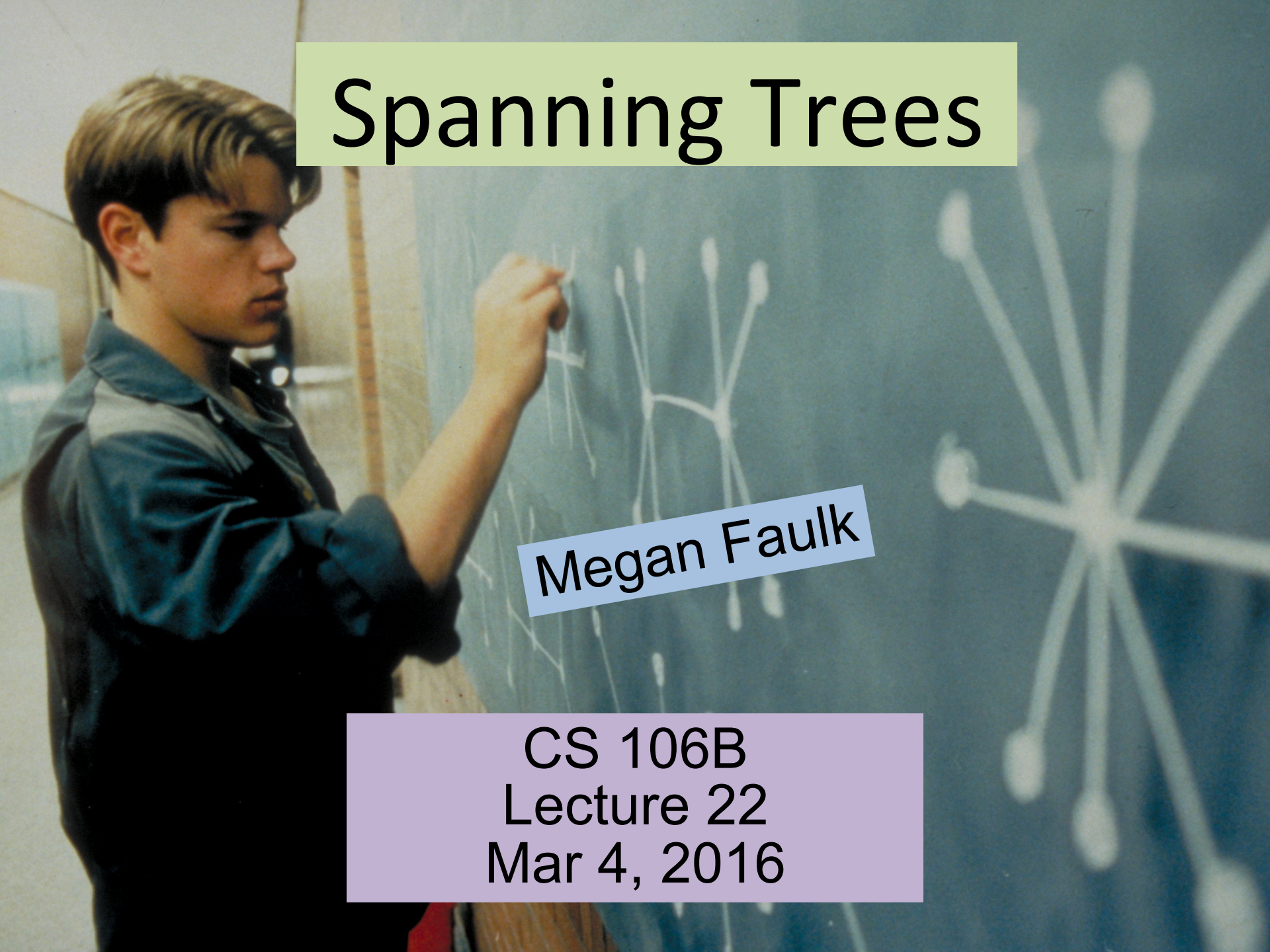


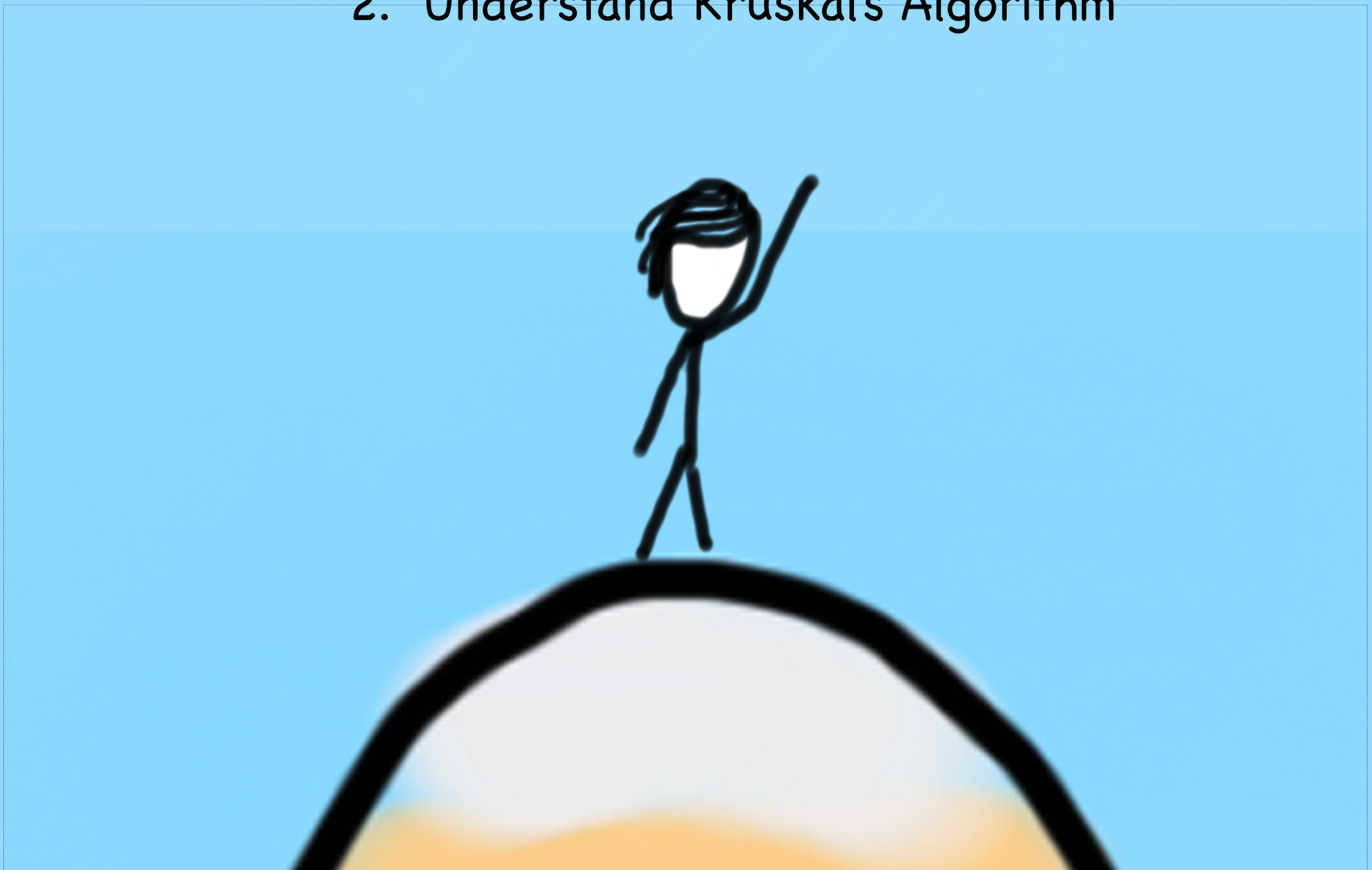
Spanning Trees

Megan Faulk

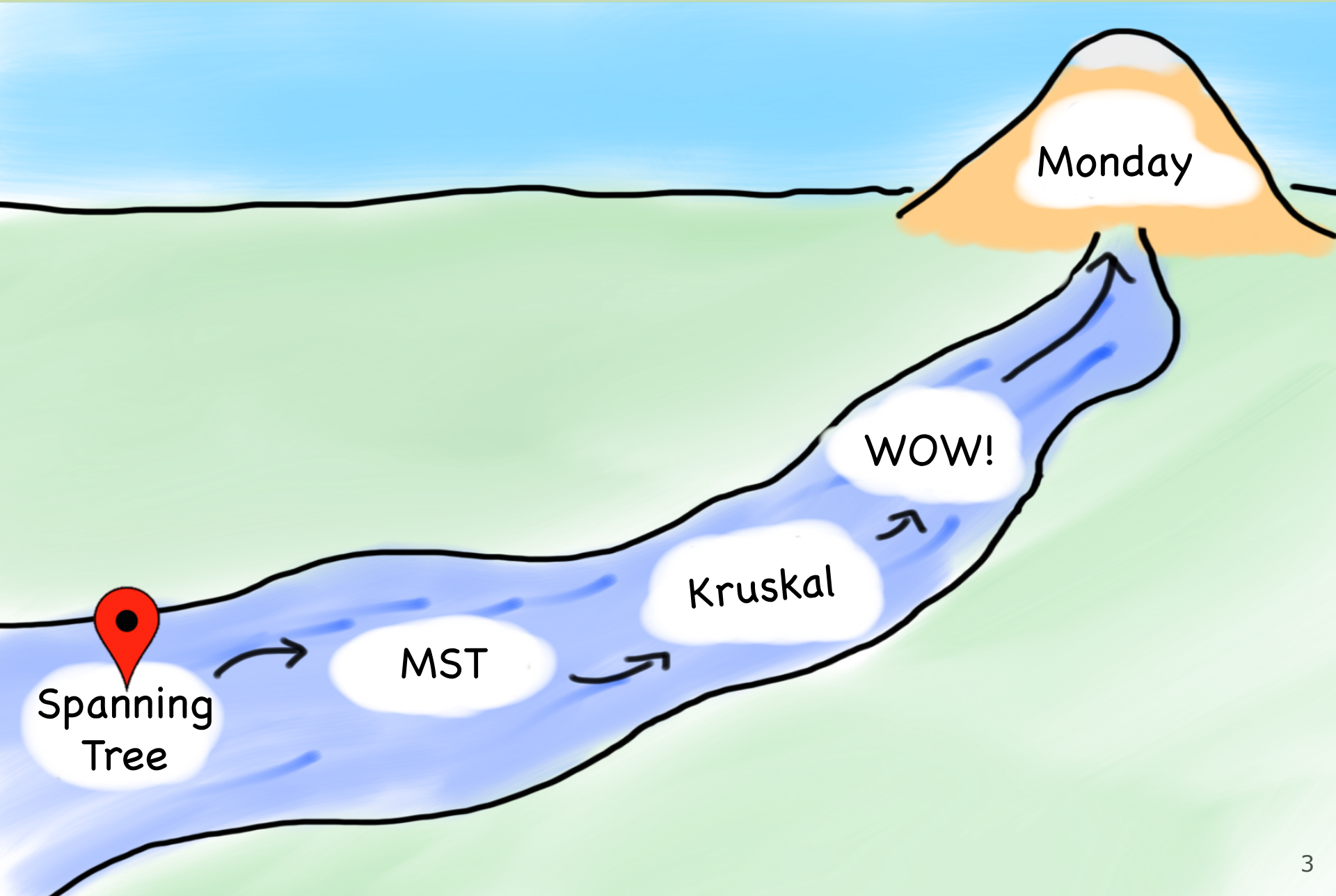
CS 106B
Lecture 22
Mar 4, 2016

Today's Goal

1. Learn what a Spanning Tree is
2. Understand Kruskal's Algorithm

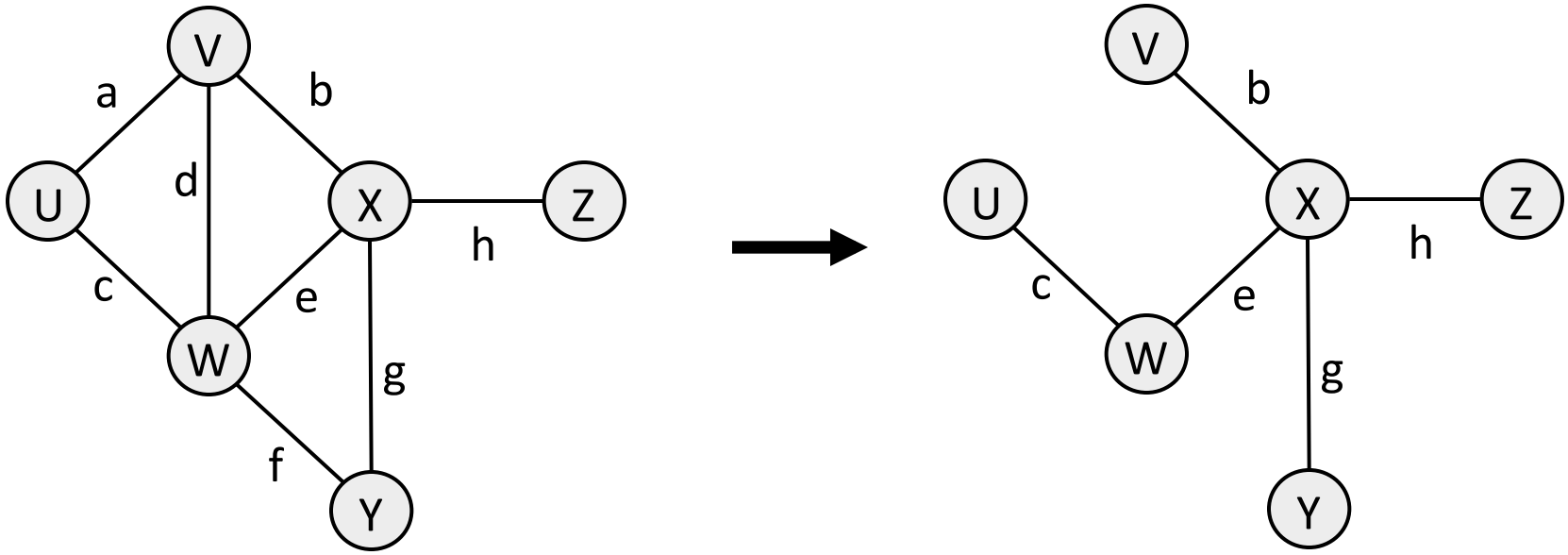


Today's Route



Spanning trees

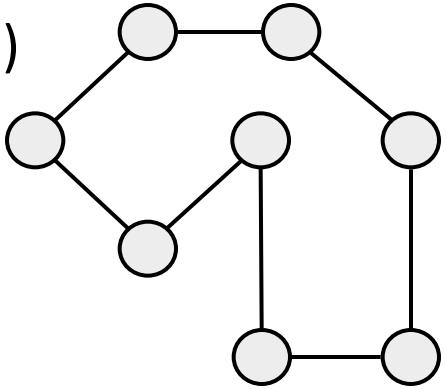
- A **spanning tree** of a graph is a set of edges that connects all vertices in the graph with no cycles.
  - What is a spanning tree for the graph below?



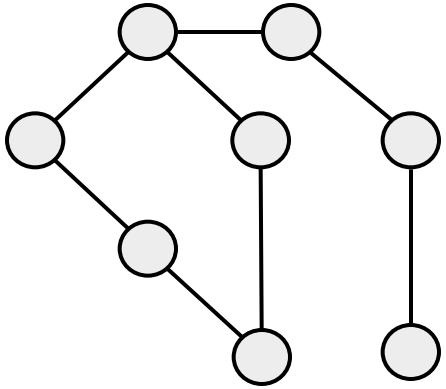
Span tree examples

- **Q:** How many of the graphs shown are legal spanning trees?

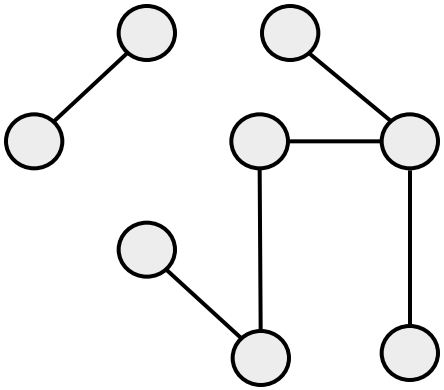
(1)



(2)



(3)



