PROBLEM SET #6 DUE THURSDAY, NOVEMBER 3
(Problems are from *Vector Calculus* by Marsden and Tromba, sixth edition.)

1

Let S be the entire surface of the solid hall ball $x^2 + y^2 + z^2 \leq 1$ with $z \geq 0$ (i.e., S is the union of the top hemisphere of the unit sphere and the unit disk in the xy-plane), and orient S by the outward-pointing normal. Let $F(x, y, z) = (x+3y^5)\vec{i}+(y+10xz)\vec{j}+(z-xy)\vec{k}$. Calculate $\iint_S \vec{F} \cdot d\vec{S}$.

2

Let $D = \left[0, \frac{\pi}{2}\right] \times \left[0, \frac{\pi}{2}\right]$, $P(x, y) = \sin x$, and $Q(x, y) = \cos y$. Compute (without using Green's theorem) both $\int_{\partial D} P\,dx + Q\,dy$ and $\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$.

3

Let C be the closed curve formed by the quadrilateral with vertices $(-2, 1)$, $(-2, -3)$, $(1, -1)$, and $(1, 5)$. Use Green's theorem to compute $\int_{C^+} 2xy\,dx + xy^2\,dy$.