100 Binomial Problems

Just for fun (and to give you a lot of practice) I wrote a generative probabilistic program which could sample binomial distribution problems. Here are 100 binomial questions:

Questions

Question 1: Laura is running a server cluster with 50 computers. The probability of a crash on a given server is 0.5. What is the standard deviation of crashes?

Answer 1:

Let X be the number of crashes. $X \sim \text{Bin}(n = 50, p = 0.5)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{50 \cdot 0.5 \cdot (1 - 0.5)}$$
$$= 3.54$$

Question 2: You are showing an online-ad to 30 people. The probability of an ad ignore on each ad shown is 2/3. What is the expected number of ad clicks?

Answer 2:

Let X be the number of ad clicks. $X \sim \text{Bin}(n = 30, p = 1/3)$

$$\text{E}[X] = np$$
$$= 30 \cdot 1/3$$
$$= 10$$

Question 3: A machine learning algorithm makes binary predictions. The machine learning algorithm makes 50 guesses where the probability of a incorrect prediction on a given guess is 19/25. What is the probability that the number of correct predictions is greater than 0?

Answer 3:

Let X be the number of correct predictions. $X \sim \text{Bin}(n = 50, p = 6/25)$

$$\text{P}(X > 0) = 1 - \text{P}(0 <= X <= 0)$$
$$= 1 - \binom{n}{0} p^0 (1-p)^{n-0}$$

Question 4: Wind blows independently across 50 locations. The probability of no wind at a given location is 0.5. What is the expected number of locations that have wind?

Answer 4:

Let X be the number of locations that have wind. $X \sim \text{Bin}(n = 50, p = 0.5)$

$$\text{E}[X] = np$$
$$= 50 \cdot 0.5$$
$$= 25.0$$

Question 5: Wind blows independently across 30 locations. What is the standard deviation of locations that have wind? the probability of wind at each location is 0.6.

Answer 5:

Let X be the number of locations that have wind. $X \sim \text{Bin}(n = 30, p = 0.6)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{30 \cdot 0.6 \cdot (1 - 0.6)}$$
$$= 2.68$$

Question 6: You are trying to mine bitcoins. There are 50 independent attempts where the probability of a mining a bitcoin on a given attempt is 0.6. What is the expectation of bitcoins mined?

Answer 6:

Let X be the number of bitcoins mined. $X \sim \text{Bin}(n = 50, p = 0.6)$

$$\text{E}[X] = np$$
$$= 50 \cdot 0.6$$
$$= 30.0$$

Question 7: You are testing a new medicine on 40 patients. What is P(X is exactly 38)? The number of cured patients can be represented by a random variable X. $X \sim \text{Bin}(40, 3/10)$.

Answer 7:

Let X be the number of cured patients. $X \sim \text{Bin}(n = 40, p = 3/10)$

$$P(X = 38) = \binom{n}{38} p^{38} (1-p)^{n-38}$$
$$= \binom{40}{38} 3/10^{38} (1 - 3/10)^{40-38}$$
$$< 0.00001$$

Question 8: You are manufacturing chips and are testing for defects. There are 50 independent tests and 0.5 is the probability of a defect on each test. What is the standard deviation of defects?

Answer 8:

Let X be the number of defects. $X \sim \text{Bin}(n = 50, p = 0.5)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{50 \cdot 0.5 \cdot (1 - 0.5)}$$
$$= 3.54$$

Question 9: Laura is flipping a coin 12 times. The probability of a tail on a given coin-flip is 5/12. What is the probability that the number of tails is greater than or equal to 2?

Answer 9:

Let X be the number of tails. $X \sim \text{Bin}(n = 12, p = 5/12)$

$$P(X >= 2) = 1 - P(0 <= X <= 1)$$
$$= 1 - \sum_{i=0}^{1} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 10: You are asking a survey question where responses are "like" or "dislike". There are 30 responses. You can assume each response is independent where the probability of a dislike on a given response is 1/6. What is the probability that the number of likes is greater than 28?

Answer 10:

Let X be the number of likes. $X \sim \text{Bin}(n = 30, p = 5/6)$

$$P(X > 28) = P(29 <= X <= 30)$$
$$= \sum_{i=29}^{30} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 11: A ball hits a series of 50 pins where it can bounce either right or left. The probability of a left on a given pin hit is 0.4. What is the standard deviation of rights?

Answer 11:

Let X be the number of rights. $X \sim \text{Bin}(n = 50, p = 3/5)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{50 \cdot 3/5 \cdot (1 - 3/5)}$$
$$= 3.46$$

Question 12: You are sending a stream of 30 bits to space. The probability of a no corruption on a given bit is 1/3. What is the probability that the number of corruptions is 10?

Answer 12:

Let X be the number of corruptions. $X \sim \text{Bin}(n = 30, p = 2/3)$

$$P(X = 10) = \binom{n}{10} p^{10} (1-p)^{n-10}$$
$$= \binom{30}{10} 2/3^{10} (1 - 2/3)^{30-10}$$
$$= 0.00015$$

Question 13: Wind blows independently across locations. The probability of wind at a given location is 0.9. The number of independent locations is 20. What is the probability that the number of locations that have wind is not less than 19?

Answer 13:

Let X be the number of locations that have wind. $X \sim \text{Bin}(n = 20, p = 0.9)$

$$P(X >= 19) = P(19 <= X <= 20)$$
$$= \sum_{i=19}^{20} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 14: You are sending a stream of bits to space. There are 30 independent bits where 5/6 is the probability of a no corruption on each bit. What is the probability that the number of corruptions is 21?

