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CS109

Section #3
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Section #3: Random Variables

1. **Website Visits**: On average, visitors leave your website after 5 minutes. Assume that the length of stay is exponentially distributed. What is the probability that a user stays more than 10 minutes?

2. **Approximating Normal**: Your website has 100 users and each day each user independently has a 20% chance of logging into your website. Use a normal approximation to estimate the probability that more than 21 users who log in.

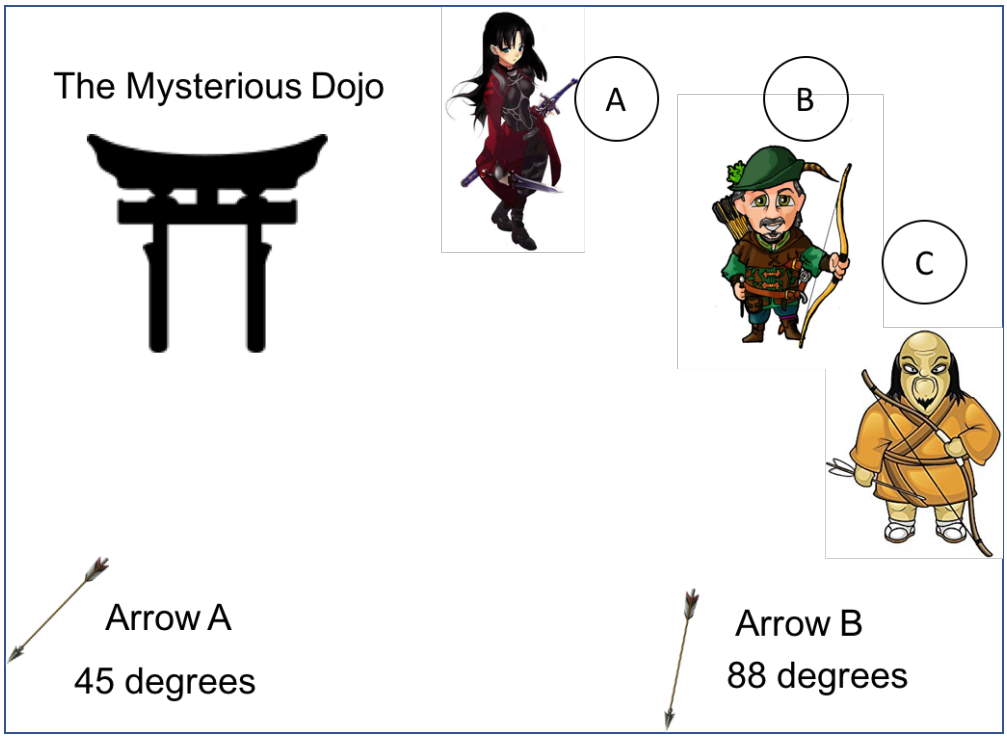
3. **Continuous Random Variable**: Let X be a continuous random variable with the following probability density function:

$$f_X(x) = \begin{cases} c(e^{x-1} + e^{-x}) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   a. Find the value of c that makes f_X a valid probability distribution.

   b. What is $P(X > 0.75)$?

4. **Who did it?**: You are on a forensic team investigating an attempted murder in an archery dojo. During the murder attempt several arrows were fired, but while we know the location of the people in the dojo, we don't know who did it! Two of the arrows were found in the Dojo wall at 45 degrees and 88 degrees angles (all angles are relative to the far wall). Can you figure out which of the three suspect best explains the data?



Suspect	Angle to Arrow A	Angle to Arrow B	Distance to Arrow A	Distance to Arrow B
Suspect A	60	94	18	10
Suspect B	50	86	20	8
Suspect C	44	84	22	6

Our arrow expert says that the angle of the arrow in the wall is normally distributed given the angle from the shooter to the target. Specifically, let A be the angle an arrow hits, let T be the angle between the shooter and the target and let D be the distance between the shooter and the target:

$$A \sim N\left(\mu = T, \sigma^2 = \frac{D}{2}\right)$$

Each arrow is independent. Which suspect makes the observed arrows the most likely?

Hint: Although the density of a continuous random variable is not a probability, we can multiply it by epsilon to get an approximate probability.