Problem Set #7 Due Thursday, November 10

(Problems are from *Vector Calculus* by Marsden and Tromba, sixth edition.)

1

Let $P(x,y) = 2x^3 - y^3$ and $Q(x,y) = x^3 + y^3$. Define the annulus $D = \left\{(x,y) \in \mathbb{R}^2 : a^2 \leq x^2 + y^2 \leq b^2\right\}$, where the outer circle oriented counter-clockwise and the inner circle oriented clockwise. Without using Green's theorem, compute both $\int_{\partial D} P\,dx + Q\,dy$ and $\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\,dx\,dy$.

2

Suppose D is a simple region or can be partitioned into simple regions. Suppose f is harmonic, i.e., $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. Show that $\int_{\partial D} \frac{\partial f}{\partial y}\,dx - \frac{\partial f}{\partial x}\,dy = 0$.

3

In class we used Green's theorem to show that the area of a region D, which is either simple or can be partitioned into simple regions, can be computed by $A = \frac{1}{2}\int_{\partial D} x\,dy - y\,dx$. Use this fact to compute the area inside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.