

$$\begin{aligned}
 p(x, 1|N) &= \alpha \begin{bmatrix} .05 & .05 & .05 \\ .9 & .9 & .9 \\ .9 & & .05 \end{bmatrix} \times \left\{ \begin{array}{l} \begin{bmatrix} 1 & .9 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \frac{1}{8} = \begin{bmatrix} .0125 & .1125 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ + \begin{bmatrix} 0 & .1 & .9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \frac{1}{8} = \begin{bmatrix} 0 & .0125 & .1125 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \frac{1}{8} = \begin{bmatrix} 0 & 0 & .125 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ + \begin{bmatrix} 0 & 0 & 0 \\ .1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \frac{1}{8} = \begin{bmatrix} 0 & 0 & 0 \\ .0125 & .1125 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ + \begin{bmatrix} 0 & 0 & 0 \\ 0 & .1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \frac{1}{8} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & .0125 & .1125 \\ 0 & 0 & 0 \end{bmatrix} \\ + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \times \frac{1}{8} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & .125 \\ 0 & 0 & 0 \end{bmatrix} \\ + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \frac{1}{8} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & .125 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \right. \\
 &\quad \downarrow \left. \begin{array}{l} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \times \frac{1}{8} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ .125 & 0 & 0 \end{bmatrix} \\ + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \frac{1}{8} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & .125 \end{bmatrix} \end{array} \right\}
 \end{aligned}$$

$$= \alpha \begin{bmatrix} .05 & .05 & .05 \\ .9 & .9 & .9 \\ .9 & & .05 \end{bmatrix} \times \begin{bmatrix} .0125 & .125 & .2375 \\ .0125 & .125 & .2375 \\ .125 & & .125 \end{bmatrix}$$

$$= \alpha \begin{bmatrix} .000625 & .0025 & .01875 \\ .0125 & .1125 & .21375 \\ .1125 & & .00625 \end{bmatrix}$$

$$= \begin{bmatrix} .00131 & .0135 & .025 \\ .02368 & .2368 & .45 \\ .2368 & & .013 \end{bmatrix}$$

$$P(X_2|NN) = \alpha \begin{bmatrix} .05 & .05 & .05 \\ .9 & .9 & .9 \\ .9 & .9 & .05 \end{bmatrix} \times \left\{ \begin{array}{l} \begin{bmatrix} 1 & .9 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times .00131 = \begin{bmatrix} 0.00131 & .001179 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & .1 & .9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times .0135 = \begin{bmatrix} 0 & .00135 & .01835 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times .025 = \begin{bmatrix} 0 & 0 & .025 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ .1 & 9 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times .02368 = \begin{bmatrix} 0 & 0 & 0 \\ .002368 & .0131 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & .9 \\ 0 & 0 & 0 \end{bmatrix} \times .2368 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & .02368 & .21312 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \times .45 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & .45 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \times .2368 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ .2368 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times .013 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & .013 \end{bmatrix} \end{array} \right\}$$

$$= \alpha \begin{bmatrix} .05 & .05 & .05 \\ .9 & .9 & .9 \\ .9 & .9 & .05 \end{bmatrix} \times \begin{bmatrix} .000131 & .002494 & .002835 \\ .002368 & .04499 & .66812 \\ .2368 & .013 & 0 \end{bmatrix}$$

$$= \alpha \begin{bmatrix} 6.55 \times 10^{-6} & 1.247 \times 10^{-4} & .00284 \\ .002131 & .0405 & .5968 \\ .2131 & & 6.5 \times 10^{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 7.66 \times 10^{-6} & 1.46 \times 10^{-4} & .0021 \\ .0025 & .0474 & .698 \\ .249 & & 7.6 \times 10^{-4} \end{bmatrix}$$

$$P(X_3 | N, N, H) = \alpha \begin{bmatrix} .9 & .9 & .05 \\ .05 & .05 & .05 \\ .05 & .05 & .9 \end{bmatrix} \times \left( \begin{bmatrix} .1 & 0 & 0 \\ .9 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 7.66 \times 10^{-6} = \begin{bmatrix} 7.66 \times 10^{-7} & 0 & 0 \\ 6.894 \times 10^{-6} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right)$$