A. none
B. one
C. two
D. all three

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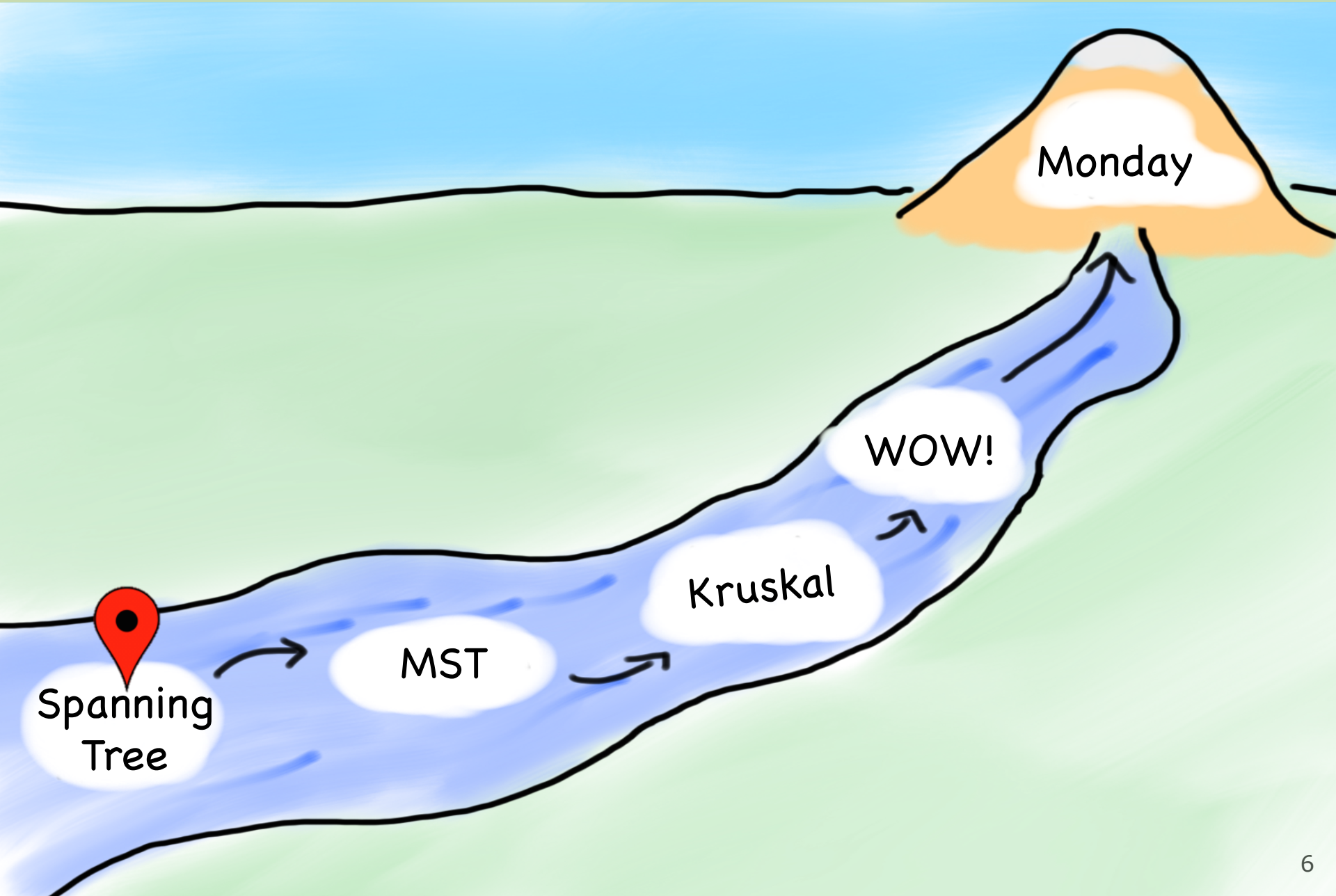
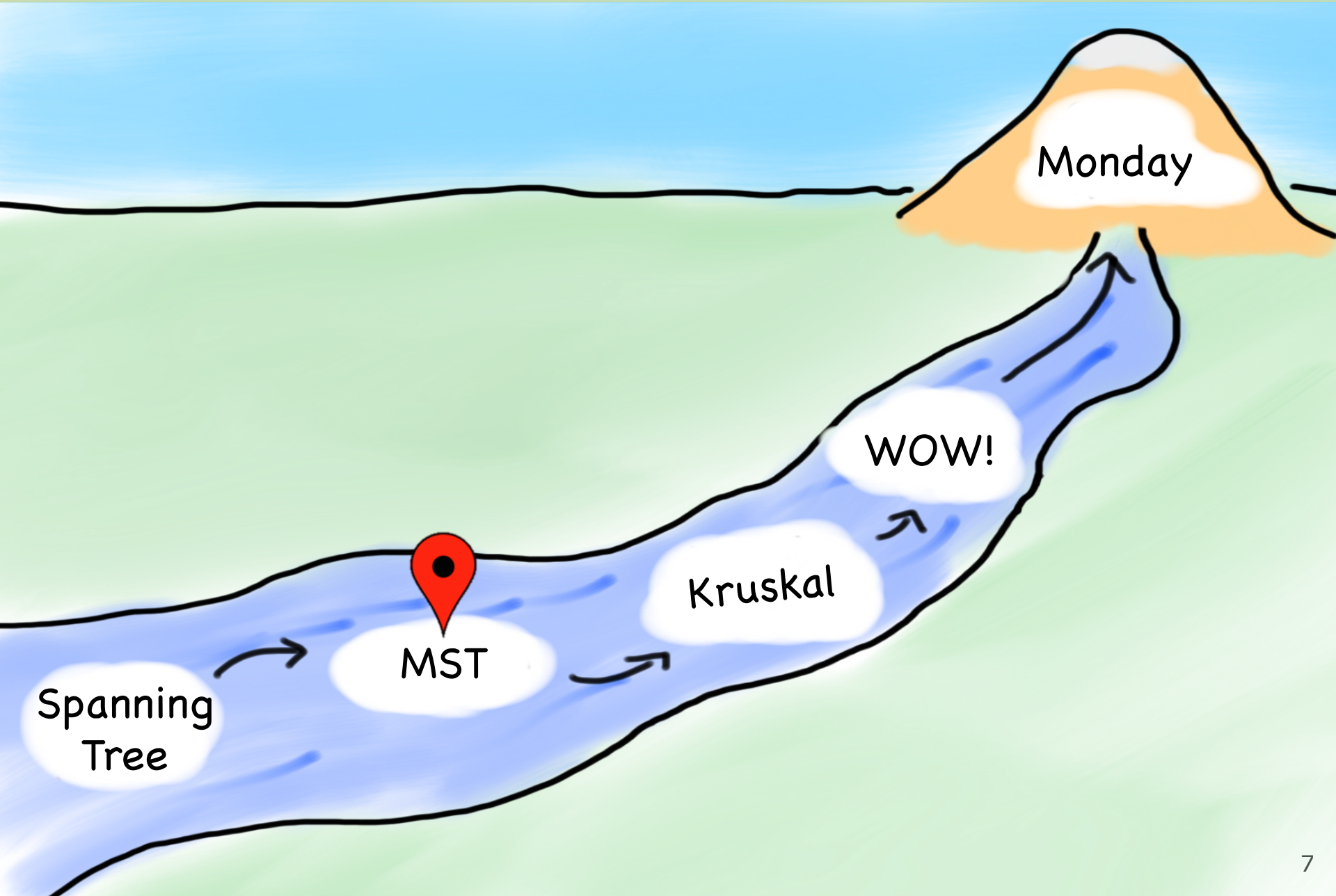
Today's Route

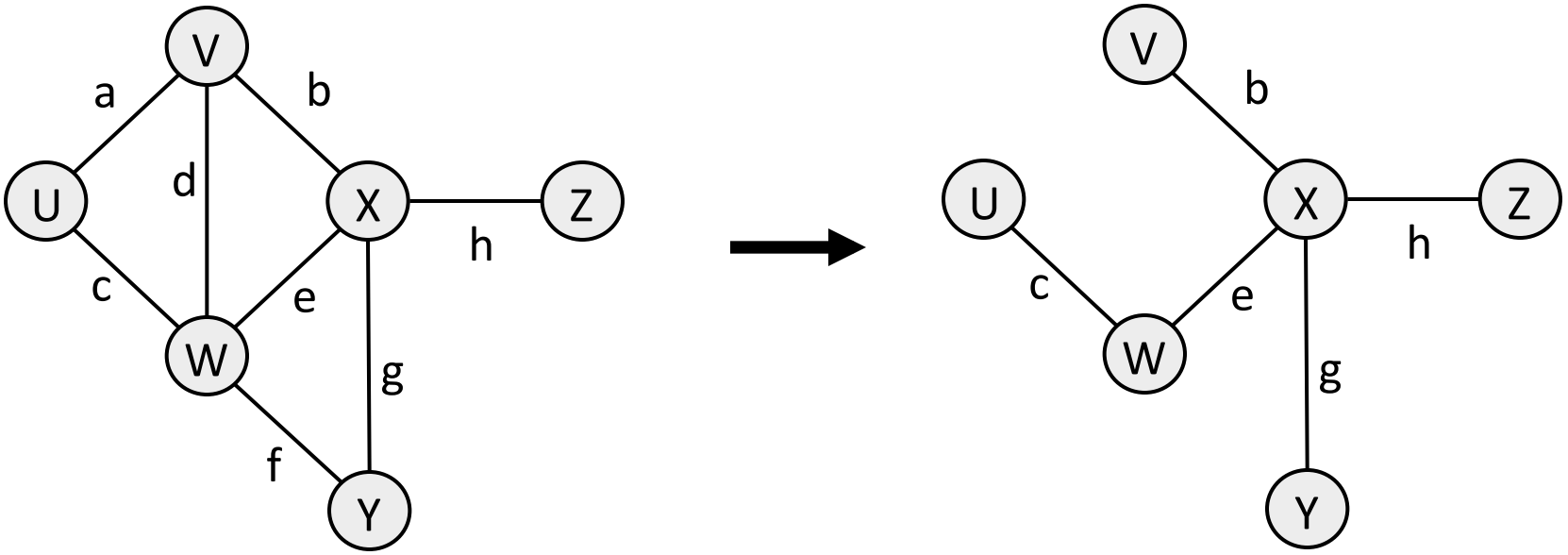


Today's Route



Minimum spanning tree

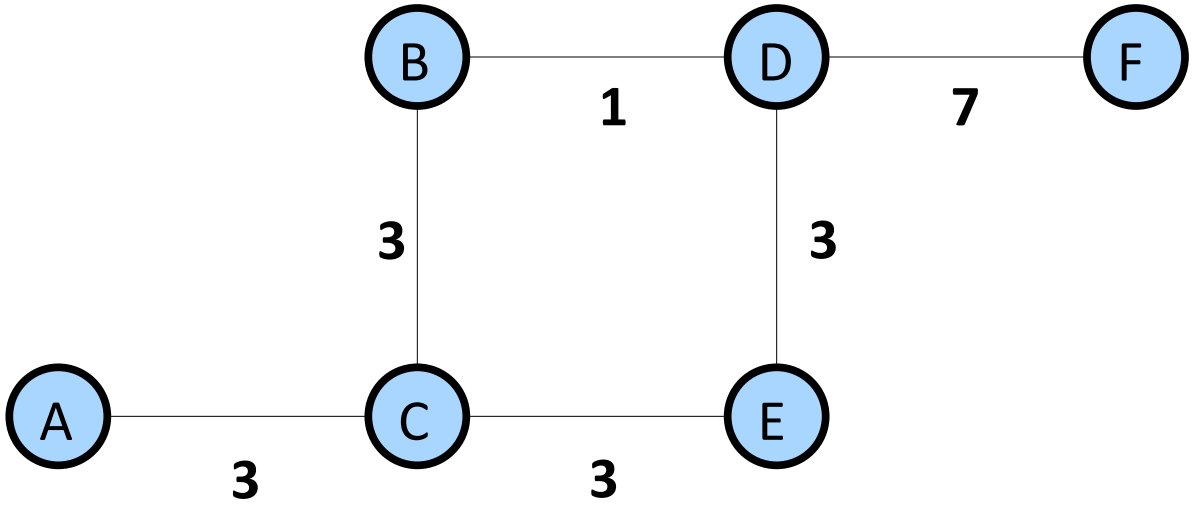
- **minimum spanning tree** (MST): A spanning tree that has the lowest combined edge weight (cost), or minimum # edges if unweighted.



MST examples

- **Q:** How many minimum spanning trees does this graph have?

A. 0-1
B. 2-3
C. 4-5
D. 6-7
E. > 7



(question courtesy Cynthia Lee)

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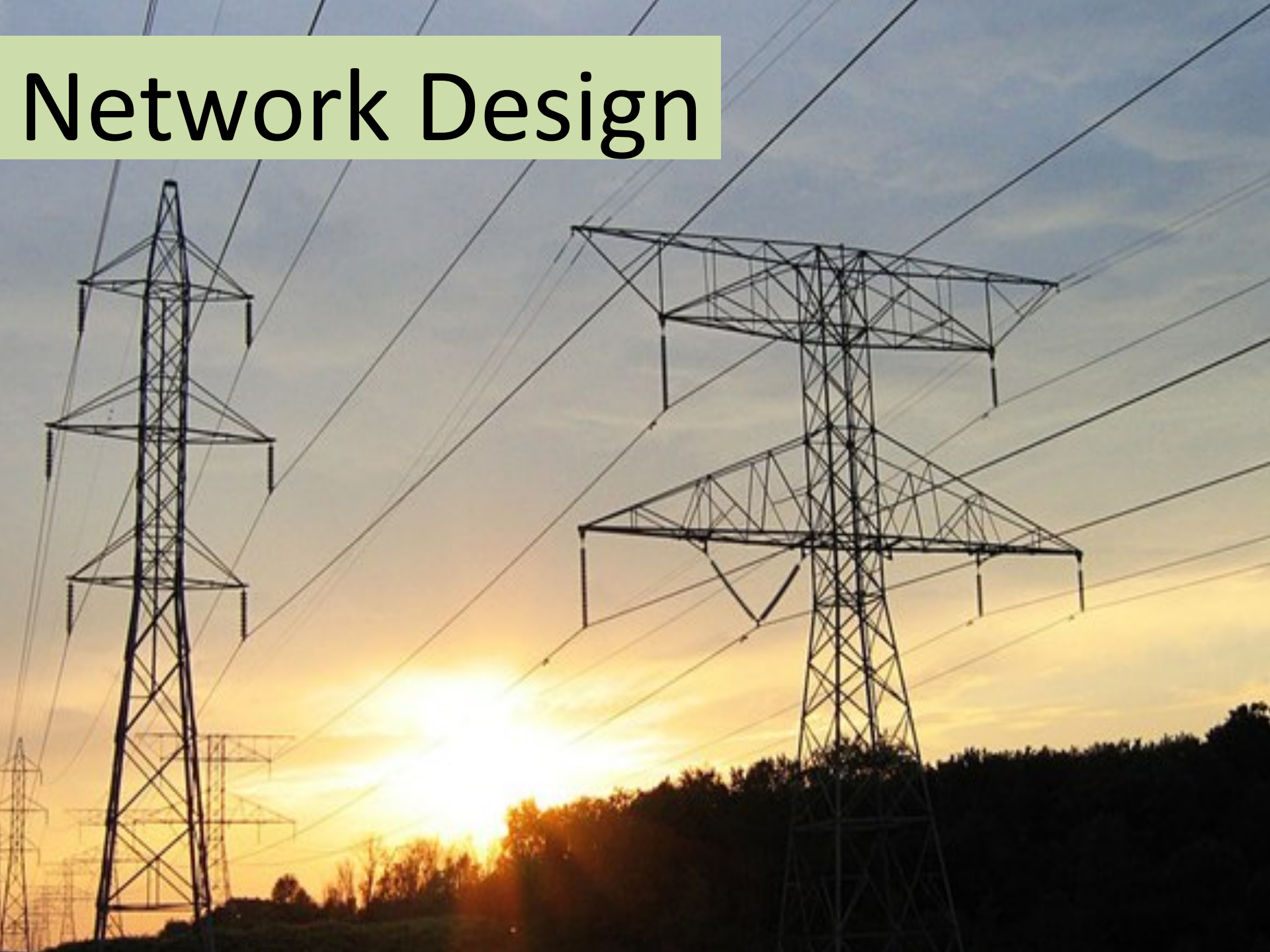
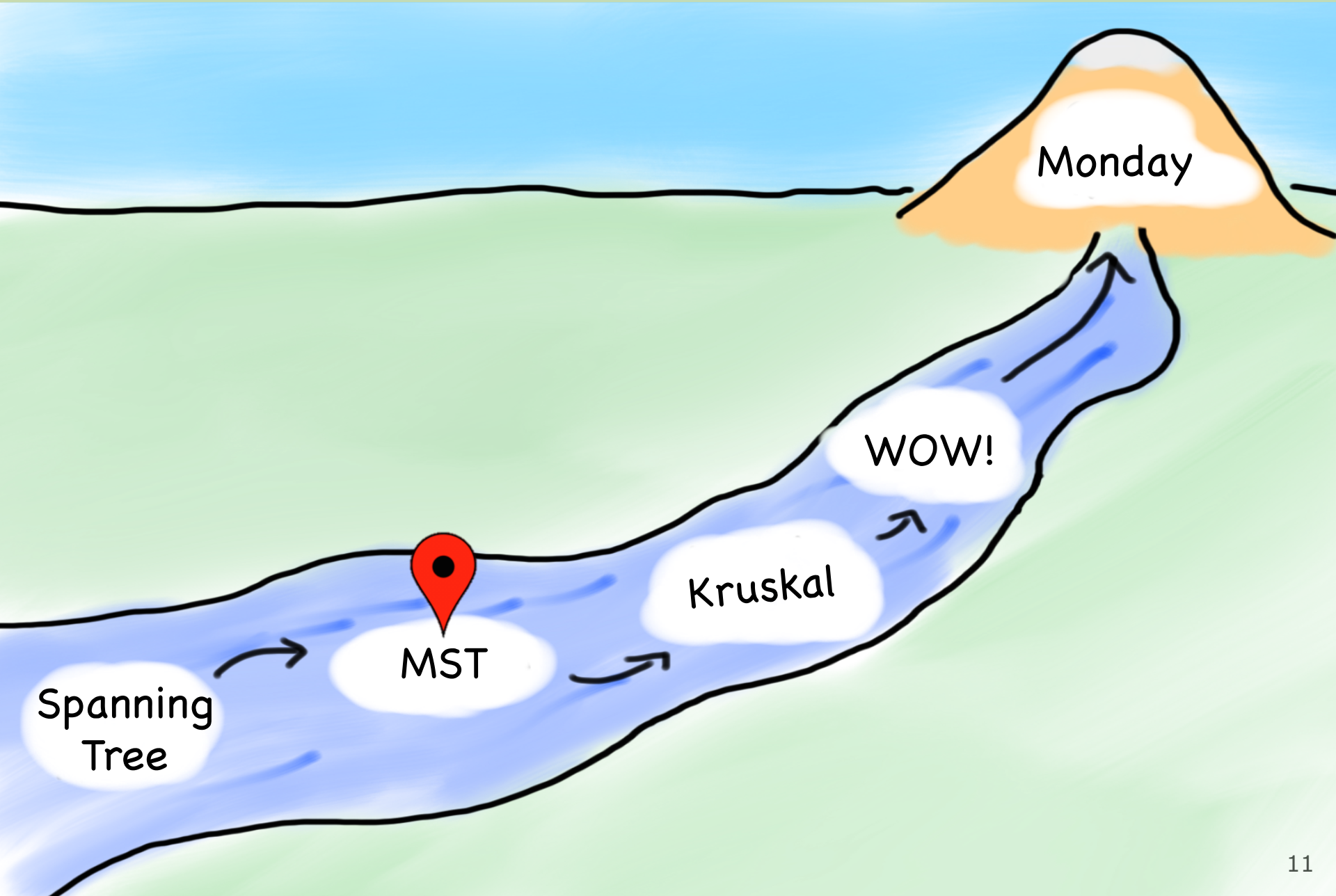
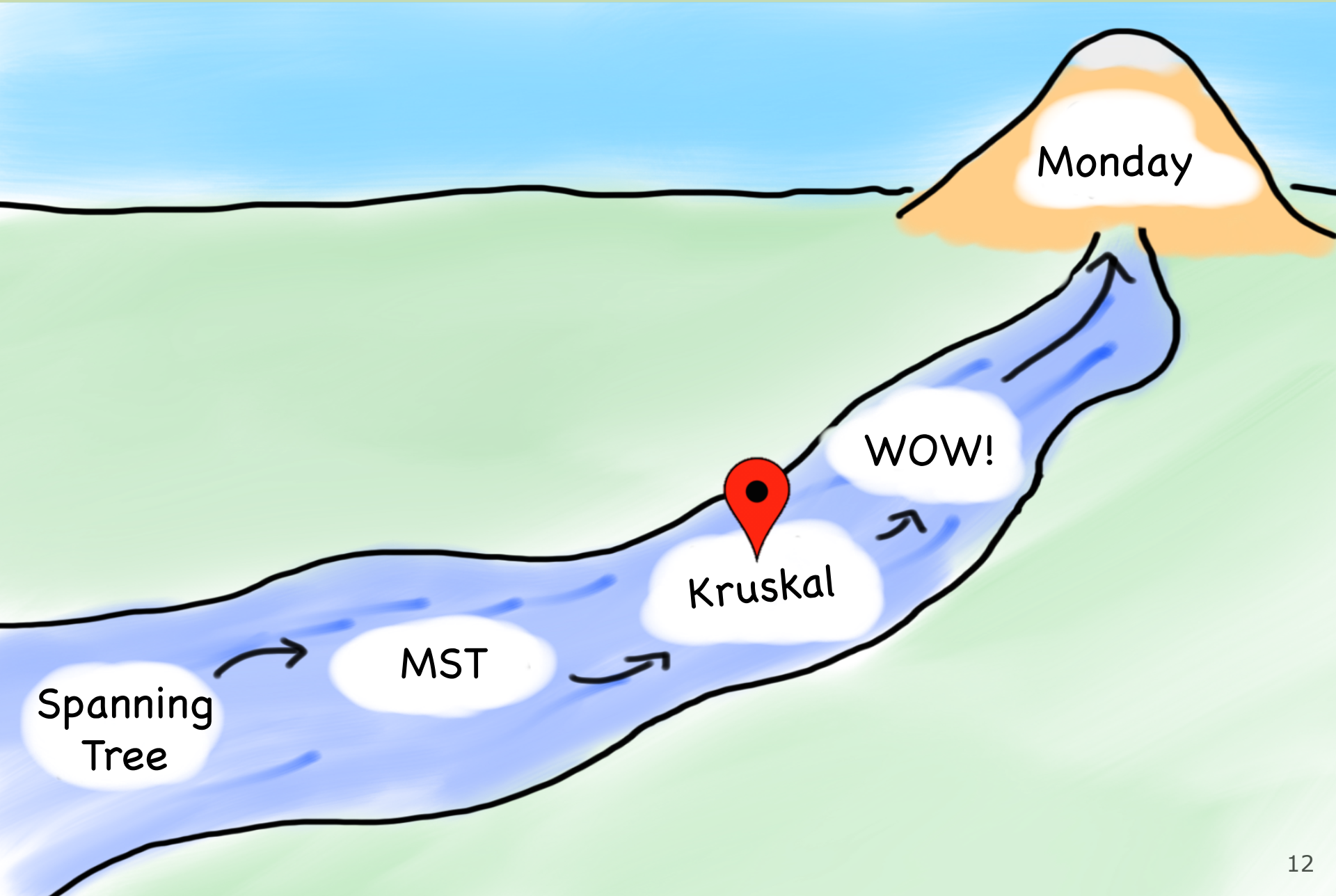
Network Design