Answer 14:

Let X be the number of corruptions. $X \sim \text{Bin}(n = 30, p = 1/6)$

$$P(X = 21) = \binom{n}{21} p^{21} (1-p)^{n-21}$$
$$= \binom{30}{21} 1/6^{21} (1 - 1/6)^{30-21}$$
$$< 0.00001$$

Question 15: Cody generates random bit strings. There are 20 independent bits. Each bit has a 1/4 probability of resulting in a 1. What is the probability that the number of 1s is 11?

Answer 15:

Let X be the number of 1s. $X \sim \text{Bin}(n = 20, p = 1/4)$

$$
\begin{aligned}
P(X = 11) &= \binom{n}{11} p^{11} (1-p)^{n-11} \\
&= \binom{20}{11} 1/4^{11} (1 - 1/4)^{20-11} \\
&= 0.00301
\end{aligned}
$$

Question 16: In a restaurant some customers ask for a water with their meal. A random sample of 40 customers is selected where the probability of a water requested by a given customer is 9/20. What is the probability that the number of waters requested is 16?

Answer 16:

Let X be the number of waters requested. $X \sim \text{Bin}(n = 40, p = 9/20)$

$$
\begin{aligned}
P(X = 16) &= \binom{n}{16} p^{16} (1-p)^{n-16} \\
&= \binom{40}{16} 9/20^{16} (1 - 9/20)^{40-16} \\
&= 0.10433
\end{aligned}
$$

Question 17: A student is guessing randomly on an exam with 12 questions. What is the expected number of correct answers? the probability of a correct answer on a given question is 5/12.

Answer 17:

Let X be the number of correct answers. $X \sim \text{Bin}(n = 12, p = 5/12)$

$$
\begin{aligned}
E[X] &= np \\
&= 12 \cdot 5/12 \\
&= 5
\end{aligned}
$$

Question 18: Laura is trying to mine bitcoins. The number of bitcoins mined can be represented by a random variable X. X ~ Bin(n = 100, p = 1/2). What is P(X is equal to 53)?

Answer 18:

Let X be the number of bitcoins mined. $X \sim \text{Bin}(n = 100, p = 1/2)$

$$
\begin{aligned}
P(X = 53) &= \binom{n}{53} p^{53} (1-p)^{n-53} \\
&= \binom{100}{53} 1/2^{53} (1 - 1/2)^{100-53} \\
&= 0.06659
\end{aligned}
$$

Question 19: You are showing an online-ad to customers. The add is shown to 100 people. The probability of an ad ignore on a given ad shown is 1/2. What is the standard deviation of ad clicks?

Answer 19:

Let X be the number of ad clicks. $X \sim \text{Bin}(n = 100, p = 0.5)$

$$
\begin{aligned}
\text{Std}(X) &= \sqrt{np(1-p)} \\
&= \sqrt{100 \cdot 0.5 \cdot (1 - 0.5)} \\
&= 5.00
\end{aligned}
$$

Question 20: You are running a server cluster with 40 computers. 5/8 is the probability of a computer continuing to work on each server. What is the expected number of crashes?

Answer 20:
Let X be the number of crashes. $X \sim \text{Bin}(n = 40, p = 3/8)$

$$E[X] = np$$
$$= 40 \cdot 3/8$$
$$= 15$$

Question 21: You are hashing 100 strings into a hashtable. The probability of a hash to the first bucket on a given string hash is 3/20. What is the probability that the number of hashes to the first bucket is greater than or equal to 97?

Answer 21:
Let X be the number of hashes to the first bucket. $X \sim \text{Bin}(n = 100, p = 3/20)$

$$P(X >= 97) = P(97 <= X <= 100)$$
$$= \sum_{i=97}^{100} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 22: You are running in an election with 50 voters. 6/25 is the probability of a vote for you on each vote. What is the probability that the number of votes for you is less than 2?

Answer 22:
Let X be the number of votes for you. $X \sim \text{Bin}(n = 50, p = 6/25)$

$$P(X < 2) = P(0 <= X <= 1)$$
$$= \sum_{i=0}^{1} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 23: Irina is sending a stream of 40 bits to space. The probability of a corruption on each bit is 3/4. What is the probability that the number of corruptions is 22?

Answer 23:
Let X be the number of corruptions. $X \sim \text{Bin}(n = 40, p = 3/4)$

$$P(X = 22) = \binom{n}{22} p^{22} (1-p)^{n-22}$$
$$= \binom{40}{22} 3/4^{22} (1 - 3/4)^{40-22}$$
$$= 0.00294$$

Question 24: You are hashing 100 strings into a hashtable. The probability of a hash to the first bucket on a given string hash is 9/50. What is the probability that the number of hashes to the first bucket is greater than 97?

Answer 24:
Let X be the number of hashes to the first bucket. $X \sim \text{Bin}(n = 100, p = 9/50)$

$$P(X > 97) = P(98 <= X <= 100)$$
$$= \sum_{i=98}^{100} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 25: You generate random bit strings. There are 100 independent bits. The probability of a 1 at a given bit is 3/25. What is the probability that the number of 1s is less than 97?

Answer 25:

Let X be the number of 1s. $X \sim \text{Bin}(n = 100, p = 3/25)$

$$P(X < 97) = 1 - P(97 <= X <= 100)$$
$$= 1 - \sum_{i=97}^{100} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 26: You are manufacturing toys and are testing for defects. What is the probability that the number of defects is greater than 1? the probability of a non-defect on a given test is 16/25 and you test 50 objects.

Answer 26:

Let X be the number of defects. $X \sim \text{Bin}(n = 50, p = 9/25)$

$$P(X > 1) = 1 - P(0 <= X <= 1)$$
$$= 1 - \sum_{i=0}^{1} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 27: Laura is sending a stream of 40 bits to space. The number of corruptions can be represented by a random variable X. X is a Binomial with n = 40 and p = 3/4. What is P(X = 25)?

