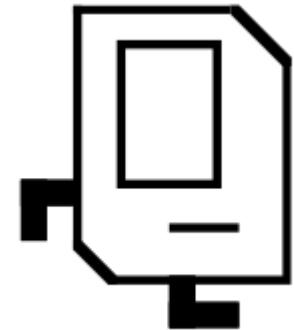


Before We Get Started

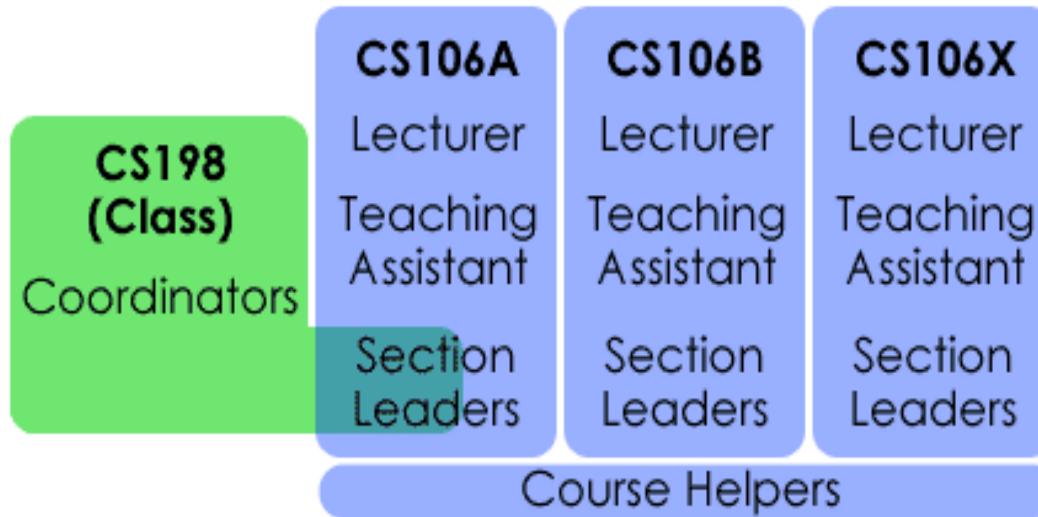


CS198 Section Leading



cs198@cs.stanford.edu

What is Section Leading?



Section leaders help with the teaching and grading of CS106A/B /X

What do Section Leaders do?

- Teach a weekly 50 minute section
- Help students in the LaIR
- Grade CS106 assignments and exams
- Hold IGs with students
- Have fun!

Prerequisites

For this round of applications, we are looking for applicants who are currently in CS106B/X... and that's you!

We are looking for section leaders from **any major** that can relate to students and clearly explain concepts.

Times and Requirements

You'll need to:

- Section lead for **two quarters!**
- Take CS198 for 3-4 units (1st quarter only)
- Attend staff meetings (Monday, 4:30-6:00PM)
- Attend workshops (Mon/Wed evenings)
- Fulfill all teaching, LaIR, and grading responsibilities

Why Section Lead?

- Teaching is an incredible experience!
- Work directly with students
- Participate in fun events
- Join an amazing group of people
- Leave your mark on campus

Participate in fun events



SLs keeping it classy for “LaIR Formal”



Welcome breakfast for new SLs

Participate in fun events



SEs at an “Iron Chef” event
with Google



SEs on a SF Bay boat cruise
with Microsoft

Join an amazing group of people



Mehran Sahami
Professor
Stanford



Marissa Mayer
CEO
Yahoo



Mike Schroepfer
CTO
Facebook

Leave your mark on campus

Section Leading is operated by current undergraduate and MS students.

Section Leading is a great way to make a substantive contribution to a meaningful program on campus.

Apply Now

Applications are open now!

Round 2 deadline: Friday, February 19th

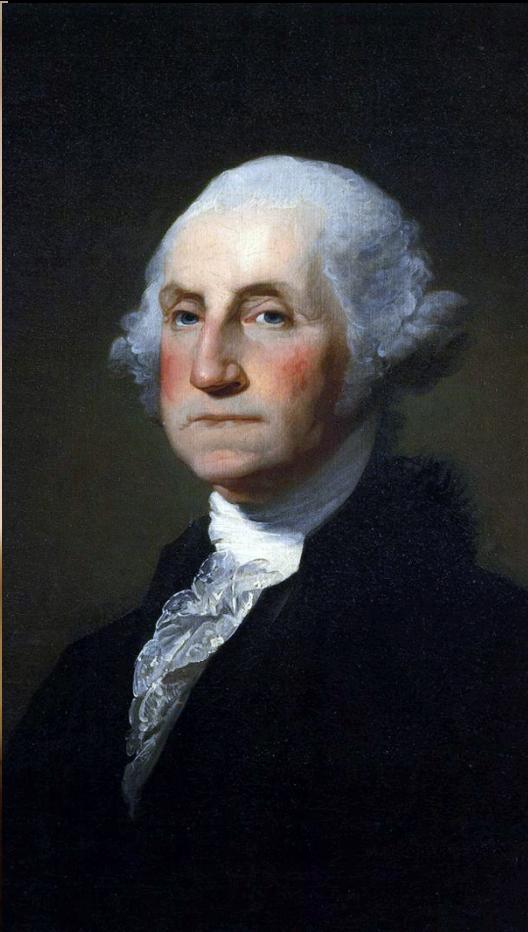
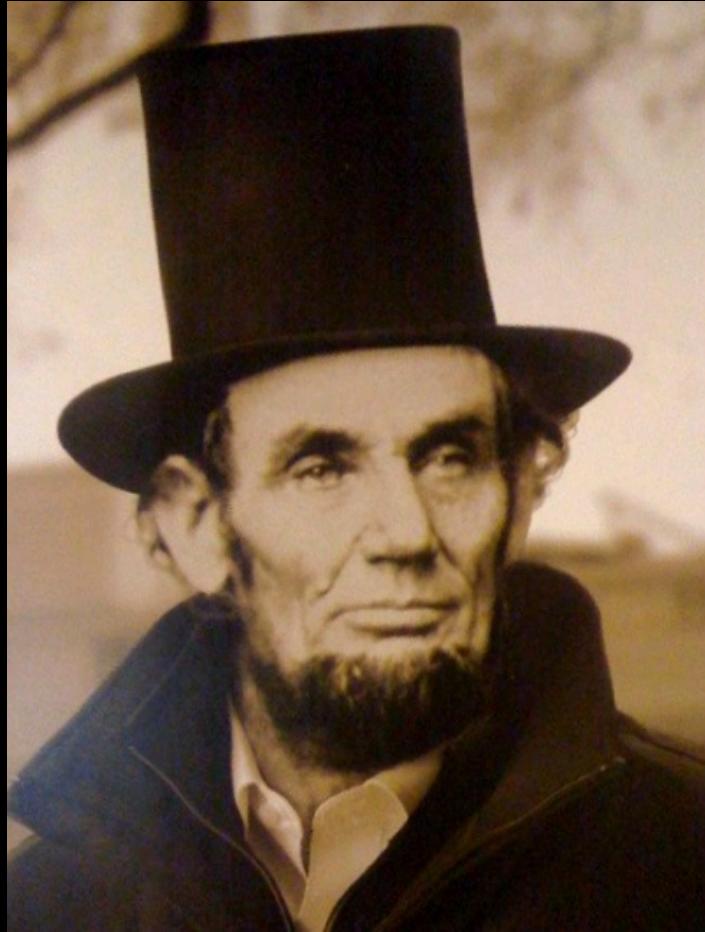
Online application: cs198.stanford.edu

Contact us: cs198@cs.stanford.edu

Thanks!

Chris Announcements

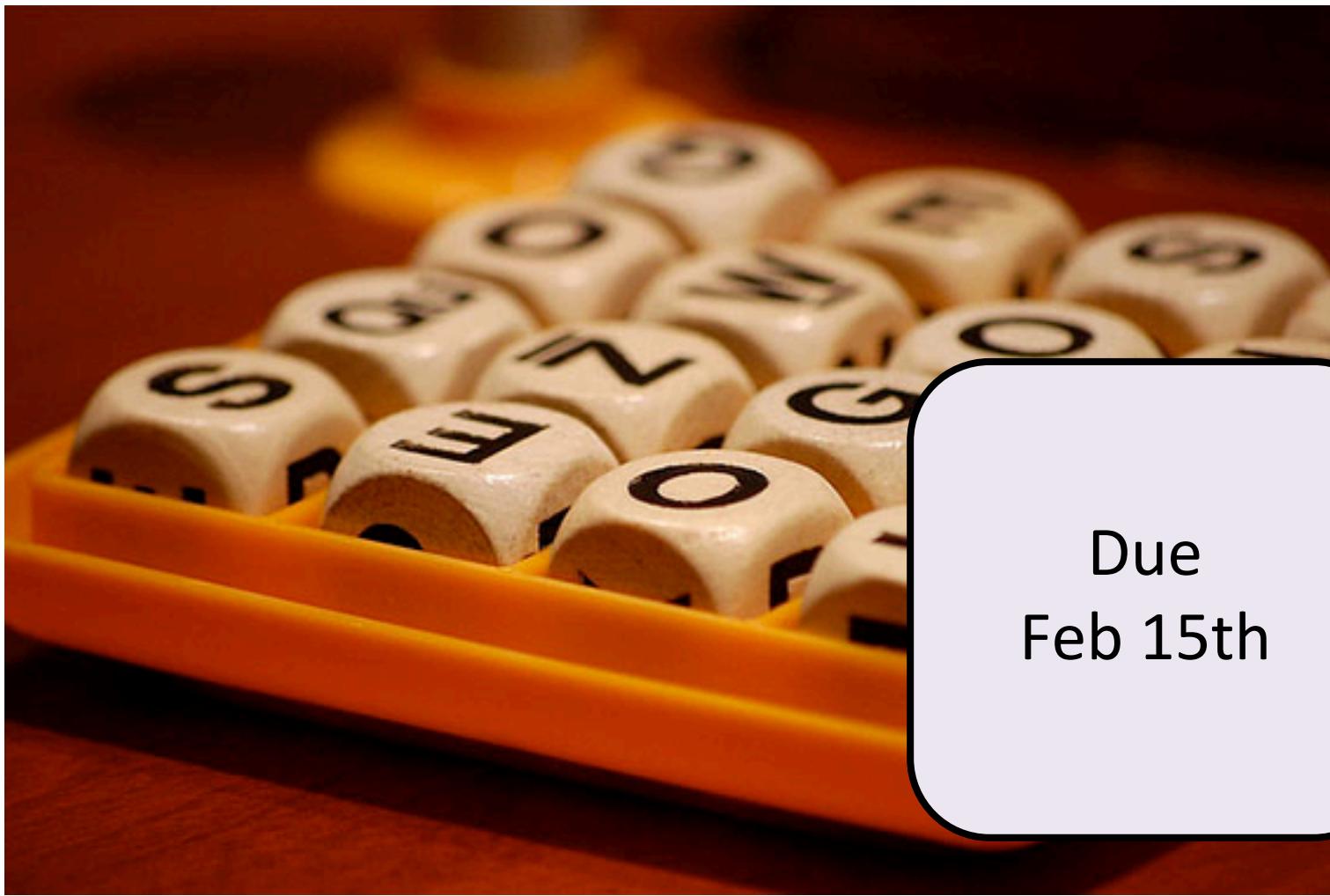
President's Day



Gravitational Waves Observed



Boggle



Due
Feb 15th

Priority Queue



Start
Feb 15nd

Due
Feb 24th

Priority Queue



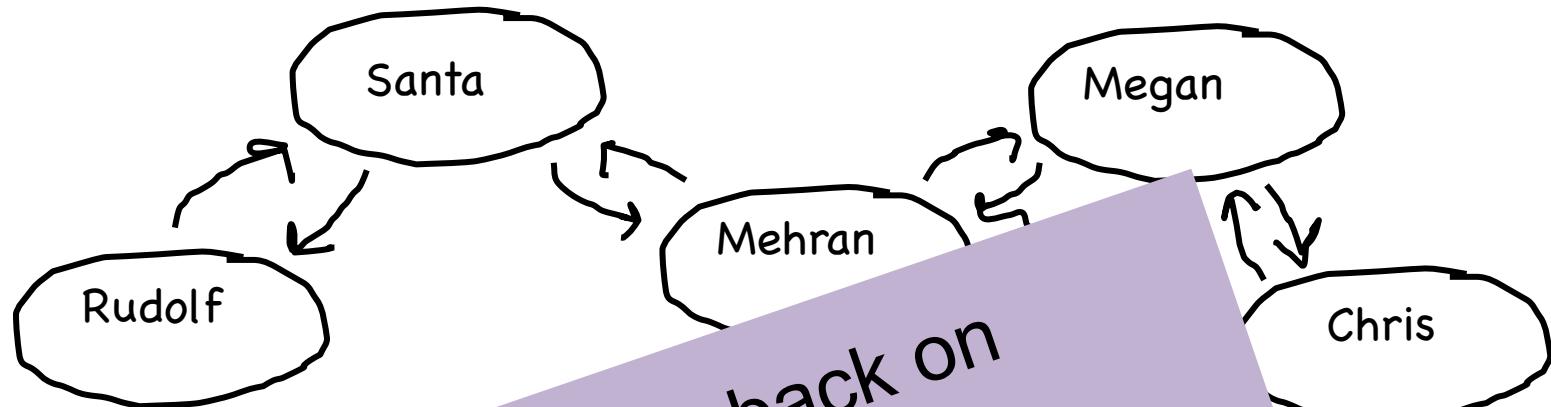
Concepts:

- Classes
- Dynamic Alloc.
- Linked Lists
- Heaps

Start
Feb 15nd

Due
Feb 24th

Midterm

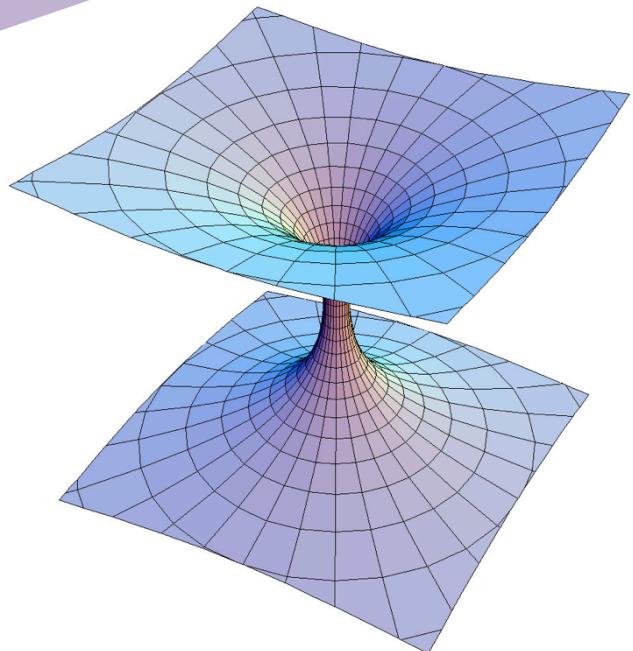


PANDORA
internet radio

Tell us your favorite song and we'll create a station that explores that part of the music universe.

Enter Song

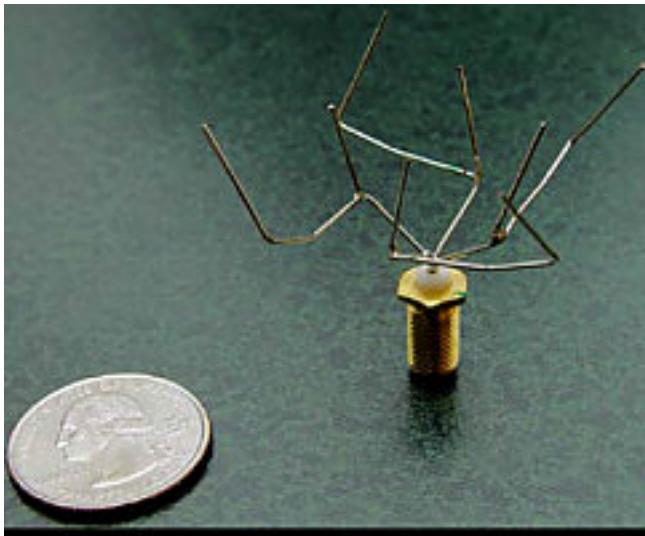
Create



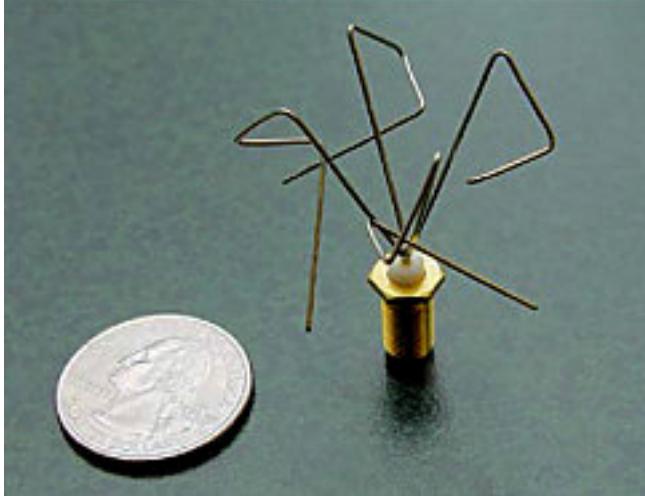
Something cool.

