Review Problems for Exam 3

(Note that all problems are odd-numbered problems from the textbook, so the answers are in the back of the book.)

Antiderivatives

Find the most general antiderivative of the following functions:

3.7.3 $f(x) = 7x^{\frac{2}{5}} + 8x^{-\frac{4}{5}}$

3.7.5 $f(x) = 3\sqrt{x} - 2\sqrt[3]{x}$

3.7.11 $f(x) = 2\sec t \tan t + \frac{1}{2}t^{-\frac{1}{2}}$

3.7.15 $f(x) = \dfrac{x^5 - x^4 + 2x}{x^4}$

3.7.19 Find the most general form of f where $f''(t) = \frac{2}{3}x^{\frac{2}{3}}$.

3.7.21 Find the most general form of f where $f'''(t) = \cos t$.

3.7.31 Find f where $f''(\theta) = \sin\theta + \cos\theta$, $f(0) = 3$, and $f'(0) = 4$.

3.7.31 Find f where $f''(x) = \dfrac{1}{x^2}$, $x > 0$, $f(1) = 0$, and $f(2) = 0$.

3.7.41 A particle is moving with acceleration $a(t) = 10\sin t + 3\cos t$, such that $s(0) = 0$ and $s(2\pi) = 12$. Find the position function $s(t)$.

Integration with Riemann sums

Note that the following formulas will be provided on the exam:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

4.2.15 Express $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{1 - x_i^2}{4 + x_i^2}\Delta x$ on the interval $[2, 6]$ as a definite integral.

4.2.17 Express $\lim_{n\to\infty} \sum_{i=1}^{n} \left(5(x_i^*)^3 - 4x_i^*\right)\Delta x$ on the interval $[2, 7]$ as a definite integral.

4.2.19 Evaluate $\displaystyle\int_2^5 (4 - 2x)\,dx$ using the definition of a definite integral, i.e., with a Riemann sum.

4.2.21 Evaluate $\int_{-2}^{0} (x^2 + x)\, dx$ using the definition of a definite integral, i.e., with a Riemann sum.

4.2.23 Evaluate $\int_{0}^{1} (x^3 - 3x^2)\, dx$ using the definition of a definite integral, i.e., with a Riemann sum.

Evaluate the following integrals by interpreting them as areas of regions under curves and then using basic geometry.

4.2.31 $\int_{-2}^{1} (1 - x)\, dx$

4.2.35 $\int_{-2}^{1} |x|\, dx$

Integration with the evaluation theorem

4.3.3 $\int_{-2}^{0} \left(\frac{1}{2}t^4 + \frac{1}{4}t^3 - t \right)\, dt$

4.3.7 $\int_{0}^{\pi} (4\sin\theta - 3\cos\theta)\, d\theta$

4.3.11 $\int_{0}^{1} x\left(\sqrt[3]{x} + \sqrt[4]{x} \right)\, dx$

4.3.21 $\int_{1}^{64} \frac{1 + \sqrt[3]{x}}{\sqrt{x}}\, dx$

4.3.41 $\int x\sqrt{x}\, dx$

4.3.43 $\int (x^2 + x^{-2})\, dx$

4.3.45 $\int (u + 4)(2u + 1)\, du$

Fundamental theorem of calculus

4.4.5 Evaluate $\frac{d}{dx} \int_{1}^{x} \frac{1}{t^3 + 1}\, dt$.

4.4.7 Evaluate $\frac{d}{dx} \int_{5}^{s} (t - t^2)^8\, dt$.

4.4.9 Evaluate $\dfrac{d}{dx}\displaystyle\int_2^{\frac{1}{x}} \sin^4 t\, dt.$

4.4.11 Evaluate $\dfrac{d}{dx}\displaystyle\int_0^{\tan x} \sqrt{t+\sqrt{t}}\, dt.$

4.4.15 Find the average value of $g(x) = \sqrt[3]{x}$ on the interval $[1, 8]$.

4.4.17 Find the average value of $g(x) = \cos x$ on the interval $[0, \frac{\pi}{2}]$.

Substitution rule (u-substitution)

4.5.7 $\displaystyle\int (1-2x)^9\, dx$

4.5.15 $\displaystyle\int \frac{a+bx^2}{\sqrt{3ax+bx^3}}\, dx$

4.5.19 $\displaystyle\int (x^2+1)(x^3+3x)^4\, dx$

4.5.21 $\displaystyle\int \frac{\cos x}{\sin x}\, dx$

4.5.21 $\displaystyle\int \sec^3 x \tan x\, dx$

4.5.31 $\displaystyle\int_0^1 \cos\frac{\pi t}{2}\, dt$

4.5.35 $\displaystyle\int_0^{\pi} \sec^2\frac{t}{4}\, dt$

4.5.41 $\displaystyle\int_{\frac{1}{2}}^1 \frac{\cos x^{-2}}{x^3}\, dx$

4.5.43 $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^3 + x^4\tan x)\, dx$