Answer 27:

Let X be the number of corruptions. $X \sim \text{Bin}(n = 40, p = 3/4)$

$$P(X = 25) = \binom{n}{25} p^{25} (1-p)^{n-25}$$
$$= \binom{40}{25} 3/4^{25} (1 - 3/4)^{40-25}$$
$$= 0.02819$$

Question 28: 100 trials are run. What is the probability that the number of successes is 78? 1/2 is the probability of a success on each trial.

Answer 28:

Let X be the number of successes. $X \sim \text{Bin}(n = 100, p = 1/2)$

$$P(X = 78) = \binom{n}{78} p^{78} (1-p)^{n-78}$$
$$= \binom{100}{78} 1/2^{78} (1 - 1/2)^{100-78}$$
$$< 0.00001$$

Question 29: You are flipping a coin. You flip the coin 20 times. The probability of a tail on a given coin-flip is 1/10. What is the standard deviation of heads?

Answer 29:

Let X be the number of heads. $X \sim \text{Bin}(n = 20, p = 0.9)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{20 \cdot 0.9 \cdot (1 - 0.9)}$$
$$= 1.34$$

Question 30: Irina is showing an online-ad to 12 people. 5/12 is the probability of an ad click on each ad shown. What is the probability that the number of ad clicks is less than or equal to 11?

Answer 30:

Let X be the number of ad clicks. $X \sim \text{Bin}(n = 12, p = 5/12)$

$$P(X <= 11) = 1 - P(12 <= X <= 12)$$
$$= 1 - \binom{n}{12} p^1 2 (1-p)^{n-12}$$

Question 31: You are flipping a coin 50 times. 19/25 is the probability of a head on each coin-flip. What is the standard deviation of tails?

Answer 31:

Let X be the number of tails. $X \sim \text{Bin}(n = 50, p = 6/25)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{50 \cdot 6/25 \cdot (1 - 6/25)}$$
$$= 3.02$$

Question 32: You are running in an election with 100 voters. The probability of a vote for you on each vote is 1/4. What is the probability that the number of votes for you is less than or equal to 97?

Answer 32:

Let X be the number of votes for you. $X \sim \text{Bin}(n = 100, p = 1/4)$

$$P(X <= 97) = 1 - P(98 <= X <= 100)$$
$$= 1 - \sum_{i=98}^{100} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 33: You are running a server cluster with 40 computers. What is the probability that the number of crashes is less than or equal to 39? 3/4 is the probability of a computer continuing to work on each server.

Answer 33:

Let X be the number of crashes. $X \sim \text{Bin}(n = 40, p = 1/4)$

$$P(X <= 39) = 1 - P(40 <= X <= 40)$$
$$= 1 - \binom{n}{40} p^4 0 (1-p)^{n-40}$$

Question 34: Waddie is sending a stream of bits to space. Waddie sends 100 bits. The probability of a corruption on each bit is 1/2. What is the standard deviation of corruptions?

Answer 34:

Let X be the number of corruptions. $X \sim \text{Bin}(n = 100, p = 1/2)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{100 \cdot 1/2 \cdot (1 - 1/2)}$$
$$= 5.00$$

Question 35: A student is guessing randomly on an exam with 100 questions. Each question has a 0.5 probability of resulting in a incorrect answer. What is the probability that the number of correct answers is greater than 97?

Answer 35:

Let X be the number of correct answers. $X \sim \text{Bin}(n = 100, p = 1/2)$

$$P(X > 97) = P(98 <= X <= 100)$$
$$= \sum_{i=98}^{100} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 36: You are testing a new medicine on patients. 0.5 is the probability of a cured patient on each trial. There are 10 independent trials. What is the expected number of cured patients?

Answer 36:

Let X be the number of cured patients. $X \sim \text{Bin}(n = 10, p = 0.5)$

$$E[X] = np$$
$$= 10 \cdot 0.5$$
$$= 5.0$$

Question 37: A ball hits a series of pins where it can either go right or left. The number of independent pin hits is 100. The probability of a right on each pin hit is 0.5. What is the standard deviation of rights?

Answer 37:

Let X be the number of rights. $X \sim \text{Bin}(n = 100, p = 0.5)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{100 \cdot 0.5 \cdot (1 - 0.5)}$$
$$= 5.00$$

Question 38: You are flipping a coin 40 times. The probability of a head on a given coin-flip is 1/2. What is the probability that the number of heads is 38?

Answer 38:

Let X be the number of heads. $X \sim \text{Bin}(n = 40, p = 1/2)$

$$P(X = 38) = \binom{n}{38} p^{38} (1-p)^{n-38}$$
$$= \binom{40}{38} 1/2^{38} (1 - 1/2)^{40-38}$$
$$< 0.00001$$

Question 39: 100 trials are run and the probability of a success on a given trial is 1/2. What is the standard deviation of successes?

Answer 39:

Let X be the number of successes. $X \sim \text{Bin}(n = 100, p = 1/2)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{100 \cdot 1/2 \cdot (1 - 1/2)}$$
$$= 5.00$$

Question 40: You are trying to mine bitcoins. There are 40 independent attempts. The probability of mining a bitcoin on each attempt is 3/10. What is the probability that the number of bitcoins mined is 19?

Answer 40:

Let X be the number of bitcoins mined. $X \sim \text{Bin}(n = 40, p = 3/10)$

$$
\begin{aligned}
P(X = 19) &= \binom{n}{19} p^{19}(1-p)^{n-19} \\
&= \binom{40}{19} 3/10^{19}(1 - 3/10)^{40-19} \\
&= 0.00852
\end{aligned}
$$

Question 41: 20 trials are run. 0.5 is the probability of a failure on each trial. What is the probability that the number of successes is 6?

