Problem Set #4 Due Tuesday, September 27

(Problems are from *Vector Calculus* by Marsden and Tromba, sixth edition.)

1

Find a parametrization of the surface $z = 3x^2 + 8xy$ and use it to find the tangent plane at $x = 1$, $y = 0$, $z = 3$.

2

Let $D = [0,1] \times [0,\pi]$ and define $\vec{\Phi}(x,y,z) : D \to \mathbb{R}^3$ by $(u,v) \mapsto (e^u \cos v, e^u \sin v, v)$. Denote the image of $\vec{\Phi}$ by S.

(a) Find $\vec{T}_u \times \vec{T}_v$.

(b) Find the equation for the tangent plane to S at $\vec{\Phi}\left(0, \frac{\pi}{2}\right)$.

(c) Find the area of S. Hint: You'll probably need to use the following fact:

$$\int \sqrt{u^2+1}\, du = \frac{u}{2}\sqrt{u^2+1} + \frac{1}{2}\log\left|u + \sqrt{u^2+1}\right| + c.$$

3

Let D be the unit disk in $\mathbb{R}^2$ and define $\vec{\Phi} : D \to \mathbb{R}^3$ by $(u,v) \mapsto (u-v, u+v, uv)$. Find the area of $\vec{\Phi}(D)$.