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ORIGINAL ARTICLE

From local to global probabilistic modeling of concrete cracking

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Abstract The description of cracks in concrete is crucial when dealing with life expectancy of structures such as dams, nuclear power plants vessels, waste (nuclear or not) storage structures, tunnels, etc. The main objective is not only to describe the growth of a preexisting flaw, but also to predict the genesis and formation of cracks in an initially flaw-free structure (at least at the macroscopic level) subjected to tension. The presented paper provides a macroscopic model for tensile cracking (i.e., a model adequate for describing the behavior at the structure level), capable at the same time of providing information on the local response (i.e., cracks). The model takes into account scale effects as well as the heterogeneous nature of concrete via appropriate, experimentally validated, size effect laws and via a statistical distribution of mechanical properties. Results are provided and validated via a 2D comparison with an original experimental test.

1 Introduction

Cracks form a barrier for heat conduction and create preferential flow paths for fluids, gas and pollutants, i.e., their description is crucial in predicting the life expectancy of structures such as dams, nuclear power plants vessels, waste (nuclear or not) storage structures, tunnels, etc. A critical point is to predict the genesis and formation of cracks in an initially flaw-free structure (at least at the macroscopic level) and not only describing the growth of a preexisting flaw. In the literature a number of approaches for describing the nucleation/evolution of cracks can be found: however, results are rarely predictive [1], especially when crack openings and spacings are concerned. The experimental analyses held at LCPC since more than 20 years [2, 3, 4, 5, 6, 7] have lead to the observation that these phenomena can be correctly described by explicitly taking into account concrete heterogeneity (which is at the origin of volume effects) in the frame of a probabilistic approach. An original model based on these concepts has been first proposed by [8, 9] and more recently by [10].

The final objective of the research held at LCPC on concrete cracking mechanisms is to develop a macroscopic continuum 3D model, integrating heterogeneity and volume effects via a probabilistic approach, adapted for the cracking analysis of full-scale civil engineering structures.

In such structural analyses, 3D modeling turns out to be necessary [7]. However, as a first step, the proposed approach has been developed in 2D (plane stresses) and has been validated on a simple but significative experimental test in order to evaluate the pertinence of the proposed analysis.

The choice in favor of a continuum approach is justified for different reasons. Firstly, the objective is to develop a model which is suitable for real structures. Macroscopic continuum models are often preferred by engineers due to the existence of a well-established theoretical framework (thermodynamics of irreversible processes) and the possibility to use measurable macroscopic parameters/variables and evolution laws. Moreover, the numerical implementation in the context of the finite element method is quite natural and robust solving algorithms are widely available. A continuum model has also some advantages over discrete models such as interface models [8, 4] and lattice models [11]. Indeed, continuum approaches are less demanding as

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far as computational effort is considered (even if computational costs are getting smaller and smaller nowadays) and can be relatively easily enhanced by including multiphysics effects (e.g., Thermo-Hydraulic [12] and creep).

The probabilistic aspect of the model is mainly based on the experimental analysis of the cracking processes originally proposed by [4]. This analysis has put in evidence the key role of heterogeneity and volume effects on cracking processes arising in concrete failure. Therefore, the probabilistic approach requires an investigation on the material random mechanical properties in order to define distribution laws (integrating heterogeneity and volume effects). In this paper an original strategy is presented in order to define the appropriate laws adapted for the continuum model. A numerical experimentation, based on a discrete approach integrating volume effects [8], is here exploited for defining the distribution laws feeding the continuum probabilistic approach.

These ideas are developed in the first part of the paper.

The second part of the paper presents the numerical modeling of the cracking behavior of concrete. Two numerical strategies will be introduced: one based on a probabilistic discrete approach, the second on an original continuum, probabilistic model. Both take into account volume effects, they are naturally oriented towards structural reliability analyses and only a few easy-to-measure parameters are required as input data for the model: to our knowledge, no similar approach is available in the existing literature. Despite the macroscopic nature of the model, local quantitative information is provided on crack patterns, spacings and openings. It should be underlined that both models do not deal with crack propagation at least in the sense of fracture mechanics (i.e., a propagation criterion is not required) but deals only with a probabilistic creation of elementary cracks. Apparent macro-crack propagation is then the consequence of aleatory crack creation; in other words, the cracking of successive elements can be considered, at a macroscopic level, as a (macro) crack propagation or cracking localization. A further discussion will be presented in Sect. 4.

The results of the probabilistic model are finally compared with an original experimental 4-points bending test. Similar results are also available in the existing literature, however the proposed experimental test providing local/global information (i.e., crack openings, displacements, …) necessary to validate the model.

2 Cracking processes in concrete

Concrete is a porous multiphase material where the solid matrix is formed by cement paste and aggregates and where voids are filled with liquid and gas. In other words, concrete is, by nature, an heterogeneous material which contains inner defects such as pores and cracks, consequences of hydration of the cement paste and restrained shrinkages at early ages. Initial conditions are also responsible of internal auto-balanced stresses (even under no external load) which determine the long-term concrete behavior [13]. Heterogeneity, inner defects, applied loads and boundary conditions, provoke stress gradients in concrete and, of course, cracking. Thus, the global mechanical response of the structure is generally a consequence of the combined effect of these stress gradients and localized cracks propagating during the loading.

Moreover, a further, well-known problem arises when dealing with concrete: size effect [14]. Heterogeneity and size effect are aspects which are strictly correlated and that should be specifically taken into account when dealing with concrete modeling.

Concerning the first aspect, heterogeneity can be taken into account by introducing statistical distributions of local material characteristics, in particular of the Young's modulus and the tensile strength [8]. This technique gives also a first hint to the size effect problem if one assumes that there is an equivalence between the finite elements of the mesh and a volume of material: the distribution function relating the material characteristics and the size has then to be experimentally determined.

Previous works [4, 9] have highlighted that it is possible to establish a link between tensile strength f_t or Young's modulus E and the volume of the tensile specimen for concretes having a compressive strength between 35 and 130 MPa.

