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CS109

Section #4
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Section #4: Joint Random Variables

1. **Are we due for an earthquake?**: After the class where we talked about the probability of Earthquakes at Stanford, a student asked a question: "Doesn't the probability of an earthquake happening change based on the fact that we haven't had one for a while?" Let's explore! Recall the USGS rate of earthquakes of magnitude 8+ is λ =0.002 earthquakes per year.

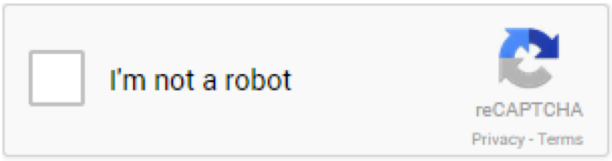
   a. What is the probability of no 8+ earthquakes in four years after the 1908 earthquake (recall that earthquakes are exponentially distributed)?

   b. What is the probability of no 8+ earthquakes in the 113 years between the 1908 earthquake and four years from now?

   c. What is the probability of no 8+ earthquakes in the 113 years between the 1908 earthquake and four years from now *given* that there have been no earthquakes in the last 109 years?

   d. Did you notice anything interesting? Would this work for any value of λ?

2. **ReCaptcha**: Based on browser history, Google believes that there is a 0.2 probability that a particular visitor to a website is a robot. They decide to give the visitor a recaptcha:



   Google presents the visitor with a box, 10 pixels wide by 10 pixels tall. The visitor must click inside the box to show that they are not a robot. You have observed that robots click uniformly in the box. However, the distance D of a human click from the center of the box, in pixels, is distributed by a *Rayliegh Distribution* with parameter $\theta = 2$. A Rayliegh random variable is parameterized by a single scale parameter θ and has the following probability density function and cumulative density function: [1]

$$f_X(x) = \begin{cases} \frac{x}{\theta} e^{-x^2/2\theta} & x \geq 0 \\ 0 & else \end{cases}$$

$$F_X(x) = \begin{cases} 1 - e^{-x^2/2\theta} & x \geq 0 \\ 0 & else \end{cases}$$

[1] Recaptcha uses more sophisticated statistics of natural human house gestures and clicks, but this problem covers the central idea behind the new click based recaptchas. It was also a midterm question last Spring.

a. What the the probability density function of a robot clicking $X = x$ pixels from the left of the box and $Y = y$ pixels from the top of the box?

b. What is the probability that a human clicks on a pixel that has a distance from the center of the box which is greater than or equal to 1.2 pixels?

c. The visitor clicks in the box at pixel ($x = 1.414$, $y = 1.414$) which has a distance of 2 pixels from the center. What is Google's new belief that the visitor is a robot?

3. It's Complicated

This probability table shows the joint distribution between two random variables: the year of the student at Stanford (Y) and their relationship status (R). The data was volunteered last year by over 200 anonymous students:

	Single	**In a Relationship**	**It's Complicated**
Freshman	0.12	0.07	0.02
Sophomore	0.17	0.12	0.02
Junior	0.10	0.11	0.02
Senior	0.01	0.07	0.00
5+	0.04	0.10	0.03

a. What is the marginal probability distribution for relationship status at Stanford (R)? Provide your result as a mapping between the values that R can take on and the corresponding probabilities.

b. What is the conditional probability of relationship status (R) given that a student is a Senior (Y = Senior)? Provide your result as a mapping between the values that R can take on and the corresponding probabilities.

c. What is the conditional probability that someone is "In a Relationship" given their year in school, $P(R = \text{In a Relationship}|Y)$? Give your answer as a mapping between the values that Y can take on and the corresponding probabilities.