Thinking Recursively

Chris Piech

CS 106B
Lecture 6
Jan 20, 2016

Women in CS Dinner

Today

5:30 – 7:30pm

Women's Community Center

Socrative.com

STUDENT

Room: 106BWIN16

Today is an exciting day

Learn a new way to *think*

Course Syllabus

Towers of Hanoi

Towers of Hanoi

Towers of Hanoi

Towers of Hanoi

Towers of Hanoi

Towers of Hanoi

Towers of Hanoi

Towers of Hanoi

Qt

By the end of today we will write code that solves this problem

We've Gotten Ahead of Ourselves

Source: The Hobbit

Start at the Beginning

Source: The Hobbit

Today's Goal

1. Be able to think about problems recursively

Today's Route

Today's Route

Pedagogy

Start Simple

Build Complexity

Recursion

Define an operation in terms of **itself**.

Understanding a Word

Operation: Understand Word

me·lod·ic

/məˈlädik/

adjective

of, having, or producing melody.
"melodic and rhythmic patterns"

- pleasant-sounding; melodious.
  "his voice was deep and melodic"

Translations, word origin, and more definitions

How Many Students in Col?

How many students total are directly behind you in your "column" of the classroom?

How Many Students in Col?

How many students total are behind you in your "column" of the classroom?

1. You can see only the people right next to you. So you can't just look back and count.

2. But you are allowed to ask questions of the person next to you.

3. How can we solve this problem (*recursively*)

How Many Students in Col?

```
int numStudentsBehind(Student curr) {
    if(lastInRow(curr)) {
        return 0;
    } else {
        Student next = curr.getBehind();
        return numStudentsBehind(next) + 1;
    }
}
```

Anatomy of Recursion

```
int numStudentsBehind(Student curr) {
    if(lastInRow(curr)) {
        return 0;
    } else {
        Student next = curr.getBehind();
        return numStudentsBehind(next) + 1;
    }
}
```

Base Case

```
int numStudentsBehind(Student curr) {
    if(lastInRow(curr)) {
        return 0;
    } else {
        Student next = curr.getBehind();
        return numStudentsBehind(next) + 1;
    }
}
```

Recursive Case

```
int numStudentsBehind(Student curr) {
    if(lastInRow(curr)) {
        return 0;
    } else {
        Student next = curr.getBehind();
        return numStudentsBehind(next) + 1;
    }
}
```

Recursive Call

```
int numStudentsBehind(Student curr) {
    if(lastInRow(curr)) {
        return 0;
    } else {
        Student next = curr.getBehind();
        return numStudentsBehind(next) + 1;
    }
}
```

Recursive Call

```
int numStudentsBehind(Student curr) {
    if(lastInRow(curr)) {
        return 0;
    } else {
        Student next = curr.getBehind();
        return numStudentsBehind(next) + 1;
    }
}
```

Progress towards the base case

Three Musts of Recursion

1. Your code must have a case for all valid inputs.

2. You must have a base case (makes no recursive calls).

3. When you make a recursive call it should be to a simpler instance (forward progress towards base case)

Today's Route

Today's Route

Powers

Write a recursive function that takes in a base and an exponent and returns the result of $base^{exp}$

Powers

Write a recursive function that takes in a base and an exponent and returns the result of $base^{exp}$

The term “base” here is the term for the bottom of a power.
Not to be confused with “base case”

Recursive Insight

$$x^0 = 1$$

$$x^n = x \cdot x^{n-1}$$

Qt

The Call Stack

Each previous call waits for the next call to

```
// first call:   5        3
int power(int base, int exp) {
    if (exp == 1) {
    } 
    }
}
```

```
// second call:  5        2
int power(int base, int exp) {
    if (exp == 1) {
    }
    }
}
```

```
// third call:   5        1
int power(int base, int exp) {
    if (exp == 1) {
        return base;   // 5
    } else {
        return base * power(base, exp - 1);
    }
}
```

Even Cooler Recursive Insight

$$x^n = \begin{cases} 1, & \text{if } n = 0 \\ \frac{1}{x}^{-n}, & \text{if } n < 0 \\ x \cdot \left(x^{\frac{n-1}{2}}\right)^2, & \text{if } n \text{ is odd} \\ \left(x^{\frac{n}{2}}\right)^2, & \text{if } n \text{ is even} \end{cases}$$

Exponentiation by squaring

Not important right now

Today's Route

Today's Route

What Does This Code Do?

```
int mystery(int n) {
    if(n < 10) {
        return n;
    } else {
        int toAdd = n % 10;
        int theRest = mystery(n / 10);
        return theRest + toAdd;
    }
}
```

A: 5

B: 6

C: 7

D: 8

What is the result of `mystery(123)`?