An experimental scale effect law has been then established for the mean tensile strength $m(f_t)$ and the standard deviation $s(f_t)$ as as functions of easily measurable quantities such as the volume of the specimen V_s and the volume of the coarsest grain of the concrete V_g (which can be related to the size of the major heterogeneity) and the compressive strength of concrete f_c (an indicator of the quality of the cement paste).

$$m(f_t) = F_\alpha\left(\frac{V_s}{V_g}, f_c\right) \tag{1}$$

$$s/m(f_t) = F_\beta\left(\frac{V_s}{V_g}, f_c\right) \tag{2}$$

These laws are given in Fig. 1. Experimental tests have also shown that the Young's modulus of concrete is not affected by a meaningful scale effect, or at least, the mean value of the Young's modulus appears to be independent of the volume of the specimen [4]. On the other hand, the dispersion depends on the specimen's volume. This dispersion is related to the fact that the Young's moduli of the cement paste and of the grains are different, and that the

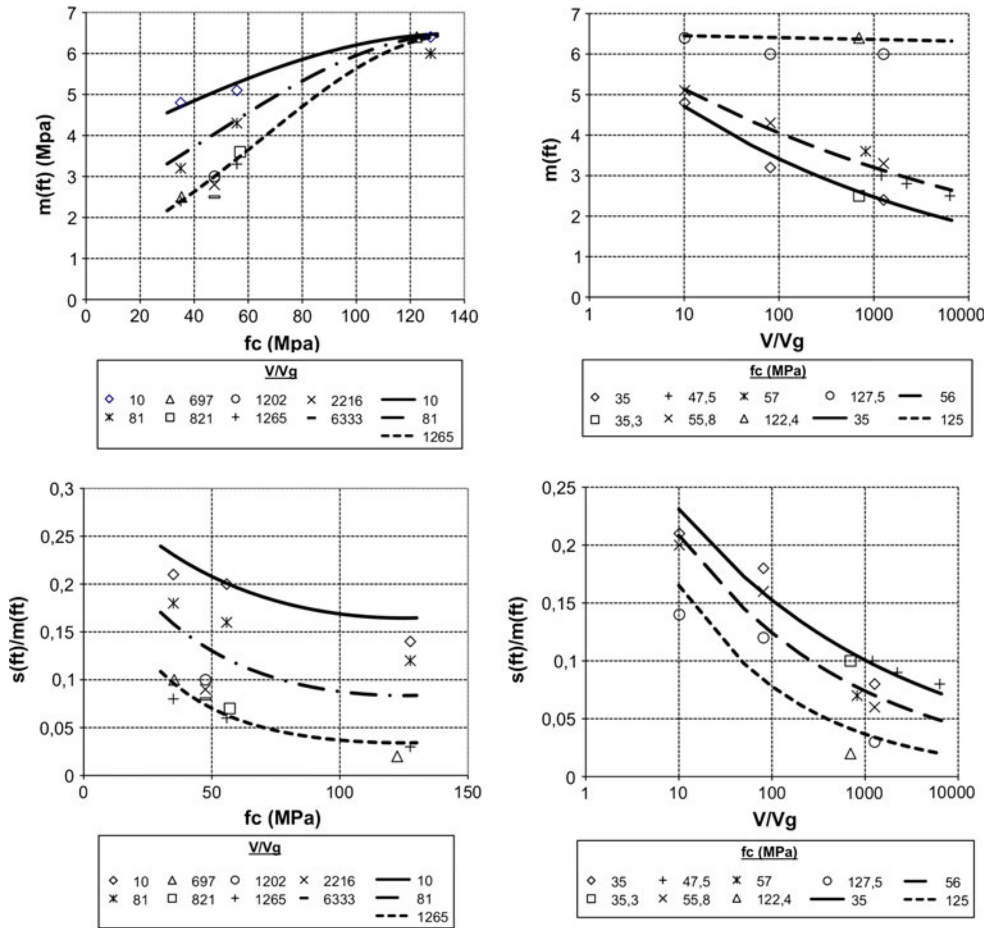

Fig. 1 Tensile strength mean value/dispersion evolutions: experimental data and fitting curves [4]

smaller the specimen dimension, the closer the Young's modulus becomes to that of the cement paste or to that of the grain, alone. The ratio $\frac{V_s}{V_g}$ governs this dispersion, and the closer is the modulus of the matrix to that of the grain, the less the ratio $\frac{V_s}{V_g}$ will influence the dispersion of the grain. In conclusion, [4] states that the Young's modulus mean value depends only on the compressive strength of concrete (no scale effect), while the dispersion is described by a law similar to the one given for the tensile strength. These scale laws have been later used as input data in a numerical model based on a probabilistic approach which allowed to obtain encouraging results [8].

3 Discrete probabilistic modeling: numerical experimentation

Rossi [8] originally presents a probabilistic model taking into account the heterogeneity of the material and the size effect via experimental results [3, 4] which are extrapolated to different sizes of concrete volumes and, in particular, to the size of the finite elements used in the finite element mesh of a numerical analysis. From a numerical point of view, the model is implemented via a discrete approach in which interface elements are used to describe the discontinuities. The mechanical properties of the interface elements (Young modulus and tensile strength) are considered as randomly distributed variables. The volume of the massive elements which are adjacent to the considered interface element, acts as the reference (material) volume. Distribution characteristics[1] can be obtained from an extrapolation of the empirical formulas given in Sect. 1.

The model is currently implemented in 2D in the finite element code CESAR-LCPC [15]. Locally, volumes of concrete associated to interface elements have an elastic-purely brittle behavior while massive elements representing the uncracked concrete remain elastic. Contact elements follow some Tresca criteria as well in tension as in shear.

Finally, one can summarize the main characteristics of this model by underlining that:

- the model is aimed at explicitly representing localized crack patterns in concrete taking into account volume effects;

[1] Mean value and standard deviation.

- the model is considered as probabilistic, but after the random distribution of mechanical properties over the mesh, the computation remains deterministic. It is then necessary to perform a large number of computations to statistically validate the results (following a Monte Carlo method);
- the model is auto-coherent in the sense that data at the local scale are coherent with results at the global scale since a generic law taking into account volume effects can define concrete mechanical properties at each scale;
- although locally no energy is dissipated (the failure of the elementary volume remains elastic-perfectly brittle) the model allows to statistically represent a global dissipation of energy through inelastic residual strains, softening behaviors, …
- the model does not deal in a strict sense with crack propagation, but it represents it by a succession of crack planes creations.

This approach, in particular for what concerns the random distribution of properties and the use of a size effect law on the tensile strength, proved to be quite effective under many circumstances. Nevertheless, some shortcomings of the original basic model should be pointed out.

