Problem Set 3 — Solutions

$\boxed{1}$ Note this is the integral of ∇f along γ where $f(x,y,z) = x^2yz + c$ for some $c \in \mathbb{R}$.

$$\int_\gamma \nabla f \, d\vec{s} = f(1,2,4) - f(1,1,1) = (8+c) - (1+c) = 7$$

$\boxed{2}$ First we determine $f(x,y,z) = yze^{x^2} + c$ for some $c \in \mathbb{R}$.

$$5 = f(0,0,0) = 0 + c \Rightarrow f(x,y,z) = yze^{x^2} + 5 \Rightarrow f(1,1,2) = 2e + 5$$

$\boxed{3}$
$$\left.\begin{aligned} \vec{T}_u &= \frac{\partial \vec{\Phi}}{\partial u} = 1\vec{i} + 1\vec{j} + 2v\vec{k} \\ \vec{T}_v &= \frac{\partial \vec{\Phi}}{\partial v} = -1\vec{i} + 1\vec{j} + 2u\vec{k} \end{aligned}\right\} \Rightarrow \vec{T}_u \times \vec{T}_v = (2u - 2v)\vec{i} - (2u + 2v)\vec{j} + (1+1)\vec{k}$$

So $\vec{T}_u \times \vec{T}_v$ is never zero since coefficient of $\vec{k}$ is never zero.

$\boxed{4}$
$$\left.\begin{aligned} \vec{T}_u &= (2u, 0, 2u) \\ \vec{T}_v &= (0, 2v, 2v) \end{aligned}\right\} \Rightarrow \vec{T}_u \times \vec{T}_v = (-4uv)\vec{i} - (4uv)\vec{j} + (4uv)\vec{k}$$

$$\Rightarrow \vec{T}_u \times \vec{T}_v \Big|_{(1,1)} = -4\vec{i} - 4\vec{j} + 4\vec{k}$$

$$\vec{\Phi}(1,1) = (1,1,2)$$

$\Rightarrow$ equation of tangent plane is $-4(x-1) - 4(y-1) + 4(z-2) = 0$.