Today's Route



Today's Route



Kruskal's algorithm

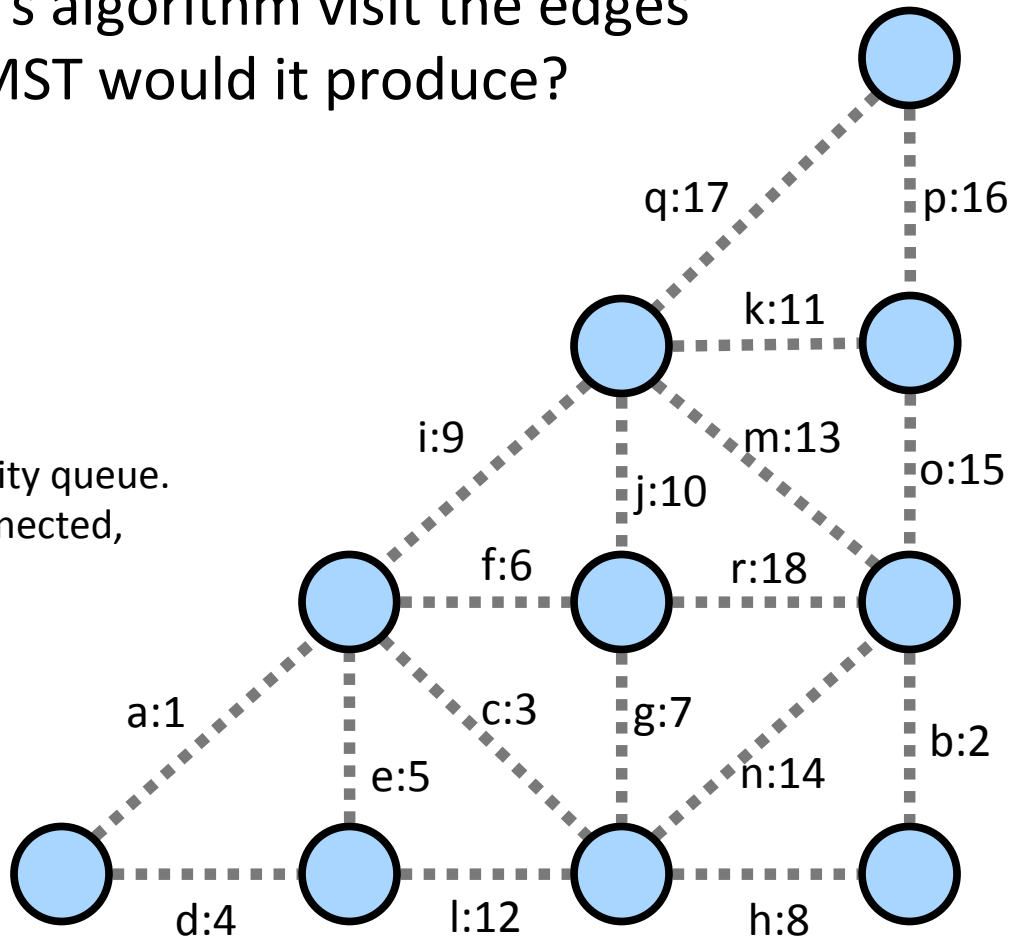
- **Kruskal's algorithm**: Finds a MST in a given graph.

```
function kruskal(graph):
    Remove all edges from the graph.
    Place all edges into a priority queue based on their weight (cost).
    While the priority queue is not empty:
        Dequeue an edge e from the priority queue.
        If e's endpoints aren't already connected to one another,
                add that edge into the graph.
        Otherwise, skip the edge.
```

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.



pq = {a:1, b:2, c:3, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
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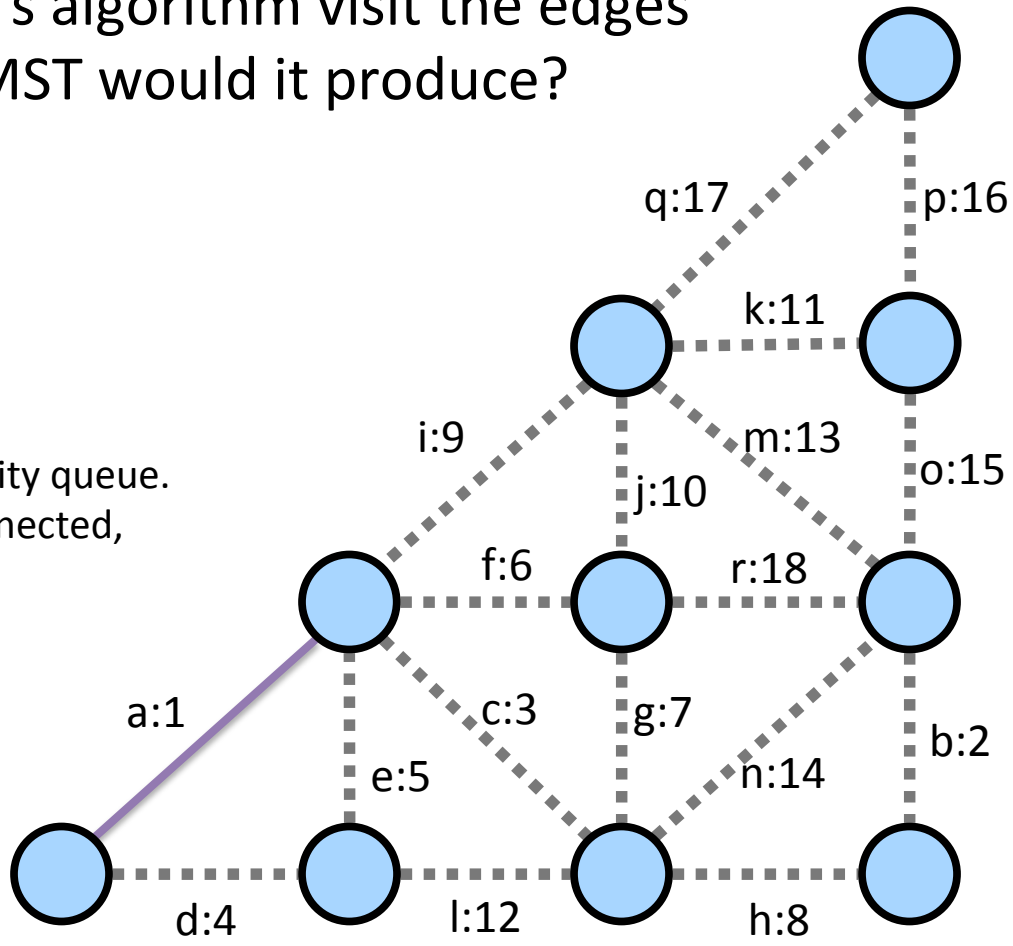
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.



pq = {**a:1**, b:2, c:3, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

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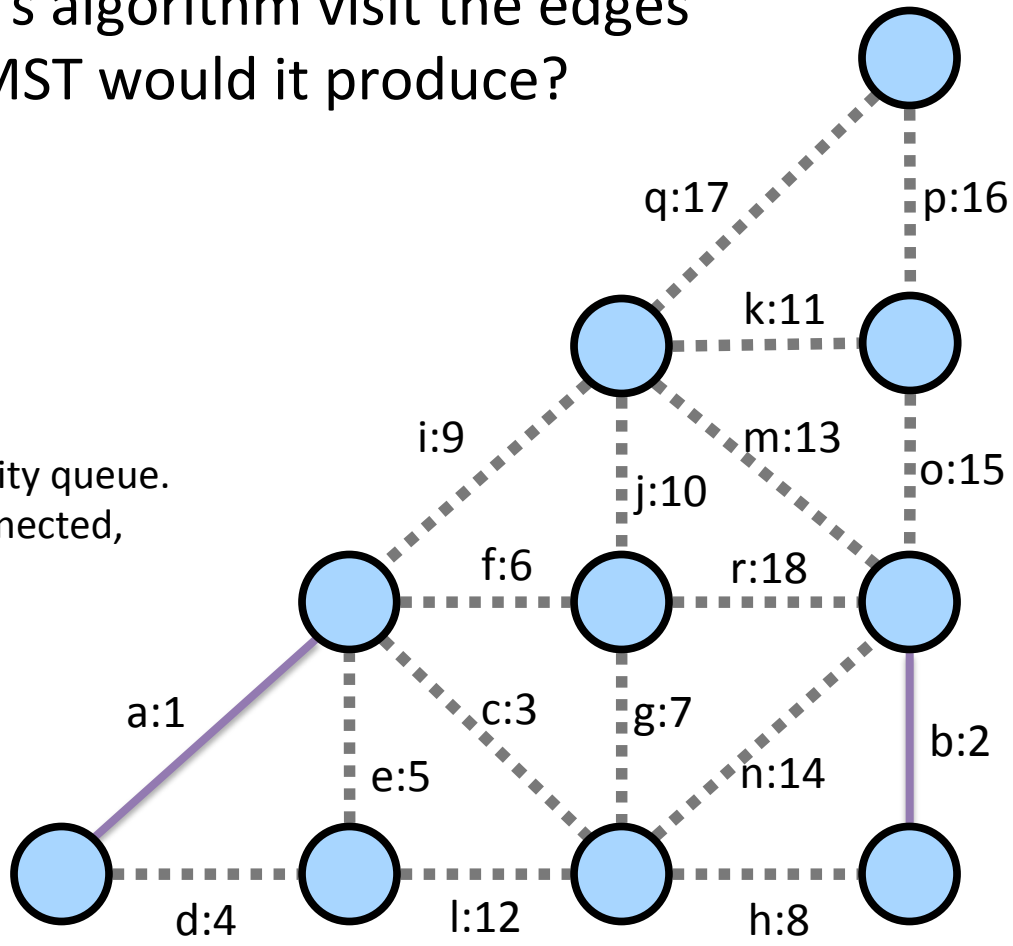
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.

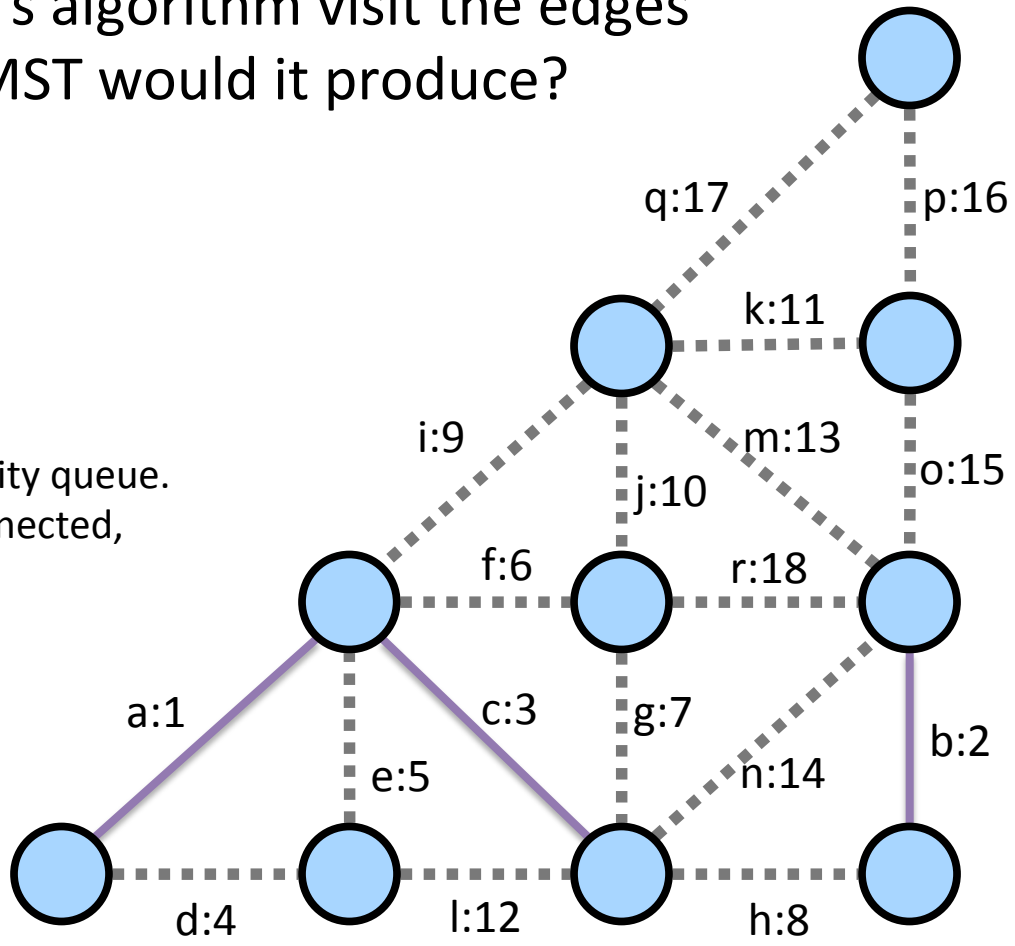


pq = {**b:2**, c:3, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.



pq = {**c:3**, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

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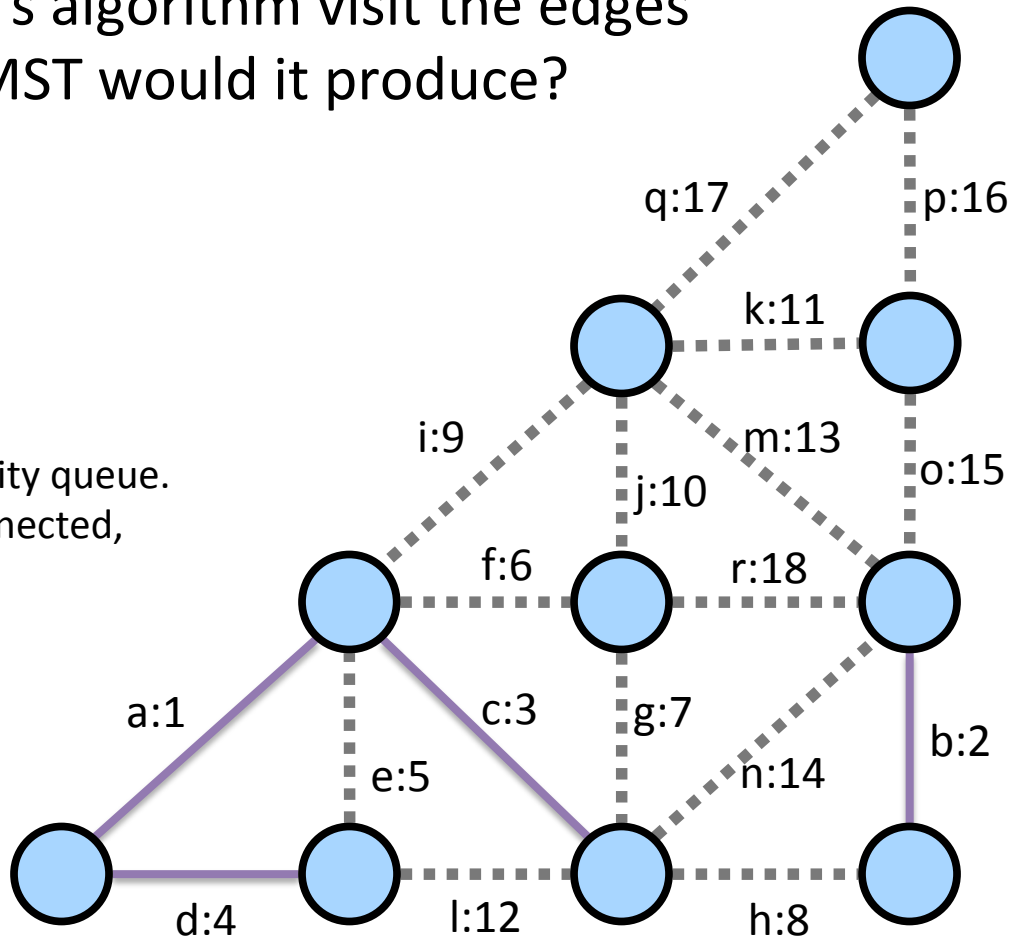
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.

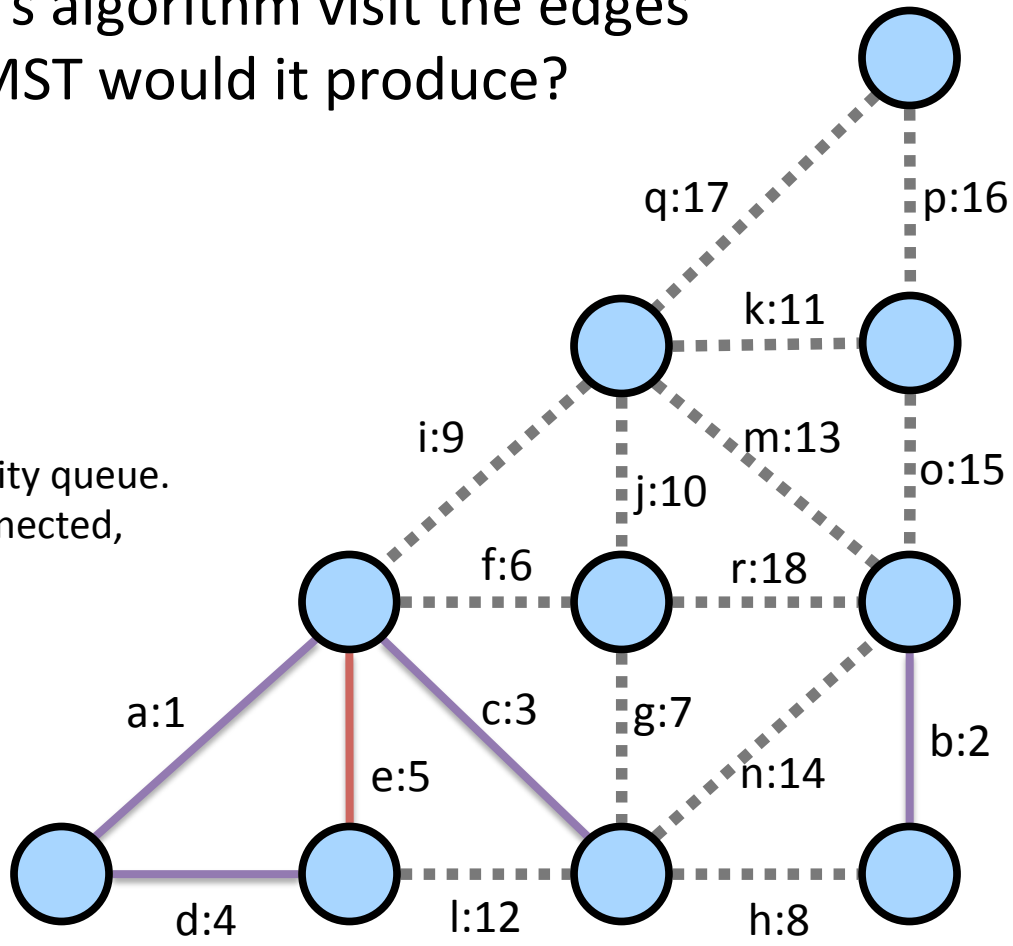


pq = {**d:4**, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.



pq = {**e:5**, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

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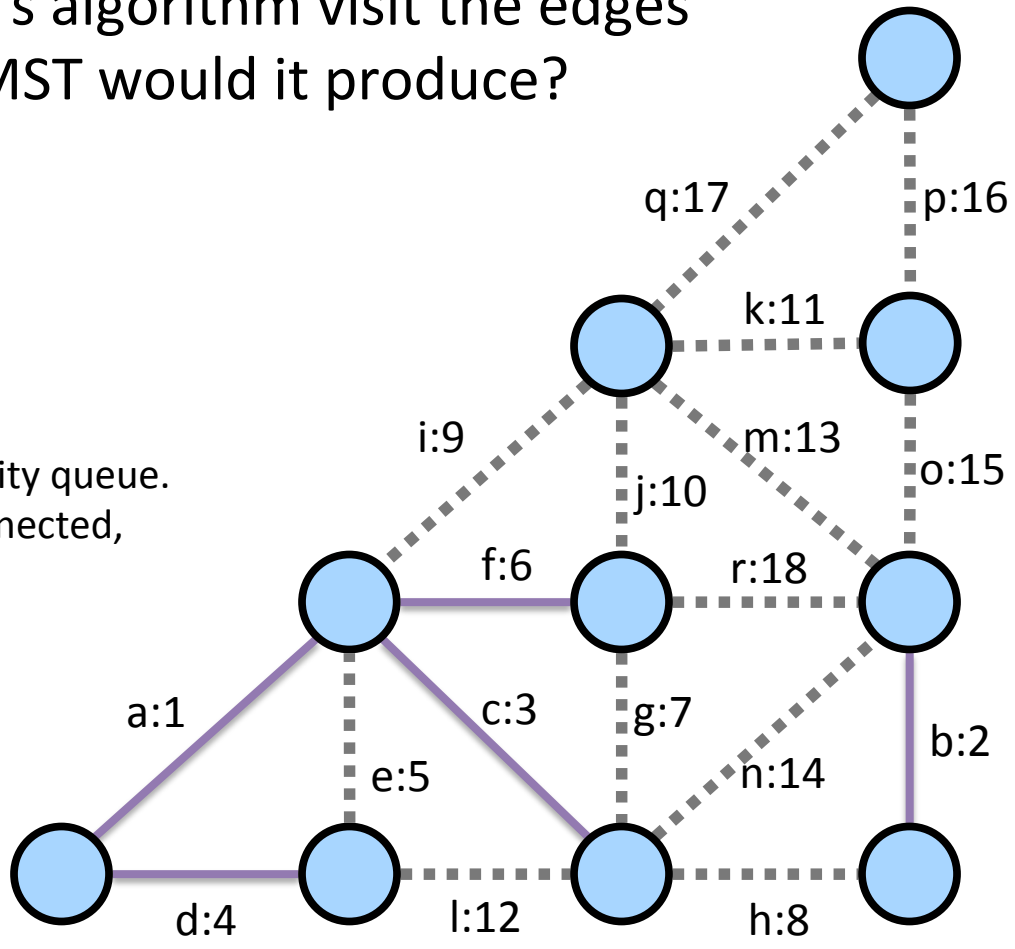
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.