According to the local and probabilistic character of the approach, the volume of the element has to be sufficiently small when compared to the volume of the meshed structure or to the zone size where stress gradients can develop (i.e., the fracture process zone [14]). This can lead to very small ratios $\frac{V_s}{V_g}$ which fall out of the domain of validity supported by the experimental campaign [4]. More recently, [16] have shown that the evolutions of the mean values and the standard deviations given by the empirical formulas with respect to the compressive strength become meaningless for ratios $\frac{V_s}{V_g} < 1$. An inverse analysis has then been proposed to determine the extrapolation of the empirical formulas to the small ratios $\frac{V_s}{V_g}$ domain. Note that a similar work was performed in [17], but for larger ratios $\frac{V_s}{V_g}$. In this paper, the extrapolations issued from the inverse analysis are taken into account for $\frac{V_s}{V_g} < 1$. The original size-effect law is therefore updated and will be used in the finite element analysis.

Moreover, the applicability of the discrete-explicit model is limited by some intrinsic limits. For more global approaches, at the scale of a whole structure for example, such model leads to important computational costs as the use of contact elements doubles the number of nodes. This is even more sensitive in the case of 3D modeling. It can be argued that computational costs are getting smaller and smaller but the above considerations limit the spread and the use of such models in the professional circle. Moreover, multi-physics remains difficult to be coupled to these approaches.

The existing modeling strategy needs therefore an enhancement towards a continuum based approach. Such a model seems more adequate in many situations and in particular when dealing with real structures. If compared to a discrete model, a continuum model does not require contact elements, i.e., no pre-oriented cracks, and any direction crack is favored. Moreover, in a continuum model the cracking of a finite element corresponds to the cracking of a volume of material.

4 Continuum probabilistic approach of concrete cracking

As shown in the previous section, the probabilistic discrete cracking model is relatively well suited for modeling cracking patterns in approaches where obtaining local information on cracks (such as crack opening and/or cracks spacing) is of importance. However, it seems necessary to enhance this approach in order to model real structures by means of the finite element method and to include, in future works, thermo-hydraulic analysis and/or creep.

When dealing with continuum approaches applied to an heterogeneous quasi-brittle material, two important facts have to be pointed out:

- Usually, the identification of the material behavior is performed on laboratory samples which size has to be sufficiently large for minimizing the effect of the heterogeneity. Nevertheless, it is well-known that, even at this scale, the material remains sensitive to volume effects [8, 4]. It is therefore impossible to directly identify the behavior of the finite element representing some volume of material in the whole modeled structure. This evidence requires to take into account scale changes, i.e., volume effects must be considered in the model.
- The localization of cracks occurring at the peak has to be carefully taken into account. Before the load peak, the material can be severely damaged and micro-cracked but no localization occurs. After the peak, cracks localize and material integrity fails such that it is impossible to consider the post-peak softening behavior as representative of the behavior of the material. In other words, after the peak we shift from a material behavior to a structural behavior [6]. Theoretically, localization is also associated to the bifurcation of the solution [18] and the violation of the Drucker's principle. Numerical translations of these problems are mostly leading to strong mesh sensitivities and non objective responses [19].

Additional assumptions are required to properly take into account these two observations.

- It is assumed that it is possible to define macroscopic quantities whatever the size of the finite element, whether it is material representative or not. It is then supposed that the mechanical behavior of the finite element depends on its size, i.e., the behavior of each finite element is prone to random variations, thus taking into account the material heterogeneity.
- For what concerns the cracking process, the model does not explicitly treat crack propagation (i.e., a propagation law like in fracture mechanics is not required) but deals only with an aleatory creation of elementary cracks. Apparent macro-crack propagation is then the consequence of aleatory crack creation; in other words, the cracking of successive elements can be considered, at a macroscopic level, as a (macro) crack propagation. In this context, the total energy is dissipated at two different scales:
    1. Local scale (scale of the finite element): at this scale, an energy (which corresponds to the material behavior) is locally dissipated
    2. Global scale (scale of the structure): at this scale, a crack appearance energy (which corresponds to a crack localization in each element) is dissipated. This dissipation leads to a decrease of the structure compliance.

It is assumed that, for a given volume, one can define a (local) material behavior equivalent to the real material in terms of deformation energy (see Fig. 2). Strictly and theoretically speaking, no continuum model is physically pertinent for taking into account a softening post-peak branch when dealing with a material behavior. The dissipative mechanism in the equivalent (local) material is therefore represented via a material behavior and perfect plasticity has been chosen. This choice is justified by the simplicity of the approach together with the well established theoretical and numerical framework (e.g., [20]). Plasticity with hardening can also be adopted but perfect plasticity requires less parameters to identify, thus simplifying the inverse analysis (see next paragraphs). Moreover,

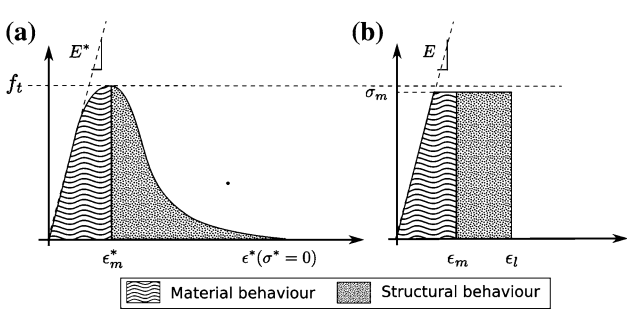

Fig. 2 Deformation energy equivalence for a uniaxial tensile behavior

no numerical instabilities have been observed so far using the perfect plasticity model.

The principle of the energy equivalence is depicted in Fig. 2.

Figure 2a schematizes the real experimental behavior of a given volume of concrete which, for the sake of simplicity is experimentally obtained by means of a uniaxial tensile test. Figure 2b represents the elastic-perfectly plastic behavior which must be identified as energetically equivalent to the real behavior. The identifiable parameters are then the maximum stress and the length of the plastic plateau which can be fixed by a limit strain or a threshold of deformation energy.

