Probability of and

The probability of the *and* of two events, say E and F, written P(E and F), is the probability of both events happening. You might see equivalent notations P(EF), $P(E \cap F)$ and P(E,F) to mean the probability of and. How you calculate the probability of event E and event F happening depends on whether or not the events are "independent". In the same way that mutual exclusion makes it easy to calculate the probability of the *or* of events, independence is a property that makes it easy to calculate the probability of the *and* of events.

And with Independent Events

If events are **independent** then calculating the probability of **and** becomes simple multiplication:

Definition: Probability of and for independent events.

If two events: E, F are independent then the probability of E and F occurring is:

$$P(E \text{ and } F) = P(E) \cdot P(F)$$

This property applies regardless of how the probabilities of E and F were calculated and whether or not the events are mutually exclusive.

The independence principle extends to more than two events. For n events $E_1, E_2, \ldots E_n$ that are **mutually** independent of one another -- the independence equation also holds for all subsets of the events.

$$\mathrm{P}(E_1 \, \mathrm{and} \, E_2 \, \mathrm{and} \dots \mathrm{and} \, E_n) = \prod_{i=1}^n \mathrm{P}(E_i)$$

We can prove this equation by combining the definition of conditional probability and the definition of independence.

Proof: If E is independent of F then $P(E \text{ and } F) = P(E) \cdot P(F)$

$$\begin{split} \mathrm{P}(E|F) &= \frac{\mathrm{P}(E \, \mathrm{and} \, F)}{\mathrm{P}(F)} &\qquad \mathrm{Definition \, of \, conditional \, probability} \\ \mathrm{P}(E) &= \frac{\mathrm{P}(E \, \mathrm{and} \, F)}{\mathrm{P}(F)} &\qquad \mathrm{Definition \, of \, independence} \\ \mathrm{P}(E \, \mathrm{and} \, F) &= \mathrm{P}(E) \cdot \mathrm{P}(F) &\qquad \mathrm{Rearranging \, terms} \end{split}$$

See the chapter on independence to learn about when you can assume that two events are independent

And with Dependent Events

Events which are not independent are called *dependent* events. How can you calculate the probability of the **and** of dependent events? If your events are mutually exclusive you might be able to use a technique called DeMorgan's law, which we cover in a later chapter. For the probability of and in dependent events there is a direct formula called the chain rule which can be directly derived from the definition of conditional probability:

Definition: The chain rule.

The formula in the definition of conditional probability can be re-arranged to derive a general way of calculating the probability of the *and* of any two events:

$$P(E \text{ and } F) = P(E|F) \cdot P(F)$$

Of course there is nothing special about ${\cal E}$ that says it should go first. Equivalently:

$$P(E \text{ and } F) = P(F \text{ and } E) = P(F|E) \cdot P(E)$$

We call this formula the "chain rule." Intuitively it states that the probability of observing events E and F is the probability of observing F, multiplied by the probability of observing E, given that you have observed F. It generalizes to more than two events: