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#### Redirected from "n-point of view".

<u>Wikipedia</u> enforces its entries to adopt an <u>NPOV</u> – a *neutral point* of view . This is appropriate for an encyclopedia.

However, the nLab is not Wikipedia, nor is it an encyclopedia, although it does aspire to provide a useful reference in many areas (among its other purposes). In particular, the nLab has a particular point of view, which we may call the nPOV or the n- categorical point of view.

Context

**Category theory** 

**Higher category theory** 

To some extent the nPOV is just the observation that <u>category theory</u> and <u>higher category theory</u>, hence in particular of <u>homotopy theory</u>, have a plethora of useful applications.

### nPOV

- 1. Idea
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## 1. Idea

Around the <u>nLab</u> it is believed that <u>category theory</u> and <u>higher category theory</u> provide a point of view on <u>Mathematics</u>, <u>Physics</u> and <u>Philosophy</u> which is a valuable unifying point of view for the understanding of the concepts involved.

So at the nLab, we don't care so much about being neutral. Although we don't want to offend people unnecessarily, we are also not ashamed about writing from this particular point of view. There are certainly other valid points of view on mathematics, but describing them and being neutral towards them is not the purpose of the nLab. Rather, the nLab starts from the premise that category theory and higher category theory are a true and useful point of view, and one of its aims is to expose this point of view generally and in a multitude of examples, and thereby accumulate evidence for it.

If you feel skeptical about the n-point of view, you may want to ignore the nLab. Or you may want to take its content as a contribution to a discussion on what is behind the claim that

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category theory is the right language to describe the world, or at least the world of mathematical ideas.

As recalled in parts on the page on <u>category theory</u>, category theorists have early on, beginning in the 1960s, proclaimed the advantages of category theory over other points of view. It has been observed that this claim, or at least the way it has been put forward, has contributed to a certain alienation of category theory in parts of the mathematical community. That may be true and is understandable. But since a claim is not false *just* because it is put forward with possibly unpleasant boldness, all evidence for the claim deserves to be collected and exposed. We hope the nLab to play a role in this effort.

In particular, there have been dramatic developments since the 1960s. Back then promoting category theory may have been as visionary as the invention of <u>complex numbers</u> was in the 16th century. But just as the early rejections of the complex numbers appear strangely out of place from today's perspective, where their ubiquity proves their reality to the point that it is hard to imagine how life must have been before their conception, so developments of category theory and its applications in the last years have in many areas brought it to the point that rejecting its prevalence amounts (we believe) to rejecting the obvious and ubiquitous. But also mathematics as a whole has drastically grown since then, and while category theory has become an entirely obvious ingredient in areas such as <u>homotopy theory</u>, <u>homological algebra</u>, <u>algebraic geometry</u> and even fields like <u>topological quantum field theory</u>, its similar role to be played in many other areas has often not found wide recognition yet. But this is gradually changing.

## 2. The role of the *n*POV

Practitioners of category theory have often attempted to express the striking power of category theory (or general conceptual methods), sometimes through aphorism, sometimes through metaphor. Early on, <u>Peter Freyd</u> wrote

Perhaps the purpose of categorical algebra is to show that which is trivial is trivially trivial.

This can be taken to mean that one thing category theory does is help make the softer bits seem utterly natural and obvious, so as to quickly get to the heart of the matter, isolating the hard nuggets, which one may then attack with abandon. This is an invaluable service category theory performs for mathematics; therefore, category theory is plain good pragmatics.

However, it is also possible to take it a step beyond the pragmatic attitude, and see category theory (and now higher category theory) as exemplifying a style for doing even hard mathematics, as in the style for which Grothendieck is renowned. Paraphrasing from Colin McLarty's <u>excellent essay</u>, let us regard the aforementioned pragmatic attitude as leading up to the *hammer-and-chisel principle*: if you think of a theorem to be proved as a nut to be opened, so as to reach "the nourishing flesh protected by the shell" (Grothendieck), then one thing to do

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is "put the cutting edge of the chisel against the shell and strike hard. If needed, begin again at many different points until the shell cracks – and you are satisfied." Grothendieck points to Serre as a master of this technique. He then says:

I can illustrate the second approach with the same image of a nut to be opened. The first analogy which came to my mind is of immersing the nut in some softening liquid, and why not simply water? From time to time you rub so the liquid penetrates better, and otherwise you let time pass. The shell becomes more flexible through weeks and months – when the time is ripe, hand pressure is enough, the shell opens like a perfectly ripened avocado!

A different image came to me a few weeks ago. The unknown thing to be known appeared to me as some stretch of earth or hard marl, resisting penetration... the sea advances insensibly in silence, nothing seems to happen, nothing moves, the water is so far off you hardly hear it... yet it finally surrounds the resistant substance. (Translated from the French by McLarty)

This arresting metaphor of "la mer qui monte" ("the rising sea"), which over time changes the very form of the resistant substance, is very much in the style of Grothendieck himself. McLarty quotes Deligne as saying that a typical Grothendieck proof consists of a long series of trivial steps where "nothing seems to happen, and yet at the end a highly non-trivial theorem is there."

# 3. Examples

For an (incomplete) list of examples of topics for which the nPOV has proven to be a useful perspective see

• <u>applications of (higher) category theory</u>

category: meta

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