Problem Set #5 Due Thursday, October 27

(Problems are from *Vector Calculus* by Marsden and Tromba, sixth edition.)

1

Let S be the surface parametrized by $\vec{\Phi} : [0,4] \times [0,\pi] \to \mathbb{R}^3$, $(u,v) \mapsto (2u\cos v, 2u\sin v, u)$. Evaluate the integral of $f(x,y,z) = x + y$ over S.

2

Evaluate $\iint_S (x+y+z)\,dS$ where S is the unit sphere, i.e., $S = \big\{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 = 1\big\}$.

3

Let S be the part of the cylinder $x^2+y^2=4$ that is bound between the planes $z=0$ and $z=1$. Orient S so that the outside of the cylinder is the positive side. Determine a parametrization $\vec{\Phi}$ of S and compute $\iint_{\vec{\Phi}} \vec{F} \cdot d\vec{S}$ where $\vec{F} : (x,y,z) \mapsto (2x, -2y, z^2)$.

4

Compute the surface integral of the vector field $\vec{V} : (x,y,z) \mapsto (3xy^2, 3x^2y, z^3)$ pointing out of the unit sphere (i.e., so that the outside of the unit sphere is the positive side).