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Macro Element Methods in FEM for 3-D Electromagnetic Radiation Problems

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***Abstract* — For the first time, macro-element technology is introduced into the vector finite element method with the perfect matching layer for analyzing 3-D electromagnetic radiation problems. By defining a macro element domain and non-macro element domain inside the finite element domain and transferring the information from the non-macro element domain onto the surface of the macro element domain, the number of unknowns in the matrix equation of the original problem and calculation time can be greatly reduced in cases where there is repetitive structural variation or when optimizing the parameters inside the macro element part. The correctness and efficiency of the proposed method are demonstrated through the numerical experiments of several typical antennas.**

***Index Terms* — Electromagnetic Radiation, Finite Element Method, Perfect Matching Layer, Macro Element Method.**

I. INTRODUCTION

In the design and optimization of modern electronic and communication equipments or systems modularization technology is being used increasingly. The application of modularization technology in the finite element method (FEM) is called macro element (ME) technology. This name may come from the fact that the defined module often contains many micro FEM mesh elements, in this case it can be considered to be a relatively “macro” part

The ME technology in FEM was firstly adopted in analyzing electric potential of the high voltage insulating system with repetitive substructures [1]. If these same substructures are defined as macro elements and the same meshing and numerical processes are applied to them, then the overall amounts of computation are reduced greatly. This method has also been used in the analysis of transmission and reflection of a hollow rectangular waveguide with dielectric posts [2]. The dielectric posts, which need finer meshing were defined as ME and the other domain needing a relatively sparse meshing was defined as non-ME. Therefore the final number of unknowns is reduced to the order of the ME part. However, the research on FEM with ME technology for 3-D radiation and scattering analysis is rarely reported.

This paper reports research on ME technology based on FEM with PML (Perfect Matched Layers) for the electromagnetic radiation analysis. Firstly the relatively small domain containing the radiation body and the feed is defined as the ME and the other region including the PML layers is defined as the non-ME part, then the computational information inside the non-ME part is transferred onto the surface of the ME part and saved for repetitive use. So the number of unknowns in the matrix equation of the original problem and the calculation time can be greatly reduced in the case of repetitive varying the structure or optimizing the parameters inside the macro element part. Finally, the accuracy and efficiency of this method are confirmed by its application to several typical antennas.

The new contributions of this paper are: ①The macro-element technology is introduced into the vector FEM-PML for analyzing 3-D electromagnetic radiation problems for the first time and the corresponding formulas are presented. ②Detailed comparisons are given of computational complexity and time between the direct FEM-PML method and the FEM-ME-PML method using several typical antennas. Further application of the ME technology in the FEM is expected to lead to additional benefits.

II. FOUNDATIONAL THEORY AND FORMULAS

As shown in Fig.1, the region Ω_1, which needs repetitive changes in the designing procedure, is defined as the ME part with boundary $\mathbf{S}_1$; the region Ω_0, which has constant parameters in geometry and dielectric characteristics is defined as the non-ME part with inner bounday $\mathbf{S}_1$ and outer bounday $\mathbf{S}_0$. $\mathbf{S}_0$ is also the outer boundary of general computational region.

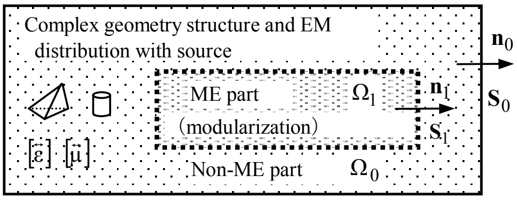

Fig.1 The schematic figure of FEM macro element theory

For part Ω_1, from the *Maxwell* equation

$$\nabla\times\mathbf{E} = -\mathbf{J}_{1m} - \mathbf{j}\omega\mu_0\hat{\mu}_r\mathbf{H} \quad (1)$$

$$\nabla\times\mathbf{H} = \mathbf{J}_{1e} + \mathbf{j}\omega\varepsilon_0\hat{\varepsilon}_r\mathbf{E} \quad (2)$$

in which $\mathbf{J}_{1e}$, $\mathbf{J}_{1m}$ are the electronic and magnetic current source impressed in the ME part Ω_1. Expend the electronic field with *Whitney* base function [3]

$$\mathbf{E} = \sum_{j=1}^{N_1}\mathbf{E_j W_j^e} = \sum_{j=1}^{N_1^I}\mathbf{E_j W_j^e} + \sum_{j=1}^{N_1^S}\mathbf{E}_{N_1^I+j}\mathbf{W}^e_{N_1^I+j} \quad (3)$$