Computer Invented This

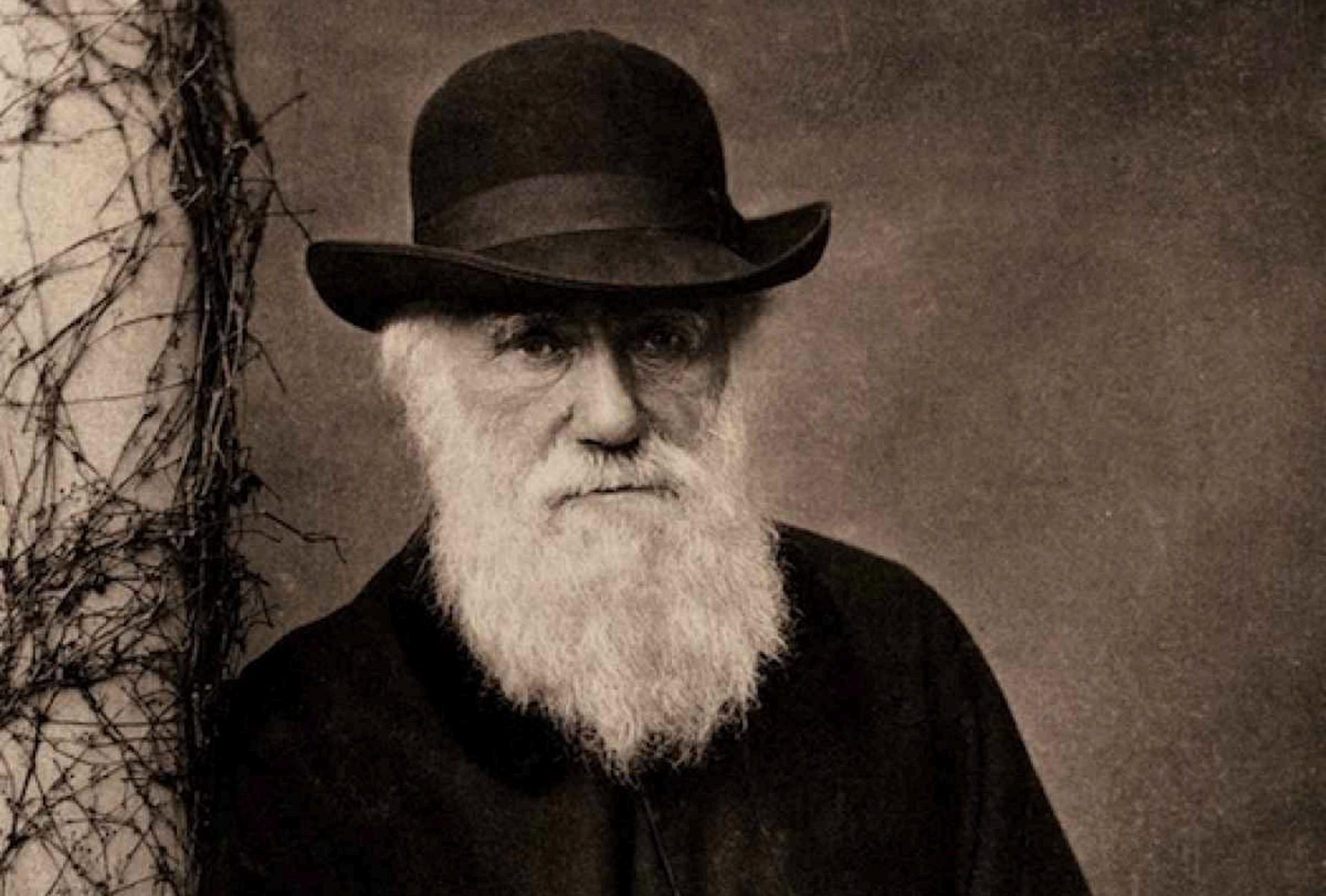
Antennae for microsatellites ($\sim 25\text{kg}$)



2006



Inspired by Biological Evolution

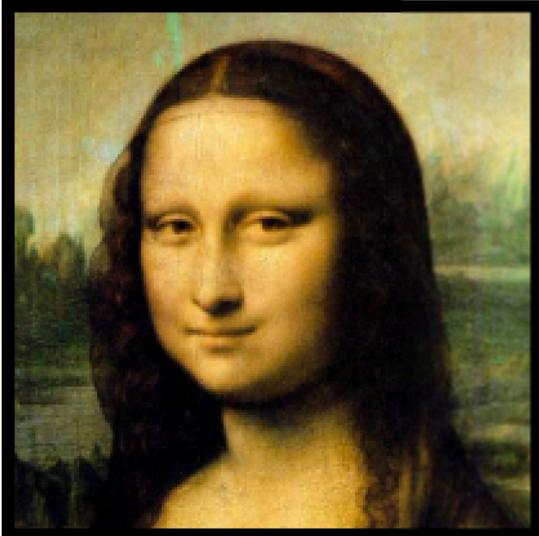


Evolving Mona Lisa

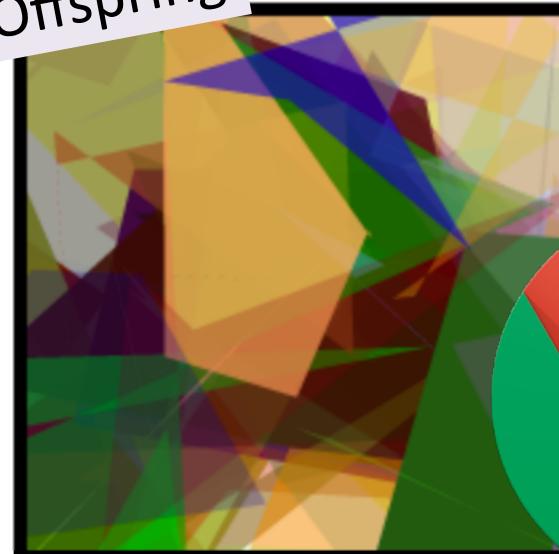
Best



Goal



Last Offspring



How do Evolutionary Algorithms work?

Short answer: PQueues

Priority Queues + Heaps

IT'S A CHRISTMAS TREE WITH A
HEAP OF PRESENTS UNDERNEATH!

... WE'RE NOT INVITING
YOU HOME NEXT YEAR.

Chris Piech

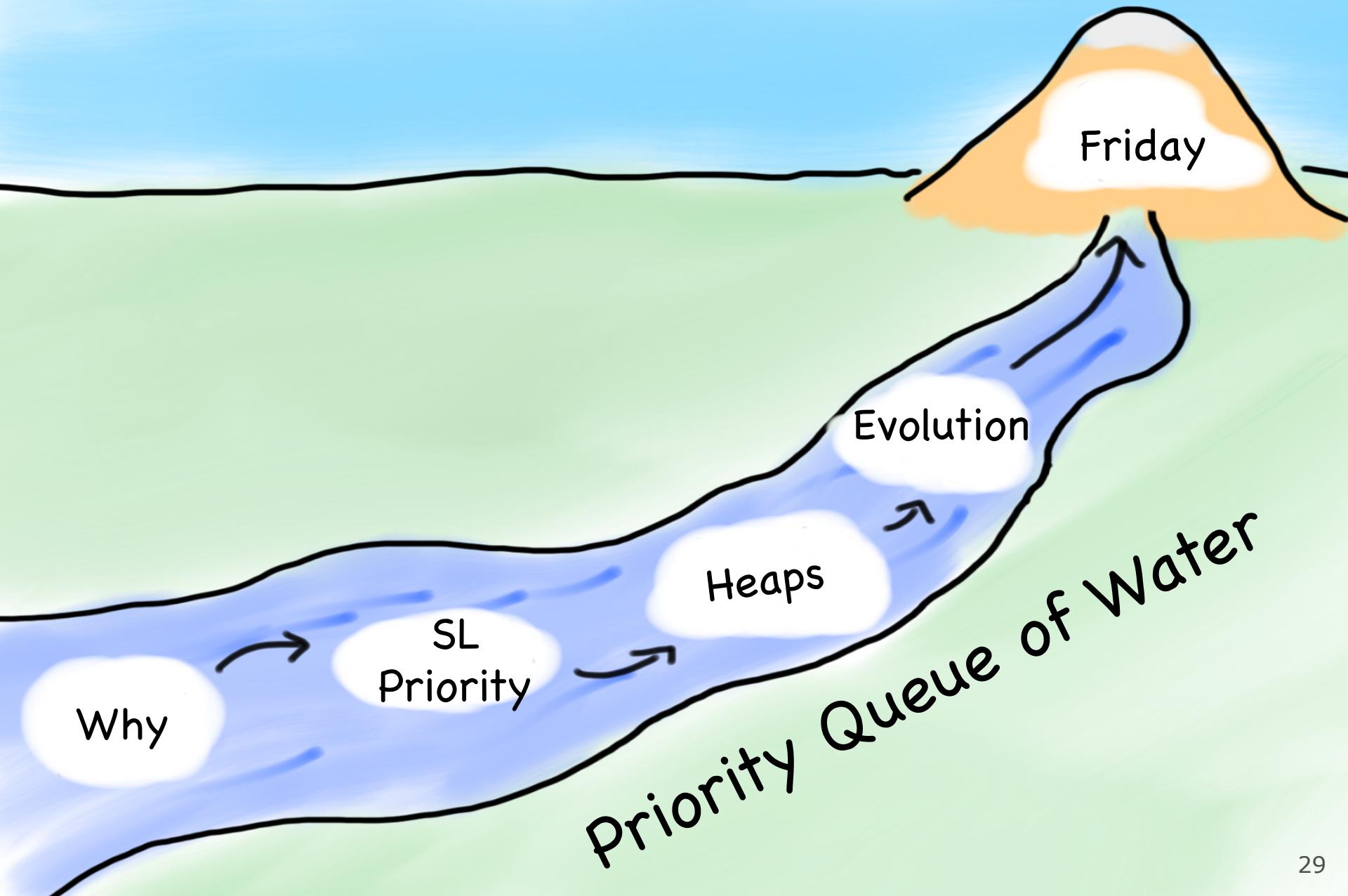
CS 106B
Lecture 14
Feb 8, 2016

Today's Goals

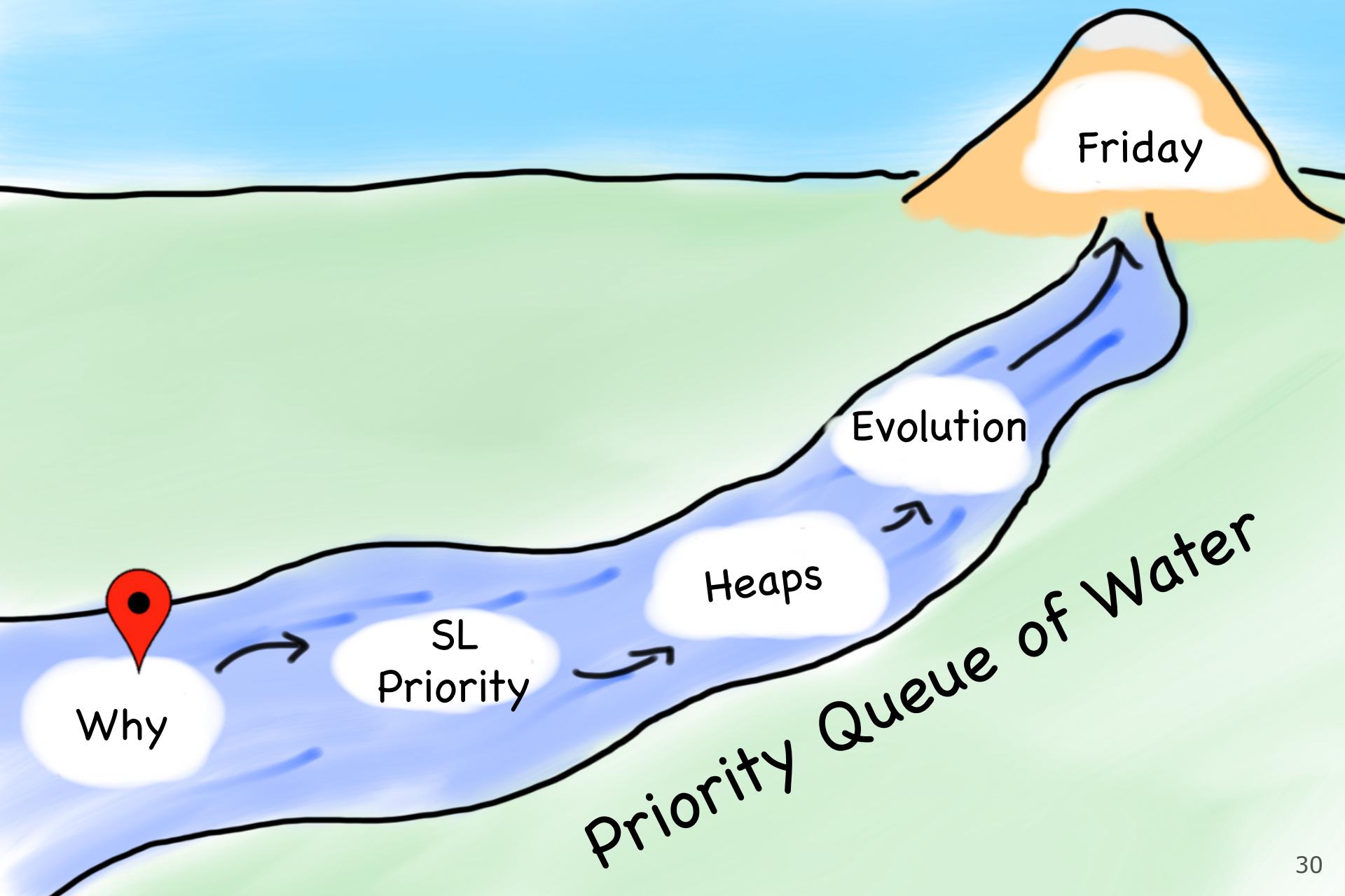
1. Care About Priority Queues
2. Learn about Heaps



Today's Goals



Today's Goals



Imagine you worked in the ER

Would look something like this...



Prioritization problems

ER scheduling: Scheduling patients for treatment in the ER. A gunshot victim should be treated sooner than a guy with a cold, regardless of arrival time. How do we always choose the most urgent case when new patients continue to arrive?

key operations we want:

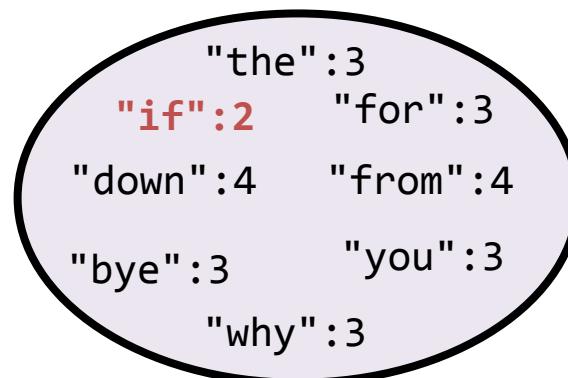
- ***add an element (print job, patient, etc.)***
- ***Get/remove the most "important" or "urgent" element***

Priority Queue ADT

priority queue: A collection of ordered elements that provides fast access to the highest-priority element.

- enqueue: adds an element with a given priority
- peek: returns highest-priority value
- dequeue: removes/returns highest-priority value

```
pq.enqueue("if", 2);  
pq.enqueue("from", 4);  
...  
—————>
```



```
—————> pq.dequeue()
```

"if"

priority queue (using lengths as priorities, in this case)

PriorityQueue members

```
#include "pqueue.h"
```

| | |
|---------------------------------------|---|
| <i>pq.enqueue(value, pri);</i> | adds value to queue, with the given priority number |
| <i>pq.dequeue()</i> | returns the value in the queue with highest (minimum) priority; throws an error if queue is empty |
| <i>pq.peek()</i> | returns highest (minimum) priority element without removing it; throws error if queue is empty |
| <i>pq.peekPriority()</i> | returns <i>priority</i> of highest (minimum) priority element; throws error if queue is empty |
| <i>pq.isEmpty()</i> | returns true if queue contains no elements (size 0) |
| <i>pq.size()</i> | returns the number of elements in the queue |
| <i>pq.toString()</i> | returns a string such as "{3, 42, -7, 15}" |
| <i>pq.changePriority(value, pri);</i> | alters an existing element's priority |
| <i>pq.clear();</i> | removes all elements of the set |

```
PriorityQueue<string> faculty;  
faculty.enqueue("Chris", 5);      // low priority  
faculty.enqueue("Eric", 2);       // high priority  
...
```

Seen PQueues Before



Duo Slango

good times

buenos tiempos

la mapa

el vecino

que honda

A screenshot of a computer application window titled "Duo Slango". The window has a light blue background. In the top-left corner, there is a small green cartoon owl. To the right of the owl, the text "good times" is displayed in a large, white, sans-serif font. Below this, there are four green rectangular boxes, each containing a Spanish word in black text. The words are arranged in a 2x2 grid. The top-left box contains "buenos tiempos", the top-right box contains "la mapa", the bottom-left box contains "el vecino", and the bottom-right box contains "que honda".

Exercise: SL scheduling

- Every quarter the SLs choose their LaIR hours.
 - SLs with more seniority (quarters worked) get to choose first.
 - Each line of the input file contains the year the SL began working with us, and the quarter (1=fall, 2=winter, 3=spring, 4=summer).
 - Write code to read the list of SLs and output the order of choosing.
 - Input file format:

| | |
|--------------------------|--------------|
| <i>name year quarter</i> | Zack 2014 2 |
| <i>name year quarter</i> | Sara 2012 4 |
| <i>name year quarter</i> | Tyler 2013 1 |
| ... | |

And... Here it is!

Exercise solution

```
PriorityQueue<string> SLs;    // read the contents of sls.txt
ifstream input;                // into a priority queue
input.open("sls.txt");
string slName;
int year;
int quarter;
while (input >> slName >> year >> quarter) {
    // store with year,quarter as priority so that the SLs
    // come out of the PQ in descending order of seniority
    // (e.g. year=2013, qtr=4 => priority = 1973)
    int priority = year - (10 * quarter);
    SLs.enqueue(slName, priority);
}

// pull the SLs out of the PQ from most to least seniority
while (!SLs.isEmpty()) {
    string sl = SLs.dequeue();
    cout << sl << " picks next." << endl;
}
```

Exercise solution

```
PriorityQueue<string> SLs;    // read the contents of sls.txt
ifstream input;                // into a priority queue
input.open("sls.txt");
string slName;
int year;
int quarter;
while (input >> slName >> year >> quarter) {
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    SLs.enqueue(slName, priority);
}

// pull the SLs out of the PQ from most to least seniority
while (!SLs.isEmpty()) {
    string sl = SLs.dequeue();
    cout << sl << " picks next." << endl;
}
```

Remember that
low priorities
come out first

What a beautiful ADT!