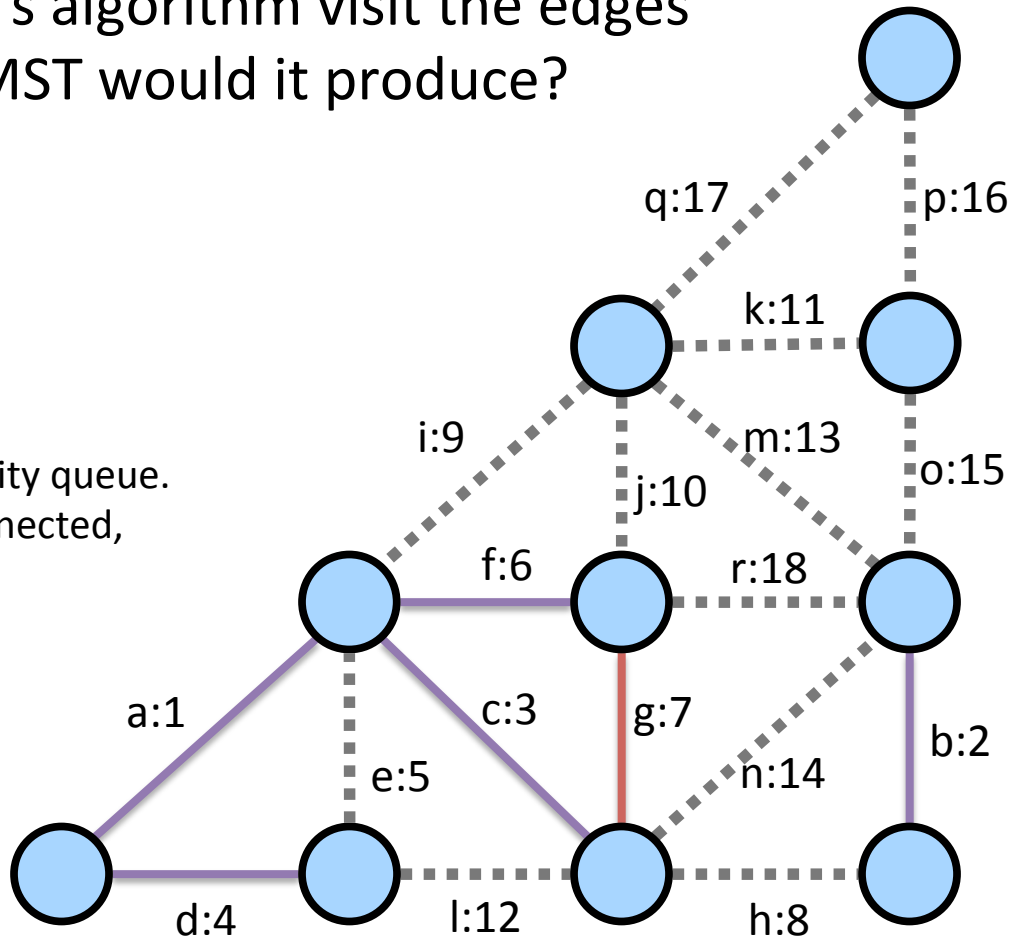


pq = {**f:6**, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.



pq = {**g:7**, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

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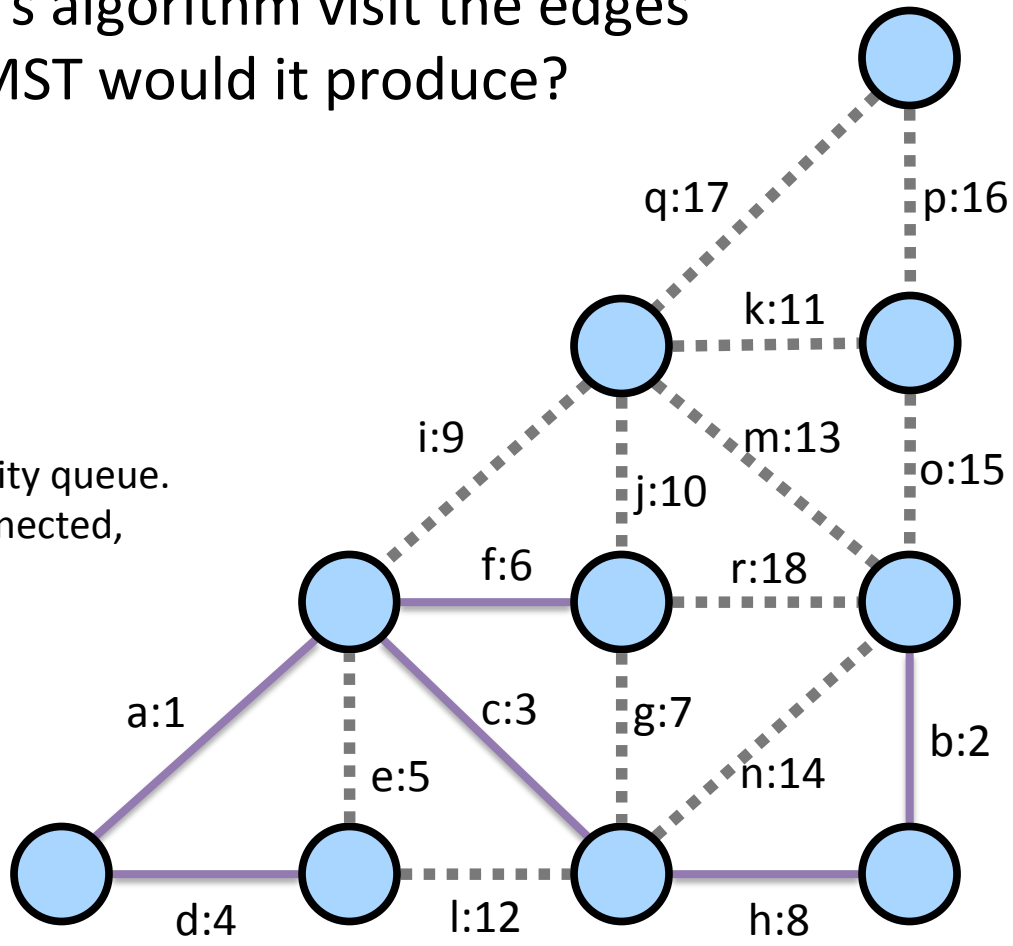
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.

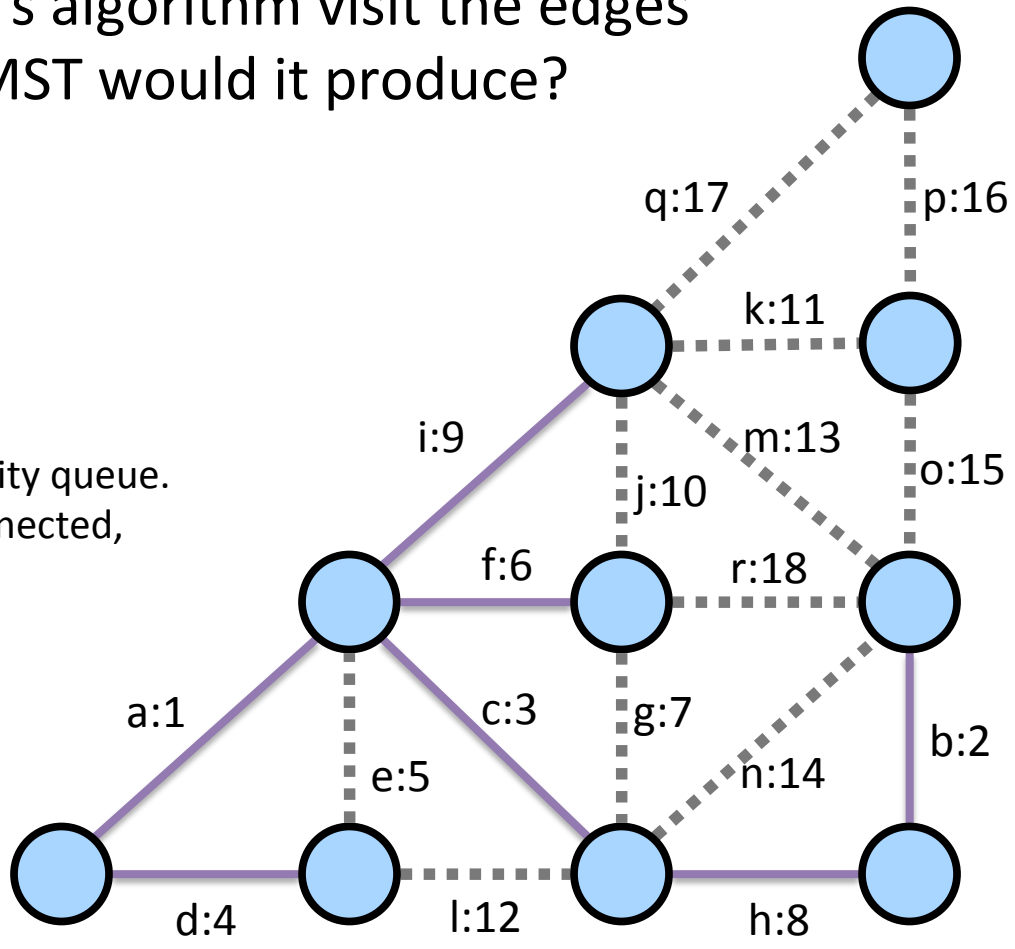


pq = {**h:8**, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.

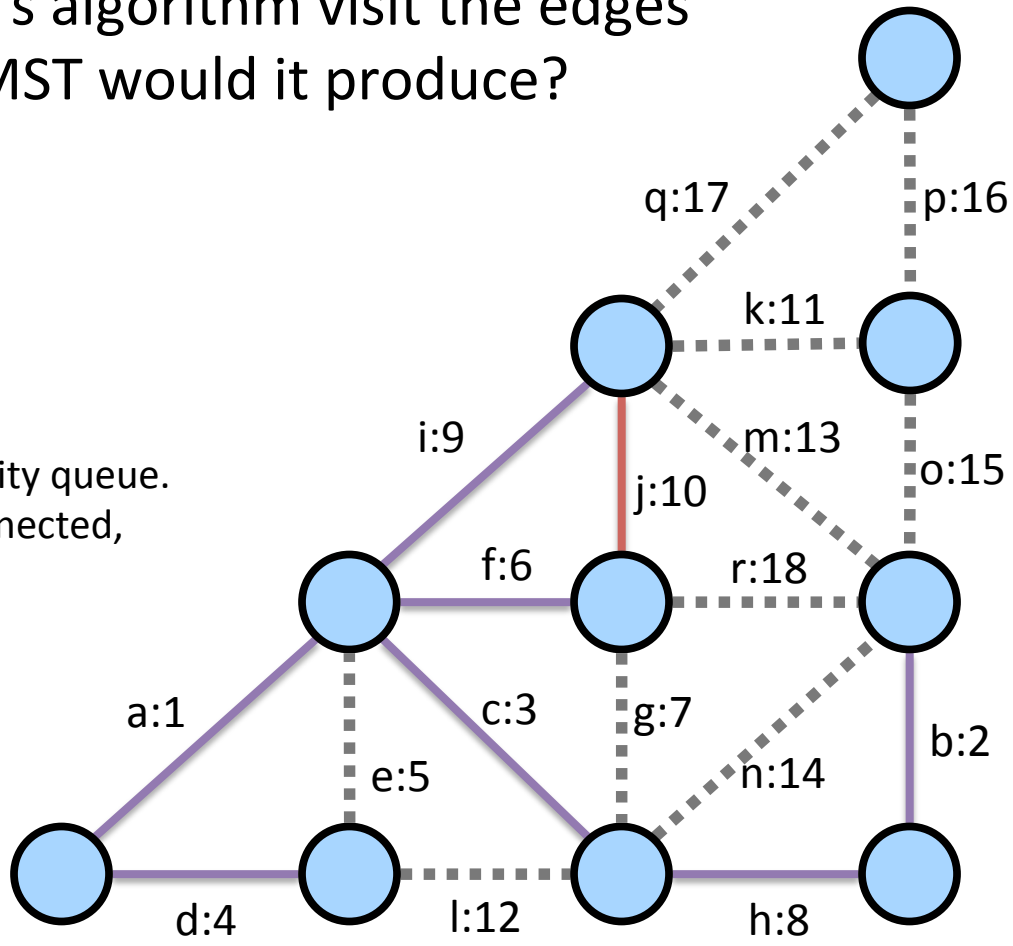


pq = {**i:9**, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.



pq = {**j:10**, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
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  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
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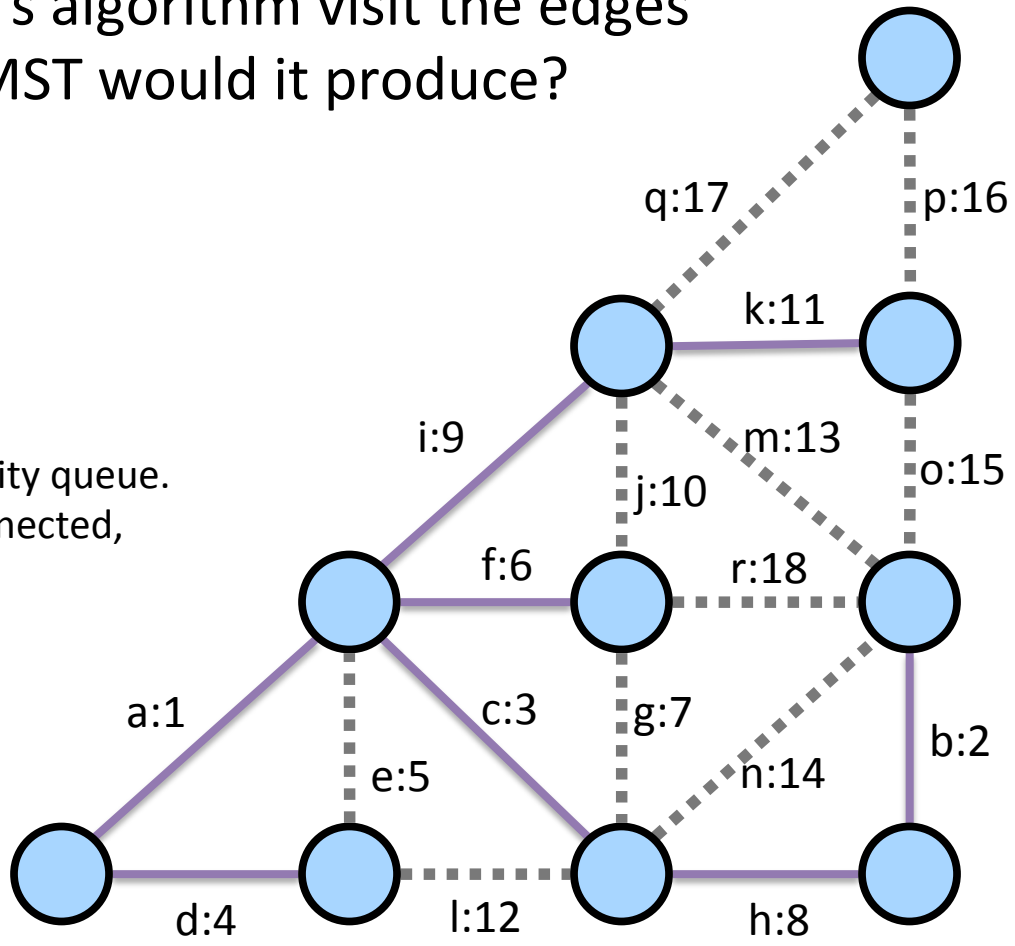
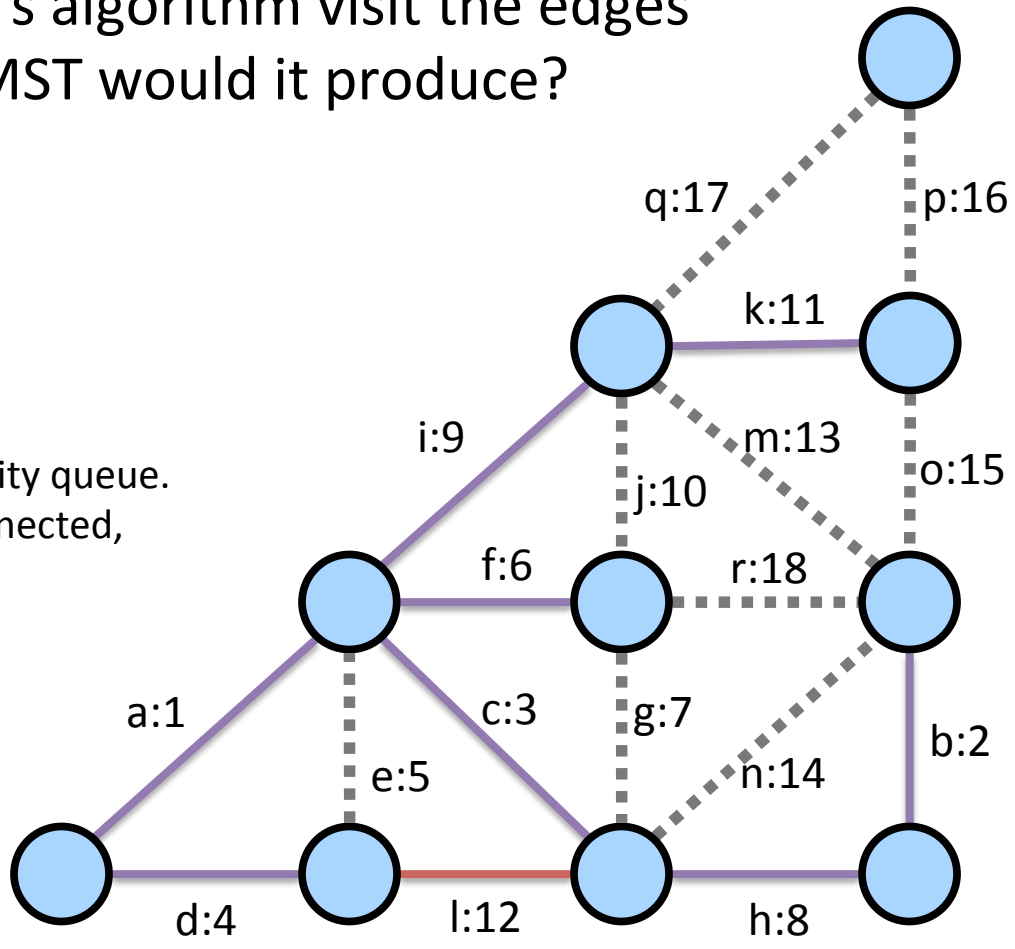


pq = {**k:11**, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.