in which $\mathbf{N}_1$ is the total base function number, $\mathbf{N_1^I}$ is the inside base function number (excluding $\mathbf{S}_1$), $\mathbf{N_1^S}$ is the surface base function number on $\mathbf{S}_1$. Adopting $\nabla\times\mathbf{W_i^e}$ and $\mathbf{W_i^e}$ as the weight function to (1) and (2) and implement the method of weighting residue, the equations on electric coefficient $[\mathbf{E}_1]$ can be got

$$\{[\mathbf{D}]-\mathbf{k}_0^2[\mathbf{S}]\}[\mathbf{E}_1] = -[\mathbf{J}_{1m}] - \mathbf{jk}_0\mathbf{Z}_0\{[\mathbf{J^S}]+[\mathbf{J}_{1e}]\} \quad (4)$$

in which $\mathbf{k}_0 = (\omega/\mathbf{c})$ and $\mathbf{Z}_0$ are the wave number and wave impedance in free space repectively; The matrix elements are

$$\begin{aligned}
\mathbf{D_{ij}} &= \int_{\Omega_1}\nabla\times\mathbf{W_i^e}\cdot\bar{\mu}_r^{-1}\nabla\times\mathbf{W_j^e}\mathbf{dV} \\
\mathbf{S_{ij}} &= \int_{\Omega_1}\mathbf{W_i^e}\cdot\hat{\varepsilon}_r^{-1}\mathbf{W_j^e}\mathbf{dV} \\
\mathbf{J_{1mi}} &= \int_{\Omega_1}\nabla\times\mathbf{W_i^e}\cdot\bar{\mu}_r^{-1}\mathbf{J_{1m}dV} \quad (5)\\
\mathbf{J_i^S} &= \oint_{\mathbf{S}}\mathbf{W_i^e}\cdot(-\mathbf{n}_1)\times\mathbf{H}_1\mathbf{dS} \\
\mathbf{J_{1ei}} &= \int_{\Omega_1}\mathbf{W_i^e}\cdot\mathbf{J_{1e}dV}.
\end{aligned}$$

Expend the tangent magnetic field $\mathbf{H}_1$ with $\mathbf{W_j^e}$ on $\mathbf{S}_1$

$$(-\mathbf{n}_1)\times\mathbf{H}_1 = \sum_{j=1}^{N_1^S}\mathbf{H^S}_{N_1^I+j}\mathbf{W^e}_{N_1^I+j}. \quad (6)$$

Substitute (6) into $\mathbf{J_i^S}$ in (5) and distinguish the inside and outside electronic field coefficient in ME part, the (4) can be rewritten as

$$\begin{bmatrix}\mathbf{K_1^{II}} & \mathbf{K_1^{IS}}\\ \mathbf{K_1^{SI}} & \mathbf{K_1^{SS}}\end{bmatrix}\begin{bmatrix}\mathbf{E_1^I}\\ \mathbf{E_1^S}\end{bmatrix} = -\begin{bmatrix}\mathbf{J_{1m}^I}\\ \mathbf{J_{1n}^S}\end{bmatrix} - \mathbf{jk}_0\mathbf{Z}_0\begin{bmatrix}0\\ \mathbf{P_1^{SS}}\cdot\mathbf{H_1^S}\end{bmatrix} - \mathbf{jk}_0\mathbf{Z}_0\begin{bmatrix}\mathbf{J_{1e}^I}\\ \mathbf{J_{1e}^S}\end{bmatrix} \quad (7)$$

in which

$$\mathbf{P_{1ij}^{SS}} = \oint_{S_1}\mathbf{W}_{N_1^I+i}\cdot\ \mathbf{W}_{N_1^I+j}\mathbf{dS} \quad (8)$$

and the superscript $\mathbf{I}$ and $\mathbf{S}$ indicate the inside or outside quantities. Define $[\mathbf{K}]=[\mathbf{D}]-\mathbf{k}_0^2[\mathbf{S}]$. For single frequency simulation, matrix $[\mathbf{K}]$ can be stored and for wide band simulation the $[\mathbf{D}]$ and $[\mathbf{S}]$ can be stored respectively. If the magnetic field $[\mathbf{H_1^S}]$ is known, then the $[\mathbf{E_1^I}]$ and $[\mathbf{E_1^S}]$ can be solved. The $[\mathbf{H_1^S}]$ is unknown by now, however it can be expressed by the field in region Ω_0.