Answer 41:

Let X be the number of successes. $X \sim \text{Bin}(n = 20, p = 0.5)$

$$
\begin{aligned}
P(X = 6) &= \binom{n}{6} p^{6}(1-p)^{n-6} \\
&= \binom{20}{6} 0.5^{6}(1 - 0.5)^{20-6} \\
&= 0.03696
\end{aligned}
$$

Question 42: You are flipping a coin. What is the probability that the number of tails is 0? there are 30 independent coin-flips where the probability of a head on a given coin-flip is 5/6.

Answer 42:

Let X be the number of tails. $X \sim \text{Bin}(n = 30, p = 1/6)$

$$
\begin{aligned}
P(X = 0) &= \binom{n}{0} p^{0}(1-p)^{n-0} \\
&= \binom{30}{0} 1/6^{0}(1 - 1/6)^{30-0} \\
&= 0.00421
\end{aligned}
$$

Question 43: In a restaurant some customers ask for a water with their meal. A random sample of 20 customers is selected and each customer has a 1/4 probability of resulting in a water not requested. What is the probability that the number of waters requested is 14?

Answer 43:

Let X be the number of waters requested. $X \sim \text{Bin}(n = 20, p = 3/4)$

$$
\begin{aligned}
P(X = 14) &= \binom{n}{14} p^{14}(1-p)^{n-14} \\
&= \binom{20}{14} 3/4^{14}(1 - 3/4)^{20-14} \\
&= 0.16861
\end{aligned}
$$

Question 44: A student is guessing randomly on an exam. 3/8 is the probability of a incorrect answer on each question. The number of independent questions is 40. What is the probability that the number of correct answers is less than or equal to 37?

Answer 44:

Let X be the number of correct answers. $X \sim \text{Bin}(n = 40, p = 5/8)$

$$
\begin{aligned}
P(X <= 37) &= 1 - P(38 <= X <= 40) \\
&= 1 - \sum_{i=38}^{40} \binom{n}{i} p^{i}(1-p)^{n-i}
\end{aligned}
$$

Question 45: You are running in an election with 30 voters. 3/5 is the probability of a vote for you on each vote. What is the standard deviation of votes for you?

Answer 45:

Let X be the number of votes for you. $X \sim \text{Bin}(n = 30, p = 3/5)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{30 \cdot 3/5 \cdot (1 - 3/5)}$$
$$= 2.68$$

Question 46: Charlotte is flipping a coin 100 times. The probability of a tail on each coin-flip is 0.5. What is the probability that the number of tails is greater than 2?

Answer 46:

Let X be the number of tails. $X \sim \text{Bin}(n = 100, p = 0.5)$

$$P(X > 2) = 1 - P(0 <= X <= 2)$$
$$= 1 - \sum_{i=0}^{2} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 47: You are trying to mine bitcoins. You try 50 times. 3/5 is the probability of a not mining a bitcoin on each attempt. What is the probability that the number of bitcoins mined is 14?

Answer 47:

Let X be the number of bitcoins mined. $X \sim \text{Bin}(n = 50, p = 2/5)$

$$P(X = 14) = \binom{n}{14} p^{14} (1-p)^{n-14}$$
$$= \binom{50}{14} 2/5^{14} (1 - 2/5)^{50-14}$$
$$= 0.02597$$

Question 48: You are testing a new medicine on 100 patients. The probability of a cured patient on a given trial is 3/25. What is the probability that the number of cured patients is not less than 97?

Answer 48:

Let X be the number of cured patients. $X \sim \text{Bin}(n = 100, p = 3/25)$

$$P(X >= 97) = P(97 <= X <= 100)$$
$$= \sum_{i=97}^{100} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 49: Wind blows independently across 40 locations. What is the probability that the number of locations that have wind is 40? 11/20 is the probability of no wind at each location.

Answer 49:

Let X be the number of locations that have wind. $X \sim \text{Bin}(n = 40, p = 9/20)$

$$P(X = 40) = \binom{n}{40} p^{40} (1-p)^{n-40}$$
$$= \binom{40}{40} 9/20^{40} (1 - 9/20)^{40-40}$$
$$< 0.00001$$

Question 50: You are showing an online-ad to 30 people. 1/6 is the probability of an ad click on each ad shown. What is the probability that the number of ad clicks is less than or equal to 28?

Answer 50:

Let X be the number of ad clicks. $X \sim \text{Bin}(n = 30, p = 1/6)$

$$P(X <= 28) = 1 - P(29 <= X <= 30)$$
$$= 1 - \sum_{i=29}^{30} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 51: You are flipping a coin. You flip the coin 40 times and 7/8 is the probability of a head on each coin-flip. What is the standard deviation of tails?

Answer 51:

Let X be the number of tails. $X \sim \text{Bin}(n = 40, p = 1/8)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{40 \cdot 1/8 \cdot (1 - 1/8)}$$
$$= 2.09$$

Question 52: Cody is sending a stream of bits to space. 2/5 is the probability of a no corruption on each bit and there are 20 independent bits. What is the expectation of corruptions?

Answer 52:

Let X be the number of corruptions. $X \sim \text{Bin}(n = 20, p = 3/5)$

$$E[X] = np$$
$$= 20 \cdot 3/5$$
$$= 12$$

Question 53: You are running in an election. There are 12 independent votes and 5/6 is the probability of a vote for you on each vote. What is the probability that the number of votes for you is greater than or equal to 9?