Equation 3 defines the deformation energy W_d for a given volume Ω of material. This energy is the summation over the domain of energy density, denoted here w_d and which expression is given by Eq. 4 where the superscript index $(.)^*$ denotes quantities determined experimentally.

$$W_d = \int_{\Omega} w_d d\Omega \tag{3}$$

$$w_d = w_{ma} + w_{pp} = \int_0^{\epsilon_m^*} \sigma^* \epsilon^* d\epsilon + \int_{\epsilon_m^*}^{\epsilon_l^*} \sigma^* \epsilon^* d\epsilon \tag{4}$$

Eq. 4 also gives the possible splitting of w_d in w_{ma}, the energy density of the pre-peak part of the (material) behavior, and w_{pp}, the structural post-peak part. These two parts can be identified from both Fig. 2a and b as the areas under the curves (ϵ^*, σ^*) and (ε, σ) leads to:

$$w_{ma} = \int_0^{\epsilon_m^*} \sigma^* \epsilon^* d\epsilon = \sigma_m \epsilon_m - \frac{1}{2}\frac{\sigma_m^2}{E} \tag{5}$$

$$w_{pp} = \int_{\epsilon_m^*}^{\epsilon^*(\sigma^*=0)} \sigma^* \epsilon^* d\epsilon = \sigma_m(\epsilon_l - \epsilon_m) \tag{6}$$

Where the equality [7] is imposed:

$$\epsilon_m = \epsilon_m^* \tag{7}$$

The parameter σ_m can thus be identified from Eq. (5) and ε_l from Eq. (6) or in a more simple way from Eq. (8):

$$w_d = \int_0^{\epsilon^*(\sigma^*=0)} \sigma^* \epsilon^* d\epsilon = \sigma_m \epsilon_l - \frac{1}{2}\frac{\sigma_m^2}{E} \tag{8}$$

As far as the uniaxial behavior depends on the stressed volume of material and presents some randomness, the area under the curves is also a random quantity influenced by volume effects. Consecutively, σ_m and w_d can be considered as random parameters of the elastic-plastic energetically equivalent model, also influenced by volume effect.

The general laws defining the characteristics of the probabilistic distributions for σ_m and w_d have now to be then identified. Here again, we assume that cracking processes are entirely driven by the heterogeneity of the material and the quality of the cement paste. Therefore, mean values and standard deviations of σ_m and w_d are considered as functions of $\frac{V_s}{V_g}$, ratio between the volume of material and the volume of the coarsest grain, and f_c, the compressive strength (see Eq. (9) and (10)):

$$\begin{aligned} m_{\sigma_m} &= f_m\left(\frac{V_m}{V_g}, f_c\right) \\ s_{\sigma_m} &= f_s\left(\frac{V_m}{V_g}, f_c\right) \end{aligned} \tag{9}$$

$$\begin{aligned} m_{w_d} &= g_m\left(\frac{V_m}{V_g}, f_c\right) \\ s_{w_d} &= g_s\left(\frac{V_m}{V_g}, f_c\right) \end{aligned} \tag{10}$$

These functions have to be identified via an experimental campaign, investigating different types of concrete and different sizes of specimens. Although this study is restricted to pure tension, such a campaign represents a considerable work. Given the considerations made in Sect. 3, this task can be then replaced by an original numerical campaign using the preceding discrete-explicit approach (see Sect. 3). In other words, the proposed approach make use in an original way of the available numerical tools for developing the new continuum approach.

The identification is then performed following these steps:

1. Choice of one type of concrete (i.e., V_g and f_c are fixed)
2. Choice of one mesh size (this fixes the ratio $\frac{V_s}{V_g}$ where V_s is the volume of the specimen represented by the chosen mesh)
3. Execution of n different computations
4. Identification of the pre-peak behavior (material behavior) on each of the n computations followed by the computation of m_{σ_m} and s_{σ_m} for the chosen ratio $\frac{V_s}{V_g}$
5. Identification of the mean value of w_d on the mean curve of the n computations for the chosen ratio $\frac{V_s}{V_g}$, according to the principle depicted figure [2].
6. Change the size of mesh, i.e., go to point (2)
7. Change the type of concrete, i.e., go to point (1)

The inverse analysis is physically justified on what has been stated concerning the close correlation between heterogeneity and scale/volume effects. The experimental campaign has allowed to observe that the physical processes leading to failure are the same at different scales. This concept has then been extrapolated by assuming that a finite volume (in the numerical analysis) is representative of a volume of material. In other words, this means that the physical processes related to the cracking mechanisms observed in the experimentation have been extended to the scale of the finite volume.

It is clear that the proposed inverse analysis, which is necessary for affecting to a given element size the correct distribution laws, is a time consuming procedure. However, this step has to be performed only once in order to compute the distribution law versus element size: in other words, once obtained the distribution laws for a reasonable range of finite volume sizes, the inverse analysis is no longer required.

Finally, the functions f_m, f_s and g_m (in Eqs. 9 and 10) can be directly determined as functions of $\frac{V_s}{V_g}$ and f_c. These functions are given in Fig. 3.

Concerning the standard deviation of the energy g_s, some questions arise. The cracking process in the post-peak phase has a 3D character due to the heterogeneity of concrete [21, 22]. A plane, 2D analysis using the discrete-explicit approach does not properly describe this phase as phenomena such as bridging, inter-locking, frictions between cement paste and aggregates, … can not be captured in the third dimension. This weakness of the 2D analysis plays a role in the determination of s_{wd} and a full 3D analysis is preferable (but is not available for this approach). Nevertheless, a more correct 2D representation of the post-peak cracking behavior via the numerical testing campaign could be obtained only when using very fine meshes: the disadvantage is that this rapidly increase the computational costs. For this reason, the dispersion is not directly examined on these results.

The post-peak energy dispersion (and then the standard deviation on w_d) identification, can be alternatively achieved via an inverse analysis on the equivalent model. Modeling the same experiment, it is possible to find the standard deviation on w_d using a minimization process for a given objective function. The chosen objective function can be a measure of the deviation between the mean curves obtained by both the discrete-explicit approach and the continuum equivalent approach. Performing this inverse analysis for different ratios $\frac{V_s}{V_g}$ and different types of concretes allows the explicit determination of the function g_s. This kind of analysis is quite demanding and has not been obtained for all the $\frac{V_s}{V_g}$ and f_c. An example will be given in Sect. 6.