How does it work?

Implementing PQ

There are several ways one might implement a priority queue.

PQ as Vector

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|---|---|----|---|---|---|---|
| <i>value</i> | x | b | a | m | q | t | | | | |
| <i>priority</i> | 5 | 4 | 8 | 5 | 5 | 2 | | | | |
| <i>size</i> | 6 | <i>capacity</i> | | | | 10 | | | | |

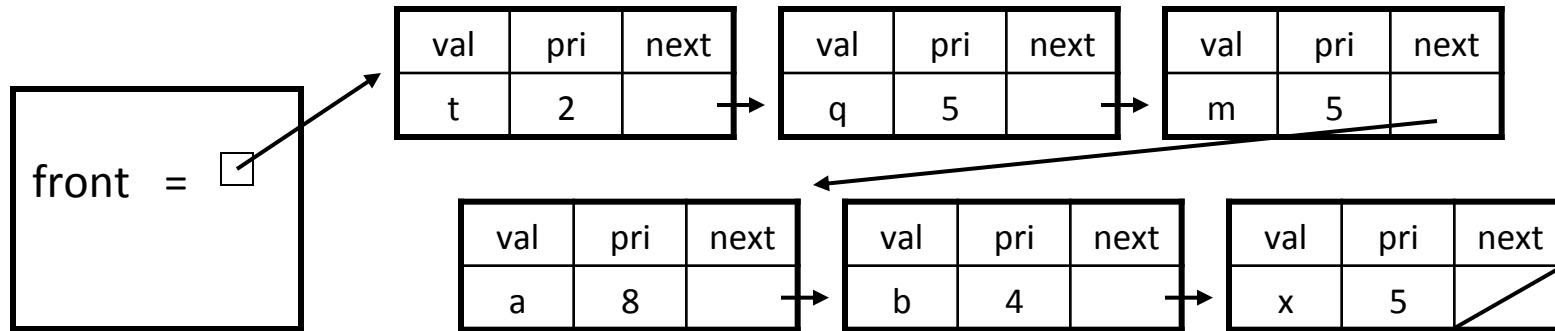
- enqueue by adding to *end*
- dequeue by searching for / removing highest-priority element
 - What is the big-Oh of enqueue? dequeue? peek?

PQ as Sorted Vector

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|---|---|----|---|---|---|---|
| <i>value</i> | t | b | m | q | x | a | | | | |
| <i>priority</i> | 2 | 4 | 5 | 5 | 5 | 8 | | | | |
| <i>size</i> | 6 | <i>capacity</i> | | | | 10 | | | | |

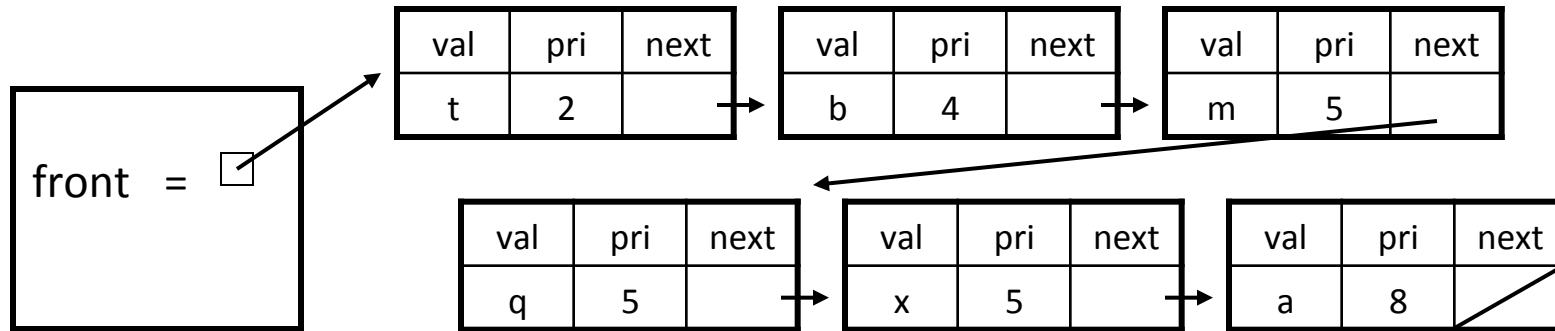
- enqueue by adding to proper place to preserve sorted order
- dequeue the *first* element
 - What is the big-Oh of enqueue? dequeue? Peek?

PQ as Linked List



- enqueue by adding to *front*
- dequeue by searching for / removing highest-priority element
 - What is the big-Oh of enqueue? dequeue? peek?

PQ as Sorted Linked List



- enqueue by adding to proper place to preserve sorted order
- dequeue the *first* element
 - What is the big-Oh of enqueue? dequeue? peek?

Sorted = fast dequeue / peek
Unsorted = fast enqueue

Best of both worlds?

Max Heaps

Tree

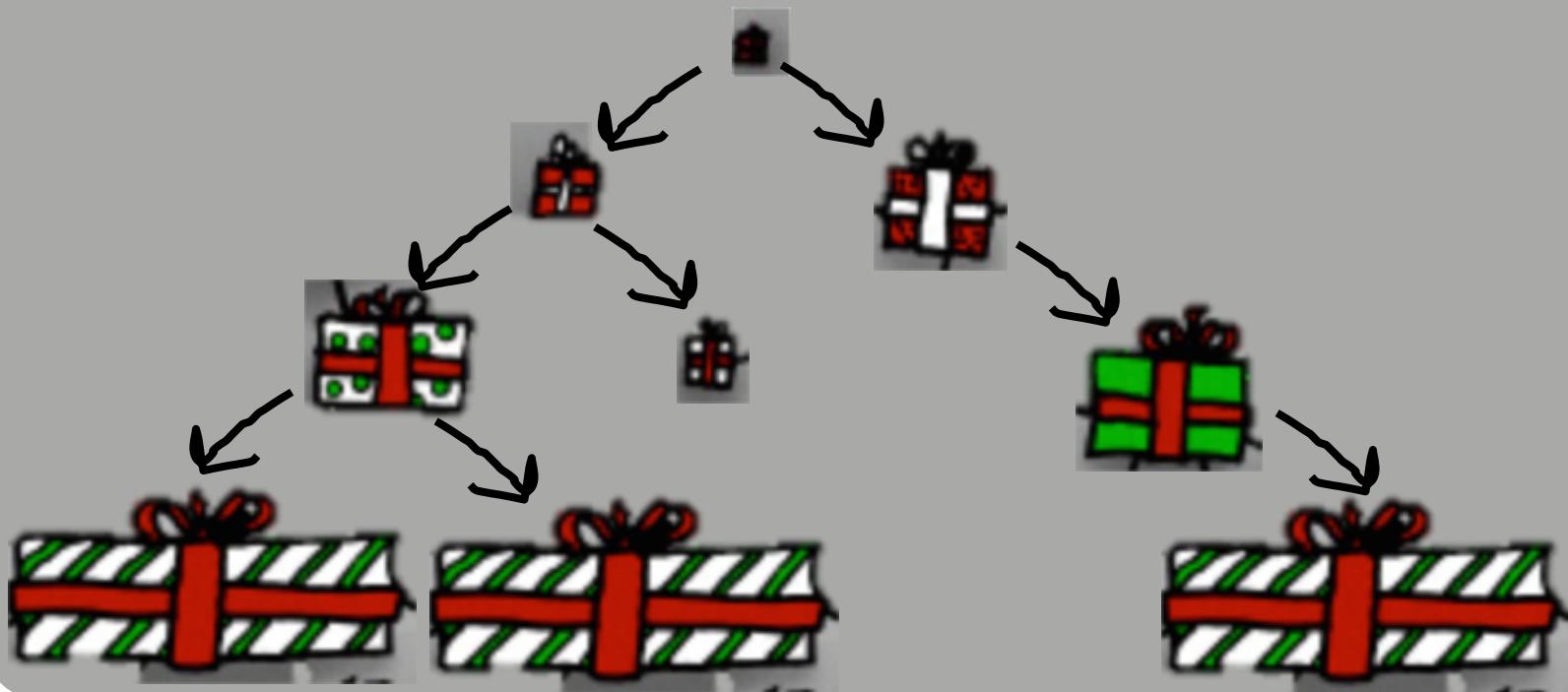
Every branch takes you to a “smaller” node



Min Heaps

Tree

Every branch takes you to a “larger” node



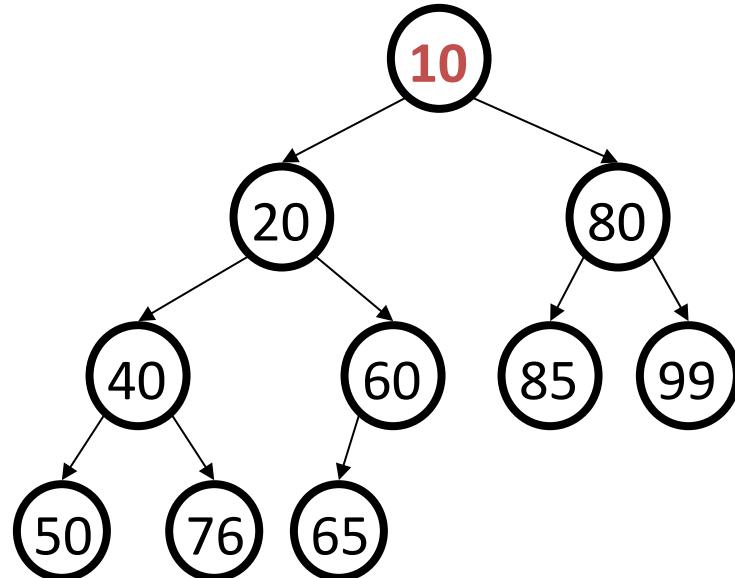
Priority Heaps

Tree

Heap Ordered: Every branch takes you to a “greater” node.

Binary: Every node has at most two children.

Complete: There are no “gaps” in the tree.



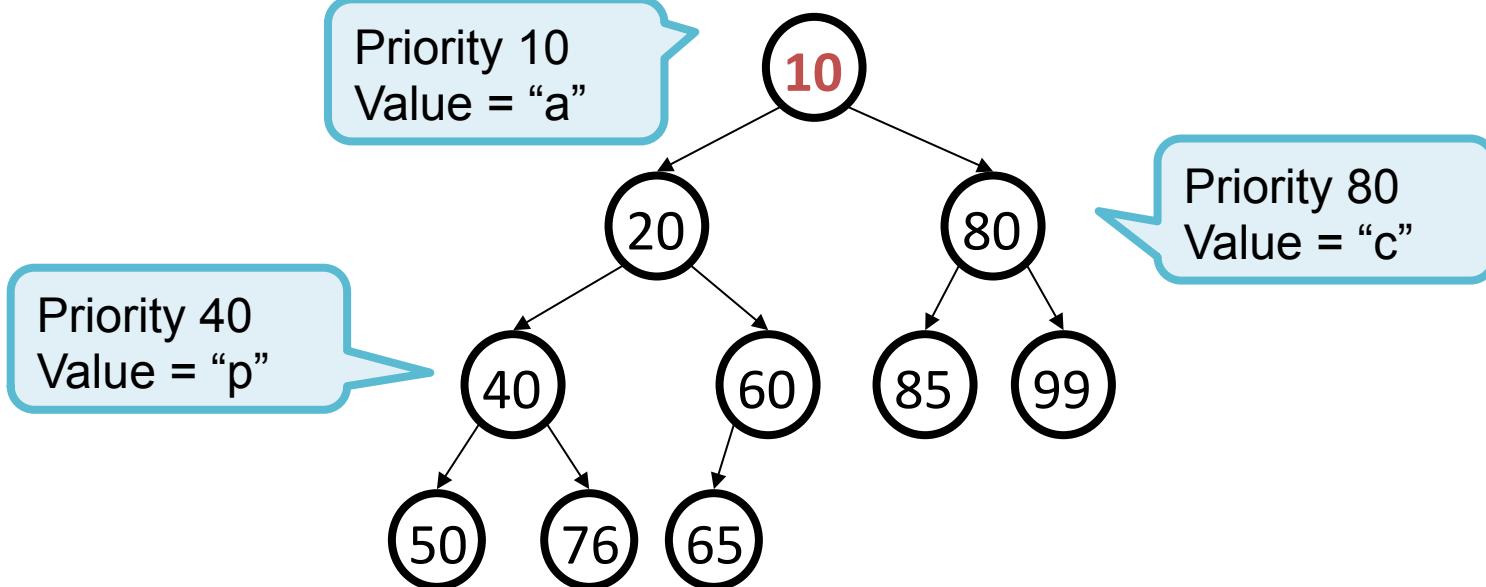
Priority Heaps

Tree

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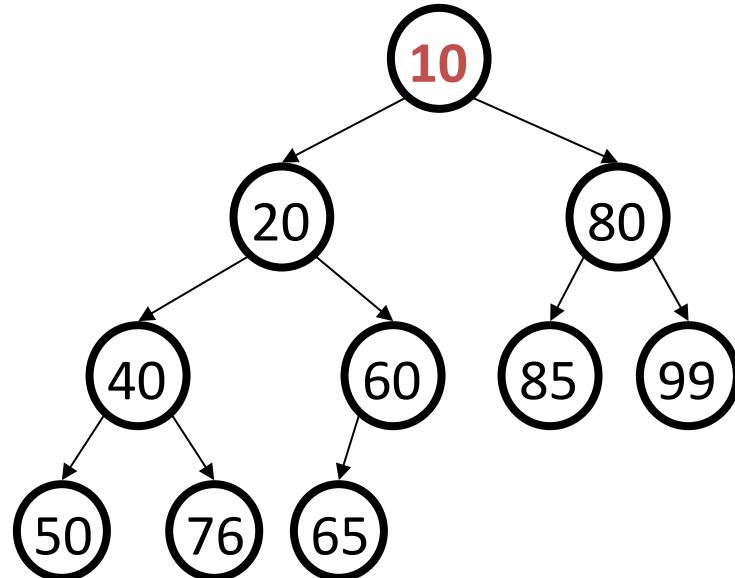
Priority Heaps

Tree

Heap Ordered: Every branch takes you to a “greater” node.

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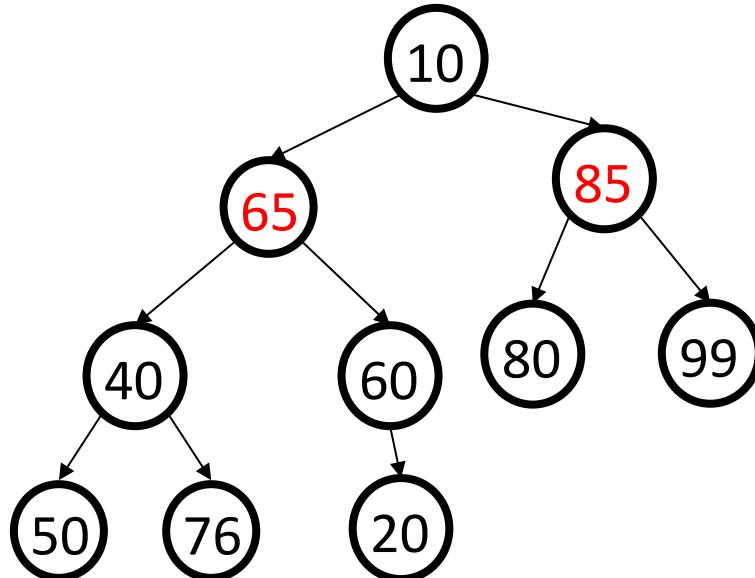
Not Priority Heaps

Tree

Not Heap Ordered: Every branch takes you to a “greater” node.

Binary: Every node has at most two children.

Complete: There are no “gaps” in the tree.



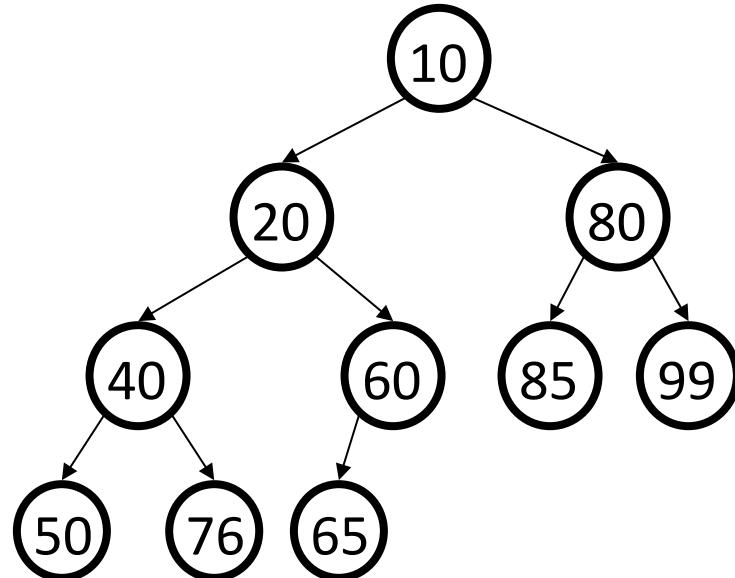
Priority Heaps

Tree

Heap Ordered: Every branch takes you to a “greater” node.

Binary: Every node has at most two children.

Complete: There are no “gaps” in the tree.



