

## 4.2.1

$\neg\neg p \rightarrow p$

|                                 |                     |
|---------------------------------|---------------------|
| 1 $F(\neg\neg p \rightarrow p)$ | ✓                   |
| 2 $T(\neg p \rightarrow p)$     | 1, $T\neg$ ✓        |
| 3 $T\neg p$                     | 2, $F\rightarrow$ ✓ |
| 4 $Fp$                          | 2, $F\rightarrow$   |
| 5 $Fp$                          | 3, $T\neg$          |

This is not valid because we can falsify it

$((p \rightarrow q) \rightarrow p) \rightarrow p$

|   |   |                             |
|---|---|-----------------------------|
|   | 1. $F((p \rightarrow q) \rightarrow p) \rightarrow p$ ✓ |                             |
|   | 1, $F\rightarrow$ ✓                                     |                             |
|   | 3. $Fp$ , 1, $F\rightarrow$                             |                             |
|   |   |                             |
| 4. $F(p \rightarrow q)$ , 2, $T\rightarrow$ ✓ |   | 5. $Tp$ , 2, $T\rightarrow$ |
| 6. $T(p)$ , 4, $F\rightarrow$                 |   |                             |
| 7. $F(q)$ , 4, $F\rightarrow$                 |   |                             |

Unable to falsify, both branches require P to be true

$(\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p)$

|                                      |   |                                      |
|--------------------------------------|---|--------------------------------------|
|                                      | 1. $F((\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p))$ ✓ |                                      |
|                                      | 2. $T(\neg p \rightarrow \neg q)$ , 1, $F\rightarrow$ ✓             |                                      |
|                                      | 3. $F(q \rightarrow p)$ , 1, $F\rightarrow$ ✓                       |                                      |
|                                      | 4. $T(q)$ , 2, $F\rightarrow$                                       |                                      |
|                                      | 5. $F(p)$ , 2, $F\rightarrow$                                       |                                      |
|                                      |   |                                      |
| 6. $F(\neg p)$ , 4, $T\rightarrow$ ✓ |   | 7. $T(\neg q)$ , 4, $T\rightarrow$ ✓ |
| 8. $T(p)$ , 6, $F\rightarrow$        |   | 9. $F(q)$ , 7, $T\rightarrow$        |

Unable to falsify

$p \rightarrow (p \wedge (q \vee p))$

|                          |   |                                   |
|--------------------------|---|-----------------------------------|
|                          | 1. $F(p \rightarrow (p \wedge (q \vee p)))$ ✓     |                                   |
|                          | 2. $T(p)$ , 1, $F\rightarrow$                     |                                   |
|                          | 3. $F(p \wedge (q \vee p))$ , 1, $F\rightarrow$ ✓ |                                   |
|                          |   |                                   |
| 4. $F(p)$ , 3, $F\wedge$ |   | 5. $F(q \vee p)$ , 3, $F\wedge$ ✓ |
|                          |   | 6. $F(q)$ , 5, $F\vee$            |
|                          |   | 7. $F(p)$ , 5, $F\vee$            |

Unable to falsify, P is once false and once true

$$(p \vee (q \wedge r)) \rightarrow ((p \vee q) \wedge (p \vee r))$$

|   |   |   |   |
|---|---|---|---|
|   |   | 1. $F((p \vee (q \wedge r)) \rightarrow ((p \vee q) \wedge (p \vee r))) \checkmark$ |   |
|   |   | 2. $T(p \vee (q \wedge r))$ 1, $F \rightarrow \checkmark$                           |   |
|   |   | 3. $F((p \vee q) \wedge (p \vee r))$ 1, $F \rightarrow \checkmark$                  |   |
|   |   |   |   |
| 4. $T(p)$ , 2, $T \vee$                     |   |   | 5. $T(q \wedge r)$ , 2, $T \vee \checkmark$     |
| 8. $F(p \vee q)$ , 3, $F \wedge \checkmark$ | 9. $F(p \vee r)$ , 3, $F \wedge \checkmark$ | 6. $T(q)$ , 5, $T \wedge$   |   |
| 10. $F(p)$ , 8, $F \vee$                    | 12. $F(p)$ , 9, $F \vee$                    | 7. $T(r)$ , 5, $T \wedge$   |   |
| 11. $F(q)$ , 8, $F \vee$                    | 13. $F(r)$ , 9, $F \vee$                    | 14. $F(p \vee q)$ , 3, $F \wedge \checkmark$  | 15. $F(p \vee r)$ ,<br>3, $F \wedge \checkmark$ |
|   |   | 16. $F(p)$ , 14, $F \vee$   | 18. $F(p)$ , 15, $F \vee$                       |
|   |   | 17. $F(q)$ , 14, $F \vee$   | 19. $F(r)$ , 15, $F \vee$                       |