Answer 53:

Let X be the number of votes for you. $X \sim \text{Bin}(n = 12, p = 5/6)$

$$P(X >= 9) = P(9 <= X <= 12)$$
$$= \sum_{i=9}^{12} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 54: You are flipping a coin. The number of tails can be represented by a random variable X. X is a Bin(n = 30, p = 5/6). What is the probability that X = 1?

Answer 54:

Let X be the number of tails. $X \sim \text{Bin}(n = 30, p = 5/6)$

$$P(X = 1) = \binom{n}{1} p^1 (1-p)^{n-1}$$
$$= \binom{30}{1} 5/6^1 (1 - 5/6)^{30-1}$$
$$< 0.00001$$

Question 55: In a restaurant some customers ask for a water with their meal. A random sample of 100 customers is selected where 0.3 is the probability of a water requested by each customer. What is the expected number of waters requested?

Answer 55:

Let X be the number of waters requested. $X \sim \text{Bin}(n = 100, p = 0.3)$

$$\begin{aligned} \text{E}[X] &= np \\ &= 100 \cdot 0.3 \\ &= 30.0 \end{aligned}$$

Question 56: You are hashing strings into a hashtable. 30 strings are hashed. The probability of a hash to the first bucket on each string hash is 1/6. What is the expected number of hashes to the first bucket?

Answer 56:

Let X be the number of hashes to the first bucket. $X \sim \text{Bin}(n = 30, p = 1/6)$

$$\begin{aligned} \text{E}[X] &= np \\ &= 30 \cdot 1/6 \\ &= 5 \end{aligned}$$

Question 57: You are flipping a coin 100 times. What is the probability that the number of tails is greater than or equal to 98? 19/20 is the probability of a head on each coin-flip.

Answer 57:

Let X be the number of tails. $X \sim \text{Bin}(n = 100, p = 1/20)$

$$\begin{aligned} \text{P}(X >= 98) &= P(98 <= X <= 100) \\ &= \sum_{i=98}^{100} \binom{n}{i} p^i (1-p)^{n-i} \end{aligned}$$

Question 58: Irina is running a server cluster. What is the probability that the number of crashes is less than 99? the server has 100 computers which crash independently and the probability of a computer continuing to work on a given server is 22/25.

Answer 58:

Let X be the number of crashes. $X \sim \text{Bin}(n = 100, p = 3/25)$

$$\begin{aligned} \text{P}(X < 99) &= 1 - \text{P}(99 <= X <= 100) \\ &= 1 - \sum_{i=99}^{100} \binom{n}{i} p^i (1-p)^{n-i} \end{aligned}$$

Question 59: You are manufacturing chairs and are testing for defects. You test 100 objects. 1/2 is the probability of a non-defect on each test. What is the probability that the number of defects is not greater than 97?

Answer 59:

Let X be the number of defects. $X \sim \text{Bin}(n = 100, p = 1/2)$

$$\begin{aligned} \text{P}(X <= 97) &= 1 - \text{P}(98 <= X <= 100) \\ &= 1 - \sum_{i=98}^{100} \binom{n}{i} p^i (1-p)^{n-i} \end{aligned}$$

Question 60: In a restaurant some customers ask for a water with their meal. There are 50 customers. You can assume each customer is independent. 0.2 is the probability of a water requested by each customer. What is the expected number of waters requested?

Answer 60:

Let X be the number of waters requested. $X \sim \text{Bin}(n = 50, p = 0.2)$

$$\begin{aligned} \mathrm{E}[X] &= np \\ &= 50 \cdot 0.2 \\ &= 10.0 \end{aligned}$$

Question 61: You are showing an online-ad to 40 people. 1/4 is the probability of an ad ignore on each ad shown. What is the probability that the number of ad clicks is 9?

Answer 61:

Let X be the number of ad clicks. $X \sim \text{Bin}(n = 40, p = 3/4)$

$$\begin{aligned} \mathrm{P}(X = 9) &= \binom{n}{9} p^9 (1 - p)^{n-9} \\ &= \binom{40}{9} 3/4^9 (1 - 3/4)^{40-9} \\ &< 0.00001 \end{aligned}$$

Question 62: 100 trials are run. Each trial has a 22/25 probability of resulting in a failure. What is the standard deviation of successes?

Answer 62:

Let X be the number of successes. $X \sim \text{Bin}(n = 100, p = 3/25)$

$$\begin{aligned} \mathrm{Std}(X) &= \sqrt{np(1 - p)} \\ &= \sqrt{100 \cdot 3/25 \cdot (1 - 3/25)} \\ &= 3.25 \end{aligned}$$

Question 63: A machine learning algorithm makes binary predictions. There are 12 independent guesses where the probability of a incorrect prediction on a given guess is 1/6. What is the expected number of correct predictions?

Answer 63:

Let X be the number of correct predictions. $X \sim \text{Bin}(n = 12, p = 5/6)$

$$\begin{aligned} \mathrm{E}[X] &= np \\ &= 12 \cdot 5/6 \\ &= 10 \end{aligned}$$

Question 64: Waddie is showing an online-ad to customers. 1/2 is the probability of an ad click on each ad shown. The add is shown to 100 people. What is the average number of ad clicks?

Answer 64:

Let X be the number of ad clicks. $X \sim \text{Bin}(n = 100, p = 1/2)$

$$\begin{aligned} \mathrm{E}[X] &= np \\ &= 100 \cdot 1/2 \\ &= 50 \end{aligned}$$

Question 65: Charlotte is testing a new medicine on 50 patients. The probability of a cured patient on a given trial is 1/5. What is the probability that the number of cured patients is 12?

