Review Problems for Exam 1

(Note that all problems are odd-numbered problems from the textbook, so the answers are in the back of the book.)

1 Limits and continuity

Evaluate the following limits or show that they do not exist:

1.4.15 $\lim_{t \to -3} \frac{t^2 - 9}{2t^2 + 7t + 3}$

1.4.17 $\lim_{h \to 0} \frac{(-5+h)^2 - 25}{h}$

1.4.21 $\lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}$

1.4.49 $\lim_{x \to 0} \frac{\sin 3x}{x}$

1.4.33 If $4x - 9 \leq f(x) \leq x^2 - 4x + 7$ for $x \geq 0$, find $\lim_{x \to 4} f(x)$.

1.4.35 Show that $\lim_{x \to 0} x^4 \cos \frac{2}{x} = 0$.

1.5.31 Find the numbers at which the following function is discontinuous:

$$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \leq x \leq 1 \\ 2 - x & \text{if } x > 1 \end{cases}.$$

1.5.29 Show that the following function is continuous:

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}.$$

1.5.39 Use the Intermediate Value Theorem to show that $f(x) = x^4 + x - 3$ has a root in the interval $(1, 2)$.

1.5.41 Use the Intermediate Value Theorem to show that $f(x) = \cos x - x$ has a root in the interval $(0, 1)$.

1.6.3 Sketch a graph of a function satisfying all the following criteria:

$$\lim_{x \to 0} f(x) = -\infty \qquad \lim_{x \to -\infty} f(x) = 5 \qquad \lim_{x \to \infty} f(x) = -5.$$

Find the following limits or say why they don't exist:

1.6.19 $\lim_{x \to \infty} \frac{3x - 2}{2x + 1}$

1.6.23 $\lim_{x \to \infty} \frac{(2x^2 + 1)^2}{(x-1)^2(x^2 + x)}$

1.6.27 $\lim_{x \to \infty} \cos x$

2 Derivatives

Find the derivative of the following functions using the definition of the derivative, i.e., as a limit of something:

2.2.19 $f(x) = \frac{1}{2}x - \frac{1}{3}$

2.2.23 $g(x) = \sqrt{9 - x}$

2.2.25 $G(t) = \frac{1 - 2t}{3 + t}$

Compute the derivatives of the following without using the definition of the derivative:

2.3.19 $y = \frac{x^2 + 4x + 3}{\sqrt{x}}$

2.4.7 $f(x) = \sin x + \frac{1}{2} \cot x$

2.4.13 $y = \frac{x^3}{1 - x^2}$

2.4.17 $f(t) = \frac{2t}{2 + \sqrt{t}}$

2.5.5 $y = \sqrt{\sin x}$

2.5.9 $F(x) = \sqrt{1 - 2x}$

2.5.19 $h(t) = (t + 1)^{\frac{2}{3}}(2t^2 - 1)^3$

2.5.29 $f(x) = \sin(\tan 2x)$

2.5.33 $y = \left(\frac{1 - \cos 2x}{1 + \cos 2x} \right)^4$

2.5.43 Find the second derivative of $y = \cos(x^2)$.

2.5.45 Find the second derivative of $H(t) = \tan 3t$.

2.3.27 Find the equation of the tangent line to $y = 6\cos x$ at the point $(\frac{\pi}{3}, 3)$.

2.4.27 Find the equation of the tangent line to $y = \frac{x^2-1}{x^2+x+1}$ at the point $(1, 0)$.

2.3.37 For what values of x does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent line?

2.3.39 Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with a slope of 4.

2.5.55 (*Important!*) Here are some values of f, g, f', and g' at certain values of x:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

Find $(f \circ g)'(1)$ and $(g \circ f)'(1)$.