5 Numerical modeling of discontinuities

The literature proposes a wide variety of models which can be roughly classified in two families, based on an implicit or explicit description of the kinematics discontinuity. In the former, one can enumerate anisotropic/isotropic

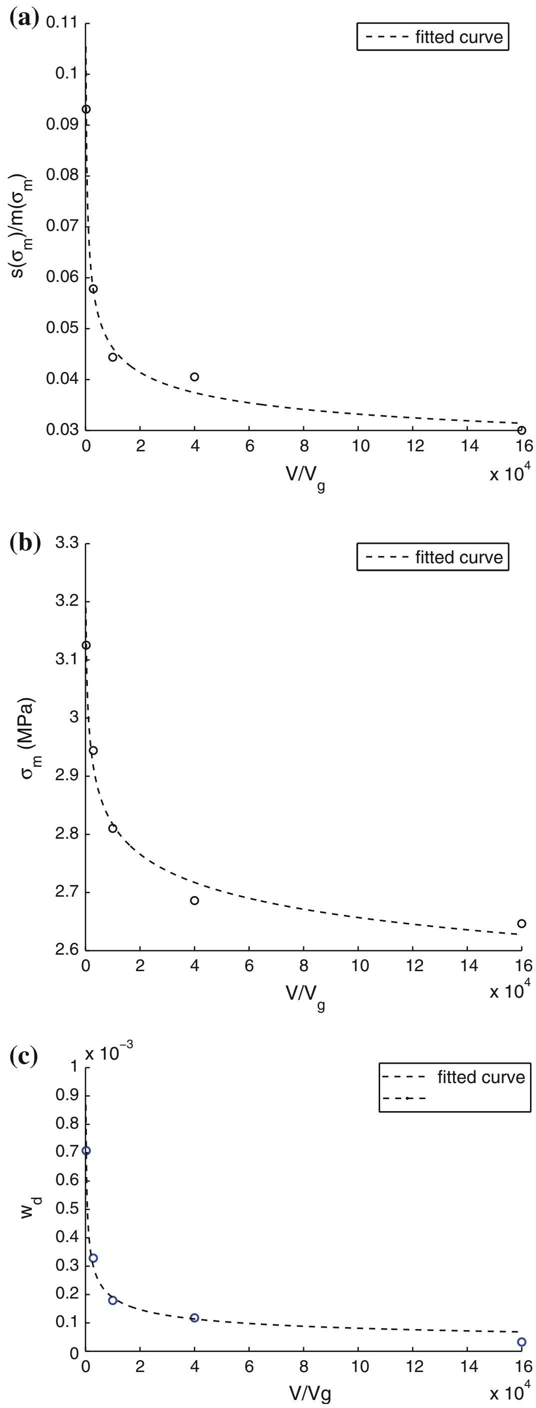

Fig. 3 Parameters identification

damage models [23, 24], smeared crack-like approaches [25, 26], enriched kinematics models [27, 28], plasticity models [20], … On the other hand, the latter includes models such as the lattice model [29, 11], discrete approaches (in which interface elements are inserted to model the discontinuities) [30, 31, 8] or, more recently, enhanced finite elements formulations such as [32, 33] and partition of unity [34] based models (GFEM) [35, 36], …

In this study, the objective is to formulate a macroscopic continuum model capable of describing the global behavior of a structure subjected to tension as well as providing fine information about crack opening, direction, propagation … The choice of a numerical support is therefore important as it should combine the relative simplicity of implicit models (which are particularly suitable for being used in the description of large structures) together with the capacity of giving some extra information necessary for a proper crack description.

5.1 Modeling strategy

Three finite element approaches are therefore considered for the study: a Rashid-like [25] model, a fixed crack model [37] and an embedded formulation [38].

5.1.1 Rashid-like formulation

This approach consists of reducing to zero the stiffness of the element in which an energy threshold is reached (i.e., local crack initialization): this method allows to obtain a global softening answer. The limits of this method in the numerical analysis are well-known as results are strongly dependent on the size of the elements of the finite element mesh (mesh dependence) [39].

5.1.2 Fixed crack model

The fixed crack model (originally proposed by [37]), considers concrete as a perfectly brittle material for which the normal stress falls to zero after the formation of the crack [40]. The main drawback of this model is represented by its poor kinematic representation (i.e., stress locking [41]) due to a spurious stress transfer across a widely open crack [42]. Later improvements (rotating crack, use of a retention factor, …), could not completely avoid stress locking. Moreover, in our model, a rotating crack is meaningless as the kinematic discontinuity appears only at the end of the cracking process, i.e., when the FE volume is cracked. For these reason, the original (simple) fixed crack formulation has been retained.

5.1.3 Embedded discontinuity (EFEM)

In the original formulation, a localization zone is embedded into the finite element when the crack initiates. The discontinuity can be weak [32] or strong [33], but, in any case, objectivity is achieved. The choice in favor of EFEM over e.g., XFEM has been made as the proposed macroscopic probabilistic model is formulated at the scale of a finite element volume, i.e., an enrichment at the level of the finite element is closer to the model's philosophy rather

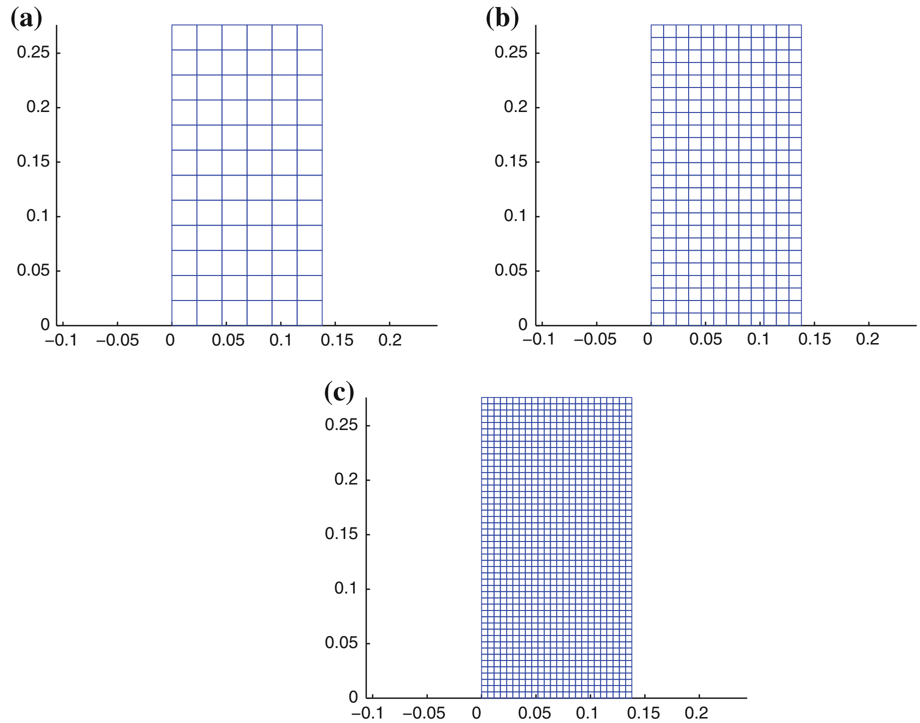

Fig. 4 Coarse (**a**), normal (**b**) and fine (**c**) mesh for the traction test

than a nodal enrichment. On the other hand, stress locking is still observed and some crack adaptation techniques are proposed [38, 43]. However, a basic formulation [40] is retained in our study.

5.2 Models comparison

The three models have been tested on different configurations in order to evaluate the eventual mesh dependence and stress locking. For each example, the mean value of 10 probabilistic computations is presented. Data used in the analysis correspond to an ordinary concrete with $f_c = 45$ MPa and 2 cm diameter of coarsest aggregate.

5.2.1 Traction test

A traction test has been performed using different meshes (72, 288, 1,152 elements, see Fig. 4) in order to evaluate the Rashid-like model objectivity when coupled with the probabilistic approach.

Results show that the probabilistic approach regularize the solution, i.e., no mesh dependence is observed (see Fig. 5) on sufficiently refined meshes.

5.2.2 Notched beam bending test

The second validation test deals with a four-point bending test on a notched beam. The geometry and the boundary conditions applied on the beam are given in Fig. 9. It should be noted that two finite elements have been erased from the mesh for creating the notch (see Fig. 6).

This test compares the three retained approaches in order to underline the eventual stress locking that can occur when simulating large deformations in a structure (see Fig. 7). The notch introduced in the mesh provokes a rotating crack, i.e., the test case is more discriminating than an unnotched beam.

The Rashid like model is the sole model which does not exhibit stress locking. This model is therefore used for a further test aiming at comparing the answer of the notched beam, meshed with regular T3 and Q4 elements and a coarse T3 mesh (see Fig. 8).

Figure 8 underlines that the results are basically mesh independent.

5.3 Conclusion

According to our test results, the Rashid-like model did not exhibit stress locking and together with the proposed probabilistic approach proved to be mesh independent. For these reasons, this model has been retained for the further probabilistic analysis.

6 Validation example

The modeling strategy presented in the previous sections is here compared to an original experimental test performed

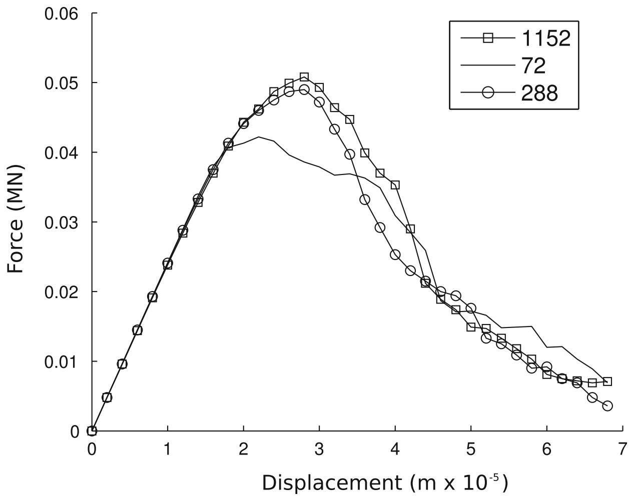

Fig. 5 Traction test: Rashid model with different mesh

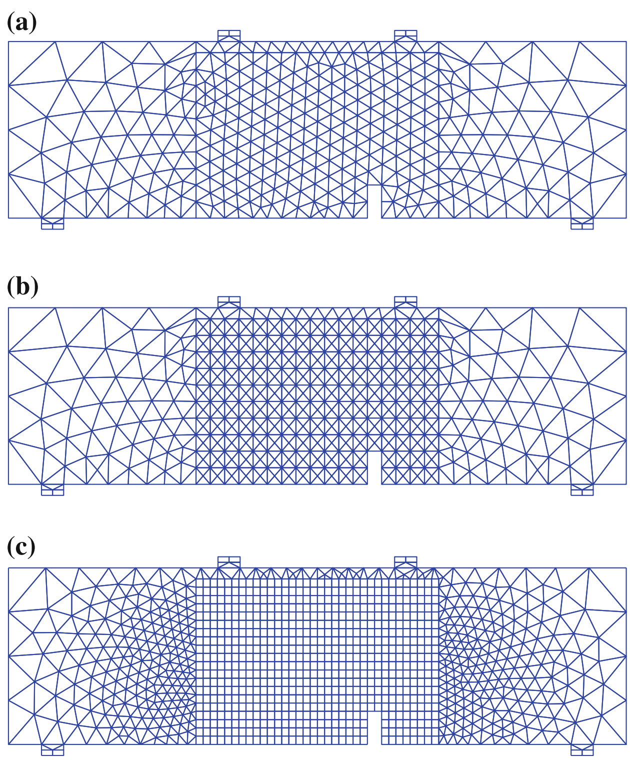

Fig. 6 Notched beam: (**a**) coarse T3 mesh, (**b**) T3 and (**c**) Q4 regular mesh

at LCPC. The experiment consists of a four point displacement-controlled bending test on a plain concrete beam. The beam geometry is given in Fig. 9, the concrete used is an ordinary concrete (E = 35GPa, f_c = 50 MPa, f_t = 3 MPa, values experimentally determined). Displacements are measured on the front face via 6 LVDTs. This feature, useful for validating the model, is generally not provided by other set up available in the literature. Moreover, it has been underlined that one of the model's

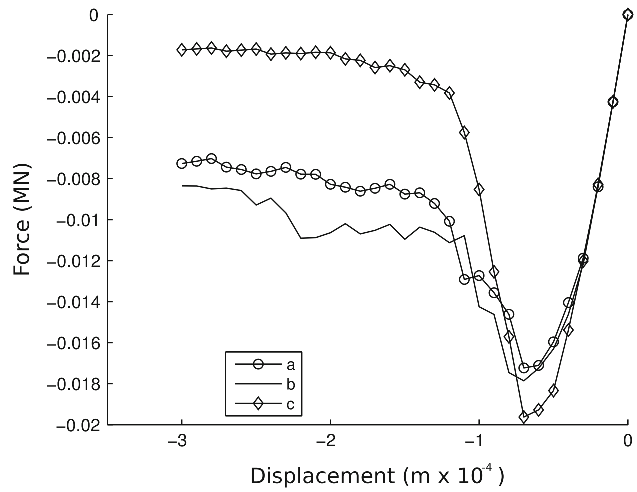

Fig. 7 Notched beam: (**a**) EFEM (**b**)smeared (**c**) Rashid model

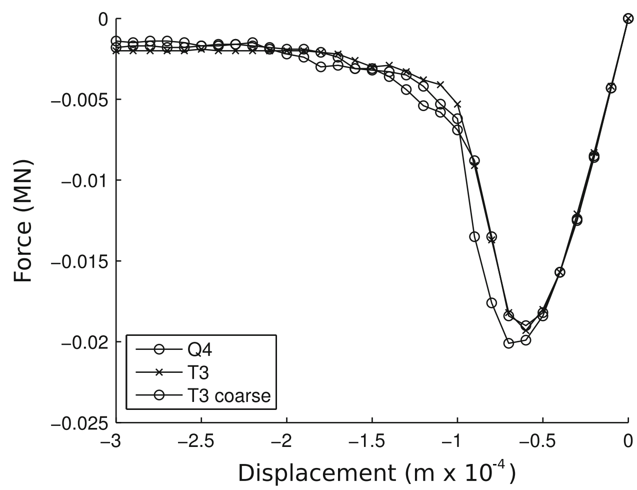

Fig. 8 Notched beam: Rashid model with different mesh

features is to require only a restricted number of parameters, but at this step of the development, these parameters are only available for the ordinary concrete used in the experimental test which is, generally speaking, different from the the concrete used in other tests available in the literature.

This set-up allowed to obtain a result which is particularly clean in particular in the post-peak phase (see Fig. 11).

The numerical–experimental probabilistic approach follows these steps:

- As the mechanical behavior of the concrete is not known in advance, 30 computations are performed with the discrete approach for simulating the uniaxial tensile behavior of the concrete. A mean behavior is deduced from these results (dashed line in Fig. 10) and is considered as the "experimental" behavior.

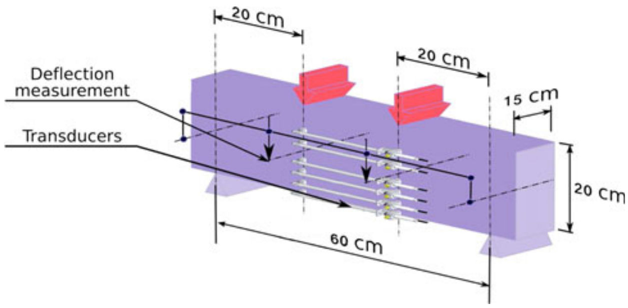

Fig. 9 Experimental set-up

- m_{σ_m} , s_{σ_m} , m_{w_d} are obtained via Eqs. (9) and (10), while s_{w_d} is given via an inverse analysis. The parameters identification allows to obtain a mean curve (plain line in Fig. 10) which is compared to the "numerical–experimental" behavior. The numerical/experimental deviation has been minimized using a simple least squares method.
- These parameters are used for modeling the bending behavior. Again 30 computations are performed.

It can be argued that 30 computations constitutes an important numerical effort. However, one should not forget that the proposed model is probabilistic and it requires, consequently, a large number of computations for obtaining a reliable answer. It should also be noted that an experimental campaign is meaningless if an insufficient number of tests are performed and that a numerical campaign is intrinsically less demanding than an equivalent experimental campaign.

Moreover, it has to be said that the number of computations depends on the test-type that is being performed. In the case of the proposed beam (small volume and fragile fracture), a large number of tests is required for obtaining a statistically acceptable result. On a larger problem such as an industrial structure (often in RC) the dispersion of the results is smaller and, consequently, the number of the required computations for obtaining a significant result is smaller.

The beam has been modeled via T3 regular elements, only the central part is subjected to a statistical distribution of the mechanical properties (see Fig. 12). Given the results obtained in Sect. 5, the Rashid-like model has been used. Results are given in Fig. 11.

The correlation between the experimental result and the mean curve is quite good as the experimental result is contained in the set of the numerical answers and is close to the mean answer.

The post-peak phase is described in a satisfactory manner, though the numerical answer is less gradual than the experimental behavior. Such a behavior is expected in a 2D analysis as the spatial nature of cracks (leading to

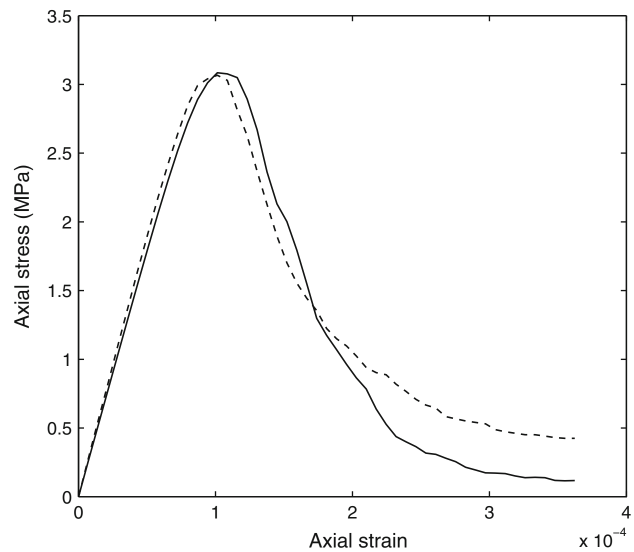

Fig. 10 Traction test for parameters identification (via the inverse analysis)

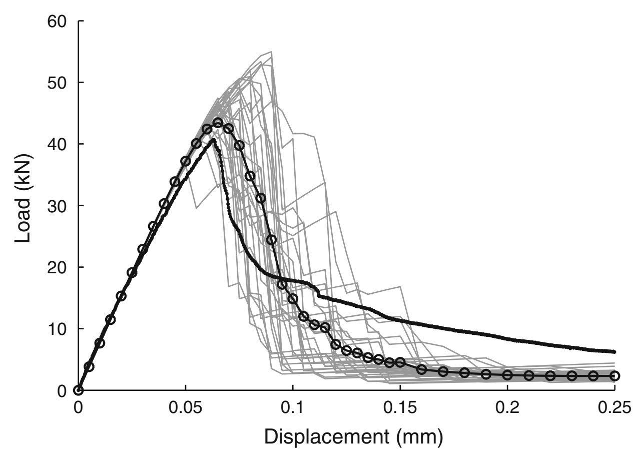

Fig. 11 Global behavior: experimental (bold), numerical answer (grey) and mean (circles)

bridging, friction, …) is not taken into account (see also the remarks about energy dispersion in Sect. 4). A similar evidence has been also observed in [7]: in conclusion, a proper description of the post-peak behavior can be achieved only via a full 3D analysis and will be presented in upcoming papers.

The macroscopic model provides not only a global answer but also some local information on cracks opening and distribution. In Figs. 12 and 14 some typical crack patterns and local displacements are presented. It is worth noting that the crack openings can be simply obtained by removing the elastic part of the local displacements: in other words, the numerical model correctly describes the crack openings. It is also interesting to observe that not only a main macro crack is represented but also the multi-cracking character of the global failure is represented (Fig. 13).