B. The FEM formulas in non-ME domain and the admittance matrix

With the same procedure, the formulas in non-ME domain can be deduced similarly as (1) – (8), in which the $\mathbf{J_i^S}$ is adjusted to

$$\begin{aligned}
\mathbf{J_i^{S_1+S_0}} &= \oint_{S_1}\mathbf{W_i^e}\cdot\mathbf{n}_1\times\mathbf{HdS} + \oint_{S_0}\mathbf{W_i^e}\cdot(-\mathbf{n}_0)\times\mathbf{HdS} \quad (9)\\
&= \mathbf{J_i^{S_1}} + \mathbf{J_i^{S_0}}.
\end{aligned}$$

When applying the PML method as boundary truncating condition, $\mathbf{S}_0$ is PEC surface and $[\mathbf{E_0^{S_0}}]=0$, so (7) for region Ω_0 will be

$$\begin{bmatrix}\mathbf{K_0^{II}} & \mathbf{K_0^{IS}}\\ \mathbf{K_0^{SI}} & \mathbf{K_0^{SS}}\end{bmatrix}\begin{bmatrix}\mathbf{E_0^I}\\ \mathbf{E_0^S}\end{bmatrix} = -\begin{bmatrix}\mathbf{J_{0m}^I}\\ \mathbf{J_{0m}^S}\end{bmatrix} - \mathbf{jk}_0\mathbf{Z}_0\begin{bmatrix}0\\ \mathbf{P_0^{SS}}\cdot\mathbf{H_0^S}\end{bmatrix} - \mathbf{jk}_0\mathbf{Z}_0\begin{bmatrix}\mathbf{J_{0e}^I}\\ \mathbf{J_{0e}^S}\end{bmatrix} \quad (10)$$

in which $\mathbf{J_{0e}}$ and $\mathbf{J_{0m}}$ are the electronic and magnetic current source impressed in the non-ME part Ω_0

If the mesh data between the ME domain and the non-ME domain are identical on $\mathbf{S}_1$, the FEM coefficents on $\mathbf{S}_1$ meet the relation of $[\mathbf{E_1^S}]=[\mathbf{E_0^S}]$ and $[\mathbf{H_1^S}]=-[\mathbf{H_0^S}]$. So (7) and (10) can be combined into a FEM matrix the same with the equation of direct FEM-PML method.

From (10), the admittance relation between $[\mathbf{H_0^S}]$ and $[\mathbf{E_0^S}]$ can be constructed as

$$[\mathbf{H}_0^{\mathbf{S}}]=[\mathbf{Y}_0][\mathbf{E}_0^{\mathbf{S}}]+[\mathbf{H}_{0\mathbf{c}}^{\mathbf{S}}] \quad (11)$$

in which

$$[\mathbf{Y}_0]=-\frac{1}{\mathbf{jk}_0\mathbf{Z}_0}[\mathbf{P}_0^{\mathbf{SS}}]^{-1}\left([\mathbf{K}_0^{\mathbf{SS}}]-[\mathbf{K}_0^{\mathbf{SI}}][\mathbf{K}_0^{\mathbf{II}}]^{-1}[\mathbf{K}_0^{\mathbf{IS}}]\right) \quad (12)$$

is the admittance matrix equations.

$$[\mathbf{H}_{0\mathbf{c}}^{\mathbf{S}}]=[\mathbf{P}_0^{\mathbf{SS}}]^{-1}\Big\{[\mathbf{J}_{0\mathbf{m}}^{\mathbf{S}}]+\mathbf{jk}_0\mathbf{Z}_0[\mathbf{J}_{0\mathbf{e}}^{\mathbf{S}}]-$$

$$[\mathbf{K}_0^{\mathbf{SI}}][\mathbf{K}_0^{\mathbf{II}}]^{-1}\Big([\mathbf{J}_{0\mathbf{m}}^{\mathbf{I}}]+\mathbf{jk}_0\mathbf{Z}_0[\mathbf{J}_{0\mathbf{e}}^{\mathbf{I}}]\Big)\Big\}. \quad (13)$$

(13) reflects the contribution of $\mathbf{J}_{0\mathbf{e}}$ and $\mathbf{J}_{0\mathbf{m}}$ in non-ME parts onto the surface of ME outer boundary. $[\mathbf{Y}_0]$ and $[\mathbf{H}_{0\mathbf{c}}^{\mathbf{S}}]$ are only decided by the information of Ω_0 domain , and can be saved beforehand for repetitively application.

