \tfrac{1}{2} S^T \dot{H} S$$

$$= S^T(\tau - Kq - BS - B\dot{q}_r - H\ddot{q}_r) + \tfrac{1}{2} S^T \dot{H} S$$

$$= S^T(\tau - Kq - B\dot{q}_r - H\ddot{q}_r) + \tfrac{1}{2} S^T \cancelto{0}{(\dot{H} - 2B)} S$$

∴ $S = \dot{q} - \dot{q}_r$

$\hookrightarrow \dot{q} = S + \dot{q}_r$

$\hookrightarrow$ $\dot{H} - 2B$ is skew symmetric

$\rightarrow \dot{H}-2B$ is skew symmetric

$= S^T(\tau - Ya)$ & choose $\tau = Y\hat{a} - KS$

$= S^T(Y\hat{a} - Ya - KS)$

$= -S^T KS + \underbrace{S^T Y\tilde{a} + \dot{\hat{a}}^T P^{-1}\tilde{a}}_{} \rightarrow \dot{\hat{a}} = -PYS$

$\therefore \dot{V} = -S^T KS$

Form Regressor Matrix

$Ya = Kq + B\dot{q}_r + H\ddot{q}_r$

$$\dot{q}_r = \begin{bmatrix}\dot{q}_{r1}\\ \dot{q}_{r2}\end{bmatrix} \qquad \ddot{q}_r = \begin{bmatrix}\ddot{q}_{r1}\\ \ddot{q}_{r2}\end{bmatrix}$$

$$= \begin{bmatrix} k_{12} & -k_{12} \\ -k_{12} & k_{12} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} (b_1+b_{12}) & -b_{12} \\ -b_{12} & (b_{12}+b_2) \end{bmatrix}\begin{bmatrix} \dot{q}_{d1} + A_1\tilde{q}_1 \\ \dot{q}_{d2} + A_2\tilde{q}_2 \end{bmatrix} + \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{q}_{d1} + A_1\dot{\tilde{q}}_1 \\ \ddot{q}_{d2} + A_2\dot{\tilde{q}}_2 \end{bmatrix}$$

$$= \begin{bmatrix} \underbrace{K_{12}}_{a_1} q_1 - \underbrace{K_{12}}_{a_1} q_2 + \underbrace{(b_1+b_{12})}_{a_2}\dot{q}_{r1} - \underbrace{b_{12}}_{a_3}\dot{q}_{r2} + \underbrace{m_1}_{a_4}\ddot{q}_{r1} \\ \underbrace{-K_{12}}_{a_1} q_1 + \underbrace{K_{12}}_{a_1} q_2 - \underbrace{b_{12}}_{a_3}\dot{q}_{r1} + \underbrace{(b_{12}+b_2)}_{a_5}\dot{q}_{r2} + \underbrace{m_2}_{a_6}\ddot{q}_{r2} \end{bmatrix}$$

$$\left[\begin{array}{c|c|c|c|c|c} (q_1-q_2) & \dot{q}_{r1} & -\dot{q}_{r2} & \ddot{q}_{r1} & 0 & 0 \\ \hline (-q_1+q_2) & 0 & -\dot{q}_{r1} & 0 & \dot{q}_{r2} & \ddot{q}_{r2} \end{array}\right] \begin{bmatrix} K_{12} \\ b_1+b_{12} \\ b_{12} \\ m_1 \\ b_{12}+b_2 \\ m_2 \end{bmatrix}$$