

# Bacteria Evolution

A wonderful property of modern life is that we have anti-biotics to kill bacterial infections. However, we only have a fixed number of anti-biotic medicines, and bacteria are evolving to become resistant to our anti-biotics. In this example we are going to use probability to understand evolution of anti-biotic resistance in bacteria.

Imagine you have a population of 1 million infectious bacteria in your gut, 10% of which have a mutation that makes them slightly more resistant to anti-biotics. You take a course of anti-biotics. The probability that bacteria with the mutation survives is 20%. The probability that bacteria without the mutation survives is 1%.

What is the probability that a randomly chosen bacterium survives the anti-biotics?

Let  $E$  be the event that our bacterium survives. Let  $M$  be the event that a bacteria has the mutation. By the [Law of Total Probability](#) (LOTP):

$$\begin{aligned} P(E) &= P(E \text{ and } M) + P(E \text{ and } M^c) && \text{LOTP} \\ &= P(E|M) P(M) + P(E|M^c) P(M^c) && \text{Chain Rule} \\ &= 0.20 \cdot 0.10 + 0.01 \cdot 0.90 && \text{Substituting} \\ &= 0.029 \end{aligned}$$

What is the probability that a surviving bacterium has the mutation?

Using the same events in the last section, this question is asking for  $P(M|E)$ . We aren't giving the conditional probability in that direction, instead we know  $P(E|M)$ . Such situations call for [Bayes' Theorem](#):

$$\begin{aligned} P(M|E) &= \frac{P(E|M) P(M)}{P(E)} && \text{Bayes} \\ &= \frac{0.20 \cdot 0.10}{P(E)} && \text{Given} \\ &= \frac{0.20 \cdot 0.10}{0.029} && \text{Calculated} \\ &\approx 0.69 \end{aligned}$$

After the course of anti-biotics, 69% of bacteria have the mutation, up from 10% before. If this population is allowed to reproduce you will have a much more resistant set of bacteria!