1 Flow lines

1.1 Show that $\vec{c}(t) = \big(\sin t, \cos t, e^t\big)$ is a flow line for $\vec{F}(x, y, z) = (y, -x, z)$.

1.2 Show that $\vec{c}(t) = \left(\frac{1}{1-t}, 0, \frac{e^t}{1-t}\right)$ is a flow line for $\vec{F}(x, y, z) = (x^2, yx^2, z + zx)$.

1.3 Show that $\vec{c}(t) = \big(t^2, 2t - 1, \sqrt{t}\big)$ where $t > 0$ is a flow line for $\vec{F}(x, y, z) = \big(y + 1, 2, \frac{1}{2z}\big)$.

2 Gradient vector fields

Find $f : \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla f(x, y, z) = (x, y, z)$.

3 Divergence

3.1 Compute the divergence of $\vec{F}(x, y) = \sin(xy)\vec{\imath} - \cos(x^2 y)\vec{\jmath}$.

3.2 Compute the divergence of $\vec{F}(x, y) = xe^y\vec{\imath} - \frac{y}{x+y}\vec{\jmath}$.

3.3 Compute the divergence of $\vec{F}(x, y) = x^3\vec{\imath} = x\sin(xy)\vec{\jmath}$.

4 Curl

4.1 Compute the curl of $\vec{F}(x, y, z) = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$.

4.2 Compute the curl of $\vec{F}(x, y, z) = yz\vec{\imath} + xz\vec{\jmath} + xy\vec{k}$.

5 Type

Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ and $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ are C^2. Determine which of the following expressions make sense. For those that make sense, identify whether they are scalar fields or vector fields. For those that are nonsene, explain why (e.g., if the expression is $\nabla(\nabla \times \vec{F})$, then "$\nabla \times \vec{F}$ is a vector field, and you can't take the gradient of a vector field" is an explanation).

5.1 $\nabla \times (\nabla \cdot (\nabla \times \vec{F}))$

5.2 $\nabla \times (\nabla \times (\nabla \times \vec{F}))$

5.3 $\nabla \cdot (\nabla(\nabla \cdot \vec{F}))$

5.4 $\nabla(\nabla \times (\nabla \cdot \vec{F}))$

5.5 $\nabla \times (\nabla(\nabla f))$

5.6 $\nabla(\nabla\cdot(\nabla f))$

5.7 $\nabla(\nabla\cdot(\nabla\times f))$

5.8 $\nabla\times(\nabla\times(\nabla f))$

6 Path integrals

6.1 Compute the path integral of $f(x,y,z)=z$ over the path $\vec{c}:[0,t_0]\to\mathbb{R}^3$ given by $t\mapsto(t\cos t, t\sin t, t)$.

6.2 Compute the path integral of $f(x,y,z)=x+\cos^2 z$ over the path $\vec{c}:[0,2\pi]\to\mathbb{R}^3$ given by $t\mapsto(\cos t,\sin t,t)$.

6.3 Compute the path integral of $f(x,y,z)=x+y+z$ over the path $\vec{c}:[0,1]\to\mathbb{R}^3$ given by $t\mapsto\left(t,t^2,\frac{2}{3}t^3\right)$.

7 Line integrals

7.1 Compute the line integral $\int_{\vec{c}}(\sin z)\,dx+(\cos z)\,dy-(xy)^{\frac{1}{3}}\,dz$ where $\vec{c}:\left[0,\frac{7\pi}{2}\right]\to\mathbb{R}^3$ is given by $\theta\mapsto(\cos^3\theta,\sin^3\theta,\theta)$.

7.2 Let $\vec{F}(x,y,z)=x\vec{i}+y\vec{j}+z\vec{k}$ and define $\vec{c}:[0,1]\to\mathbb{R}^3$ by $t\mapsto(e^t,t,t^2)$. Compute $\int_{\vec{c}}\vec{F}\cdot d\vec{s}$.

8 Parametrized surfaces

8.1 Define $\vec{\Phi}:\mathbb{R}^2\to\mathbb{R}^3$ by $(u,v)\mapsto(u^2-v^2,u^2+v^2,v)$. Find all points $\vec{\Phi}(u_0,v_0)$ where the surface parametrized by $\vec{\Phi}$ is not regular.

8.2 Find an equation of the tangent plane for the surface parametrized by $\vec{\Phi}:\mathbb{R}^2\to\mathbb{R}^3$, $(u,v)\mapsto(2u,u^2+v,v^2)$ at the point $(0,1,1)$. At what points is this surface regular?

8.3 Let $\vec{\Phi}(u,v)=(u^2\cos v,u^2\sin v,u)$. Compute the equation of the tangent plane at $\vec{\Phi}(1,0)$.

9 Area of a surface

9.1 Find the area of the surface S parametrized by $\vec{\Phi}:[0,1]\times[0,2\pi]\to\mathbb{R}^3$, $(r,\theta)\mapsto(r\cos\theta,2r\cos\theta,\theta)$.

9.2 Find the area of the surface S parametrized by $\vec{\Phi}:[0,1]\times[0,1]\to\mathbb{R}^3$, $(u,v)\mapsto(u+v,u,v)$.