Review Problems for Exam I

1 Limits and continuity

Evaluate the following limits or show that they do not exist:

1.1 $\displaystyle\lim_{h\to 0} \frac{(2+h)^3-8}{h}$

1.2 $\displaystyle\lim_{x\to -1} \frac{x^2+2x+1}{x^4-1}$

1.3 $\displaystyle\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}$

1.4 $\displaystyle\lim_{x\to 5} \frac{x^2-6x+5}{x-5}$

1.5 $\displaystyle\lim_{x\to 0} \frac{\sin 3x}{x}$ (Hint: Multiply and divide by something to put it in the form $c\dfrac{\sin\theta}{\theta}$. Or see example 10 on pages 42–43.)

1.6 $\displaystyle\lim_{x\to 0} \frac{\sin 3x \sin 5x}{x^2}$

1.7 $\displaystyle\lim_{x\to 0} \frac{\sin 4x}{\sin 6x}$

1.8 Show that $\displaystyle\lim_{x\to 0} x^4 \cos\frac{2}{x} = 0$.

1.9 If $4x-9 \le f(x) \le x^2-4x+7$ for $x \ge 0$, find $\displaystyle\lim_{x\to 4} f(x)$.

1.10 Find the numbers at which the following function is discontinuous:

$$f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } x > 1 \end{cases}.$$

1.11 Show that the following function is continuous:

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \ge 1 \end{cases}.$$

1.12 Use the Intermediate Value Theorem to show that $\sqrt[3]{x} = 1-x$ has a root in the interval $(0,1)$.

1.13 Use the Intermediate Value Theorem to show that $\sin x = x^2 - x$ has a root in the interval $(0, 1)$.

1.14 Sketch a graph of a function satisfying all the following criteria:

$$\lim_{x\to 0} f(x) = -\infty \qquad \lim_{x\to -\infty} f(x) = 5 \qquad \lim_{x\to \infty} f(x) = -5.$$

1.15 Sketch a graph of a function satisfying all the following criteria:

$$\lim_{x\to 2} f(x) = \infty \qquad \lim_{x\to -2^+} f(x) = \infty \qquad \lim_{x\to -2^-} f(x) = -\infty \qquad \lim_{x\to -\infty} f(x) = 0 \qquad \lim_{x\to \infty} f(x) = 0.$$

Find the following limits or say why they don't exist:

1.16 $\displaystyle\lim_{x\to 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4}$

1.17 $\displaystyle\lim_{x\to \infty} \frac{x^2}{\sqrt{x^4 + 1}}$

1.18 $\displaystyle\lim_{x\to \infty} \cos x$

2 Derivatives

Find the derivative of the following functions using the definition of the derivative:

2.1 $f(x) = \frac{1}{2}x - \frac{1}{3}$

2.2 $f(x) = \dfrac{1 - 2t}{3 + t}$

2.3 $f(x) = \sqrt{9 - x}$

Compute the derivatives of the following without appealing to the definition of the derivative:

2.4 $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

2.5 $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$

2.6 $f(x) = \sin x + \frac{1}{2}\cot x$

2.7 $f(x) = \sin(\tan 2x)$

2.8 $y = 3\cot n\theta$ where n is a constant

2.9 $f(x) = 2\sec x - \csc x$

2.10 $f(\theta) = \sin\theta\cos\theta$

2.11 $y = \dfrac{x^3}{1-x^2}$

2.12 $f(t) = (3t-1)^4(2t+1)^{-3}$

2.13 $y = \dfrac{\sqrt{x}+x}{x^2}$

2.14 $y = \sqrt{\sin x}$

2.15 $f(x) = \sqrt{x}\sin x$

2.16 Find the second derivative of $f(x) = x^4 - 3x^3 + 16x$.

2.17 Find the second derivative of $y = \cos^2 x$.

2.18 Find the second derivative of $f(x) = \tan 3x$.

2.19 Find the equation of the tangent line to $y = 6\cos x$ at the point $(\frac{\pi}{3}, 3)$.

2.20 Find the equation of the tangent line to $y = \frac{x^2-1}{x^2+x+1}$ at the point $(1,0)$.

2.21 Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$ where the tangent line is horizontal.

2.22 Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with a slope of 4.

2.23 Here are some values of f, g, f', and g' at certain values of x:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

Find $(f\circ g)'(1)$, $(g\circ f)'(2)$, and $(f\circ f)'(3)$.