Problem Set #2 — Due Thursday, September 15

(Problems are from *Vector Calculus* by Marsden and Tromba, sixth edition.)

1

Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ is given by $(x, y, z) \mapsto x \cos z$ and $\vec{c} : [0, 1] \to \mathbb{R}^3$ is given by $t \mapsto t\vec{i} + t^2\vec{j}$. Compute the path integral of f along $\vec{c}$.

2

Given a path $\vec{c}$, define $\ell(c) = \int_{\vec{c}} \|\vec{c}\,'(t)\|\, dt$ to be the length of the path $\vec{c}$. Given $f : \mathbb{R}^3 \to \mathbb{R}$, define

$$\frac{\int_{\vec{c}} f(x, y, z)\, ds}{\ell(\vec{c})}$$

to be the average value of f along $\vec{c}$.

Compute the length of the semicircle parametrized by $\vec{c} : [0, \pi] \to \mathbb{R}^3$, $\theta \mapsto (0, a \sin\theta, a\cos\theta)$ where $a > 0$. Find the average y coordinate of the points on the semicircle. (Hint: the y coordinate of a point is given by the function $f : (x, y, z) \mapsto y$.)

3

Evaluate the line integral of $\vec{F} : \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto y^2\vec{i} + 2xy\vec{j}$ along the curve $\vec{c} : [0, 2\pi] \to \mathbb{R}^2$, $\theta \mapsto (\cos\theta, \sin\theta)$.

4

Suppose $\vec{c} : [a, b] \to \mathbb{R}^3$, $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$, and that $\vec{F}(\vec{c}(t))$ is perpendicular to $\vec{c}\,'(t)$ for all $t \in [a, b]$. Show that the line integral of $\vec{F}$ along $\vec{c}$ is zero. (Hint: what does perpendicularity have to do with dot products?)