C. The information transmission from non-ME domain to ME domain

Parameters in the ME domain presented in this paper are changeable. The mesh data on $\mathbf{S}_1$ surface may be different between the non-ME domain and ME domain. The eletric and magnetic fields relationship on $\mathbf{S}_1$ between these two domain can be calculated by interpolation method

$$\begin{aligned}[\mathbf{E}_0^{\mathbf{S}}]_{\mathbf{N}_0^s\times 1}&=[\mathbf{T}_{01}]_{\mathbf{N}_0^s\times\mathbf{N}_1^s}[\mathbf{E}_1^{\mathbf{S}}]_{\mathbf{N}_1^s\times 1}\\ [\mathbf{H}_1^{\mathbf{S}}]_{\mathbf{N}_1^s\times 1}&=-[\mathbf{T}_{10}]_{\mathbf{N}_1^s\times\mathbf{N}_0^s}[\mathbf{H}_0^{\mathbf{S}}]_{\mathbf{N}_0^s\times 1}\end{aligned} \quad (14)$$

in which $[\mathbf{T}_{01}]$ and $[\mathbf{T}_{10}]$ are the interpolation transmission matrix as in reference [2]. Apply (14) into (12) and (13), (7) can be rewritten as

$$\begin{bmatrix}\mathbf{K}_1^{\mathbf{II}} & \mathbf{K}_1^{\mathbf{IS}}\\ \mathbf{K}_1^{\mathbf{SI}} & \widetilde{\mathbf{K}}_1^{\mathbf{SS}}\end{bmatrix}\begin{bmatrix}\mathbf{E}_1^{\mathbf{I}}\\ \mathbf{E}_1^{\mathbf{S}}\end{bmatrix}=-\begin{bmatrix}\mathbf{J}_{1\mathbf{m}}^{\mathbf{I}}\\ \mathbf{J}_{1\mathbf{m}}^{\mathbf{S}}\end{bmatrix}-\mathbf{jk}_0\mathbf{Z}_0\begin{bmatrix}0\\ \mathbf{J}_{1\mathbf{c}}^{\mathbf{S}}\end{bmatrix}-$$

$$\mathbf{jk}_0\mathbf{Z}_0\begin{bmatrix}\mathbf{J}_{1\mathbf{e}}^{\mathbf{I}}\\ \mathbf{J}_{1\mathbf{e}}^{\mathbf{S}}\end{bmatrix} \quad (15)$$

in which

$$[\widetilde{\mathbf{K}}_1^{\mathbf{SS}}]=[\mathbf{K}_1^{\mathbf{SS}}]-\mathbf{jk}_0\mathbf{Z}_0[\mathbf{P}_1^{\mathbf{SS}}][\mathbf{T}_{10}][\mathbf{Y}_0][\mathbf{T}_{01}] \quad (16)$$

$$[\mathbf{J}_{1\mathbf{c}}^{\mathbf{S}}]=[\mathbf{P}_1^{\mathbf{SS}}][\mathbf{T}_{10}][\mathbf{H}_{0\mathbf{c}}^{\mathbf{S}}] \quad (17)$$

(15) is the final matrix equations for FEM-ME-PML method, in which all the information in the non-ME domain are transmitted onto the surface of the ME domain by $[\mathbf{Y}_0]$ and $[\mathbf{H}_{0\mathbf{c}}^{\mathbf{S}}]$.

The FEM-ME method will reduced the unknowns greatly due to the final solving domain being defined in the ME domain. Although the transmission procedure in (16) and (17) needs extra computation costs, the overall efficiency can be realized and compensated when repetitively optimizing the parameters inside the ME domain.

III. NUMERICAL RESULTS

Firstly, a dipole antenna model is adopted to test the correctness of the FEM-ME-PML method with the PML parameter from reference [4].

