Review Problems for Exam 3

(Note that all problems are odd-numbered problems from the textbook, so the answers are in the back of the book.)

## Antiderivatives

Find the most general antiderivative of the following functions:

**3.7.3** $f(x) = 7x^{\frac{2}{5}} + 8x^{-\frac{4}{5}}$

**3.7.5** $f(x) = 3\sqrt{x} - 2\sqrt[3]{x}$

**3.7.11** $f(x) = 2\sec t \tan t + \frac{1}{2}t^{-\frac{1}{2}}$

**3.7.15** $f(x) = \dfrac{x^5 - x^4 + 2x}{x^4}$

**3.7.19** Find the most general form of $f$ where $f''(t) = \frac{2}{3}x^{\frac{2}{3}}$.

**3.7.21** Find the most general form of $f$ where $f'''(t) = \cos t$.

**3.7.31** Find $f$ where $f''(\theta) = \sin\theta + \cos\theta$, $f(0) = 3$, and $f'(0) = 4$.

**3.7.31** Find $f$ where $f''(x) = \dfrac{1}{x^2}$, $x > 0$, $f(1) = 0$, and $f(2) = 0$.

**3.7.41** A particle is moving with acceleration $a(t) = 10\sin t + 3\cos t$, such that $s(0) = 0$ and $s(2\pi) = 12$. Find the position function $s(t)$.

## Integration with Riemann sums

Note that the following formulas will be provided on the exam:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

**4.2.15** Express $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{1 - x_i^2}{4 + x_i^2}\Delta x$ on the interval $[2, 6]$ as a definite integral.

**4.2.17** Express $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(5(x_i^*)^3 - 4x_i^*\right)\Delta x$ on the interval $[2, 7]$ as a definite integral.

**4.2.19** Evaluate $\displaystyle\int_2^5 (4 - 2x)\,dx$ using the definition of a definite integral, i.e., with a Riemann sum.

**4.2.21** Evaluate $\int_{-2}^{0}(x^2+x)\,dx$ using the definition of a definite integral, i.e., with a Riemann sum.

**4.2.23** Evaluate $\int_{0}^{1}(x^3-3x^2)\,dx$ using the definition of a definite integral, i.e., with a Riemann sum.

Evaluate the following integrals by interpreting them as areas of regions under curves and then using basic geometry.

**4.2.31** $\int_{-2}^{1}(1-x)\,dx$

**4.2.33** $\int_{-2}^{1}|x|\,dx$

**4.2.35** $\int_{0}^{10}|x-5|\,dx$

## Integration with the evaluation theorem

**4.3.3** $\int_{-2}^{0}\left(\frac{1}{2}t^4+\frac{1}{4}t^3-t\right)\,dt$

**4.3.7** $\int_{0}^{\pi}(4\sin\theta-3\cos\theta)\,d\theta$

**4.3.11** $\int_{0}^{1}x\left(\sqrt[3]{x}+\sqrt[4]{x}\right)\,dx$

**4.3.21** $\int_{1}^{64}\frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx$

**4.3.41** $\int x\sqrt{x}\,dx$

**4.3.43** $\int (x^2+x^{-2})\,dx$

**4.3.45** $\int (u+4)(2u+1)\,du$

## Fundamental theorem of calculus

**4.4.5** Evaluate $\frac{d}{dx}\int_{1}^{x}\frac{1}{t^3+1}\,dt$.

**4.4.7** Evaluate $\dfrac{d}{dx}\displaystyle\int_5^s (t-t^2)^8\,dt$.

**4.4.9** Evaluate $\dfrac{d}{dx}\displaystyle\int_2^{\frac{1}{x}} \sin^4 t\,dt$.

**4.4.11** Evaluate $\dfrac{d}{dx}\displaystyle\int_0^{\tan x} \sqrt{t+\sqrt{t}}\,dt$.

**4.4.15** Find the average value of $g(x) = \sqrt[3]{x}$ on the interval $[1, 8]$.

**4.4.17** Find the average value of $g(x) = \cos x$ on the interval $[0, \frac{\pi}{2}]$.

## Substitution rule ($u$-substitution)

**4.5.7** $\displaystyle\int (1-2x)^9\,dx$

**4.5.15** $\displaystyle\int \frac{a+bx^2}{\sqrt{3ax+bx^3}}\,dx$

**4.5.19** $\displaystyle\int (x^2+1)(x^3+3x)^4\,dx$

**4.5.21** $\displaystyle\int \frac{\cos x}{\sin x}\,dx$

**4.5.21** $\displaystyle\int \sec^3 x \tan x\,dx$

**4.5.31** $\displaystyle\int_0^1 \cos\frac{\pi t}{2}\,dt$

**4.5.35** $\displaystyle\int_0^\pi \sec^2\frac{t}{4}\,dt$

**4.5.41** $\displaystyle\int_{\frac{1}{2}}^1 \frac{\cos x^{-2}}{x^3}\,dx$

**4.5.43** $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^3 + x^4\tan x)\,dx$