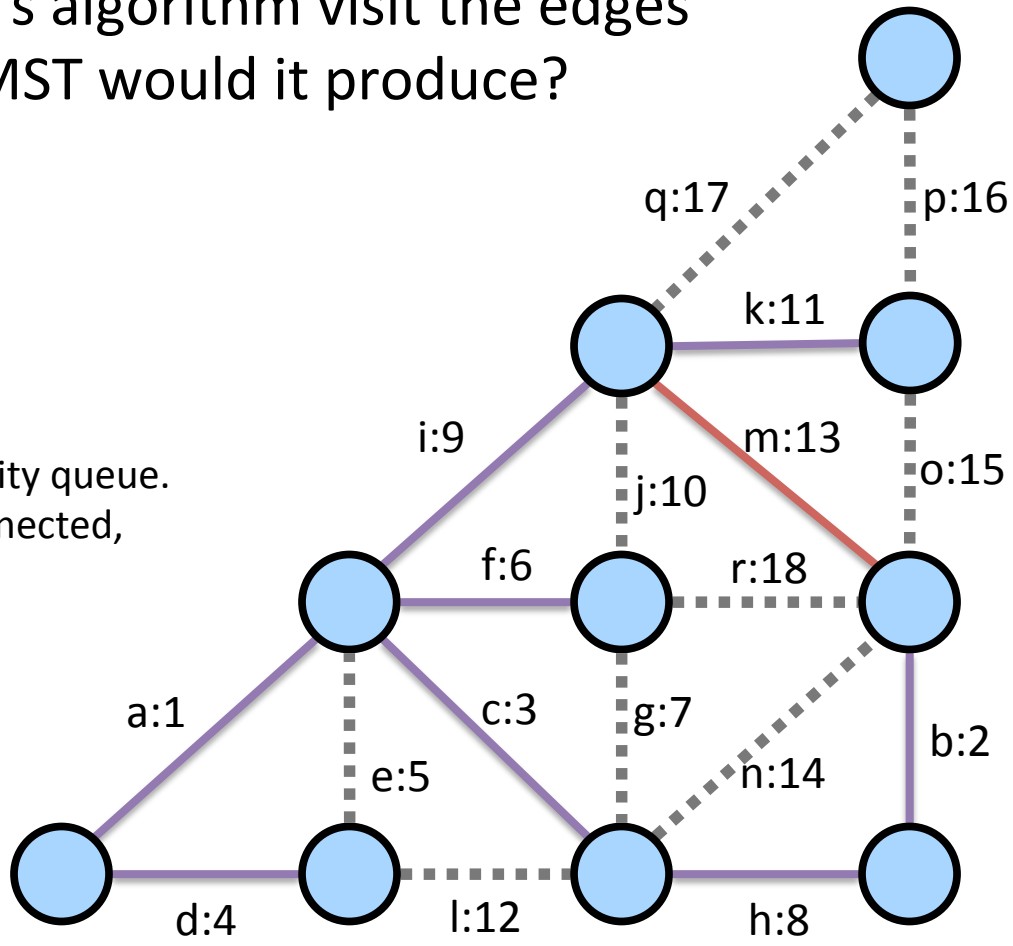


pq = {**l:12**, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.

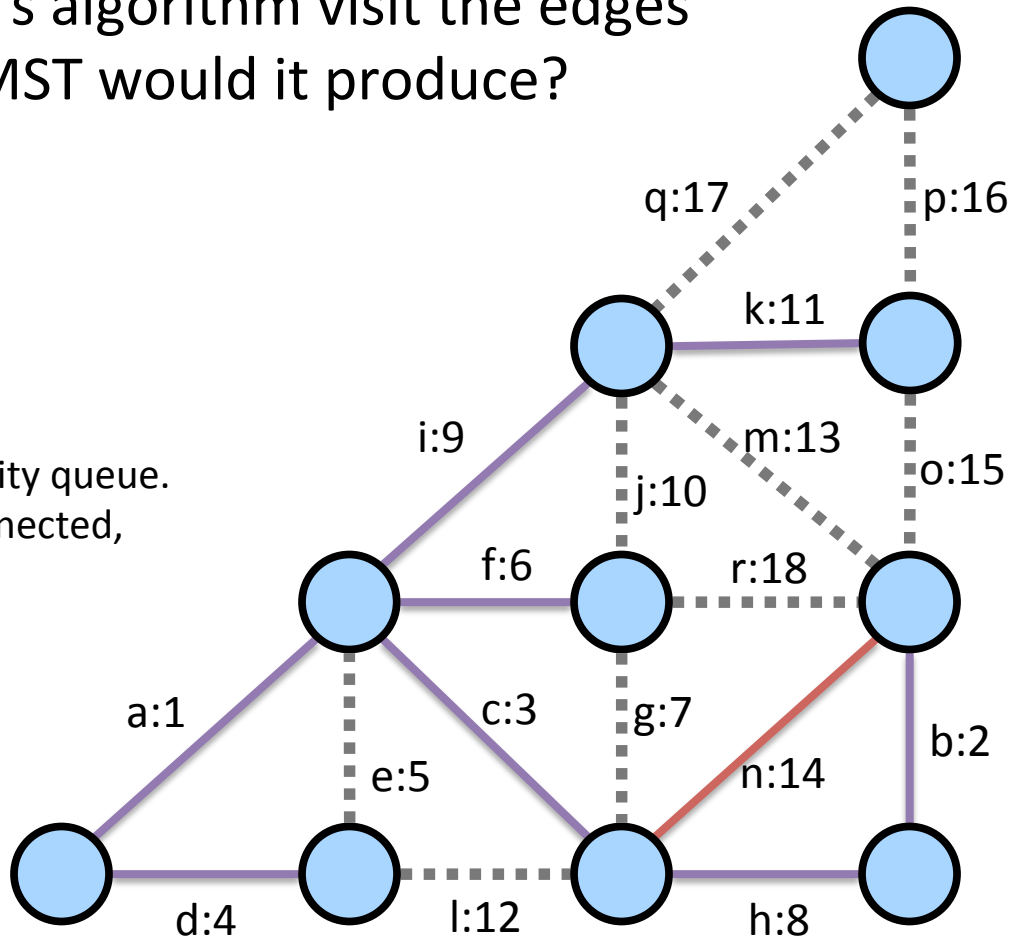


pq = {**m:13**, n:14, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.



pq = {**n:14**, o:15, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
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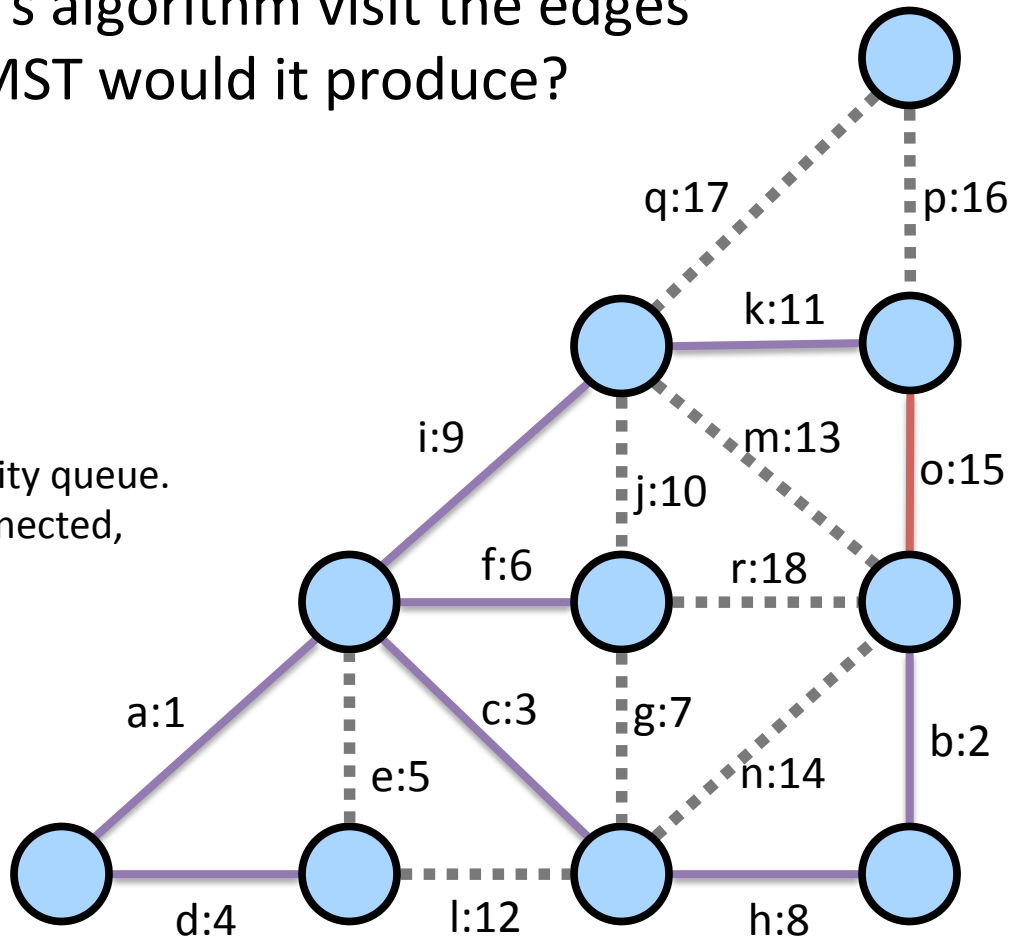
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.



pq = {**o:15**, p:16, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
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  While the priority queue is not empty:
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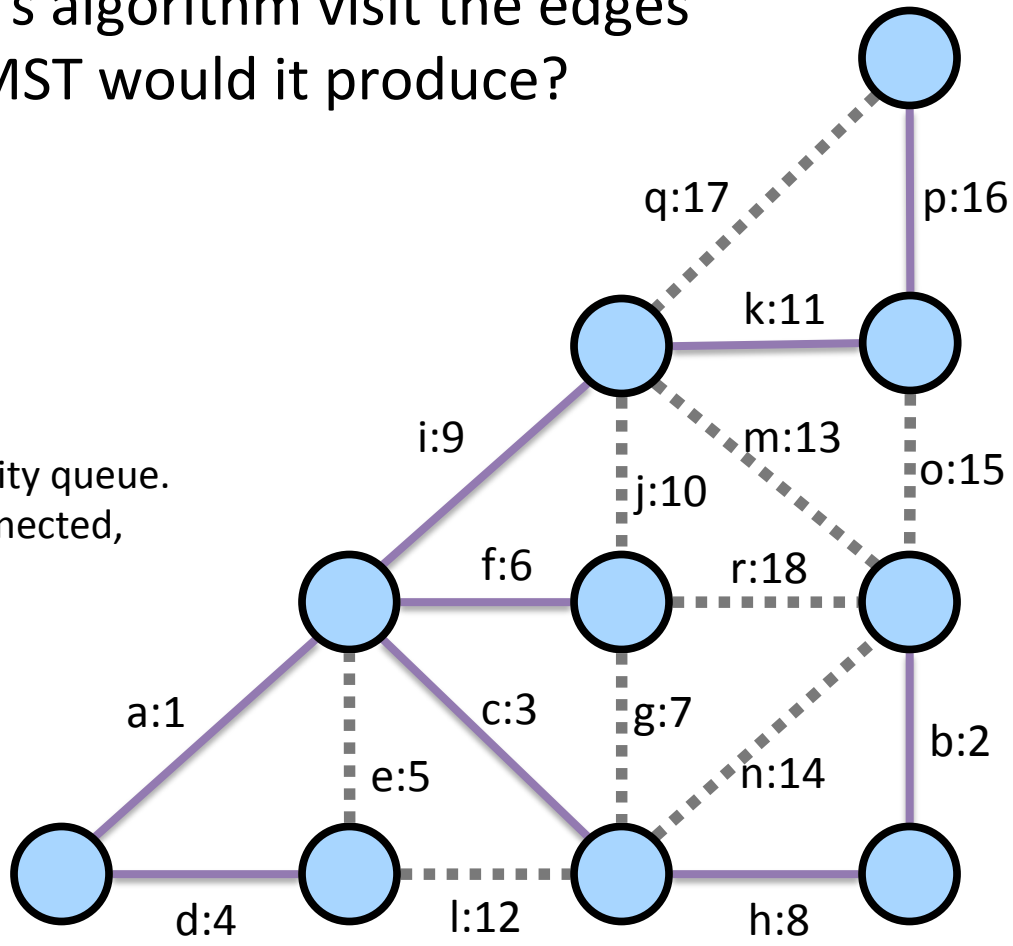
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.



pq = {**p:16**, q:17, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
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  While the priority queue is not empty:
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    If *e*'s endpoints aren't already connected, add that edge into the graph.
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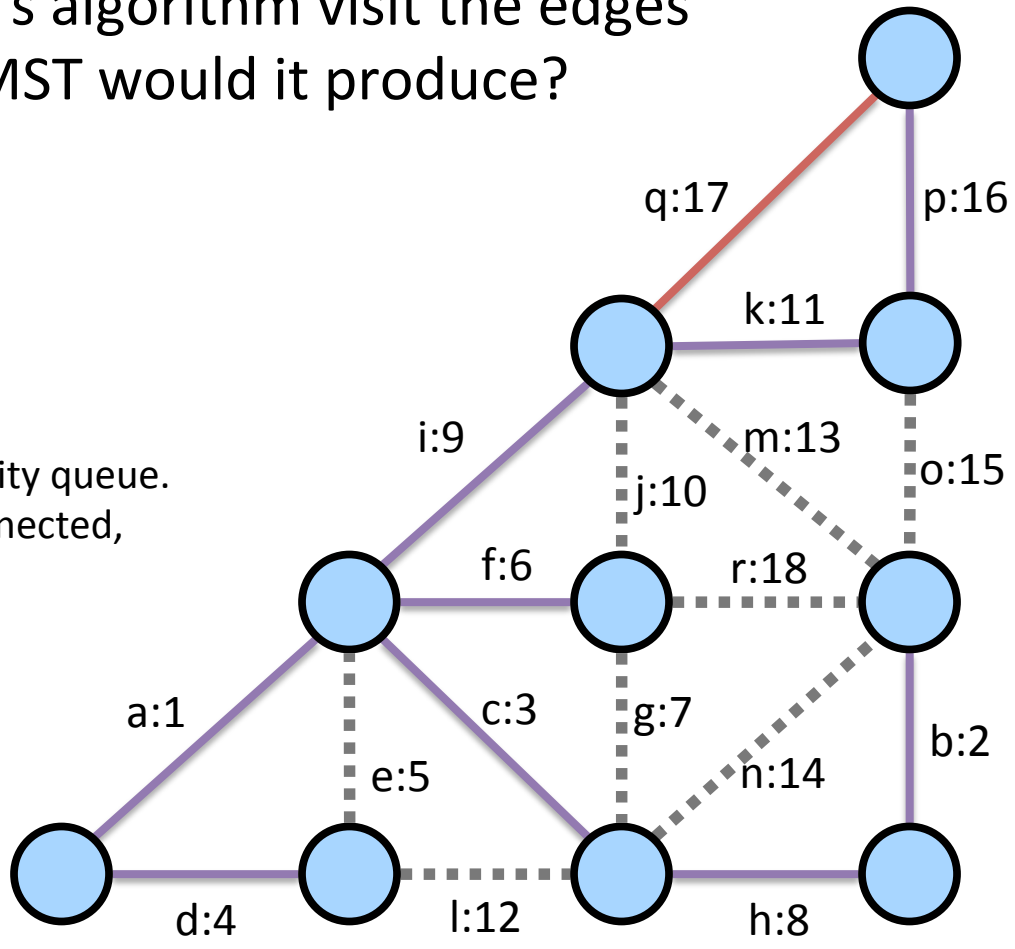

pq = {**q:17**, r:18}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
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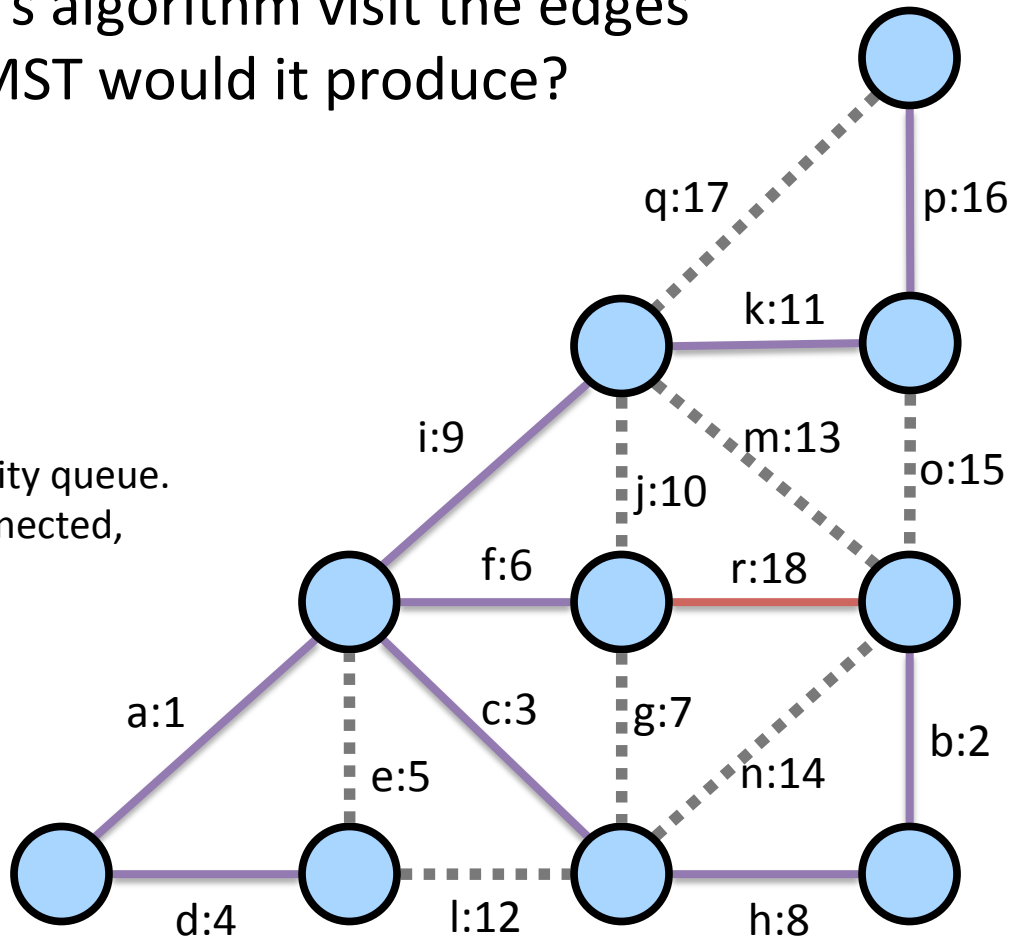
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.



