**4.2.1**

¬¬p→p

|  |  |
| --- | --- |
| 1 F(¬¬p→p) | ✓ |
| 2 T(¬p→p) | 1,T¬ ✓ |
| 3 T¬p | 2,F→ ✓ |
| 4 Fp | 2,F→ |
| 5 Fp | 3,T¬ |

This is not valid because we can falsify it

((p→q)→p)→p

|  |  |  |
| --- | --- | --- |
|  | 1. F((p→q)→p)→p ✓ |  |
|  | 1,F→ ✓ |  |
|  | 3. Fp, 1,F→ |  |
|  | |  |
| 4. F(p→q), 2,T→ ✓ | | 5. Tp , 2,T→ |
| 6. T(p), 4, F→ |  |  |
| 7. F(q), 4F→ |  |  |

Unable to falsify , both branches require P to be true

(¬p→¬q)→(q→p)

|  |  |  |
| --- | --- | --- |
|  | 1. F((¬p→¬q)→(q→p)) ✓ |  |
|  | 2. T(¬p→¬q), 1,F→✓ |  |
|  | 3. F (q→p), 1,F→✓ |  |
|  | 4. T(q), 2,F→ |  |
|  | 5. F(p), 2,F→ |  |
|  |  |  |
| 6. F(¬p), 4,T→✓ |  | 7. T(¬q), 4,T→✓ |
| 8. T(p), 6, F¬ |  | 9. F(q), 7, T¬ |

Unable to falsify

p→(p∧(q∨p))

|  |  |  |
| --- | --- | --- |
|  | 1. F(p→(p∧(q∨p))) ✓ |  |
|  | 2. T(p), 1,F→ |  |
|  | 3. F(p∧(q∨p)), 1,F→✓ |  |
|  |  |  |
| 4. F(p), 3,F∧ |  | 5. F(q∨p), 3,F∧✓ |
|  |  | 6. F(q) , 5,F∨ |
|  |  | 7. F(p) , 5,F∨ |

Unable to falsify, P is once false and once true

(p∨(q∧r))→((p∨q)∧(p∨r))

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | 1. F((p∨(q∧r))→((p∨q)∧(p∨r)) ✓ | |  |
|  |  | 2. T(p∨(q∧r)) 1,F→ ✓ |  |  |
|  |  | 3. F((p∨q)∧(p∨r)) 1,F→ ✓ |  |  |
|  | |  |  | |
| 4. T(p) , 2,T∨ | |  | 5. T(q∧r), 2, T∨✓ | |
| 8. F(p∨q), 3,F∧ ✓ | 9. F(p∨r), 3,F∧ ✓ | | 6. T(q) , 5, T∧ |  |
| 10. F(p) , 8, F∨ | 12. F(p) , 9, F∨ |  | 7. T( r), 5, T∧ |  |
| 11. F(q), 8,F∨ | 13. F(r) , 9, F∨ |  | 14. F(p∨q), 3,F∧✓ | 15. F(p∨r), 3,F∧✓ |
|  |  |  | 16. F(p) , 14, F∨ | 18. F(p) , 15, F∨ |
|  |  |  | 17. F(q), 14,F∨ | 19. F(r) , 15, F∨ |

Unable to falsify

**4.4.1**

|  |  |
| --- | --- |
| ¬(¬¬p→p) | Negate |
| p→p | Convert to NNF |
| ¬p ∨ p | Convert to NNF |
| [¬p ,p] | CNF |

|  |  |
| --- | --- |
| ¬ (((p→q)→p)→p) | Negate |
| ((p→q)→p)∧¬p | Convert to NNF |
| (¬(p→q)∨p)∧¬p | Convert to NNF |
| (p∧¬q∨p)∧¬p | Convert to NNF |
| ((p∨p)∧(¬q∨p))∧¬p | Convert to CNF |
| [[p,p],[¬q,p],[¬p]] | CNF |

