Review Problems for the Final Exam

(Note that all problems are odd-numbered problems from the textbook, so the answers are in the back of the book. Also note that *all previous sets of review problems, i.e, for the first three exams, should also be considered review problems for the final.*)
The following formulas will be provided on the exam:

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \tan x = \frac{\sin x}{\cos x}$$
$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \cot x = \frac{\cos x}{\sin x}$$
$$\frac{d}{dx}\cot x = -\csc^2 x \qquad \sec x = \frac{1}{\cos x}$$
$$\frac{d}{dx}\csc x = -\csc x \cot x \qquad \csc x = \frac{1}{\sin x}$$

## Theory

Decide whether the following statements are true or false:

1. If the domain of $f(x)$ is $(-\infty, 1) \cup (1, \infty)$, then $\lim_{x\to 1} f(x)$ does not exist.
2. The function $f(x) = \begin{cases} 1, & x < 0 \\ \cos x, & x \geq 0 \end{cases}$ is continuous.
3. If a function is differentiable, then it is continuous.
4. If a function is continuous, then it is differentiable.
5. If a function is continuous, the it is integrable.
6. If $f'(x) = g'(x)$, then $f(x) = g(x)$.
7. If $f'(a) = 0$, then there is a local minimum or local maximum at $x = a$.
8. If $f$ is an even function, then $\int_{-a}^{a} f(x)\,dx = 0$.
9. If $f$ is integrable, then $f$ has infinitely many antiderivatives.
10. There exist functions $f$ such that $\int_a^a f(x)\,dx \neq 0$.
11. For any function $f$, $\int_a^b f(x)\,dx = \int_a^b f(p)\,dp$.
12. For any function $f$, $\frac{d}{dx}\int_a^b f(x)\,dx = 0$.

Answers: false, true, true, false, true, false, false, false, true, false, true, true.

## Limits

Evaluate the following:

**1.4.13** $\lim_{x\to 5} \frac{x^2-5x+6}{x-5}$

**1.4.17** $\lim_{h\to 0} \frac{(-5+h)^2-25}{h}$

**1.4.21** $\lim_{h\to 0} \frac{(x+h)^3-x^3}{h}$

**1.6.13** $\lim_{x\to -3^+} \frac{x+2}{x+3}$

**1.6.15** $\lim_{x\to 1} \frac{2-x}{(x-1)^2}$

**1.6.21** $\lim_{t\to \infty} \frac{\sqrt{t}+t^2}{2t-t^2}$

**1.6.27** $\lim_{x\to \infty} \frac{x^4-3x^2+x}{x^3-x+2}$

## Derivatives

Differentiate the following:

**2.2.21** $f(x) = x^2 - 2x^3$ (using the definition of the derivative)

**2.3.9** $g(x) = x^2(1-2x)$

**2.3.19** $y = \frac{x^2+4x+3}{\sqrt{x}}$

**2.4.15** $y = \frac{v^3 - 3v\sqrt{v}}{v}$

**2.4.19** $y = \frac{x}{2-\tan x}$

**2.4.21** $f(\theta) = \frac{\sec\theta}{1+\sec\theta}$

**2.4.23** $y = \frac{t\sin t}{1+t}$

**2.5.7** $F(x) = (x^4+3x^2-2)^5$

**2.5.13** $y = \cos(a^3+x^3)$

**2.5.17** $f(x) = (2x-3)^4(x^2+x+1)^5$

**2.5.21** $y = \left(\dfrac{x^2+1}{x^2-1}\right)^3$

**2.5.25** $y = \dfrac{r}{\sqrt{r^2+1}}$

**2.5.27** $y = \sin\sqrt{1+x^2}$

**2.5.31** $y = \sec^2 x + \tan^2 x$

**2.5.43** $y = \cos(x^2)$ (also find second derivative)

**Chapter 2 review, 17** $y = x^2 \sin \pi x$

**Chapter 2 review, 21** $y = \tan\sqrt{1-x}$

**Chapter 2 review, 27** $y = (1-x^{-1})^{-1}$

**Chapter 2 review, 29** $y = \sin(xy) = x^2 - y$

**Chapter 2 review, 35** $y = \tan^2(\sin\theta)$

**2.3.29** Find an equation of the tangent line to the curve $y = 3x^2 - x^3$ at the point $(1, 2)$.

**Chapter 2 review, 45** Find an equation of the tangent line to the curve $y = 4\sin^2 x$ at the point $(\frac{\pi}{6}, 1)$.

**2.4.35** If $H(\theta) = \theta\sin\theta$, find $H'(\theta)$ and $H''(\theta)$.

**2.6.5** Find $\frac{dy}{dx}$ where $x^2 + xy - y^2 = 4$.

**2.6.7** Find $\frac{dy}{dx}$ where $y\cos x = x^2 + y^2$.

**2.6.15** Find $\frac{dy}{dx}$ where $y\cos x = 1 + \sin(xy)$.

**2.7.7** Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm$^2$?

**2.7.21** At noon, ship $A$ is 100 km west of ship $B$. Ship $A$ is sailing south at 35 km/hr and ship $B$ is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00pm?

