PROBLEM SET #9 DUE TUESDAY, NOVEMBER 28

(Problems are from *Vector Calculus* by Marsden and Tromba, sixth edition.)

1

Determine whether $\vec{F}(x,y) = (x^2 + y^2)\vec{i} + 2xy\vec{j}$ is a conservative field. If it is, find its potential function (i.e., some function f such that $\nabla f = \vec{F}$).

2

Let $\vec{F}(x,y,z) = (e^x \sin y, e^x \cos y, z^2)$. Determine whether $\vec{F}$ is a conservative field. Also determine whether there exists a vector field $\vec{G}$ such that $\nabla \times \vec{G} = \vec{F}$.

3

Let $\vec{F}(x,y,z) = (2xyz + \sin x)\vec{i} + x^2 z\vec{j} + x^2 y\vec{k}$. Find its potential function.

4

(a) Let C be the unit circle in $\mathbb{R}^2$. Show that $\int_C \frac{x\,dy - y\,dx}{x^2+y^2} = 2\pi$. Is the vector field conservative?

(b) Show that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ for this vector field. Why does this not contradict the proposition about conservative fields in $\mathbb{R}^2$ that we covered in class?