Answer 65:

Let X be the number of cured patients. $X \sim \text{Bin}(n = 50, p = 1/5)$

$$P(X = 12) = \binom{n}{12} p^{12} (1-p)^{n-12}$$
$$= \binom{50}{12} 1/5^{12} (1 - 1/5)^{50-12}$$
$$= 0.10328$$

Question 66: You are running in an election. The number of votes for you can be represented by a random variable X. X is a Bin(n = 50, p = 0.4). What is P(X is exactly 8)?

Answer 66:

Let X be the number of votes for you. $X \sim \text{Bin}(n = 50, p = 0.4)$

$$P(X = 8) = \binom{n}{8} p^{8} (1-p)^{n-8}$$
$$= \binom{50}{8} 0.4^{8} (1 - 0.4)^{50-8}$$
$$= 0.00017$$

Question 67: Irina is flipping a coin 100 times. The probability of a head on a given coin-flip is 1/2. What is the probability that the number of tails is less than or equal to 99?

Answer 67:

Let X be the number of tails. $X \sim \text{Bin}(n = 100, p = 0.5)$

$$P(X <= 99) = 1 - P(100 <= X <= 100)$$
$$= 1 - \binom{n}{100} p^{1}00 (1-p)^{n-100}$$

Question 68: You are manufacturing airplanes and are testing for defects. You test 30 objects and the probability of a defect on a given test is 5/6. What is the probability that the number of defects is 14?

Answer 68:

Let X be the number of defects. $X \sim \text{Bin}(n = 30, p = 5/6)$

$$P(X = 14) = \binom{n}{14} p^{14} (1-p)^{n-14}$$
$$= \binom{30}{14} 5/6^{14} (1 - 5/6)^{30-14}$$
$$< 0.00001$$

Question 69: You are flipping a coin 20 times. The number of heads can be represented by a random variable X. X is a Binomial with 20 trials. Each trial is a success, independently, with probability 1/4. What is the standard deviation of X?

Answer 69:

Let X be the number of heads. $X \sim \text{Bin}(n = 20, p = 1/4)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{20 \cdot 1/4 \cdot (1 - 1/4)}$$
$$= 1.94$$

Question 70: You are giving a survey question where responses are "like" or "dislike" to 100 people. What is the probability that X is equal to 4? The number of likes can be represented by a random variable X. X is a Bin(100, 0.5).

Answer 70:

Let X be the number of likes. $X \sim \text{Bin}(n = 100, p = 0.5)$

$$\begin{aligned} \text{P}(X = 4) &= \binom{n}{4} p^4 (1-p)^{n-4} \\ &= \binom{100}{4} 0.5^4 (1 - 0.5)^{100-4} \\ &< 0.00001 \end{aligned}$$

Question 71: You are flipping a coin. There are 20 independent coin-flips where the probability of a tail on a given coin-flip is 0.9. What is the standard deviation of tails?

Answer 71:

Let X be the number of tails. $X \sim \text{Bin}(n = 20, p = 0.9)$

$$\begin{aligned} \text{Std}(X) &= \sqrt{np(1-p)} \\ &= \sqrt{20 \cdot 0.9 \cdot (1 - 0.9)} \\ &= 1.34 \end{aligned}$$

Question 72: You are flipping a coin. There are 50 independent coin-flips. The probability of a tail on a given coin-flip is 4/5. What is the expectation of heads?

Answer 72:

Let X be the number of heads. $X \sim \text{Bin}(n = 50, p = 1/5)$

$$\begin{aligned} \text{E}[X] &= np \\ &= 50 \cdot 1/5 \\ &= 10 \end{aligned}$$

Question 73: You are giving a survey question where responses are "like" or "dislike" to 100 people. What is the standard deviation of likes? the probability of a dislike on each response is 41/50.

Answer 73:

Let X be the number of likes. $X \sim \text{Bin}(n = 100, p = 9/50)$

$$\begin{aligned} \text{Std}(X) &= \sqrt{np(1-p)} \\ &= \sqrt{100 \cdot 9/50 \cdot (1 - 9/50)} \\ &= 3.84 \end{aligned}$$

Question 74: In a restaurant some customers ask for a water with their meal. 0.6 is the probability of a water requested by each customer and there are 30 independent customers. What is the expected number of waters requested?

Answer 74:

Let X be the number of waters requested. $X \sim \text{Bin}(n = 30, p = 0.6)$

$$\begin{aligned} \text{E}[X] &= np \\ &= 30 \cdot 0.6 \\ &= 18.0 \end{aligned}$$

Question 75: There are 40 independent trials and 0.5 is the probability of a failure on each trial. What is the expectation of successes?

Answer 75:

Let X be the number of successes. $X \sim \text{Bin}(n = 40, p = 1/2)$

$$
\begin{aligned}
\mathrm{E}[X] &= np \\
&= 40 \cdot 1/2 \\
&= 20
\end{aligned}
$$

Question 76: Imran is showing an online-ad to 30 people. 5/6 is the probability of an ad click on each ad shown. What is the standard deviation of ad clicks?

Answer 76:

Let X be the number of ad clicks. $X \sim \text{Bin}(n = 30, p = 5/6)$

$$
\begin{aligned}
\mathrm{Std}(X) &= \sqrt{np(1-p)} \\
&= \sqrt{30 \cdot 5/6 \cdot (1 - 5/6)} \\
&= 2.04
\end{aligned}
$$

Question 77: You are running a server cluster. What is the probability that the number of crashes is 1? the server has 30 computers which crash independently and each server has a 1/3 probability of resulting in a crash.