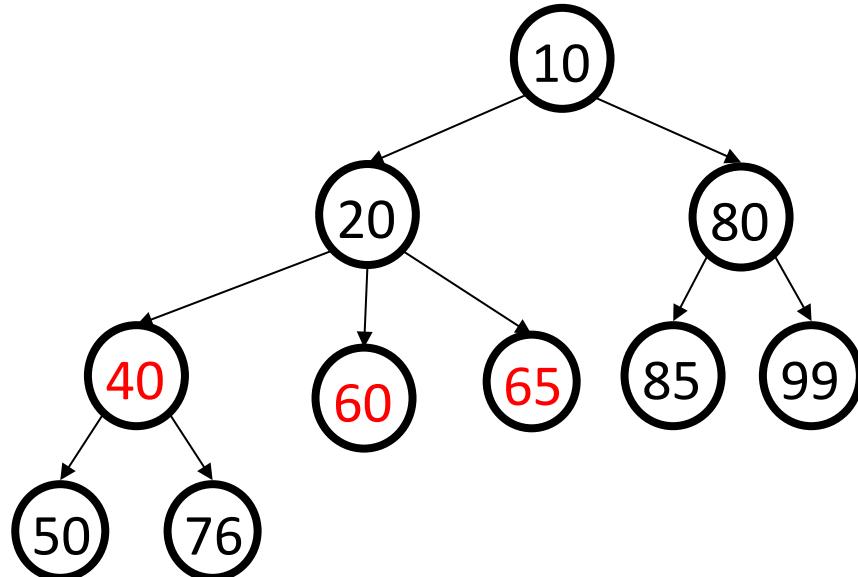
Not Priority Heaps

Tree

Heap Ordered: Every branch takes you to a “greater” node.

Not Binary: Every node has at most two children.

Complete: There are no “gaps” in the tree.



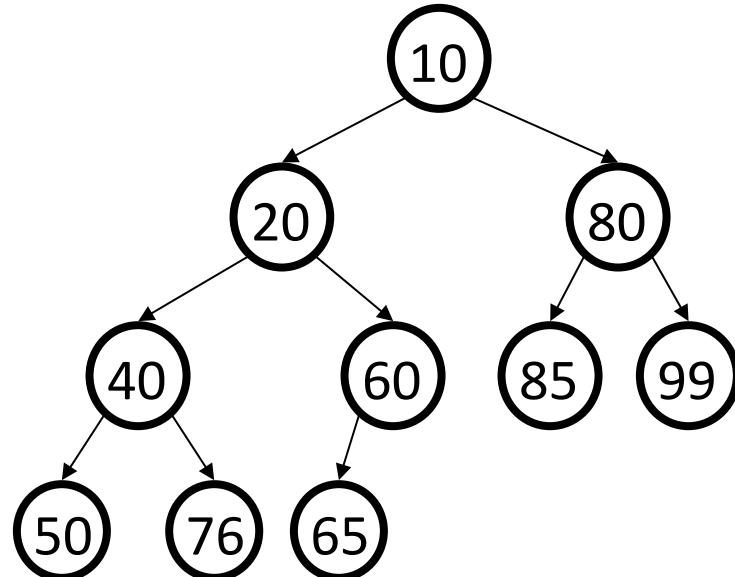
Priority Heaps

Tree

Heap Ordered: Every branch takes you to a “greater” node.

Binary: Every node has at most two children.

Complete: There are no “gaps” in the tree.



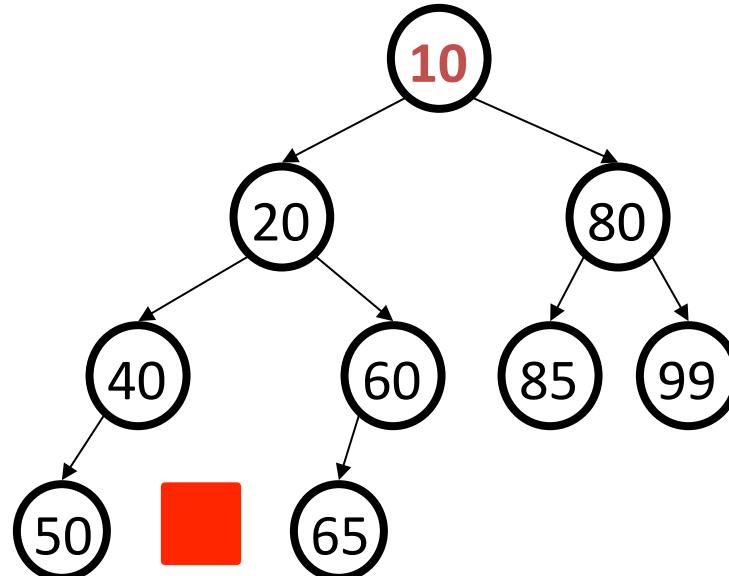
Priority Heaps

Tree

Heap Ordered: Every branch takes you to a “greater” node.

Binary: Every node has at most two children.

Not Complete: There are no “gaps” in the tree.



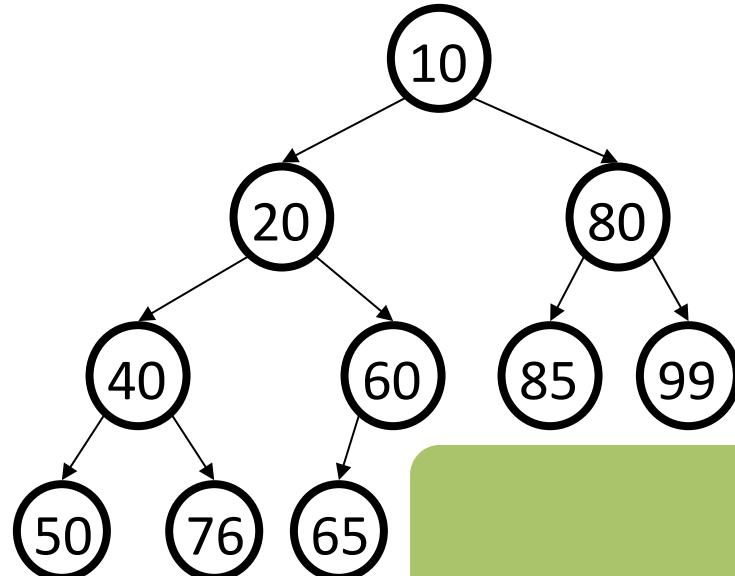
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Tree

Heap Ordered: Every branch takes you to a “greater” node.

Binary: Every node has at most two children.

Complete: There are no “gaps” in the tree.



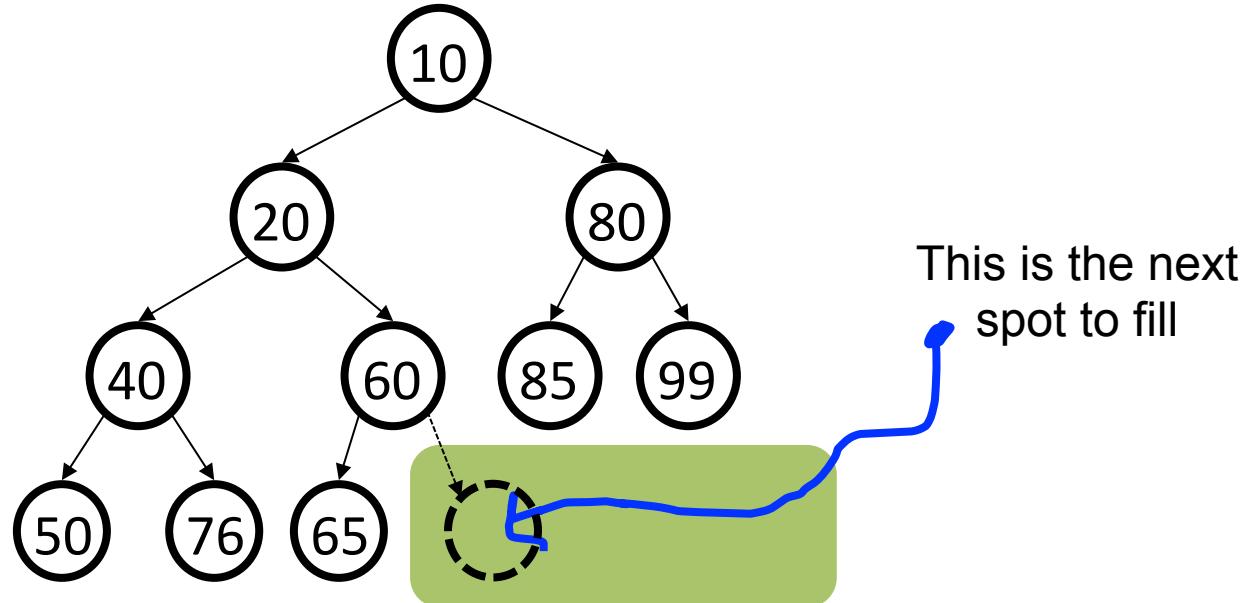
Priority Heaps

Tree

Heap Ordered: Every branch takes you to a “greater” node.

Binary: Every node has at most two children.

Complete: There are no “gaps” in the tree.



How do we represent that?

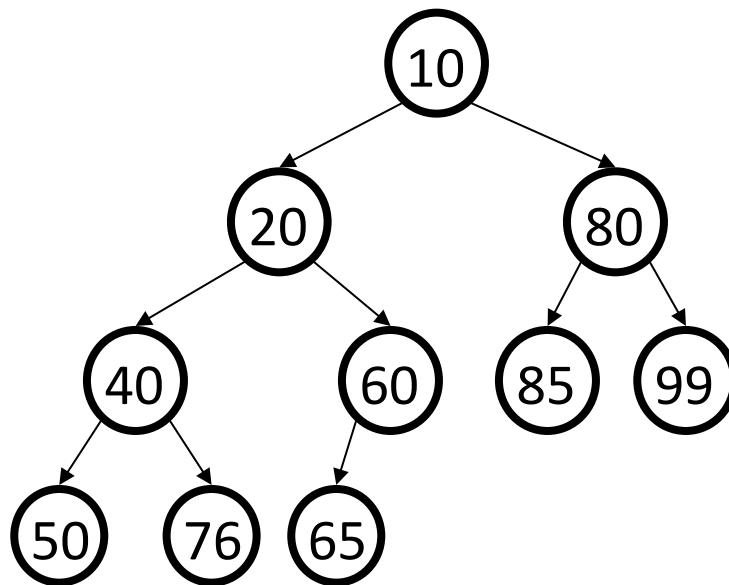
Heaps

Since it's binary and complete, it turns out we can use an array!

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
|-----------------|---|-----------------|----|---|---|---|---|---|---|---|--|
| <i>value</i> | | t | m | b | x | q | a | | | | |
| <i>priority</i> | | 2 | 5 | 4 | 5 | 5 | 8 | | | | |
| <i>size</i> | 6 | <i>capacity</i> | 10 | | | | | | | | |

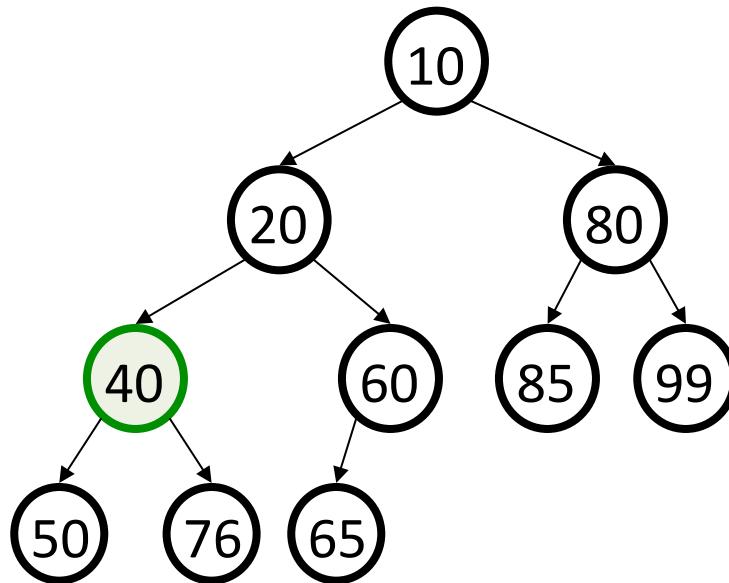
- The "start" or "root" index is 1.
- Every index i has a "parent" index: $i/2$
- Every index has two "child" indexes: $i*2, i*2 + 1$

Heap as Tree



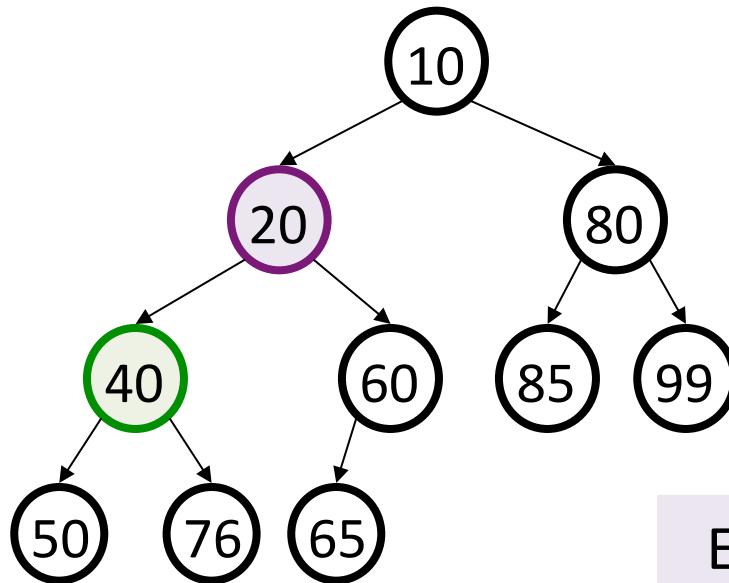
| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 65 | 0 | 0 |
| <i>size</i> | 10 | capacity | 13 | | | | | | | | | | |

Heap as Tree



| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 65 | 0 | 0 |
| <i>size</i> | 10 | capacity | | 13 | | | | | | | | | |

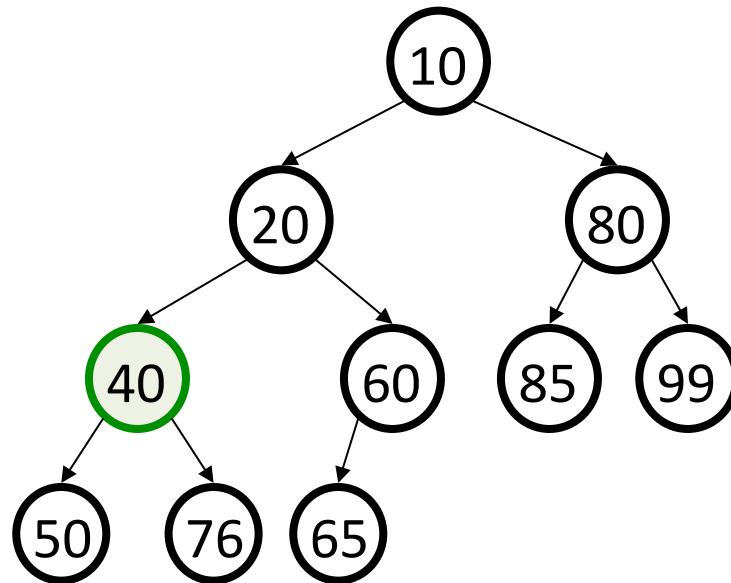
Heap as Tree



Every index i has a "parent" index: $i/2$

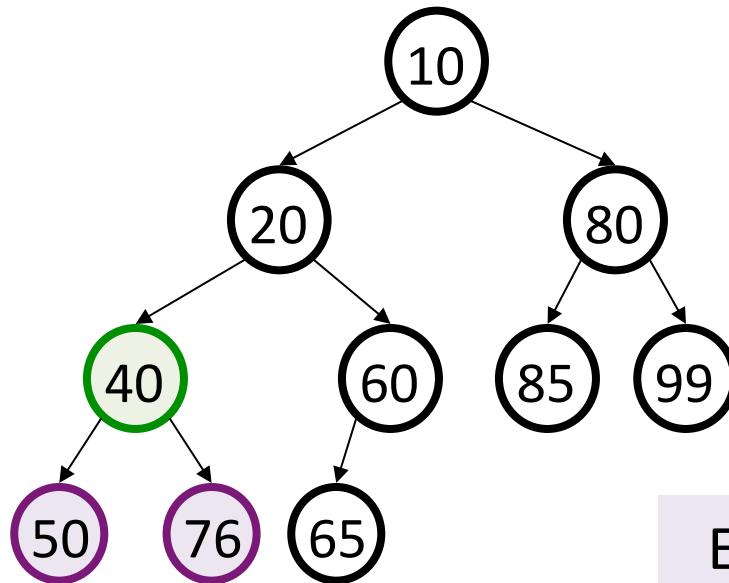
| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| value | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 65 | 0 | 0 |
| size | 10 | capacity | 13 | | | | | | | | | | |

Heap as Tree



| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 65 | 0 | 0 |
| <i>size</i> | 10 | capacity | | 13 | | | | | | | | | |

Heap as Tree



Every index i has two children at index $2 * i$ and $2 * i + 1$

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| value | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 65 | 0 | 0 |
| size | 10 | capacity | | 13 | | | | | | | | | |

Heap add (enqueue)

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|---|----|---|---|---|---|---|
| <i>value</i> | | t | m | b | x | q | a | | | |
| <i>priority</i> | | 2 | 5 | 4 | 5 | 5 | 8 | | | |
| <i>size</i> | 6 | <i>capacity</i> | | | 10 | | | | | |

```
pq.enqueue("k", 1);
```

Heap add (enqueue)

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|---|----|---|---|---|---|---|
| <i>value</i> | | t | m | b | x | q | a | k | | |
| <i>priority</i> | | 2 | 5 | 4 | 5 | 5 | 8 | 1 | | |
| <i>size</i> | 7 | <i>capacity</i> | | | 10 | | | | | |

```
    pq.enqueue("k", 1);
```

1. Place it in the first empty index

Heap add (enqueue)

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|----------|-----------------|---|----|---|---|---|----------|---|---|
| <i>value</i> | | t | m | b | x | q | a | k | | |
| <i>priority</i> | | 2 | 5 | 4 | 5 | 5 | 8 | 1 | | |
| <i>size</i> | 7 | <i>capacity</i> | | 10 | | | | | | |

```
pq.enqueue("k", 1);
```

1. Place it in the first empty index
2. Bubble up to conserve “heap” property

Bubble Up!

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|---|----|---|---|---|---|---|
| <i>value</i> | | t | m | b | x | q | a | k | | |
| <i>priority</i> | | 2 | 5 | 4 | 5 | 5 | 8 | 1 | | |
| <i>size</i> | 7 | <i>capacity</i> | | | 10 | | | | | |

```
    pq.enqueue("k", 1);
```

1. Check priority against its parent.
2. If priority is smaller: swap the two.
3. Repeat until the priority of the parent is smaller or you reach the top of the tree

Bubble Up!

