(Problems are from *Vector Calculus* by Marsden and Tromba, sixth edition.)

1

Show that $\vec{c}(t) = \left(\frac{1}{t^3}, e^t, \frac{1}{t}\right)$ is a flow line for $\vec{F}(x,y,z) = (-3z^4, y, -z^2)$.

2

Compute the divergence of $\vec{V}(x,y,z) = x\vec{i} + (y + \cos x)\vec{j} + (z + e^{xy})\vec{k}$.

3

Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ and $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ are C^2. Which of the following expressions make sense and which are nonsense? For those that make sense, decide whether the expression gives a scalar function or a vector field. (You do not have to show your work for this problem. Just write the answer.)

1. $\nabla \times (\nabla f)$
2. $\nabla(\nabla \times f)$
3. $\nabla \cdot (\nabla f)$
4. $\nabla(\nabla \cdot \vec{F})$
5. $\nabla \times (\nabla \cdot \vec{F})$
6. $\nabla \cdot (\nabla \times \vec{F})$

4

Suppose $\vec{F} = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$ is C^2. Show that $\nabla \cdot (\nabla \times \vec{F}) = 0$.

5

Verify that $\nabla \times (\nabla f) = \vec{0}$ for $f(x,y,z) = x^2y^2 + y^2z^2$.

6

Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ and $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ are C^2. Let $\vec{F} = F_1\vec{i} + F_2\vec{j} + F_3\vec{k}$. Prove the following identity:

$$\nabla \times (f\vec{F}) = (f)(\nabla \times \vec{F}) + (\nabla f) \times \vec{F},$$

where "$f\vec{F}$" denotes multiplication by scalar, i.e, $(f\vec{F})(x,y,z) = f(x,y,z)\left(F_1\vec{i} + F_2\vec{j} + F_3\vec{k}\right)$. (Just compute both sides of the identity and conclude they're equal.)