Unable to falsify

## 4.4.1

|                                  |                |
|----------------------------------|----------------|
| $\neg(\neg\neg p \rightarrow p)$ | Negate         |
| $p \rightarrow p$                | Convert to NNF |
| $\neg p \vee p$                  | Convert to NNF |
| $[\neg p, p]$                    | CNF            |

|   |                |
|---|----------------|
| $\neg(((p \rightarrow q) \rightarrow p) \rightarrow p)$ | Negate         |
| $((p \rightarrow q) \rightarrow p) \wedge \neg p$       | Convert to NNF |
| $(\neg(p \rightarrow q) \vee p) \wedge \neg p$          | Convert to NNF |
| $(p \wedge \neg q \vee p) \wedge \neg p$                | Convert to NNF |
| $((p \vee p) \wedge (\neg q \vee p)) \wedge \neg p$     | Convert to CNF |
| $[[p, p], [\neg q, p], [\neg p]]$                       | CNF            |

|   |                |
|---|----------------|
| $\neg((\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p))$ | Negate         |
| $(\neg p \rightarrow \neg q) \wedge \neg(q \rightarrow p)$        | Convert to NNF |
| $(\neg p \rightarrow \neg q) \wedge (q \wedge \neg p)$            | Convert to NNF |
| $(\neg\neg p \vee \neg q) \wedge (q \wedge \neg p)$               | Convert to NNF |
| $(p \vee \neg q) \wedge (q \wedge \neg p)$                        | Convert to CNF |
| $[[p, \neg q], [q], [\neg p]]$                                    | CNF            |

|   |                |
|---|----------------|
| $\neg(p \rightarrow (p \wedge (q \vee p)))$                   | Negate         |
| $p \wedge \neg(p \wedge (q \vee p))$                          | Convert to NNF |
| $p \wedge (\neg p \vee \neg(q \vee p))$                       | Convert to NNF |
| $p \wedge (\neg p \vee (\neg q \wedge \neg p))$               | Convert to NNF |
| $p \wedge ((\neg p \vee \neg q) \wedge (\neg p \vee \neg p))$ | Convert to CNF |
| $[[p], [\neg p, \neg q], [\neg p, \neg p]]$                   | CNF            |

|  |                |
|--|----------------|
| $\neg((p \vee (q \wedge r)) \rightarrow ((p \vee q) \wedge (p \vee r)))$   | Negate         |
| $(p \vee (q \wedge r)) \wedge \neg((p \vee q) \wedge (p \vee r))$  | Convert to NNF |
| $(p \vee (q \wedge r)) \wedge (\neg(p \vee q) \vee \neg(p \vee r))$  | Convert to NNF |
| $(p \vee (q \wedge r)) \wedge ((\neg p \wedge \neg q) \vee (\neg p \wedge \neg r))$  | Convert to NNF |
| $((p \vee q) \wedge (p \vee r)) \wedge ((\neg p \wedge \neg q) \vee (\neg p \wedge \neg r))$   | Convert to CNF |
| $((p \vee q) \wedge (p \vee r)) \wedge ((\neg p \vee (\neg p \wedge \neg r)) \wedge (\neg q \vee (\neg p \wedge \neg r)))$                         | Convert to CNF |
| $((p \vee q) \wedge (p \vee r)) \wedge ((\neg p \vee \neg p) \wedge (\neg p \vee \neg r) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee \neg r))$ | Convert to CNF |
| $[[p, q], [p, r], [\neg p, \neg p], [\neg p, \neg r], [\neg q, \neg p], [\neg q, \neg r]]$   | CNF            |

## 4.4.5

|               |   |
|---------------|---|
| $[\neg p, p]$ | Set   |
| $[\neg p, p]$ | Resolution using p, not possible to create empty set. |

|                                   |                   |
|-----------------------------------|-------------------|
| $[[p, p], [\neg q, p], [\neg p]]$ | CNF               |
| $[[p], [\neg q, p], [\neg p]]$    | SET               |
| $[[], [\neg q, p], []]$           | Resolution with p |

|                                |                    |
|--------------------------------|--------------------|
| $[[p, \neg q], [q], [\neg p]]$ | CNF already in set |
| $[[], [\neg q, p], []]$        | Resolution with p  |

|   |                   |
|---|-------------------|
| $[[p], [\neg p, \neg q], [\neg p, \neg p]]$ | CNF               |
| $[[p], [\neg p, \neg q], [\neg p]]$         | SET               |
| $[[], [\neg p, \neg q], []]$                | Resolution with p |

|  |                   |
|--|-------------------|
| $[[p, q], [p, r], [\neg p, \neg p], [\neg p, \neg r], [\neg q, \neg p], [\neg q, \neg r]]$ | CNF               |
| $[[p, q], [p, r], [\neg p], [\neg p, \neg r], [\neg q, \neg p], [\neg q, \neg r]]$         | SET               |
| $[[q], [p, r], [], [\neg p, \neg r], [\neg q, \neg p], [\neg q, \neg r]]$                  | Resolution with p |

**Formulate axioms in first-order logic that describes the properties of the spatial relations used in the image models.**

$$\forall x \forall y (\text{SUPPORTS}(x,y) \rightarrow \text{TOUCH}(x,y))$$

$$\forall x \forall y (\text{TOUCH}(x,y) \rightarrow \text{TOUCH}(y,x))$$

$$\forall x \forall y (\text{NEAR}(x,y) \rightarrow \text{NEAR}(y,x))$$

$$\exists x \exists y (\text{TOUCH}(x,y) \rightarrow \text{SUPPORTS}(x,y))$$

$$\forall x \forall y (\text{TOUCH}(x,y) \rightarrow \neg \text{PART\_OF}(x,y))$$

$$\forall x \forall y (\text{PART\_OF}(x,y) \rightarrow \neg (\text{SUPPORT}(x,y) \wedge \text{TOUCH}(x,y) \wedge \text{NEAR}(x,y) ))$$

$$\forall x \forall y (\text{NEAR}(x,y) \rightarrow \neg (\text{PART\_OF}(x,y) \wedge \text{TOUCH}(x,y)))$$

$$\forall x \forall y (\text{PART\_OF}(x,y) \rightarrow \neg \text{PART\_OF}(y,x))$$