_____

What Does This Code Do?

```
int mystery(int n) {
    // first call: 123
    int mystery(int n) {
        if (n < 10) {
            return n;
        } else {
            return (n % 10) + mystery(n / 10);
        }
    }
    }
}
```

A: 5

B: 6

C: 7

D: 8

What is the result of `mystery(123)`?

_____

What Does This Code Do?

```
int mystery(int n) {
    // first call: 123
    int mystery(int n) {
        if (n < 10) {
            // second call: 12
            int mystery(int n) {
                if (n < 10) {
                    return n;
                } else {
                    return (n % 10) + mystery(n / 10);
                }
            }
        }
    }
}
```

A: 5

B: 6

C: 7

D: 8

What is the result of `mystery(123)`?

_____

What Does This Code Do?

```
int mystery(int n) {
    // first call: 123
    int mystery(int n) {
        if (n < 10) {
            // second call: 12
            int mystery(int n) {
                if (n < 10) {
                    // third call: 1
                    int mystery(int n) {
                        if (n < 10) {
                            return n; 1
                        } else {
                            return (n % 10) + mystery(n / 10);
                        }
                    }
            }
        }
    }
}
```

A: 5

B: 6

C: 7

D: 8

What is the result of `mystery(123)`?

_____

What Does This Code Do?

```
int mystery(int n) {
    // first call: 123
    int mystery(int n) {
        if (n < 10) {
            // second call: 12
            int mystery(int n) {
                if (n < 10) {
                    return n;
                } else {
                    return (n % 10) + mystery(n / 10);
                }
            }
        }
    }
}
```

2 1

A: 5

B: 6

C: 7

D: 8

What is the result of `mystery(123)`?

_____

What Does This Code Do?

```
int mystery(int n) {
    // first call: 123
    int mystery(int n) {
        if (n < 10) {
            return n;
        } else {
            return (n % 10) + mystery(n / 10);
        }
    }
    }
}
```

3 3

A: 5

B: 6

C: 7

D: 8

What is the result of `mystery(123)`?

_____

Is Palindrome

Write a recursive function `isPalindrome(string s)` accepts a `string` and returns `true` if it reads the same forwards as backwards.

```
isPalindrome("madam")                          → true
isPalindrome("racecar")                        → true
isPalindrome("step on no pets")                → true
isPalindrome("able was I ere I saw elba")      → true
isPalindrome("Q")                              → true
isPalindrome("Java")                           → false
isPalindrome("rotater")                        → false
isPalindrome("byebye")                         → false
isPalindrome("notion")                         → false
```

What is a good base case?

Is Palendrome

```
// Returns true if the given string reads the same
// forwards as backwards.
// Trivially true for empty or 1-letter strings.
bool isPalindrome(string s) {
    if (s.length() < 2) {   // base case
        return true;
    } else {                // recursive case
        if (s[0] != s[s.length() - 1]) {
            return false;
        }
        string middle = s.substr(1, s.length() - 2);
        return isPalindrome(middle);
    }
}
```

Hailstone

Print the sequences of numbers that you take to get from N until 1, using the Hailstone (Collatz) production rules:

If n == 1, you are done.

If n is odd your next number is 3*n + 1.

If n is even your next number is n / 2.