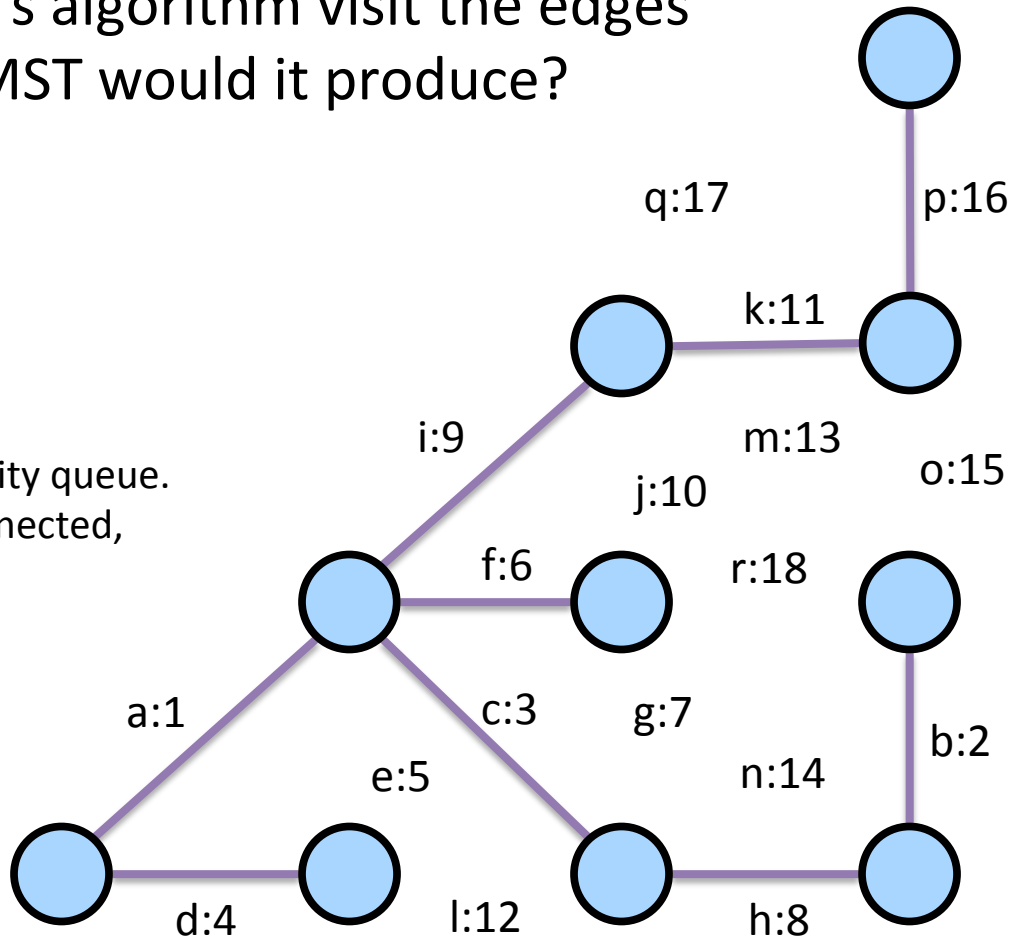
pq = {**r:18**}

Kruskal example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
  Remove all edges from the graph.
  Place all edges into a priority queue based on their weight (cost).
  While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected, add that edge into the graph.
    Otherwise, skip the edge.

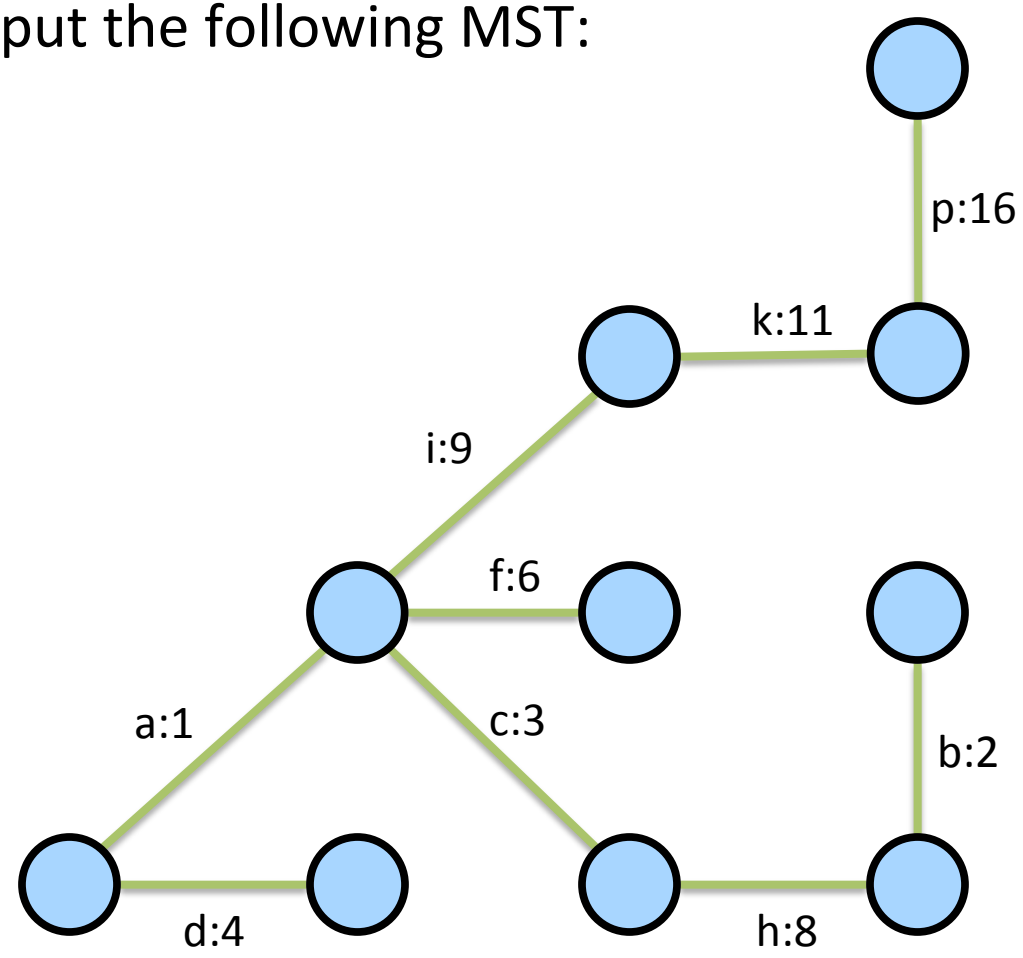
pq = {}



Kruskal example

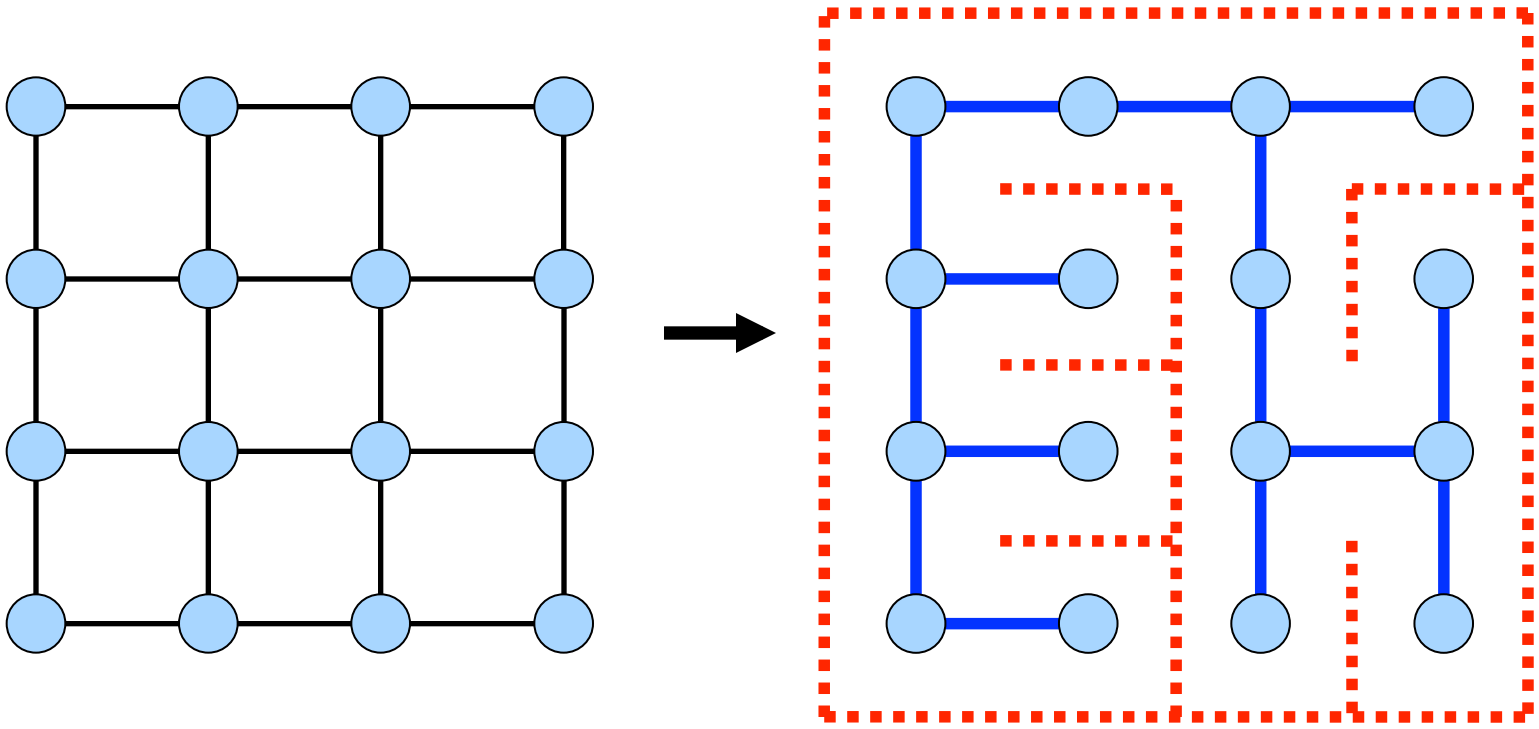
- Kruskal's algorithm would output the following MST:
  - {a, b, c, d, f, h, i, k, p}
- The MST's total cost is:

  1+2+3+4+6+8+9+11+16 = 60



Mazes with Kruskal's

- Start with a fully connected maze, and then run Kruskal's to add walls (edges) back in until you have a MST:



Implementing Kruskal

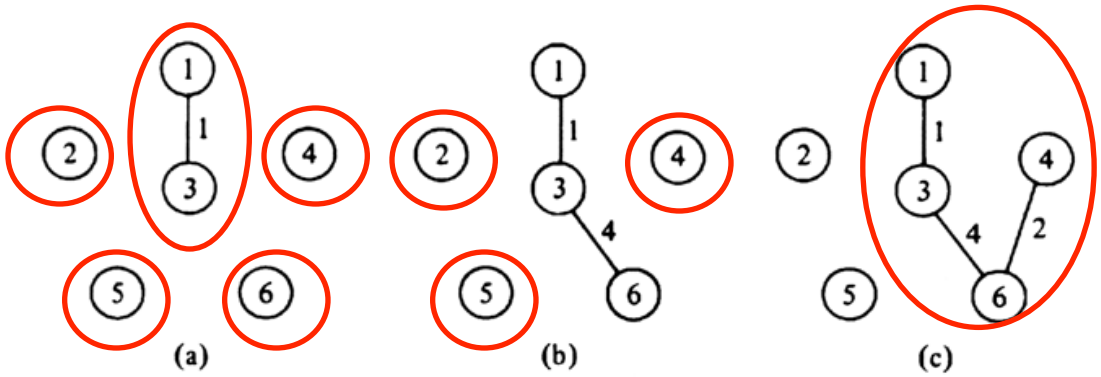
- What data structures should we use to implement this algorithm?

function **kruskal**(graph):
    Remove all edges from the graph.
    Place all edges into a **priority queue**
        based on their weight (cost).
    While the priority queue is not empty:
        Dequeue an edge *e* from the priority queue.
        If *e*'s endpoints aren't already connected,
            add that edge into the graph.
        Otherwise, skip the edge.

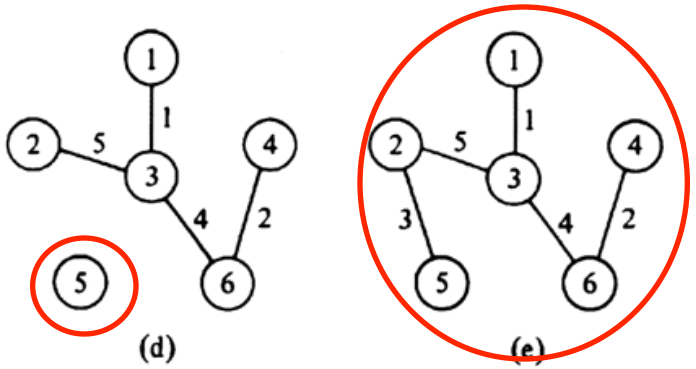
Vertex clusters

- Need some way to identify which vertexes are "connected" to which other ones
  - we call these "**clusters**" of vertices

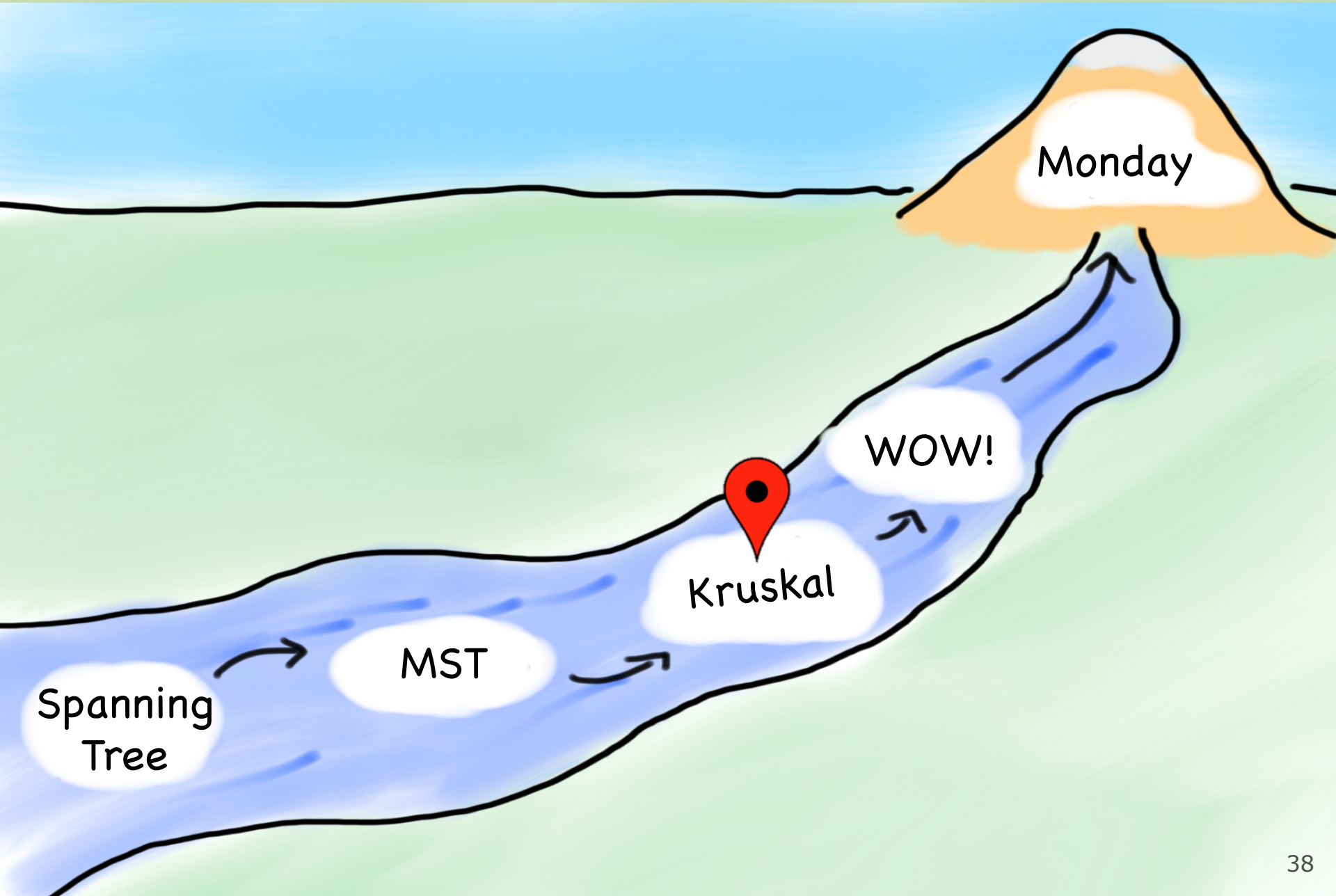
- Also need an efficient way to figure out which cluster a given vertex is in.



- Also need to **merge clusters** when adding an edge.