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|----|---|---|---|---|---|---|
| value | | t | m | b | x | q | a | k | | |
| priority | | 2 | 5 | 4 | 5 | 5 | 8 | 1 | | |
| size | 7 | capacity | | 10 | | | | | | |

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3. Repeat until the priority of the parent is smaller or you reach the top of the tree

Bubble Up!

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|---|----------|---|----|---|---|---|---|---|
| value | | t | m | b | x | q | a | k | | |
| priority | | 2 | 5 | 4 | 5 | 5 | 8 | 1 | | |
| size | 7 | | capacity | | 10 | | | | | |

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3. Repeat until the priority of the parent is smaller or you reach the top of the tree

Bubble Up!

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|---|----------|----|---|---|---|---|---|---|
| value | | t | m | k | x | q | a | b | | |
| priority | | 2 | 5 | 1 | 5 | 5 | 8 | 4 | | |
| size | 7 | | capacity | 10 | | | | | | |

```
pq.enqueue("k", 1);
```

1. Check priority against its parent.
2. If priority is smaller: swap the two.
3. Repeat until the priority of the parent is smaller or you reach the top of the tree

Bubble Up!

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|---|----|---|---|---|---|---|
| value | | t | m | k | x | q | a | b | | |
| priority | | 2 | 5 | 1 | 5 | 5 | 8 | 4 | | |
| size | 7 | capacity | | | 10 | | | | | |

1. Check priority against its parent.
2. If priority is smaller: swap the two.
- 3. Repeat until the priority of the parent is smaller or you reach the top of the tree**

Bubble Up!

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|---|----|---|---|---|---|---|
| value | | t | m | k | x | q | a | b | | |
| priority | | 2 | 5 | 1 | 5 | 5 | 8 | 4 | | |
| size | 7 | capacity | | | 10 | | | | | |

1. Check priority against its parent.
2. If priority is smaller: swap the two.
3. Repeat until the priority of the parent is smaller or you reach the top of the tree

Bubble Up!

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|----|---|---|---|---|---|---|
| <i>value</i> | | t | m | k | x | q | a | b | | |
| <i>priority</i> | | 2 | 5 | 1 | 5 | 5 | 8 | 4 | | |
| <i>size</i> | 7 | <i>capacity</i> | | 10 | | | | | | |

1. Check priority against its parent.
2. If priority is smaller: swap the two.
3. Repeat until the priority of the parent is smaller or you reach the top of the tree

Bubble Up!

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|---|----|---|---|---|---|---|
| value | | t | m | k | x | q | a | b | | |
| priority | | 2 | 5 | 1 | 5 | 5 | 8 | 4 | | |
| size | 7 | capacity | | | 10 | | | | | |

1. Check priority against its parent.
2. If priority is smaller: swap the two.
3. Repeat until the priority of the parent is smaller or you reach the top of the tree

Bubble Up!

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|----------|---|---|----|---|---|---|---|---|
| <i>value</i> | | k | m | t | x | q | a | b | | |
| <i>priority</i> | | 1 | 5 | 2 | 5 | 5 | 8 | 4 | | |
| <i>size</i> | 7 | capacity | | | 10 | | | | | |

1. Check priority against its parent.
2. If priority is smaller: swap the two.
3. Repeat until the priority of the parent is smaller or you reach the top of the tree

Bubble Up!

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|----|---|---|---|---|---|---|
| <i>value</i> | | k | m | t | x | q | a | b | | |
| <i>priority</i> | | 1 | 5 | 2 | 5 | 5 | 8 | 4 | | |
| <i>size</i> | 7 | <i>capacity</i> | | 10 | | | | | | |

1. Check priority against its parent.
2. **If priority is smaller: swap the two.**
3. Repeat until the priority of the parent is smaller or you reach the top of the tree

Bubble Up!

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|----|---|---|---|---|---|---|
| <i>value</i> | | k | m | t | x | q | a | b | | |
| <i>priority</i> | | 1 | 5 | 2 | 5 | 5 | 8 | 4 | | |
| <i>size</i> | 7 | <i>capacity</i> | | 10 | | | | | | |

1. Check priority against its parent.
2. If priority is smaller: swap the two.
- 3. Repeat until the priority of the parent is smaller or you reach the top of the tree**

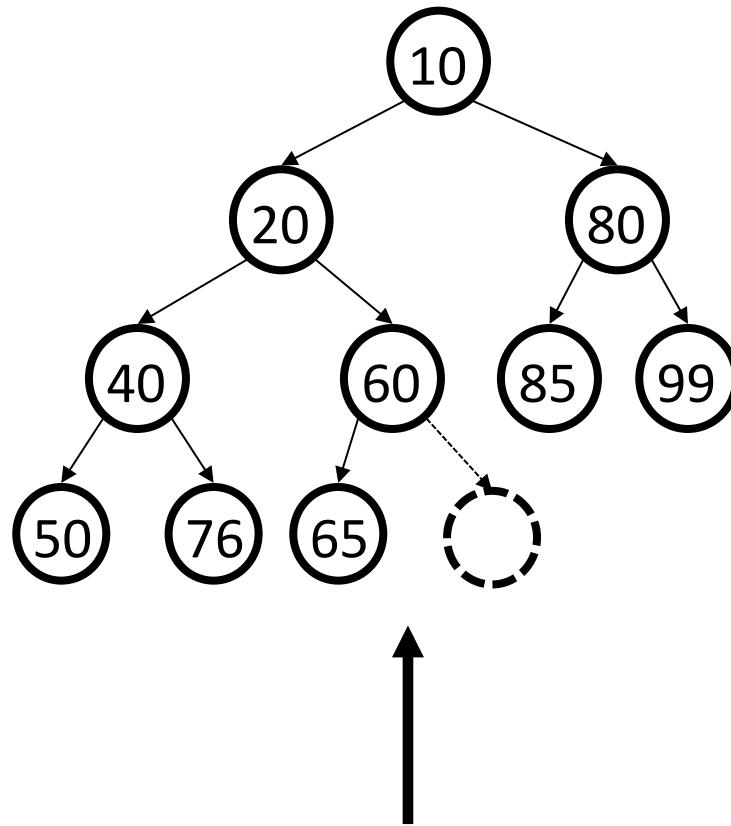
Bubble Up!

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|---|----|---|---|---|---|---|
| <i>value</i> | | k | m | t | x | q | a | b | | |
| <i>priority</i> | | 1 | 5 | 2 | 5 | 5 | 8 | 4 | | |
| <i>size</i> | 7 | <i>capacity</i> | | | 10 | | | | | |

1. Check priority against its parent.
2. If priority is smaller: swap the two.
- 3. Repeat until the priority of the parent is smaller or you reach the top of the tree**

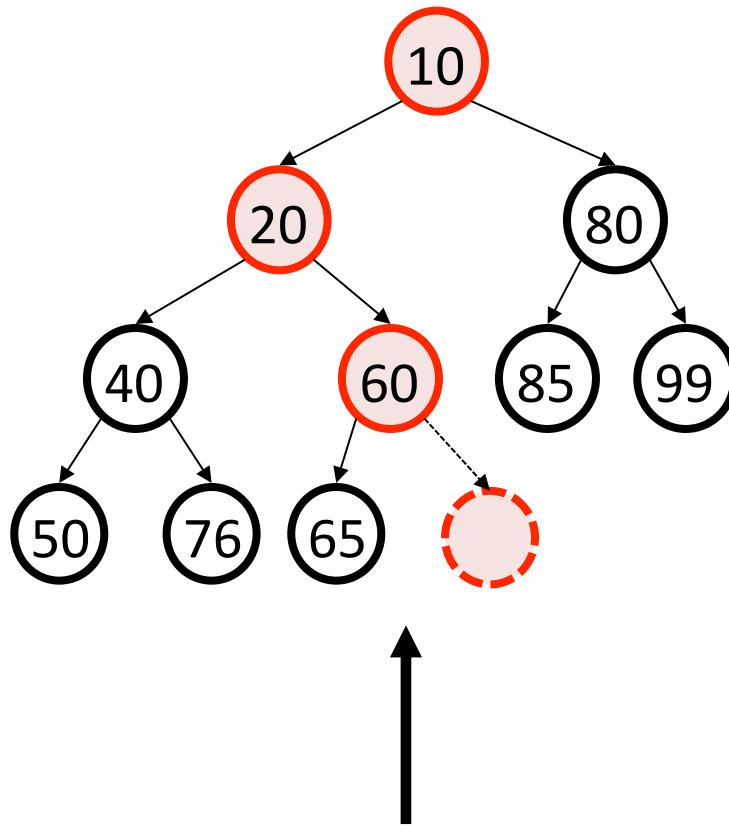
Big O?

Worst Case



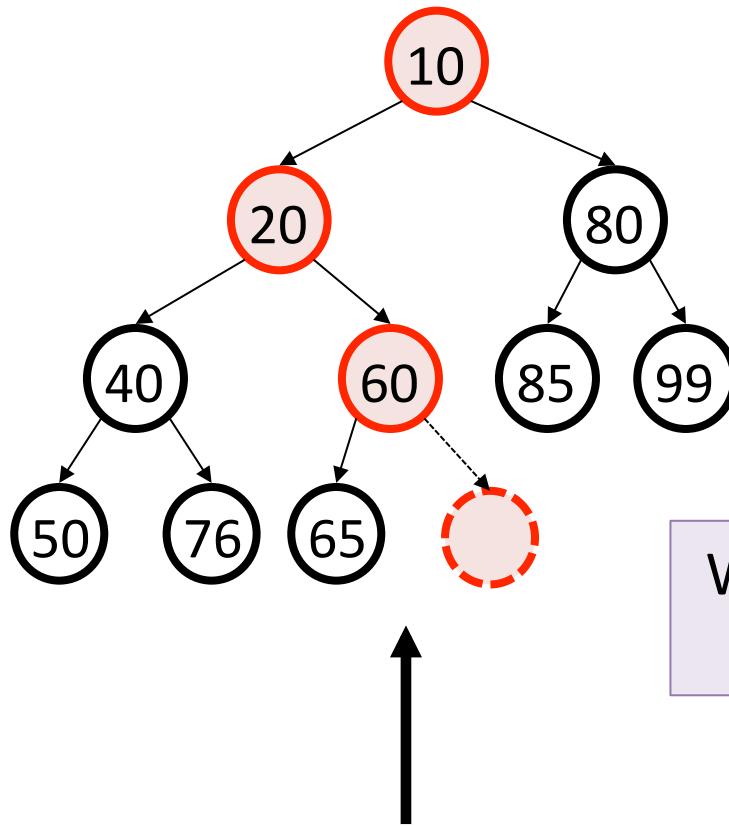
| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 65 | 0 | 0 |
| <i>size</i> | 10 | capacity | 13 | | | | | | | | | | |

Worst Case



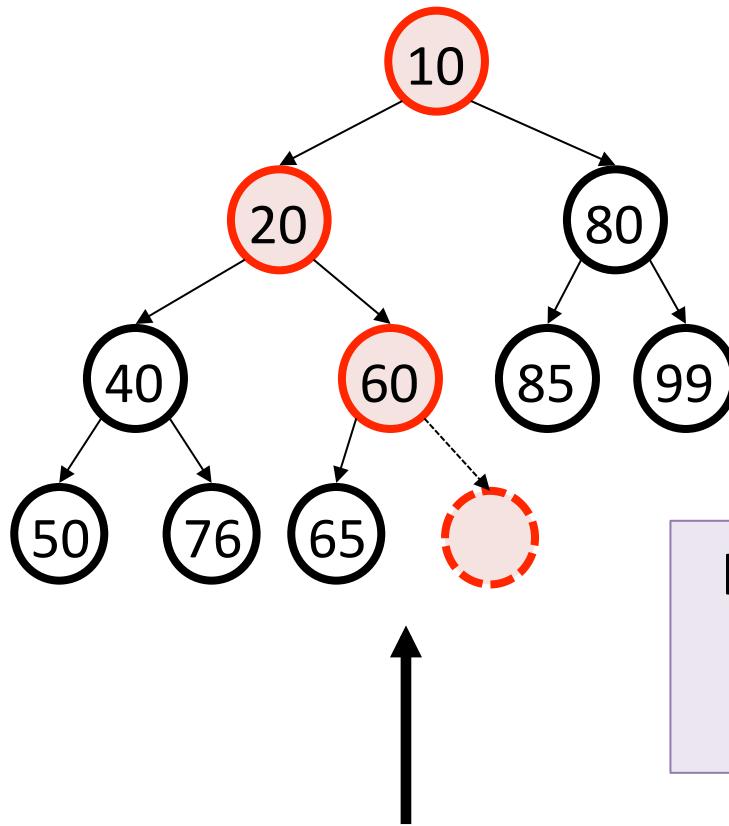
| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 65 | 0 | 0 |
| <i>size</i> | 10 | capacity | | 13 | | | | | | | | | |

Worst Case



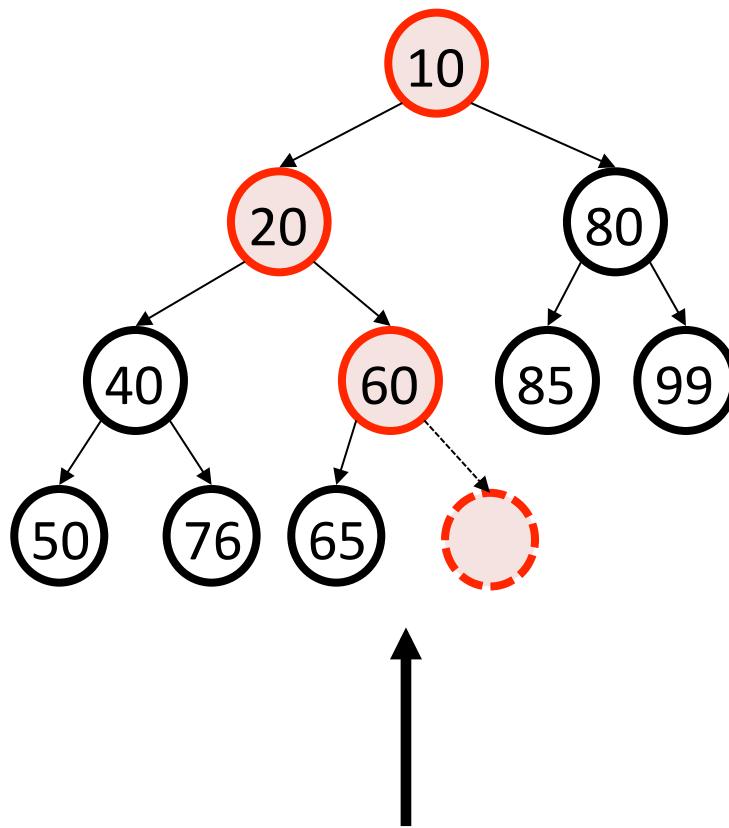
| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 65 | 0 | 0 |
| <i>size</i> | 10 | capacity | 13 | | | | | | | | | | |

Worst Case



| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 65 | 0 | 0 |
| <i>size</i> | 10 | capacity | 13 | | | | | | | | | | |

Worst Case



| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 65 | 0 | 0 |
| <i>size</i> | 10 | capacity | 13 | | | | | | | | | | |

Enqueue Big O?

$O(\log n)$

Peek

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|---|----|---|---|---|---|---|
| <i>value</i> | | k | m | t | x | q | a | b | | |
| <i>priority</i> | | 1 | 5 | 2 | 5 | 5 | 8 | 4 | | |
| <i>size</i> | 7 | <i>capacity</i> | | | 10 | | | | | |

`pq.peek()` --> "k"

`pq.peekPriority()` --> 1

It is trivial to peek at the min-priority element in a heap.
It is always located in index 1! What is the Big-Oh?

Peek Big O?

$O(1)$

Heap remove (dequeue)

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|---|----|---|---|---|---|---|
| <i>value</i> | | k | c | f | p | e | v | y | | |
| <i>priority</i> | | 1 | 2 | 4 | 7 | 5 | 8 | 6 | | |
| <i>size</i> | 7 | <i>capacity</i> | | | 10 | | | | | |

`pq.dequeue()` --> "k"

1. Save the first value
2. Move the last element to the first
3. Bubble up the smaller of the children.
4. Repeat step 3 until you get to the bottom of the tree.

Heap remove (dequeue)

| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|---|----|---|---|---|---|---|
| <i>value</i> | | k | c | f | p | e | v | y | | |
| <i>priority</i> | | 1 | 2 | 4 | 7 | 5 | 8 | 6 | | |
| <i>size</i> | 7 | <i>capacity</i> | | | 10 | | | | | |

`pq.dequeue()` --> "k"

1. Save the first value
2. Move the last element to the first
3. Bubble up the smaller of the children.
4. Repeat step 3 until you get to the bottom of the tree.

Heap remove (dequeue)

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|----|---|---|---|---|---|---|
| value | | y | c | f | p | e | v | y | | |
| priority | | 6 | 2 | 4 | 7 | 5 | 8 | 6 | | |
| size | 6 | capacity | | 10 | | | | | | |

`pq.dequeue() --> "k"`

1. Save the first value
2. **Move the last element to the first**
3. Bubble up the smaller of the children.
4. Repeat step 3 until you get to the bottom of the tree.

Heap remove (dequeue)

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|----|---|---|---|---|---|---|
| value | | y | c | f | p | e | v | | | |
| priority | | 6 | 2 | 4 | 7 | 5 | 8 | | | |
| size | 6 | capacity | | 10 | | | | | | |

pq.dequeue() --> "k"

1. Save the first value
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- 3. Bubble up the smaller of the children.**
4. Repeat step 3 until you get to the bottom of the tree.

Heap remove (dequeue)

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|---|----|---|---|---|---|---|
| value | | y | c | f | p | e | v | | | |
| priority | | 6 | 2 | 4 | 7 | 5 | 8 | | | |
| size | 6 | capacity | | | 10 | | | | | |

pq.dequeue() --> "k"

1. Save the first value
2. Move the last element to the first
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4. Repeat step 3 until you get to the bottom of the tree.