Answer 77:

Let X be the number of crashes. $X \sim \text{Bin}(n = 30, p = 1/3)$

$$
\begin{aligned}
\mathrm{P}(X = 1) &= \binom{n}{1} p^1 (1-p)^{n-1} \\
&= \binom{30}{1} 1/3^1 (1 - 1/3)^{30-1} \\
&= 0.00008
\end{aligned}
$$

Question 78: Cody is running a server cluster with 40 computers. What is P(X <= 39)? The number of crashes can be represented by a random variable X. X is a Bin(n = 40, p = 3/4).

Answer 78:

Let X be the number of crashes. $X \sim \text{Bin}(n = 40, p = 3/4)$

$$
\begin{aligned}
\mathrm{P}(X <= 39) &= 1 - \mathrm{P}(40 <= X <= 40) \\
&= 1 - \binom{n}{40} p^4 0 (1-p)^{n-40}
\end{aligned}
$$

Question 79: You are hashing strings into a hashtable. 5/6 is the probability of a hash to the first bucket on each string hash. There are 30 independent string hashes. What is the probability that the number of hashes to the first bucket is greater than or equal to 29?

Answer 79:

Let X be the number of hashes to the first bucket. $X \sim \text{Bin}(n = 30, p = 5/6)$

$$
\begin{aligned}
\mathrm{P}(X >= 29) &= P(29 <= X <= 30) \\
&= \sum_{i=29}^{30} \binom{n}{i} p^i (1-p)^{n-i}
\end{aligned}
$$

Question 80: Irina is flipping a coin. Irina flips the coin 30 times and the probability of a head on each coin-flip is 0.4. What is the probability that the number of tails is 19?

Answer 80:
Let X be the number of tails. $X \sim \text{Bin}(n = 30, p = 0.6)$

$$P(X = 19) = \binom{n}{19} p^{19}(1-p)^{n-19}$$
$$= \binom{30}{19} 0.6^{19}(1-0.6)^{30-19}$$
$$= 0.13962$$

Question 81: You are asking a survey question where responses are "like" or "dislike". The probability of a like on a given response is 1/2. You give the survey to 100 people. What is the probability that the number of likes is not less than 2?

Answer 81:
Let X be the number of likes. $X \sim \text{Bin}(n = 100, p = 1/2)$

$$P(X >= 2) = 1 - P(0 <= X <= 1)$$
$$= 1 - \sum_{i=0}^{1} \binom{n}{i} p^i (1-p)^{n-i}$$

Question 82: Wind blows independently across locations. The number of independent locations is 100. The probability of wind at a given location is 3/20. What is the probability that the number of locations that have wind is 93?

Answer 82:
Let X be the number of locations that have wind. $X \sim \text{Bin}(n = 100, p = 3/20)$

$$P(X = 93) = \binom{n}{93} p^{93}(1-p)^{n-93}$$
$$= \binom{100}{93} 3/20^{93}(1-3/20)^{100-93}$$
$$< 0.00001$$

Question 83: You are flipping a coin. 0.9 is the probability of a tail on each coin-flip. You flip the coin 50 times. What is the expected number of heads?

Answer 83:
Let X be the number of heads. $X \sim \text{Bin}(n = 50, p = 0.1)$

$$E[X] = np$$
$$= 50 \cdot 0.1$$
$$= 5.0$$

Question 84: A machine learning algorithm makes binary predictions. What is the probability that the number of correct predictions is less than or equal to 0? the probability of a incorrect prediction on a given guess is 1/4. The number of independent guesses is 40.

Answer 84:
Let X be the number of correct predictions. $X \sim \text{Bin}(n = 40, p = 3/4)$

$$P(X <= 0) = P(0 <= X <= 0)$$
$$= \binom{n}{0} p^0 (1-p)^{n-0}$$

Question 85: Wind blows independently across 20 locations. 1/2 is the probability of wind at each location. What is the standard deviation of locations that have wind?

Answer 85:

Let X be the number of locations that have wind. $X \sim \text{Bin}(n = 20, p = 1/2)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{20 \cdot 1/2 \cdot (1 - 1/2)}$$
$$= 2.24$$

Question 86: 7/10 is the probability of a failure on each trial and the number of independent trials is 100. What is the probability that the number of successes is 7?

Answer 86:

Let X be the number of successes. $X \sim \text{Bin}(n = 100, p = 0.3)$

$$P(X = 7) = \binom{n}{7}p^7(1-p)^{n-7}$$
$$= \binom{100}{7}0.3^7(1-0.3)^{100-7}$$
$$< 0.00001$$

Question 87: You generate random bit strings. What is the expectation of 1s? there are 100 independent bits and 0.1 is the probability of a 1 at each bit.

Answer 87:

Let X be the number of 1s. $X \sim \text{Bin}(n = 100, p = 0.1)$

$$E[X] = np$$
$$= 100 \cdot 0.1$$
$$= 10.0$$

Question 88: You are testing a new medicine on patients. 3/5 is the probability of a cured patient on each trial. There are 30 independent trials. What is the probability that the number of cured patients is greater than or equal to 1?

Answer 88:

Let X be the number of cured patients. $X \sim \text{Bin}(n = 30, p = 3/5)$

$$P(X >= 1) = 1 - P(0 <= X <= 0)$$
$$= 1 - \binom{n}{0}p^0(1-p)^{n-0}$$

Question 89: A student is guessing randomly on an exam. 0.9 is the probability of a correct answer on each question and the test has 20 questions. What is the standard deviation of correct answers?

Answer 89:

Let X be the number of correct answers. $X \sim \text{Bin}(n = 20, p = 0.9)$

$$\text{Std}(X) = \sqrt{np(1-p)}$$
$$= \sqrt{20 \cdot 0.9 \cdot (1 - 0.9)}$$
$$= 1.34$$

Question 90: A student is guessing randomly on an exam with 40 questions. What is the probability that the number of correct answers is 32? 0.5 is the probability of a correct answer on each question.

