


# A simple illustration

Example: Nonlinear PDE

$$\frac{\partial^2 u}{\partial x^2} + u^2 = \lambda \quad + \text{b.c}$$
$$\lambda = a$$

$$u(a) = u_0 + au_1 + a^2u_2 + \dots + a^Nu_N$$

$$\lambda(a) = \lambda_0 + a\lambda_1 + a^2\lambda_2 + \dots + a^N\lambda_N$$


$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u_0}{\partial x^2} + a \frac{\partial^2 u_1}{\partial x^2} + a^2 \frac{\partial^2 u_1}{\partial x^2} + \dots + a^N \frac{\partial^2 u_N}{\partial x^2}$$

Exemple: EDP non linéaire

$$\left\{ \begin{array}{l} \frac{\partial^2 u}{\partial x^2} + u^2 = \lambda \\ \lambda = a \end{array} \right. \left\{ \begin{array}{l} \frac{\partial^2 u_0}{\partial x^2} + a \frac{\partial^2 u_1}{\partial x^2} + a^2 \frac{\partial^2 u_2}{\partial x^2} + \dots + a^N \frac{\partial^2 u_N}{\partial x^2} \\ + u_0^2 + a u_0 u_1 + a^2 u_0 u_2 + a^3 u_0 u_3 + \dots + a^N u_0 u_N \\ + a u_1 u_0 + a^2 u_1 u_1 + a^3 u_1 u_2 + \dots + \\ + a^2 u_2 u_0 + a^3 u_2 u_1 + \dots \\ + a^3 u_3 u_0 + \dots \\ + \dots \\ = \lambda_0 + a \lambda_1 + a^2 \lambda_2 + \dots + a^N \lambda_N \\ \lambda_0 + a \lambda_1 + a^2 \lambda_2 + \dots + a^N \lambda_N = a \end{array} \right.$$

Identification des même puissances  
de «  $a$  »:

$$\begin{aligned}
 & \left\{ \begin{aligned}
 & \frac{\partial^2 u_0}{\partial x^2} + a \frac{\partial^2 u_1}{\partial x^2} + a^2 \frac{\partial^2 u_2}{\partial x^2} + \dots + a^N \frac{\partial^2 u_N}{\partial x^2} \\
 & + u_0^2 + a u_0 u_1 + a^2 u_0 u_2 + a^3 u_0 u_3 + \dots + a^N u_0 u_N \\
 & + a u_1 u_0 + a^2 u_1 u_1 + a^3 u_1 u_2 + \dots + \\
 & + a^2 u_2 u_0 + a^3 u_2 u_1 + \dots \\
 & + a^3 u_3 u_0 + \dots \\
 & + \dots \\
 & = \lambda_0 + a \lambda_1 + a^2 \lambda_2 + \dots + a^N \lambda_N \\
 & \lambda_0 + a \lambda_1 + a^2 \lambda_2 + \dots + a^N \lambda_N = a
 \end{aligned} \right.
 \end{aligned}$$

Ordre 0

$$\begin{aligned}
 & \frac{\partial^2 u_0}{\partial x^2} + u_0^2 = \lambda_0 \\
 & \lambda_0 = 0
 \end{aligned}$$

Ordre 0

$$\frac{\partial^2 u_0}{\partial x^2} + u_0^2 = \lambda_0$$

$$\lambda_0 = 0$$

On suppose  $u_0$  connue, exemple:

$$u_0 = 0$$

$$\lambda_0 = 0$$

Identification des même puissances  
de a:

$$\left\{ \begin{aligned}
 & \frac{\partial^2 u_0}{\partial x^2} + a \frac{\partial^2 u_1}{\partial x^2} + a^2 \frac{\partial^2 u_2}{\partial x^2} + \dots + a^N \frac{\partial^2 u_N}{\partial x^2} \\
 & + u_0^2 + a u_0 u_1 + a^2 u_0 u_2 + a^3 u_0 u_3 + \dots + a^N u_0 u_N \\
 & + a u_1 u_0 + a^2 u_1 u_1 + a^3 u_1 u_2 + \dots + \\
 & + a^2 u_2 u_0 + a^3 u_2 u_1 + \dots \\
 & + a^3 u_3 u_0 + \dots \\
 & + \dots \\
 & = \lambda_0 + a \lambda_1 + a^2 \lambda_2 + \dots + a^N \lambda_N \\
 & \lambda_0 + a \lambda_1 + a^2 \lambda_2 + \dots + a^N \lambda_N = a \cdot 1
 \end{aligned} \right.$$

Ordre 1

$$\begin{aligned}
 & \frac{\partial^2 u_1}{\partial x^2} + 2u_0 u_1 = \lambda_1 \\
 & \lambda_1 = 1
 \end{aligned}$$

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$$\lambda_1 = 1$$

$$\frac{\partial^2 u_1}{\partial x^2} = \lambda_1$$

$$\lambda_1 = 1$$

On trouve

$u_1$

$$\lambda_1 = 1$$

Identification des même puissances  
de a:

$$\left\{ \begin{aligned}
 & \frac{\partial^2 u_0}{\partial x^2} + a \frac{\partial^2 u_1}{\partial x^2} + a^2 \frac{\partial^2 u_2}{\partial x^2} + \dots + a^N \frac{\partial^2 u_N}{\partial x^2} \\
 & + u_0^2 + a u_0 u_1 + a^2 u_0 u_2 + a^3 u_0 u_3 + \dots + a^N u_0 u_N \\
 & + a u_1 u_0 + a^2 u_1 u_1 + a^3 u_1 u_2 + \dots + \\
 & + a^2 u_2 u_0 + a^3 u_2 u_1 + \dots \\
 & + a^3 u_3 u_0 + \dots \\
 & + \dots \\
 & = \lambda_0 + a \lambda_1 + a^2 \lambda_2 + \dots + a^N \lambda_N \\
 & \lambda_0 + a \lambda_1 + a^2 \lambda_2 + \dots + a^N \lambda_N = a.1
 \end{aligned} \right.$$

Ordre 2

$$\begin{aligned}
 & \frac{\partial^2 u_2}{\partial x^2} + 2u_0 u_2 + u_1^2 = \lambda_2 \\
 & \lambda_2 = 0
 \end{aligned}$$

Ordre 2

$$\frac{\partial^2 u_2}{\partial x^2} + 2u_0 u_2 + u_1^2 = \lambda_2$$
$$\lambda_2 = 0$$

$$\frac{\partial^2 u_2}{\partial x^2} = -u_1^2$$
$$\lambda_2 = 0$$

On trouve

$$u_2$$
$$\lambda_1 = 1$$

Etc...