Heap remove (dequeue)

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|----|---|---|---|---|---|---|
| value | | c | y | f | p | e | v | | | |
| priority | | 2 | 6 | 4 | 7 | 5 | 8 | | | |
| size | 6 | capacity | | 10 | | | | | | |

`pq.dequeue() --> "k"`

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Heap remove (dequeue)

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|----|---|---|---|---|---|---|
| value | | c | y | f | p | e | v | | | |
| priority | | 2 | 6 | 4 | 7 | 5 | 8 | | | |
| size | 6 | capacity | | 10 | | | | | | |

`pq.dequeue() --> "k"`

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Heap remove (dequeue)

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|---|----|---|---|---|---|---|
| value | | c | y | f | p | e | v | | | |
| priority | | 2 | 6 | 4 | 7 | 5 | 8 | | | |
| size | 6 | capacity | | | 10 | | | | | |

`pq.dequeue() --> "k"`

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- 3. Bubble up the smaller of the children.**
4. Repeat step 3 until you get to the bottom of the tree.

Heap remove (dequeue)

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|---|----|---|---|---|---|---|
| value | | c | y | f | p | e | v | | | |
| priority | | 2 | 6 | 4 | 7 | 5 | 8 | | | |
| size | 6 | capacity | | | 10 | | | | | |

pq.dequeue() --> "k"

1. Save the first value
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Heap remove (dequeue)

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|----------|---|---|----|---|---|---|---|---|
| value | | c | e | f | p | y | v | | | |
| priority | | 2 | 5 | 4 | 7 | 6 | 8 | | | |
| size | 6 | capacity | | | 10 | | | | | |

`pq.dequeue() --> "k"`

1. Save the first value
2. Move the last element to the first
- 3. Bubble up the smaller of the children.**
4. Repeat step 3 until you get to the bottom of the tree.

Heap remove (dequeue)

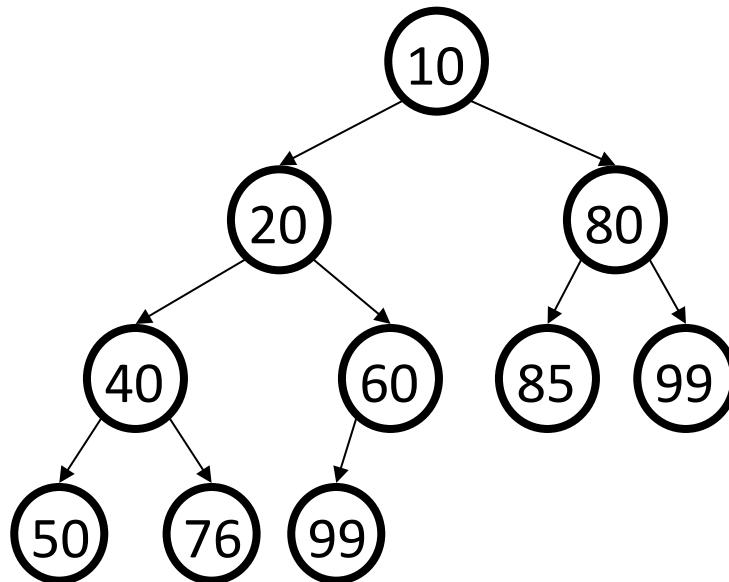
| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------------|---|-----------------|---|---|---|---|---|---|---|---|
| <i>value</i> | | c | e | f | p | y | v | | | |
| <i>priority</i> | | 2 | 5 | 4 | 7 | 6 | 8 | | | |
| <i>size</i> | 6 | <i>capacity</i> | | | | | | | | |

`pq.dequeue() --> "k"`

1. Save the first value
2. Move the last element to the first
3. Bubble up the smaller of the children.
- 4. Repeat step 3 until you get to the bottom of the tree.**

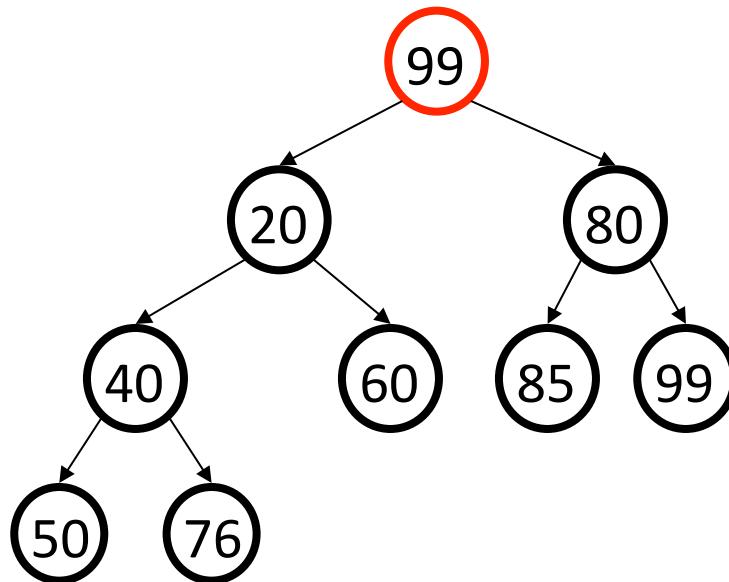
Big O?

Worst Case



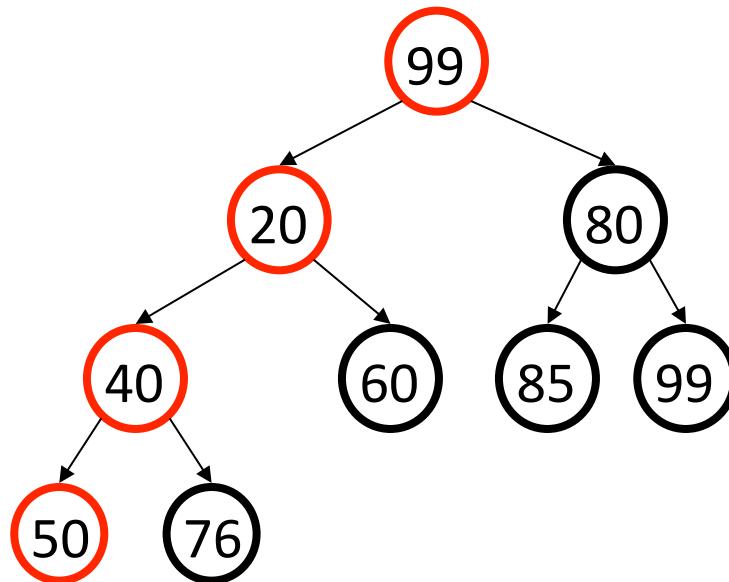
| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 99 | 0 | 0 |
| <i>size</i> | 10 | capacity | | 13 | | | | | | | | | |

Worst Case



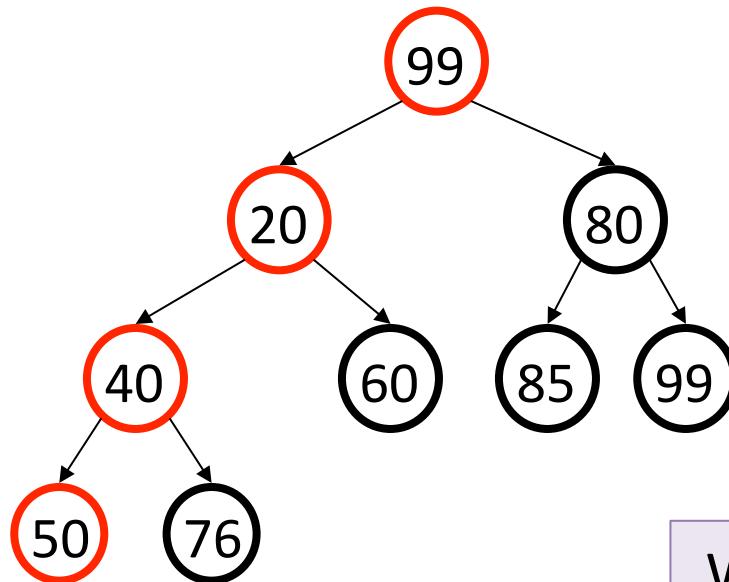
| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 99 | 0 | 0 |
| <i>size</i> | 10 | capacity | 13 | | | | | | | | | | |

Worst Case



| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 99 | 0 | 0 |
| <i>size</i> | 10 | capacity | | 13 | | | | | | | | | |

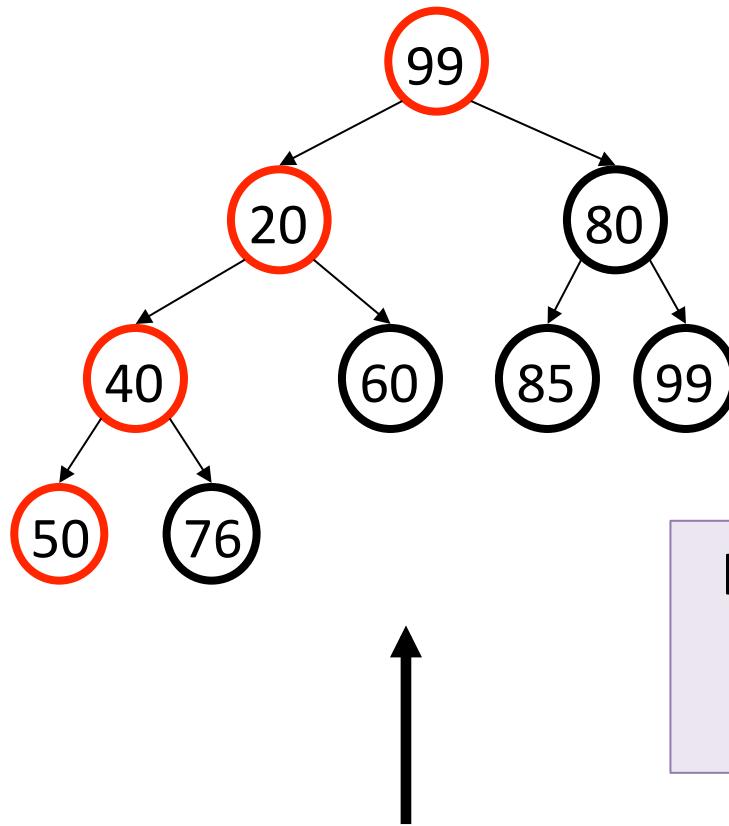
Worst Case



What is the depth
of the tree?

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| value | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 99 | 0 | 0 |
| size | 10 | capacity | 13 | | | | | | | | | | |

Worst Case



| <i>index</i> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|----|----------|----|----|----|----|----|----|----|----|----|----|----|
| <i>value</i> | 0 | 10 | 20 | 80 | 40 | 60 | 85 | 99 | 50 | 76 | 99 | 0 | 0 |
| <i>size</i> | 10 | capacity | | 13 | | | | | | | | | |

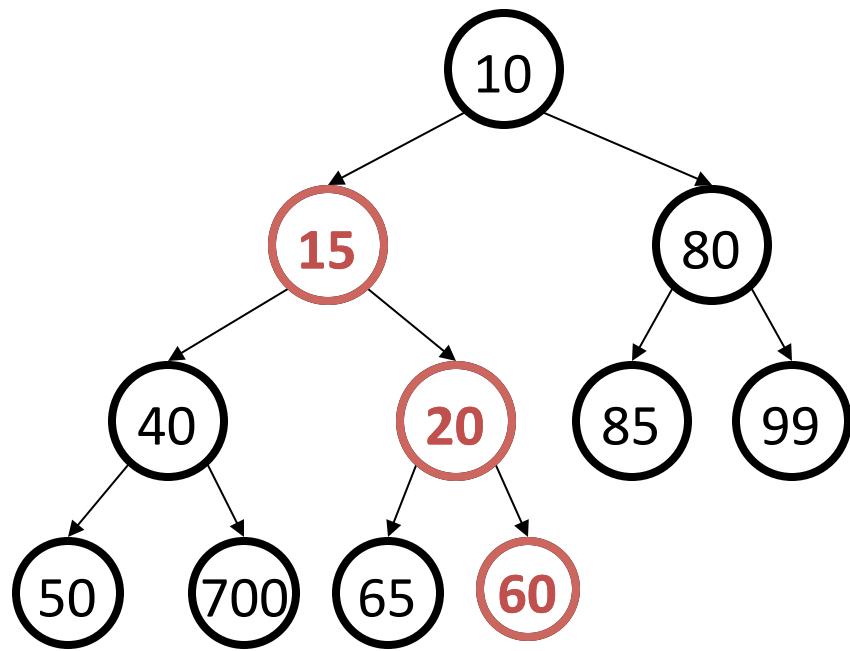
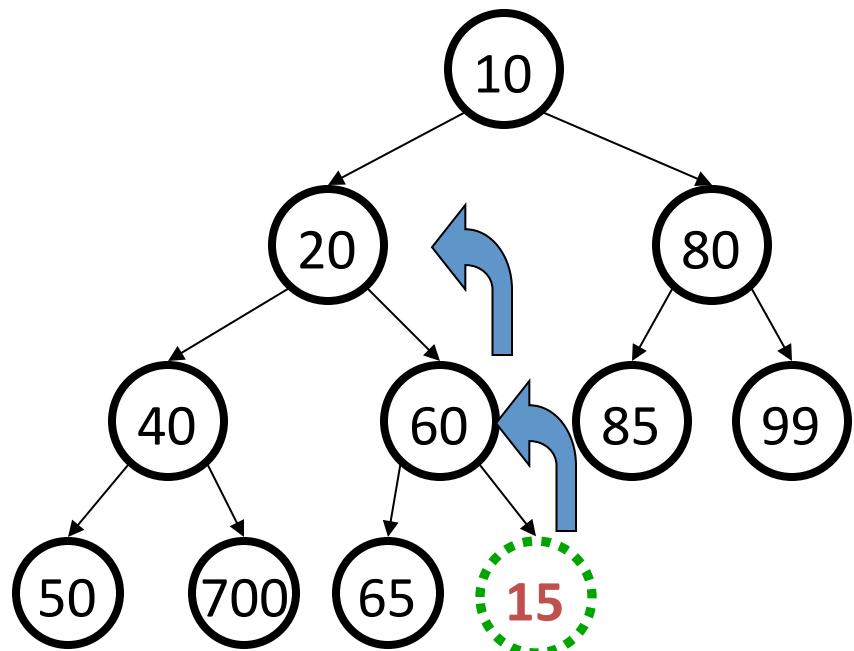
Enqueue Big O?

$O(\log n)$

See Enqueue / Dequeue
as Trees

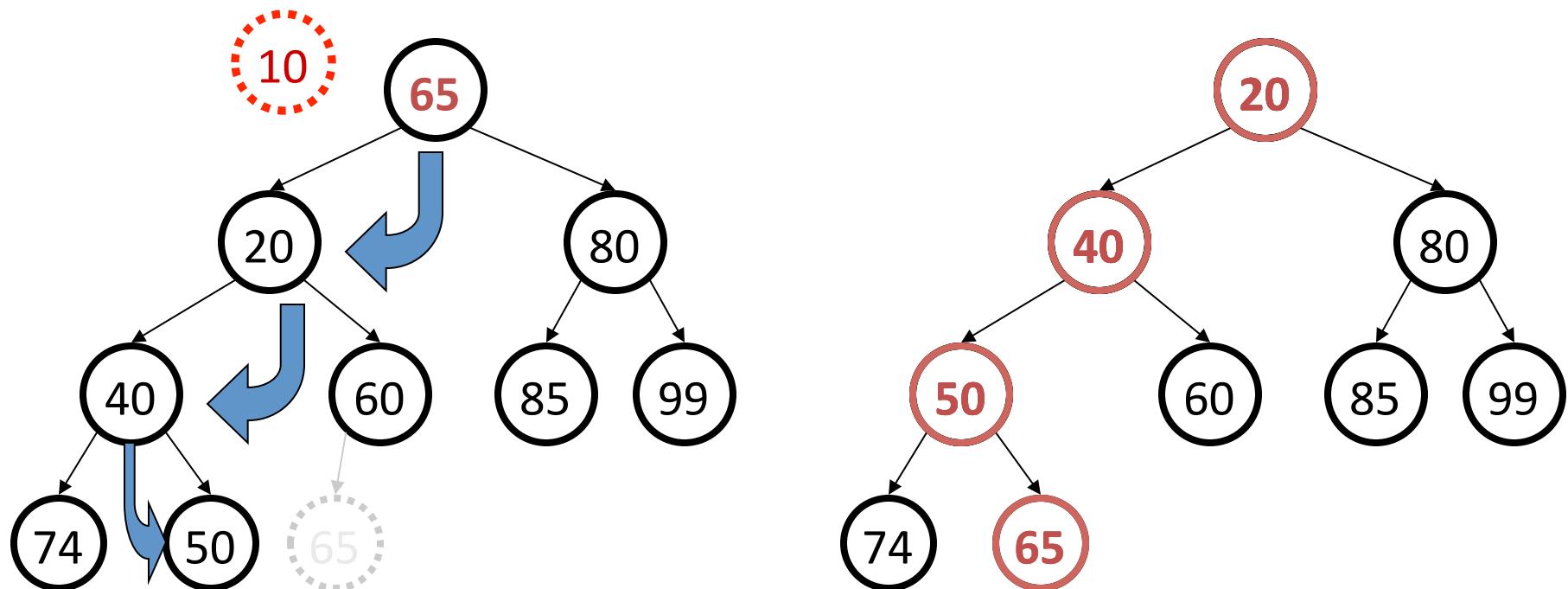
Enqueue

- When adding to a heap, the value is first placed at bottom-right.
 - To restore heap ordering, the newly added element is shifted ("bubbled") up the tree until it reaches its proper place.
 - Enqueue 15 at bottom-right; bubble up until in order.

