Bacteria Evolution

A wonderful property of modern life is that we have anti-biotics to kill bacterial infections. However, we only have a fixed number of anti-biotic medicines, and bacteria are evolving to become resistent to our anti-biotics. In this example we are going to use probability to understand evolution of anti-biotic resistence in bacteria.

Imagine you have a population of 1 million infectious bacteria in your gut, 10% of which have a mutation that makes them slightly more resistant to anti-biotics. You take a course of anti-biotics. The probability that bacteria with the mutation survives is 20%. The probability that bacteria without the mutation survives is 1%.

What is the probability that a randomly chosen bacterium survives the anti-biotics?

Let E be the event that our bacterium survives. Let M be the event that a bacteria has the mutation. By the <u>Law of Total Probability</u> (LOTP):

$$\begin{split} \mathbf{P}(E) &= \mathbf{P}(E \, \text{and} \, M) + \mathbf{P}(E \, \text{and} \, M^{\, \mathrm{C}}) & \text{LOTP} \\ &= \mathbf{P}(E|M) \, \mathbf{P}(M) + \mathbf{P}(E|M^{\, \mathrm{C}}) \, \mathbf{P}(M^{\, \mathrm{C}}) & \text{Chain Rule} \\ &= 0.20 \cdot 0.10 + 0.01 \cdot 0.90 & \text{Substituting} \\ &= 0.029 \end{split}$$

What is the probability that a surviving bacterium has the mutation?

Using the same events in the last section, this question is asking for P(M|E). We aren't giving the conditional probability in that direction, instead we know P(E|M). Such situations call for <u>Bayes'</u> Theorem:

$$P(M|E) = \frac{P(E|M) P(M)}{P(E)}$$
 Bayes
$$= \frac{0.20 \cdot 0.10}{P(E)}$$
 Given
$$= \frac{0.20 \cdot 0.10}{0.029}$$
 Calculated ≈ 0.69

After the course of anti-biotics, 69% of bacteria have the mutation, up from 10% before. If this population is allowed to reproduce you will have a much more resistent set of bacteria!