1 Surface integrals

1.1 (pg 424, #11) Let $f(x,y,z) = x^2 + y^2 + z^2$ and let $\vec{\Phi} : [0,1] \times [0,1] \to \mathbb{R}^3$ be given by $(u,v) \mapsto (u+v, u, v)$. Compute $\iint_{\vec{\Phi}} f\, dS$.

1.2 (pg 424, #14) Compute the integral of $f(x,y,z) = x + y$ over the unit sphere.

1.3 (pg 425, #22) Let $S = \{(x,y,z) \in \mathbb{R}^2 : z^2 = x^2 + y^2, \quad 1 \le z \le 2\}$, i.e., S is the part of the cone $z^2 = x^2 + y^2$ between $z = 1$ and $z = 2$,

oriented such that the normal vector points out of the cone. Let $\vec{F}(x,y,z) = (x^2, y^2, z^2)$. Compute $\iint_S \vec{F} \cdot d\vec{S}$.

1.4 (pg 425, #23) Let $\vec{F} = x\vec{i} + x^2\vec{j} + yz\vec{k}$ and let $S = [0,1] \times [0,1] \times \{0\} \subseteq \mathbb{R}^3$, oriented as a graph. Compute $\iint_S \vec{F} \cdot d\vec{S}$.

2 Green's theorem

2.1 State the equation for Green's theorem.

2.2 (pg 490, #4) Let D be the region between the curves $y = x$ and $y = x^3$. Verify Green's theorem for the line integral $\int_{\partial D} x^2 y\, dx + y\, dy$.

2.3 (pg 437, #9) Let $D = [-1,1] \times [-1,1] \subset \mathbb{R}^2$. Compute $\int_{\partial D} y\, dx - x\, dy$.

2.4 (pg 437, #11a) Let D be the disk with radius R centered at the origin. Verify Green's theorem where $P(x,y) = xy^2$, and let $Q(x,y) = -yx^2$.

3 Stokes' theorem

3.1 State the equation for Stokes' theorem (not the differential forms version).

3.2 (pg 491, #19) Evaluate $\int_C (x+y)\, dx + (2x - z)\, dy + (y+z)\, dz$ where C is the perimeter of the triangle connecting $(2,0,0)$ to $(0,3,0)$ to $(0,0,6)$, oriented as a graph.

3.3 (pg 451, #15) Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ where S is the portion of the unit sphere defined by $x^2 + y^2 + z^2 = 1$ and $x + y + z \ge 1$ (oriented as a graph) and where $\vec{F} = \vec{r} \times (\vec{i} + \vec{j} + \vec{k})$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

3.4 (pg 451, #19) Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ where $\vec{F}(x,y,z) = (y, -x, zx^3y^2)$ and S is the unit lower hemisphere oriented so that the normal vector points away from the origin [note Marsden and Tromba don't specify an orientation].

4 Conservative fields

4.1 (pg 490, #5) Show that $\vec{F}(x, y, z) = (x^3 - 2xy^3)\vec{i} - 3x^2y^2\vec{j}$ is a conservative field and evaluate the line integral of $\vec{F}$ along the curve $\vec{c}\colon [0, \frac{\pi}{2}] \to \mathbb{R}^2$ given by $\theta \mapsto (\cos^3\theta, \sin^3\theta)$.

4.2 (pg 490, #7) Show that $\vec{F}(x, y, z) = 6xy(\cos z)\vec{i} + 3x^2(\cos z)\vec{j} - 3x^2y(\sin z)\vec{k}$ is a conservative field without finding its potential function. Then find its potential function.

4.3 (pg 491, #19) Which of the following are conservative fields?

- $\vec{F}(x, y, z) = 3x^2y\vec{i} + x^3\vec{j} + 5\vec{k}$
- $\vec{F}(x, y, z) = (x + z)\vec{i} - (y + z)\vec{j} + (x - y)\vec{k}$
- $\vec{F}(x, y, z) = 2xy^3\vec{i} + x^2z^3\vec{j} + 3x^2yz^2 2veck$

5 Gauss's theorem

5.1 State the equation for Gauss's theorem.

5.2 (pg 474, #9a) Let $\vec{F}(x, y, z) = (y, z, xz)$ and $W = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq z \leq 1\right\}$. Evaluate $\iint_{\partial W} \vec{F} \cdot d\vec{S}$.

5.3 (pg 490, #3) Let $\vec{F}(x, y, z) = (x^2y, z^8, -2xyz)$ and let W be the unit cube $[0, 1] \times [0, 1] \times [0, 1] \in \mathbb{R}^3$. Evaluate $\iint_{\partial W} \vec{F} \cdot d\vec{S}$.

5.4 (pg 475, #28) Let S be a closed surface and suppose $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ is C^2. Use Gauss's theorem to show that $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = 0$.

6 Differential forms

6.1 State the equation for Stokes' theorem (the differential forms version).

6.2 (pg 489, #1a) Evaluate $\omega \wedge \eta$ where $\omega = 2x\,dx + y\,dy$ and $\eta = x^3\,dx + y^2\,dy$.

6.3 (pg 489, #1d) Evaluate $\omega \wedge \eta$ where $\omega = xy\,dy\,dz + x^2\,dx\,dy$ and $\eta = dx + dy$.

6.4 (pg 489, #3b) Evaluate $d(\omega)$ where $\omega = xy\,dx + y^2 \cos x\,dy + dz$.

6.5 (pg 489, #3h) Evaluate $d(\omega)$ where $\omega = x^2y\,dy\,dz$.

6.6 (pg 489, #10) Let $\omega = (x + y)\,dz + (y + z)\,dx + (x + z)\,dy$ and let S be the upper unit hemisphere. Evaluate $\int_{\partial S} \omega$ both directly and by using Stokes' theorem.

6.7 (pg 489, #12) Let $\omega = z\,dx\,dy + x\,dy\,dz + y\,dz\,dx$ and let S be the upper sphere. Evaluate $\iint_S \omega$ both directly and by using Stokes' theorem.