PROBLEM SET #4 Due Thursday, September 29 (Problems are from $Vector\ Calculus$ by Marsden and Tromba, sixth edition.)

1

Find a parametrization of the surface $z = 3x^2 + 8xy$ and use it to find the tangent plane at x = 1, y = 0, z = 3.

$\mathbf{2}$

Let $D = [0,1] \times [0,\pi]$ and define $\vec{\Phi}(x,y,z) : D \to \mathbb{R}^3$ by $(u,v) \mapsto (e^u \cos v, e^u \sin v, v)$. Denote the image of $\vec{\Phi}$ by S.

- (a) Find $\vec{T}_u \times \vec{T}_v$.
- (b) Find the equation for the tangent plane to S at $\vec{\Phi}(0, \frac{\pi}{2})$.
- (c) Find the area of S.

3

Let D be the unit disk in \mathbb{R}^2 and define $\vec{\Phi}: D \to \mathbb{R}^3$ by $(u, v) \mapsto (u - v, u + v, uv)$. Find the area of $\vec{\Phi}(D)$.

4

Let S be the surface parametrized by $\vec{\Phi}: [0,4] \times [0,\pi] \to \mathbb{R}^3, (u,v) \mapsto (2u\cos v, 2u\sin v, u)$. Evaluate the integral of f(x,y,z) = x+y over S.