A multi-scale finite element model for inelastic behavior of heterogeneous structures and its parallel computing implementation

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Abstract

In this work we consider a strongly coupled multi-scale problem in the context of the inelastic structural mechanics. Supposing the significant but not neglecting ratio between scales, we replace the somehow more standard phenomenological and analytical homogenization approaches by a lower level numerical description of the micro-structural behavior. More particularly, we take continuum damage and plasticity finite element method model to describe the matrix-inclusion type of micro-structure (e.g. porous or hard inclusion composite). The micro FEM model is then coupled to the macro FEM model through the localized Lagrange multiplier approach as introduced in [5]. This multi-scale strategy being very well adapted to parallel computing, we parallelized the algorithm using the *Communication Template Library CTL*, a recent development of the institute of Scientifc Computing in Braunschweig, Germany. The efficiency of the implementation is shown on numerical large scale examples.

1 Introduction

Composite material have always had an important role in structural engineering. Either involuntarily, because the material processing cannot always

avoid some final defaults like porosity or voluntarily because one wants to improve material resistance, like in hard inclusion composites (e.g. metal composites, concrete). Unfortunately, heterogeneous nature of materials induces more significant heterogeneous field solutions in a structural loading framework and consequently makes modeling in general more difficult.

The homogenization techniques, which are able to model the composite materials can be divided into two groups: *analytical* and *numerical*. In the first approach one does some physically justified assumptions on the heterogeneity of the micro deformation and stress fields and defines the macroscopic response through the constituent behaviors analytically (see e.g. [1], [2] or [8]). On the other hand, in numerical homogenization, the micro response is obtained by a numerical method, like the *finite element method* (FEM). When the scales are separated enough the micro-level FE computation are carried out for each integration point of the macro FE computation, in order to obtain constitutive stress-strain relation (e.g. see [3]).

Since in many civil and mechanical engineering applications the scales are strongly coupled, we have proposed a multi-scale strategy, where the fine microstructure representation is not introduced at the level of the Gauss point, but on the level of the whole element (see [4] and [5]). We apply the localized Lagrangian multiplier method to couple the two scales ([7] and [6]). In this context, the macro scale is the frame situated between different sub-domain, which can be interpreted as micro scale, because of its much finer FE representation. Thus, for each macro element, we have a choice using either a macroscopic model, obtained in the most convenient way, or a very fine FE model, which replaces the less exact constitutive relations.

An important characteristics of our approach (see [4] and [5]) is the local nature of micro calculations. The microstructure representing FE models communicate between them exclusively by the macro degrees of freedom. Hence, the substructuring does by no means affect the macro resolution procedure. In addition, the independency of micro calculations renders the micro-macro strategy easily parallelizable.

2 Formulation of the strong coupling multi-scale interface

2.1 Concept of strongly coupled scales

We consider a general class of heterogeneous structure problems submitted to an arbitrary loading and obeying non-linear physical laws resulting in phenomena like damage and plasticity. It is assumed that the scales are strongly coupled and that their evolution has to be calculated simultaneously. The FEM has shown undoubted efficiency in solid and structural

analyses and we thus decide to use it on both scales. Therefore, the structure is meshed with a macro FE mesh and the microstructure representing volume similarly with a micro FE mesh (see Figure 1).

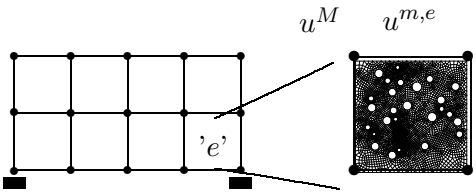

Figure 1: *Micro-macro finite element model of a simple structure. Each macro FE equally represents a meso scale window containing the microstructure information, which is again modeled by the FE mesh*

By the appropriate formulation the macro mesh element quantities (residual, stiffness etc.) are obtained from the micro FE calculation, replacing in this way the macroscopic constitutive law at the finite element level rather than at the level of a Gauss numerical integration point. The microstructure window (see Figure 1) is chosen such that its dimensions match those of the corresponding macro finite element.

2.2 Variational formulation

The coupling of the scales in our multi-scale FE model is obtained through the framework of localized Lagrange multipliers (e.g. see [7] and [6]). The macro mesh plays the role of a frame which is connected to the micro mesh through the Lagrange multipliers.

