## Sketches of an Elephant A Topos Theory Compendium VOLUME 1

## PETER T. JOHNSTONE

Reader in the Foundations of Mathematics University of Cambridge

CLARENDON PRESS • OXFORD 2002

## CONTENTS VOLUME 1

A '	TOPOS	SES AS CATEGORIES.	1
<b>A</b> 1	Regul	ar and cartesian closed categories	3
	A1.1	Preliminary assumptions	3
	A1.2	Cartesian categories	11
	A1.3	Regular categories	18
	A1.4	Coherent categories	30
	A1.5	Cartesian closed categories	44
	A1.6	Subobject classifiers	57
<b>A2</b>	Topos	ses – basic theory	68
	A2.1	Definition and examples	68
	A2.2	The Monadicity Theorem	85
	A2.3	The Fundamental Theorem	90
	A2.4	Effectiveness, positivity and partial maps	97
	A2.5	Natural number objects	107
	A2.6	Quasitoposes	119
<b>A3</b>	Allege	ories	130
	A3.1	Relations in regular categories	130
	A3.2	Allegories and tabulations	136
	A3.3	Splitting symmetric idempotents	144
	A3.4	Division allegories and power allegories	153
<b>A4</b>	Geometric morphisms - basic theory		161
	A4.1	Definition and examples	161
	A4.2	Surjections and inclusions	172
	A4.3	Cartesian reflectors and sheaves	184
	A4.4	Local operators	195
	A4.5	Examples of local operators	204
	A4.6	The hyperconnected-localic factorization	223

**General Index** 

В	2-CAT	EGORICAL ASPECTS OF TOPOS THEORY		233
<b>B</b> 1	Index	ted categories and fibrations	}	- 235
	B1.1	Review of 2-categories		235
	B1.2	Indexed categories		259
	B1.3	Fibred categories	•	264
	B1.4	Limits and colimits		278
	B1.5	Descent conditions and stacks		293
B2	Inter	nal and locally internal categories		304
	B2.1	Review of enriched categories		304
	B2.2	Locally internal categories		310
	B2.3	Internal categories and diagram categories		317
	B2.4	The Indexed Adjoint Functor Theorem		332
	B2.5	Discrete opfibrations		339
	B2.6	Filtered colimits	•	350
	B2.7	Internal profunctors		359
B3	Toposes over a base			368
	B3.1	$\mathcal{S}$ -toposes as $\mathcal{S}$ -indexed categories		368
	B3.2	Diaconescu's Theorem		378
	B3.3	Giraud's Theorem		389
	B3.4	Colimits in Top		401
B4	BTo	$\mathfrak{p}/\mathcal{S}$ as a 2-Category		415
	B4.1	Finite weighted limits		415
	B4.2	Classifying toposes via weighted limits		424
	B4.3	Some exponentiable toposes	· ·	432
	B4.4	Fibrations and partial products		438
	B4.5	The symmetric monad	\$	456
	Bibliography		follows p.	467
	Index	c of notation	;	[55]

[61]