Answer 90:

Let X be the number of correct answers. $X \sim \text{Bin}(n = 40, p = 0.5)$

$$
\begin{aligned}
P(X = 32) &= \binom{n}{32} p^{32}(1-p)^{n-32} \\
&= \binom{40}{32} 0.5^{32}(1-0.5)^{40-32} \\
&= 0.00007
\end{aligned}
$$

Question 91: In a restaurant some customers ask for a water with their meal. A random sample of 40 customers is selected where the probability of a water not requested by a given customer is 1/4. What is the standard deviation of waters requested?

Answer 91:

Let X be the number of waters requested. $X \sim \text{Bin}(n = 40, p = 3/4)$

$$
\begin{aligned}
\text{Std}(X) &= \sqrt{np(1-p)} \\
&= \sqrt{40 \cdot 3/4 \cdot (1 - 3/4)} \\
&= 2.74
\end{aligned}
$$

Question 92: A machine learning algorithm makes binary predictions. The number of correct predictions can be represented by a random variable X. X is a Bin(n = 30, p = 2/5). What is P(X < 27)?

Answer 92:

Let X be the number of correct predictions. $X \sim \text{Bin}(n = 30, p = 2/5)$

$$
\begin{aligned}
P(X < 27) &= 1 - P(27 <= X <= 30) \\
&= 1 - \sum_{i=27}^{30} \binom{n}{i} p^i (1-p)^{n-i}
\end{aligned}
$$

Question 93: Irina is flipping a coin. The probability of a tail on each coin-flip is 3/4. The number of independent coin-flips is 40. What is the probability that the number of tails is greater than 0?

Answer 93:

Let X be the number of tails. $X \sim \text{Bin}(n = 40, p = 3/4)$

$$
\begin{aligned}
P(X > 0) &= 1 - P(0 <= X <= 0) \\
&= 1 - \binom{n}{0} p^0 (1-p)^{n-0}
\end{aligned}
$$

Question 94: Waddie is sending a stream of 50 bits to space. The probability of a no corruption on a given bit is 1/2. What is the expectation of corruptions?

Answer 94:

Let X be the number of corruptions. $X \sim \text{Bin}(n = 50, p = 0.5)$

$$
\begin{aligned}
E[X] &= np \\
&= 50 \cdot 0.5 \\
&= 25.0
\end{aligned}
$$

Question 95: You are hashing strings into a hashtable. There are 30 independent string hashes where the probability of a hash to the first bucket on each string hash is 5/6. What is the probability that the number of hashes to the first bucket is 24?

Answer 95:

Let X be the number of hashes to the first bucket. $X \sim \text{Bin}(n = 30, p = 5/6)$

$$
\begin{aligned}
P(X = 24) &= \binom{n}{24} p^{24} (1-p)^{n-24} \\
&= \binom{30}{24} 5/6^{24} (1 - 5/6)^{30-24} \\
&= 0.16009
\end{aligned}
$$

Question 96: Charlotte is hashing strings into a hashtable. 100 strings are hashed and the probability of a hash to the first bucket on a given string hash is 1/5. What is the probability that the number of hashes to the first bucket is greater than or equal to 1?

Answer 96:

Let X be the number of hashes to the first bucket. $X \sim \text{Bin}(n = 100, p = 1/5)$

$$
\begin{aligned}
P(X >= 1) &= 1 - P(0 <= X <= 0) \\
&= 1 - \binom{n}{0} p^0 (1-p)^{n-0}
\end{aligned}
$$

Question 97: You are flipping a coin. Each coin-flip has a 3/10 probability of resulting in a head and there are 100 coin-flips. You can assume each coin-flip is independent. What is the probability that the number of heads is 0?

Answer 97:

Let X be the number of heads. $X \sim \text{Bin}(n = 100, p = 3/10)$

$$
\begin{aligned}
P(X = 0) &= \binom{n}{0} p^0 (1-p)^{n-0} \\
&= \binom{100}{0} 3/10^0 (1 - 3/10)^{100-0} \\
&< 0.00001
\end{aligned}
$$

Question 98: Chris is sending a stream of 50 bits to space. 16/25 is the probability of a no corruption on each bit. What is the probability that the number of corruptions is greater than or equal to 47?

Answer 98:

Let X be the number of corruptions. $X \sim \text{Bin}(n = 50, p = 9/25)$

$$
\begin{aligned}
P(X >= 47) &= P(47 <= X <= 50) \\
&= \sum_{i=47}^{50} \binom{n}{i} p^i (1-p)^{n-i}
\end{aligned}
$$

Question 99: You are flipping a coin 30 times. What is the probability that the number of tails is less than 29? the probability of a tail on a given coin-flip is 2/3.

Answer 99:

Let X be the number of tails. $X \sim \text{Bin}(n = 30, p = 2/3)$

$$
\begin{aligned}
P(X < 29) &= 1 - P(29 <= X <= 30) \\
&= 1 - \sum_{i=29}^{30} \binom{n}{i} p^i (1-p)^{n-i}
\end{aligned}
$$

Question 100: You are manufacturing chips and are testing for defects. There are 40 independent tests. The probability of a non-defect on a given test is 5/8. What is the probability that the number of defects is 10?

Answer 100:

Let X be the number of defects. $X \sim \text{Bin}(n = 40, p = 3/8)$

$$
\begin{aligned}
P(X = 10) &= \binom{n}{10} p^{10}(1-p)^{n-10} \\
&= \binom{40}{10} 3/8^{10}(1 - 3/8)^{40-10} \\
&= 0.03507
\end{aligned}
$$