We can write the elastic potential, whose stationary value will lead to the solution of the problem, as following:

$$\Pi = \int_{\Omega^m} \Psi^m(\epsilon, \xi_k)\, dV + \int_{\Gamma} \lambda(u_m - u_M)\, dS - \int_{\partial_\sigma \Omega} u_M t\, dS - \int_{\Omega} u_m f\, dV \mapsto stat., \tag{1}$$

where u^M is macro displacement field (coarse discretization), u^m is micro displacement field (fine discretization). Accordingly, Ψ^m is micro scale free energy, ϵ is the deformation field at the micro scale, ξ^k are the internal variables at micro scale, and λ is the Lagrange multiplier field glueing two different scales. We also note that Ω defines the total domain, $\partial_\sigma \Omega$ is the part of the boundary, where tractions are applied and Γ denotes the total interface surface between the two scales. Using the stationarity condition of the potential,

$$\delta\Pi = \frac{\partial \Pi}{\partial u_m}\delta u_m + \frac{\partial \Pi}{\partial u_M}\delta u_M + \frac{\partial \Pi}{\partial \lambda}\delta\lambda = 0;\ \forall \delta u_m,\ \delta u_M,\ \delta\lambda \tag{2}$$

we can get the weak formulation in terms of three equations: *macro equilibrium* (δu_M), *micro equilibrium* (δu_m) and *micro-macro* compatibility ($\delta\lambda$).

From the present variational formulation we considered two special choices, the *displacement based* and *the force based* interface conditions. They both result from a particular choice of the Lagrange multiplier interpolation. In the displacement based formulation the displacement field is assumed linear on the interfaces, whereas in the case of force based interface the forces are linearly distributed over the interfaces. In general the displacement based interface choice supplies always a stiffer response and in most cases the force based interface give more accurate approximation (see [5]).

3 Parallel solving of equations

The applied formulation of micro-macro analyses enables a very efficient computer implementation using the possibility of distributed computing. In our approach the micro resolution of a sub-domain comunicates exclusively with the macro scale and not with other micro sub-domain. Therefore, the micro calculation can be done simultaneously on different computer processors.

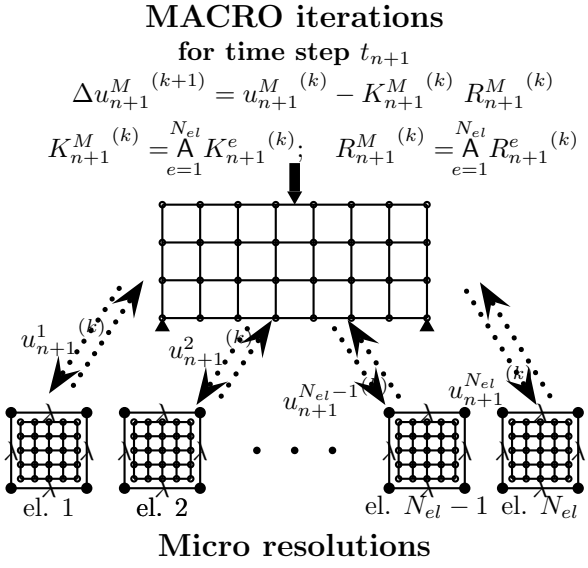

Figure 2: *Micro-macro computation parallelizable sheme. At the upper level the standard FE resolution and assembling (operator A above) procedure is carried out. The element stiffnesses and residuals are obtained through the lower level FE analysis for given nodal displacement values.*

At the computer implementation level, the micro and macro computations are carried out by two different instances of the *Finite Element Analysis Program* (*FEAP*, developped at the University of California, Berkeley) and they communicate through the *Communication Template Library CTL*, a recent development of the institute of Scientifc Computing in Braunschweig, Germany. The computational sheme is shown in the Figure 2.

4 Conclusion

We apply a micro-macro computational strategy based on the scale decomposition using the formulation principle of the localized Lagrange multipliers. The scales are strongly coupled and we use two different micro-macro interface conditions, displacement based and force based. The latter shows in most cases a better approximation, but the first enables a simpler and more robust numerical implementation. Both of them are implemented for a parallel computing architecture.

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