Problem Set #8 Due Thursday, November 17

(Problems are from *Vector Calculus* by Marsden and Tromba, sixth edition.)

1

Let $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1,\ z \geq 0\}$, oriented as a graph. What is ∂S as a set? Let $\vec{F}(x, y, z) = (y, z, x)$. Compute, without using Stokes' theorem, both $\iint (\nabla \times \vec{F}) \cdot d\vec{S}$ and $\int_{\partial S} \vec{F} \cdot d\vec{s}$.

2

Let C be the triangle in $\mathbb{R}^3$ formed by traveling in straight lines between the points $(0, 0, 0)$, $(2, 1, 5)$, $(1, 1, 3)$, and back to the origin, in that order. Use Stokes' theorem to evaluate $\int_C (xyz)\, dx + (xy)\, dy + (x)\, dz$.

3

Calculate (feel free to use Stokes' theorem) the surface integral $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ where $\vec{F}(x, y, z) = x^3\vec{i} - y^3\vec{j}$ and S is the hemisphere defined by $x^2 + y^2 + z^2 = 1$ and $x \geq 0$.

4

Suppose C is a closed curve that is the boundary of some surface S. Let $\vec{v}$ be a constant vector. Show that $\int_C \vec{v} \cdot d\vec{s} = 0$.