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$$+ \begin{bmatrix} 0 & .1 & 0 \\ 0 & .9 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 1.46 \times 10^{-4} = \begin{bmatrix} 0 & 1.46 \times 10^{-5} & 0 \\ 0 & 1.314 \times 10^{-4} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & .1 \\ 0 & 0 & .9 \\ 0 & 0 & 0 \end{bmatrix} \times .0021 = \begin{bmatrix} 0 & 0 & .00021 \\ 0 & 0 & .00189 \\ 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ .1 & 0 & 0 \\ .9 & 0 & 0 \end{bmatrix} \times .0025 = \begin{bmatrix} 0 & 0 & 0 \\ .00025 & 0 & 0 \\ .00225 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times .0474 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & .0474 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & .1 \\ 0 & 0 & .9 \end{bmatrix} \times .698 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & .0698 \\ 0 & 0 & .6282 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \times .249 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ .249 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times 7.6 \times 10^{-4} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7.6 \times 10^{-4} \end{bmatrix}$$

$$\alpha \begin{bmatrix} .9 & .9 & .05 \\ .05 & .05 & .05 \\ .05 & .05 & .9 \end{bmatrix} \times \begin{bmatrix} 7.66 \times 10^{-7} & 1.46 \times 10^{-5} & .00021 \\ 2.57 \times 10^{-4} & .0475 & .0717 \\ .25125 & .629 & 0 \end{bmatrix}$$

$$= \alpha \begin{bmatrix} 6.894 \times 10^{-7} & 1.314 \times 10^{-5} & 1.05 \times 10^{-5} \\ 1.285 \times 10^{-5} & .002375 & .003585 \\ .01256 & .5661 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.78 \times 10^{-6} & 2.247 \times 10^{-5} & 1.796 \times 10^{-5} \\ 2.198 \times 10^{-5} & .00406 & .00613 \\ .02148 & .9682 & 0 \end{bmatrix}$$

$$P(X_4 | N, N, H, H) = \alpha \begin{bmatrix} .9 & .9 & .05 \\ .05 & .05 & .05 \\ .05 & .05 & .9 \end{bmatrix} \times \left( \begin{array}{l} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times 1.78 \times 10^{-6} = \begin{bmatrix} 1.78 \times 10^{-7} & 0 & 0 \\ 1.602 \times 10^{-6} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \times 2.247 \times 10^{-5} = \begin{bmatrix} 0 & 2.247 \times 10^{-6} & 0 \\ 0 & 2.022 \times 10^{-5} & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 1.796 \times 10^{-5} = \begin{bmatrix} 0 & 0 & 1.796 \times 10^{-6} \\ 0 & 0 & 1.616 \times 10^{-5} \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times 2.198 \times 10^{-5} = \begin{bmatrix} 0 & 0 & 0 \\ 2.198 \times 10^{-6} & 0 & 0 \\ 1.918 \times 10^{-5} & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times .00406 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & .00406 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \times .00613 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & .000613 \\ 0 & 0 & .00552 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \times .02148 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ .02148 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times .9682 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & .9682 \end{bmatrix} \end{array} \right)$$

$$= \alpha \begin{bmatrix} .9 & .9 & .05 \\ .05 & .05 & .05 \\ .05 & .05 & .9 \end{bmatrix} \times \begin{bmatrix} 1.78 \times 10^{-7} & 2.247 \times 10^{-6} & 1.796 \times 10^{-6} \\ 3.8 \times 10^{-6} & .00406 & 6.29 \times 10^{-4} \\ .0215 & & .97372 \end{bmatrix}$$

$$= \alpha \begin{bmatrix} 1.602 \times 10^{-7} & 2.022 \times 10^{-6} & 8.98 \times 10^{-8} \\ 1.9 \times 10^{-7} & 2.04 \times 10^{-4} & 3.145 \times 10^{-5} \\ .00107 & & .87635 \end{bmatrix}$$

$$= \begin{bmatrix} 1.825 \times 10^{-7} & 2.304 \times 10^{-6} & 1.023 \times 10^{-7} \\ 2.165 \times 10^{-7} & 2.924 \times 10^{-4} & 3.523 \times 10^{-5} \\ .00122 & & .9985 \end{bmatrix}$$