**2.7.27** Gravel is being dumped from a conveyor belt at a rate of 30 $\text{ft}^3/\text{min}$, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

**2.7.29** Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 radian/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\frac{\pi}{3}$.

**3.1.35** Find the absolute maximum and the absolute minimum of $f(x) = 12 + 4x - x^2$ on $[0, 5]$.

**3.1.47** Find the absolute maximum and the absolute minimum of $f(x) = x^5 - x^3 + 2$ on $[-1, 1]$.

**3.1.49** Find the absolute maximum and the absolute minimum of $f(x) = x\sqrt{x - x^2}$ on $(-\infty, \infty)$.

**3.3.21** For $f(x) = x^3 - 12x + 2$, find the intervals where $f$ is increasing/decreasing, the local maximum and minimum values, the intervals where $f$ is concave up/down, and the inflection points.

**3.3.25** For $h(x) = (x + 1)^5 - 5x - 2$, find the intervals where $h$ is increasing/decreasing, the local maximum and minimum values, the intervals where $h$ is concave up/down, and the inflection points.

**3.3.29** For $C(x) = x^{\frac{1}{3}}(x + 4)$, find the intervals where $C$ is increasing/decreasing, the local maximum and minimum values, the intervals where $C$ is concave up/down, and the inflection points.

**3.5.19** Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triange of fixed side length $L$ if one side of the rectangle lies on the base of the triangle.

**3.5.25** A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted (i.e., greatest area).

**3.5.27** A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How show the wire be cut so that the total area enclosed is (a) maximized? (b) minimized?

**3.5.39** Find an eqution of the line through the point $(3, 5)$ that cuts off the least area from the first quadrant.

**3.7.17** Find the most general form of $f$ such that $f''(x) = 20x^3 - 12x^2 + 6x$.

**3.7.25** Find $f$ such that $f''(x) = \sqrt{x}(6 + 5x)$ and $f(1) = 10$.

## Integrals

Evaluate the following:

**4.3.9** $\displaystyle\int_1^4 \left(\frac{4+6u}{\sqrt{u}}\right) du$

**4.3.13** $\displaystyle\int_1^4 \sqrt{\frac{5}{x}}\, dx$

**4.3.47** $\displaystyle\int \frac{\sin x}{1-\sin^2 x}\, dx$

**4.3.47** $\displaystyle\int \frac{\sin x}{1-\sin^2 x}\, dx$

**4.5.17** $\displaystyle\int \sec^2\theta \tan^3\theta\, d\theta$

**4.5.23** $\displaystyle\int \sqrt{\cot x}\csc^2 x\, dx$

**4.5.27** $\displaystyle\int \frac{z^2}{\sqrt[3]{1+z^3}}\, dz$

**4.5.29** $\displaystyle\int x(2x+5)^8\, dx$

**4.5.33** $\displaystyle\int_0^\pi \sqrt[3]{1+7x}\, dx$

**4.5.33** $\displaystyle\int_0^a x\sqrt{x^2+a^2}\, dx$ where $a>0$

**4.5.53** If $f$ is continuous and $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$.

**7.1.7** Find the area of the region enclosed by the curvesw$y=(x-2)^2$ and $y=x$.

**7.1.9** Find the area of the region enclosed by the curves $x=1-y^2$ and $x=y^2-1$.

**7.1.11** Find the area of the region enclosed by the curves $y=12-x^2$ and $y=x^2-6$.

**7.1.15** Find the area of the region enclosed by the curves $x=2y^2$ and $x=4+y^2$.

**7.1.17** Find the area of the region enclosed by the curves $y=\cos\pi x$ and $y=4x^2-1$.

**7.2.1** Let $R$ be the region enclosed by the curves $y = 2 - \frac{1}{2}x$, $y = 0$, $x = 1$, and $x = 2$. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

**7.2.5** Let $R$ be the region enclosed by the curves $y = x^3$, $y = x$, and $x \geq 0$. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

**7.2.7** Let $R$ be the region enclosed by the curves $y^2 = x$ and $x = 2y$. Find the volume of the solid obtained by rotating $R$ about the $y$-axis.

**7.2.9** Let $R$ be the region enclosed by the curves $y = x$ and $y = \sqrt{x}$. Find the volume of the solid obtained by rotating $R$ about the line $y = 1$.

**7.2.17** Let $R$ be the region enclosed by the curves $y = x^3$ and $y = \sqrt{x}$. Find the volume of the solid obtained by rotating $R$ about the line $x = 1$.