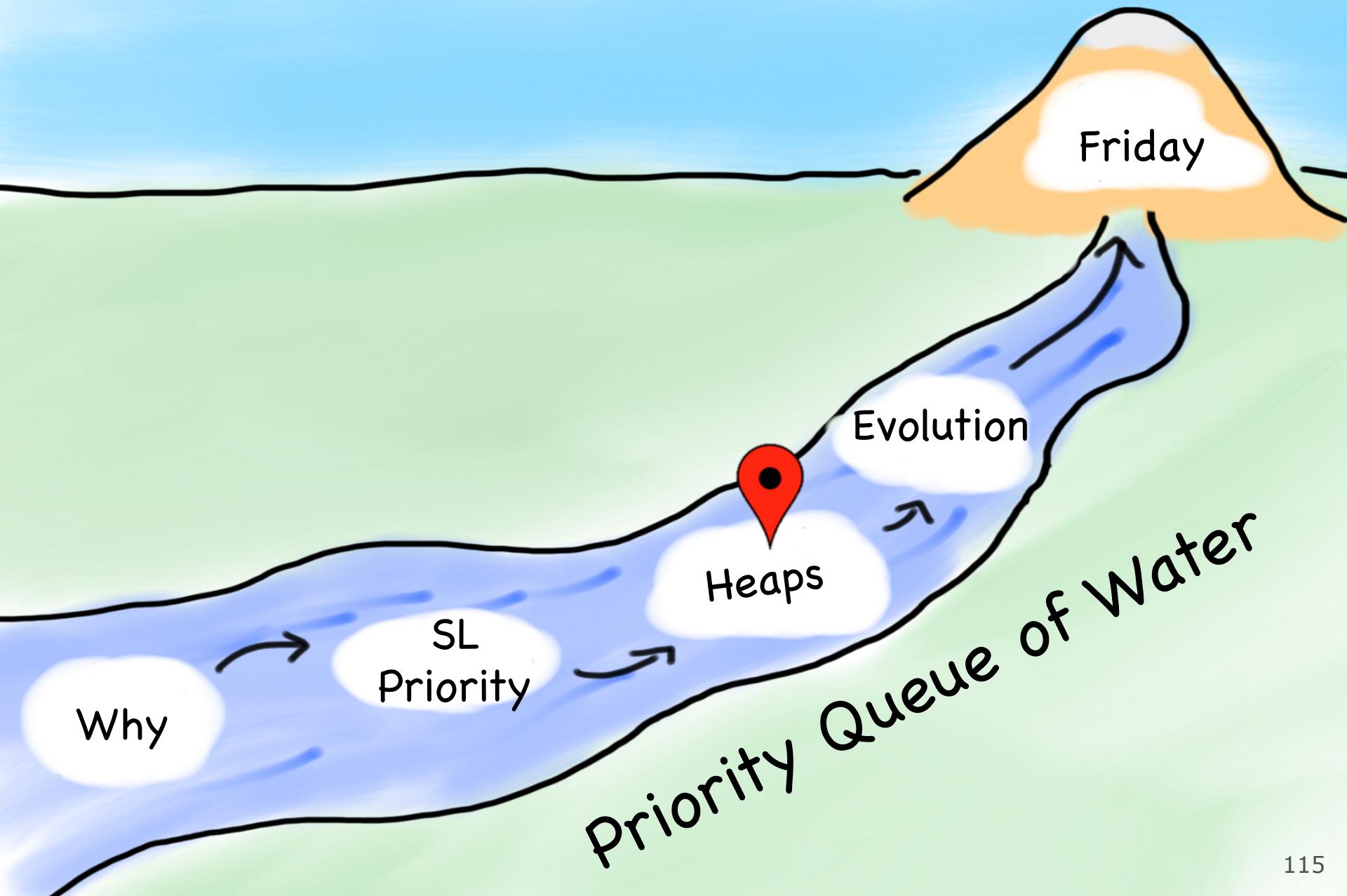


Dequeue

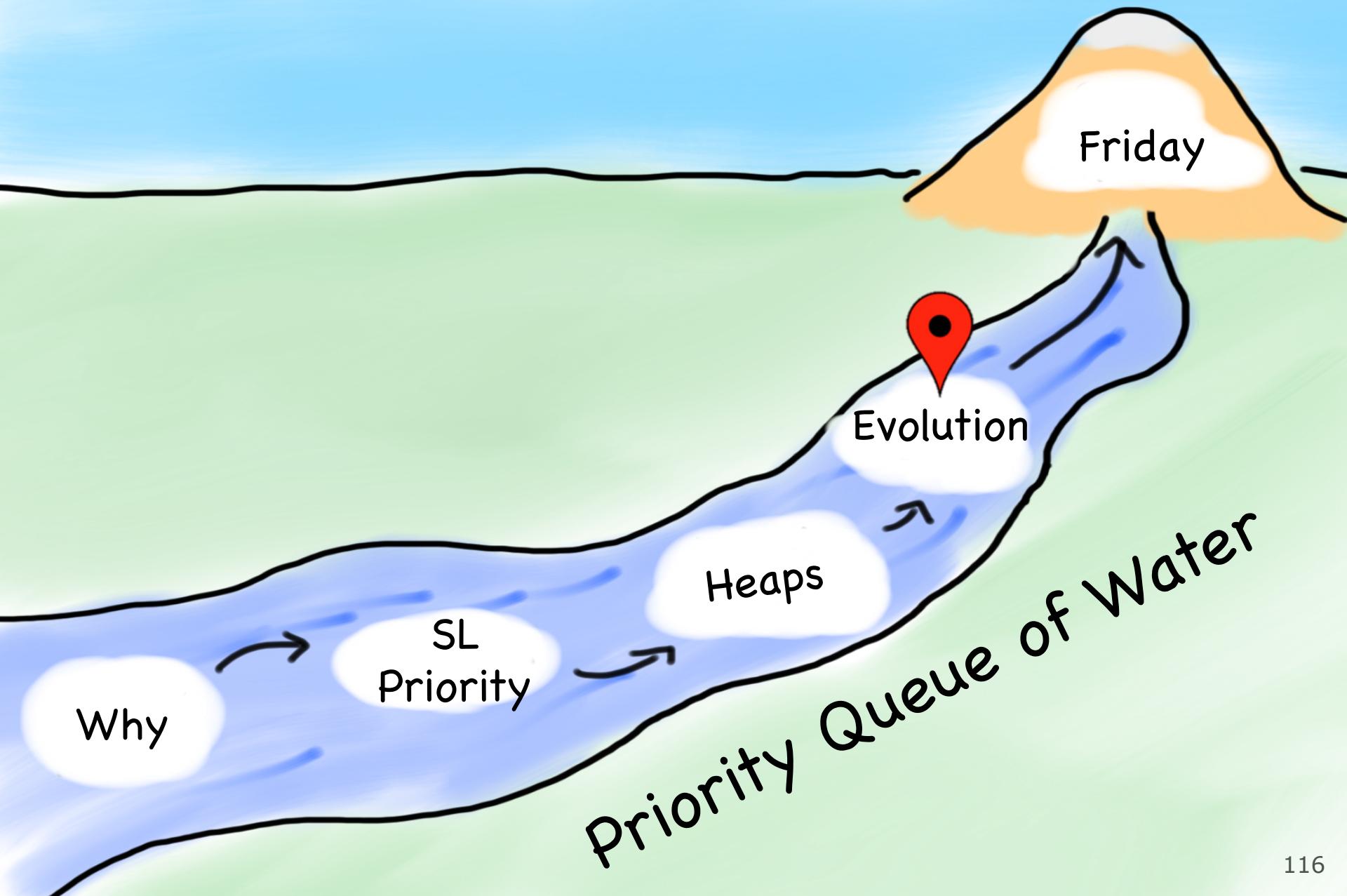
- To restore heap order, the improper root is shifted ("bubbled") down the tree by swapping with its smaller (higher-pri) child.
 - dequeue min of 10; swap up bottom-right leaf of 65; bubble down.



Today's Goals

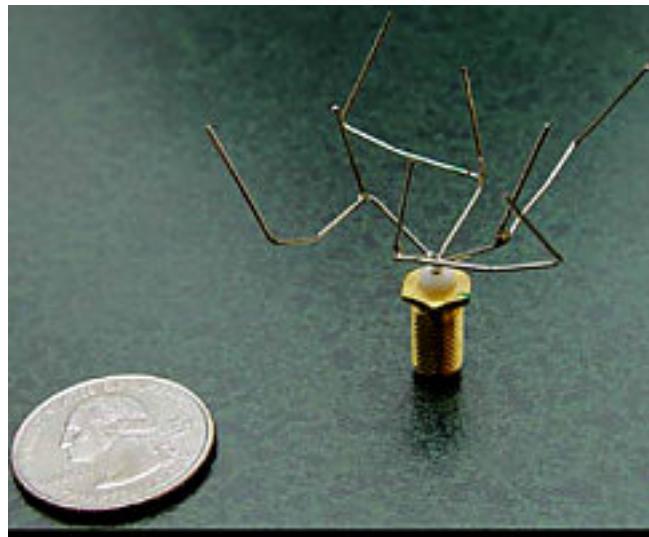


Today's Goals

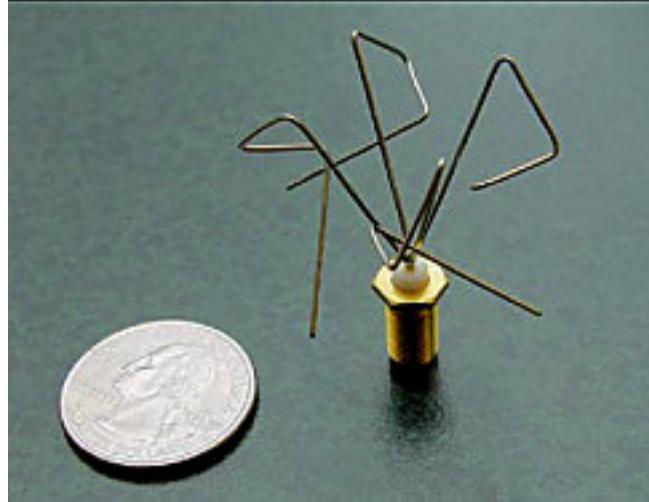


Evolutionary Algorithms

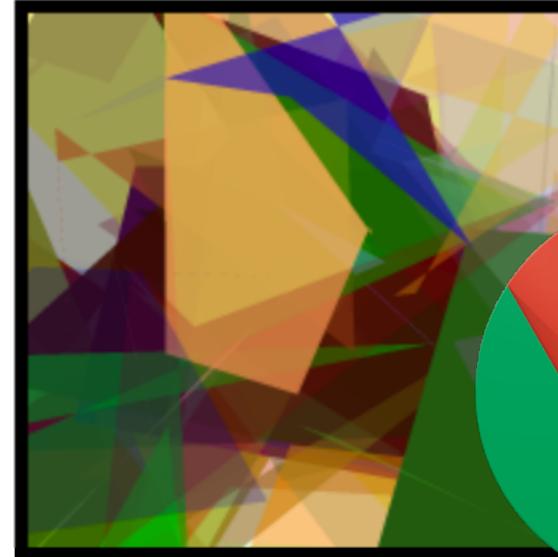
Antennae for microsatellites ($\sim 25\text{kg}$)



2006

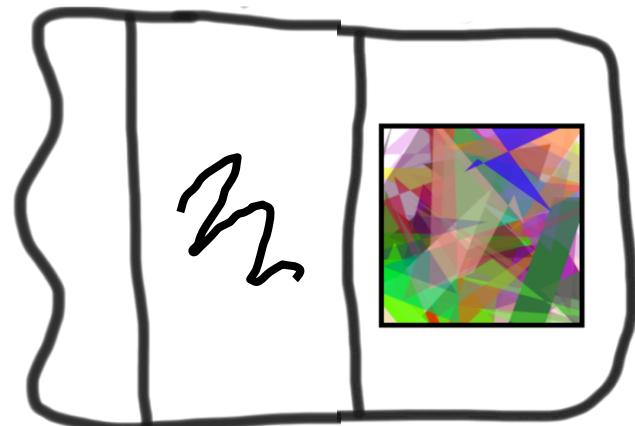
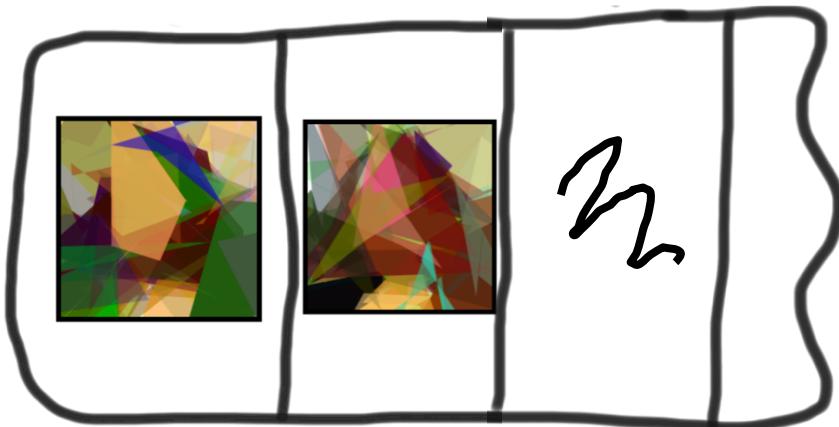


Evolving Mona Lisa



Evolution PQueue

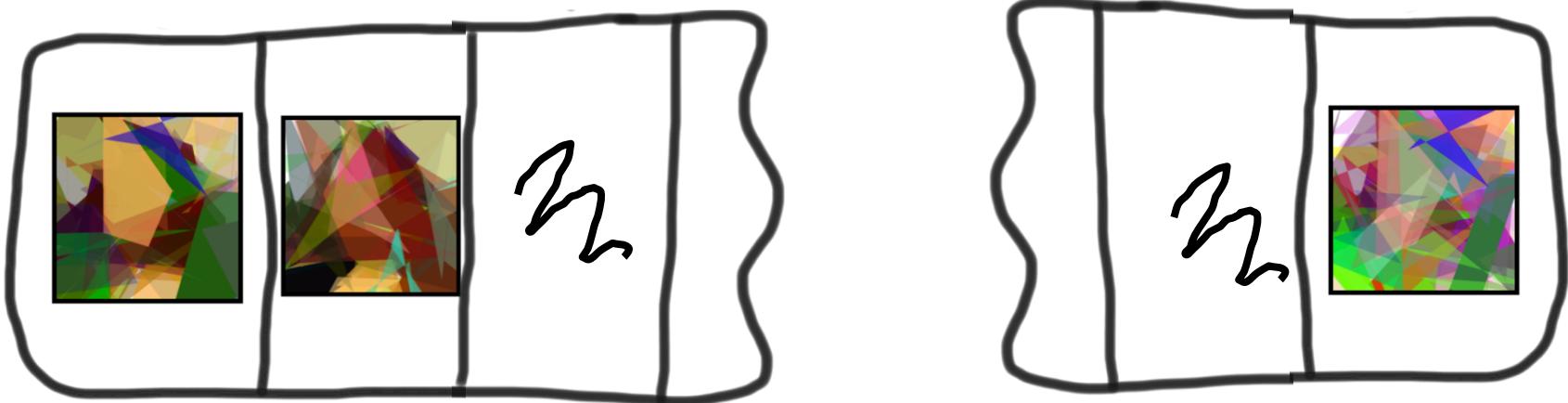
`PQueue<Species * > generation`



Priority is how different the image is from the target

Natural Selection

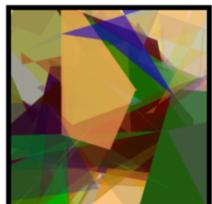
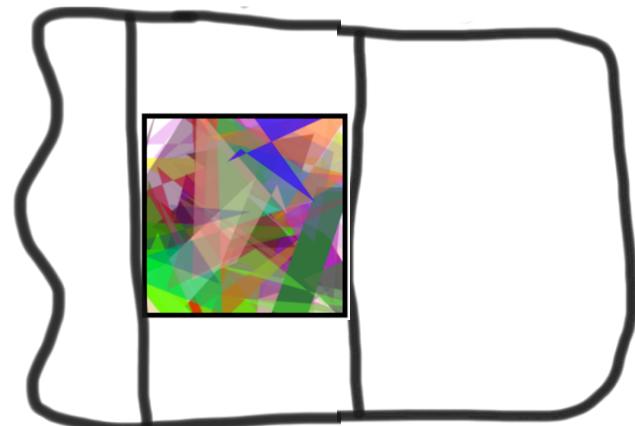
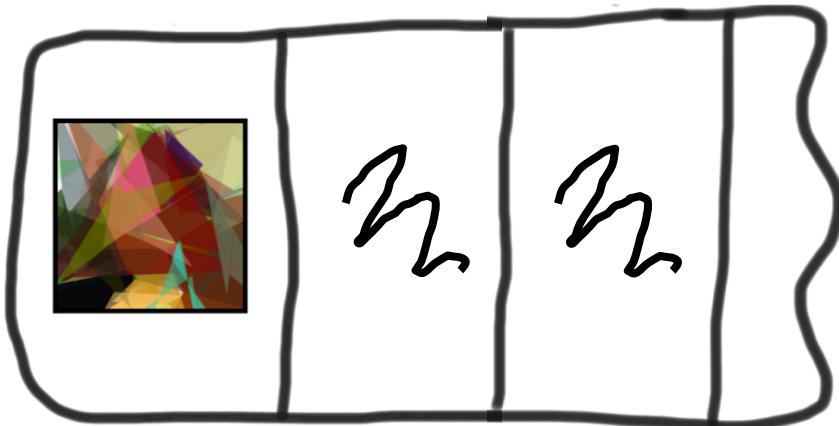
PQueue<Species * > generation