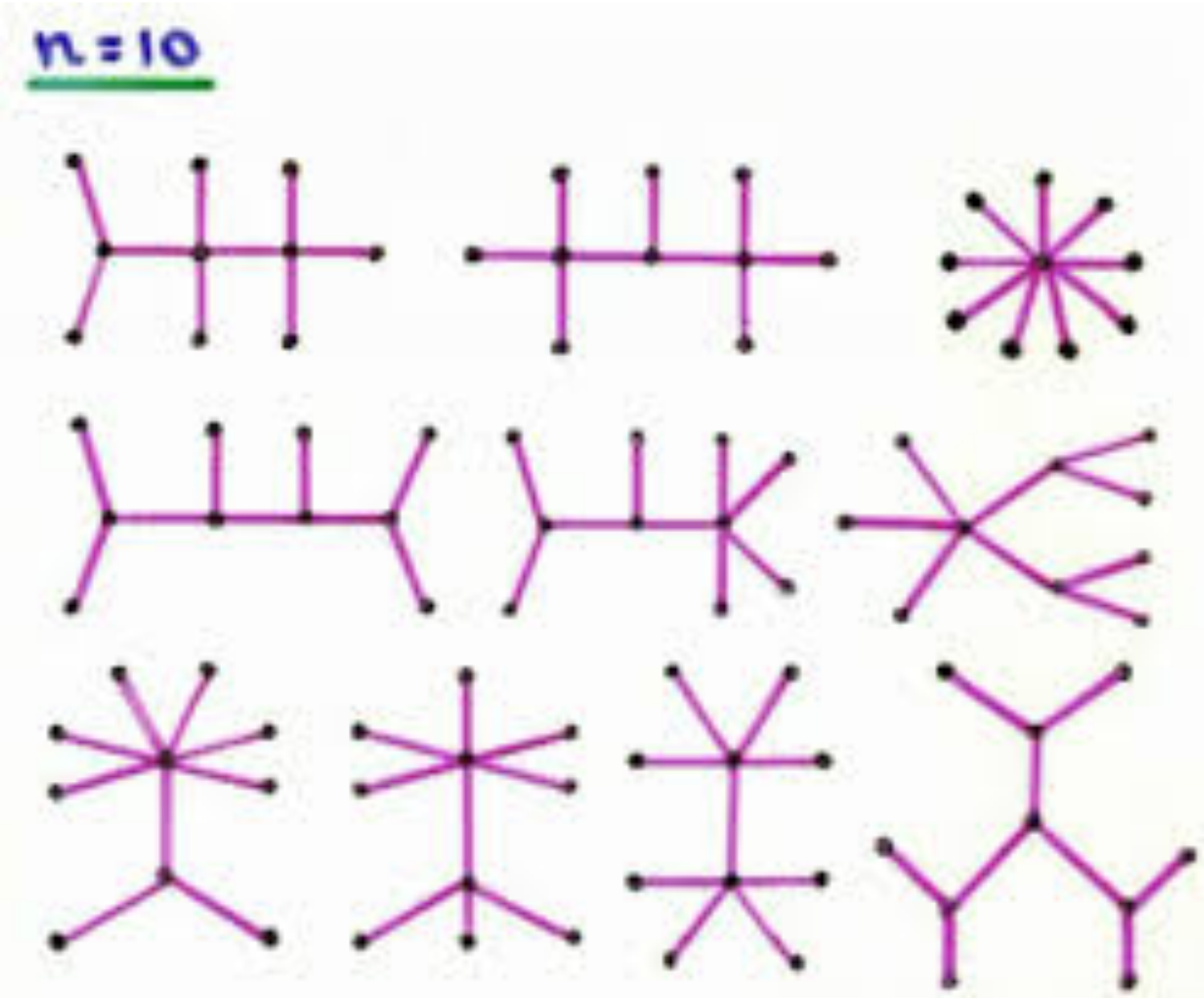
Today's Route



Today's Route

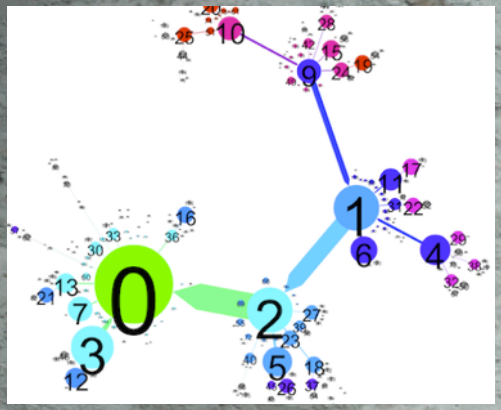

These are spanning trees!
…well homeomorphically irreducible trees

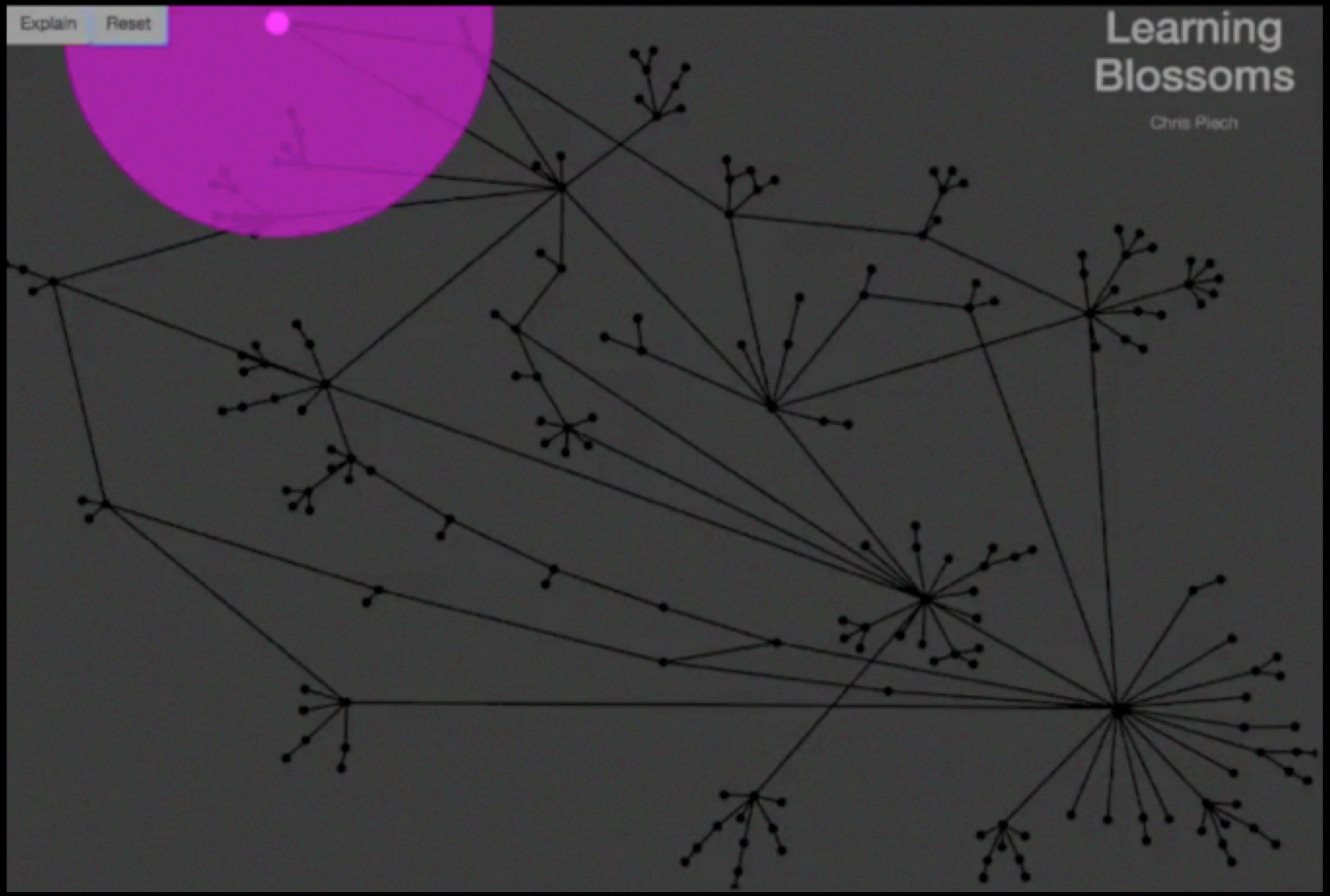
n = 10

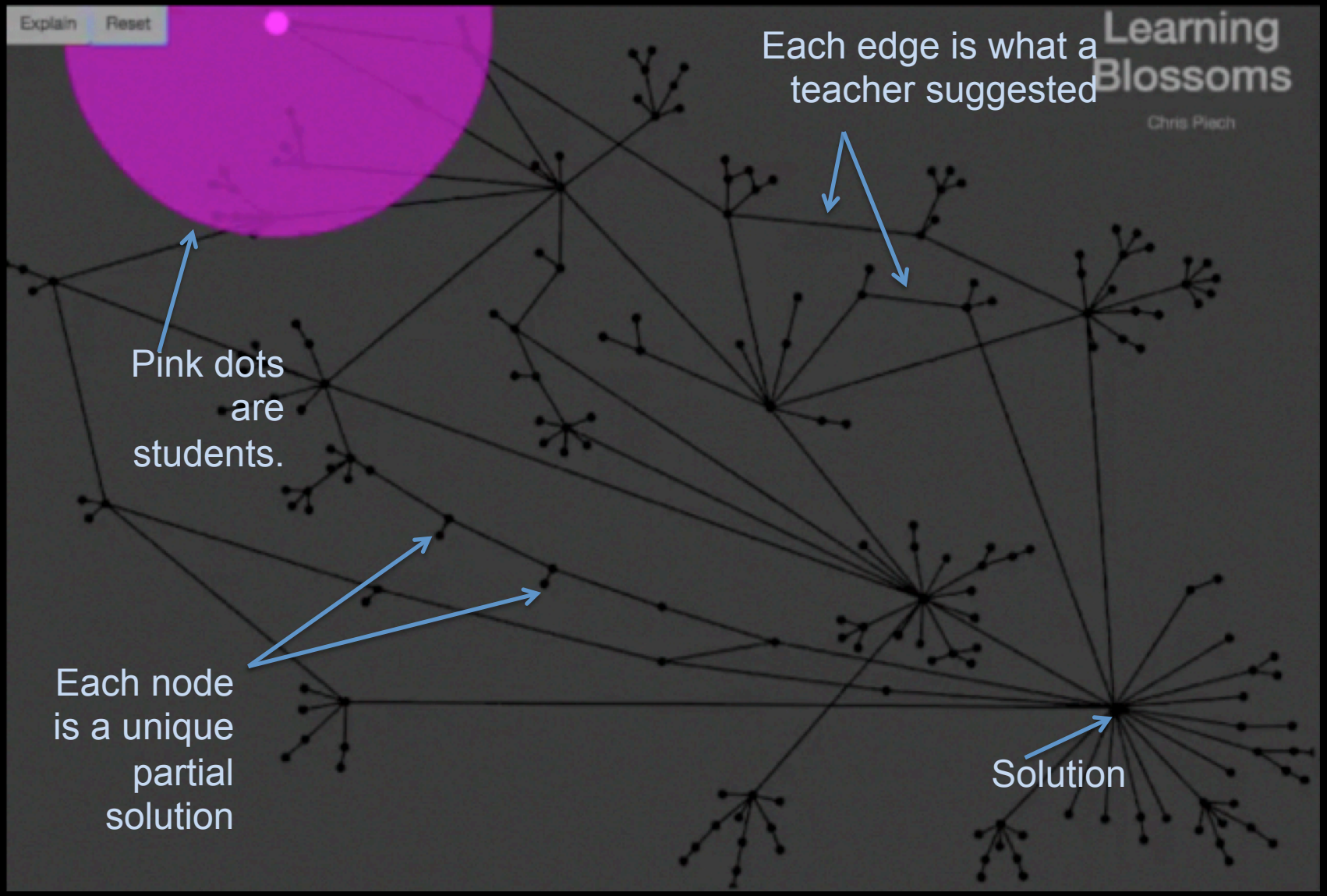


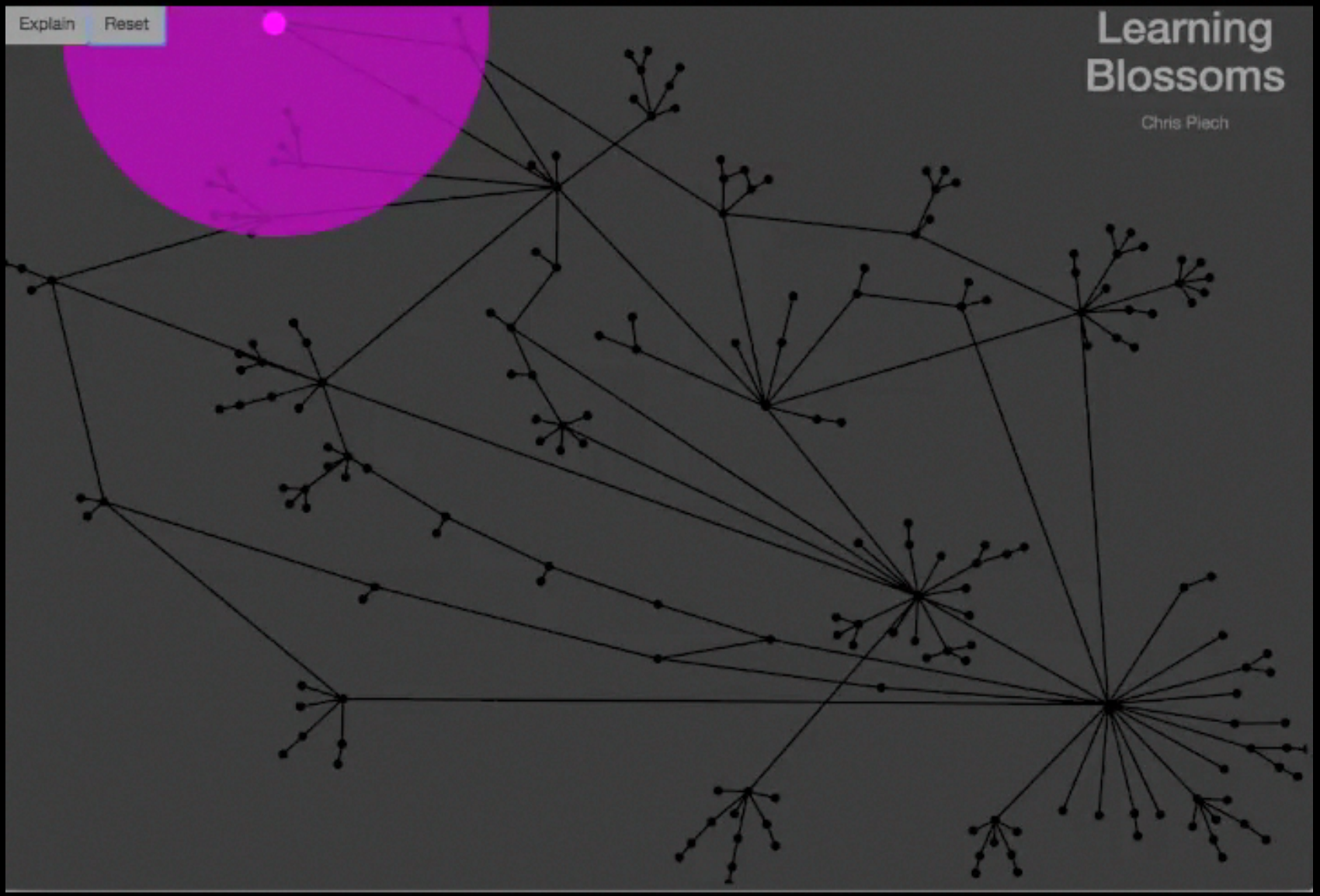
Did you know Chris is awesome?

Autonomously Generating Hints by Inferring Problem Solving Policies

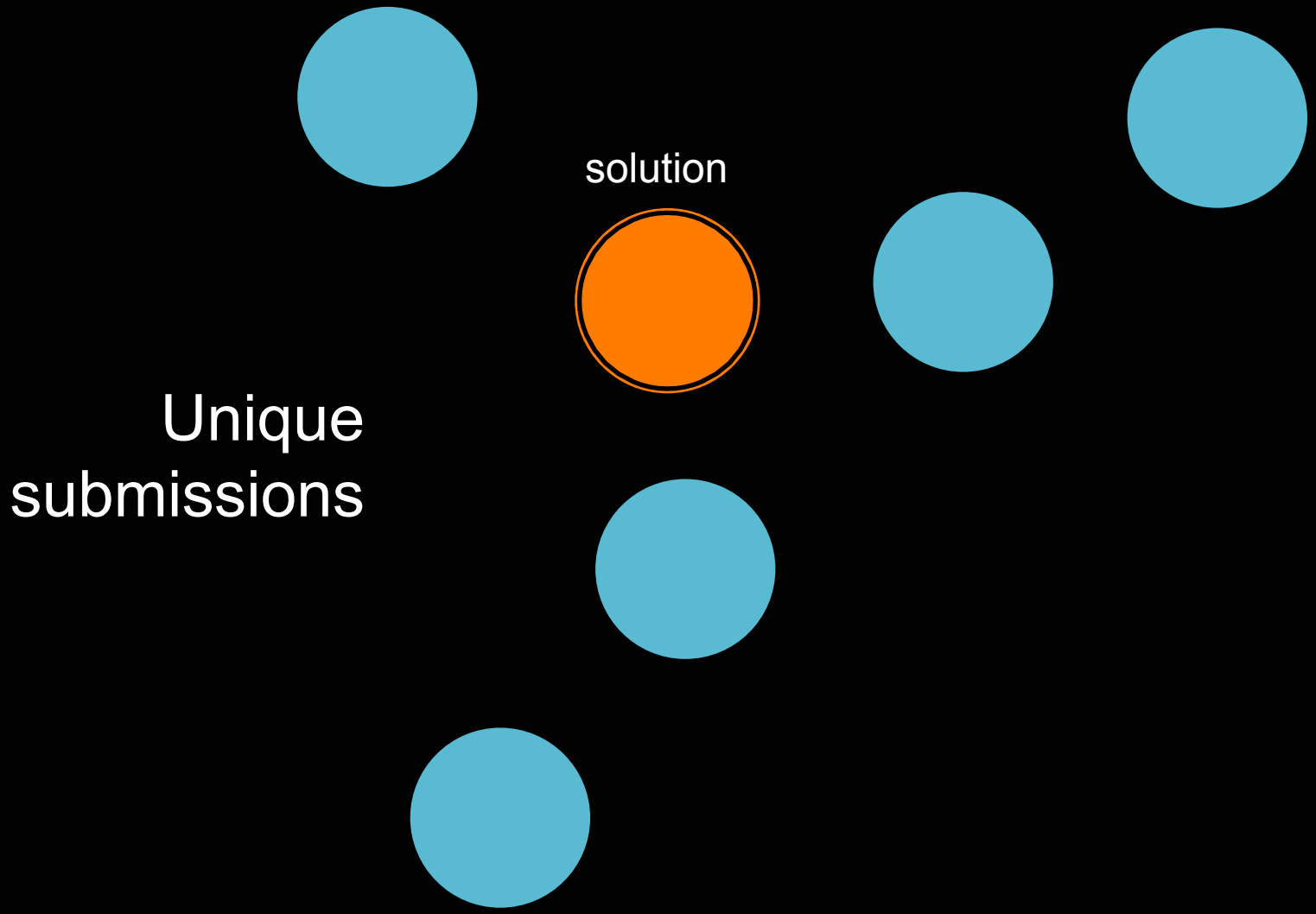



Explain
Reset
Learning
Blossoms
Chris Piech

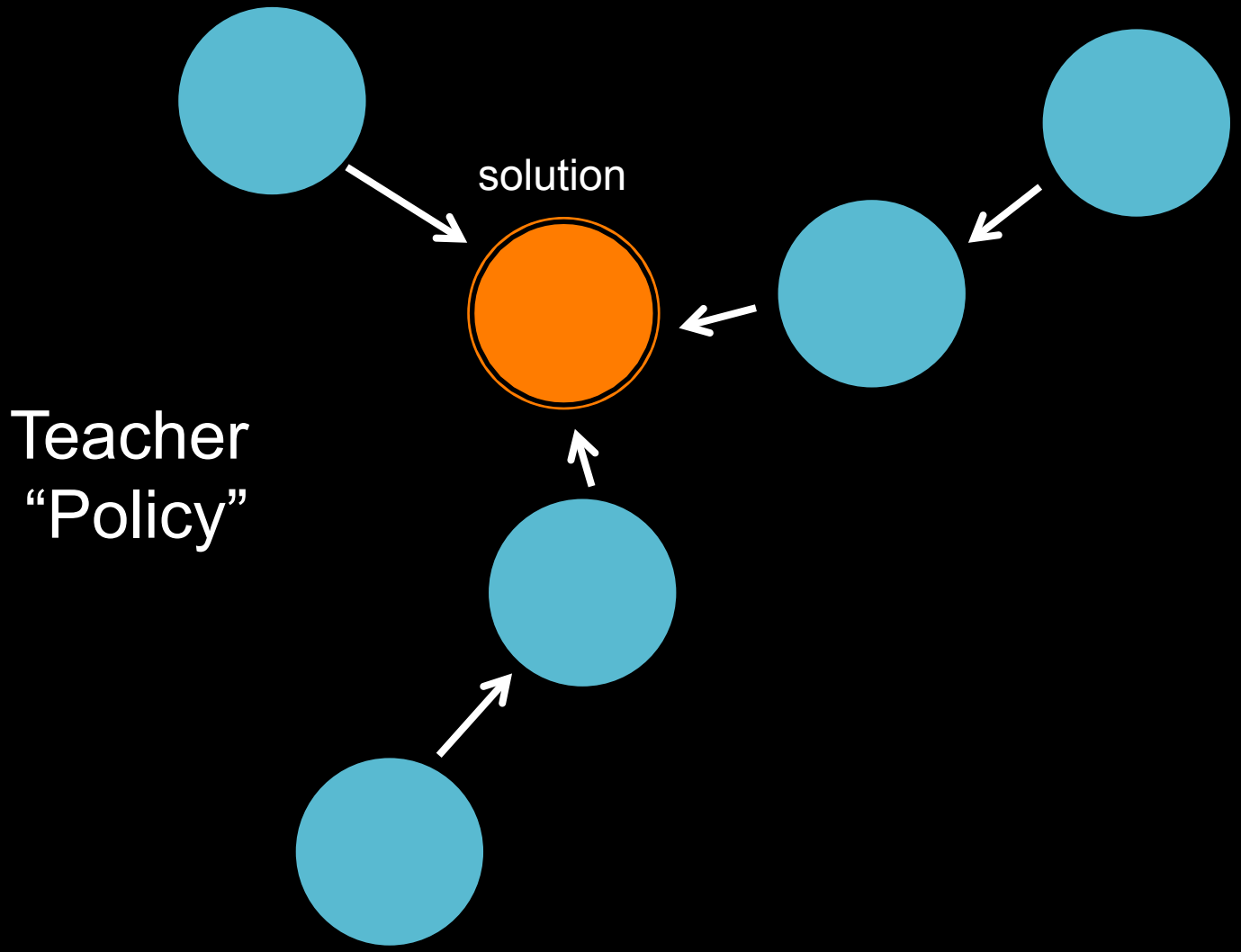

Explain
Reset
Learning Blossoms
Chris Piech
Each edge is what a teacher suggested
Pink dots are students.
Each node is a unique partial solution
Solution