|  |  |
| --- | --- |
| ¬((¬p→¬q)→(q→p)) | Negate |
| (¬p→¬q)∧¬(q→p) | Convert to NNF |
| (¬p→¬q)∧(q∧¬p) | Convert to NNF |
| (¬¬p∨¬q)∧(q∧¬p) | Convert to NNF |
| (p∨¬q)∧(q∧¬p) | Convert to CNF |
| [[p,¬q],[q],[¬p]] | CNF |

|  |  |
| --- | --- |
| ¬( p→(p∧(q∨p))) | Negate |
| p ∧ ¬ (p∧(q∨p))) | Convert to NNF |
| p∧(¬p∨¬(q∨p)) | Convert to NNF |
| p∧(¬p∨(¬q∧¬p)) | Convert to NNF |
| p∧((¬p∨¬q)∧(¬p∨¬p)) | Convert to CNF |
| [[p],[¬p,¬q],[¬p,¬p]] | CNF |

|  |  |
| --- | --- |
| ¬((p∨(q∧r))→((p∨q)∧(p∨r))) | Negate |
| (p∨(q∧r))∧(¬((p∨q)∧(p∨r))) | Convert to NNF |
| (p∨(q∧r))∧(¬(p∨q)∨¬(p∨r)) | Convert to NNF |
| (p∨(q∧r))∧((¬p∧¬q)∨(¬p∧¬r)) | Convert to NNF |
| ((p∨q)∧(p∨r))∧((¬p∧¬q)∨(¬p∧¬r)) | Convert to CNF |
| ((p∨q)∧(p∨r))∧((¬p∨(¬p∧¬r))∧(¬q∨(¬p∧¬r))) | Convert to CNF |
| ((p∨q)∧(p∨r))∧((¬p∨¬p)∧(¬p∨¬r)∧(¬q∨¬p)∧(¬q∨¬r)) | Convert to CNF |
| [[p,q],[p,r],[¬p,¬p],[¬p,¬r],[¬q,¬p],[¬q,¬r]] | CNF |

**4.4.5**

|  |  |
| --- | --- |
| [¬p ,p] | Set |
| [¬p ,p] | Resolution using p, not possible to create empty set. |

|  |  |
| --- | --- |
| [[p,p],[¬q,p],[¬p]] | CNF |
| [[p],[¬q,p],[¬p]] | SET |
| [[],],[¬q,p],[]] | Resolution with p |

|  |  |
| --- | --- |
| [[p,¬q],[q],[¬p]] | CNF allready in set |
| [[],[¬q,p],[]] | Resolution with p |

|  |  |
| --- | --- |
| [[p],[¬p,¬q],[¬p,¬p]] | CNF |
| [[p],[¬p,¬q],[¬p]] | SET |
| [[],[¬p,¬q],[]] | Resolution with p |

|  |  |
| --- | --- |
| [[p,q],[p,r],[¬p,¬p],[¬p,¬r],[¬q,¬p],[¬q,¬r]] | CNF |
| [[p,q],[p,r],[¬p],[¬p,¬r],[¬q,¬p],[¬q,¬r]] | SET |
| [[q],[p,r],[],[¬p,¬r],[¬q,¬p],[¬q,¬r]] | Resolution with p |

**Formulate axioms in first-order logic that describes the properties of the spatial relations used in the image models.**

∀x∀y(SUPPORTS(x,y) → TOUCH(x,y))

∀x∀y(TOUCH(x,y) → TOUCH(y,x))

∀x∀y(NEAR(x,y) → NEAR(y,x))

∃x∃y(TOUCH(x,y) -> SUPPORTS(x,y)

∀x∀y(TOUCH(x,y) → ¬PART\_OF(x,y)

∀x∀y(PART\_OF(x,y) → ¬(SUPPORT(x,y) ∧TOUCH(x,y) ∧NEAR(x,y) ))

∀x∀y(NEAR(x,y) → ¬(PART\_OF(x,y) ∧TOUCH(x,y)))

∀x∀y(PART\_OF(x,y) → ¬PART\_OF(y,x))