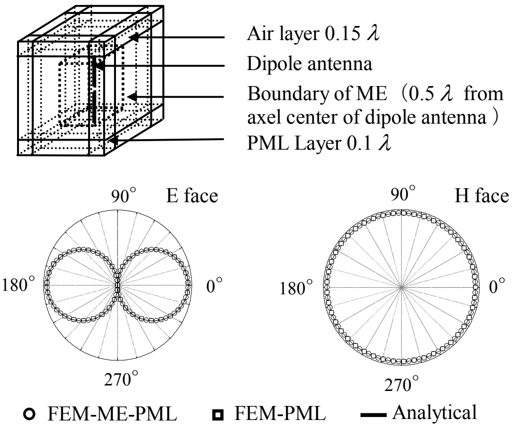

Fig.2. Dipole antenna model and simulation results for FEM-ME-PML and FEM-PML methods

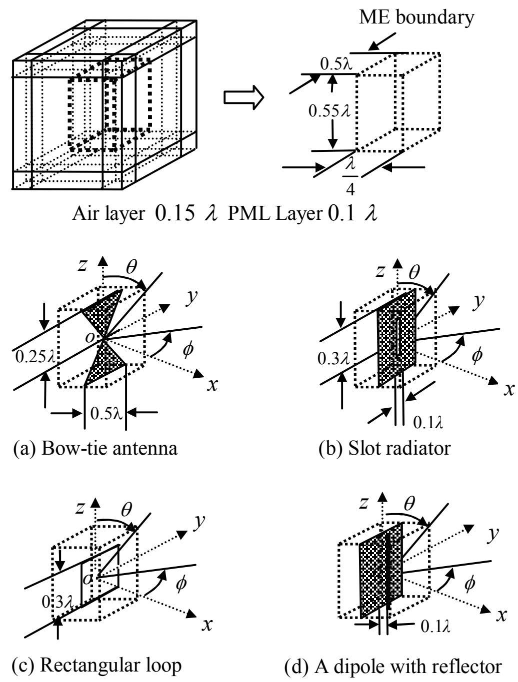

Fig.3 Models for testing FEM-ME-PML method

Then four typical antennas are adopted to research the computational efficiency of FEM-ME-PML method. Here, the relatively small region containing

TABLE I
Comparison of calculation time by models in Fig.4 (the FEM-PML method vs. the FEM-ME-PML method)

	FEM-PML (s) (Total time)	FEM-ME-PML (s)		
		Transmission Matrix	Matrix solving	Total time
Bow-tie antenna	97.0	787.3	11.9	799.2
Slot radiator	105.5	0	11.3	11.3
Rectangular loop	132.2	0	11.4	11.4
A dipole with reflector	107.6	0	11.0	11.0

the antenna body is defined as the ME part (different antennas are located inside this region) and the other part including the PML region is defined as the non-ME part, as shown in Fig.3. By using the FEM-PML direct method codes and FEM ME-PML codes, the identical radiation pattern are obtained as shown in Fig.4. The results for the bow-tie antenna agree with those in reference [5], which also validates the FEM-ME-PML codes.

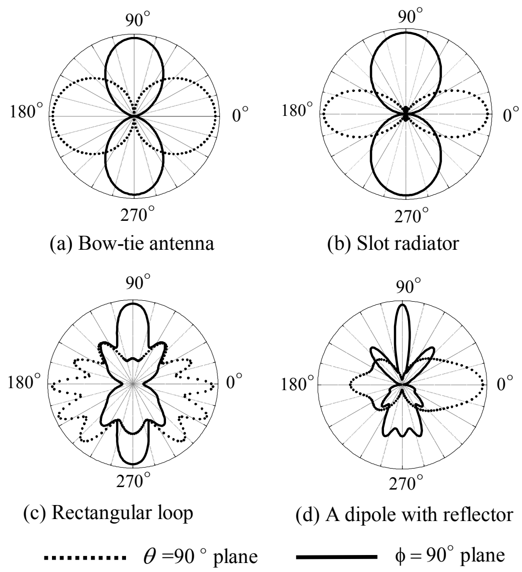

Fig.4 Result of FEM-ME-PML method for models in Fig.3

Then the efficiency of the FEM-ME-PML is analyzed. The comparison of the computation time between the direct FEM-PML method and the FEM-ME-PML method for the multi-time applications is shown in Table 1. The GMRES solver from reference [6] is selected on a P4-820, 2GRAM PC.

From Table 1, it can be seen that although the process of transmission matrix (16) and (17) needs very long time (787.3 s) for the first application of ME (bow-tie antennas), these results can be saved and reused in the following applications of ME, therefore the total time is greatly reduced. The efficiency of the method is evident.

IV. CONCLUSION

This paper introduces the macro element (ME) technology into the vector FEM with PML for analyzing problems of 3-D electromagnetic radiation. The correctness and efficiency of the proposed method are demonstrated through the numerical experiments of several typical antennas.

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