PROBLEM SET #10 DUE TUESDAY, DECEMBER 6
(Problems are from *Vector Calculus* by Marsden and Tromba, sixth edition.)

1

Let B be the unit ball, i.e., $B = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1\}$, and let $\vec{F}(x, y, z) = -y\vec{i} + x\vec{j} + z\vec{k}$. Without using Gauss's theorem, compute $\iint_{\partial B} \vec{F} \cdot d\vec{S}$ and $\iiint_B (\nabla \cdot \vec{F})\, dV$.

2

Let $\vec{F}(x, y, z) = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$. Compute (using Gauss's theorem if you feel like it) the surface integral of $\vec{F}$ over the unit sphere S.

3

Let $W \subset \mathbb{R}^3$ be the solid enclosed by the surfaces $x = y^2$, $x = 9$, $z = 0$, and $x = z$. Use Gauss's theorem to compute the flux $\iint_{\partial W} \vec{F} \cdot d\vec{S}$ where $\vec{F}(x, y, z) = (3x - 5y)\vec{i} + (4z - 2y)\vec{j} + (8yz)\vec{k}$.

4

Let W be the solid cylinder $W = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 1, \quad 0 \leq z \leq 1\}$. Compute $\iint_{\partial W} \vec{F} \cdot \vec{n}\, dS$ where $\vec{F}(x, y, z) = \vec{i} + \vec{j} + z(x^2 + y^2)^2\vec{k}$.