

Core Probability Reference

Definition: Empirical Definition of Probability

The probability of any event E can be defined as:

$$P(E) = \lim_{n \rightarrow \infty} \frac{\text{count}(E)}{n}$$

Where $\text{count}(E)$ is the number of times that E occurred in n experiments.

Definition: Core Identities

For an event E and a sample space S

$$0 \leq P(E) \leq 1$$

All probabilities are numbers between 0 and 1.

$$P(S) = 1$$

All outcomes must be from the Sample Space.

$$P(E) = 1 - P(E^c)$$

The probability of an event from its complement.

Definition: Probability of Equally Likely Outcomes

If S is a sample space with equally likely outcomes, for an event E that is a subset of the outcomes in S :

$$P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in } S} = \frac{|E|}{|S|}$$

Definition: Conditional Probability.

The probability of E given that (aka conditioned on) event F already happened:

$$P(E|F) = \frac{P(E \text{ and } F)}{P(F)}$$

Definition: Probability of **or** with Mutually Exclusive Events

If two events E and F are mutually exclusive then the probability of E **or** F occurring is:

$$P(E \text{ or } F) = P(E) + P(F)$$

For n events E_1, E_2, \dots, E_n where each event is mutually exclusive of one another (in other words, no outcome is in more than one event). Then:

$$P(E_1 \text{ or } E_2 \text{ or } \dots \text{ or } E_n) = P(E_1) + P(E_2) + \dots + P(E_n) = \sum_{i=1}^n P(E_i)$$

Definition: General Probability of **or** (Inclusion-Exclusion)

For any two events E and F :

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

For three events, E , F , and G the formula is:

$$\begin{aligned} P(E \text{ or } F \text{ or } G) &= P(E) + P(F) + P(G) \\ &\quad - P(E \text{ and } F) - P(E \text{ and } G) - P(F \text{ and } G) \\ &\quad + P(E \text{ and } F \text{ and } G) \end{aligned}$$

For more than three events see the chapter of [probability of or](#).

Definition: Probability of **and** for Independent Events.

If two events: E , F are independent then the probability of E **and** F occurring is:

$$P(E \text{ and } F) = P(E) \cdot P(F)$$

For n events E_1, E_2, \dots, E_n that are independent of one another:

$$P(E_1 \text{ and } E_2 \text{ and } \dots \text{ and } E_n) = \prod_{i=1}^n P(E_i)$$

Definition: General Probability of **and** (The Chain Rule)

For any two events E and F :

$$P(E \text{ and } F) = P(E|F) \cdot P(F)$$

For n events E_1, E_2, \dots, E_n :

$$P(E_1 \text{ and } E_2 \text{ and } \dots \text{ and } E_n) = P(E_1) \cdot P(E_2|E_1) \cdot P(E_3|E_1 \text{ and } E_2) \dots P(E_n|E_1 \dots E_{n-1})$$

Definition: The Law of Total Probability

For any two events E and F :

$$\begin{aligned} P(E) &= P(E \text{ and } F) + P(E \text{ and } F^C) \\ &= P(E|F) P(F) + P(E|F^C) P(F^C) \end{aligned}$$

For [mutually exclusive](#) events: B_1, B_2, \dots, B_n such that every outcome in the sample space falls into one of those events:

$$\begin{aligned} P(E) &= \sum_{i=1}^n P(E \text{ and } B_i) && \text{Extension of our observation} \\ &= \sum_{i=1}^n P(E|B_i) P(B_i) && \text{Using chain rule on each term} \end{aligned}$$

Definition: Bayes' Theorem

The most common form of Bayes' Theorem is **Bayes' Theorem Classic**:

$$P(B|E) = \frac{P(E|B) \cdot P(B)}{P(E)}$$

Bayes' Theorem combined with the Law of Total Probability:

$$P(B|E) = \frac{P(E|B) \cdot P(B)}{P(E|B) \cdot P(B) + P(E|B^C) \cdot P(B^C)}$$