Dequeue (and save) the top
M species

Natural Selection

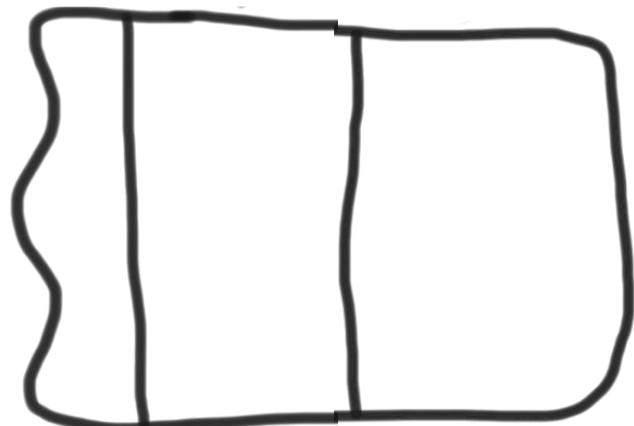
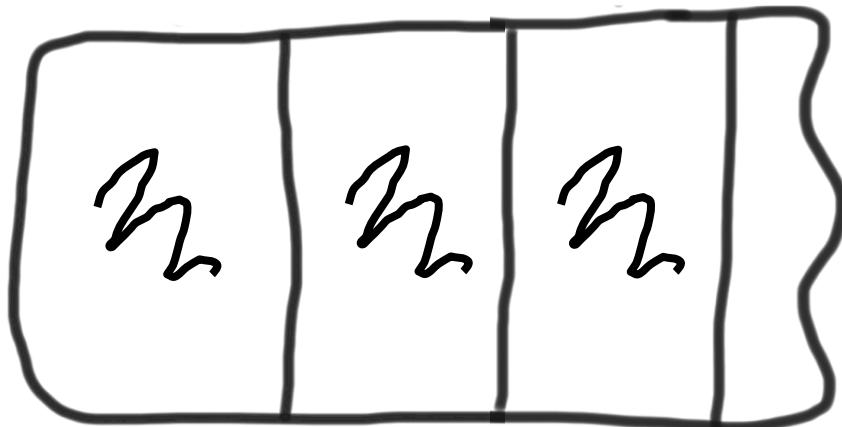
PQueue<Species * > generation



Dequeue (and save) the top
M species

Natural Selection

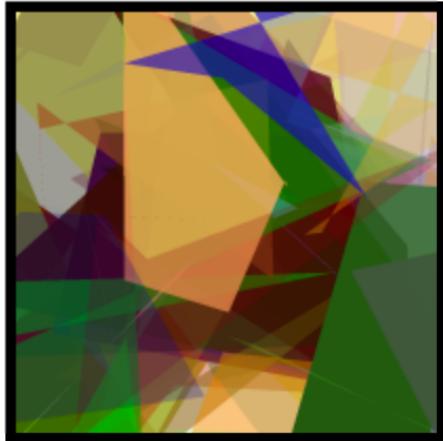
PQueue<Species * > generation



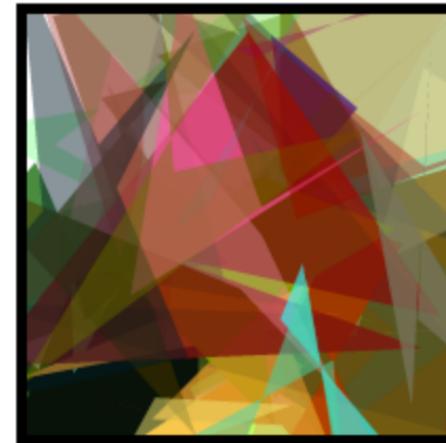
Dequeue (and save) the top
M species

Mate

Species * parent1



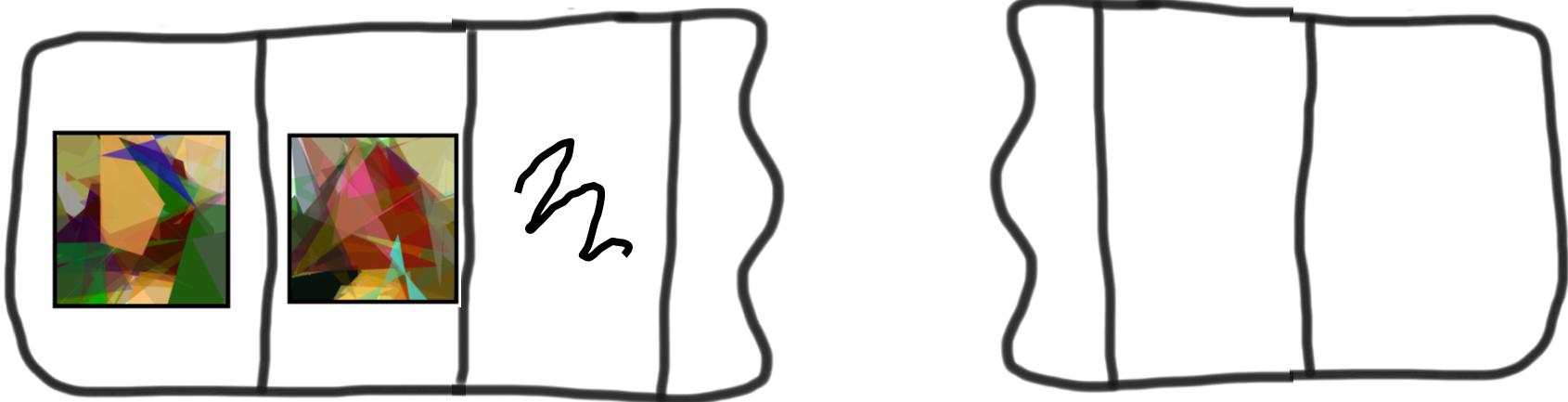
Species * parent2



Species * child

Repopulate

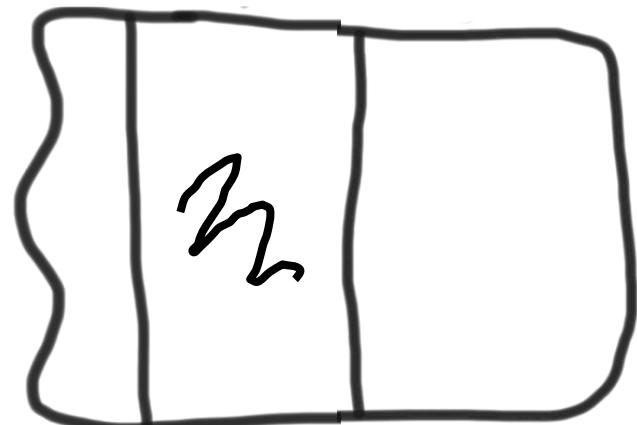
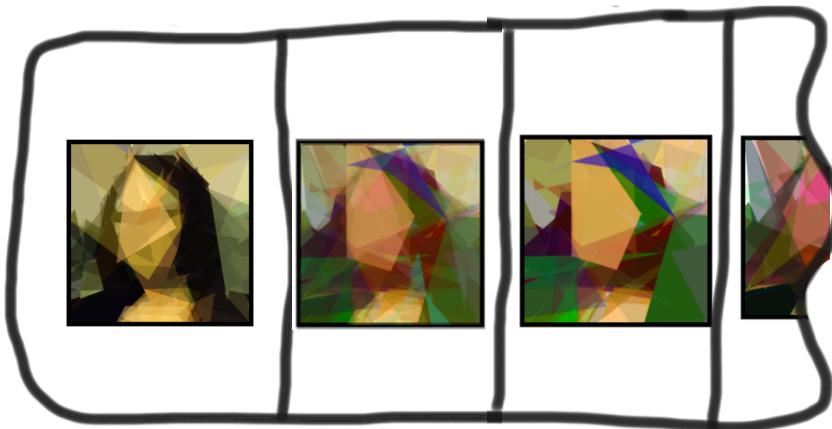
PQueue<Species *> generation



Enqueue all parents

Repopulate

PQueue<Species * > generation



Enqueue all children

Repeat

Evolutionary Algorithms

```
Pqueue<Species *> alive;
```

populate pqueue with random species

```
while(true){
```

dequeue the best species as parents

mate the parents to produce new children

start a new generation

enqueue the parents and the children

```
}
```

Evolutionary Algorithms

```
Pqueue<Species *> alive;
for(int i = 0; i < N_POP; i++) {
    Species * starter = new Species();
    alive->enqueue(starter, starter->getFitness());
}

while(true){
    Vector<Species *> parents;
    for(int i = 0; i < N_SURVIVORS; i++) {
        parents.add(alive->dequeue());
    }
    Vector<Species *> children = mate(parents, N_CHILDREN);
    alive.clear();
    for(Species * p : parents) {
        alive.enqueue(p, p->getFitness());
    }
    for(Species * c : children) {
        alive.enqueue(c, c->getFitness());
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}
```

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```

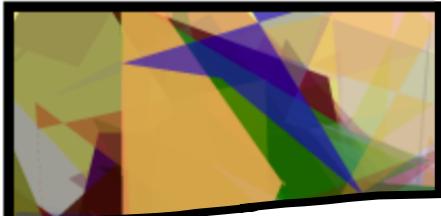
Evolutionary Algorithms

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    }
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        alive.enqueue(c, c->getFitness());
    }
}
```

Mate

Species * parent1



010101111101110110101



Species * parent2

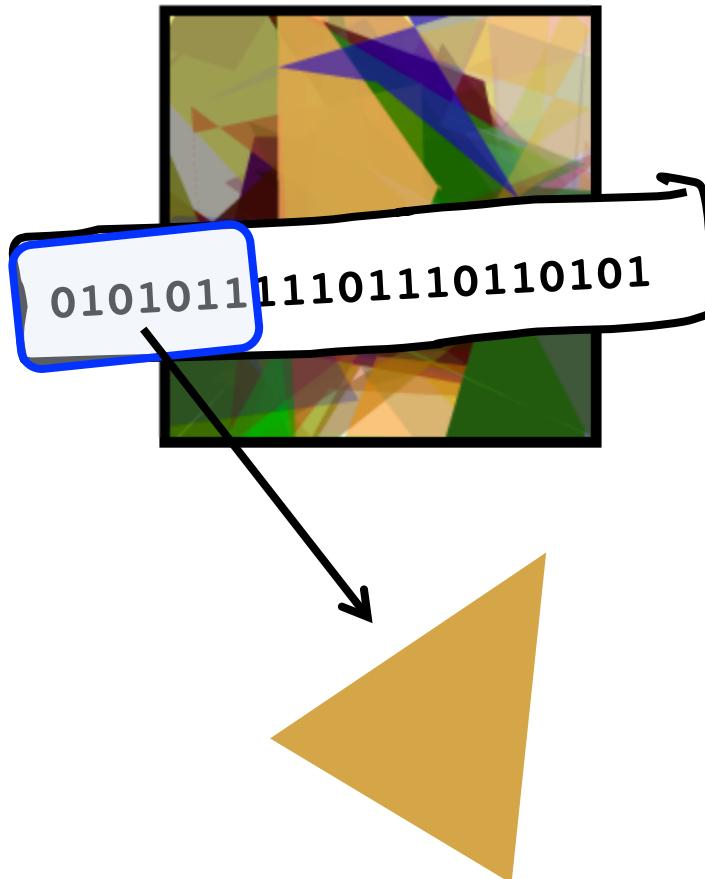


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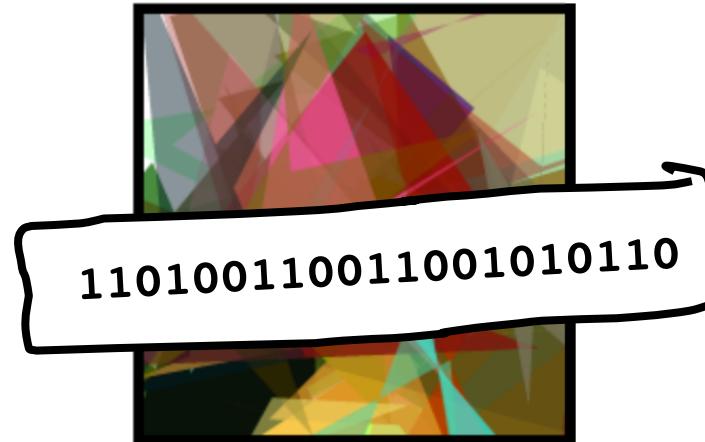


DNA

Species * parent1



Species * parent2

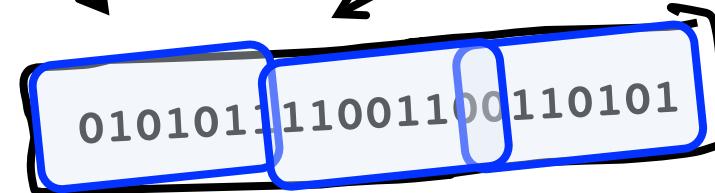


Crossover

Species * parent1



Species * parent2

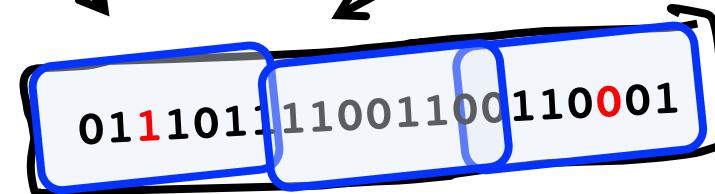
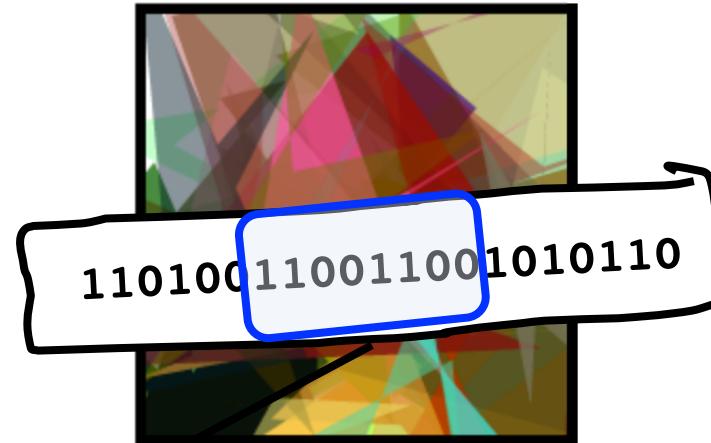


Mutation

Species * parent1

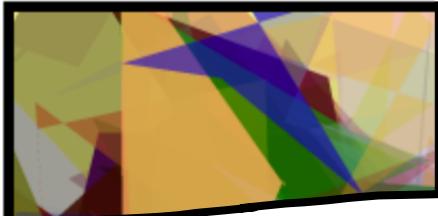


Species * parent2



Mate

Species * parent1



01010111101110110101



Species * parent2



110100110011001010110



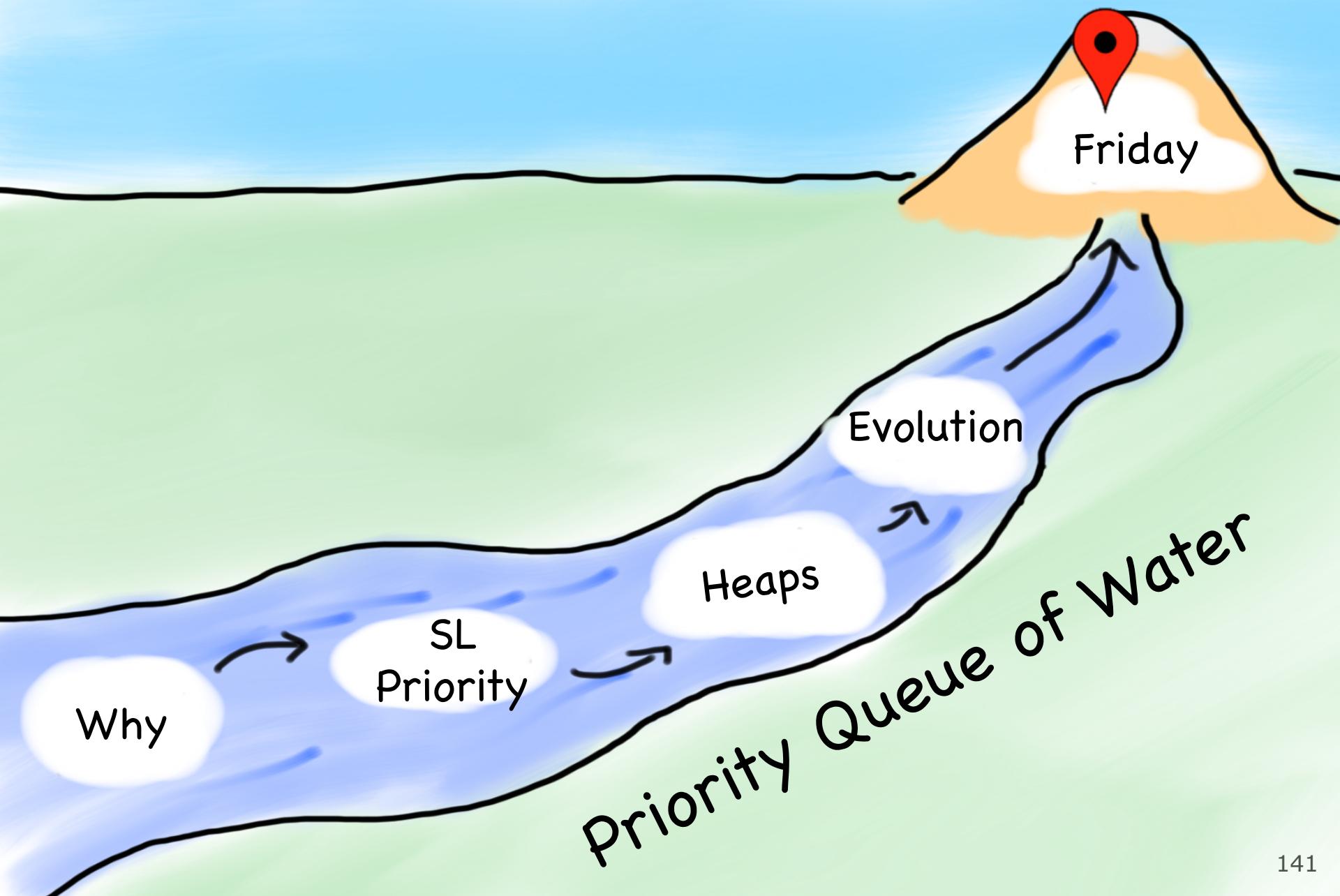
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Species * child

Today's Goals



Today's Goals



Today's Goals

1. Care About Priority Queues
2. Learn about Heaps