Hailstone

```
// Couts the sequence of numbers from n to one
// produced by the Hailstone (aka Collatz)
// procedure
void hailstone(int n) {
   cout << n << endl;
   if(n == 1) {
      return;
   } else {
      if(n % 2 == 0) {
         // n is even so we repeat with n/2
         hailstone(n / 2);
      } else {
         // n is odd so we repeat with 3 * n + 1
         hailstone(3 * n + 1);
      }
   }
}
```

Are The Recursive Calls Simpler?

```
// Couts the sequence of numbers from n to one
// produced by the Hailstone (aka Collatz)
```

3. When you make a recursive call it should be to a simpler instance (forward progress towards base case)

Well that seems to be true…

```
}
```

Works for numbers up to 5×10^{18}

Reward for proof

$1,400

Today's Route

Today's Route

And Here We Are

Tower of Hanoi

Tower of Hanoi Insight

Tower of Hanoi Insight

Tower of Hanoi Insight

Tower of Hanoi Insight

Tower of Hanoi Insight

Tower of Hanoi Insight

Tower of Hanoi Insight

Tower of Hanoi Insight

Base Case

- We need to find a very simple case that we can solve directly in order for the recursion to work.
- If the tower has size one, we can just move that single disk from the source to the destination.

Qt

```
int main() {
    moveTower(3, 'a', 'c', 'b');
}
```

```
int main() {
    moveTower(3, 'a', 'c', 'b');
}
```

```
int main() {
    moveTower(3, 'a', 'b', 'c');
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
3	a	c	b

```
int main() {
    moveTower(3, 'a', 'b', 'c');
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
3	a	c	b

```
int main() {
    moveTower(3, 'a', 'b', 'c');
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
3	a	c	b

```
int main() {
    moveTower(3, 'a', 'b', 'c');
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
3	a	c	b

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	a	b	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	a	b	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	a	b	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	a	b	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	a	c	b

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	a	c	b

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	a	c	b

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	a	c	b

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	a	b	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	a	b	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	a	b	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	a	b	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	c	b	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	c	b	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	c	b	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	c	b	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	a	b	c

```
int main() {
    moveTower(3, 'a', 'b', 'c');
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
3	a	c	b

```
int main() {
    moveTower(3, 'a', 'b', 'c');
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
3	a	c	b

```
int main() {
    moveTower(3, 'a', 'b', 'c');
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
3	a	c	b

```
int main() {
    moveTower(3, 'a', 'b', 'c');
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
3	a	c	b

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	b	c	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	b	c	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	b	c	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	b	c	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	b	a	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	b	a	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	b	a	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	b	a	c

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	b	c	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	b	c	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	b	c	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	b	c	a

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	a	c	b

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	a	c	b

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	a	c	b

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
1	a	c	b

```
int main() {
}
```

```
void moveTower(int n, char from, char to, char temp) {
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
2	b	c	a

```
int main() {
    moveTower(3, 'a', 'b', 'c');
}
```

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

n	from	to	temp
3	a	b	c

```
int main() {
    moveTower(3, 'a', 'b', 'c');
}
```

Recursive Leap of Faith

Recursive Leap of Faith

The Solution

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

Three Musts of Recursion

1. Your code must have a case for all valid inputs.

2. You must have a base case.

3. When you make a recursive call it should be to a simpler instance (forward progress towards base case)

The Solution

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 1) {
        moveSingleDisk(from, to);
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

Another Solution

```
void moveTower(int n, char from, char to, char temp) {
    if (n == 0) {
        return;
    } else {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

Even Simpler Solution

```
void moveTower(int n, char from, char to, char temp) {
    if (n > 0) {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

Even Simpler Solution

```
void moveTower(int n, char from, char to, char temp) {
    if (n > 0) {
        moveTower(n – 1, from, temp, to);
        moveSingleDisk(from, to);
        moveTower(n – 1, temp, to, from);
    }
}
```

You can do recursion.

Your Brain is Recursive

There is a pathway in your brain for imagination

Step 1: Imagine your life at the end of CS106B

Step 1: Imagine your life at the end of CS106B

Step 2: Imagine your life after Stanford

Step 2: Imagine your life after Stanford

Executes the same "function" with different inputs

Step 2: Imagine your life after Stanford

Executes the same "function" with different inputs

Corballis, M.C. The Recursive Mind. Princeton University Press, 2011.

Today's Goal

1. Be able to think about problems recursively

Three Musts of Recursion

1. Your code must have a case for all valid inputs.

2. You must have a base case.

3. When you make a recursive call it should be to a simpler instance (forward progress towards base case)