

- 1. Website Visits:** Let X be the number of minutes that a user stays. $X \sim \text{Exp}(\lambda = \frac{1}{5})$.

$$\begin{aligned} P(X > 10) &= 1 - F_X(10) \\ &= 1 - (1 - e^{\lambda 10}) = e^{-2} \approx 0.1353 \end{aligned}$$

- 2. Continuous Random Variable:** The number of users that log in B is binomial: $B \sim \text{Bin}(n = 100, p = 0.2)$. It can be approximated with a normal that matches the mean and variance. Let C be the normal that approximates B .

$$E[B] = np = 20.$$

$$\text{Var}(B) = np(1 - p) = 16$$

$$C \sim N(\mu = 20, \sigma^2 = 16).$$

$$\begin{aligned} P(B > 21) &\approx P(C > 20.5) \\ &= P\left(\frac{C - 20}{\sqrt{16}} > \frac{20.5 - 20}{\sqrt{16}}\right) \\ &= P(Z > 0.125) \\ &= 1 - P(Z < 0.125) \\ &= 1 - \phi(0.125) = 1 - 0.5478 = 0.4522 \end{aligned}$$

- 3. Continuous Random Variable:**

- a. We need $\int_{-\infty}^{\infty} dx f_X(x) = 1$.

$$\begin{aligned} 1 &= \int_{-\infty}^{\infty} f_X(x) dx = \int_0^1 c(e^{x-1} + e^{-x}) dx \\ &= c [e^{x-1} - e^{-x}]_{x=0}^1 \\ 1 &= c(e^{1-1} - e^{-1} - (e^{0-1} - e^{-0})) \\ c &= \frac{1}{1 - e^{-1} - (e^{-1} - 1)} = \frac{1}{2 - \frac{2}{e}} \end{aligned}$$

- b.

$$\begin{aligned} P(X > 0.75) &= \int_{0.75}^1 dx c(e^{x-1} + e^{-x}) \\ &= -c [e^{x-1} - e^{-x}]_{x=0.75}^1 \\ &= \boxed{-c (e^{1-1} - e^{-1} - (e^{0.75-1} - e^{-0.75}))} \\ &= -c (1 - e^{-1} - e^{-0.25} + e^{-0.75}) = \frac{1 - e^{-1} - e^{-0.25} + e^{-0.75}}{2 - \frac{2}{e}} \end{aligned}$$

4. Who did it?

We want to compare $P(\text{Arrows} | \text{Suspect A})$, $P(\text{Arrows} | \text{Suspect B})$, $P(\text{Arrows} | \text{Suspect C})$.
Let A_1 be the observation of arrow 1 and A_2 be the observation of arrow 2.

Suspect A

$$A_1 | \text{Suspect A} \sim N(\mu = 45, \sigma^2 = 9)$$

$$A_2 | \text{Suspect A} \sim N(\mu = 88, \sigma^2 = 5)$$

$$\begin{aligned} P(\text{Arrows} | \text{Suspect A}) &= P(A_1 | \text{Suspect A})P(A_2 | \text{Suspect A}) \\ &= \epsilon \cdot f_{A_1}(60) \cdot \epsilon \cdot f_{A_2}(94) \\ &= \epsilon^2 \cdot \frac{1}{\sqrt{2\pi \cdot 9}} e^{-\frac{(60-45)^2}{2 \cdot 9}} \frac{1}{\sqrt{2\pi \cdot 5}} e^{-\frac{(94-88)^2}{2 \cdot 5}} \\ &\approx \epsilon^2 \cdot 0 \end{aligned}$$

Suspect B

$$A_1 | \text{Suspect B} \sim N(\mu = 45, \sigma^2 = 10)$$

$$A_2 | \text{Suspect B} \sim N(\mu = 88, \sigma^2 = 4)$$

$$\begin{aligned} P(\text{Arrows} | \text{Suspect B}) &= P(A_1 | \text{Suspect B})P(A_2 | \text{Suspect B}) \\ &= \epsilon \cdot f_{A_1}(50) \cdot \epsilon \cdot f_{A_2}(86) \\ &= \epsilon^2 \cdot \frac{1}{\sqrt{2\pi \cdot 10}} e^{-\frac{(50-45)^2}{2 \cdot 10}} \frac{1}{\sqrt{2\pi \cdot 4}} e^{-\frac{(86-88)^2}{2 \cdot 4}} \\ &\approx \epsilon^2 \cdot 0.0044 \end{aligned}$$

Suspect C

$$A_1 | \text{Suspect C} \sim N(\mu = 45, \sigma^2 = 11)$$

$$A_2 | \text{Suspect C} \sim N(\mu = 88, \sigma^2 = 3)$$

$$\begin{aligned} P(\text{Arrows} | \text{Suspect C}) &= P(A_1 | \text{Suspect C})P(A_2 | \text{Suspect C}) \\ &= \epsilon \cdot f_{A_1}(44) \cdot \epsilon \cdot f_{A_2}(84) \\ &= \epsilon^2 \cdot \frac{1}{\sqrt{2\pi \cdot 11}} e^{-\frac{(44-45)^2}{2 \cdot 11}} \frac{1}{\sqrt{2\pi \cdot 3}} e^{-\frac{(84-88)^2}{2 \cdot 3}} \\ &\approx \epsilon^2 \cdot 0.0018 \end{aligned}$$

Suspect A certainly did not do it. The locations of the arrows are 2.3 times as likely assuming that Suspect B was the culprit than if Suspect C was the culprit.