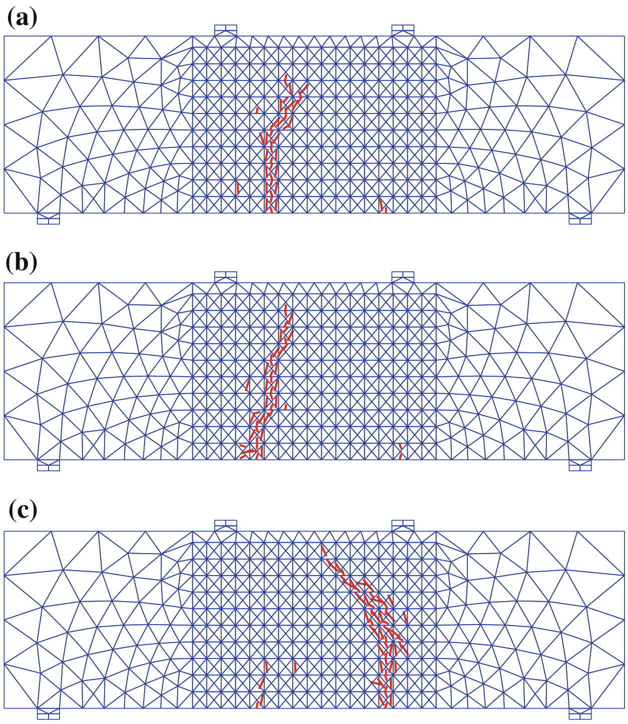

Fig. 12 Typical crack patterns

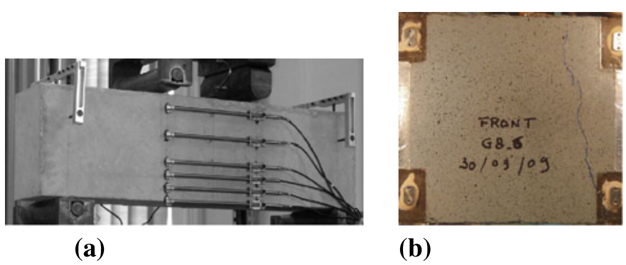

Fig. 13 **a** Experimental set-up and **b** typical crack pattern (courtesy of C.Boulay)

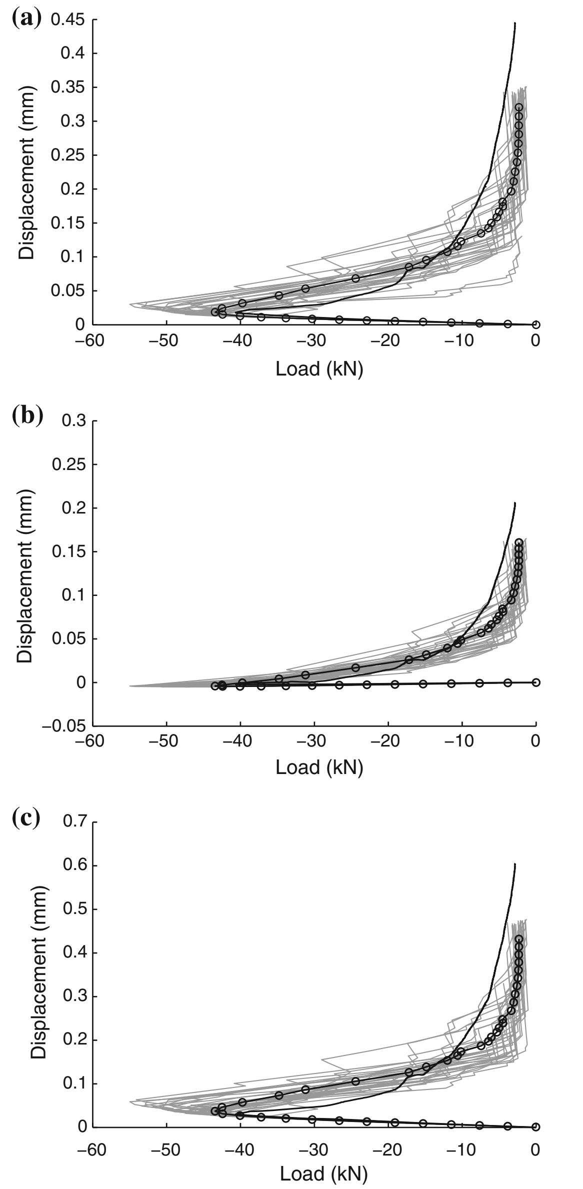

Fig. 14 Local displacements (numerical -grey-, mean answer -circles- and experimental -bold-)

7 Conclusions

In this paper, a model representing cracking processes and localized crack patterns in concrete (subjected to tension) through a robust continuum approach coupled with a simple numerical modeling of discontinuities is presented. One should not forget that the simplicity of the numerical modeling is meaningful if and only if some extra information are provided. First of all, the concrete is locally (at the scale of the finite element) replaced by an energetically equivalent material in which cracks are diffused until the local failure is reached. Later, strength and (possible) dissipation in the equivalent material are taken into account as random variables, functions of the volume and the heterogeneity of the material (volume effect laws). These aspects require a considerable numerical/experimental effort. An original strategy making use of existing numerical tools has been then presented and exploited to reduce the need of experimental analysis. It should also be pointed out that the model is probabilistic in the sense that mechanical properties are random variables, but the computations themselves remain deterministic. In fact, the spirit of the proposed approach makes that a single calculation is not pertinent and useful and that it is necessary to perform repeated computations (like in Montecarlo methods) for obtaining a meaningful answer. Consequently, the proposed approach is naturally oriented towards structural reliability analysis.

The model allows to properly take into account scale effects as well as the heterogeneous nature of concrete, providing a consistent global answer as well as local information such as crack patterns and local displacements/crack openings. These information turns out to be particularly useful for the evaluation of structures life expectancy.

However, further numerical–experimental comparisons seem necessary in order to fully validate the proposed approach. Moreover, the enhancement of the model towards 3D seems an inevitable enhancement to take into account the complex nature and geometry of cracks and giving a satisfactory description of the local-global behavior of a structure.

Finally, it is important to remind that the model presented in this paper deals only with tensile cracking behavior. In order to provide a more general description of the cracking behavior, it is then necessary to introduce in the model a cracking criterion in shear, as already proposed in the discrete approach [9]. This further enhancement will be the object of forthcoming works.

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