Explain
Reset
Learning
Blossoms
Chris Piech

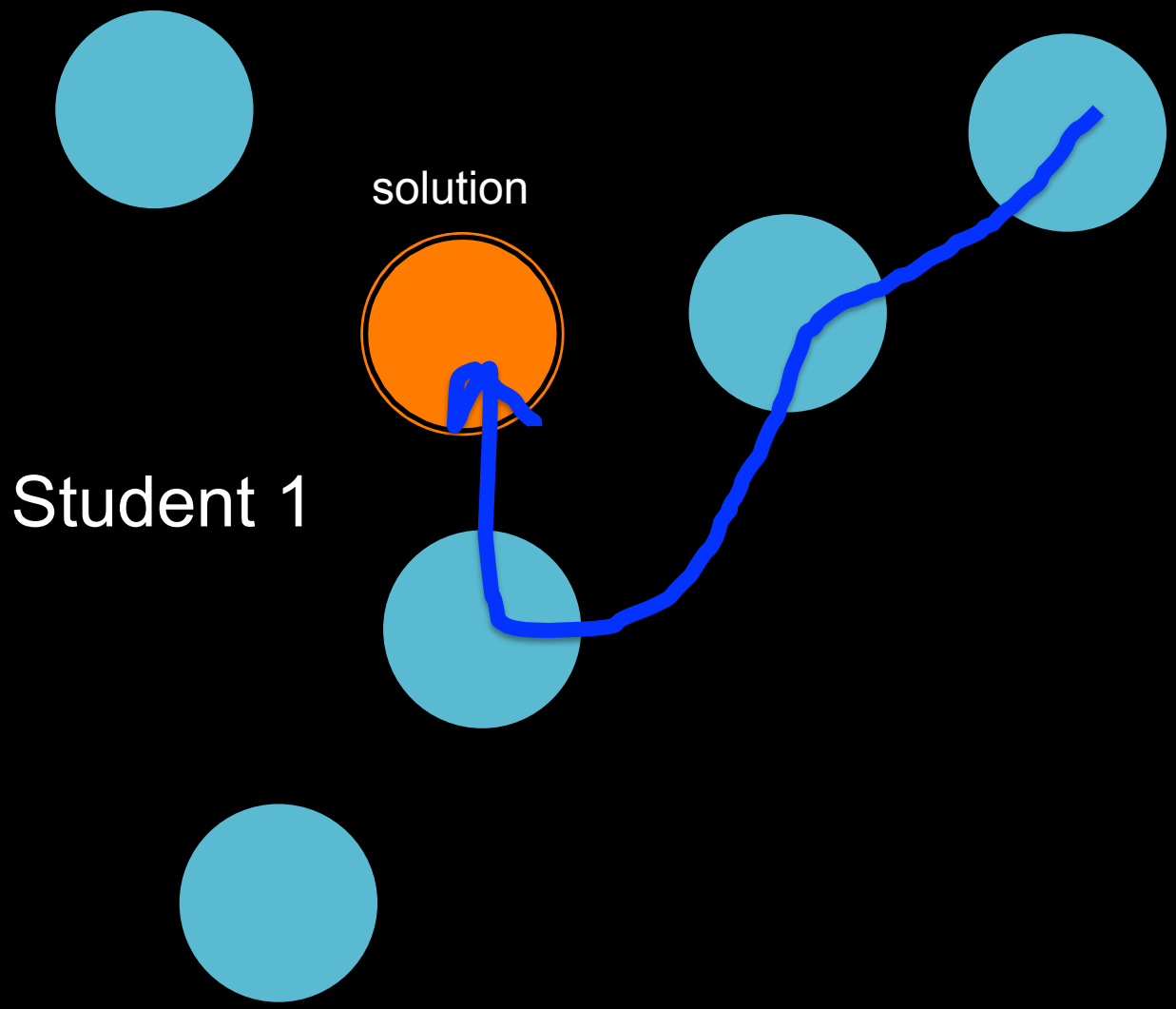
Machine Learning Problem



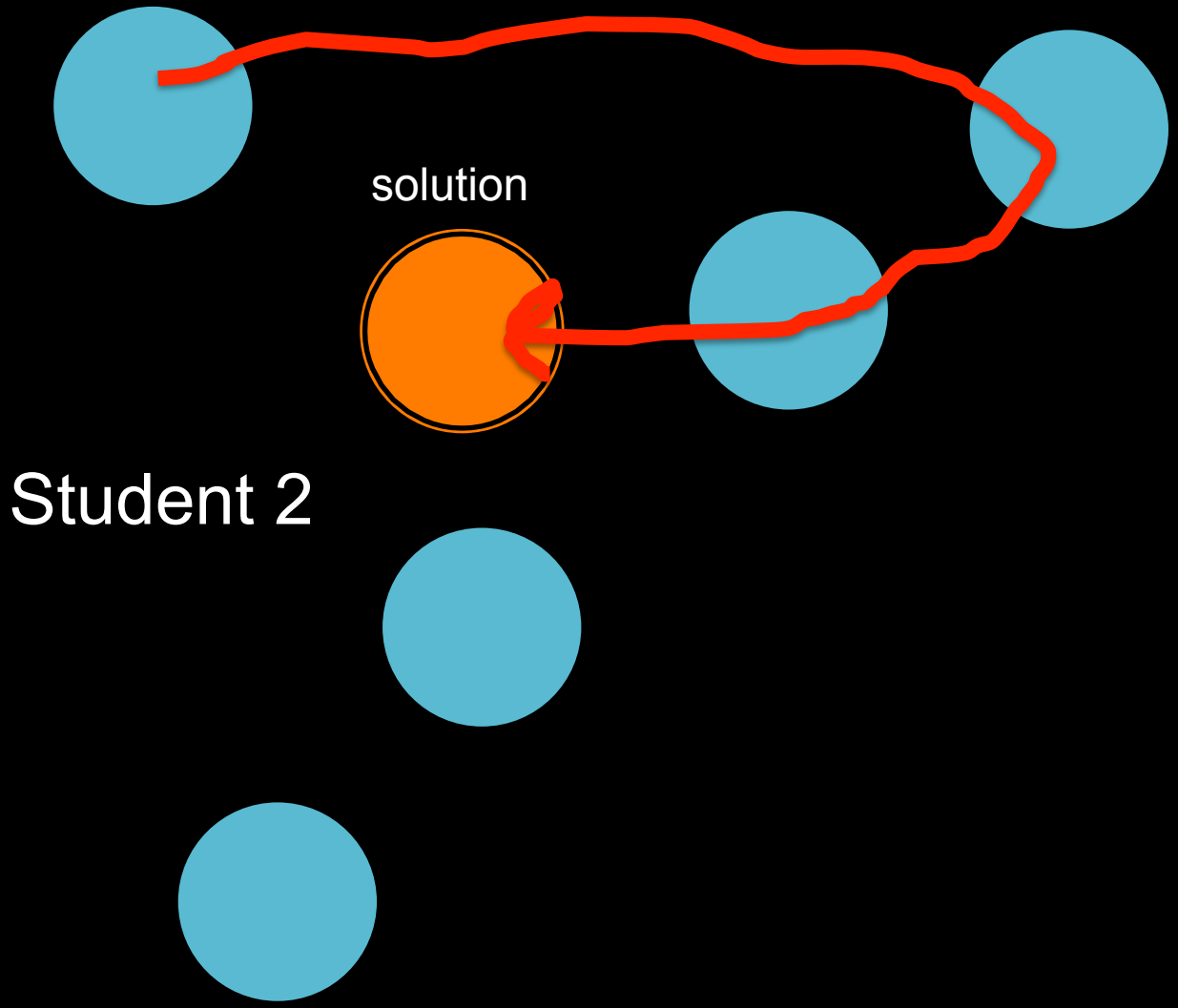
Machine Learning Problem



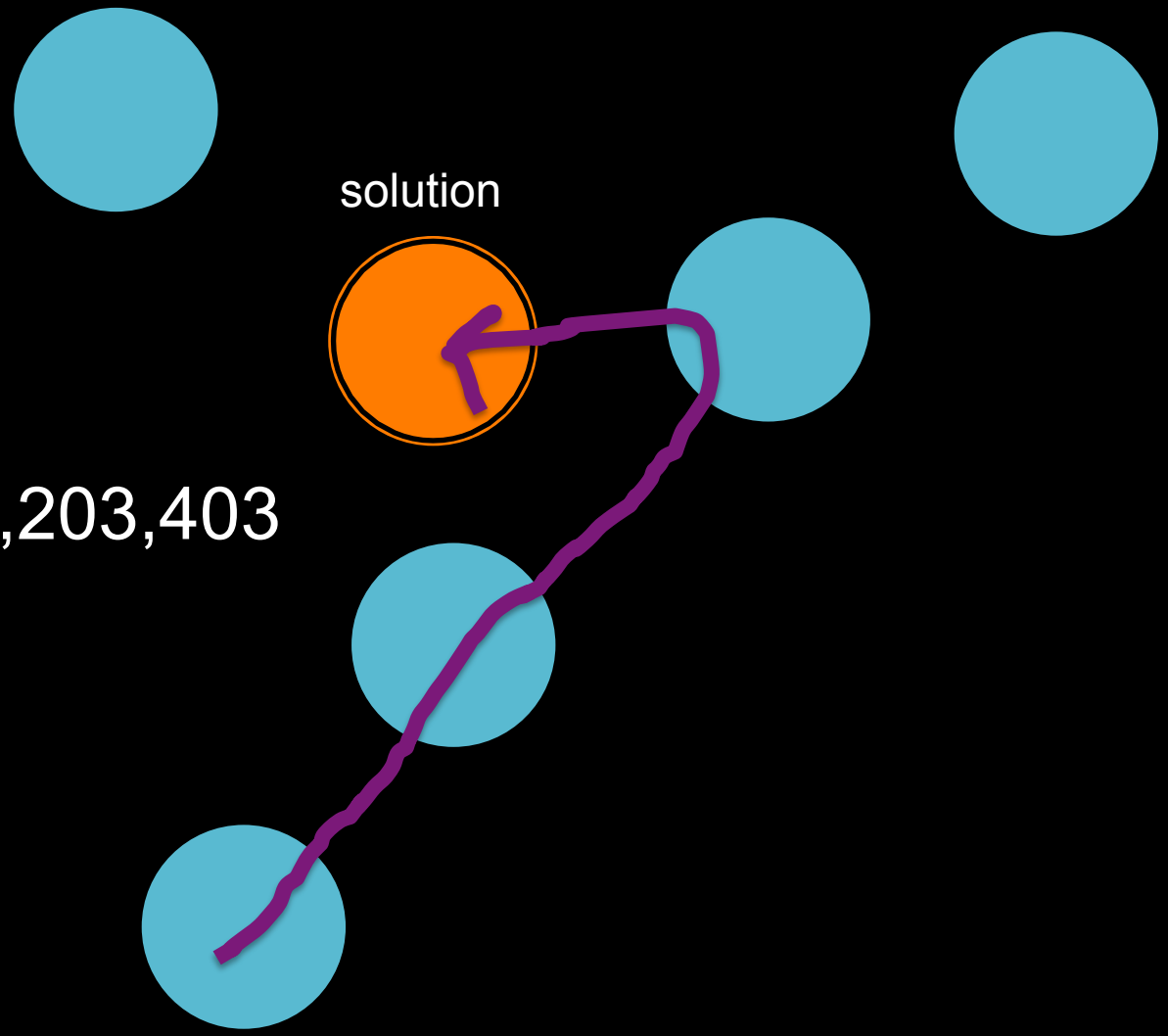
Example Student



Example Student

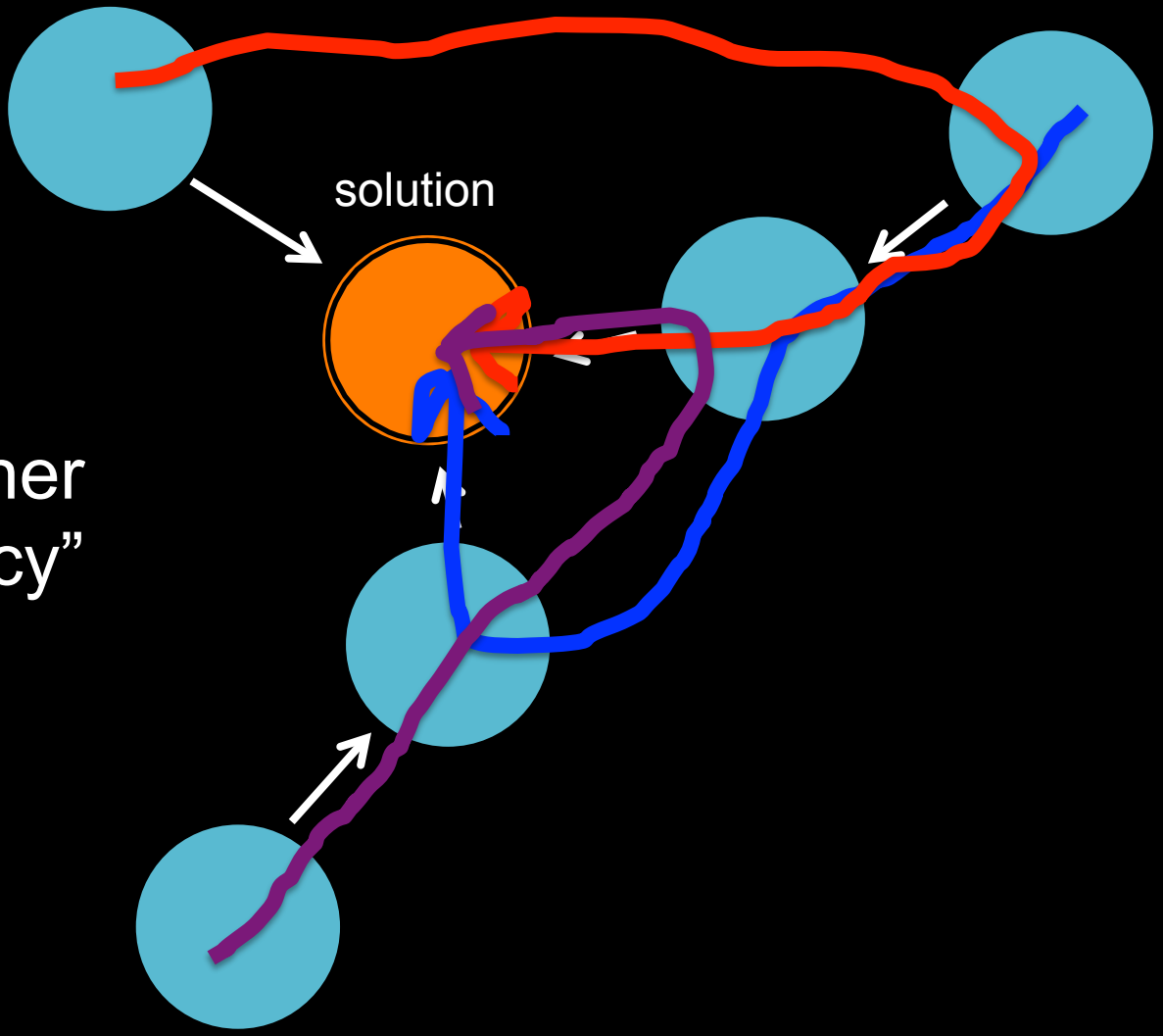


Example Student

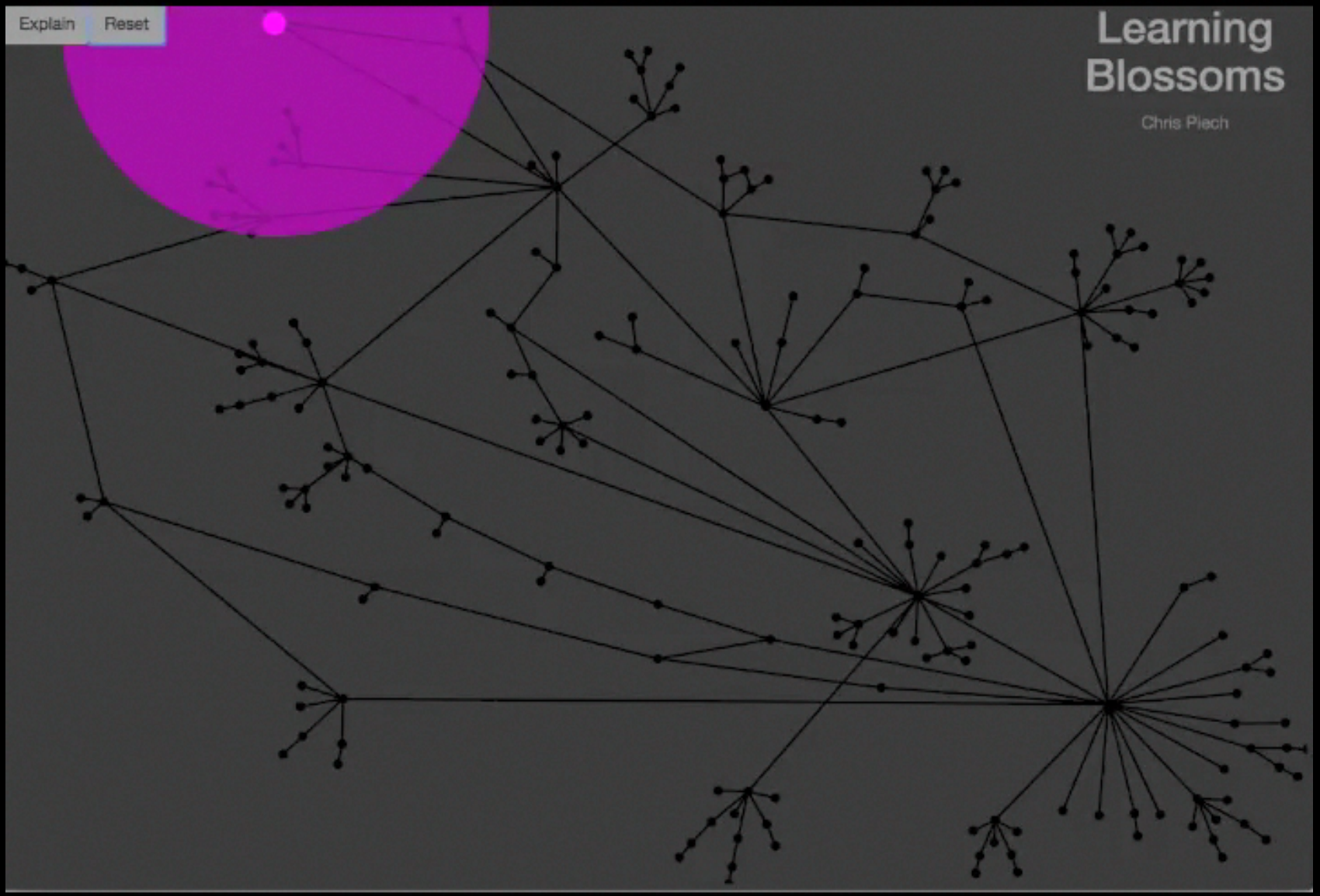


Student 1,203,403

Machine Learning Problem



Teacher
"Policy"


Explain
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Chris Piech