Problem Set 2 – Solutions

1 $\vec{z}'(t) = (1, 2t, 0)$

$\|\vec{z}'(t)\| = \sqrt{1+4t^2}$

$f(\vec{z}(t)) = t\cos 0 = t$

$$\int_{\vec{z}} f\,ds = \int_0^1 t\sqrt{1+4t^2}\,dt \qquad u = 1+4t^2,\ du = 8t\,dt$$

$$= \frac{1}{8}\int_1^5 \sqrt{u}\,du$$

$$= \frac{1}{8}\frac{2}{3}u^{3/2}\Big|_1^5 = \frac{1}{12}\left(5^{3/2}-1\right)$$

2 $$\ell(c) = \int_{\vec{z}} \|\vec{z}'(t)\|\,dt = \int_0^{\pi} \sqrt{a^2\cos^2\theta + a^2\sin^2\theta}\,d\theta = a\pi$$

$$\int_{\vec{z}} f\,ds = \int_0^{\pi} a\sin\theta \cdot a\,d\theta = -a^2\cos\theta\Big|_0^{\pi} = -a^2(-1-1) = 2a^2$$

$$\text{avg value} = \frac{2a^2}{a\pi} = \frac{2a}{\pi}$$

3 Let $f(x,y) = xy^2$. Then $\nabla f = (y^2, 2xy) = \vec{F}(x,y)$. So $\int_{\vec{z}} \vec{F}\cdot d\vec{s} = 0$.

4 $\vec{F}(\vec{z}(t))\cdot\vec{z}'(t) = 0 \ \ \forall t \Rightarrow \int_{\vec{z}} \vec{F}\cdot d\vec{s} = 0$